A Modified Group Chain Sampling Plan for lifetime following Kumaraswamy Generalized Power Weibull Distribution with Minimum Angle Approach

Majid Liaqat[1], Nadia Saeed[2], Kanwal Saleem[3], Muhammad Aslam[*4], Rehan Ahmad Khan Sherwani[2]

[1]Department of Social Welfare and Bait ul Maal, Lahore-54000, Pakistan;
majidliaqat85@gmail.com
[2]College of Statistical Sciences, University of the Punjab Lahore-54000, Pakistan;
nadia.stat@pu.edu.pk; rehan.stat@pu.edu.pk
[3]National University of Computer and Emerging Sciences, Lahore-54000, Pakistan;
kanwal.saleem@nu.edu.pk

[4]Department of Statistic, Faculty of Science, King Abdulaziz University, Jeddah 21551, Saudi Arabia;
aslam_ravian@hotmail.com, Tel: 00966593329841

*corresponding author

Abstract

The present study proposes a new modified group chain sampling plan for truncated life test when the lifetime of products follow a Kumaraswamy Generalized Power Weibull (KGPW) distribution. The results of optimal group size, mean ratio of true mean to the specified mean, operating characteristic values, minimum angles, acceptance quality level, lower quality level are obtained against the specified producer's, consumer's risk, test termination time and mean ratios. The performance of the proposed chart is also monitored through a real life dataset of 63 single carbon fibers' measurements with specified gauge length. Control limits are constructed to check the quality of strength of a single carbon fibers at gauge length of 20-mm. From the results, it is observed that when the test termination time increases the operating characteristic and mean ratio of proposed plan also increase disproportionately.

Keywords: Optimal group size, producer's risk, consumer's risk, mean ratio, test termination time, acceptance quality level.

Mathematics Subject Classification: 90B25· 60K10 62N05

1. Introduction

Acceptance sampling plan is considered as a well-known quality control tool, which is adopted to ensure the quality of product and services. It comprises a set of techniques which are implemented for the inspection of incoming products of lots and a decision is made about the lot's acceptance or rejection. This technique facilitates in such a way that instead of inspecting the entire lot, decision can be made based on a sample drawn from the entire lot. It is obvious that if the sample items are supposed to be good, the lot will be accepted accordingly. Whereas, the decision will lead us to the rejection of lot if the sample items are not as much good and it has to be returned back to the supplier. The specified sampling plan is used to decrease the expenses related to the inspection process and helps in identifying whether the products are good for marketing or not. In addition to this it protects both producers and consumers from future lose by minimizing the producer’s and consumer’s risks where producer’s risk is the risk of rejecting the good quality lot whereas, consumer’s risk is the risk of accepting the poor quality lot. Actual and expected difference of supplied quality products can also be figured out by using the sampling plan. Acceptance sampling plan is a cost-effective evaluation of many units in which all measurement equipment are capable and critical characteristics are being monitored.

The particular sampling plan has an application in manufacturing unit and industries where inspection and quality assurance of every item is a necessity. Cost and time are the two major constraints which makes it impossible to conduct 100% inspection. The acceptance and rejection of items in inspection process is based on the samples drawn from submitted lot at regular intervals during manufacturing process.

The basic concept of chain sampling inspection plan was actually proposed by Dodge [1] aimed to overcome the shortfalls of single acceptance sampling plan with the help of cumulative information of different random samples, not completely depends upon one sample anymore. Rosaiah et al. [2] considered the well-known Rayleigh distribution to model the lifetime of products based on acceptance sampling. Jun et al. [3] proposed single and double sampling plans for the lot of acceptance when the products followed the Weibull distribution. It was observed that in chain sampling plan, current submitted lot is accepted given that one defective item is found in the sample and further considering that all other samples have one defective product. For the testing of the group containing number of items, an acceptance sampling plan was proposed by Aslam and Jun [4]. The SKSP-3 plan was introduced by Balamurali and Subramani [5] which was the extension of the skip-lot sampling plan. The new proposed plan worked better than the skip-lot plan in terms of reduced inspection costs. According to the proposed plan, Weibull distribution was used for the lifetime of the products. Rao [6] also explored a sampling plan which comprises the lifetime of products following the generalized exponential distribution. Ramaswamy and Jayasri, [7] worked on time truncated chain sampling plans based on generalized exponential distribution. Many studies about time truncated chain sampling plans for generalized exponential distribution, Marshall-Olkin extended exponential, log-logistic and inverse Rayleigh distributions have also been investigated by Ramaswamy and Jayasri [8-10]. Ramaswamy and Jayasri [11] also worked on the concept of chain sampling plans for time truncated based on Weibull distribution.

Group chain sampling plan is an extensive idea from ordinary group sampling plan. The purpose of both plans is to reduce the inspection cost and time by placing groups of items where each group consists of a number of items for the considered procedure of inspection. To the best of our understanding, Aslam et al. [12] has designed a group sampling plan based on truncated life test for the items following gamma distribution. Under various distributions, Ramaswamy and Sutharani [13] designed a sampling plan based on time truncated life test with the help of minimum angle method.

Mughal et al. [14] worked on time truncated group chain sampling strategy when the lifetime of items follow Pareto distribution of the second kind. Jamaludin et al. [15] developed a modified group chain sampling plans for lifetime following a Rayleigh distribution. Teh et al. [16] also proposed group chain sampling plans based on truncated life tests for log-logistic and Rayleigh distributions.

Teh et al. [17] worked on group chain sampling plans based on truncated life tests for exponential distribution. the comparison of group chain acceptance plan was made with group acceptance sampling plan and performance of group chain sampling plan was found better in term of having minimum size of group, cost, Labor and time efficient. In the next year, Teh et al. [18] proposed another approach in finding number of optimal groups for group chain acceptance sampling plans by using minimum angle method. By using this approach, the optimal group size was obtained.

Many researchers consider group acceptance plans under different distributions in the recent years such as Sivakumar et al. [19] worked on a group acceptance sampling plan (GASP) when the lifetime of the products follows odd generalized exponential log-logistic distribution. Khan and Alqarni [20] proposed group acceptance sampling plan for inverse Weibull distribution, In these studies, design parameters values such as acceptance number, minimum group size, operating characteristic (OC) values and minimum mean ratios were calculated under various quality levels. Hafeez et.al [21] used bayesian group chain sampling plan for Poisson distribution with gamma prior to minimize the average number of defective items.

Recently, Teh et al. [22] established new group chain acceptance sampling plans using minimum angle method for generalized exponential distribution. It functions with four acceptance criteria. The study suggested a balanced approach between group and modified group of chain acceptance sampling plans by minimizing both producer's and consumer's risks. Aziz et al [23] introduced two-sided group chain sampling plans based on generalized exponential distribution. The findings showed that proposed plan could reduce the inspection time, cost and resources using smaller number of groups by providing the desired consumer's protection. Therefore, in this study, a group chain sampling plan with minimum angle approach is developed to sentence the submitted lot when the lifetime of items following Kumaraswamy generalized power Weibull distribution. Tables and graphs are also constructed for selected designed parameters using two-point approach along with respective consumer's and producer's risks. Control charts on different subgroups samples are also constructed on real life data set of strength of a single carbon fibers at gauge length of 20-mm.

The main conditions for a lot formation under the group chain sampling plan (GCSP) can be described that the lot should be taken from lots of a sequential streams series and the required quality level of lot should be same. Hence by using the GCSP under Kumaraswamy generalized power Weibull distribution, we will be able to find optimal group size for truncated life test by satisfying the consumer's and producer's risks. Also the mean ratios and probability of acceptance by using optimal number of groups will be calculated. The parameter estimation through minimum angle method will also be provided. Finally, real life application will justify the findings.

1.1 Kumaraswamy Generalized Power Weibull (KGPW) Distribution

Kumaraswamy generalized power Weibull distribution is the extension of generalized power Weibull (GPW) distribution that was developed by Selim and Badr [24]. It is widely used for constructing accelerated failures times models that identify the dependence of the lifetime distribution on explanatory variables. It also provides a good fit to the well-known randomly censored survival time's data for patients at arm-A of the head-and-neck cancer clinical trial. The cdf and pdf of GPW distribution is:

$$F_{gpw}(t,\gamma,\lambda,\vartheta)=1-\exp\left\{1-\left(1+\left(\frac{t}{\lambda}\right)^{\gamma}\right)^{\vartheta}\right\} \qquad \gamma,\lambda,\vartheta>0, t>0 \qquad (1)$$

$$f_{gpw}(t,\gamma,\lambda,\vartheta)=\frac{\gamma\vartheta}{\lambda^{\gamma}}t^{\gamma-1}\left(1+\left(\frac{t}{\lambda}\right)^{\gamma}\right)^{\vartheta-1}\exp\left\{1-\left(1+\left(\frac{t}{\lambda}\right)^{\gamma}\right)^{\vartheta}\right\} \qquad \gamma,\lambda,\vartheta>0, \qquad t>0 \qquad (2)$$

Kumaraswamy [25] proposed a two-parameter distribution on (0, 1) called Kumaraswamy distribution, it is denoted by Kum(a, b). The cumulative distribution function (cdf) of Kumaraswamy distribution is described as

$$F_{KUM}(t)=1-\left[1-t^{\delta}\right]^{b} \quad , \quad 0<t<1$$

and the probability density function (pdf) of Kumaraswamy distribution is

$$f_{KUM}(t)=\delta b t^{\delta-1}\left[1-t^{\delta}\right]^{b-1} \ , \ 0<t<1$$

By substituting the pdf and cdf of GPW distribution in cdf of Kumaraswamy distribution, the pdf of KGPW distribution can be obtained as:

$$f_{Kgpw}(t,\gamma,b,\delta,\lambda,\vartheta)=\frac{\delta b\gamma\vartheta}{\lambda^{\gamma}}t^{\gamma-1}\left(1+\left(\frac{t}{\lambda}\right)^{\gamma}\right)^{\vartheta-1}e^{1-\left(1+\left(\frac{t}{\lambda}\right)^{\gamma}\right)^{\vartheta}}\left[1-e^{1-\left(1+\left(\frac{t}{\lambda}\right)^{\gamma}\right)^{\vartheta}}\right]^{\delta-1}\left\{1-\left[1-e^{1-\left(1+\left(\frac{t}{\lambda}\right)^{\gamma}\right)^{\vartheta}}\right]^{\delta}\right\}^{b-1} ,\delta,b,\gamma,\lambda,\vartheta>0,t>0 \qquad (3)$$

2. Methodology

The cumulative distribution function (cdf) of Kumaraswamy generalized power Weibull (KGPW) distribution is

$$F(t,\gamma,b,\delta,\lambda,\vartheta)=1-\left[1-\left[1-e^{1-\left(1+\left(\frac{t}{\lambda}\right)^{\gamma}\right)^{\vartheta}}\right]^{\delta}\right]^{b} \qquad (4)$$

where $\lambda,\delta,b,\gamma,\vartheta>0$ and $t>0$. Furthermore $\delta,b,\gamma,\vartheta$ are referred as shape parameters while λ is scale parameter.

The r^{th} order moment about mean life of KGPW distribution is given by

$$\mu = E\left(t^r\right) = \lambda^r \sum_{i=1}^{b}\sum_{j=1}^{\delta i}\sum_{k=0}^{\frac{r}{\gamma}} (-1)^{i+j+\frac{r}{\gamma}-k} \binom{b}{i}\binom{\delta i}{j}\binom{\frac{r}{\gamma}}{k} e^{j} \frac{\Gamma\left(\frac{k}{\vartheta}+1, j\right)}{j^{\frac{k}{\vartheta}}} \tag{5}$$

The scale parameter for $r=1$ and $(\delta, b, \gamma, \vartheta) = (2,1,1,2)$ can be expressed as

$$\mu = \lambda(0.489)$$

Moreover, the scale parameter λ can be written as

$$\lambda = \frac{\mu}{0.489} \tag{6}$$

The time termination t_0 is multiple of pre-assumed constant a and specified mean life μ_0 i.e.

$$t_0 = a\mu_0 \tag{7}$$

To find out the probability of defective items, as lifetime distribution function always depends only on termination time t_0 and scale parameter λ, thus the probability of defective items is given by

$$p = F\left(t_0; \lambda\right) \tag{8}$$

According to Equation 7 and Equation 8 as function of pre-assumed constant a and mean ratio of true mean to the specified mean μ / μ_0, the probability of defective can be expressed as

$$p = F\left(a\mu_0; \mu / \mu_0\right) \tag{9}$$

Therefore by substituting fixed values of shape parameters as $(\delta, b, \gamma, \vartheta) = (2,1,1,2)$ and using Equation 6; the defective probability KGPW distribution through Equation 4; can be written as:

$$p = p_{KGPW} = F\left(a\mu_0; \frac{\mu}{\mu_0}\right) = \left[1 - e^{1-\left(1+\left(\frac{a(0.489)}{\frac{\mu}{\mu_0}}\right)^{1}\right)^{2}}\right]^{2} \tag{10}$$

Operating characteristic curve is considered to reveal the practical performance of various sampling plans. To determine the basic ideas of modified group chain acceptance sampling plan (GCASP), suppose lot size N, from which sample size $n = rg$ is randomly taken from whole lot. The r items in g groups are placed for the purpose of testing. It is stated that current lot will be accepted if the true mean to the specified mean is greater than or equal to one i.e. $\mu \geq \mu_0$ and hence the quality of the current lot is considered to be good. On the other hand, quality of the lot will be bad.

The probability of acceptance is defined as a function of deviation of specified mean from true mean. This function is known as operating characteristic (OC) function. Minimum group size can be obtained using the OC values of the given lot. Therefore, by substituting the value of the probability of defective in the OC function, the OC function of KGPW distribution, in case of group chain acceptance sampling plan (GCASP) can be calculated using the following equation:

$$\mathrm{L}\left(p_{KGPW}\right) = \left(1-p_{KGPW}\right)^{gr} + \left(rgp_{KGPW}\right)\left(1-p_{KGPW}\right)^{(gr)-1}\left(1-p_{KGPW}\right)^{gri} \tag{11}$$

Hence the procedure of group chain acceptance sampling plan (GCASP) can easily be understood by Figure 1.

The foremost purpose of the GCASP is to find out its design parameters. Some mathematical formulas of probability of acceptance are shown, with the help of two-point approach, design parameters, minimum producer's and consumer's risks by satisfying the following inequalities:

$$\mathrm{L}\left(p_1 = AQL\right) = \left(1-p_1\right)^{gr} + \left(rgp_1\right)\left(1-p_1\right)^{(gr)-1}\left(1-p_1\right)^{gri} \geq 1-\alpha \tag{12}$$

$$\mathrm{L}\left(p_2 = LQL\right) = \left(1-p_2\right)^{gr} + \left(rgp_2\right)\left(1-p_2\right)^{(gr)-1}\left(1-p_2\right)^{gri} \leq \beta \tag{13}$$

where $\mathrm{L}\left(p\right)$ denotes probability of acceptance under $\mathrm{AQL}\left(p_1\right)$ and $\mathrm{LQL}\left(p_2\right)$. The AQL is defined as the Acceptance Quality Level which provides maximum number of defective per hundred items. The LQL is the Lower or Limiting Quality Level which provides percentage of various items a lot contains. Moreover, LQL provides minimum consumer's risk which ultimately protects the consumers.

By using the different levels of β, number of testers r and producer's risk $\alpha = 0.05$, the design parameters of proposed plan can be determined.

A useful discriminating plan known as minimum angle technique with minimum angle which provides the tangent of angle between the lines joining the points $\left(p_1 = \text{AQL}, 1-\alpha\right), (p_2 = \text{LQL}, \beta)$. It is shown in the Figure 2.

Tangent angle will be made by two lines AB and AC i.e.

$$tan\theta = \frac{BC}{AC} = \frac{p_2 - p_1}{L\left(p_1\right) - L\left(p_2\right)} \tag{14}$$

If the *tanθ* results in smaller value, the angle θ will be close to zero. Thus the chord AB approaches to AC, which is the ideal condition, through $\left(\text{AQL}, 1-\alpha\right)$. Both consumer and producer favor this criterion because this approach minimizes producer's and consumer's risks simultaneously. In this paper, by using this minimum angle method, various parameters are designed by satisfying condition $\beta \le 0.10$ with producer's risk $\alpha = 0.05$, $\mu/\mu_0 = 4,6,8,10,12$ and $a = 0.7, 0.8, 1.0, 1.2, 1.5, 2.0$ for the GCSP based on truncated life tests following KGPW distribution.

3. Analysis and Interpretation

The analysis section is comprised of four tables (Tables 1-4) and four figures (Figures 3-6). The Table 1 shows optimal group size of the proposed plan using fixed values of shape parameters while Table 2 addresses minimum ratio $\left(\mu/\mu_0\right)$ of proposed plan. Table 3 and Table 4 are based on the operating characteristic (OC) values and minimum angle for proposed plan respectively. Similarly in Figure 3, the line chart of optimum number of groups of proposed plan is constructed. In Figure 4, mean ratio curves versus items in each group are compared and in Figure 5, OC curves regarding probability of acceptance of proposed plan are constructed. Finally, Figure 6 is constructed on the basis of minimum angle of proposed plan under different values of fraction defectives.

The Illustration of the tables and figures can better be understood by the following interpretation:

Suppose an electrical device is manufactured by the investigator in a factory with the aim to ensure the lifetime of the electrical device. It is assumed that true unknown average life of the electrical device is at least 1200 hours with $\beta = 0.10$ (Consumer's risk), but it is stated that experiment will be stopped at 700 hours. It is also assumed that the number of preceding samples $i = 1$. Then minimum group size $g = 4$ can be determined based on β and a by satisfying the condition that $\mathrm{L}(p) \le \beta$. If no defective items are observed in $i = 1$ for previous lot and for each group having $r = 2$ number of items is tested, then the experiment can assert with 0.90 confidence level that true average life is at least 1200 hours. While if one defective item found, it will provide that $i = 1$ preceding samples $n = rg = 8$ are free from defective then lot will be accepted otherwise lot will be rejected. Thus, the required design of parameters of the proposed plan will be $(a, r, i, g) = (0.7, 2, 1, 4)$. For the parameters $(\delta, b, \gamma, \vartheta) = (2, 1, 1, 2)$ with $\beta = 0.01$, the line chart of optimum number of groups of proposed plan is shown in Figure 3. From this figure, we understand that optimum groups g are decreased as the value of $r = (2, 3, 4, 5, 6, 7, 8)$ and $i = (1, 2, 3, 4, 5, 6, 7)$ are increased to a certain increase in test termination time multiplier a. Also, we observe that the proposed plan requires larger value of g when the value of a is small. From Table 2, suppose $r = 8$, $i = 7$ and if experiment time is taken 700 ($a = 0.7$) hours with $\beta = 0.01$ then it will provide the minimum mean ratio $\mu/\mu_0 = 8.598$ under the same design of parameters, while if the specified time changes from 700 to 2000 then μ/μ_0 also increases from 8.598 to 17.378. From Figure 4, it is important to note that the values of mean ratio μ/μ_0 are increased to their corresponding number of tester or items with $\beta = 0.25$ when the test termination time a is high.

For $\mu/\mu_0 = 2$, test termination time 700 hours $(a = 0.7)$ with $g = 4$, $r = 4$ and $\beta = 0.01$ under the fixed values of consumer's risk, the probability of accepting the lot will be 0.201 and if average mean ratio $\mu/\mu_0 = 12$, then the probability of accepting the lot will be $L(p) =$ 0.992 which is approximately equals to 1 (Table 3).

Therefore, it can be concluded from Table 3 and Figure 5 that probability of accepting of lot also increases as the mean ratio increases from 2 to 12.

From Table 4 and Figure 6, let's we take $a = 0.7$ and $\mu / \mu_0 = 6$, each group having $r = 3$ items and $i = 2$ preceding lots with minimum producer's and consumer's risks $(\alpha = 0.025, \beta = 0.038)$ satisfy the condition of inequality $L(p_1) \geq 0.95, L(p_2) \leq 0.10$ with $p_1 = \text{AQL} = 0.012$ and $p_2 = \text{LQL} = 0.304$. The minimum producer's and consumer's risks will be found by taking $g = 3$ corresponding to minimum angle $\theta = 17.311$. Thus, the required sampling plan has parameters $(n, i) = (9, 2)$.

4. Application

In this section, real data set regarding strength of data which is measured in GPA is taken from Bader and Priest [26]. They used different measurements of single carbon fibers at gauge lengths of 1, 10, and 20 and 50mm under tension for their experiments. Whereas, 63 observations of a sample contain only measurements of a single carbon fibers at gauge length of 20-mm is considered. The observations are listed below in increasing order.

1.901, 2.132, 2.203, 2.228, 2.257, 2.350, 2.361, 2.396, 2.397, 2.445, 2.454, 2.474, 2.518, 2.522, 2.525, 2.532, 2.575, 2.614, 2.616, 2.618, 2.624, 2.659, 2.675, 2.738, 2.740, 2.856, 2.917, 2.928, 2.937, 2.937, 2.977, 2.996, 3.030, 3.125, 3.139, 3.145, 3.220, 3.223, 3.235, 3.243, 3.264, 3.272, 3.294, 3.332, 3.346, 3.377, 3.408, 3.435, 3.493, 3.501, 3.537, 3.554, 3.562, 3.628, 3.852, 3.871, 3.886, 3.971, 4.024, 4.027, 4.225, 4.395, 5.020.

By using the scheme of modified group chain sampling, to check whether the quality of real data set as strength of a single carbon fibers at gauge length of 20-mm is satisfactory or not, the lower, upper and central control limits are constructed based on the estimates of mean and standard deviation by using different optimal subgroups or group sizes and these subgroups are drawn randomly without replacement from the lot. The control limits structure is defined under different parameters of group chain sampling plan such as number of preceding samples or subgroups (i), group size (g) and item in each group (r). The required results under the optimal parameters are described in Tables 5-7. Moreover, for the final decision, the control limits are constructed by using the following expression

$$\bar{X} \pm A_1 \bar{S} \tag{15}$$

where $A_1 = \frac{3}{\left(\sqrt{n}\right) c_2} = \frac{A}{c_2}$ and c_2 depends upon subgroup size n i.e. $c_2 = \sqrt{\frac{2}{n}} \frac{\left(\frac{n-2}{2}\right)!}{\left(\frac{n-3}{2}\right)!}$. Further details are provided in the book by Agerwal [27].

Tables 5-7 demonstrate different results of upper, central and lower control limits which are based on different subgroups. In order to ensure the quality of 20-mm gauge length strength of single carbon fibers, the means of each group are placed along y-axis against x-axis which shows the subgroups.

Then central limits are parallel to x-axis, whereas upper and lower control limits are parallel to abscissa drawn with smooth line. Thus it is observed from control charts that all sample means lie within the control limits under 7 and 3 optimal groups each having 9 and 21 items (Tables 5-6 and Figure 7, panels a-b) so it can be concluded that the quality strength of single carbon fibers is good. Taking subgroups of items can reduce cost and save the time that consume in inspection. Furthermore, greater efficiency can be seen due to little inspection time.

Figure 7 (panels a-b) shows that the manufacturing products are under control but if all the 63 observations of carbon fibers are considered (Table 7 and Figure 7, panel-c), then some observations are above the upper control limit due to deterioration and some are below the lower control limits due to slackness. Such types of measurements are occurred due to faulty process.

5. Conclusion

In this paper, a group chain acceptance sampling plan based on truncated life tests is presented. Assuming lifetime of items follows Kumaraswamy Generalized Power Weibull distribution. The minimum number of groups, probability of acceptance, minimum mean ratio and minimum angle are calculated. It is observed from the table and figure of operating characteristic that as the test termination time increases, operating characteristic and mean ratio values of proposed plan also increase disproportionately even operating characteristic values reaches at maximum to 1. It can be suggested that the proposed sampling plan can be used in real practical situations for the industrial purposes to save time, to reduce cost and labor of the life test experiment.

Glossary of Symbols and Abbreviations

GCAS	Group chain acceptance sampling	$L(p_{KGPW})$	probability of acceptance of KGPW
KGPW	Kumaraswamy generalized power Weibull	$\delta, b, \gamma, \varphi$	shape parameters
GPW	Generalized Power Weibull	f_{Kgpw}	pdf of KGPW distribution
f_{Kgpw}	pdf of GPW distribution	F_{Kgpw}	cdf of KGPW distribution
F_{gpw}	cdf of GPW distribution	r	Each groups having items
t_0	termination time	λ	scale parameter
g	Number of groups	d	Number of defectives
c	Acceptance number	θ	Minimum Angle
a	Test termination time multiplier	$p_1 = AQL$	Acceptance Quality level (AQL)
α	Producer's risk	$p_2 = LQL$	Lower Quality level (LQL)
β	Consumer's risk	$L(p_1)$	AQL, $(1-\alpha)$
p	Probability of failure	$L(p_2)$	LQL, (β)
$L(p)$	Operating characteristics function	LCL	Lower Control Limit
$\left(\mu/\mu_0\right)$	Ratio of True average life μ to the specified average life μ_0	CL	Central Limit
μ_0	Specified mean	UCL	Upper Control Limit
μ	True mean	$\bar{X}$	Estimated Mean
p_{KGPW}	probability of Failure under KGPW	s_i	Estimated Standard Deviation

i	The number of preceding samples	n	Random sample size

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Table Captions

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Table 2. Minimum ratio $\left(\mu / \mu_0\right)$ of proposed plan with shape parameter $(\delta, b, \gamma, \vartheta) = (2,1,1,2)$

Table 3. Operating Characteristic of proposed plan with $r = 4, i = 3$ under shape parameter $(\delta, b, \gamma, \vartheta) = (2,1,1,2)$

Table 4. Minimum angle for proposed plan with $r = 3, i = 2$

Table 5. Control limits for mean chart with $n = rg = 63,\ r = 9, g = 7$

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Table 1. Optimal group size of the proposed plan with shape parameter $(\delta, b, \gamma, \vartheta) = (2,1,1,2)$

β	r	i	a					
			0.7	**0.8**	**1.0**	**1.2**	**1.5**	**2.0**
0.25	2	1	3	2	2	1	1	1
	3	2	2	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
	6	5	1	1	1	1	1	1
	7	6	1	1	1	1	1	1
	8	7	1	1	1	1	1	1
0.10	2	1	4	3	2	2	1	1
	3	2	3	1	1	1	1	1
	4	3	2	2	1	1	1	1
	5	4	2	1	1	1	1	1
	6	5	2	1	1	1	1	1
	7	6	1	1	1	1	1	1
	8	7	1	1	1	1	1	1
0.05	2	1	5	4	3	2	1	1

	3	2	3	3	2	2	1	1
	4	3	3	2	2	1	1	1
	5	4	2	2	1	1	1	1
	6	5	2	2	2	1	1	1
	7	6	2	1	1	1	1	1
	8	7	1	1	1	1	1	1
0.01	2	1	7	6	4	3	1	1
	3	2	5	4	3	2	2	1
	4	3	4	3	2	2	1	1
	5	4	3	2	2	1	1	1
	6	5	2	2	2	1	1	1
	7	6	2	2	1	1	1	1
	8	7	2	2	1	1	1	1

Table 2. Minimum ratio $\left(\mu / \mu_0\right)$ of proposed plan with shape parameter $\left(\delta, b, \gamma, \vartheta\right) = \left(2, 1, 1, 2\right)$

β	r	i	a					
			0.7	**0.8**	**1.0**	**1.2**	**1.5**	**2.0**
0.25	2	1	3.491	3.241	3.991	3.291	4.121	5.421
	3	2	3.989	3.159	3.919	4.709	5.889	7.861
	4	3	3.611	3.989	4.979	5.975	7.475	9.965
	5	4	4.365	4.765	5.985	7.385	8.996	11.958
	6	5	4.885	5.484	6.885	8.583	10.280	13.880
	7	6	5.477	6.257	7.757	9.258	11.659	15.438
	8	7	5.998	6.968	8.768	10.578	12.778	17.378
0.10	2	1	4.278	3.978	3.991	4.978	4.121	5.421
	3	2	4.975	3.159	3.919	4.709	5.889	7.861
	4	3	5.178	5.878	4.979	5.975	7.475	9.965
	5	4	5.978	4.765	5.985	7.385	8.996	11.985

	6	5	6.888	5.484	6.885	8.583	10.280	13.880
	7	6	5.477	6.257	7.757	9.258	11.658	15.438
	8	7	5.998	6.968	8.768	10.578	12.778	17.378
0.05	2	1	4.628	4.718	4.984	4.978	4.121	5.421
	3	2	4.975	5.684	5.674	6.814	5.889	7.861
	4	3	6.214	5.878	7.204	5.975	7.475	9.965
	5	4	5.978	6.875	5.985	7.385	8.996	11.985
	6	5	6.888	7.864	9.854	8.583	10.280	13.880
	7	6	7.754	6.257	7.757	9.258	11.659	15.438
	8	7	5.998	6.968	8.768	10.578	12.778	17.378
0.01	2	1	5.454	5.756	5.864	5.984	4.121	5.421
	3	2	6.484	6.554	7.253	6.814	8.514	7.861
	4	3	7.214	7.111	7.204	8.611	7.475	9.965
	5	4	7.410	6.875	8.611	7.385	8.996	11.985
	6	5	6.888	7.864	9.854	8.583	10.280	13.880
	7	6	7.754	8.801	7.757	9.258	11.659	15.438
	8	7	8.598	9.788	8.768	10.578	12.778	17.378

Table 3. Operating Characteristic of proposed plan with $r=4, i=3$ under shape parameter $(\delta, b, \gamma, \vartheta) = (2,1,1,2)$

β	g	a	μ/μ_0					
			2	**4**	**6**	**8**	**10**	**12**
0.25	1	0.7	0.751	0.969	0.993	0.997	0.999	0.999
	1	0.8	0.664	0.951	0.988	0.996	0.998	0.999
	1	1.0	0.493	0.900	0.973	0.990	0.996	0.998
	1	1.2	0.349	0.832	0.951	0.982	0.992	0.996
	1	1.5	0.197	0.708	0.900	0.960	0.982	0.990
	1	2.0	0.065	0.493	0.779	0.900	0.951	0.973
0.10	2	0.7	0.478	0.898	0.973	0.990	0.996	0.998

	2	0.8	0.371	0.849	0.958	0.984	0.993	0.997
	1	1.0	0.493	0.900	0.973	0.990	0.996	0.998
	1	1.2	0.349	0.832	0.951	0.982	0.992	0.996
	1	1.5	0.196	0.708	0.900	0.960	0.982	0.990
	1	2.0	0.065	0.493	0.779	0.900	0.951	0.973
0.05	3	0.7	0.306	0.812	0.945	0.980	0.991	0.996
	2	0.8	0.371	0.849	0.958	0.985	0.994	0.997
	2	1.0	0.211	0.731	0.912	0.966	0.985	0.992
	1	1.2	0.350	0.833	0.951	0.982	0.992	0.996
	1	1.5	0.197	0.708	0.901	0.961	0.982	0.990
	1	2.0	0.065	0.493	0.779	0.901	0.951	0.973
0.01	4	0.7	0.201	0.726	0.911	0.966	0.984	0.992
	3	0.8	0.213	0.737	0.915	0.967	0.985	0.992
	2	1.0	0.211	0.730	0.912	0.966	0.985	0.992
	2	1.2	0.112	0.601	0.849	0.937	0.971	0.984
	1	1.5	0.196	0.708	0.900	0.960	0.982	0.990
	1	2.0	0.065	0.493	0.779	0.900	0.951	0.973

Table 4. Minimum angle for proposed plan with $r=3, i=2$

a	μ/μ_0	g	p_1	$L(p_1)$	p_2	$L(p_2)$	$tan\theta$	θ
0.7	6	3	0.012	0.975	0.304	0.038	0.312	17.311
	8	4	7.01×10^{-3}	0.985	0.304	0.013	0.306	17.001
	10	4	4.52×10^{-3}	0.993	0.304	0.013	0.306	16.993
	12	5	3.16×10^{-3}	0.995	0.304	4.34×10^{-3}	0.304	16.903
0.8	6	2	0.016	0.981	0.369	0.064	0.385	21.070
	8	2	9.091×10^{-3}	0.993	0.369	0.064	0.387	21.178
	10	2	5.880×10^{-3}	0.997	0.369	0.064	0.389	21.272
	12	3	4.112×10^{-3}	0.997	0.369	0.016	0.372	20.414

1.0	6	2	0.024	0.959	0.496	0.017	0.500	26.573
	8	2	0.014	0.985	0.496	0.017	0.497	26.438
	10	3	9.091×10^{-3}	0.986	0.496	2.115×10^{-3}	0.495	26.318
	12	3	6.369×10^{-3}	0.993	0.496	2.115×10^{-3}	0.494	26.283
1.2	6	1	0.034	0.978	0.610	0.060	0.627	32.081
	8	2	0.020	0.971	0.610	3.532×10^{-3}	0.610	31.365
	10	3	0.013	0.972	0.610	2.099×10^{-4}	0.614	31.547
	12	4	9.091×10^{-3}	0.975	0.610	1.248×10^{-5}	0.616	31.625
1.5	6	1	0.052	0.953	0.749	0.016	0.743	36.616
	8	1	0.031	0.983	0.749	0.016	0.743	36.614
	10	2	0.020	0.971	0.749	2.509×10^{-4}	0.751	36.898
	12	3	0.014	0.968	0.749	3.973×10^{-6}	0.759	37.201
2.0	8	1	0.052	0.953	0.894	1.181×10^{-3}	0.884	41.488
	10	1	0.034	0.978	0.894	1.181×10^{-3}	0.880	41.349
	12	2	0.024	0.959	0.894	1.394×10^{-6}	0.908	42.225

Table 5. Control limits for mean chart with $n = rg = 63,\ r = 9,\ g = 7$

Subgroups (i)	**Total** $\sum X_i$	**Estimated Means** $(\bar{X})$	**Estimated Subgroup SD** (s_i)
1	29.416	3.269	0.600
2	27.658	3.074	0.723
3	25.966	2.885	0.735
4	27.880	3.098	0.507
5	29.240	3.249	0.799
6	25.342	2.816	0.419
7	27.234	3.026	0.546
Total	192.736	21.415	4.331

Mean	3.059	3.059	0.619
Control Limit	**LCL**	**CL**	**UCL**
	2.382	3.060	3.737

Table 6. Control limits for mean chart with $n = rg = 63, r = 21$ and $g = 3$

Subgroups (i)	**Total** $\sum X_i$	**Estimated Means** $(\bar{X})$	**Estimated Subgroup SD** (s_i)
1	66.979	3.190	0.644
2	61.518	2.930	0.574
3	64.239	3.059	0.646
Total	192.736	9.178	1.863
Mean	3.059	3.059	0.621
Control Limit	**LCL**	**CL**	**UCL**
	2.638	3.059	3.481

Table 7. Control limits for mean chart with $n = rg = 63, r = 63$ and $g = 1$

Subgroups (i)	**Total** $\sum X_i$	**Estimated Means** $(\bar{X})$	**Estimated Subgroup SD** (s_i)
1	94.930	1.507	0.325
Control Limit	**LCL**	**CL**	**UCL**
	2.825	3.059	3.294

Figure 1. Group Chain Acceptance Sampling Plan

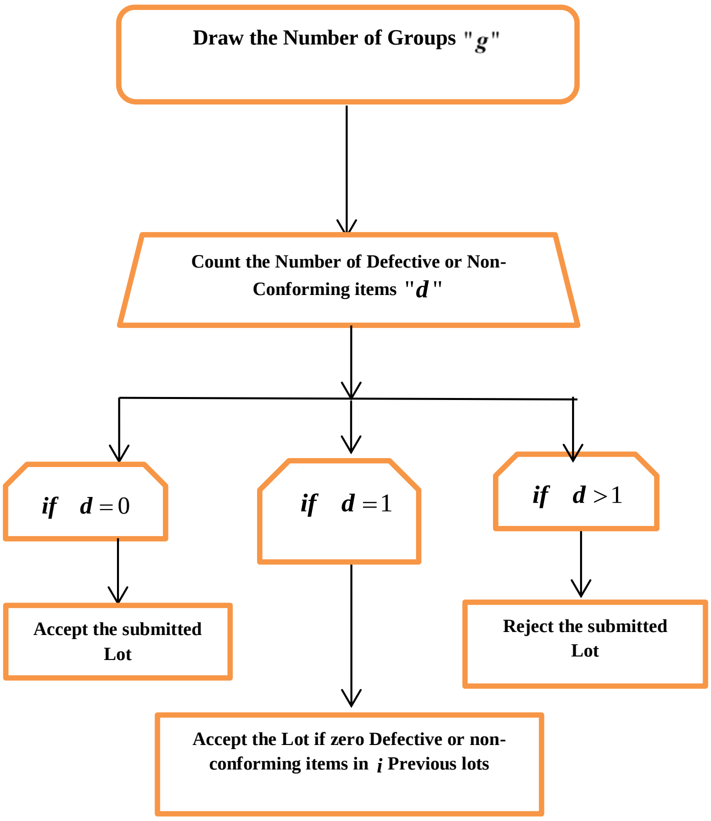


Figure 2. Minimum angle for p_1 and p_2

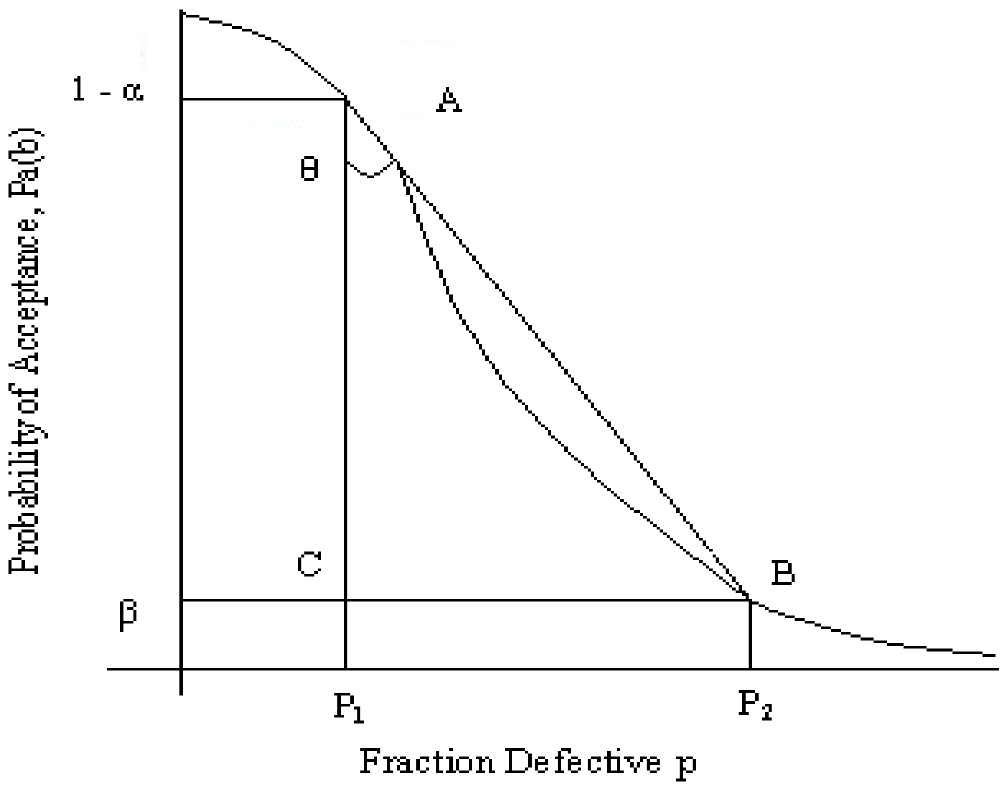


Figure 3. Line chart of optimum number of groups of proposed plan

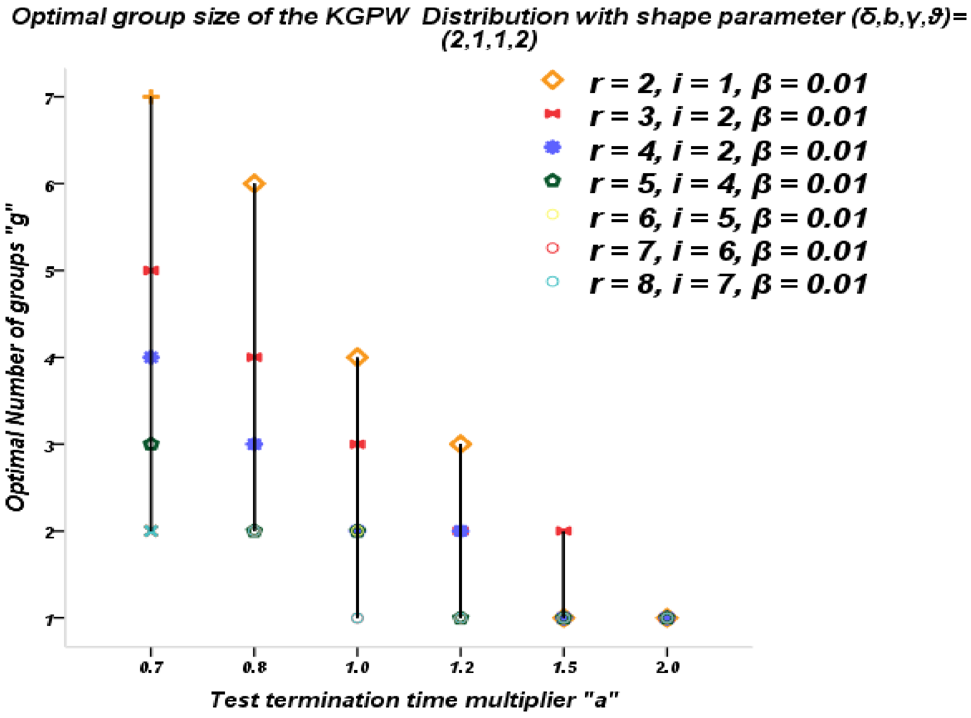


Figure 4. Mean ratio curve in respect different number of testers or items in each groups of proposed plan

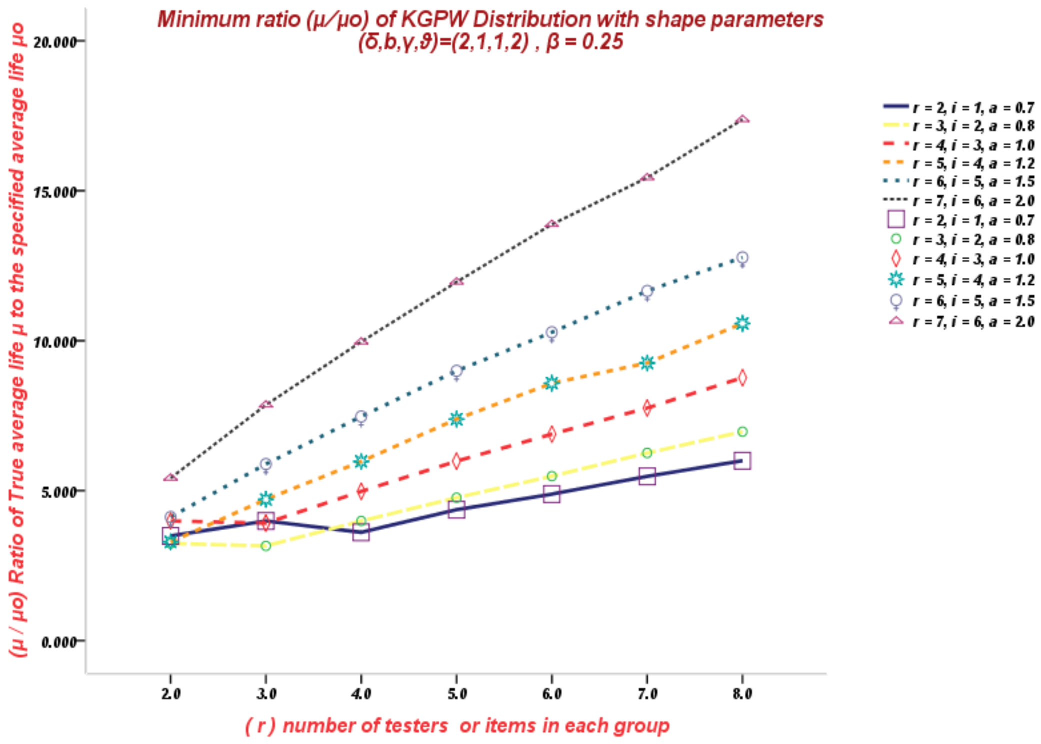


Figure 5. OC curve regarding probability of acceptance of proposed plan

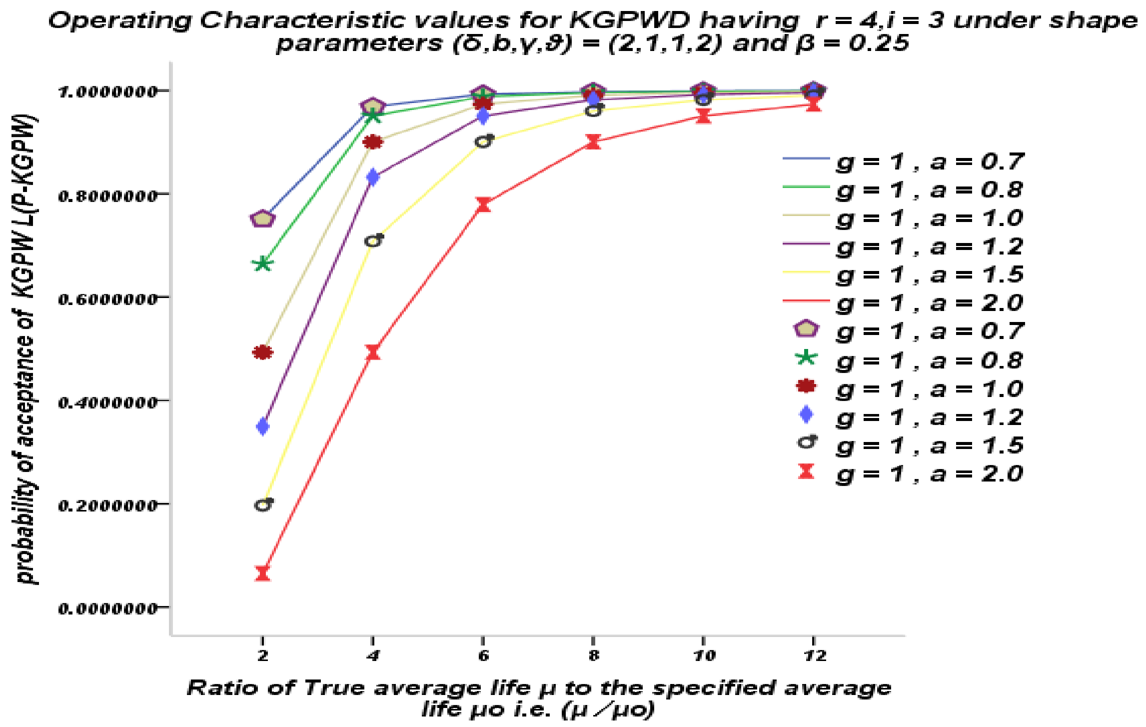


Figure 6. Minimum angle of proposed plan under different values fraction defectives

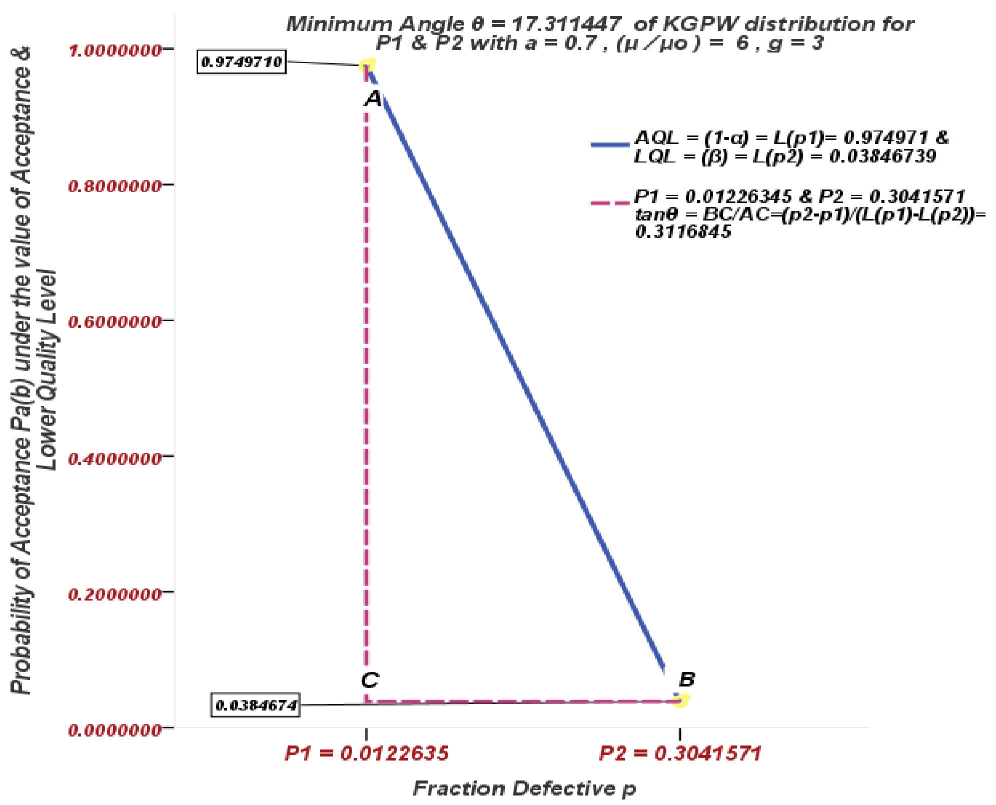
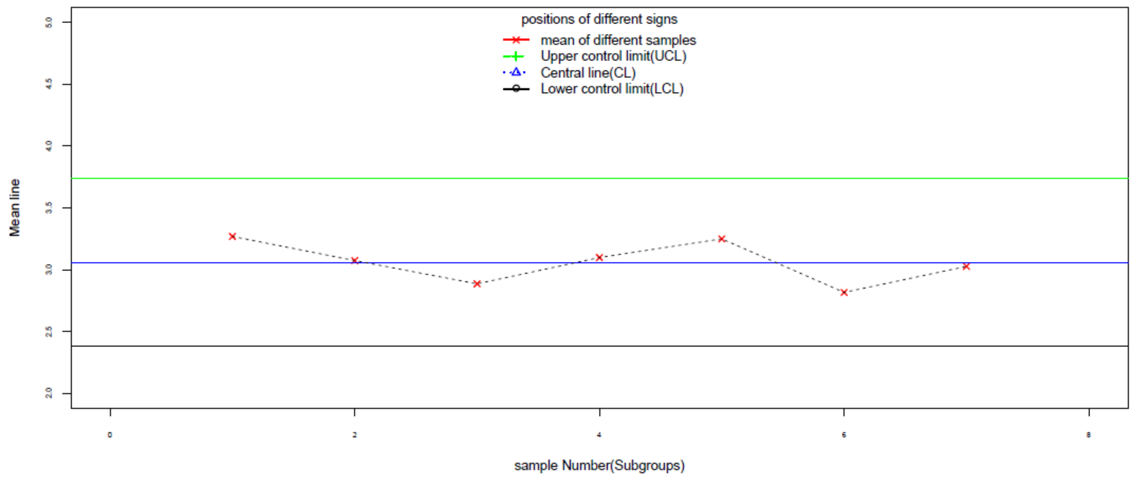
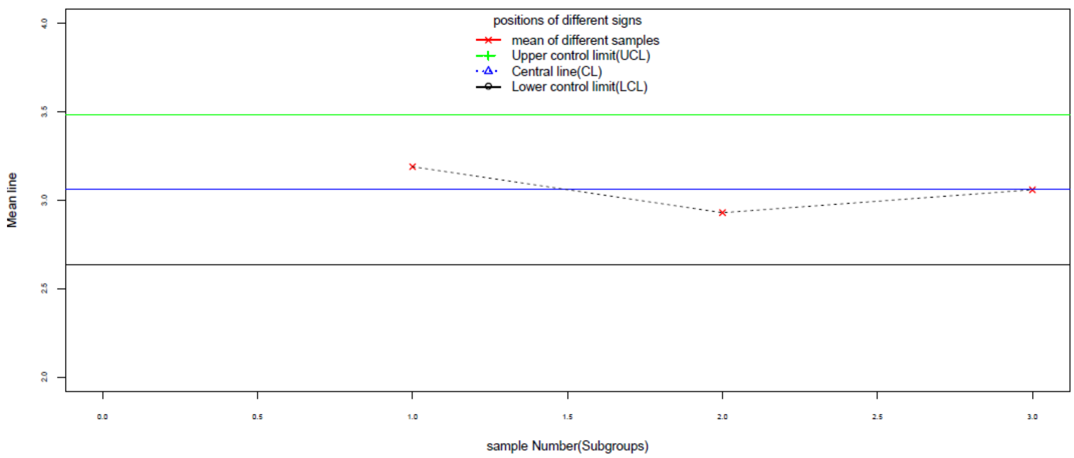


Figure 7. Control charts for process mean under different group sizes

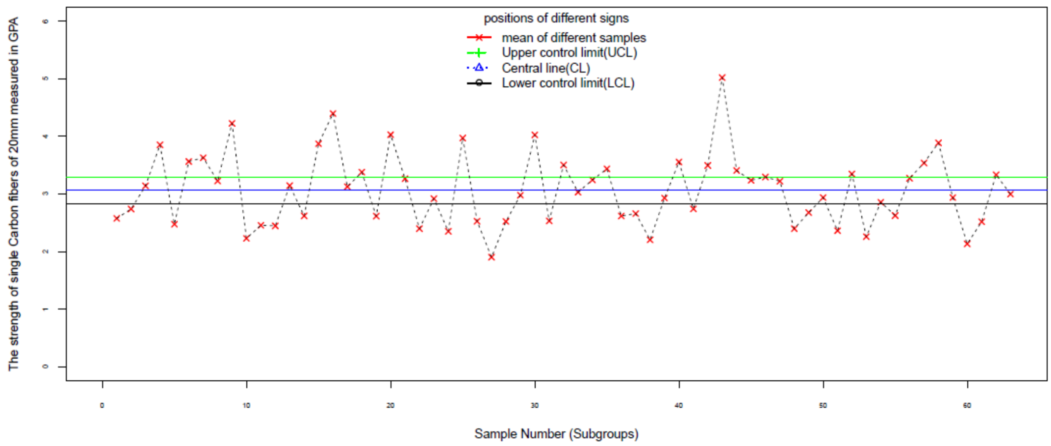
a. $r = 9, g = 7$



b. $r = 21, g = 3$



c. $r = 63, g = 1$



Biographies

Mr. Majid Liaqat is a Lecturer (Statistics) at College Shalimar Graduate Baghbanpura, Higher Education Department, Government of the Punjab, Lahore. Majid has done M.Phil (Statistics), from University of the Punjab, Lahore. He has over Nine years teaching and research experience. He has also worked as Research Officer in Directorate General Social Welfare & Bait-ul-Maal Government of the Punjab, Lahore w.e.f 2018 to 2021. He has more than three year working experience in Research Officer. He has one year experience as a Statistical Assistant for international journal of Linguistics, Social and Natural science, Mehria International publishers. The areas of his research interest include Statistical Process Control and Probability Distributions.

Dr. Nadia Saeed is an Associate Professor at College of Statistical Sciences, University of the Punjab, Lahore. Dr. Nadia has done PhD (Statistics), from University of the Punjab, Lahore in 2020. She has over 15 years teaching and research experience. Nadia's publications have appeared in national and international journals. The areas of her research interest include Statistical Quality Control, Survey Research Methods and Statistical Inference.

Ms. Kanwal Saleem is serving as a Lecturer (Statistics) at National University of Computer and Emerging Sciences (NUCES-FAST), Lahore, Pakistan. She has done M.Phil (Statistics) from University of the Punjab, Lahore in 2018. She has over four years of teaching and research experience. She has also served as an Assistant Editor of Pakistan Journal of Statistics & Operations Research (PJSOR) – a journal run by College of Statistical Sciences, University of the Punjab, Lahore – for almost four years. She has publications in national and international journals as well. Her research interest includes Statistical Quality Control, Meta-Analysis, Multilevel Modeling and Applied Statistics.

Dr. Muhammad Aslam did PhD (Statistics) from NCBA & E, Pakistan. Prof. Muhammad Aslam is the founder of Neutrosophic Inferential Statistics (NIS), Neutrosophic Circular Statistics (NCS), Neutrosophic Applied Statistics (NAS), and Neutrosophic Statistical Quality Control (NSQC). He is the author of three books. He is listed in, top 2% of scientists of the world in the list released by Standford University, USA, and at rank 35/93 among the King Abdulaziz University scientists.

Dr. Rehan Ahmad Khan Sherwani is working as a Professor in Statistics at the College of Statistical Sciences, University of the Punjab, Pakistan. His teaching, mentoring, and tutoring experiences, along with his research background, make him a strong candidate to teach both theoretical and applied courses from the field of statistics and to make substantial contributions to the academic environment of Punjab University. His areas of specialization are Applied Statistics, including Mixed Models, Statistical Process Control, Neutrosophic Theories and their applications. Dr. Rehan is an HEC approved supervisor and have numerous publications in peer-reviewed national and international research journals. He has an editorial and peer-reviewed experience of several national and international research journals. He is member of Boards of Studies of different Universities in Pakistan and also worked as a Member Punjab Technical Committee for Census 2017; Member Dean, Faculty of Science, Purchase Committee; Focal Person QEC Faculty of Science, University of the Punjab, Lahore; Coordinator M.Sc. Biostatistics Programme; Coordinator M.Sc. Business Statistics and Management Programme. He is also a member of the National Curriculum Review Committee by HEC for Statistics.