Improved Control Charts for Phase II Monitoring of Simple Linear Profiles Using Auxiliary Information

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Abstract

In some manufacturing or non-manufacturing systems, the process outcome is better characterized by a relationship between a main variable and some associated supporting variables. The monitoring of this functional relationship over time is termed as profile monitoring. This study aims to improve Phase II monitoring of simple linear profiles by using auxiliary information which is correlated with the main variable. To accomplish that, using a mean estimator, two auxiliary information-based (AIB) control charts namely AIB-MEWMA and AIB-DMEWMA charts are proposed for detecting different shifts in model parameters. Using two numerical examples based on simulation studies, the sensitivity of the proposed control charts is evaluated and compared with the existing MEWMA and DMEWMA charts in terms of the average run length (*ARL*) metric. The results of simulations reveal that the proposed charts perform better than the existing MEWMA and DMEWMA charts. The applicability of the proposed control charting methods is demonstrated using a real-life data example from the cylinder production process.

Keywords: Statistical process monitoring, Mean estimator, Simple linear profile, Auxiliary information, Phase II.

1. Introduction

In many statistical process monitoring (SPM) applications, control charts are established to monitor the quality of a single or multiple quality characteristics. In some manufacturing or non-manufacturing systems, however, the main (study) variable is associated with one or several

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supporting (explanatory) variables. Monitoring this relationship among the study and explanatory variables over time in SPM context is referred as profile monitoring. Depending on the nature of the process, different profile models can be categorized into simple linear, multiple linear, polynomial, nonlinear, and complicated profiles. Among them, the simple linear profile is the most common type because of its easier implementation as well as its coverage in real-world applications. In a simple linear profile, the value of study variable is dependent on one explanatory variable by a linear regression model. For example, in a photo-voltaic (PV) system, the output voltage is the main variable which is linearly associated with the capacitance level of capacitors, used in the Z-source inverter (Mahmood et al [1]). Different profile monitoring methods can be classified into Phase I and Phase II ones. The purpose of Phase I analysis is to evaluate the process stability and estimate the regression parameters while Phase II analysis aims to detect the process disturbances as quickly as possible. Monitoring simple linear profiles has gained increasing attention in recent years. For example, Haq [2] employed adaptive features for enhancing the detectability of MEWMA control chart in Phase II monitoring of simple linear profiles in both univariate and multivariate cases. Furthered examples include Kazemzadeh et al. [3], Riaz et al. [4], Mahmoodet al. [5], Riaz et al. [6], Chang and Chen [7], Haq et al. [8], Khalafi et al. [9], Yeganeh et al. [10] and the references cited therein. Further information concerning the simple linear profile monitoring approaches can be found in Maleki et al. [11].

Sometimes, the auxiliary variables are available to quality engineering practitioners along with the original variable of interest. In such cases, direct measurements of the main variable can be performed alongside with another variable. According to Ahmad et al. [12], the industrial systems with auxiliary variables which are correlated with the main variable can be categorized into non-cascading and cascading processes. In a non-cascading process, the study variable can endure a shift in mean and/or variability parameters without affecting the auxiliary variables. For example in platinum refinery operation, the amount of platinum metal as the study variable is generally correlated with the magnitude of other metals as the auxiliary ones. In this process, as stated by Hawkins [13], the quantity of the other metals will not be affected by the shift in the concentration of platinum metal during the production cycle. In contrast, in cascading processes, the shift in the study variable directly affects the auxiliary variable(s). For example, in the production process of shaft, any change in inner diameter will directly deviate from its outer diameter. In other words, the outer diameter of the shaft increases as the inner diameter increases and vice versa. In this condition, the distribution parameters of outer diameter are not fixed when the assignable cause affects the inner diameter of shaft. Hence, as mentioned by Saleh et al. [14], concurrent changes in both inner and outer diameters as the study and auxiliary variables should be taken into account.

Utilizing auxiliary or supplementary variables which are correlated with the main variable can improve the efficiency of control charts by reducing the variance of parameter estimators. In recent years, considering the auxiliary variable(s) to establish different control charts have received much attention by the researchers. More specifically, considering the auxiliary variables enhances the sensitivity of different control charting methods to detect small and moderate shifts. As a pioneer work regarding the incorporation of auxiliary information and control charting structures, Riaz [15] used a single auxiliary variable X for better monitoring of variable Y and proposed a Shewhart control chart for detecting the process mean shifts. Then, Riaz [16] proposed an auxiliary information-based (AIB) control chart for monitoring the process variability. Abbas et al. [17] employed single and two auxiliary variables to enhance the sensitivity of progressive mean control chart in detecting small mean shifts. Based on auxiliary variables, Chiang et al. [18] proposed an improved Hotelling T^2 control charting method in which the normality assumption of quality characterises is relaxed. Recently AIB control charts have been well documented in the literature. Some of the recently published ones include Adegoke et al. [19], Arshad et al. [20], Haq and Khoo [21], Sanusi et al. [22], Noor-ul-Amin et al. [23], Saha et al. [24], Saghir et al. [25], Noor-ul-Amin et al. [26], and Anwar et al. [27].

To the best of the authors' knowledge, employing the auxiliary information for enhancing the detectability of profile monitoring control charts has been clearly neglected in the literature. In other words, despite the improved efficiency of AIB control charts, no research has adopted the use of auxiliary information to design control charting methods in profile monitoring context. Due to efficiency of AIB control charts in detecting process disturbances, in this paper we employ the auxiliary information and design two memory-type control charts including AIB-MEWMA and AIB-DMEWMA charts for Phase II monitoring of simple linear profiles. To accomplish that we consider the non-cascading condition under the assumption of stable auxiliary characteristics and develop mean estimators in place of ordinary least estimators to obtain the regression model parameters. Through Mont Carlo simulations, the run length performance of the proposed control charts are evaluated and compared with two efficient control charts. The rest of this article is organized as follows: The background of linear profile model along with the mean estimator is discussed in Section 2. The proposed control charts based on auxiliary characteristics are presented in Section 3. Extensive simulation studies are presented in Section 4 to evaluate the performance of the proposed AIB control charts. The applicability of the proposed control charts is demonstrated using a real-life data example from the cylinder production process in Section 5. Finally, the conclusion remarks along with some recommendations for future studies are given in Section 6.

2. Problem definition

2.1. One auxiliary profile

Consider a process where the process outcome is characterized by a bivariate linear profile model. The first variable is the main variable while the second one is considered as the auxiliary variable. For t^{th} random sample collected over time, we have n observations given as $(x_i, y_{i1t}, y_{i2t}); i=1,2,...,n$, $t=1,2,...$ where x_i denotes the value of explanatory variable at i^{th} experimental setting. Moreover, for t^{th} random sample, y_{i1t} and y_{i2t} are the i^{th} value of the study and auxiliary response variables in t^{th} profile, respectively. When the process is statistically in-control, the regression model that associates the bivariate response variables with the explanatory variable is given as:

$$\mathbf{Y}_t = \mathbf{XB} + \mathbf{E}_t, \tag{1}$$

where, $\mathbf{B} = \begin{bmatrix} \beta_{01} & \beta_{02} \\ \beta_{11} & \beta_{12} \end{bmatrix}$ while $\mathbf{Y}_t$ is a $n \times 2$ matrix and contain the observations of bivariate response variables. Obviously, the first column in $\mathbf{Y}_t$ indicates the observations of the study variable while the second column contains the observed values of the auxiliary variable. Moreover, $\mathbf{E}_t$ is a $n \times 2$ matrix of error terms which is expressed as:

$$\mathbf{E}_t = \begin{bmatrix} \varepsilon_{11t} & \varepsilon_{12t} \\ \varepsilon_{21t} & \varepsilon_{22t} \\ \vdots & \vdots \\ \varepsilon_{n1t} & \varepsilon_{n2t} \end{bmatrix} \tag{2}$$

It is assumed that each row of matrix $\mathbf{E}_t$ follows a bivariate normal distribution with mean vector $\mathbf{0}$ and the following covariance matrix:

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} \tag{3}$$

For t^{th} random sample, the vector of estimated regression parameters for the study profile through the mean estimator (MS) method can be obtained by retrieving from Riaz et al. [15] as:

$$\hat{\boldsymbol{\beta}}_1^{MS} = \begin{pmatrix} \hat{\beta}_{01} \\ \hat{\beta}_{11} \end{pmatrix} + \Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}\hat{\boldsymbol{\beta}}_2^{OLS}} \Sigma^{-1}_{\hat{\boldsymbol{\beta}}_2^{OLS}} \left[\begin{pmatrix} \beta_{02} \\ \beta_{12} \end{pmatrix} - \begin{pmatrix} \hat{\beta}_{02} \\ \hat{\beta}_{12} \end{pmatrix} \right], \tag{4}$$

where $\hat{\boldsymbol{\beta}}_1^{OLS} = \begin{pmatrix} \hat{\beta}_{01} \\ \hat{\beta}_{11} \end{pmatrix}$ and $\hat{\boldsymbol{\beta}}_2^{OLS} = \begin{pmatrix} \hat{\beta}_{02} \\ \hat{\beta}_{12} \end{pmatrix}$ are the estimated regression parameters for the study and auxiliary profiles, respectively obtained by ordinary least square (OLS) method. The expected value and covariance matrix of $\hat{\boldsymbol{\beta}}_1^{MS}$ when the process is statistically in-control can be obtained according to Equations (5) and (6), respectively:

$$\mathrm{E}\left(\hat{\boldsymbol{\beta}}_1^{MS}\right) = \begin{pmatrix} \beta_{01} \\ \beta_{11} \end{pmatrix}, \mathbf{\Sigma}_{\hat{\boldsymbol{\beta}}_1^{MS}} = Var\left(\hat{\boldsymbol{\beta}}_1^{MS}\right) = \Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}} - \Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}\hat{\boldsymbol{\beta}}_2^{OLS}} \Sigma^{-1}_{\hat{\boldsymbol{\beta}}_2^{OLS}} \Sigma_{\hat{\boldsymbol{\beta}}_2^{OLS}\hat{\boldsymbol{\beta}}_1^{OLS}} \tag{5}$$

Moreover, according to Noorossana et al. [28] we have $\hat{\boldsymbol{\beta}}^{OLS} = \left(\hat{\beta}_{01}, \hat{\beta}_{11}, \hat{\beta}_{02}, \hat{\beta}_{12}\right)^T$ and:

$$\Sigma_{\hat{\boldsymbol{\beta}}^{OLS}} = \begin{pmatrix} \Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}} & \Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}\hat{\boldsymbol{\beta}}_2^{OLS}} \\ \Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}\hat{\boldsymbol{\beta}}_2^{OLS}} & \Sigma_{\hat{\boldsymbol{\beta}}_2^{OLS}} \end{pmatrix}_{4\times 4} \tag{6},$$

where $\Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}} = \sigma_1^2 \left(\mathbf{X}^T\mathbf{X}\right)^{-1}$ and $\Sigma_{\hat{\boldsymbol{\beta}}_2^{OLS}} = \sigma_2^2 \left(\mathbf{X}^T\mathbf{X}\right)^{-1}$. Furthermore, the covariance between the estimated parameters (the elements of $\Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}\hat{\boldsymbol{\beta}}_2^{OLS}}$) can be obtained as:

$$\mathbf{Cov}(\hat{\beta}_{01}, \hat{\beta}_{02}) = \sigma_{12}\left(\frac{1}{n} + \frac{\overline{X}^2}{S_{xx}}\right), \tag{7}$$

$$\mathbf{Cov}(\hat{\beta}_{11}, \hat{\beta}_{12}) = \frac{\rho_{12}\sigma_1\sigma_2}{S_{xx}} \tag{8}$$

$$\mathbf{Cov}(\hat{\beta}_{0u}, \hat{\beta}_{1v}) = -\rho_{uv}\sigma_u\sigma_v \frac{\overline{X}}{S_{xx}}; u, v = 1, 2 \tag{9}$$

2.2. Two auxiliary profiles

The regression model considering two auxiliary profiles is given as:

$$\begin{bmatrix} y_{11t} & y_{12t} & y_{13t} \\ y_{21t} & y_{22t} & y_{23t} \\ \vdots & \vdots & \vdots \\ y_{n1t} & y_{n2t} & y_{n3t} \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} \beta_{01} & \beta_{02} & \beta_{03} \\ \beta_{11} & \beta_{11} & \beta_{11} \end{bmatrix} + \begin{bmatrix} \varepsilon_{11t} & \varepsilon_{12t} & \varepsilon_{13t} \\ \varepsilon_{21t} & \varepsilon_{22t} & \varepsilon_{23t} \\ \vdots & \vdots & \vdots \\ \varepsilon_{n1t} & \varepsilon_{n2t} & \varepsilon_{n3t} \end{bmatrix} \tag{10}$$

The mean estimator for t^{th} random sample is defined as follows:

$$\hat{\boldsymbol{\beta}}_1^{MS} = \begin{pmatrix} \hat{\beta}_{01} \\ \hat{\beta}_{11} \end{pmatrix} + \Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}\hat{\boldsymbol{\beta}}_2^{OLS}} \Sigma^{-1}_{\hat{\boldsymbol{\beta}}_2^{OLS}} \left[\begin{pmatrix} \beta_{02} \\ \beta_{12} \end{pmatrix} - \begin{pmatrix} \hat{\beta}_{02} \\ \hat{\beta}_{12} \end{pmatrix}\right] + \Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}\hat{\boldsymbol{\beta}}_3^{OLS}} \Sigma^{-1}_{\hat{\boldsymbol{\beta}}_3^{OLS}} \left[\begin{pmatrix} \beta_{03} \\ \beta_{13} \end{pmatrix} - \begin{pmatrix} \hat{\beta}_{03} \\ \hat{\beta}_{13} \end{pmatrix}\right], \tag{11}$$

In the case of two auxiliary profiles, we have: $\hat{\boldsymbol{\beta}}^{OLS} = \left(\hat{\beta}_{01}, \hat{\beta}_{11}, \hat{\beta}_{02}, \hat{\beta}_{12}, \hat{\beta}_{03}, \hat{\beta}_{13}\right)^T$ and:

$$\Sigma_{\hat{\boldsymbol{\beta}}^{OLS}} = \begin{pmatrix} \Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}} & \Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}\hat{\boldsymbol{\beta}}_2^{OLS}} & \Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}\hat{\boldsymbol{\beta}}_3^{OLS}} \\ \Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}\hat{\boldsymbol{\beta}}_2^{OLS}} & \Sigma_{\hat{\boldsymbol{\beta}}_2^{OLS}} & \Sigma_{\hat{\boldsymbol{\beta}}_2^{OLS}\hat{\boldsymbol{\beta}}_3^{OLS}} \\ \Sigma_{\hat{\boldsymbol{\beta}}_1^{OLS}\hat{\boldsymbol{\beta}}_3^{OLS}} & \Sigma_{\hat{\boldsymbol{\beta}}_2^{OLS}\hat{\boldsymbol{\beta}}_3^{OLS}} & \Sigma_{\hat{\boldsymbol{\beta}}_3^{OLS}} \end{pmatrix}_{6\times 6}, \tag{12}$$

The elements of $\Sigma_{\hat{\boldsymbol{\beta}}^{OLS}}$ are computed similar to the previous subsection.

3. Proposed control charts based on auxiliary information

In this section, two improved control charts based on auxiliary information for minoring simple linear profiles are presented. The proposed AIB-MEWMA and AIB-DMEWMA control charts are presented in Section 3.1 and 3.2, respectively.

3.1. AIB-MEWMA control chart

The MEWMA statistic has been first developed by Zou et al. [29] for monitoring general linear profiles. The MEWMA control chart is an efficient monitoring scheme to detect different shifts in slope, intercept and error variance parameters especially when the shift magnitude is small and moderate. Here, we introduce an auxiliary information based monitoring scheme termed as AIB-MEWMA control chart to further enhance the detection capability of MEWMA procedure.

Let $\mathbf{A}_t$ be the EWMA sequence for t^{th} sample which is computed based on the auxiliary variable(s) along with the estimated regression parameters (obtained by mean estimator):

$$\mathbf{A}_t = \lambda\left(\hat{\boldsymbol{\beta}}_{1,t}^{MS} - \begin{bmatrix}\beta_{01}\\ \beta_{11}\end{bmatrix}\right) + (1-\lambda)\mathbf{A}_{t-1}, \tag{13}$$

where $\hat{\boldsymbol{\beta}}_{1,t}^{MS}$ represents the estimated regression parameters of the study profile at t^{th} profile obtained by mean estimator method. Moreover, $\lambda \in (0.1]$ denotes the smoothing parameter while the starting value of chart statistic is:

$$\mathbf{A}_0 = E\left(\hat{\boldsymbol{\beta}}_1^{MS} - \begin{bmatrix}\beta_{01}\\ \beta_{11}\end{bmatrix}\right) = \begin{pmatrix}0\\0\end{pmatrix}, \tag{14}$$

To AIB-MEWMA plotting statistic can be obtained according to Equation (15):

$$AIB-MEWMA_t = \mathbf{A}_t^T \mathbf{\Sigma}_{\mathbf{A}}^{-1} \mathbf{A}_t, \tag{15}$$

where:

$$\Sigma_{\mathbf{A}} = \frac{\lambda}{2-\lambda} \Sigma_{\hat{\boldsymbol{\beta}}_1^{MS}}, \tag{16}$$

The chart triggers an out-of-control signal for any value of $AIB-MEWMA_t > h$ where h is determined to obtain a pre-determined in-control average run length (ARL_0).

3.2. AIB-DMEWMA control chart

The double multivariate exponentially weighted moving average (DMEWMA) chart as an extension of MEWMA technique by performing the exponential smoothing twice. This chart has been shown to be more sensitive to react to small process disturbances. Here, the AIB-DMEWMA monitoring scheme based on the combination of auxiliary characteristics and DMEWMA statistic is presented. To obtain the AIB-DMEWMA statistic for t^{th} sample, first we define:

$$\mathbf{B}_t = \lambda \left(\hat{\boldsymbol{\beta}}_{1,t}^{MS} - \begin{bmatrix} \beta_{01} \\ \beta_{11} \end{bmatrix} \right) + (1-\lambda) \mathbf{B}_{t-1}, \tag{17}$$

$$\mathbf{C}_t = \lambda \mathbf{B}_t + (1-\lambda) \mathbf{C}_{t\text{-}1}, \tag{18}$$

where $\lambda \in (0,1]$ and $\mathbf{B}_0 = \mathbf{C}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. The AIB-DMEWMA chart statistic at t^{th} sampling point will be given as:

$$AIB-DMEWMA_t = \mathbf{C}_t^{\mathrm{T}} \mathbf{\Sigma}_{\mathbf{C}}^{-1} \mathbf{C}_t, \tag{19}$$

where

$$\mathbf{\Sigma}_{\mathbf{C}} \simeq \frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3} \Sigma_{\hat{\boldsymbol{\beta}}_1^{MS}}, \tag{20}$$

The process is declared out-of-control if $AIB-DMEWMA_t > h'$ where h' is computed to have a pre-determined ARL_0 value.

4. Performance evaluation and comparison

This section presents two numerical examples to evaluate the capability of proposed AIB-MEWMA and AIB-DMEWMA control charts in Phase II monitoring of simple linear profiles in terms of out-of-control average run length (ARL_1) metric. For this purpose, the detection capability of the proposed control charts is compared with classical MEWMA and DMEWMA charts through 10000 simulation replicates in MATLAB software. Note that, the average run length is defined as the average number of successive samples taken until the chart issues an out-of-control signal. This metric should be as large as possible when the process is in-control to ensure optimal chart efficiency. In contrast, under out-of-control scenarios, the *ARL* should be large enough to ascertain the detection capability of the control chart. In order to have fair comparisons among the proposed and existing methods, all control charts are designed to obtain $ARL_0 = 200$.

To conduct simulation studies, we select different values for parameter λ to analyze the sensitivity of proposed control charts on smoothing parameter. Moreover, in order to assess the effect of correlation between the main and auxiliary response variables, different values for parameter ρ is considered in simulation experiments. Note that, all simulations are conducted for especial cases of $p = 2$ and $p = 3$.

4.1 Numerical example 1

In this subsection, the capability of the proposed control charts in detecting different step shifts in regression model parameters is evaluated for especial case of $p = 2$. Without loss of generality, we assume a bivariate linear profile as $Y_1 = 3 + 2x + \varepsilon_1$ and $Y_2 = 2 + x + \varepsilon_2$ where Y_1 and Y_2 are the study and auxiliary response variables, respectively. Accordingly, the vector of random errors $(\varepsilon_1, \varepsilon_2)$ follows a bivariate normal distribution with mean vector **0** and covariance matrix $\mathbf{\Sigma} = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_2\sigma_1 & \sigma_2^2 \end{bmatrix}$. Besides, according to SPM literature, we fix the values of explanatory variable at $\mathbf{X} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 6 & 8 \end{bmatrix}$. To generate random profiles and establish chart statistics, we consider $\sigma_1^2 = \sigma_2^2 = 1$ and $\lambda \in \{0.05, 0.1, 0.2\}$. Moreover, we choose the correlation coefficient as $\rho \in \{0.1, 0.5, 0.9\}$. The *ARL*s under $\rho \in \{0.1, 0.5, 0.9\}$ and $\lambda = 0.2$ when the intercept parameter of the study profile changes from β_{01} to $\beta_{01} + \delta_0 \sigma_1$ for $\delta_0 \in \{0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2\}$ are reported in Table 1. Note that, in the first two rows of Table 1, the performance of the MEWMA and DMEWMA control charts for monitoring the stability of the study profile without using auxiliary profile(s) is evaluated. Therefore, for these control charts, the sign "-" is used in the column ρ due to existence of only one profile. However, in the other rows of Table 1, the *ARL* values of the proposed AIB-MEWMA and AIB-DMEWMA charts in the presence of one auxiliary profile are reported under different values of ρ.

It can be seen from Table 1 that for all out-of-control scenarios (small, moderate and large shifts), the proposed auxiliary information based control charts outperform the existing MEWMA and DMEWMA charts. In other words, using auxiliary information to construct the chart statistics, improves the capability of both MEWMA and DMEWMA monitoring schemes to detect different step shifts in intercept parameter of the study profile. The results of Table 1 also indicate that for any values of ρ, the AIB-MEWMA chart performs better than AIB-DMEWMA chart except when $\delta_0 = 0.2$. Moreover, for both AIB-MEWMA and AIB-DMEWMA control charts, the *ARL* values decrease rapidly when ρ increases.

[Please insert Table 1 about here]

Table 2 gives the *ARL*s of the proposed AIB-MEWMA and AIB-DMEWMA charts at fix value of $\rho = 0.5$ when $\lambda \in \{0.05, 0.1, 0.2\}$. Similar to Table 1, The resulting *ARL* values reported in Table 2 show that using auxiliary variable along with the study response variable enhances the capability of both MEWMA and DMEWMA charts in detecting different shifts in intercept parameter. Besides, as the smoothing parameter increases, the capability of the proposed control charts to detect intercept shifts improves.

[Please insert Table 2 about here]

The *ARL* curves of the proposed AIB-MEWMA and AIB-DMEWMA charts under different values of correlation coefficient $\rho \in \{0.1, 0.5, 0.9\}$ are illustrated in Figures 1-a and 1-b when

$\lambda = 0.2$. Figures 1-a and 1-b confirms that the *ARL* performance of both proposed AIB charts is better than the existing MEWMA and DMEWMA charts. As the value of ρ increases, the detection capability of the proposed charts specially for small intercept shifts enhances.

[Please insert Figure 1 about here]

The *ARL* comparisons when the slope parameter of the study profile changes from β_{11} to $\beta_{11} + \delta_1\sigma_1$ for $\delta_1 \in \{0.025, 0.050, 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250\}$ are given in Table 3. As seen, similar to Tables 1-2, utilizing one auxiliary profile which is correlated with the main profile, causes an increase in the efficiency of both MEWMA and DMEWMA control charts to react to different slope shifts. The results of Table 3 reveal that the AIB-DMEWMA chart has the lowest *ARL* values as compared to the other control charts when $\rho = 0.1$. However, for $\rho = 0.5$ and $\rho = 0.9$, the AIB-MEWMA chart outperforms the AIB-DMEWMA chart under all shift magnitudes except $\delta_1 = 0.025$. In addition, at the small amount of slope shifts (ranges from 0.025 to 0.125), the DMEWMA chart has a better detection capability than the MEWMA chart.

[Please insert Table 3 about here]

Table 4 contains the *ARL*s when $\lambda \in \{0.05, 0.1, 0.2\}$ and correlation coefficient is fixed at $\rho = 0.5$. As it can be observed, with some exceptions, as λ increases, the efficiency of the control charts to detect slope shifts improves. In other words, an increase in parameter λ causes a reduction in the *ARL* of all control charts. Moreover, for any value of smoothing parameter, using auxiliary response variable along with the main variable, improves the performance of both MEWMA and DMEWMA charts to detect different slope shifts.

[Please insert Table 4 about here]

Next, we compare the performance of the proposed AIB control charts with existing MEWMA and DMEWMA charts to detect the error variance shifts. Table 5 reports the resulting *ARL* values when σ_1 goes to its out-of-control value denoted by $\gamma\sigma_1; \gamma \in \{1.2, 1.4, 1.6, 1.8, 2, 2.2, 2.4, 2.6, 2.8, 3\}$. It can be remarkable from Table 5 that the obtained *ARL*s of both control charts are highly dependent on the value of ρ. In fact, the larger the value of correlation coefficient ρ, the better the efficiency of proposed AIB control charts over the classical MEWMA and DMEWMA charts. The results also indicate the better performance of MEWMA chart over DMEWMA chart under each shift magnitude.

[Please insert Table 5 about here]

The *ARL* values of the proposed AIB control charts along with the existing charts under different values of smoothing parameter $\lambda \in \{0.05, 0.1, 0.2\}$ when $\rho = 0.5$ are summarized in Table 6. As seen, the sensitivity of all control charts to react to error variance shifts depends on the value of smoothing parameter. It is remarkable from Table 6 that, for all shifts, the AIB-MEWMA chart has its best performance when $\lambda = 0.2$. For small and moderate shifts in parameter σ_1 (ranging from 1,2 to 1.8). the proposed AIB-DMEWMA chart has its best detecting capability at $\lambda = 0.2$ while under large shifts ($\gamma \in \{2, 2.2, 2.4, 2.6, 2.8, 3\}$**,** this chart has its best performance when λ is selected equal to 0.1.

[Please insert Table 6 about here]

Table 7 gives the *ARL*s for joint intercept and slope shifts in the study profile. In this case, β_{01} and β_{11} change to $\beta_{01} + \tau\sigma_1$ and $\beta_{11} + \eta\sigma_1$, respectively where $\tau = -\eta\bar{x}; \bar{x} = 5$ for $\eta \in \{0.04, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18, 0.20\}$ which is equivalent to $\tau \in \{-0.2, -0.3, -0.4, -0.5, -0.6, -0.7, -0.8, -0.9, -1\}$. As shown in Table 7, using auxiliary one profile to construct the chart statistic causes both the MEWMA and DMEWMA control charts more sensitive to detect the concurrent shifts in intercept and slope parameters.

[Please insert Table 7 about here]

Table 8 displays the *ARL*s at different values of λ using $\rho = 0.5$ when both intercept and slope parameters deviate from their corresponding target values. It is remarkable that for all values of intercept parameter, using the auxiliary information has enhanced the performance of both MEWMA and DMEWMA charts in detection of concurrent shifts in intercept and slope parameters. Furthermore, under all scenarios, the AIB-MEWMA chart is performing better than the other charts.

[Please insert Table 8 about here]

4.2 Numerical example 2

In this subsection, the sensitivity of the proposed control charts in detecting different shifts in profile parameters is evaluated using two auxiliary profiles. To do that, we consider a multivariate linear profile as $Y_1 = 3 + 2x + \varepsilon_1$, $Y_2 = 2 + x + \varepsilon_2$, and $Y_3 = 1 + x + \varepsilon_3$ where Y_1 denotes the main response variables while Y_2 and Y_3 are the auxiliary response variables. Similar to numerical example 1, we use sample size of $n = 4$ and the fixed values of explanatory variable as $\mathbf{X} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 6 & 8 \end{bmatrix}$. Furthermore, it is assumed that the vector of random errors, $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$, follows a multivariate Normal distribution with mean vector $\mathbf{0}$ and covariance matrix $\mathbf{\Sigma} = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \rho\sigma_1\sigma_3 \\ \rho\sigma_2\sigma_1 & \sigma_2^2 & \rho\sigma_2\sigma_3 \\ \rho\sigma_3\sigma_1 & \rho\sigma_3\sigma_2 & \sigma_3^2 \end{bmatrix}$. Then, the random profiles are generated by considering $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 1$ and $\lambda \in \{0.05, 0.1, 0.2\}$. In order to evaluate the impact of correlation coefficient, we select two values of $\rho \in \{0.1, 0.5\}$. The results for different intercept shifts, when $\lambda = 0.2$ are summarized in Table 9. As it can be seen from Table 9, employing two auxiliary profiles enhances the sensitivity of both MEWMA and DMEWMA control charts to react to intercept shifts, especially for larger value of parameter ρ. For both values of parameter ρ, the detection performance of the AIB-DMEWMA chart is better than the AIB-MEWMA under small intercept shifts ($\delta_0 \in \{0.2, 0.4\}$). Conversely, under moderate and large intercept disturbances $\left(\delta_0 \in \{0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2\}\right)$, the AIB-MEWMA control chart outperforms the AIB-DMEWMA chart.

[Please insert Table 9 about here]

The *ARL* values for the proposed AIB-MEWMA and AIB-DMEWMA control charts at fixed $\rho = 0.5$ when $\lambda \in \{0.1, 0.2\}$ are provided in Table 10. We can see from the resulting *ARL*s that for small intercept shifts ($\delta_0 \in \{0.2, 0.4\}$), the *ARL* properties of the AIB-MEWMA chart is better when $\lambda = 0.1$, while for moderate and large shifts i.e., $\delta_0 \in \{0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2\}$, the chart performs better when $\lambda = 0.2$. It is also remarkable from Table 10 that under all out-of-control scenarios except $\delta_0 = 0.2$, the AIB-DMEWMA chart is more capable to detect intercept shifts when $\lambda = 0.2$.

[Please insert Table 10 about here]

The *ARL* values of the proposed AIB charts for different slope shifts when $\lambda = 0.2$ and $p = 3$ are summarized in Table 11. It is obvious from Table 11 that using two auxiliary profiles improves the capability of both MEWMA and DMEWMA charts to detect different step changes in slope parameter. For weak positive correlation ($\rho = 0.1$), the AIB-DMEWMA chart outperform other charts when $\delta_1 \in \{0.025, 0.050, 0.075\}$. In this scenario, the AIB-MEWMA performs better than the other charts when $\delta_1 \in \{0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250\}$. A similar trend can be observed when $\rho = 0.5$ expect that the AIB-MEWMA chart gives the smallest *ARL* when $\delta_1 = 0.075$.

[Please insert Table 11 about here]

The comparisons of all the charts using $\lambda \in \{0.05, 0.1, 0.2\}$ when $\rho = 0.5$ and $p = 3$ are provided in Table 12 and Figures 2-a, 2-b, 2-c and 3-a, 3-b, 3-c. From the findings of Table 12 and Figures 2-a, 2-b, and 2-c, it is concluded that for most values of δ_1, the proposed AIB-MEWMA chart has its best performance in detecting slope shifts when $\lambda = 0.2$. It can be clearly seen from Figures 3-a, 3-b, and 3-c that when $\lambda = 0.05$ and $\rho = 0.5$, the AIB-DMEWMA and DMEWMA charts produce almost the same *ARL* values. Moreover, it is remarkable from Table 12 that, for any values of smoothing parameter, the AIB-MEWMA is outperforming the classical MEWMA chart.

[Please insert Table 12 about here]

[Please insert Figure 2 about here]

[Please insert Figure 3 about here]

The resulting *ARL*s of proposed AIB charts are compared with existing MEWMA and DMEWMA charts under different error variance shifts in Table 13 at fixed value of $\lambda = 0.2$. The results of Table 13 show that for both values of correlation coefficient, the proposed AIB charts are more efficient in detecting error variance shifts than the existing MEWMA and DMEWMA charts. In general, the proposed AIB-MEWMA control chart is the best method among the other charts to detect error variance shifts.

[Please insert Table 13 about here]

The *ARL* values for AIB-MEWMA and AIB-DMEWMA charts with different values of λ when $\rho = 0.5$ and $p = 3$ are presented in Table 14. It can be seen from Table 14 that for all shifts except $\gamma = 1.2$, the AIB-MEWMA scheme gives its best *ARL*s when the smoothing parameter is selected as $\lambda = 0.2$. Moreover, it can be observed that for small and moderate shifts ($\gamma \in \{1.2, 1.4, 1.6\}$), the AIB-DMEWMA chart has its best performance at $\lambda = 0.05$ while this chart is more efficient to detect large shifts ($\gamma \in \{1.8, 2, 2.2, 2.4, 2.6, 2.8, 3\}$) when $\lambda = 0.2$.

[Please insert Table 14 about here]

The simulated *ARL* values under concurrent shifts in intercept and slope parameters when $\lambda = 0.2$ and $p = 3$ are presented in Table 15. The resulting *ARL*s show that the use of two auxiliary profiles considerably improves the efficiency of the proposed MEWMA and DMEWMA charts to detect concurrent shifts. Moreover, Table 15 indicates that the proposed AIB-MEWMA scheme is superior to other control charts when the out-of-control condition affects both intercept and slope parameters, concurrently.

[Please insert Table 15 about here]

The *ARL*s for concurrent shifts are given in Table 16 where the correlation coefficient is fixed at $\rho = 0.5$. It is remarkable from Table 16 that except $\tau = -0.2$, the AIB-MEWMA chart has its best performance at $\lambda = 0.2$. For AIB-DMEWMA chart, the best performance for $\tau \in \{-0.2, -0.3\}$ is obtained when smoothing parameter is selected as $\lambda = 0.1$ while for other out-of-control scenarios, this chart performs better when $\lambda = 0.2$.

[Please insert Table 16 about here]

5. Real-life example

In this section, the applicability of the proposed control charting method is investigated using a real-life example from calibration application in cylinder production process. In this manufacturing process which is also investigated by Noorossana et al. [28], the relationships between the nominal force (x) and the real forces y_1 and y_2 in two cylinders as study and auxiliary response variables are characterized by two simple linear profile models. According to Phase I analysis, theses regression models are:

$$\begin{cases} y_1 = -8.5 + 0.87x + \varepsilon_1 \\ y_2 = -5.8 + 0.95x + \varepsilon_2 \end{cases}, \tag{21}$$

where $(\varepsilon_1, \varepsilon_2)$ follows a bivariate normal vector of error terms with mean vector of zero and the following covariance matrix:

$$\mathbf{\Sigma} = \begin{bmatrix} 80.0 & 89.6 \\ 89.6 & 122.1 \end{bmatrix}. \tag{22}$$

The matrix of explanatory variable is $\mathbf{X} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 50 & 80 & 110 & 140 & 170 & 200 & 230 & 260 & 290 & 320 & 350 \end{bmatrix}$ and remains unchanged over time. In order to represent the detectability of the proposed auxiliary based monitoring scheme, out-of-control profiles under $\lambda = 0.2$ are generated in a way that the intercept parameter β_{01} of the study profile deviates from -8.5 to -7.5. The estimated regression parameters for study profile under OLS and MS methods along with the AIB-MEWMA chart are reported in Table 17. Moreover, the values of AIB-MEWMA charting scheme are illustrated in Figure 4. As it can be seen from Table 17, using auxiliary information leads to more accurate estimations of the regression parameters for the study profile. That is to say, the estimated regression parameters obtained by the mean estimator ($\hat{\boldsymbol{\beta}}_1^{MS}$) are more precise than the obtained estimations by the ordinary least square method ($\hat{\boldsymbol{\beta}}_1^{OLS}$). Moreover, it is remarkable from Figure 4 that the proposed AIB-MEWMA control charting method detects the occurrence of assignable cause at 13th sample.

[Please insert Table 17 about here]

[Please insert Figure 4 about here]

6. Conclusion Remarks

There are many strategies in the literature of SPM to improve the capability of control charts to detect different process disturbances. Examples include the use of double sampling strategies, use of adaptive sampling approaches, utilizing run rules mechanism, as well as the application of auxiliary information. Among them, using auxiliary information to formulate different chart statistics can considerably enhance the performance of control chart to react to out-of-control conditions. In this paper we proposed two auxiliary information based schemes termed as AIB-MEWMA and AIB-DMEWMA charts for Phase II monitoring of a simple linear profile. In terms of *ARL* metric, we compared the capability of the proposed charts to detect different out-of-control scenarios with existing MEWMA and DMEWMA charts. The resulting *ARL*s showed that the proposed AIB-MEWMA and AIB-DMEWMA monitoring schemes are more efficient than their comparative counterparts in detection of all regression parameters. Finally, the applicability of the proposed auxiliary based monitoring scheme was highlighted using a real-life example from calibration application. This paper developed two AIB control charts for monitoring simple linear profiles in non-cascading processes. Hence, the extension of the proposed control charts for cascading processes in which the assignable cause deviates both auxiliary and study profiles from their nominal values can be fruitful as a future direction. Moreover, the use of auxiliary information for Phase II monitoring of nonlinear profiles is recommended for future studies.

References

1. Mahmood, T., Abbasi, S. A., Riaz, M., et al. "An efficient Phase I analysis of linear profiles with application in photo-voltaic system", *Arabian Journal for Science and Engineering*, **44**(3), pp. 2699-2716 (2019)..

2. Haq, A. “Adaptive MEWMA charts for univariate and multivariate simple linear profiles”, *Communications in Statistics-Theory and Methods*, **51**(16), pp. 5383-5411 (2022).

3. Kazemzadeh, R. B., Amiri, A., and Kouhestani, B. “Monitoring simple linear profiles using variable sample size schemes”, *Journal of Statistical Computation and Simulation*, **86**(15), pp. 2923-2945 (2016).

4. Riaz, M., Mahmood, T., Abbasi, S. A., et al. “Linear profile monitoring using EWMA structure under ranked set schemes”, *The International Journal of Advanced Manufacturing Technology*, **91**(5), pp. 2751-2775 (2017).

5. Mahmood, T., Riaz, M., Omar, M. H., et al. “Alternative methods for the simultaneous monitoring of simple linear profile parameters”, *The International Journal of Advanced Manufacturing Technology*, **97**(5), pp. 2851-2871 (2018).

6. Riaz, M., Mahmood, T., Abbas, N., et al. “On improved monitoring of linear profiles under modified successive sampling”, *Quality and Reliability Engineering International*, **35**(7), pp. 2202-2227 (2019).

7. Chang, Y. C., and Chen, C. M. “A Kullback- Leibler information control chart for linear profiles monitoring”, *Quality and Reliability Engineering International*, **36**(7), pp. 2225-2248 (2020).

8. Haq, A., Bibi, M., and Shah, B. A. “A novel approach to monitor simple linear profiles using individual observations”, *Communications in Statistics-Simulation and Computation*, **51**(11), pp. 6269-6282 (2022).

9. Khalafi, S., Salmasnia, A., and Maleki, M. R. “Remedial approaches to decrease the effect of measurement errors on simple linear profile monitoring”, *International Journal for Quality Research*, **14**(4), pp. 1019-1036 (2020).

10. Yeganeh, A., Shadman, A., and Amiri, A. “A novel run rules based MEWMA scheme for monitoring general linear profiles”, *Computers & Industrial Engineering*, **152**, pp. 1-16 (2021).

11. Maleki, M. R., Amiri, A., and Castagliola, P. “An overview on recent profile monitoring papers (2008–2018) based on conceptual classification scheme”, *Computers & Industrial Engineering*, **126**, pp. 705-728 (2018).

12. Ahmad, S., Abbasi, S. A., Riaz, M., et al. “On efficient use of auxiliary information for control charting in SPC”, *Computers & Industrial Engineering*, **67**, pp. 173-184 (2014).

13. Hawkins, D. M. “Multivariate quality control based on regression-adjusted variables”, *Technometrics*, **33**(1), pp. 61-75 (1991).

14. Saleh, N. A., Mahmoud, M. A., Woodall, W. H., et al. “A review and critique of auxiliary information-based process monitoring methods”, *Quality Technology & Quantitative Management*, In Press (2023).

15. Riaz, M. “Monitoring process mean level using auxiliary information”, *Statistica neerlandica*, **62**(4), pp. 458-481 (2008).

16. Riaz, M. “Monitoring process variability using auxiliary information” *Computational statistics*, **23**(2), pp. 253-276 (2008).

17. Abbas, Z., Nazir, H. Z., Abbasi, S. A., et al. “On designing efficient memory-type charts using multiple auxiliary-information”, *Journal of Statistical Computation and Simulation*, In Press, 10.1080/00949655.2022.2116747 (2023).

18. Chiang, J. Y., Tsai, T. R., Pham, H., et al. “A new multivariate control chart for monitoring the quality of a process with the aid of auxiliary information”, *Journal of Statistical Computation and Simulation*, **92**(3), pp. 645-666 (2022).

19. Adegoke, N. A., Riaz, M., Sanusi, R. A., et al. “EWMA-type scheme for monitoring location parameter using auxiliary information”, *Computers & Industrial Engineering*, **114**, pp. 114-129 (2017).

20. Arshad, W., Abbas, N., Riaz, M., et al. “Simultaneous use of runs rules and auxiliary information with exponentially weighted moving average control charts”, *Quality and Reliability Engineering International*, **33**(2), pp. 323-336 (2017).

21. Haq, A., and Khoo, M. B. “A new double sampling control chart for monitoring process mean using auxiliary information”, *Journal of Statistical Computation and Simulation*, **88**(5), pp. 869-899 (2018).

22. Sanusi, R. A., Abbas, N., and Riaz, M. “On efficient CUSUM-type location control charts using auxiliary information”, *Quality Technology & Quantitative Management*, **15**(1), pp. 87-105 (2018).

23. Noor-ul-Amin, M., Khan, S., and Sanaullah, A. “HEWMA control chart using auxiliary information”, *Iranian Journal of Science and Technology, Transactions A: Science*, **43**(3), pp. 891-903 (2019).

24. Saha, S., Khoo, M. B., Lee, M. H., et al. “A variable sample size and sampling interval control chart for monitoring the process mean using auxiliary information”, *Quality Technology & Quantitative Management*, **16**(4), pp. 389-406 (2019).

25. Saghir, A., Ahmad, L., Aslam, M., et al. “A EWMA control chart based on an auxiliary variable and repetitive sampling for monitoring process location”, *Communications in Statistics-Simulation and Computation*, **48**(7), pp. 2034-2045 (2019).

26. Noor-ul-Amin, M., Javaid, A., Hanif, M., et al. “Performance of maximum EWMA control chart in the presence of measurement error using auxiliary information”, *Communications in Statistics-Simulation and Computation*, **51**(9), pp. 5482-5506 (2022).

27. Anwar, S. M., Aslam, M., Riaz, M., et al. “On mixed memory control charts based on auxiliary information for efficient process monitoring”, *Quality and Reliability Engineering International*, **36**(6), pp. 1949-1968, (2020).

28. Noorossana, R., Eyvazian, M., and Vaghefi, A. “Phase II monitoring of multivariate simple linear profiles”, *Computers & Industrial Engineering*, **58**(4), pp. 563-570 (2010).

29. Zou, C., Tsung, F., and Wang, Z. “Monitoring general linear profiles using multivariate exponentially weighted moving average schemes”, *Technometrics*, **49**(4), pp. 395-408 (2007).

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Figures

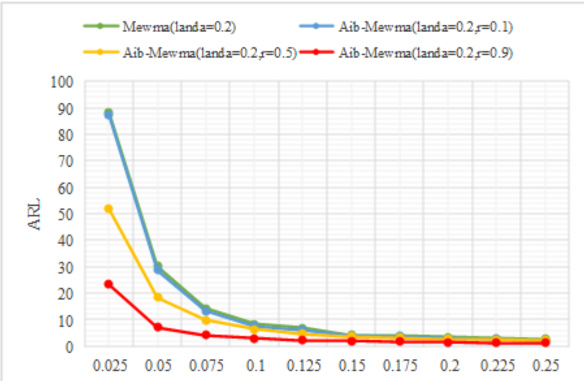

Figure 1-a. Comparison of AIB-MEWMA chart with MEWMA chart under different ρ when $\lambda = 0.2$

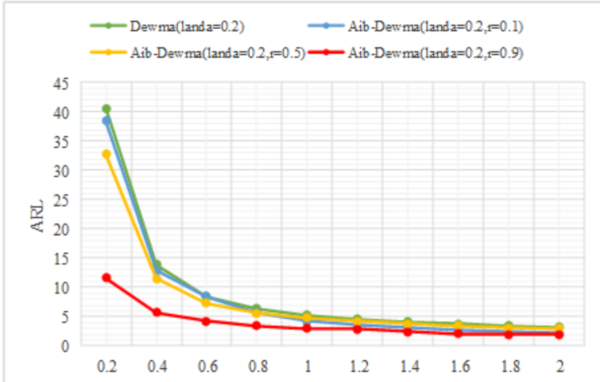

Figure 1-b. Comparison of AIB-DMEWMA chart with DMEWMA chart under different ρ when $\lambda = 0.2$

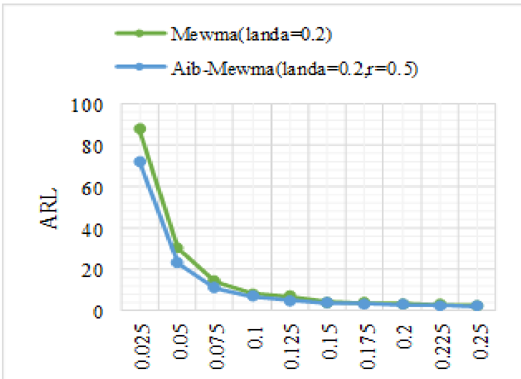

Figure 2-a. Comparison between AIB-MEWMA and MEWMA charts when $\rho = 0.5$ and $\lambda = 0.2$

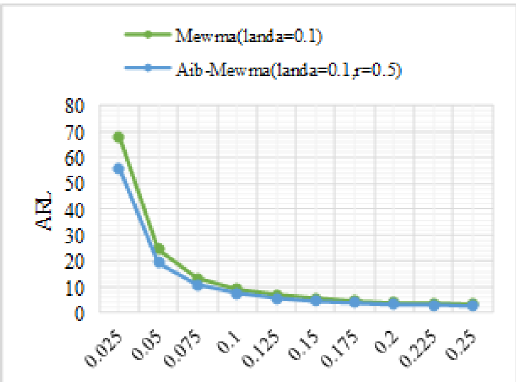

Figure 2-b. Comparison between AIB-MEWMA and MEWMA charts when $\rho = 0.5$ and $\lambda = 0.1$

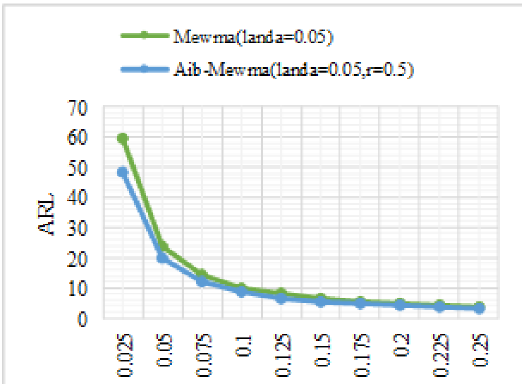

Figure 2-c. Comparison between AIB-MEWMA and MEWMA charts when $\rho = 0.5$ and $\lambda = 0.05$

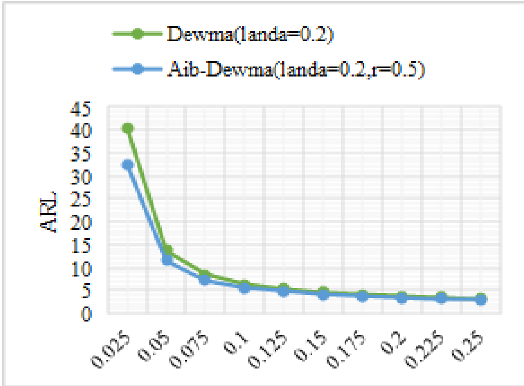

Figure 3-a. Comparison between AIB-DMEWMA and DMEWMA charts when $\rho = 0.5$ and $\lambda = 0.2$

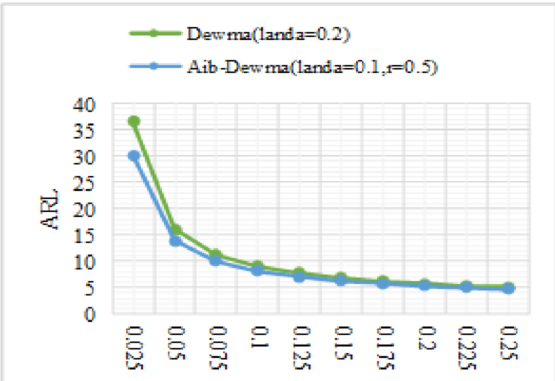

Figure 3-b. Comparison between AIB-DMEWMA and DMEWMA charts when $\rho = 0.5$ and $\lambda = 0.1$

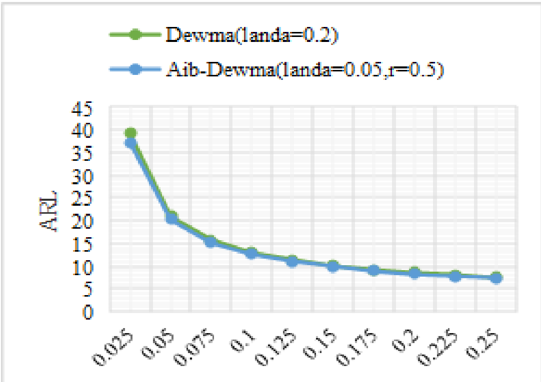

Figure 3-c. Comparison between AIB-DMEWMA and DMEWMA charts when $\rho = 0.5$ and $\lambda = 0.05$

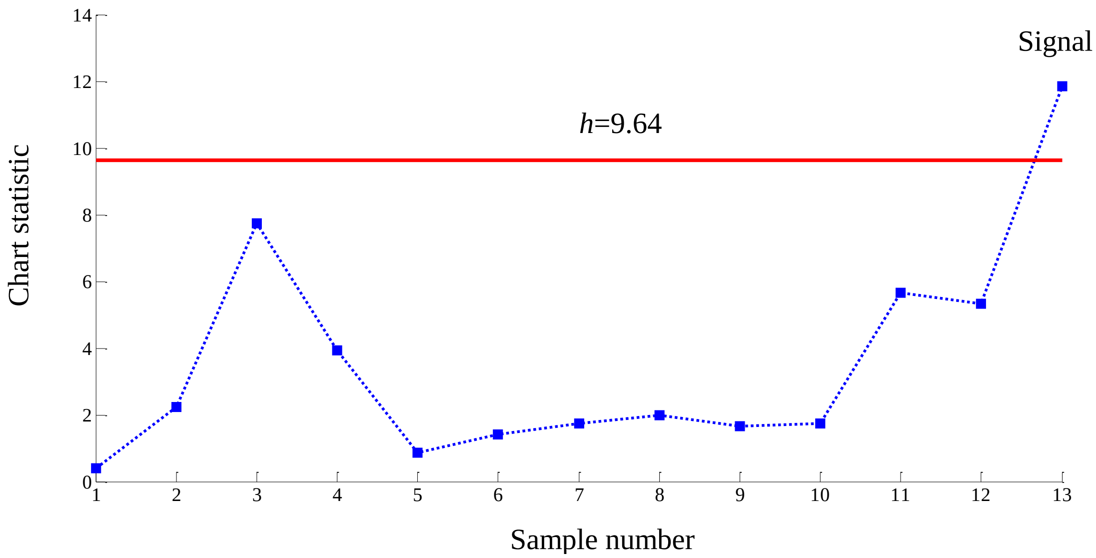

Figure 4. The values of chart statistic over time

Tables

Table 1. The *ARL*s under intercept shifts when $\lambda = 0.2$ and $p = 2$

Chart	ρ	δ_0									
		0.2	**0.4**	**0.6**	**0.8**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
MEWMA	-	51.8611	15.0036	7.5868	5.0196	3.7722	3.0705	5.5933	2.2797	2.0743	1.9139
DMEWMA		40.5407	13.8470	8.4651	6.3859	5.3150	4.6364	4.1660	3.7998	3.4878	3.2348
AIB-MEWMA	0.1	50.2950	13.9295	6.6305	4.0958	3.3610	2.8354	2.1830	2.0687	1.9541	1.6008
AIB-DMEWMA		38.4350	12.8760	8.3904	5.7642	4.2880	3.6039	3.1332	2.7922	2.4779	2.2237
AIB-MEWMA	0.5	40.4482	11.5933	6.1281	4.1920	3.1787	2.6177	2.2567	2.0189	1.8505	1.6811
AIB-DMEWMA		32.7401	11.4524	7.2623	5.6533	4.7665	4.1922	3.7775	3.4174	3.1582	3.0242
AIB-MEWMA	0.9	11.8259	4.1902	2.6410	2.0352	1.7132	1.3490	1.0898	1.0130	1.0012	1.0000
AIB-DMEWMA		11.6584	5.6756	4.2116	3.4376	3.0330	2.8491	2.4010	2.0735	2.0024	2.0000

Table 2. The *ARL*s under intercept shifts when $\rho = 0.5$ and $p = 2$

Chart	λ	δ_0									
		0.2	**0.4**	**0.6**	**0.8**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
MEWMA	0.2	51.8611	15.0036	7.5868	5.0196	3.7722	3.0705	5.5933	2.2797	2.0743	1.9139
AIB-MEWMA		40.4482	11.5933	6.1281	4.1920	3.1787	2.6177	2.2567	2.0189	1.8505	1.6811
DMEWMA		40.5407	13.8470	8.4651	6.3859	5.3150	4.6364	4.1660	3.7998	3.4878	3.2348
AIB-DMEWMA		32.7401	11.4524	7.2623	5.6533	4.7665	4.1922	3.7775	3.4174	3.1582	3.0242
MEWMA	0.1	39.4317	13.8773	7.9441	5.6635	4.4020	3.6545	3.1227	2.7555	2.4577	2.2360
AIB-MEWMA		31.7907	11.2523	6.6777	4.8067	3.8111	3.1582	2.7196	2.4004	2.1670	2.0274
DMEWMA		37.6118	16.1345	11.2632	9.0938	7.8012	6.9305	6.2859	5.7791	5.3632	5.0601
AIB-DMEWMA		30.5907	14.1701	10.1069	8.2550	7.1018	6.3337	5.7276	5.2791	4.9626	4.6622

MEWMA	0.05	36.2468	14.7987	9.1552	6.6655	5.2983	4.4196	3.8007	3.3570	3.0309	2.7657
AIB-MEWMA		29.6745	12.3119	7.8195	5.7330	4.5785	3.8293	3.3242	2.9573	2.6644	2.3934
DMEWMA		38.9977	21.0267	15.7422	13.7422	11.3704	10.1880	9.2849	8.5870	8.0271	7.5435
AIB-DMEWMA		33.6974	18.8983	14.3203	11.9525	10.4199	9.3613	8.5471	7.9091	7.3798	6.9889

Table 3. The ARLs under slope shifts when $\lambda = 0.2$ and $p = 2$

Chart	ρ	δ_1									
		0.025	**0.050**	**0.075**	**0.100**	**0.125**	**0.150**	**0.175**	**0.200**	**0.225**	**0.250**
MEWMA	-	88.1952	30.2681	14.1843	8.2841	6.9371	4.1952	3.9416	3.5620	2.9954	2.6860
DMEWMA		40.5407	13.8470	8.4651	6.3859	5.3156	4.6364	4.1660	3.7998	3.4878	3.2348
AIB-MEWMA	0.1	87.2924	28.7986	13.3095	7.6701	6.0391	3.8420	3.7556	3.0648	2.6717	2.1437
AIB-DMEWMA		38.4354	12.8769	8.3904	5.7642	4.2880	3.6039	3.1332	2.7922	2.4779	2.2237
AIB-MEWMA	0.5	51.7830	18.5411	9.8134	6.3408	4.7355	3.7967	3.2127	2.7859	2.4710	2.2477
AIB-DMEWMA		38.3939	19.8463	11.0933	7.9775	6.4411	5.5552	4.9163	4.4706	4.1112	3.8243
AIB-MEWMA	0.9	23.4328	7.0869	4.1082	2.8993	2.3129	1.9909	1.7638	1.5171	1.2733	1.1146
AIB-DMEWMA		19.8824	8.0174	5.5755	4.4994	3.8569	3.3672	3.0772	2.9544	2.7965	2.4812

Table 4. The ARLs under slope shifts when $\rho = 0.5$ and $p = 2$

Chart	λ	δ_1									
		0.025	**0.050**	**0.075**	**0.100**	**0.125**	**0.150**	**0.175**	**0.200**	**0.225**	**0.250**
MEWMA	0.2	88.1952	30.2681	14.1843	8.2841	6.9371	4.1952	3.9416	3.5620	2.9954	2.6860
AIB-MEWMA		51.7830	18.5411	9.8134	6.3408	4.7355	3.7967	3.2127	2.7859	2.4710	2.2477
DMEWMA		40.5407	13.8470	8.4651	6.3859	5.3156	4.6364	4.1660	3.7998	3.4878	3.2348
AIB-DMEWMA		38.3939	19.8463	11.0933	7.9775	6.4411	5.5552	4.9163	4.4706	4.1112	3.8243
MEWMA	0.1	68.0359	24.5624	13.3661	9.0515	6.8182	5.4921	4.6306	4.0206	3.5475	3.1944
AIB-MEWMA		56.6286	19.5035	10.8020	7.4785	5.7464	4.7065	3.9420	3.4507	3.0791	2.7803
DMEWMA		62.3517	24.8762	15.7732	12.1473	10.2065	8.9650	8.0397	7.3681	6.8116	6.3778
AIB-DMEWMA		50.6476	21.0620	13.7553	10.8558	9.1840	8.1143	7.3110	6.7064	6.2175	5.8156
MEWMA	0.05	59.2877	23.8185	14.2177	10.0931	7.9013	6.4926	5.5245	4.8138	4.3067	3.8937
AIB-MEWMA		49.3989	19.8830	12.0447	8.6159	6.7534	5.5997	4.7955	4.1764	3.7277	3.3889
DMEWMA		58.6103	28.5375	20.5820	16.7697	14.4606	12.8500	11.6652	10.7640	10.0165	9.4185
AIB-DMEWMA		50.0982	25.4254	18.5449	15.2501	13.1700	11.7245	10.7044	9.8577	9.2049	8.6467

Table 5. The ARLs under error variance shifts when $\lambda = 0.2$ and $p = 2$

Chart	ρ	γ									
		1.2	**1.4**	**1.6**	**1.8**	**2**	**2.2**	**2.4**	**2.6**	**2.8**	**3**
MEWMA	-	55.6428	26.5276	8.2618	3.2607	2.4955	2.6114	1.5530	1.2590	1.0953	1.0006
DMEWMA		78.2846	29.5940	15.0587	10.8951	8.6248	6.2517	5.2154	4.0215	3.8621	3.2519
AIB-MEWMA	0.1	55.4479	13.1039	3.9026	1.7047	1.1490	1.0288	1.0051	1.0001	1.0000	1.0000
AIB-DMEWMA		73.9130	27.4157	13.2862	8.0955	5.5660	4.1206	3.2754	2.6935	2.2576	1.9119

AIB-MEWMA	0.5	54.3852	11.9063	3.5395	1.5707	1.1154	1.0247	1.0045	1.0006	1.0002	1.0000
AIB-DMEWMA		72.7354	25.0300	12.1063	7.3362	4.9729	3.7952	2.9754	2.4478	2.0368	1.7467
AIB-MEWMA	0.9	40.5800	6.1379	1.8815	1.1708	1.0369	1.0082	1.0003	1.0002	1.0001	1.0000
AIB-DMEWMA		58.1834	15.2844	7.0254	4.2109	2.9879	2.2814	1.7788	1.4936	1.3153	1.2165

Table 6. The *ARL*s under error variance shifts when $\rho = 0.5$ and $p = 2$

Chart	λ	γ									
		1.2	**1.4**	**1.6**	**1.8**	**2**	**2.2**	**2.4**	**2.6**	**2.8**	**3**
MEWMA	0.2	55.6428	26.5276	8.2618	3.2607	2.4955	2.6114	1.5530	1.2590	1.0953	1.0006
AIB-MEWMA		54.3852	11.9063	3.5395	1.5707	1.1154	1.0247	1.0045	1.0006	1.0002	1.0000
DMEWMA		78.2846	29.5940	15.0587	10.8951	8.6248	6.2517	5.2154	4.0215	3.8621	3.2519
AIB-DMEWMA		72.7354	25.0300	12.1063	7.3362	4.9729	3.7952	2.9754	2.4478	2.0368	1.7467
MEWMA	0.1	67.2527	21.4597	9.0337	4.8442	2.9790	2.0811	1.6408	1.3919	1.2686	1.1744
AIB-MEWMA		66.0123	16.7042	5.2745	1.9991	1.2062	1.0425	1.0079	1.0009	1.0000	1.0000
DMEWMA		93.8932	37.1114	16.6010	8.2932	4.4366	2.3899	1.3168	1.0426	1.0068	1.0005
AIB-DMEWMA		93.2361	35.3347	14.8163	8.0371	4.2771	2.0482	1.2694	1.0073	1.0006	1.0000
MEWMA	0.05	81.3496	30.1615	13.3189	7.1206	4.2466	2.8344	2.0842	1.6824	1.4297	1.2985
AIB-MEWMA		80.0814	23.7323	7.7249	2.7748	1.3628	1.0653	1.0117	1.0015	1.0002	1.0000
DMEWMA		113.0677	54.1134	27.0204	14.4849	7.8450	4.2883	2.2986	1.2605	1.0310	1.0028
AIB-DMEWMA		99.2389	48.7780	20.5469	10.2801	6.2970	3.5139	2.0546	1.2473	1.0060	1.0000

Table 7. The *ARL*s under concurrent intercept and slope shifts when $\lambda = 0.2$ and $p = 2$

Chart	ρ	τ								
		-0.2	**-0.3**	**-0.4**	**-0.5**	**-0.6**	**-0.7**	**-0.8**	**-0.9**	**-1.0**
MEWMA	-	12.2850	6.3461	4.3191	3.2945	2.7046	2.3168	2.0732	1.8931	1.7428
DMEWMA		23.9336	14.4961	10.8202	9.8782	8.2883	7.8619	6.4973	5.7160	5.0638
AIB-MEWMA	0.1	11.9013	3.9655	1.7090	1.1491	1.0288	1.0065	1.0002	1.0001	1.0000
AIB-DMEWMA		11.1899	7.2721	5.1772	4.1650	3.9740	3.1477	2.9908	2.6137	2.0575
AIB-MEWMA	0.5	6.3028	2.9111	1.5123	1.1160	1.0190	1.0045	1.0003	1.0000	1.0000
AIB-DMEWMA		27.1399	10.0008	6.5598	5.1926	4.4004	3.8797	3.4793	3.1772	3.0228
AIB-MEWMA	0.9	3.1373	1.6350	1.1500	1.0330	1.0066	1.0010	1.0002	1.0000	1.0000
AIB-DMEWMA		5.2065	3.9081	3.1855	2.9400	2.5490	2.1000	2.0038	2.0000	2.0000

Table 8. The *ARL*s under concurrent intercept and slope shifts when $\rho = 0.5$ and $p = 2$

Chart	λ	τ								
		-0.2	**-0.3**	**-0.4**	**-0.5**	**-0.6**	**-0.7**	**-0.8**	**-0.9**	**-1.0**
MEWMA	0.2	12.2850	6.3461	4.3191	3.2945	2.7046	2.3168	2.0732	1.8931	1.7428
AIB-MEWMA		6.3028	2.9111	1.5123	1.1160	1.0190	1.0045	1.0003	1.0000	1.0000
DMEWMA		23.9336	14.4961	10.8202	9.8782	8.2883	7.8619	6.4973	5.7160	5.0638
AIB-DMEWMA		27.1399	10.0008	6.5598	5.1926	4.4004	3.8797	3.4793	3.1772	3.0228
MEWMA	0.1	11..9068	6.9299	4.9997	3.9357	3.2513	2.8044	2.4759	2.2187	2.0662
AIB-MEWMA		7.8180	3.9702	1.9095	1.2070	1.0402	1.0067	1.0008	1.0000	1.0000

DMEWMA		31.7659	14.5502	10.3672	8.4283	7.2588	6.4452	5.8656	5.3953	5.0442
AIB-DMEWMA		26.5557	12.7655	9.3040	7.6383	6.6077	5.8892	5.3501	4.9745	4.6492
MEWMA	0.05	12.8453	8.0657	5.9393	4.7333	3.9551	3.4379	3.0541	2.7418	2.4692
AIB-MEWMA		9.5676	5.2839	2.5459	1.3507	1.0625	1.0112	1.0012	1.0001	1.0001
DMEWMA		19.2661	14.6134	12.1811	10.6226	9.5313	8.7100	8.0647	7.5309	7.0961
AIB-DMEWMA		17.4873	13.3319	11.1427	9.7450	8.7544	8.0140	7.4149	6.9630	6.5557

Table 9. The *ARL*s under intercept shifts when $\lambda = 0.2$ and $p = 3$

Chart	ρ	δ_0									
		0.2	**0.4**	**0.6**	**0.8**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
MEWMA	-	51.8611	15.0036	7.5868	5.0196	3.7722	3.0705	2.5933	2.2797	2.0743	1.9139
DMEWMA		40.5407	13.8470	8.4651	6.3859	5.3156	4.6364	4.1660	3.7998	3.4878	3.2348
AIB-MEWMA	0.1	50.4609	14.6952	6.5194	4.9527	3.2444	3.0289	2.1584	2.0514	1.9526	1.8943
AIB-DMEWMA		39.4576	12.6451	7.9214	5.8464	5.1735	4.0877	3.9291	3.5758	3.1791	2.8189
AIB-MEWMA	0.5	40.3003	11.6925	6.1937	4.1760	3.1827	2.6166	2.2517	2.0180	1.8489	1.6757
AIB-DMEWMA		32.5562	11.5178	7.2903	5.6579	4.7629	4.1906	3.7832	3.4200	3.1624	3.0282

Table 10. The *ARL*s under intercept shifts when $\rho = 0.5$ and $p = 3$

Chart	λ	δ_0									
		0.2	**0.4**	**0.6**	**0.8**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
MEWMA	0.2	51.8611	15.0036	7.5868	5.0196	3.7722	3.0705	5.5933	2.2797	2.0743	1.9139
AIB-MEWMA		40.4482	11.5933	6.1281	4.1920	3.1787	2.6177	2.2567	2.0189	1.8505	1.6811
DMEWMA		40.5407	13.8470	8.4651	6.3859	5.3150	4.6364	4.1660	3.7998	3.4878	3.2348
AIB-DMEWMA		32.7401	11.4524	7.2623	5.6533	4.7665	4.1922	3.7775	3.4174	3.1582	3.0242
MEWMA	0.1	39.4317	13.8773	7.9441	5.6635	4.4020	3.6545	3.1227	2.7555	2.4577	2.2360
AIB-MEWMA		31.8987	11.2063	6.6637	4.7730	3.7800	3.1389	2.7198	2.3989	2.1614	2.0321
DMEWMA		36.5818	16.1331	11.2977	9.0796	7.7785	6.9070	6.2603	5.7565	5.3523	5.0488
AIB-DMEWMA		30.1377	13.8106	10.0100	8.1312	7.0148	6.2514	5.6715	5.2305	4.9204	4.5993

Table 11. The *ARL*s under slope shifts when $\lambda = 0.2$ and $p = 3$

Chart	ρ	δ_1									
		0.025	**0.050**	**0.075**	**0.100**	**0.125**	**0.150**	**0.175**	**0.200**	**0.225**	**0.250**
MEWMA	-	88.1952	30.2681	14.1843	8.2841	6.9371	4.1952	3.9416	3.5620	2.9954	2.6860
DMEWMA		71.4636	24.6512	13.5171	9.3227	7.3471	6.2382	5.4749	4.9457	4.5271	4.2221
AIB-MEWMA	0.1	87.1491	29.6465	13.1956	8.0814	6.1567	3.7690	3.5148	3.0580	2.7391	2.3213
AIB-DMEWMA		70.6009	23.5837	12.2151	9.1785	6.8990	6.1630	4.4679	3.9118	3.5155	3.1859
AIB-MEWMA	0.5	72.2077	23.0599	10.9909	7.0133	5.0830	4.0351	3.3492	2.8854	2.5385	2.3119
AIB-DMEWMA		70.0136	19.6987	11.0463	7.9692	6.4490	5.5268	4.9136	4.4695	4.1261	3.8287

Table 12. The ARLs under slope shifts when $\rho = 0.5$ and $p = 3$

Chart	λ	δ_1									
		0.025	**0.050**	**0.075**	**0.100**	**0.125**	**0.150**	**0.175**	**0.200**	**0.225**	**0.250**
MEWMA	0.2	88.1952	30.2681	14.1843	8.2841	6.9371	4.1952	3.9416	3.5620	2.9954	2.6860
AIB-MEWMA		72.2077	23.0599	10.9909	7.0133	5.0830	4.0351	3.3492	2.8854	2.5385	2.3119
DMEWMA		71.4636	24.6512	13.5171	9.3227	7.3471	6.2382	5.4749	4.9457	4.5271	4.2221
AIB-DMEWMA		70.0136	19.6987	11.0463	7.9692	6.4490	5.5268	4.9136	4.4695	4.1261	3.8287
MEWMA	0.1	68.0359	24.5624	13.3661	9.0515	6.8182	5.4921	4.6306	4.0206	3.5475	3.1944
AIB-MEWMA		55.7646	19.4204	10.7769	7.4372	5.6846	4.6368	3.9399	3.4392	3.0671	2.7727
DMEWMA		71.4636	24.6512	13.5171	9.3227	7.3471	6.2382	5.4749	4.9457	4.5271	4.2221
AIB-DMEWMA		70.6009	23.5837	12.2151	9.1785	6.8990	6.1630	4.4679	3.9118	3.5155	3.1859
MEWMA	0.05	59.2870	23.8185	14.2177	10.0931	7.9013	6.4926	5.5245	4.8138	4.3067	3.8937
AIB-MEWMA		48.1330	19.6420	11.9010	8.5094	6.7163	5.5675	4.7462	4.1654	3.7284	3.3770
DMEWMA		58.0698	28.7048	20.5890	16.7910	14.4200	12.8641	11.6846	10.7626	10.0155	9.4191
AIB-DMEWMA		57.1110	28.2997	20.5144	16.6370	14.3781	12.7739	11.0223	9.0309	7.6694	6.5304

Table 13. The ARLs under error variance shifts when $\lambda = 0.2$ and $p = 3$

Chart	ρ	γ									
		1.2	**1.4**	**1.6**	**1.8**	**2**	**2.2**	**2.4**	**2.6**	**2.8**	**3**
MEWMA	-	56.6934	15.0643	6.1197	3.2700	2.1237	1.5886	1.3573	1.2175	1.1389	1.0968
DMEWMA		78.2846	29.5940	15.0587	10.8951	8.6248	6.2517	5.2154	4.0215	3.8621	3.2519
AIB-MEWMA	0.1	42.1731	12.0000	6.1048	3.1037	2.0822	1.6811	1.3037	1.0625	1.0006	1.0000
AIB-DMEWMA		77.8553	31.2851	14.0211	7.9909	5.3160	3.9141	3.1024	2.4915	2.0867	1.7873
AIB-MEWMA	0.5	52.3642	15.5103	4.0742	1.6455	1.1388	1.0244	1.0045	1.0006	1.0000	1.0000
AIB-DMEWMA		73.1670	26.2766	13.0251	7.9426	5.4772	4.1301	3.2963	2.6947	2.2404	1.9289

Table 14. The ARLs under error variance shifts when $\rho = 0.5$ and $p = 3$

Chart	λ	γ									
		1.2	**1.4**	**1.6**	**1.8**	**2**	**2.2**	**2.4**	**2.6**	**2.8**	**3**
MEWMA	0.2	56.6934	15.0643	6.1197	3.2700	2.1237	1.5886	1.3573	1.2175	1.1389	1.0968
AIB-MEWMA		52.3642	15.5103	4.0742	1.6455	1.1388	1.0244	1.0045	1.0006	1.0000	1.0000
DMEWMA		78.2846	29.5940	15.0587	10.8951	8.6248	6.2517	5.2154	4.0215	3.8621	3.2519
AIB-DMEWMA		73.1670	26.2766	13.0251	7.9426	5.4772	4.1301	3.2963	2.6947	2.2404	1.9289
MEWMA	0.1	67.2527	21.4597	9.0337	4.8442	2.9790	2.0811	1.6408	1.3919	1.2686	1.1744
AIB-MEWMA		42.7494	21.4684	6.0351	2.1412	1.2162	1.0384	1.0062	1.0011	1.0001	1.0000
DMEWMA		112.0510	48.5924	25.9485	16.5249	10.2617	8.4168	6.8521	5.3658	4.8594	4.2648
AIB-DMEWMA		104.9965	45.8931	23.1323	14.1126	9.6987	7.2001	5.7276	4.6980	3.9459	3.3774
MEWMA	0.05	81.3496	30.1615	13.3189	7.1206	4.2466	2.8344	2.0842	1.6824	1.4297	1.2985
AIB-MEWMA		48.6811	24.8090	8.9045	2.9305	1.3851	1.0664	1.0116	1.0013	1.0003	1.0001
DMEWMA		38.6207	24.2951	18.2681	12.3118	10.1827	7.9426	7.0018	6.7429	4.8420	4.2486
AIB-DMEWMA		18.7219	13.7604	10.4628	8.5206	7.1176	6.2066	5.5398	5.0377	4.6290	4.2183

Table 15. The ARLs under concurrent intercept and slope shifts when $\lambda = 0.2$ and $p = 3$

Chart	ρ	τ								
		-0.2	**-0.3**	**-0.4**	**-0.5**	**-0.6**	**-0.7**	**-0.8**	**-0.9**	**-1.0**
MEWMA	-	12.2850	6.3461	4.3191	3.2945	2.7046	2.3168	2.0732	1.8931	1.7428
DMEWMA		23.9336	14.4961	10.8202	9.8782	8.2883	7.8619	6.4973	5.7160	5.0638
AIB-MEWMA	0.1	12.0916	3.9912	1.7188	1.1500	1.0300	1.0049	1.0006	1.0000	1.0000
AIB-DMEWMA		23.8239	10.6696	6.9934	5.3145	4.2468	3.4988	2.9245	2.5037	2.1353
AIB-MEWMA	0.5	9.6727	5.2101	3.6438	2.8216	2.3522	2.0568	1.8529	1.6815	1.4714
AIB-DMEWMA		23.4158	9.9373	6.5872	5.1709	4.4016	3.8853	3.4795	3.1707	3.0218

Table 16. The ARLs under concurrent intercept and slope shifts when $\rho = 0.5$ and $p = 3$

Chart	λ	τ								
		-0.2	**-0.3**	**-0.4**	**-0.5**	**-0.6**	**-0.7**	**-0.8**	**-0.9**	**-1.0**
MEWMA	0.2	12.2850	6.3461	4.3191	3.2945	2.7046	2.3168	2.0732	1.8931	1.7428
AIB-MEWMA		9.6727	5.2101	3.6438	2.8216	2.3522	2.0568	1.8529	1.6815	1.4714
DMEWMA		23.9336	14.4961	10.8202	9.8782	8.2883	7.8619	6.4973	5.7160	5.0638
AIB-DMEWMA		23.4158	9.9373	6.5872	5.1709	4.4016	3.8853	3.4795	3.1707	3.0218
MEWMA	0.1	11.9068	6.9299	4.9997	3.9357	3.2513	2.8044	2.4759	2.2187	2.0662
AIB-MEWMA		9.6552	5.8615	4.2513	3.3740	2.8309	2.4295	2.1763	2.0260	1.9361
DMEWMA		12.5071	9.1420	7.4993	6.4868	5.7819	5.2763	4.8904	4.5225	4.2114
AIB-DMEWMA		11.9827	8.8360	7.2474	6.2787	5.6050	5.1157	4.7510	4.3647	4.0888
MEWMA	0.05	12.8453	8.0657	5.9393	4.7333	3.9551	3.4379	3.0541	2.7418	2.4692
AIB-MEWMA		10.7364	6.8702	5.0941	4.0925	3.4339	3.0080	2.6725	2.3575	2.1431
DMEWMA		19.3848	14.6285	12.1726	10.6240	9.5217	8.7004	8.0631	7.5172	7.0897
AIB-DMEWMA		18.6184	14.1532	11.8106	10.3140	9.2461	8.4650	7.8386	7.2880	6.9398

Table 17. Estimated regression parameters of study profile and the values of AIB-MEWMA chart statistic

Profile	$\hat{\boldsymbol{\beta}}_1^{OLS}$	$\hat{\boldsymbol{\beta}}_1^{MS}$	$AIB-MEWMA$
1	$(-6.544, 0.856)^T$	$(-7.368, 0.870)^T$	0.4004
2	$(-16.474, 0.898)^T$	$(-12.856, 0.895)^T$	2.2215
3	$(-14.466, 0.921)^T$	$(-7.409, 0.880)^T$	7.7433
4	$(-0.848, 0.823)^T$	$(-3.553, 0.846)^T$	3.9207
5	$(-10.680, 0.863)^T$	$(-8.640, 0.864)^T$	0.8767
6	$(-0.028, 0.829)^T$	$(-6.615, 0.851)^T$	1.4126
7	$(-6.877, 0.878)^T$	$(-6.831, 0.862)^T$	1.7451
8	$(-2.846, 0.844)^T$	$(-6.473, 0.863)^T$	1.9872
9	$(3.742, 0.794)^T$	$(-10.575, 0.871)^T$	1.6458

10	$(-16.858, 0.887)^T$	$(-6.617, 0.862)^T$	1.7281
11	$(-0.207, 0.855)^T$	$(-4.130, 0.873)^T$	5.6607
12	$(-12.108, 0.893)^T$	$(-9.587, 0.880)^T$	5.3182
13	$(-13.034, 0.895)^T$	$(-3.1080, 0.857)^T$	11.8487