An Equitable Fuzzy Approach for Facility Delocation: A Case Study of Banks Merging

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Abstract

This paper aims to provide an equitable approach for the delocation via merging different bank branches. Due to the profit loss, some banks have resisted this change, so we developed a n/equity approach to modeling this issue to convince bank owners and employees. The proposed model is a mixed-integer programming model to have an equitable approach to fuzzy constraints based on the weighted sum of the remaining branches to the total number of branches of each type of bank. Moreover, this equitable approach was also used to avoid unemployment of the closed branches staff. The result showed that using fuzzy constraints, equity can be well modeled. Moreover, increasing the equity coefficient reduces the number of facilities remaining in the system, and as a consequence, the desired efficiency (i.e., personnel retention) is reduced. So, we can reach the non-dominated answers. Finally, the results showed that reducing the minimum distance between facilities will allow more facilities to remain in the system and retain more staff.

Keywords: Delocation; Equity; Fuzzy Constraints; Data Envelopment Analysis; Mixed Integer Programming.

1. Introduction

Under macro policy changes and new legislation, technological changes, demographic changes, the effect of epidemics, and changes in consumer behavior, some facilities need to be merged and maintained as a common brand to survive in competitive markets. Facility merging can lead to costs associated with changes in repurposing or attrition of closed facilities, costs related to layoffs and loss of customers, etc. Accordingly, some measures need to be taken to persuade stakeholders, and the context for these changes should be created. Creating an equitable approach to the merger process can be an essential persuasive factor for business owners. Moreover, as employees are reluctant to lose their jobs, systems strive to ensure maximum employee retention in their plans. Generally, it can be argued that new strategies are not always creative and developmental, and changes in industries and competitive conditions may generate adverse outcomes. In this case, facilities should be delocated through measures such as closing, merging, shrinkage, resizing, restructuring, and so on. The issue of facility location has been widely discussed in the literature. However, little attention has been paid to facility delocation.

Closing a facility can be mainly called delocation [1]. In recent years, one of the most challenging issues in the banking industry of the Islamic Republic of Iran has been the merger of the so-called Armed Forces banks in Sepah Bank. After a few years, only the term "affiliated to Sepah Bank" has been added next to the boards of these different banks, and no measures have been taken to close and fully merge branches. One of the critical constraints, in this case, is the existence of a reasonable distance of facilities from each other so that when closing facilities, this distance between the remaining facilities must be maintained in a manner that makes sense both socially and in terms of urban rules and regulations.

Furthermore, a proper evaluation of bank branches must be done first. Since branches have different sizes and are located in different geographical areas, and offer different services to different customers, it isn't easy to measure the performance of bank branches [2]. What complicates the issue of delocation or merger is the existence of stakeholders and users of facilities. If a facility decided to close a number of its branches according to its business conditions, it tended to meet the competitive conditions and not leave the market to competitors [3]. However, according to a macro-policy, some facilities operating in the same field with different brands and distinct owners are merged and reduced. Due to the sovereignty of the issue, equality and equity for the stakeholders come to the surface. At the same time, competition is no longer a priority. That is, each of the existing facilities tends to be equitably exposed to the merger, depending on their market share or the extent of their facilities. An important and effective stakeholder in the integration of facilities is the staff. Due to the volatile and challenging economic and employment conditions of some societies, job loss is very stressful for staff and should be well managed. Thus, the retention of employees in the process of merger or reduction of facilities must be taken into account. Using data envelopment analysis (DEA) and based on determined inputs and outputs, all branches with different types of services are evaluated together. The total efficiency score of each branch is measured. Afterward, a mathematical model is proposed to make the ratio of the total weight of each facility type as fair as possible in comparison with other facilities. Fuzzy constraints will be used to do this.

- **Research problem**

The Central Bank of the Islamic Republic of Iran officially announced the order to merge banks and affiliated institutions, including Ansar Bank, Hekmat Iranian, Ghavamin, Mehr Eghtesad, and Kosar Credit Institution, into Sepah Bank. According to the experts' opinion, the problem is a super-merger project. For this reason, this project has been pursued obsessively, carefully, and deliberately for several years.

- **Research necessity**

This super project is almost unique globally, and according to experts, even global models could not help such a large integration in the Iranian banking system. According to some economists, challenges such as a large number of branches, staffing, shareholder uncertainty, and depositors' confusion are some of the issues that may not be considered at first glance, but during implementation can lead to difficulties and complexities. There are many stakeholders involved in this regard, and due to the reduction of benefits, there is resistance to change. So, by using a fair approach, the conflict between the stakeholders is reduced, and public acceptance is created. Moreover, employees and owners expect branch closures happened according to a fair approach.

- **Research Motivation**

Providing a mathematical model considering stakeholder conditions while having rational and moral foundations reduces resistance to managerial change and increases trust in the organizational system for closing some branches of the bank.

- **Research question**

In short, the main question of this paper is how can we reduce the facilities of multiple stakeholders so that their interests are adjusted fairly?

- **Research novelty**

After the literature gap analysis, the research novelty will be presented at the end of the next section.

The rest of this paper is organized as follows: Section 2 reviews the literature. Section 3 describes the problem and the mathematical model. Then, numerical examples are solved in Sections 4, followed by sensitivity analysis and managerial insights. Finally, Section 5 summarizes the discussion and concludes the paper.

2. Literature review

In many contractionary and reduction policies, which often have multiple stakeholders, change is associated with resistance, and in such systems, stakeholder persuasion is difficult. Stakeholder participation can help persuade stakeholders [4,5]. Björklund [6], in their review study for the urban consolidation centers, mentioned that the stakeholders are one of the essential influential groups in this area. In designing socio-technical systems, attention to stakeholders has also been discussed [7]. One way to engage stakeholders is to use soft techniques, counseling, and dialogue [8], and the other way is to do it by providing mathematical techniques and models. In this study, based on macro policies that the delocation process has faced resistance, an appropriate response to the demands of stakeholders is provided by providing a mathematical model with a justice-oriented approach.

Based on the features of the proposed model in this paper, the literature reviews are provided in four subsections as follows.

2.1. Delocation

Facility location is a strategic decision and is an action that is typically taken in the long run, such as the construction of a hospital, a factory, a fire station, a shopping center, an emergency center, a bank, a school, etc. These facilities are often not temporary, and their construction is costly, and if such decisions are not made based on careful planning and estimation, facilities should be delocated. Facility delocation includes a merger, shrinkage, reduction, and restructuring, mainly a matter of closing the facility. Bhaumik [1] was the first one to discuss delocation for such facilities. The term had also been used by ReVelle [3] as one of the keywords of their study.

Furthermore, Ruiz-hernández [9] also used this keyword extensively. ReVelle [3] addressed two location models for ceding market share and shrinking services. The first model sought to reduce the market share competitively and the second model sought to reduce the level of service. Wang [10] discussed the opening and closing of facilities for bank branches in a New York area, given the budget-constrained location problem to minimize the distance between customers and the remaining or newly opened branches. Merger-oriented delocation was addressed by Bruno [11], who assessed the merging of schools of different levels to minimize the sum of the distances of the merged institutions. Different educational levels were considered service types, and sensitivity analysis was performed based on the number of created clusters. Monteiro [12] addressed opening and closing bank branches and assigned predefined branch sizes to the remaining or newly created banks. They also considered expenses for dismissal or hiring of staff.

Bhaumik [1] proposed a delocation problem to retailers and distribution centers that need to reduce the size or shrink their distribution chain. An essential requirement in the problem stipulated that all demand nodes had to receive their resources from their respective current distributors, except when the current resource was relocated, and only such nodes with unmet demand should be met by one of the remaining suppliers. Bhaumik [1] investigated hospital closures based on GIS and using covering and median models and showed that closing a large number of facilitates increased access distances to a great extent under the median model. Besides, the covering model indicated that the coverage tends to be zero with the closure of a large number of facilities. However, they concluded that the closure of several hospital facilities did not change the criteria for coverage, equity, and access and estimated that about 10% of the facilities could be closed. Yavari [13] addressed the restructuring hierarchical capacitated facility location problem by closing and reopening the facility and resizing at multiple levels, which used auxiliary facilities to cover the demand under budget constraints and uncertainties. Cheng [14] Proposed a bio-objective mixed-integer programming model for railway express consolidation centers in China that is similar to facility merging models. Aljohani [15] proposed a multi-criteria decision approach and GIS to freight consolidation locations. Dynamic and multi-period location problems are also commonly used by facility opening and closure at different periods. For example, Jena [16] investigated a multilevel multi-commodity multi-period location model to be applied in the field of forest management for workers' camps seeking to open and close facilities with different capacities to cover their demand in different periods. Güden [17] proposed a dynamic p-median problem with mobile facilities for a construction company engaged in different projects with some mobile and stationary concrete equipment. The problem sought to select concrete workshops in each period to reduce transportation costs and the cost of moving the workshop.

2.2. The equity and equality approach to location problems

According to Islamic teachings, the most famous and comprehensive definition of equity, which covers all forms of equity, including developmental, moral, and social, is "to put everything in its right place" and "give any right person his right". Although different in wording, these two definitions are pretty similar in terms of meaning because the real position of every object is its right, just as the real right of every object is its real position [18,19] (in Persian). Besides, some authors have considered equity as the slightest difference in terms of services between groups of people Braveman [20]. According to Karsu [21], equity and equality can be divided into two categories, vertical and horizontal. In horizontal equity, individuals, regions, groups, and demand points are considered equally without priority or superiority. In vertical equity, all stakeholders are considered based on their needs, preferences, and priorities.

The issue of equity is critical in choosing a location for facilities, especially in the public sector. There are different criteria for measuring equity, but there is little consensus over these models on how equity is measured in matters. Marsh [22] and Mesa [23] have collected different measures to assess equity in their study. Location decision-makers often manage resources and costs more effectively. However, this approach is not adequate for public location decision-making, and another criterion is needed: equity. Government-managed public goods and services are more influenced by location decisions [24].

It is sometimes necessary to balance the distribution of this load in open locations [25]. However, a solution that maximizes equity is not necessarily the least costly solution [26]. In problems related to equity, the issues of equality and efficiency are often mentioned together (Burkey et al. [27,28], Cho [29], Morrill [30], Morrill and Symons [31], Symons [32]). Equality is vital in locating public facilities and is a subjective socio-political concept that brings with its fairness and equity.

Center (P-Center) objective functions are one of the criteria that yield equity levels. Besides, Other criteria are suitable for measuring equity. Several common measures include variance, interval, mean, or total absolute weight difference, absolute maximum weight deviation, and the Gini coefficient. Lejeune and Prasad [33] discussed the equity-effectiveness approach in location models. Different criteria can be used to assess equity and fairness in the allocation of public services. These criteria can be modeled based on whether the criteria for the services provided are input or output of (potential or rendered) services. For example, the allocation of the police force can be measured by equalizing the number of police in each acre or residents (the input) in one area or based on equality in crime rate in areas (output). The measure of equity can range from zero to infinity. Infinite equity is a system in which people have public service centers in their vicinity, and zero equity is when there is only one service center in an infinitely large area [34]. Mandell points out that some authors stated that input-based models are an essential issue, while others preferred output-based models. The Gini coefficient is a good indicator for measuring equity [35]**.** Therefore, the importance of each branch in terms of inputs and outputs is assessed based on the efficiency score obtained using data envelopment analysis (DEA).

2.3. Data envelopment analysis in the banking industry

As noted in the previous section, input or output criteria are used to measure equity. The terms input and output unconsciously remind us of data envelopment analysis. Measuring the performance of bank branches is a complicated task. Branches are constructed in different sizes and offer different services to different customers while operating in different economic regions [2]. Therefore, evaluating the performance of branches via data envelopment analysis significantly contributes to setting fairness standards based on inputs and outputs. Lou [36] pointed out that extensive studies have used the DEA method in the banking industry. Accordingly, in their special issues, several journals, including Econometrics in 1990, the European Journal of Operations research in 1997, the Journal of Banking and Finance of Economics and Business in 1998, INTERFACES in 1999, and Management Science in 1999 have welcomed the use of this technique in the banking industry.

2.4. Literature gap analysis and the paper's novelty

In this paper, the problem of the delocation of the bank branches is modeled using location models, equity approach, and fuzzy constraints to fill the literature gap. Given the complexity of evaluating and determining the importance of banks importance of each branch is estimated using DEA. The proposed model is mixed-integer programming that is analyzed using numerical examples using the General Algebraic Modeling System (GAMS).

The novelties of the current paper can be summarized as follows:

- Unlike other delocation models, where the facility merging has often been competitive in nature, the equity approach has been considered.
- Modeling the delocation of banks with different brands considering an equitable approach using the empirical data,
- using the fuzzy logic and fuzzy constraints to implement the considered equitable approach,
- Using the DEA model to determine facility weight and performance score as a parameter in the delocation model.

3. Problem statement and mathematical modeling

In recent years, for some reason, one of the general banking policies of the Islamic Republic of Iran has been to merge several banks into one bank. According to location science, several branches need to be closed when the issue is addressed. Suppose banks have branches with different brands along a street, and if all of them change their name to one brand and continue to operate, it will not be optimal both in terms of urban order and rationality. Therefore, it is necessary to set a distance restriction between branches. The application of distance constraints to coverage and location problems has been extensively discussed by Berman [37]. The distance constraint between a set of facilities or between the facility and customers has been addressed in research on hazardous materials location. The considered delocation process is shown in Figure 1.

Insert Fig. 1 here

However, owners of institutions operating in the same industry are not very willing to merge because many facilities, staff, buildings, and equipment are moved and depreciated during the merger process, and many benefits are changed. Such problems are modeled following the fact that an institution reduces its facilities by taking into account the competitive conditions [3]. Therefore, the equitable approach would be that the weight ratio of each type of facility after the merger is approximately equal to their weight before the merger. The assumptions of the problem are as follows:

- According to urban order and regulation, observing the minimum distance between the facilities is necessary.
- To convince the owners of different facilities, equality and equity are followed.
- The importance of the facility is assessed based on the efficiency score obtained from data envelopment analysis.
- There is no limit to the number of final facilities remaining in the system. The model constraints determine the number of facilities remaining.
- The remaining facilities have potential operator capacity that can be activated to closed facilities staff.
- Staff is not transferred between open facilities.

3.1. Conceptual model of the study

First, based on the data envelopment analysis, all branches with different brands are evaluated together. Regardless of the type of brand, the efficiency scores are entered into the location model to measure the importance of each branch. The conceptual model used in the study is shown in Figure 2.

Insert Fig. 2 here

3.2. Determining input and output criteria

Several studies have addressed the banking performance using data envelopment analysis and using different inputs and outputs. For instance, Henriques [38] proposed a classification of inputs and outputs in the DEA literature in banking. The main inputs in the literature were the number of employees, fixed assets, deposits, property and equipment, costs and expenditure including staff costs, operating costs, interest rates, and other costs. Outputs mainly include the total number of loans, investments, securities, types of income, including net, loan income, investment income, off-balance-sheet income, income before tax, etc. In addition to financial inputs, Wu [39] cited environmental inputs such as income level, population density, and economic status in their study. Qashghaei [40] (in Persian) identified 64 input and output criteria in their evaluation of Ansar Bank branches. After reviewing their applicability and effectiveness in Iranian banks and aggregating some criteria and consensus of experts, nine criteria were finally selected as listed in Table 1. It is worth mentioning that one of the banks merging in the delocation is Ansar Bank.

Insert Table 1 here

The selected criteria are detailed as follows:

- The number of branch staff
- The branch resource supply: The total value of interest-free, current, saving, and deposit accounts, managed funds, special unallocated funds, saving deposits, short-term (regular and special), and long-term (1-year, 2-year, 3-year, 4-year, 5-year) deposit investments
- The branch resource allocation: The net sum of facilities granted at the end of a year
- The number of banking services offered by the branch: The number of banking cards issued by the branch (cash withdrawal cards, gift cards, coupon cards, and credit cards), the number of facilities granted (free-interest facilities and loans, exchange contracts, partnership contracts), and the number of short and long-term current, saving, and special interest-free accounts
- Branch claims: The Past due to bank receivables, postponed claims, and claims for guarantees paid to the private sector at the end of the year
- Total branch expenses: All operative and non-operative expenses
- The branch location: The location is scored verbally based on factors such as the building value or rental, the branch business position, regional income level, access, regional population, the branch area, etc.
- The customer satisfaction that is scored verbally
- The total branch income: The interest rate on facilities, wages, and delayed penalty guarantee

3.3. The input-oriented CCR (Charnes, Cooper and Rhodes) model

DEA has two main categories. The CCR model was proposed by Charnes, Cooper, and Rhodes [41] and had a return to a constant scale, and the other is the BCC model proposed by Banker, Charnes, and Cooper [42], which has a variable scale return. DEA is presented in different forms of input-oriented, output-oriented, primal and dual, shown in Table 2. For more descriptions of these models, the readers are encouraged to check the aforementioned references.

The terminologies which are used in this subsection are as follows:

- Indices:

  i, j : Indices of location and $i, j: 1, 2, ..., n$,

  k : Indices of facility type and $k: 1, 2, ..., K$,

- Parameters

  y_{rj} : Value of output r in DMU_j,

  x_{rj} : Value of input r in DMU_j,

- variables

  u_r : Weight of output r,

v_i : Weight of input i,

Insert Table 2 here

Since the problem inputs are controllable, the input-oriented CCR data envelopment analysis model is used. The details of the CCR model are as follows.

The objective function of the model aims to maximize the performance of the decision-making unit *p* as follows:

$$e_p = \max \sum_{r=1}^{s} u_r y_{rp} \tag{1}$$

In Equation 1, $u_r \times y_{rp}$ is the weighted output r for DMU_p. So, by summing up all outputs, we can calculate the performance of the decision-making unit *p*.

Equation (2) refers to the full use of inputs, and Equation (3) ensures that the output-to-input ratio in other decision-making units is less than 1, and Equation (4) shows the boundaries of the variables as follows:

$$\sum_{i=1}^{m} v_i x_{ip} = 1 \tag{2}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad \forall j \tag{3}$$

$$u_{r,} v_i \ge \varepsilon \tag{4}$$

Andersen [43] developed the data envelopment analysis model in which when a large number of decision-making units (DMU) have 100% efficiency, the relevant constraint for the DMU whose efficiency is calculated is removed, and the model is allowed to find the maximum efficiency for the DMU. Therefore, constraint (3) is written as Equation (5):

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad \forall j \ne p \tag{5}$$

After calculating the efficiency score of each branch, the total efficiency score of each type of branch is calculated.

3.4. Nomenclatures

In general, the indices, parameters, and variables of the proposed delocation problem are as follows:

- Indices:

i, j : Indices of location and $i, j : 1, 2, ..., n$

$k:$ Indices of facility type and $k:1,2,...,K$,

- Parameters

$d_{ij}:$ Spatial distance between facility in location i and j,

$s_l:$ Minimum acceptable distance between all types of facility in locations,

$E_{jk}:$ Binary parameter if facility type of k be in location j is one otherwise 0,

$CP_j:$ Total staff capacity (server) of a facility in location j,

$P_j:$ Number of existing staff of the facility in location j,

$TP_k:$ The total staff of facility type k,

$\alpha:$ Equality/fair coefficient,

$\beta:$ Equality/fair coefficient,

$\Delta:$ Deviation from the desirable value,

$\Delta_1:$ Deviation from the desirable value,

$TE_k:$ The total efficiency of type k facility,

$e_j:$ The efficiency of the facility on location j,

$M:$ The arbitrary very large number,

- variables

$y_j:$ Binary variable if the facility in location j stay open; otherwise, 0,

$z_{ij}:$ Integer variable assigned staffs from a closed facility in location i to open the facility in location j,

3.5. Facility delocation model

Before presenting the facility delocation model, the results from DEA are used in it, where e_i is the weight of each facility and TE_k is the total weight of the facility of type K estimated using Equation (6):

$$TE_k = \sum_{i=1}^{n} e_i . E_{ik}; \quad \forall k \tag{6}$$

The basic model of study covers the location problem. The set-covering model was proposed by Toregas [44] and the maximum covering model by Church [45].

In the following, we will complete the model step by step. Equation (7) shows an objective function that seeks to maximize staff retention as follows:

$$\max Z = \sum_{i=1}^{n} \sum_{j=1}^{n} z_{ij} \tag{7}$$

Equation (8) states that in the process of merging and reducing facilities, which ultimately leads to merging the facilities with a brand, the minimum distance must be kept between the remaining facilities.

$$d_{ij}.y_i \geq s_l - M\left(2 - y_i - y_j\right); \quad \forall i, j \tag{8}$$

In Equation (8), if both facilities i and j were established, the distance between these two locations (d_{ij}) should be more than the minimum acceptable distance between all types of facility in locations (s_l), else the Equation (8) will be relaxed ($d_{ij}.y_i \geq -\infty$).

The following Equation is related to an equitable approach to closing the facility. It was sought to establish relative equity between the types of facilities according to their previous conditions. Previous studies in equitable location approaches and equality and inequality criteria have pointed to a neutral state or similar preferences. Besides, the relationship between fair preferences is expressed as $y' \cong y''$ (Kostreva and Ogryczak [46], Ogryczak [24], Ogryczak [47]). This idea was put into practice in this study and used for the fair closure of facilities. The fair closure of facilities is ensured when the number of remaining facilities is almost equal to the initial state of each type of facility:

$$\frac{\text{the total weight of type } L \text{ facilities remained after delocation}}{\text{the total weight of type } L \text{ facilities before delocation}} \cong \frac{\text{the total weight of type } k \text{ facilities remained after delocation}}{\text{the total weight of type } k \text{ facilities before delocation}}$$

This equation evokes a fuzzy constraint, and thus the fuzzy constraint is used (9):

$$\left(y' \cong y''\right) \rightarrow \frac{\sum_{j=1}^{n} E_{jk}.y_j.e_j}{TE_k = \sum_{j=1}^{n} E_{jk}.e_j} \cong \frac{\sum_{j=1}^{n} E_{jl}.y_j.e_j}{TE_l = \sum_{j=1}^{n} E_{jl}.e_j} \qquad \forall k, l, k \neq l \tag{9}$$

Definition:

If the fuzzy constraint is defined as $g(x) \cong b$, then $g(x) \leq b$ or $g(x) \geq b$ as the membership functions for these constraints are as follows (Figure 3):

Insert Fig. 3 here

$$\mu(x) = \begin{cases} 0 & g(x) \leq b - \Delta \\ \dfrac{g(x) - (b - \Delta)}{\Delta} & b - \Delta \leq g(x) \leq b \\ \dfrac{b + \Delta - g(x)}{\Delta} & b \leq g(x) \leq b + \Delta \\ 0 & g(x) \geq b + \Delta \end{cases} \tag{10}$$

It is attempted to maximize the equilibrium state with a desired value of α so that we will have:

$$\frac{g(x)-(b-\Delta)}{\Delta} \geq \alpha \ \rightarrow \ g(x) \geq b+(\alpha-1)\Delta \tag{11}$$

$$\frac{b+\Delta-g(x)}{\Delta} \geq \alpha \ \rightarrow \ g(x) \leq b-(1-\alpha)\Delta \tag{12}$$

Equation (10) is related to the membership function of a triangular fuzzy number. Equations (11) and (12) correspond to the right and left of the membership function, respectively. Using the definition (10), (11), and (12), the fuzzy constraint (9) is expanded as a definite constraint:

$$\frac{\sum_{j=1}^{n} E_{jk}.y_j.e_j}{TE_k = \sum_{j=1}^{n} E_{jk}.e_j} - \frac{\sum_{j=1}^{n} E_{jl}.y_j.e_j}{TE_l = \sum_{j=1}^{n} E_{jl}.e_j} \cong 0; \qquad \forall k,l,k \neq l \tag{13}$$

$$\frac{\sum_{j=1}^{n} E_{jk}.y_j.e_j}{TE_k} - \frac{\sum_{j=1}^{n} E_{jl}.y_j.e_j}{TE_l} \geq (\alpha-1)\Delta; \quad \forall k,l,k \neq l \tag{14}$$

$$\frac{\sum_{j=1}^{n} E_{jk}.y_j.e_j}{TE_k} - \frac{\sum_{j=1}^{n} E_{jl}.y_j.e_j}{TE_l} \leq (1-\alpha)\Delta; \quad \forall k,l,k \neq l \tag{15}$$

Equations (14) and (15) state that any facility, types k and l should remain fairly. It should be noted that the facility at location j has weight.

The constraint on equality in the retention or dismissal of the staff is addressed here. Given that the staff's main concern as one of the beneficiaries of branches is job retention, the number of staff that each facility dismisses in proportion to the entire facility staff of type k or L should be fair. Therefore, the following constraint is applied: *The ratio of the remaining staff to the initial state in each type of facility is approximately equal*. The total value in parentheses indicates the number of staff transferred to other facilities due to the closure of a facility, and the second part shows the number of the remaining staff in open facilities type k and L:

$$\frac{\textit{number of staff in type L facilities remained after delocation}}{\textit{number of staff in type L facilities before delocation}} \cong \frac{\textit{number of staff in type k facilities remained after delocation}}{\textit{number of staff in type k facilities before delocation}}$$

$$\sum_{j=1}^{n}\sum_{i=1}^{n} \frac{(z_{ij}+y_j.P_j)E_{jk}}{TP_k} \cong \sum_{j=1}^{n}\sum_{i=1}^{n} \frac{(z_{ij}+y_j.P_j)E_{jl}}{TP_l}; \ \ \forall k,l,k \neq l \tag{16}$$

Similar to Equation (9), the fuzzy constraint (16) is extended as constraints (17) and (18).

$$\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{\left(z_{ij}+y_j.P_j\right)E_{jk}}{TP_k}-\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{\left(z_{ij}+y_j.P_j\right)E_{jl}}{TP_l}\geq\left(\beta-1\right)\Delta;\quad \forall k,l,k\neq l \tag{17}$$

$$\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{\left(z_{ij}+y_j.P_j\right)E_{jk}}{TP_k}-\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{\left(z_{ij}+y_j.P_j\right)E_{jl}}{TP_l}\leq\left(1-\beta\right)\Delta;\quad \forall k,l,k\neq l \tag{18}$$

The Equations (17) and (18) maintain fairness in the staff retention between types of facilities compared to the pre-merger stage.

$$\sum_{i=1}^{n} z_{ij}\leq\left(CP_j-P_j\right)y_j;\quad \forall j \tag{19}$$

Equation (19) states that the number of staff transferred from closed facilities cannot exceed the excess capacity of open facilities.

$$\sum_{j=1}^{n} z_{ij}\leq P_i(1-y_i);\quad \forall i \tag{20}$$

Equation (20) states that the closed facility cannot allocate more staff to the open facility than its existing staff.

$$z_{ij}\leq M\left(2-y_i-y_j\right);\quad \forall i,j,i\neq j \tag{21}$$

Equation (21) prevents the transfer of staff between open facilities.

$$y_i,y_j\in\{0,1\},z_{ij}\in z^{+} \tag{22}$$

Equation (22) marks the boundaries of variables. Mathematical relations 7, 8, 14, 15, 17, 18, 19, 20, 21, and 22 constitute the facility delocation model of this study.

4. Numerical examples

4.1. Example 1

There are three types of facilities (k=3) at different locations shown with matrix elements 0 and 1 in Table 3, other data related to the problem are shown in Table 4, and the distance matrix is displayed in Table 5.

Insert Tables 3 to 5 here

As can be seen, the developed model is linear. Therefore, GAMS software was used to solve the problem, and the results for this problem with the standard minimum distance of 300 meters are as shown in Table 6.

Insert Table 6 here

Facilities operating at locations 1, 2, 4, 11, 12, and 14 are open, and the rest are closed. Besides, following the limitation of relative equity, two facilities of each kind have remained open. The value of the objective function is 15, indicating that 15 persons of the staff of the closed facilities have been hired in the network. The staff hired by the remaining facilities is presented in Table 7.

Insert Table 7 here

4.2. Example 2

In this example, 40 facilities with five different brands are considered. The data about these facilities are given in Table 8 and Figures 4 and 5, respectively.

Insert Table 8 here

Insert Figures 4 and 5 here

The value of the objective function with a minimum standard distance of 300 $(s_l = 300)$ and 90% equity is 53. The allocation of staff from closed facilities to open facilities is as detailed in Table 9, and closed facilities without staff allocation are not included.

Insert Table 9 here

4.3. Sensitivity analysis

4.3.1. The effect of the minimum standard distance between facilities

As shown in Figures 6 and 7, if the distance between the remaining facilities increases, the value of the objective function decreases, indicating the validity of the model as by increasing the distance, fewer facilities have the chance of staying in the system, and therefore the operator's capacity to accept facilities decreases and eventually the value of the objective function decreases. Also, different fairness coefficients produce contours, and the higher these coefficients, the worse the value of the objective function. In other words, each of the curves can be equivalent to a tradeoff curve.

Insert Figures 6 and 7 here

In Table 10, based on the minimum standard distance of the facilities, the objective function and the number of open branches along with the open branch index for the case n=40 are given. Checking all branches shows that facilities such as branches number 4, 14, 19, 32, and 37 are not too sensitive to the distance limit, so it can be considered that these facilities are stable against this limit.

Insert Table 10 here

The number of open branches based on changing the minimum standard distance is presented in Figure 8. Considering that there are five different banks in the system and the equity factor is 90%, the model has tried to maintain equity using alternative solutions. At least one bank branch keeps in the system for a minimum standard distance of branches larger than 500 meters.

Insert Fig. 8 here

4.3.2. Effect of equity coefficients (the membership degree of fuzzy constraints)

This section assesses the effect of equity coefficients on the objective function $(s_l = 100, 200, 300)$. In this analysis, the values α and β are considered equal. As the equity coefficient increases, the value of the objective function decreases. Efficiency and equity are often inconsistent in terms of the location of facilities. If we considered fuzzy constraints as forms of objective functions for equity such as absolute error value, the results would be a tradeoff curve similar to Figure 9.

Insert Fig. 9 here

In Figure 9, the curve is plotted based on the changes in parameters α and β. Similar to Figures 6 and 7, better answers can be obtained when the acceptable distance between facilities is smaller. In other words, by changing the minimum acceptable distance parameter, staff retention can be maximized.

4.4. Managerial insight

The banking industry is one area with a lot of potential for development and improvement. Analyzes performed to merge several large banks with different brands showed that minimum distance constraints (like the equity coefficient) play an important role in response rates and branch retention. By increasing on equity, selecting the remaining facilities in the system followed more carefully, and the possibility of remaining in the system for many facilities will be eliminated. However, considering all aspects of the problem leads to a solution that balances the stakeholders' problem objectives and aspirations.

Considering the changes in all industries COVID-19 pandemic, change in demand, increased online services due to the growth of information technology, and the possibility of providing services in the context of social networks led most industries to change their provided services. So, delocation and merging are using fair approaches in accompanying and reaching a collective agreement between stakeholders can play an influential role. In this regard, the results of this paper showed that adopting a mathematical model concerning equity can play an important role in relocating and merging the branches in leading industries such as education, banking, hypermarkets, and hazardous industries.

5. Conclusions and suggestions

This paper addressed a strategic delocation decision to merge the branches of different banks, which was adopted following macro-national policies. One way to persuade stakeholders is to significant change using equitable approaches. With the idea of using fuzzy constraints, maintaining the interests of each stakeholder relative to the initial conditions and other stakeholders was modeled. The proposed mathematical model is linear mixed-integer programming and can achieve the optimal solution with a reasonable solution time. Similar to the equitable facility location problem in the literature, the results showed that the fuzzy constraints for the equity modeling approach, based on changes in the degree of membership of fuzzy constraints, can create a tradeoff curve with the efficiency objective function. Accordingly, decision-makers can agree on the expected goal based on the degree of membership of the fuzzy constraints that play the equity coefficient role. Therefore, it can be argued that when equity is presented as an approach and not as a goal, fuzzy constraints can be used for equity and efficiency facility location problems. For future development, customers' roles, allocation, and other assumptions can be added to the model and discuss the potential outcomes.

Conflict of interest

We have no conflicts of interest to disclose.

Data Availability Statement

Due to the nature of this research, supporting data is not available.

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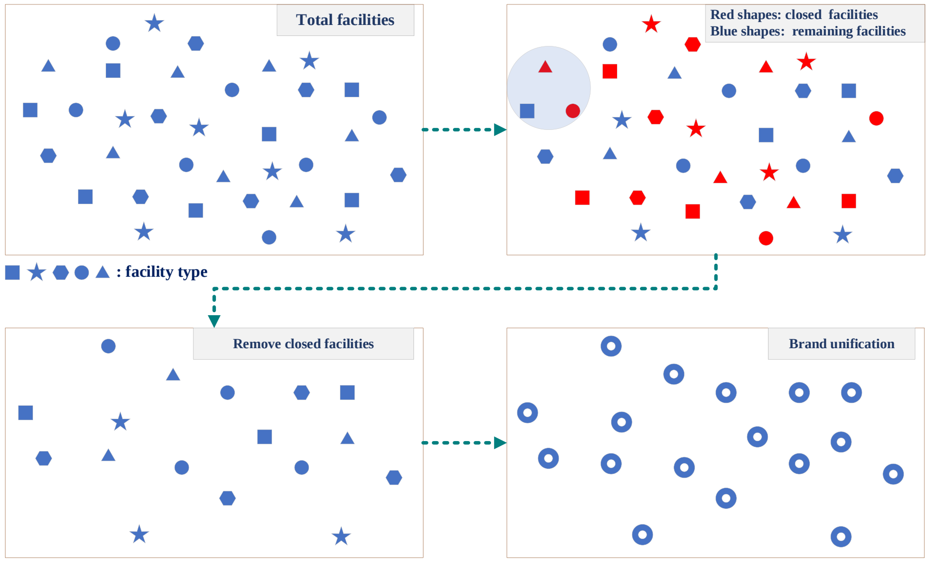

Fig. 1: An overview of the delocation problem.

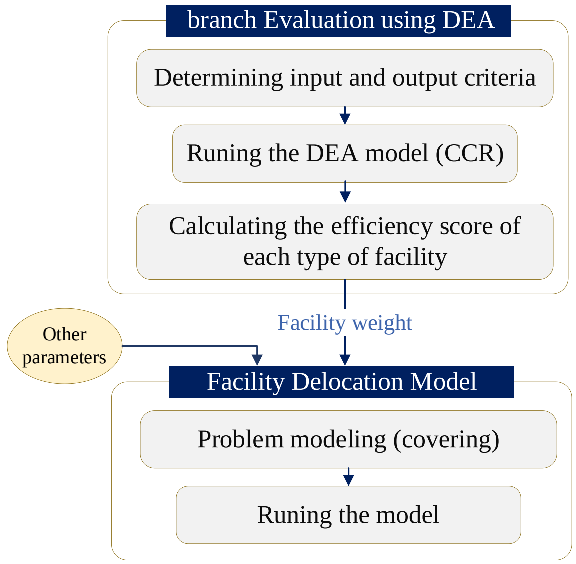

Fig. 2: The conceptual model of the study.

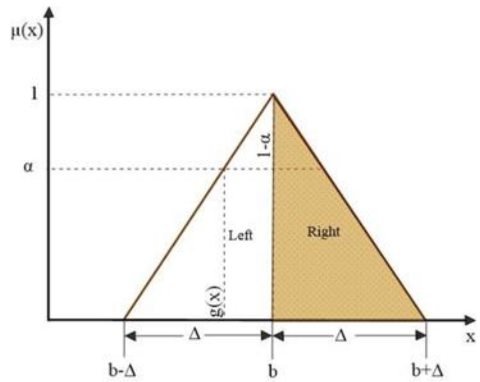

Fig. 3: The fuzzy triangular equation and the membership degree.

E_{jk}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
k=1	1	0	0	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
k=2	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
k=3	0	1	1	0	0	0	0	0	0	0	0	1	1	0	1	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
k=4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
k=5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0

Fig. 4: Parameter for determining the types of facilities in the specified locations $(n = 40, k = 5)$.

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	0	623	563	167	675	1334	671	2020	810	1202	1283	2090	1344	1935	831	1376	1821	1970	652	320	1179	914	290	1476	549	2052	1756	856	631	1905	1743	411	987	469	1351	1224	751	1768	1522	1332
2		0	756	1451	668	1754	954	611	331	1968	2041	1925	709	2053	1443	1067	1106	284	1487	312	322	1662	235	1071	1469	357	1111	1893	1961	1876	1563	291	643	354	522	1730	888	1984	1410	1199
3			0	804	1598	1505	625	1203	626	748	1711	2016	788	1797	653	1070	1067	1206	1883	890	889	328	256	1448	1869	1852	887	1183	440	334	1714	1332	1541	613	437	488	894	279	571	1652
4				0	1213	1798	1225	1546	770	810	1198	933	748	748	873	1324	435	2082	1102	625	1838	345	1776	1295	168	1327	656	1307	1450	823	1264	1946	450	1314	1388	265	1682	927	675	489
5					0	1534	930	550	1735	1333	129	1725	532	366	406	1770	1176	1465	1658	474	725	387	1266	1571	554	769	906	982	1724	147	2024	1811	804	333	1025	1664	663	629	927	1777
6						0	999	805	581	448	1552	849	1823	1343	488	2066	338	1598	1367	2007	1957	919	455	1859	1390	1872	310	511	1096	1621	159	1009	175	1840	343	1996	1250	2073	1272	266
7							0	650	1739	1831	2100	965	1257	1560	2021	1149	840	232	938	500	185	2001	862	1044	1264	150	1456	164	570	1994	360	1701	1420	1104	507	710	1808	1961	722	1019
8								0	1318	1743	290	651	1895	1465	1714	1907	1231	416	2071	688	361	1605	600	562	113	1744	1201	1709	1713	125	2004	867	1852	837	124	1006	1406	2047	1605	1057
9									0	1508	1867	879	1832	1231	1256	1994	621	549	1448	1663	1895	813	838	1453	1102	234	2048	525	124	2075	731	2096	971	1797	1115	1873	1414	779	1277	1877
10										0	1260	1120	1947	1059	172	900	855	1611	429	1794	649	1213	2080	1231	1460	326	1925	2000	446	1249	319	1454	1388	1474	1298	537	1208	247	1301	1591
11											0	1691	2034	151	766	753	461	2080	961	1673	846	875	893	1987	1906	1498	2067	1578	1135	284	1436	1585	370	711	509	286	320	637	549	883
12												0	1606	1295	305	1817	664	384	1612	703	1018	487	1335	1329	746	475	1541	1239	1832	2092	1672	2045	1845	320	1038	1955	1739	1408	1627	747
13													0	339	718	925	1899	486	1660	1390	1274	1677	1347	463	1848	793	1409	841	866	1461	1559	208	374	1844	708	1690	318	1928	582	988
14														0	1214	1202	975	128	1444	2095	1113	1905	876	1247	1149	478	322	700	1722	1593	1641	1631	1672	954	650	1926	1187	258	1927	1241
15															0	1834	1189	529	820	2077	1766	1225	1415	1736	1796	1866	1204	260	1330	544	1419	1581	1752	1936	946	499	1434	1621	774	1733
16																0	1376	1922	1082	1538	191	1794	1055	1873	1966	1391	522	607	1843	880	468	1483	1651	1237	1368	311	2006	189	1311	2021
17																	0	868	186	2099	1051	863	1392	1387	1628	1494	419	2070	260	203	154	1585	993	381	942	978	2001	477	1927	2007
18																		0	1182	1823	819	768	728	1123	191	2031	1474	702	1006	1625	329	273	606	1614	1604	1488	321	504	1680	799
19																			0	328	1173	2012	263	1575	1663	1806	365	1733	2019	371	1578	1531	663	1932	817	933	1307	1776	1990	2025
20																				0	1364	696	476	1383	951	1434	883	1863	1622	1768	1942	1564	749	1459	406	1000	601	999	1242	589
21																					0	729	972	612	1241	363	1142	744	428	1187	1588	1921	201	140	1912	758	1088	1811	609	132
22																						0	1376	1064	1378	1820	1076	1982	1995	1851	1351	521	1145	2016	1958	1588	1964	1181	1303	1617
23																							0	1926	1449	713	733	1489	523	704	1855	1489	531	610	733	1651	1754	289	576	1438
24																								0	936	1355	1843	328	622	1778	1593	1860	1556	1430	272	2084	869	1818	262	959
25																									0	1931	1578	918	479	111	489	1460	990	1485	570	592	561	1850	2000	1538
26																										0	291	1803	1992	1165	780	2055	1329	929	302	271	1796	124	662	1329
27																											0	534	1827	412	1351	1236	261	1306	630	1955	139	2004	1849	1735
28																												0	1602	1921	491	1495	1891	1902	891	2046	436	2099	475	696
29																													0	1326	1092	197	268	977	848	1864	1825	2007	955	1051
30																														0	1974	580	1241	1100	1983	1335	1767	684	786	950
31																															0	1281	1501	1706	670	774	397	883	727	1698
32																																0	159	1095	1024	233	1943	1479	1807	1128
33																																	0	206	2055	245	1375	1369	543	1314
34																																		0	1295	1640	1951	1647	1740	1276
35																																			0	1609	1322	1403	170	1126
36																																				0	675	810	1501	301
37																																					0	2077	716	629
38																																						0	882	1333
39																																							0	458
40																																								0

Fig. 5: The distance matrix $(n = 40)$.

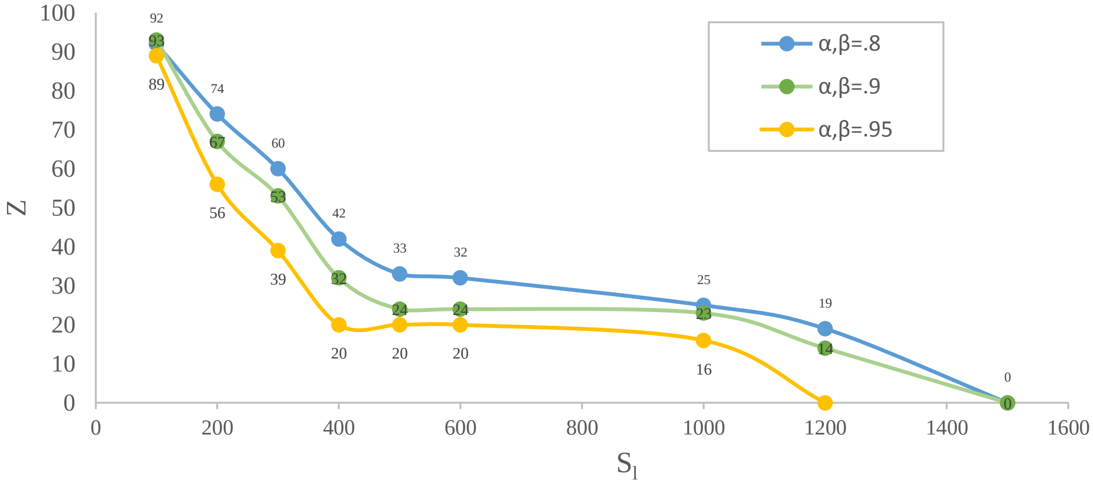

Fig. 6: The effect of minimum standard distance on the objective function $(\Delta = 1, \Delta_1 = 10, n = 40)$.

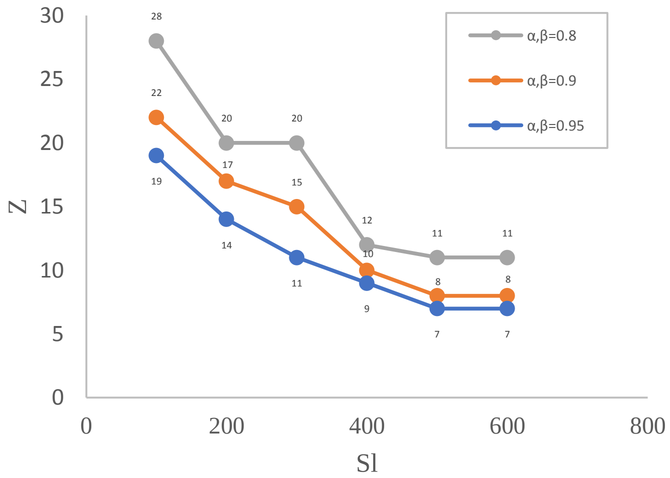

Fig. 7: The effect of minimum standard distance on the objective function $(\Delta = 1, \Delta_1 = 1, n = 15)$.

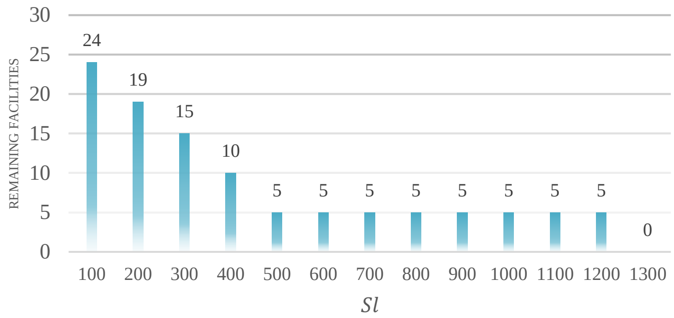

Fig. 8: The effect of minimum standard distance on remaining facilities.

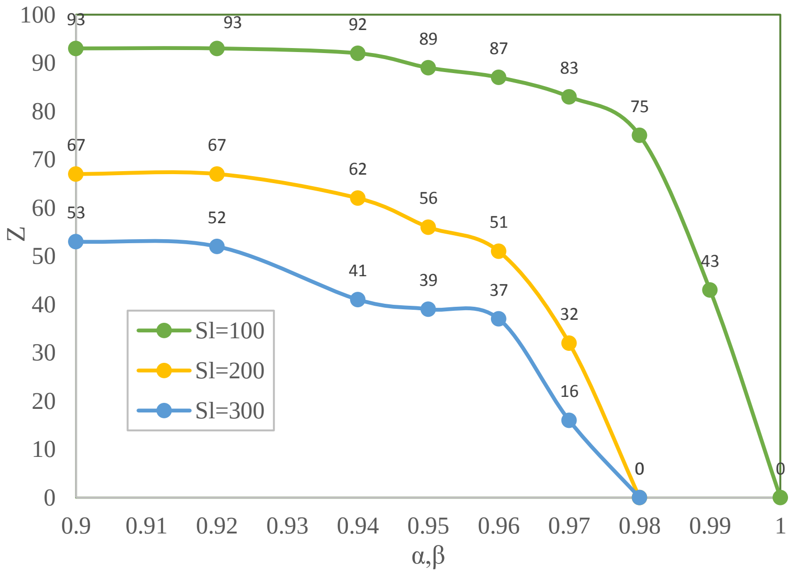

Fig. 9: The effect of equity coefficients on the objective function $(\Delta = 1, \Delta_1 = 10)$.

Table 1.
Inputs and outputs are used in data envelopment analysis.

Inputs	Outputs
Number of branch staff	Branch resource supply
Total expenses	Branch resource allocation
Location	Number of banking services
Branch claims	customer satisfaction
	Total branch income

Table 2.
The data envelopment analysis models.

	CCR		BCC	
	Primal	dual	Primal	dual
Input-Oriented	$\min \theta_p$ $\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_p x_{ip} \quad i=1,2,\ldots,m$ $\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rp} \quad r=1,2,\ldots,s$ $\lambda_j \ge 0$	$\max \sum_{r=1}^{s} u_r y_{rp}$ $\sum_{i=1}^{m} v_i x_{ip} = 1$ $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad \forall j$ $u_{r,} v_i \ge \varepsilon$	$\min \theta_p$ $\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_p x_{ip} \quad i=1,2,\ldots,m$ $\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rp} \quad r=1,2,\ldots,s$ $\sum_{j=1}^{n} \lambda_j = 1$	$\max \sum_{r=1}^{s} u_r y_{rp} + u'_p$ $\sum_{i=1}^{m} v_i x_{ip} = 1$ $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u'_p \le 0 \quad \forall j$ $u_{r,} v_i \ge \varepsilon \ , u'_p$ *free in sign*
Output-Oriented	$\max \varphi_p$ $\sum_{j=1}^{n} \lambda_j y_{rj} \ge \varphi_p y_{rp} \quad r=1,2,\ldots,s$ $\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ip} \quad i=1,2,\ldots,m$ $\lambda_j \ge 0$	$\min \sum_{i=1}^{m} v_i x_{ip}$ $\sum_{r=1}^{s} u_r y_{rp} = 1$ $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0$ $u_{r,} v_i \ge \varepsilon$	$\max \varphi_p$ $\sum_{j=1}^{n} \lambda_j y_{rj} \ge \varphi_p y_{rp} \quad r=1,2,\ldots,s$ $\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ip} \quad i=1,2,\ldots,m$ $\sum_{j=1}^{n} \lambda_j = 1$	$\min \sum_{i=1}^{m} v_i x_{ip} + v'_p$ $\sum_{r=1}^{s} u_r y_{rp} = 1$ $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - v'_p \le 0 \ \forall j$ $u_{r,} v_i \ge \varepsilon , v'_p$ *free in sign*

Table 3.
Parameter for determining the types of facilities in the specified locations $(n = 15, k = 3)$.

E_{jk}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$k = 1$	1	0	0	0	1	0	1	0	1	0	1	0	0	0	0
$k = 2$	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0
$k = 3$	0	1	1	0	0	0	0	0	0	0	0	1	1	0	1

Table 4.
The data for the problem with 15 locations and three types of facilities.

K=1				K=2				K=3			
j	e_j	p_j	CP_j	j	e_j	p_j	CP_j	j	e_j	p_j	CP_j
1	1.1	5	8	4	1	7	10	2	1.2	5	9
5	1.12	6	9	6	0.86	7	9	3	0.9	6	7
7	1	5	9	8	0.76	9	12	12	0.82	7	9
9	1.15	6	8	10	1.2	7	8	13	0.96	6	11
11	1	5	8	14	1.1	7	11	15	0.9	7	10
Sum	5.37	27			4.92	37			4.78	31	

Table 5.
The distance matrix $(n = 15)$.

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	300	1000	1100	900	600	800	700	1200	250	500	750	600	1300	1550
2		0	800	900	700	400	550	600	800	200	300	500	450	900	1000
3			0	250	150	700	450	500	700	400	860	500	900	650	700
4				0	200	800	600	650	800	500	850	500	950	910	980
5					0	730	540	650	970	490	770	530	860	680	700
6						0	390	410	500	400	120	400	180	450	500
7							0	100	350	500	500	130	400	350	350
8								0	300	510	500	100	400	310	310
9									0	600	460	400	530	100	80
10										0	400	470	460	660	690
11											0	550	100	500	540
12												0	490	310	310
13													0	400	400
14														0	100
15															0

Table 6.
Remaining and closed facilities after solving the model.

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
y_j	1	1	0	1	0	0	0	0	0	0	1	1	0	1	0
k	1	3	3	2	1	2	3	2	1	2	1	3	3	2	3

Table 7.
Staff hired by the remaining facilities $(\alpha, \beta = 0.9, \Delta, \Delta_1 = 1)$.

z_{ij}		Open facilities					
		1	2	4	11	12	14
Closed facilities with allocated staff	3					2	
	5				3		
	6			3			4
	10	3					

Table 8.
The data relating to the problem with 40 facilities which is classified into five types.

K=1				K=2				K=3				K=4				K=5			
j	P_j	CP_j	e_j	j	P_j	CP_j	e_j	j	P_j	CP_j	e_j	j	P_j	CP_j	e_j	j	P_j	CP_j	e_j
1	5	8	1.1	4	7	10	1	2	5	9	1.2	33	5	8	0.74	26	7	12	0.8
5	6	9	1.12	6	5	9	0.86	3	6	7	0.9	34	8	10	0.91	27	7	10	0.98
7	5	9	1	8	9	12	0.76	12	7	9	0.82	35	9	12	0.8	28	7	9	0.87
9	6	8	1.15	10	7	8	1.2	13	6	11	0.96	36	9	11	0.85	29	6	9	0.98
11	5	8	1	14	7	11	1.1	15	9	10	0.9	37	7	13	0.96	30	5	8	0.75
20	6	10	1	16	5	9	0.88	18	5	8	1	38	7	10	0.98	31	5	9	0.93
21	4	8	1	17	5	10	0.9	19	5	11	1.04	39	10	12	0.87	32	5	10	0.67
23	6	10	1.2	22	8	10	1.1	25	6	11	0.76	40	12	15	0.9				
24	6	9	1.12																
TP=49		TE=9.69		TP=53		TE=7.8		TP=49		TE=7.58		TP=67		TE=7.01		TP=42		TE=5.98	

Table 9.
The staff retention by open facilities $(\alpha, \beta = 0.9, \Delta = 1, \Delta_1 = 10)$

z_{ij}		Open facilities														
		3	4	7	9	12	14	16	19	20	30	31	32	34	35	37
Closed facilities with allocated staff	2											4				
	17						4		1							
	22															6
	23				1									2	3	
	24			3							3					
	25									4						
	26		3													
	27				1	2										
	33								5							
	38	1		1									5			
	39							4								

Table 10.
Minimum standard distance effect on remaining facility and objective function.

S_l	remaining facilities (open)	Objective Function	Remaining facilities indices
100	24	93	1, 2, 4, 5, 6, 13, 14, 16, 17, 18, 19, 20, 21, 23, 25, 26, 27, 31, 32, 33, 35, 36, 37, 38
200	19	67	2, 4, 5, 10, 12, 13, 14, 16, 19, 20, 23, 24, 26, 28, 29, 33, 34, 37, 40
300	15	53	3, 4, 7, 9, 12, 14, 16, 19, 20, 30, 31, 32, 34, 35, 37
400	10	32	13, 16, 20, 22, 23, 25, 29, 31, 34, 35
500	5	24	7, 14, 19, 32, 37
600	5	24	4, 19, 24, 32, 37
700	5	24	14, 19, 24, 32, 37
800	5	24	4, 7, 19, 32, 37
900	5	24	4, 7, 19, 32, 37
1000	5	23	9, 14, 19, 32, 37
1100	5	23	9, 14, 19, 32, 38
1200	5	14	12, 14, 24, 30, 36
1300	0	0	0

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