# Estimation of the Effective Moment of Inertia for Hybrid Concrete Beams Reinforced with Steel and FRP Bars

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**ABSTRACT**

The application of fiber-reinforced polymer bars is rapidly rising in concrete structures because of corrosion resistance and high tensile strength. By contrast, concrete structures reinforced with Fiber-Reinforced Polymer (FRP) bars illustrate less ductility and brittle failure without warning than reinforced concrete structures with conventional steel bars. Hybrid concrete structures with the combination of FRP and steel bars can simultaneously increase strength and ductility. This paper aims to estimate the effective moment of inertia in hybrid concrete beams by using a neuro-fuzzy technique and Artificial Neural networks. A new equation has been proposed for hybrid beams with attention to the importance of calculating the effective moment of inertia in concrete beams. The proposed equation has been considered the effect of elastic modulus and hybrid reinforcement ratio on this parameter for hybrid reinforced concrete beams having FRP bars. This equation has been presented based on the neural networks and experimental data conducted by other researchers on the simple beams to calculate the effective moment of inertia for hybrid reinforced concrete beams. The result shows that both soft computing models are highly precise compared to experimental data.

**Keywords:** Artificial neural networks; ANFIS; Effective moment of inertia; Hybrid concrete beam; FRP bars

## 1. Introduction

During recent decades, the corrosion of steel bars in Reinforced Concrete (RC) structures exposed to deicing salts and marine environments has become a significant concern. To avoid deterioration in this condition, the use of Fiber-Reinforced Polymer (FRP) bars increased because of their high strength-to-weight ratios, corrosion resistance compared to conventional steel bars, durability, and non-magnetic. However, because of linear elastic behavior up to the failure of FRP bars, concrete structure members reinforced with this reinforcement exhibit more significant crack widths and deflection than steel-reinforced concrete members [1]. Therefore, researchers a combination of FRP and steel bars suggested as an effective solution in concrete elements to solve these problems [2]. Using the additional steel bars can increase the flexural members' ductile behavior in hybrid RC members than concrete members reinforced with pure FRP bars. Thus, steel and FRP bars significantly improve ductility and strength in the hybrid beams, respectively. One of the essential factors for providing the balance between improving strength and ductility is the hybrid reinforcement ratio. Qin et al.[3] recommended this ratio within the range of 1 to 2.5 in the over-reinforcement hybrid beams. The study by Akiel et al. [4] showed that members reinforced with hybrid steel-BFRP bars have less deflection and smaller crack widths under service conditions than RC members having BFRP bars only. Sheik and Kharal [5] evaluated the behavior of GFRP-RC beams in flexural, shear, tension, and compression. The results of their studies indicate that the proposed tension-stiffening model is a significantly improves in the prediction of deflection and stiffness of the beams. Salleh et al. [6] evaluated the load-deflection behavior, ratio, and the ordinate of GFRP to steel in hybrid RC beams using ATENA software. Pang et al. [7] investigated the appropriate reinforcement ratio limits to ensure sufficient strength and ductility in hybrid FRP-steel RC beams. Refai et al. [8] proposed a bond coefficient to estimate the crack width of concrete beams reinforced with hybrid bars base on the ACI-440.1R-06 equation. Kara et al. [9] presented a numerical method using force equilibrium and strain compatibility to predict the curvature, deflection, and moment capacity of hybrid RC beams. Dunder et al. [1] proposed a numerical method to calculate the deflection of hybrid beams regardless of the reinforcement type. Al-Sunna et al. [10] experimentally evaluated deflection in RC beams and slabs having FRP bars to compare with existing equations. Naderpour et al. [11] investigated a proposed equation using the artificial neural network (ANN) to predict the FRP-confined compressive strength of concrete. Kheyroddin and Mirza [12, 13] presented a new equation to determine the effect of tension and compression reinforcement ratio, concrete compressive strength, and the form of loading on the flexural rigidity (EI) of RC beams. Bui et al. [14] investigated the ductility of the hybrid RC beams considering various factors, including the effects of the FRP/steel reinforcement ratio, the location and form of FRP bars, and the concrete compressive strength. Shield et al. [15] performed many experiments to recommend a recalibration of bond-dependent coefficients in concrete elements Reinforced with GFRP bars. Nguyen et al. [16] presented a simple equation for predicting the effective moment of inertia ($I_e$) for FRP beams using gene expression programming (GEP). Ge et al. [17] investigated the flexural behavior of concrete beams reinforced with steel-FRP composite bars. Moolaei et al. [18] experimentally evaluated the flexural behavior of beams reinforced with GFRP and steel bars and High Performance Fiber Reinforced Cementitious Composites (HPFRCC). Wang et al.[19] investigated the flexural behavior of five hybrid BFRP and steel bars RC beams subject to four-point bending tests. Arabshahi et al. [20] proposed an equation for the effective moment of inertia in concrete beams reinforced with FRP bars. Lyu et al. [21] studied the usage of back-propagation Neural Network (NN) and Genetic Algorithm (GA) for the predicting of torsional strength RC beams. Li et al.[22] used an artificial neural network and an imperialist competitive algorithm to provide an accurate method for simulating the deflection of the RC beam. Jayasinghe et al. [23] using ANN showed that the new equation for the shear strength of the RC beam in ACI 318–19 and AS 3600–2018 is more accurate compared to other provisions. Alagundi and Palanisamy [24] proposed a model of ANN for prediction of shear strength of an exterior beam-column joint. Zayan and Mahmoud [25] stated that the proposed artificial neural network can successfully evaluate the combined flexural torsional strength of prestressed concrete beams. Zhang et al. [26] application Convolutional Neural Networks (CNNs) to recognize the symmetry group and symmetry order in planar structures. Khan et al. [27] used an artificial neural network and a random forest to estimate the Flexural Strength of beams. The results of the ANN showed that the bottom flexural bars of the beam are the most effective factor in yielding flexural capacity. Peng et al. [28] proposed the Adaptive Neural Fuzzy Inference System (ANFIS) method to investigate the flexural behavior of corroded concrete beams. Because in the finite element method, many inputs are required that it was expensive to collect this amount of data. Barkhordari et al. [29] showed that the Hybrid Algorithm (PSO-ANN) for computing the shear strength of deep RC beams has high accuracy and used SHapley Additive exPlanations (SHAP) method to exhibit the effective parameters for estimating shear strength beams. In this study, due to the lack of accuracy of the existing equations in calculating the effective moment of inertia in the hybrid RC beams, equations were proposed. Machine learning and numerical studies are widely used to investigate the behavior of beams, columns, and bridges [30-32].

## 2. Research significance

The short-term deflection is estimated using the effective moment of inertia at the service load [33]. The primary purpose of this study is to investigate the effect of the elastic modulus of FRP and steel bars and hybrid reinforcement ratio, $A_f/A_s$, on the effective moment of inertia for hybrid RC beams. Besides, existing methods for calculated the effective moment of inertia are not suitable in hybrid beams. Because these equations proposed to calculate the effective moment of inertia in beams reinforced with FRP or steel bars and they do not have enough accuracy in hybrid beams. Consequently, a new model is presented based on an artificial neural network, and then this model provides a comparison of experimental data and other present equations.

## 3. Existing models

Serviceability is defined as satisfactory performance at service load levels that can be described in terms of cracking and deflection criteria. Excessive deflection is undesirable for the appearance and efficiency of the structure. Excessive crack width also seriously affects the aesthetic and durability of the structure [34]. One common and plain method for calculating deflection is the use of $I_e$. As the cracking load exceeds, flexural stiffness changes due to the existence of discrete cracks along with the member [35]. $I_e$ accounted for considering the effect of the flexural stiffness variation and concrete tension stiffening. The Eq. (1) proposed by Branson [36] is applicable to steel-reinforced concrete beams at the service loads.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g \tag{1}$$

where $I_g$ is the gross moment of inertia, $I_{cr}$ is the cracked moment of inertia, $M_{cr}$ and $M_a$ are the cracking moment and applied moments at the critical section, respectively.

The results of studies provide reveal that this equation overestimates $I_e$ in concrete beams reinforced with FRP bars, especially in beams are under reinforcement [37-40].

Bischoff [41] suggested an equation for $I_e$, which could be computed from Eqs. (2) and (3). This equation compared with experimental results illustrates that is suitable for both steel and FRP reinforced concrete beams:

$$I_e = \frac{I_{cr}}{1-\eta\left(M_{cr}/M_a\right)^2} \leq I_g \tag{2}$$

$$\eta = 1-\frac{I_{cr}}{I_g} \tag{3}$$

The ACI 440.1R-15 [34] committee offered an additional factor γ, in the equation proposed by Bischoff to consider the variety in stiffness along the length of the member, as illustrated in Eq. (4). The new expression presents a reasonable approximation of the deflection for RC beams with FRP and one-way slabs [42].

$$I_e = \frac{I_{cr}}{1-\gamma\left(\frac{M_{cr}}{M_a}\right)^2\left[1-\frac{I_{cr}}{I_g}\right]} \leq I_g \qquad where \;\; M_a \geq M_{cr} \tag{4}$$

The factor $\gamma$ is defined based on the load and boundary conditions and considers the length of the uncracked regions of the element and change in stiffness in the crack regions [34]. This factor could be computed from Eq. (5).

$$\gamma = 1.72-0.72\left(M_{cr}/M_a\right) \tag{5}$$

Benmokrane et al. recommended Eq. (6), which calibrated utilizing a few numbers of experimental data [36].

$$I_e = \alpha I_{cr} + \left(\frac{I_g}{\beta}-\alpha I_{cr}\right)\left[\frac{M_{cr}}{M_a}\right]^3 \tag{6}$$

The factor α which exhibiting the diminished composite behavior between the FRP bars and concrete is equal to o.84. The factor $\beta$ is equal to 7, which was applied to provide a faster transition from $I_g$ to $I_{cr}$.

Pirayeh Gar et al.[35] have proposed an equation to predict the deflection of FRP prestressed concrete (PSC) beams regardless of the $I_{cr}/I_g$ ratio.

Mousavi et al.[43] evaluated the effect of several parameters on the power m in the equation of Branson utilizing the genetic algorithm method. The proposed equations for $I_e$ can be determined as follows.

The objective function of model A has been described by Eq. (7)

$$e = \left|\delta_{\exp}-\delta_{cal}\right| \tag{7}$$

Model A is described by Eqs. (8) and (9) as follows.

$$\left(I_e\right)_{Model\,A} = 0.15\left(\frac{M_{cr}}{M_a}\right)^m I_g + 0.89\left[1-\left(\frac{M_{cr}}{M_a}\right)^m\right] I_{cr} \leq I_g \tag{8}$$

$$m = 0.66 - 0.3\frac{\rho_f}{\rho_{fb}} + 1.94\frac{M_{cr}}{M_a} + 4.64\frac{E_f}{E_s} \quad (9)$$

The objective function of model B has been described by Eq. (10)

$$e = \left| (I_e)_{\exp} - (I_e)_{theo} \right| \quad (10)$$

Model B is described by Eqs. (11) and (12).

$$(I_e)_{Model\ B} = 0.17\left(\frac{M_{cr}}{M_a}\right)^m I_g + 0.94\left[1 - \left(\frac{M_{cr}}{M_a}\right)^m\right] I_{cr} \le I_g \quad (11)$$

$$m = 1.69 - 0.51\frac{\rho_f}{\rho_{fb}} + 1.77\frac{M_{cr}}{M_a} + 6.67\frac{E_f}{E_s} \quad (12)$$

Where $E_f$ and $E_s$ are the elastic modulus of FRP and steel bars, respectively. The $\rho_f$ is the FRP reinforcement ratio, and $\rho_{fb}$ is the FRP balance ratio.

In FRP reinforced beams, the balance reinforcement ratio could be calculated by Eq. (13) where the rupture of FRP bars and concrete crushing occur simultaneously.

$$\rho_{fb} = 0.85\beta_1 \frac{f_c^{'}}{f_{fu}} \frac{E_f \varepsilon_{cu}}{E_f \varepsilon_{cu} + f_{fu}} \quad (13)$$

Where $f_{fu}$ is the ultimate tensile stress of FRP bars, $\beta_1$ considers between 0.85 and 0.65 with attention to concrete strength, $f_c^{'}$ and $\varepsilon_{cu}$ are the concrete compressive strength and maximum concrete compressive strain, respectively.

Kheyroddin and Maleki [44] suggested an equation for calculating $I_e$ in Hybrid RC beams using the genetic algorithm method and experimental data, the proposed expression presented in Eqs. (14) and (15).

$$(I_e)_{theo} = 0.136\left(\frac{M_{cr}}{M_a}\right)^m I_g + 1.117\left[1 - \left(\frac{M_{cr}}{M_a}\right)^m\right] I_{cr} \quad (14)$$

$$m = 0.836\frac{E_f}{E_s} + 0.208\rho_{fb}\frac{A_f}{A_s} + 3.709\frac{M_{cr}}{M_a} \quad (15)$$

**4**. **Soft Computing**

The Soft Computing (SC) method which was first introduced by Zadeh [45], mimics the ability of the human brain and how it relates to the environment of uncertainty and inaccuracy. Soft computing aims to use human knowledge to solve complex problems with acceptable precision to have high similarity with human decision making [46]. Methods in soft computing were inspired by nature and has been considered as a main technique in structural engineering fields [47-52]. One of the most important reasons for the importance of soft computing and intelligence Computation is the existence of uncertainties and ambiguities in the real world. The purpose of combining the methods of genetic algorithm, fuzzy system, and neural network in soft computing is to achieve the ability to solve problems that each method can't solve alone.

Artificial neural networks and fuzzy systems are examples of the most important soft computing models that are widely used in various sciences. In the last few years, powerful systems called the Adaptive Neuro-Fuzzy Inference System (ANFIS) have been used in various sciences. These types of systems, by taking advantage of the training power of neural networks and the linguistic advantage of fuzzy systems, have been able to benefit from These two models should be used for process analysis. As one of the common neuro-fuzzy systems, ANFIS was suggested by Jang [53] in 1993. This system uses back-propagation gradient descent and the least-squares methods for the training process.

This study was used as two methods consist of ANN and ANFIS models to determine $I_e$ in hybrid beams. To achieve the desired output, for each method select the best model, and the results are evaluated with the experimental.

**5. ANN modeling**

The artificial neural network is one of the branches of soft-computing which has been used in modeling complex nonlinear systems. In recent years, the artificial neural network has been considered a powerful computational method. That solves complex problems and can be applied to simulate, evaluate, and approximate with high accuracy.

For the correct equation of Branson, one hundred twenty-two data as follow have been applied. These data are the number of points on the curve of load-deflection hybrid FRP/steel RC beams with four-point loading. Table 1 presents the detail of the experimental specimens used in this study.

The parameters include reinforcement ratio ($A_f/A_s$), elastic modulus ratio ($E_f/E_s$), level of loading ($M_{cr}/M_a$), the gross moment of inertia ($I_g$), the balance ratio ($\rho_{fb}$),and cracked moment of inertia ($I_{cr}$) used as the six input nodes, the target node was the effective moment of inertia. Two hidden layers were utilized in this ANN modeling, where Log-Sigmoid and Purline were transfer functions, respectively. Normalization /scaling for all data was performed and for this purpose, all data were scaled between 0.1 and 0.9. Tables 2 and 3 presented the values of experimental data and normalization equations, respectively.

In the Levenberg –Marquardt method input and target nodes randomly have distributed into three sets, consist of the train, validate, and test the network. Varying the relative percentages of those three sets could slightly improve the generate method. The regression values and Mean Square Error (MSE) of the networks with the different number of hidden nodes exhibit in Fig. 1 and Fig. 2, respectively.

Network with 16 nodes of the hidden layer was chosen since it presents good results in the case of R-values and also has the least value of MSE among all networks. The results for training are summarized in Figs. 3–5.

**6. Sensitivity analysis**

To calculate the influence of the input data on the effective moment of inertia has used the weight matrix according to table 4 and the equation of Milne [54]. This study used the modified version of Grason's equation [55] because it considers the absolute values of weights. Milne's formula is presented in Eq. 16.

$$\frac{\sum_{j=1}^{nhidden} \frac{w_{ji}}{\sum_{l=1}^{ninputs} \left| w_{jl} \right|} .w_{oj}}{\sum_{k=1}^{ninputs} \left( \sum_{j=1}^{nhidden} \left| \frac{w_{jk}}{\sum_{l=1}^{ninputs} \left| w_{jl} \right|} .w_{oj} \right| \right)} \tag{16}$$

Where $w_{jl}$ and $w_{oj}$ are the weight between neuron $j$ in the hidden layer with input node l and the weight between neuron $j$ and neuron $o$ in the output layer, respectively.

In Fig. 6, the percentage influence of each of the six input parameters on the effective moment of inertia is shown. Table 4 presents the details of weights and their bias of the proposed neural network.

**7. Proposed approach**

The range and the reference value of the six inputs data are represented in Table 5. The reference numbers were considered close to the mean values. As the first step, $I_e$ is plotted against $\rho_{fb}$ while the other input parameters have the reference values, as shown in Fig. 7. To account for the effect of other factors on $I_e$, the correction function can be computed from Eq. 17.

$$F\left(\frac{M_{cr}}{M_a},\frac{A_f}{A_s},I_g,\frac{E_f}{E_s},I_{cr}\right)=C\left(\frac{M_{cr}}{M_a}\right)*C\left(\frac{A_f}{A_s}\right)*C\left(I_g\right)*C\left(\frac{E_f}{E_s}\right)*C\left(I_{cr}\right) \tag{17}$$

To draw a curve in Figure 8, reference values are considered for the input parameters except for $A_f/A_s$ and $M_{cr}/M_a$. For $A_f/A_s$ one of the values specified in the figure and for $M_{cr}/M_a$ the entire data range is applied as input. To determine $C(M_{cr}/M_a)$, the result obtained $I_e$ from the above neural network divided by the value in the case where $M_{cr}/M_a$ also has a reference value. Another 9 curves are plotted in the figure with the same method and various values of $A_f/A_s$. A similar method was applied to draw $C(M_{cr}/M_a)$ versus other input parameters including $I_g$, $E_f/E_s$, and $I_{cr}$ (Figs. 9-11). The same method has been utilized to achieve the correction factors for the other input parameters. Some of them are exhibited in Figs. 12–15.

To determine the following equation (18–22), according to 40 curves related to each correction factor, a line with the least squared error was drawn and the corresponding equations were presented:

$$C\left(\frac{M_{cr}}{M_a}\right)=0.37*\left(\frac{M_{cr}/M_a}{0.4}\right)+0.63 \tag{18}$$

$$C\left(\frac{A_f}{A_s}\right)=-0.2*\left(\frac{A_f/A_s}{1.2}\right)+1.2 \tag{19}$$

$$C\left(I_g\right)=-0.095*\ln\left(\frac{I_g}{2.7E+8}\right)+0.99 \tag{20}$$

$$C\left(\frac{E_f}{E_s}\right)=0.87*\ln\left(\frac{E_f/E_s}{0.45}\right)+1 \tag{21}$$

$$C\left(I_{cr}\right)=-0.03\left(\frac{I_{cr}}{6.9E+7}\right)^2+0.44\left(\frac{I_{cr}}{6.9E+7}\right)+0.56 \tag{22}$$

Consequently, the effective moment of inertia will be established from Eq. (23) in which, ($I_e$) curve can be read from Fig. 7.

$$I_e = (I_e)_{curve} * C\left(\frac{M_{cr}}{M_a}\right) * C\left(\frac{A_f}{A_s}\right) * C(I_g) * C\left(\frac{E_f}{E_s}\right) * C(I_{cr}) \tag{23}$$

## 8. Neuro-Fuzzy approach

The neuro-fuzzy model defined in this article has five Gaussian membership functions for each input variable. Each membership function represents a language expression for the variable. For example, membership function number 1 represents very low values and membership function number 5 represents very high values. Each of the above Gaussian functions, also shown in Fig. 16, has two parameters, including the mean value *m* as well as the value of variance *v*, the details of which can be seen in Table 6.

In adjusting the structure of the above system (Fig. 17), five fuzzy rules are considered, which will be used to estimate the output parameter. Using the data used in the neural network training process, the best neuro-fuzzy model was determined, the results of training and test datasets can be seen in Fig. 18.

As shown in Fig. 17, the proposed model has five linear functions. These functions ($F_1, \ldots, F_5$), which include a set of coefficients for each of the input variables and a constant, are shown in Eq. 24 to 28. The above coefficients (Table 7) are determined by the neuro-fuzzy optimization algorithm in the training process of the model.

$$F_1 = -2.13X_1 - 0.51X_2 + 1.92X_3 + 0.99X_4 + 1.848X_5 + 1.55X_6 - 0.9 \tag{24}$$

$$F_2 = -1.97X_1 - 25.33X_2 + 0.94X_3 - 11.74X_4 - 0.49X_5 + 15.01X_6 - 3.33 \tag{25}$$

$$F_3 = -1.75X_1 + 0.08X_2 + 0.33X_3 - 0.6X_4 + 0.63X_5 + 2.29X_6 + 0.58 \tag{26}$$

$$F_4 = -0.06X_1 + 0.01X_2 + 0.27X_3 + 3.53X_4 - 0.27X_5 - 0.08X_6 - 0.17 \tag{27}$$

$$F_5 = 3.89X_1 - 0.16X_2 + 0.11X_3 + 6.13X_4 - 0.16X_5 - 0.06X_6 - 1.63 \tag{28}$$

Using the number of input variables, the value of each of the linear functions is determined. Then, the value of the output variable is estimated by Eq. 29 .It should be noted that the output value obtained by this equation is normalized and needs to be converted to its real value by the relation provided in Table 3.

$$0.1 \leq \left( Y = \frac{\sum_{i=1}^{5} G_i W_i}{\sum_{i=1}^{5} W_i} \right) \leq 0.9 \tag{29}$$

The parameter *W* in the above equation is related to the set of fuzzy rules of the system (five rules for the proposed model) whose values can be calculated using Eq. 30 to 34 with consideration of the Gaussian membership functions provided in Table 6.

$$W_1 = G_{1,X1}\, G_{1,X2}\, G_{1,X3}\, G_{1,X4}\, G_{1,X5}\, G_{1,X6} \tag{30}$$

$$W_2 = G_{2,X1}\, G_{2,X2}\, G_{2,X3}\, G_{2,X4}\, G_{2,X5}\, G_{2,X6} \tag{31}$$

$$W_3 = G_{3,X1}\, G_{3,X2}\, G_{3,X3}\, G_{3,X4}\, G_{3,X5}\, G_{3,X6} \tag{32}$$

$$W_4 = G_{4,X1}\, G_{4,X2}\, G_{4,X3}\, G_{4,X4}\, G_{4,X5}\, G_{4,X6} \tag{33}$$

$$W_5 = G_{5,X1}\, G_{5,X2}\, G_{5,X3}\, G_{5,X4}\, G_{5,X5}\, G_{5,X6} \tag{34}$$

## 9. Comparison study

To verify the proposed ANN and ANFIS models compared with the genetic algorithm method and existing models, respectively, Fig. 19 and Fig 20 have been presented. Also, the distribution of error and percentage error for various equations is shown in Table 8 and Table 9, respectively.

At first, the error value of existing and proposed models is calculated from Eq. 35 then the calculating average error is presented in Table 10.

$$e = \left| \frac{(I_e)_{theo} - (I_e)_{\exp}}{(I_e)_{\exp}} \right| \times 100 \tag{35}$$

According to the results of the proposed methods and their comparison with existing methods, it can be seen that the use of ANFIS and ANN models have higher accuracy and lower error percentage in estimating the effective moment of inertia. The proposed equation using the ANN method reduces the error in the calculation of $I_e$ by about 51.22, 17.36, 30.33, and 3.07 percent, compared with the equations proposed by Branson, Bischoff, ACI, and Benmokrane, respectively. Also, this error reduction in the proposed equation with ANFIS is 87.77, 79.28, 82.54, and 75.70 percent respectively.

**10. Conclusions**

A wide number of experimental data for hybrid RC beams was collected. To predict the effective moment of inertia using the neural network, six related parameters were considered as network inputs. After investigating the performance ($R$ and MSE) of 17 neural networks with various numbers of nodes in the hidden layer, a network with the highest performance in the simulation was chosen. According to these correction coefficients obtained from the simulated results of the neural network, a general equation was presented to calculate $I_e$ in hybrid beams independent of the neural network. The average error of the proposed model is 30%, and more than 99% simulated result is within a 60% range of error. A comparison of the results from ANN simulation, ANFIS model, and available equations with the experimental data showed that the Soft Computing models have high accuracy. The precision of the ANN and ANFIS models was verified by existing experimental data and exhibited good agreement. Also, the comparison of two SC models revealed that the ANFIS model is less error and more accurate than ANN in predicting $I_e$ in hybrid beams.

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## Biographies

**Fahimeh Maleki** received her B.Sc. and M.Sc. degrees in Civil and Structural Engineering from Semnan University. Now she is a Ph.D. Candidate at Semnan University. She is a member of the Iranian Concrete Institute (ICI). Her study fields include the Finite Element Method, Artificial Neural Networks, Hybrid Concrete Structures, and FRP bars.

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**Masoomeh Mirrashid** is currently postdoctoral research fellow in structural engineering at Semnan University, Iran. She has taught several courses of higher education including theory of elasticity and plasticity, dynamic of structures, advanced concrete and, steel Structures. Her scientific activities include author and co-author of more than twenty ISI articles, technical committee member for more than ten international conferences, editor and, reviewer for international journals. Her fields of interests are structural engineering, earthquake, vulnerability, neural networks, Fuzzy and Neuro-Fuzzy systems, machine learning methods and also optimization algorithms.

**Figure Caption**



**Tables Caption**

Table 5. The input parameters range and reference values

Table 6. Gaussian membership function details

Table 7. Output Functions

Table 8. The errors range for various existing equations versus the ANN and ANFIS models

Table 9. The percentage errors for various existing equations versus the ANN and ANFIS model

Table 10. Comparison of average error of $I_e$ between the proposed equation and four existing models

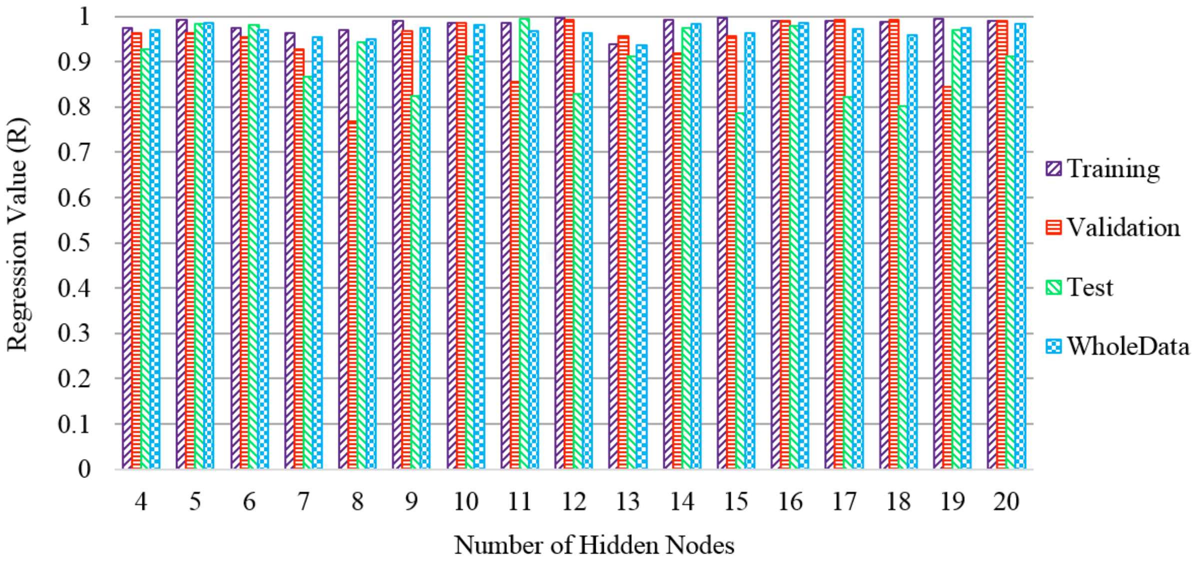


Fig. 1. R-values vs the different number of hidden nodes

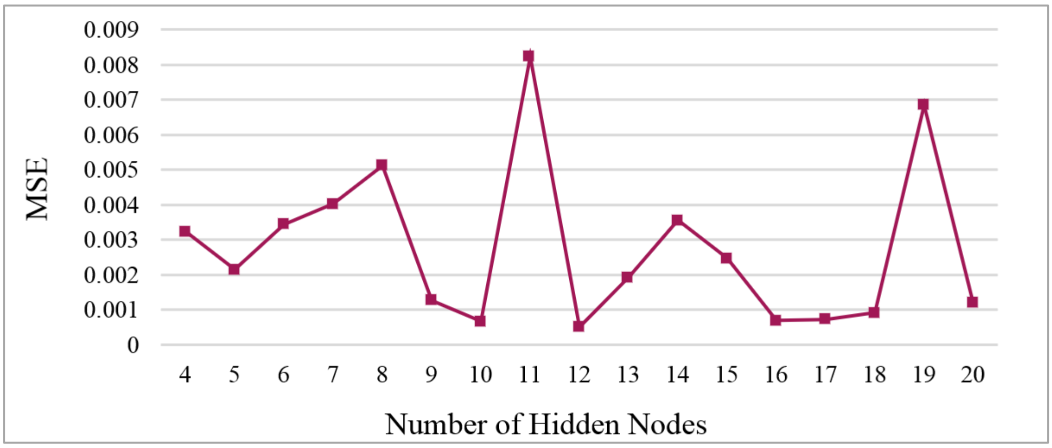


Fig. 2. MSE versus hidden nodes number

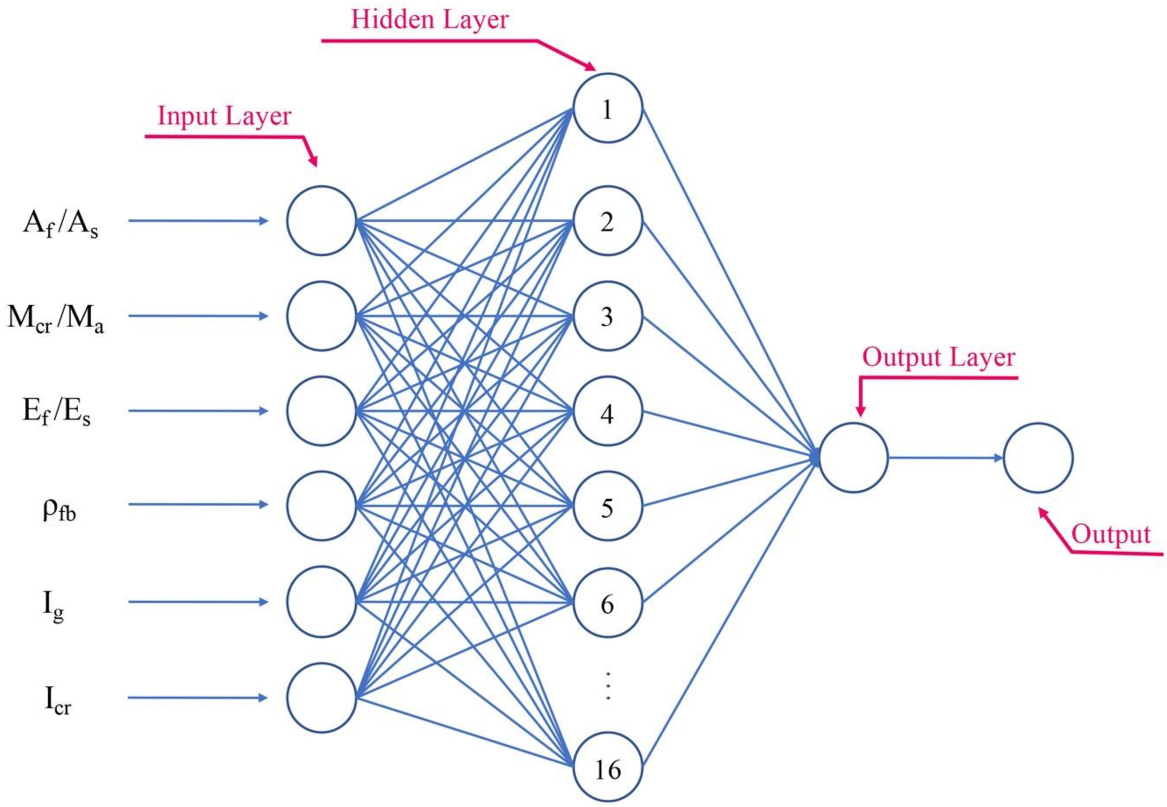


**Fig. 3.** Scheme of ANN

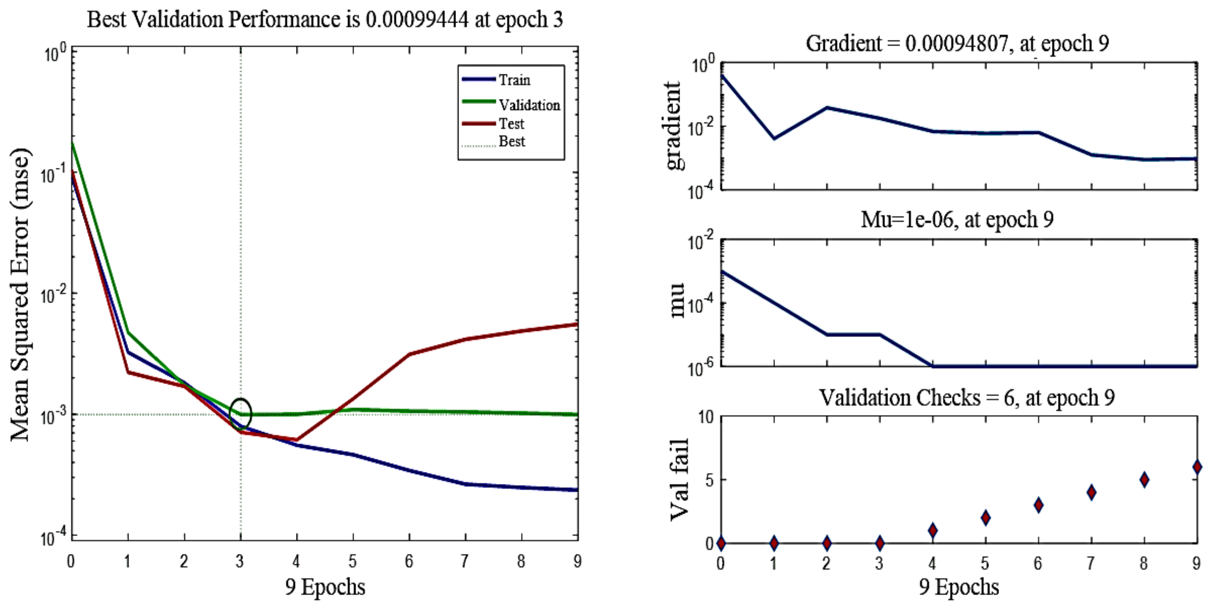


Fig. 4. Performance and Training state of ANN

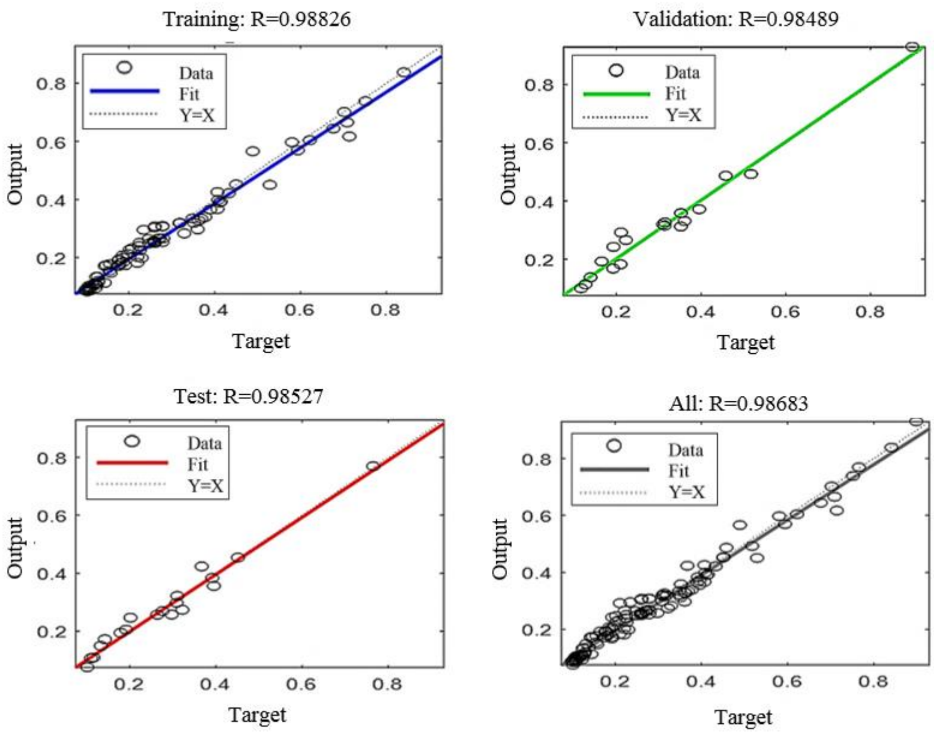


Fig. 5. Regressions of training, validation, and test data simulated by ANN

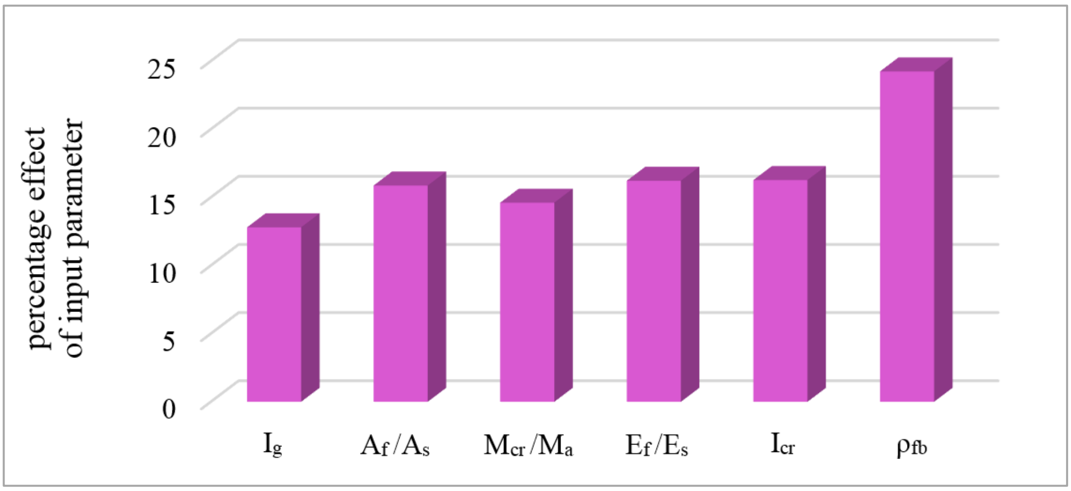


Fig. 6. Contribution of input parameters in target

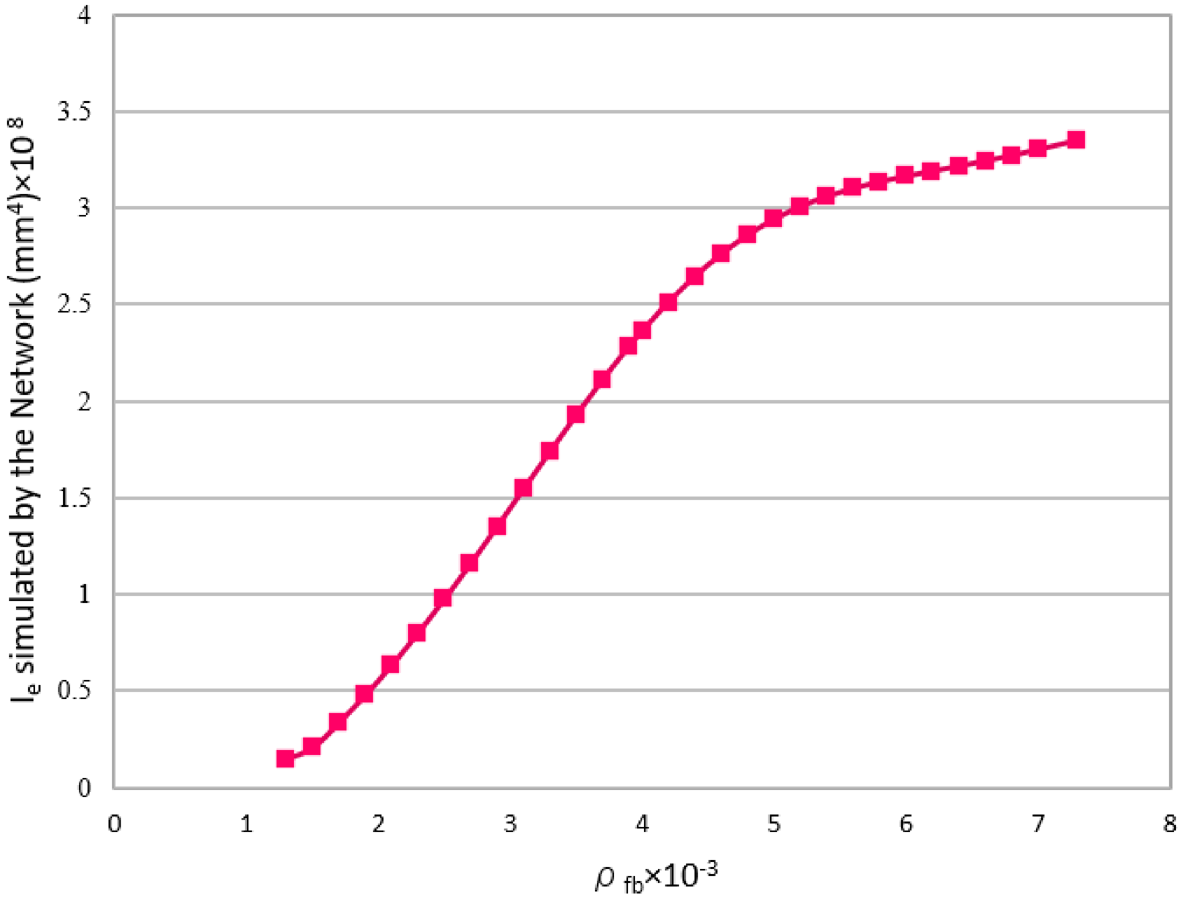


Fig. 7. Variation of $I_e$ against $\rho_{fb}$.

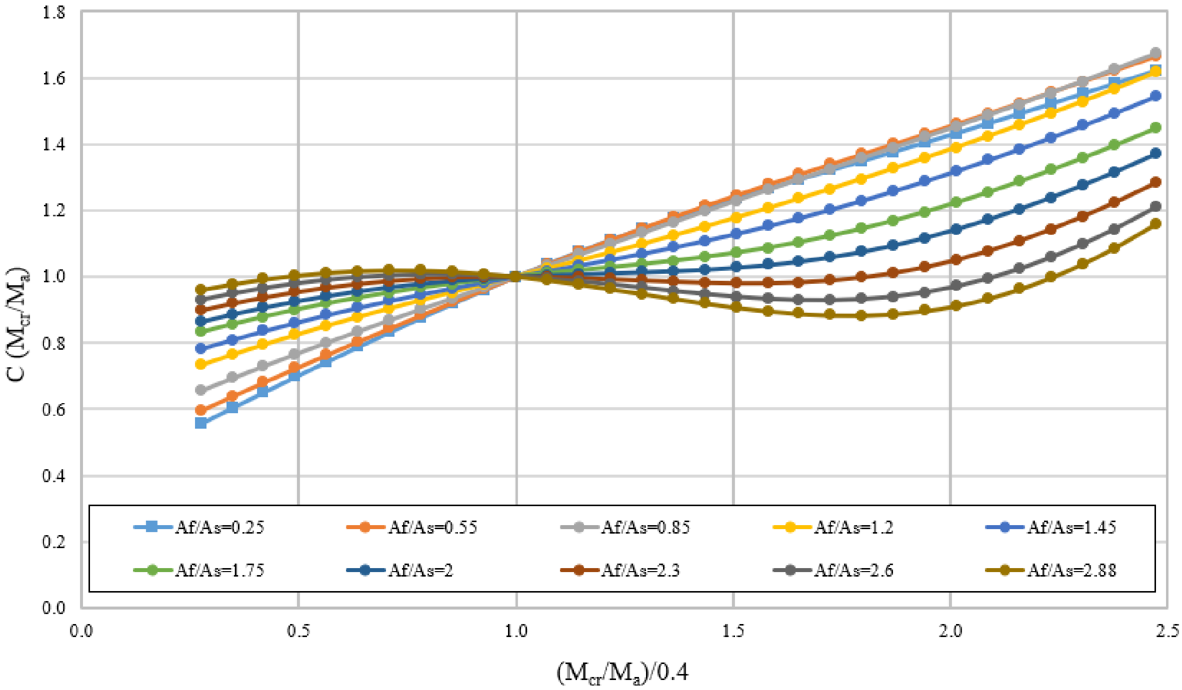


Fig. 8. Factor $C\ (M_{cr}/M_a)$ for different $(A_f/A_s)$ values.

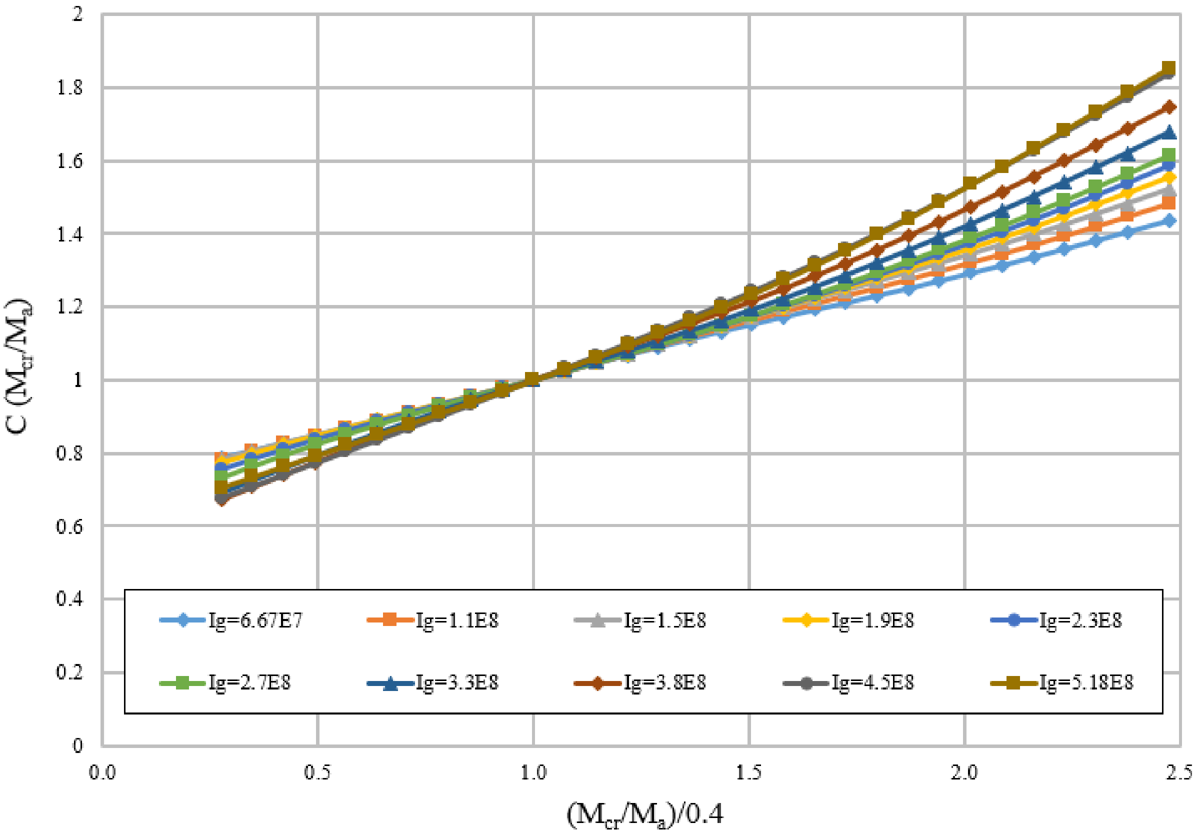


Fig. 9. Factor $C$ $(M_{cr}/M_a)$ for different $I_g$ values

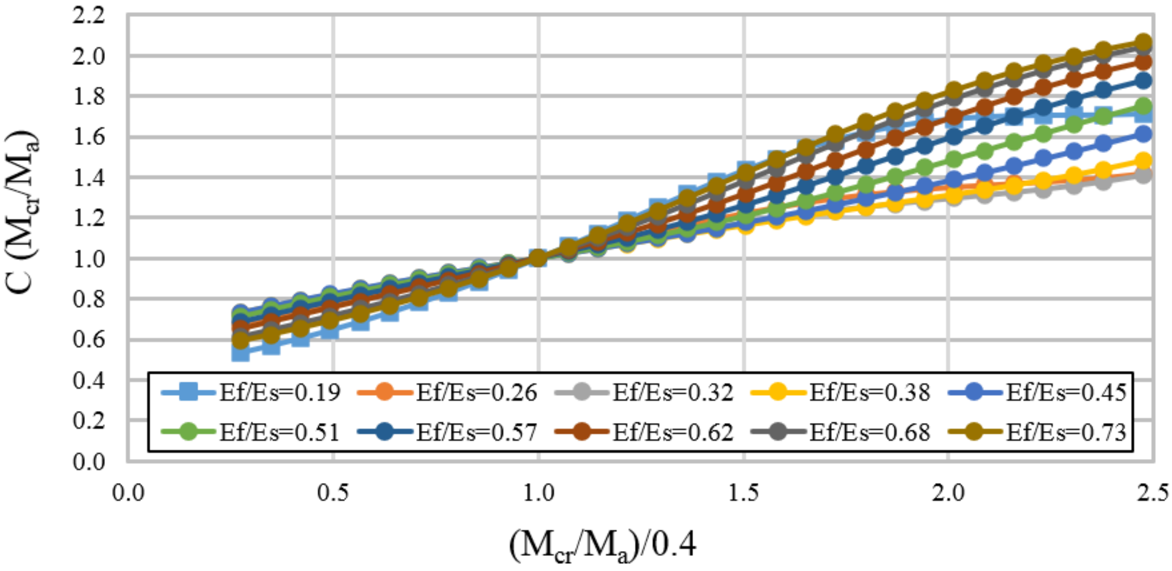


Fig. 10. Factor C ($M_{cr}/M_a$) for different ($E_f/E_s$) values

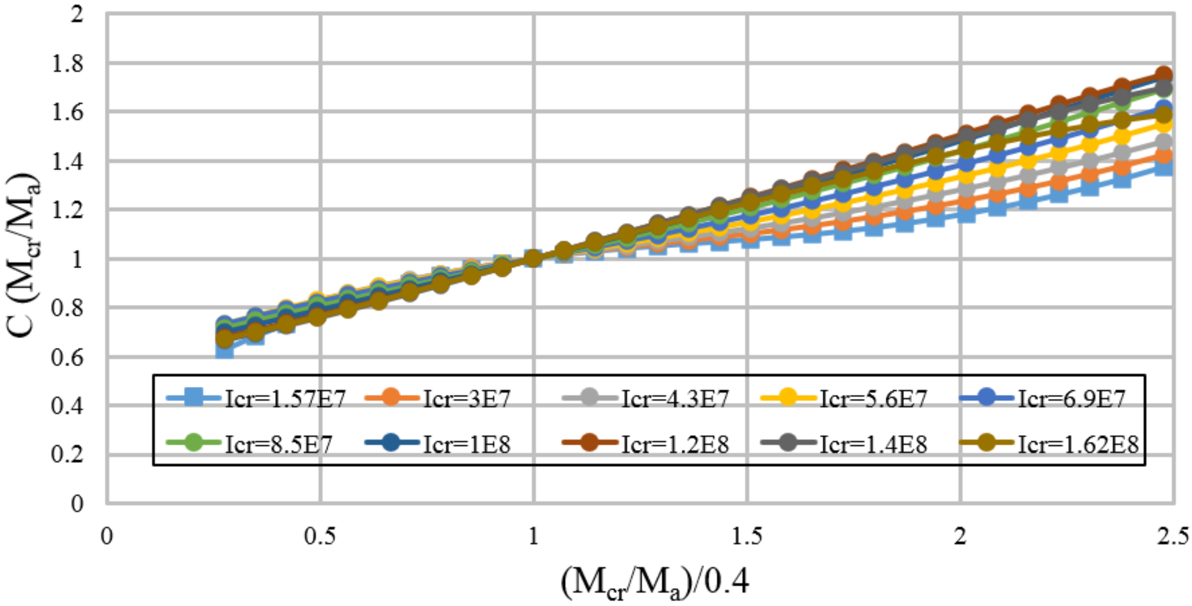


Fig. 11. Factor $C$ $(M_{cr}/M_a)$ for different $I_{cr}$ values

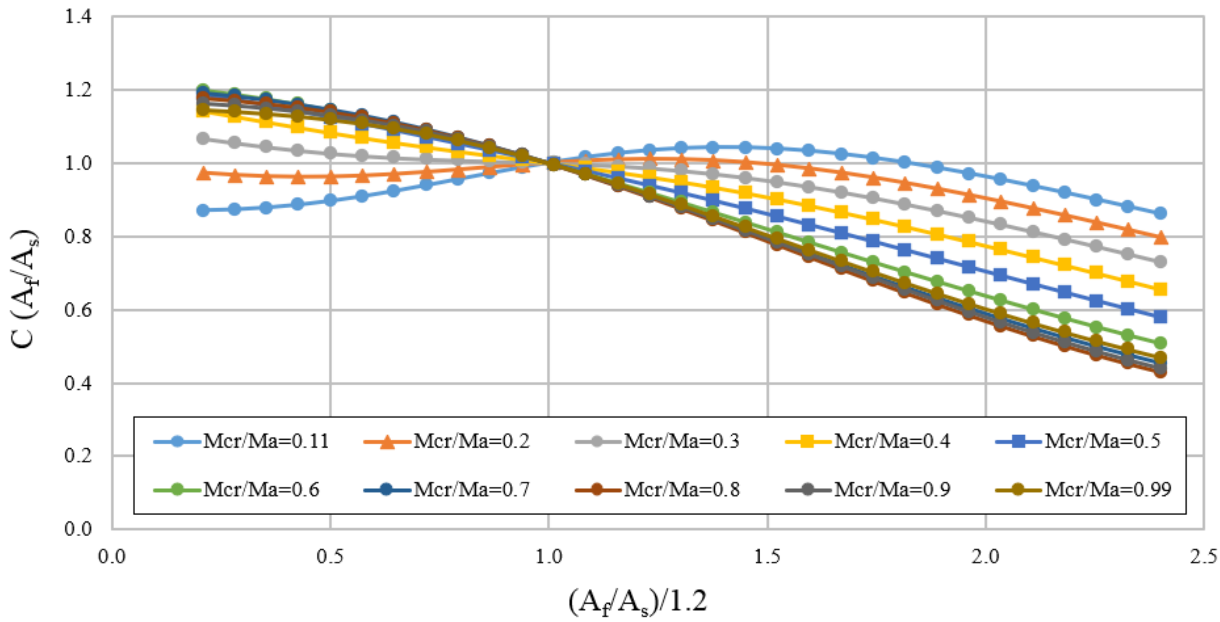


Fig. 12. Factor *C* $(A_f/A_s)$ for different $(M_{cr}/M_a)$ values

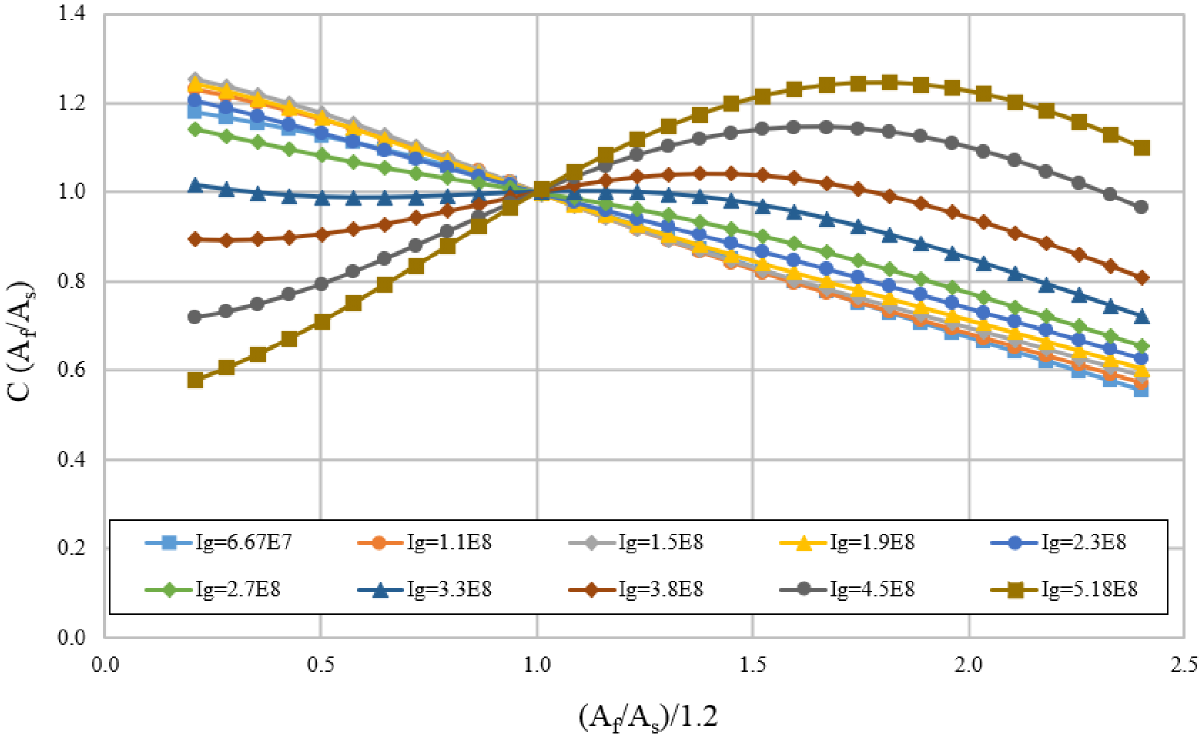


Fig. 13. Factor $C$ $(A_f/A_s)$ for different $I_g$ values

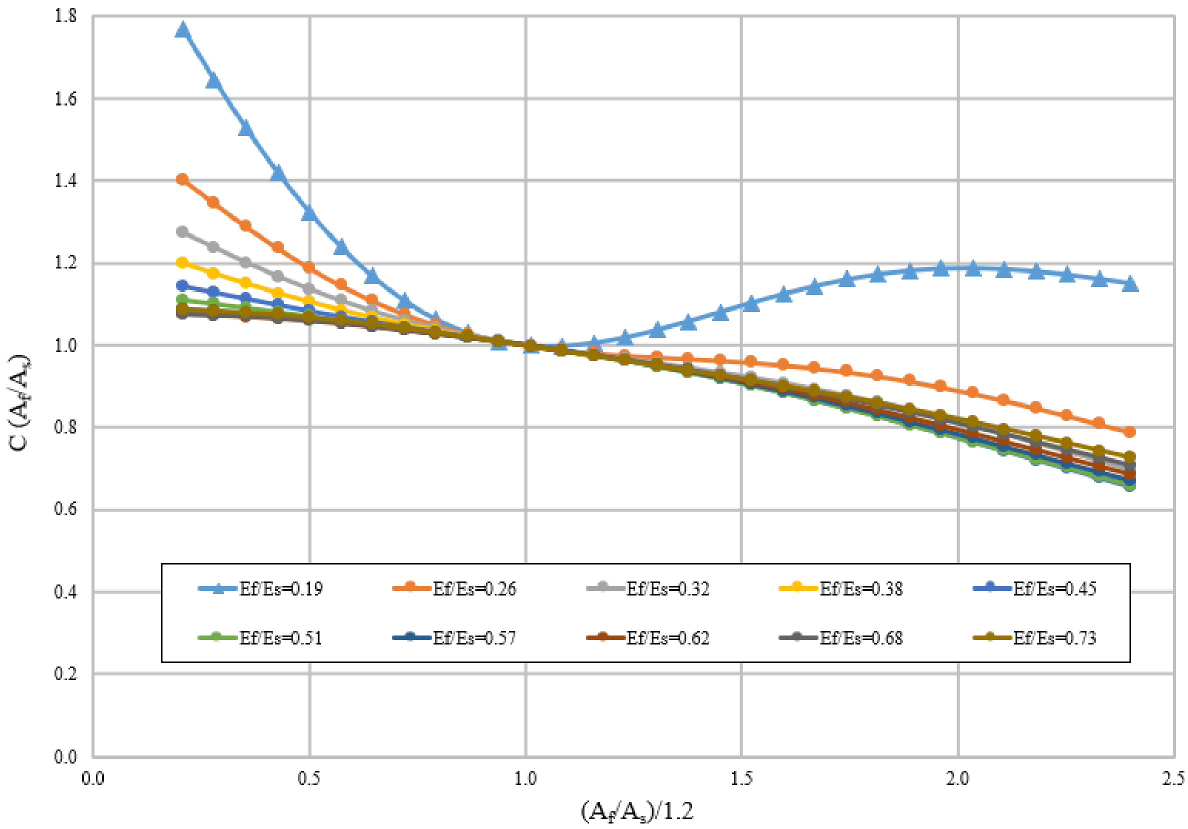


Fig. 14. Factor $C\ (A_f/A_s)$ for different $(E_f/E_a)$ values

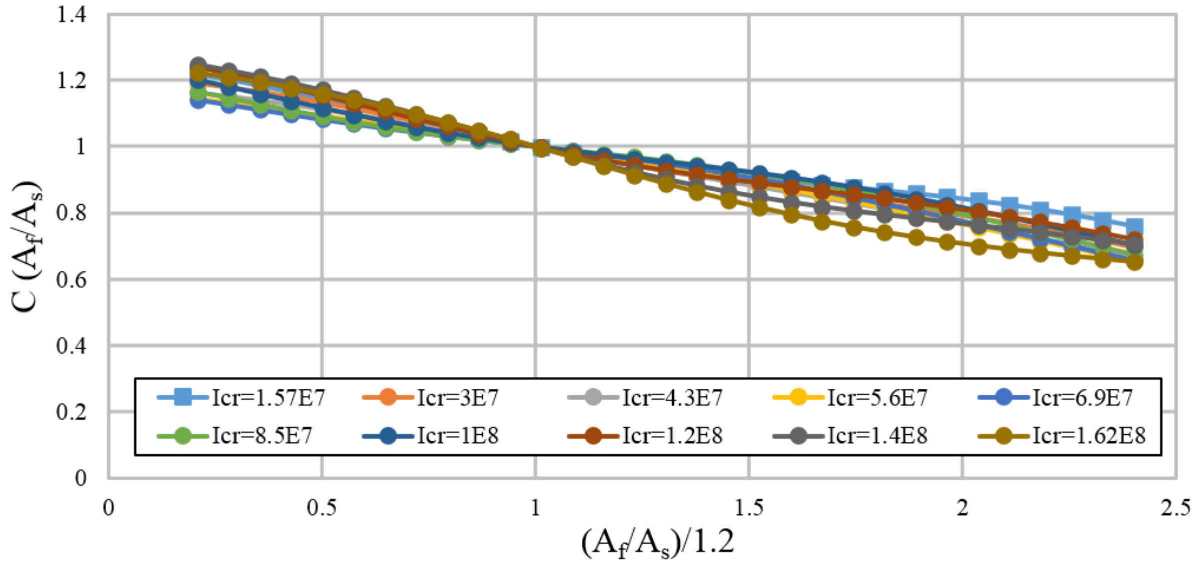


Fig. 15. Factor $C\ (A_f/A_s)$ for different $I_{cr}$ values

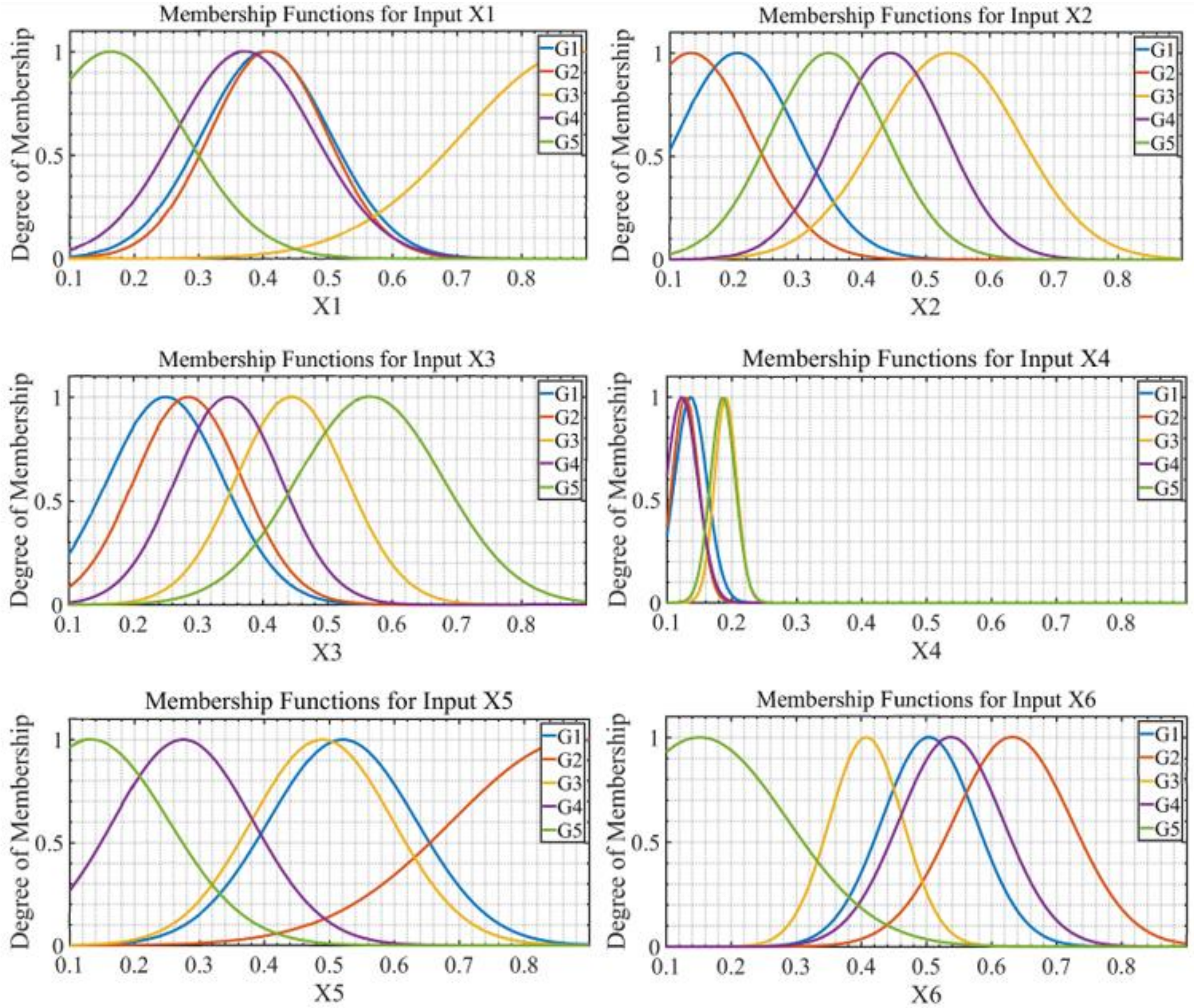


Fig. 16. Gaussian membership functions for the considered inputs

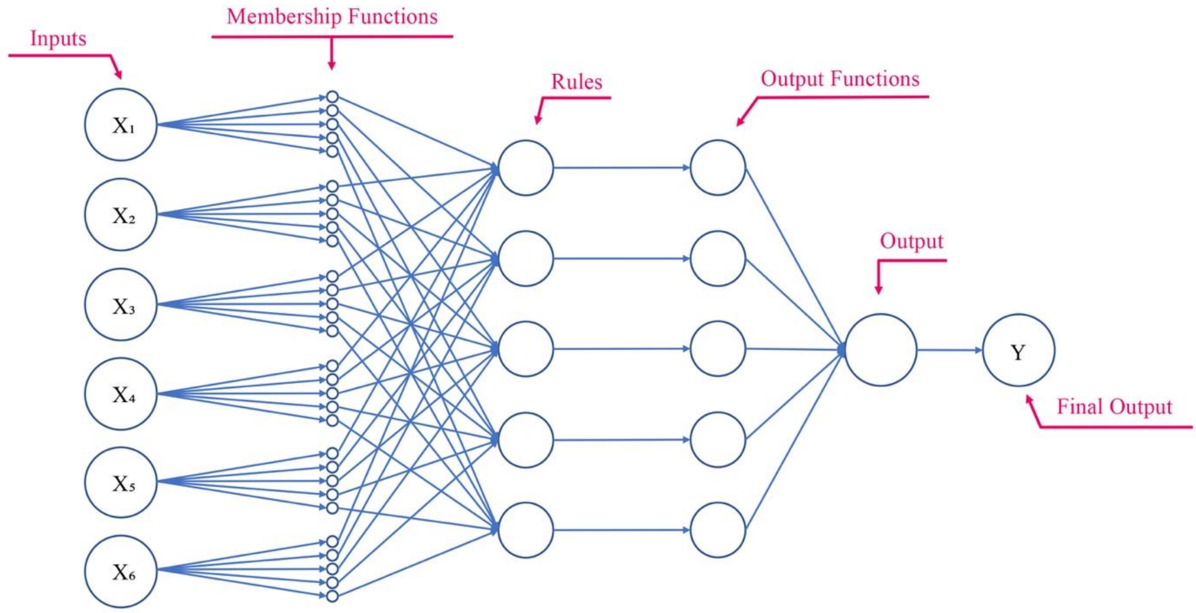


Fig. 17. Proposed ANFIS structure

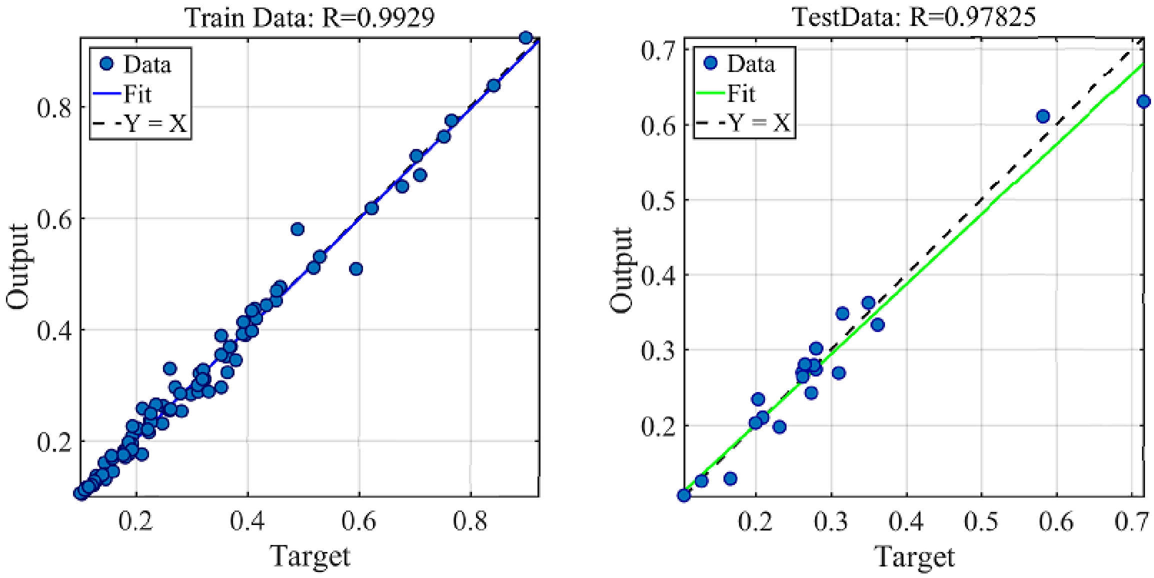


Fig. 18. Regression plots obtained form ANFIS

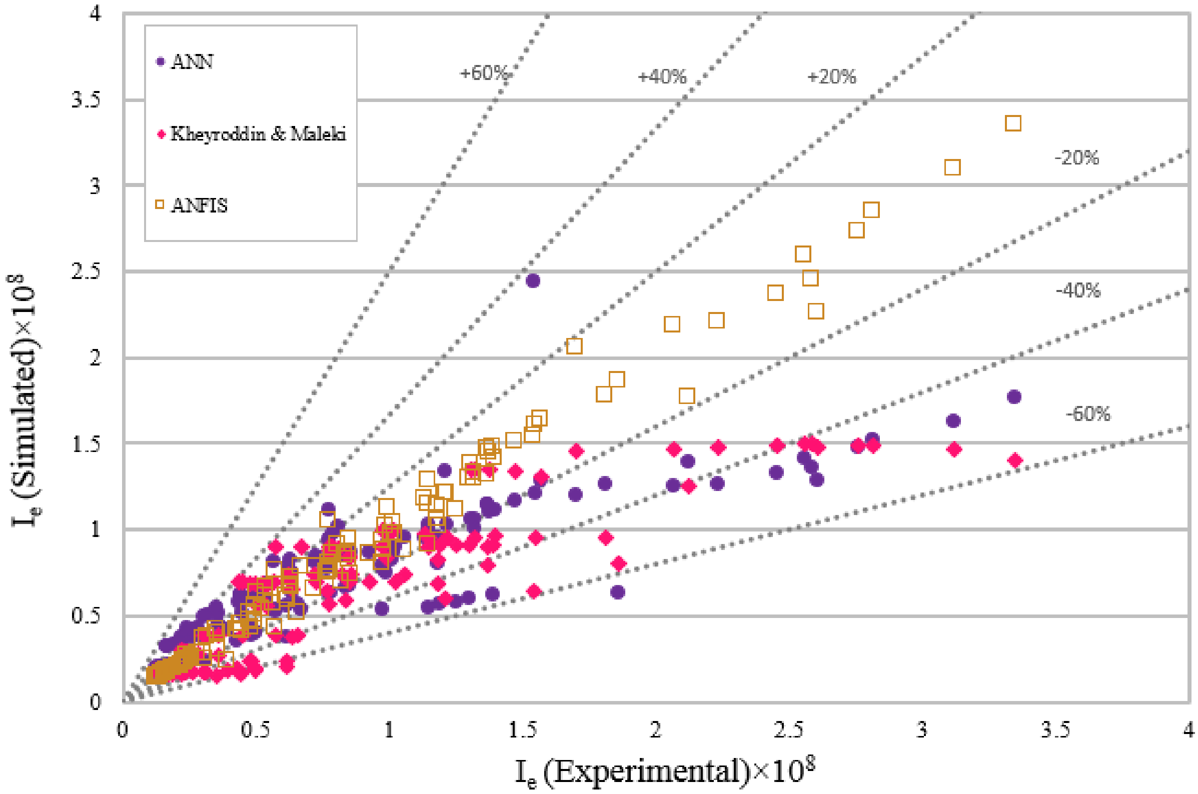


**Fig. 19.** Comparison of predicted values of $I_e$ between the proposed equations and equation using genetic algorithm

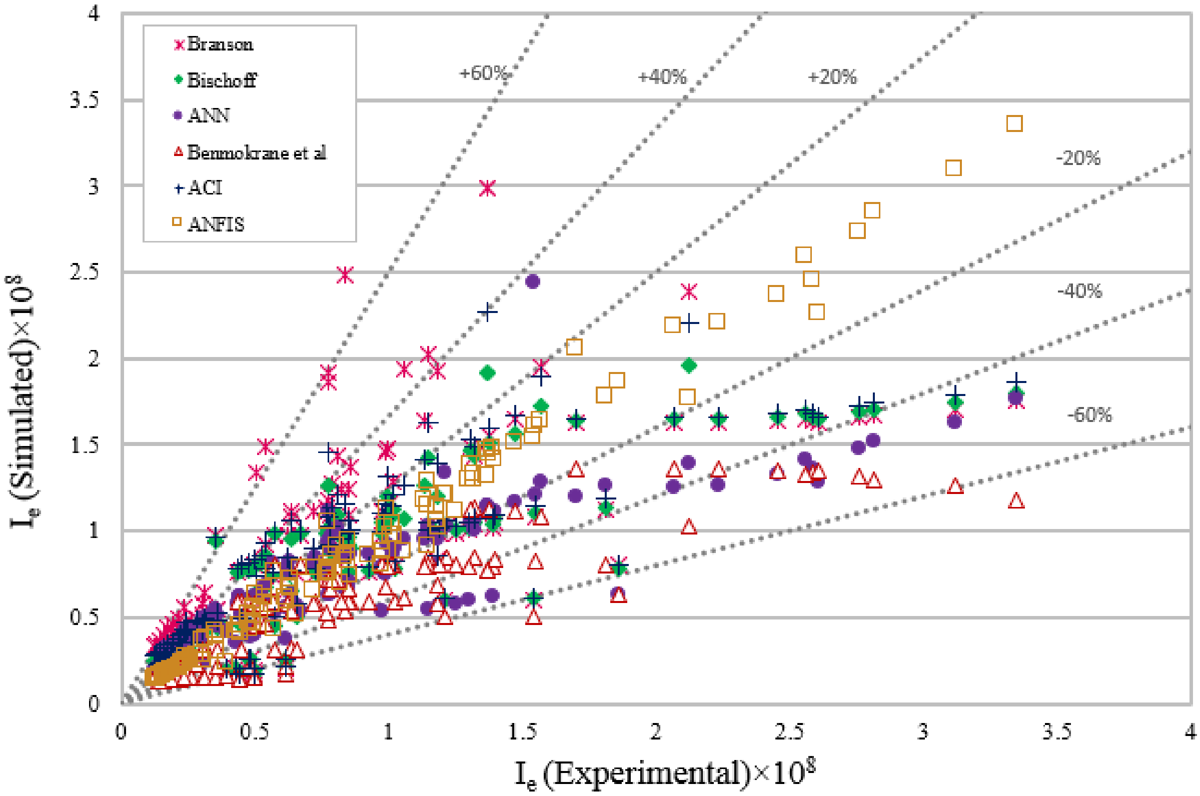


**Fig. 20.** Comparison of predicted values of $I_e$ between the proposed equations and six existing equations

Table1

Experimental studies of hybrid Steel/FRP RC beams

| Reference | Number of beam specimens | Number of data | Type of FRP bar* |
|---|---|---|---|
| Aiello and Ombres (2002) - [56] | 4 | 29 | AFRP |
| Qu et al (2009) - [57] | 6 | 46 | GFRP |
| Leung and Balendran (2003) – [58] | 4 | 6 | GFRP |
| Almusallam et al (2013) – [59] | 2 | 3 | GFRP |
| Safan (2013) – [60] | 4 | 4 | GFRP |
| Yang et al (2011) – [61] | 1 | 1 | GFRP |
| Refai et al (2015) – [8] | 6 | 33 | GFRP |
| ***AFRP** (Aramid Fiber Reinforced Polymer), **GFRP** (Glass Fiber Reinforced Polymer) | | | |

Table2

Properties of the experimental data

| Parameter | $I_g$ ($mm^4$) | $A_f/A_s$ | $M_{cr}/M_a$ | $E_f/E_s$ | $I_{cr}$ ($mm^4$) | $\rho_{fb}$ | $I_{e\text{-}exp}$ ($mm^4$) |
|---|---|---|---|---|---|---|---|
| Mean | 2.71E+8 | 1.18 | 0.44 | 0.23 | 6.86E+7 | 0.0036 | 8.8E+7 |
| Minimum | 6.67E+7 | 0.25 | 0.11 | 0.19 | 1.57E+7 | 0.0013 | 1.25E+7 |
| Maximum | 5.18E+8 | 2.88 | 0.99 | 0.73 | 1.62E+8 | 0.0073 | 3.35E+8 |
| Standard deviation | 1.66E+8 | 0.66 | 0.19 | 0.05 | 4.46E+7 | 0.00134 | 6.93E+7 |
| Coefficient of variation | 0.612 | 0.564 | 0.429 | 0.219 | 0.65 | 0.372 | 0.788 |

Table3

Scaling equation for parameters

| Parameter | Scaling equation |
|---|---|
| $I_g$ ($mm^4$) | $I_{g\text{-}s} = (0.8) \times (I_g - I_{g\text{-}min}) / (I_{g\text{-}max} - I_{g\text{-}min}) + 0.1$ |
| $A_f/A_s$ | $(A_f/A_s)_s = (0.8) \times ( (A_f/A_s) - (A_f/A_s)_{min}) / ((A_f/A_s)_{max} - (A_f/A_s)_{min}) + 0.1$ |
| $M_{cr}/M_a$ | $(M_{cr}/M_a)_s = (0.8) \times ( (M_{cr}/M_a) - (M_{cr}/M_a)_{min}) / ((M_{cr}/M_a)_{max} - (M_{cr}/M_a)_{min}) + 0.1$ |
| $E_f/E_s$ | $(E_f/E_s)_s = (0.8) \times ( (E_f/E_s) - (E_f/E_s)_{min}) / ((E_f/E_s)_{max} - (E_f/E_s)_{min}) + 0.1$ |
| $I_{cr}$ ($mm^4$) | $I_{cr\text{-}s} = (0.8) \times (I_{cr} - I_{cr\text{-}min}) / (I_{cr\text{-}max} - I_{cr\text{-}min}) + 0.1$ |
| $\rho_{fb}$ | $\rho_{fb\text{-}s} = (0.8) \times (\rho_{fb} - \rho_{fb\text{-}min}) / (\rho_{fb\text{-}max} - \rho_{fb\text{-}min}) + 0.1$ |
| $I_{e\text{-}exp}$ ($mm^4$) | $I_{e\text{-}exp\text{-}s} = (0.8) \times (I_{e\text{-}exp} - I_{e\text{-}exp\text{-}min}) / (I_{e\text{-}exp\text{-}max} - I_{e\text{-}exp\text{-}min}) + 0.1$ |

Table 4

Details of ANN

| Node | Input weights | | | | | | Layer weights | Bias to | |
|---|---|---|---|---|---|---|---|---|---|
| | Input 1 | Input 2 | Input 3 | Input 4 | Input 5 | Input 6 | | Hidden layer | Output layer |
| Node 1 | 1.0586 | -1.0264 | -1.9345 | 2.6546 | -1.8642 | -1.9836 | -1.0917 | -4.4309 | - |
| Node 2 | -1.2625 | 2.0084 | -1.5379 | 1.4949 | 2.4722 | -1.9054 | 0.64216 | 3.9394 | - |
| Node 3 | -2.9143 | -1.1867 | 0.58662 | 1.6268 | -1.995 | 2.3412 | -0.6328 | 2.946 | - |
| Node 4 | -1.35 | 1.489 | -1.8677 | -1.5295 | -1.585 | -2.9664 | -1.0239 | 2.2105 | - |
| Node 5 | 2.5372 | -0.0875 | -2.057 | -1.2016 | 0.35133 | 3.1498 | -1.8568 | -0.8106 | - |
| Node 6 | 0.25862 | 1.3789 | 1.7669 | -2.1967 | -0.1888 | -2.995 | -1.5746 | -0.5444 | - |
| Node 7 | -0.1379 | 2.4461 | 0.26625 | -0.4596 | 1.7655 | 2.8143 | 1.3585 | 0.37002 | - |
| Node 8 | -1.0545 | -0.0605 | 2.9446 | 2.8226 | 0.15898 | -1.5928 | -0.1011 | -0.1590 | - |
| Node 9 | 1.6029 | 2.1827 | -3.1115 | 0.27195 | -1.6601 | 0.30525 | 0.87172 | -0.4882 | - |
| Node 10 | 1.9747 | -1.9675 | -0.3196 | 1.3692 | 1.7427 | 2.9141 | 0.98504 | 1.1482 | - |
| Node 11 | -0.0468 | 2.1972 | 0.13591 | 1.1114 | -2.6795 | 2.7945 | 0.96851 | 1.1291 | - |
| Node 12 | -2.1131 | -2.6384 | 1.1274 | 1.1272 | 1.3603 | -1.8536 | -1.8381 | 0.7921 | - |
| Node 13 | -0.5279 | -0.3021 | 2.724 | 2.9029 | 1.2099 | 1.3633 | -2.6271 | 0.80781 | - |
| Node 14 | 0.0365 | -2.7636 | -1.7993 | 0.91934 | -1.4456 | 2.4717 | 0.1776 | 3.0041 | - |
| Node 15 | -0.9784 | 1.9368 | -0.7579 | -1.8431 | -1.362 | 2.8226 | -0.2872 | -3.9263 | - |
| Node 16 | -0.1474 | -1.7758 | -2.8125 | 0.3824 | -1.9395 | -2.2417 | 0.0681 | -4.4138 | - |
| Output | - | - | - | - | - | - | - | - | -0.2269 |

Table 5
The input parameters range and reference values

| Input parameters | Minimum | Maximum | Reference |
|---|---|---|---|
| $I_g(mm^4)$ | 6.67E+7 | 5.18E+8 | 2.7E+8 |
| $A_f/A_s$ | 0.25 | 2.88 | 1.2 |

| | | | |
|---|---|---|---|
| $M_{cr}/M_a$ | 0.11 | 0.99 | 0.40 |
| $E_f/E_s$ | 0.19 | 0.73 | 0.45 |
| $I_{cr}(mm^4)$ | 1.57E+7 | 1.62E+8 | 6.9E+7 |
| $\rho_{fb}$ | 0.0013 | 0.0073 | 0.004 |

Table 6. Gaussian membership function details

| Input | Parameter | G1 | G2 | G3 | G 4 | G5 |
|---|---|---|---|---|---|---|
| X1 | v | 0.0991 | 0. 0905 | 0.1769 | 0.1075 | 0.1155 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | m | 0.4045 | 0.4078 | 0.8869 | 0.3711 | 0.1633 |
| X2 | v | 0.0937 | 0.0954 | 0.1111 | 0.0880 | 0.0909 |
| | m | 0.2060 | 0.1333 | 0.5358 | 0.4446 | 0.3482 |
| X3 | v | 0.0891 | 0.0830 | 0.0858 | 0.0816 | 0.1131 |
| | m | 0.2501 | 0.2850 | 0.4449 | 0.3473 | 0.5661 |
| X4 | v | 0.02465 | 0.0203 | 0.01689 | 0.02401 | 0.01897 |
| | m | 0.1368 | 0.1287 | 0.1907 | 0.1236 | 0.1865 |
| X5 | v | 0.1133 | 0.1930 | 0.1085 | 0.1076 | 0.1194 |
| | m | 0.5207 | 0.8828 | 0.4896 | 0.2751 | 0.1331 |
| X6 | v | 0.0723 | 0.0906 | 0.0563 | 0.0796 | 0.1350 |
| | m | 0.5041 | 0.6325 | 0.4078 | 0.5381 | 0.1518 |

Table 7. Output Functions

| Input | F1 | F2 | F3 | F4 | F 5 |
|---|---|---|---|---|---|
| X1 | -2.1300 | -1.9720 | -1.7470 | -0.0629 | 3.8920 |

| | | | | | |
|---|---|---|---|---|---|
| X2 | -0.5103 | -25.3300 | 0.0860 | 0.0172 | -0.1555 |
| X3 | 1.9180 | 0.9402 | 0.3254 | 0.2661 | 0.1091 |
| X4 | 0.9948 | -11.7400 | -0.6004 | 3.5290 | 6.1320 |
| X5 | 1.8480 | -0.4929 | 0.6282 | -0.2726 | -0.1599 |
| X6 | 1.5500 | 15.0100 | 2.2860 | -0.0850 | -0.0603 |
| Constant | -0.8997 | -3.3320 | 0.5812 | -0.1729 | -1.6260 |

Table 8

The errors range for various existing equations versus the ANN and ANFIS models

| Range of error (%) | Branson | Bischoff | ACI | ANN model | Benmokrane et al. | ANFIS model |
|---|---|---|---|---|---|---|
| ±15 | 16 | 33 | 25 | 34 | 31 | 107 |
| ±30 | 37 | 74 | 61 | 85 | 63 | 121 |

| | | | | | | |
|---|---|---|---|---|---|---|
| ±45 | 81 | 107 | 104 | 111 | 92 | 122 |
| ±60 | 110 | 117 | 119 | 121 | 114 | 122 |
| ±75 | 122 | 122 | 122 | 122 | 122 | 122 |

Table 9
The percentage errors for various existing equations versus the ANN and ANFIS models

| Range of error (%) | Branson (%) | Bischoff (%) | ACI (%) | ANN model (%) | Benmokrane et al. (%) | ANFIS model (%) |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| **±**15 | 13.11 | 27.05 | 20.49 | 27.87 | 25.41 | 87.70 |
| **±**30 | 30.33 | 60.66 | 50 | 69.67 | 51.64 | 99.18 |
| **±**45 | 66.39 | 87.70 | 85.25 | 90.98 | 75.41 | 100 |
| **±**60 | 90.16 | 95.90 | 97.54 | 99.18 | 93.44 | 100 |
| **±**75 | 100 | 100 | 100 | 100 | 100 | 100 |

Table 10

Comparison of average error of $I_e$ between the proposed equation and four existing models

| Average error | Branson | Bischoff | ACI | ANN model | Benmokrane et al. | ANFIS model |
|---|---|---|---|---|---|---|
| | 62.08 | 34.64 | 43.46 | 30.28 | 31.24 | 7.59 |