An integrated group entropy-weighted interval type-2 fuzzy weighted aggregated sum product assessment (WASPAS) method in maritime transportation

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Abstract

This study aims to provide an integrated decision-making approach in maritime transportation problems. The duty of hatch cover is to barricade the entrance of water into the load container and insulate the material from being hurt. Hence, it has the considerable influence in efficiency of maritime transportation systems. Since each hatch cover has distinguished properties with respect to criteria than the others, the hatch cover evaluation problem (HCEP) can be considered as a multi-criteria decision-making (MCDM) problem. In this paper, interval type-2 fuzzy sets (IT2FSs) are first used to weight criteria and evaluations of hatch covers with respect to criteria. In addition, an integrated group Shannon entropy- based weighted aggregated sum product assessment (WASPAS) approach is applied to solve the HCEP using the limit distance mean (*LDM*) in which the interval type-2 fuzzy (IT2F) Shannon entropy approach is used to determine the objective weights and then they are integrated with the subjective weights. On the other hand, in order to demonstrate the effectiveness and practicability of the proposed method, it is fulfilled in an illustrative example and the ranked orders are analyzed with the others.

Keywords: Maritime transportation; Hatch cover evaluation problem; Shannon entropy method; WASPAS; Interval type-2 fuzzy sets

1. Introduction

According to the published documentations, world seaborne trade has a growth in 2017, with volumes expanding at 4 percent, the fastest growth in five years [1]. Obviously, this volume of world seaborne trade has increased maritime transportation costs. The various transportation costs can affect maritime transportation equipment. The section of such costs is depended to entrance of water into the load container, the frugality of material handling cost due to the use of below space of deck, and/or losses due to maritime accidents. Hatch covers have been designing for preventing the problems described above. Hatch cover is a mechanical device allowing hatches to be opened and closed.

The selection of unsuitable hatch cover can be caused more costs such as rearranging costs, purchasing of new hatch cover, etc. The different types of hatch covers (for example, folding, side-rolling, and lifting hatch cover, etc.) have been designed for handling goods among different distances. Each model has unique characterizations than the others such that study of the hatch cover evaluation problem (HCEP) has converted into difficult issue. Therefore, this problem can be taken into account as a multiple criteria decision-making (MCDM) problem. It can be stated as the methodology of selecting the suitable option among all proposed options with respect to a number of different criteria. In the literature, most researches have been carried out regarding lightening weight, type of consuming materials, risk assessment, watertight integrity, and strength of hatch covers. Based on our studies, there is only one study to evaluate of hatch covers using MCDM techniques. Unfortunately, it has many limitations, as explained in Section 2. There are many differences in the ranked results between the corrected and proposed Vlsekriterijumska Optimizacija I Kompromisno Resenje (VIKOR) methods (see Section 6). Based on assessing and weighting method, the various approaches exist for evaluating MCDM problems. In a more general classification, the MCDM approaches can be partitioned into two groups: utility theory and outranking methods. Weighted aggregated sum product assessment (WASPAS) is one of utility theory approaches where the weighted sum model (WSM) and weighted product model (WPM) are used for ranking alternatives with respect to a collection of criteria. The WASPAS approach is one of the most popular MCDM branches of knowledge. It consists of two aggregated parts, i.e., the WSM and WPM. The

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assessments of options with respect to criteria and weights of criteria in classical WASPAS using crisp measures are not desirable when dealing with obscure decision-making situations. Uncertainty is one of the problems occurred when decision-makers (DMs) handled realistic situations in the science and technology environment [2]. Hence, the fuzzy data have been utilized to MCDM branched of knowledge. Thus, it is widely adopted by many authors in MCDM techniques through fuzzy operators [3]. The fuzzy sets (FSs) are a new branch of knowledge for dealing with multi-attribute group decision-making (MAGDM) issues [4]. Although type-1 fuzzy appraisals and weights are adopted to type-1 fuzzy WASPAS, an expert may not be sure regarding the variety of membership function (MF). Hence, the interval type-2 fuzzy sets (IT2FSs) should be used to take into account instead. The different MFs have been proposed by the authors in the literature [5] such as Gaussian interval type-2 fuzzy sets (GIT2FSs). Gaussian MF is continuous function and has mathematical and statistical applications, due to its differentiable property. Thus, it can be used to the curved MFs. Moreover, these can be applied to the different curved probability density functions like Normal, Beta, etc. The attempt of this paper is to define verbal variables for the appraisals of hatch covers with respect to criteria as well as weights of criteria and then specify interval type-2 fuzzy numbers (IT2FNs).

The main objective of present research is to offer an integrated collective interval type-2 fuzzy (IT2F) decision-making manner for HCEP. The use of type-2 fuzzy data is more advisable instead of type-1 version based on the explanations defined above. Accordingly, the present paper presents IT2F WASPAS approach under the subjective and objective weights where the subjective weights are attained according to the DMs aggregated standpoints and the objective weights are calculated by the Shannon entropy approach. On the other hand, since choice of HCEP is a collective decision-making process, the integrated evaluation approach is handled to merge the fuzzy data of hatch covers with respect to criteria. The subjective and objective weights-based WASPAS method is then adopted to select the most desirable hatch cover. Generally, the principal contributions of this research are stated as follows:

- Since HCEP is a collective decision-making, the integrated arithmetic operations are used to synthesize the IT2F data of alternatives with respect to criteria.
- Several IT2F MCDM techniques are used to solve the HCEP.
- The simultaneous use of the subjective and objective weights is applied to determine the weights of criteria.
- The objective and subjective weights-based IT2F WASPAS approach is used to solve the HCEP.
- In order to use GIT2FSs to the MCDM techniques, their corresponding arithmetic operations are presented.
- The integration of IT2F WASPAS and Shannon entropy methods is applied to solve the HCEP.
- The new limit distance mean (*LDM*) approach is applied to determine the crisp aggregated weighted ratings.

The remainder of present research is sorted as follows: Section 2 includes the literature review for the MCDM techniques, WASPAS, and HCEP. In Section 3, Shannon entropy, WASPAS, and arithmetic calculations of T2FSs are explained. Our ranking approach is structured in Section 4. In Section 5, the proposed ranking approach is integrated with the Entropy-based WASPAS framework. An illustrative example is presented in Section 6 where the proposed approach is utilized to the WASPAS method. Lastly, the results obtained from our approach are stated in Section 7.

2. Literature review

Mardani et al. [6] listed applications of fuzzy generalizations for two new MCDM approaches including step-wise weight assessment ratio analysis (SWARA) and WASPAS. The MCDM approach in diverse areas of science and technology draws the attention of the researchers [7]. The MCDM techniques have been applied to search a suitable alternative from a feasible collection of finite alternatives based on different quantitative and qualitative criteria [8]. Dorfeshan and Mousavi [9] introduced a new IT2FSs-relative preference relation approach based on multi-attributive border approximation area comparison (MABAC) method for determining the critical path of production projects in which weights of criteria are calculated based on a novel generalized WASPAS method by using the DMs or experts' standpoints with respect to the importance of criteria and weights of DMs. Ilbahara and Kahraman [10] extended a new interval-valued Pythagorean fuzzy WASPAS method to appraise retail shops. Ramadhan et al. [11] used the WASPAS approach to choose the prospective employees for content creators. Kumar et al. [12] adopted WASPAS to evaluate different portable hard disk drive options with respect storage volume, measure, data transfer rapidity, and physical properties where the subjective weights were taken into account to choose the suitable option. Mathew et al. [13] applied different normalization approaches to WASPAS for studying the robot evaluation problem and concluded that the linear normalization (Max–Min) approach has the best results than the others. Mic and Figen-Antmen [14] adopted technique for order preference by similarity to ideal solution (TOPSIS), WASPAS, and multi-objective optimization on the basis of ratio analysis (MOORA) for solving university location selection problem. Mathew and Sahu [15] assessed a conveyor selection problem with respect to six criteria and four options through combinative distance based assessment (CODAS), evaluation based on distance from average solution (EDAS), WASPAS, and MOORA techniques. On the other hand, the automated guided vehicles selection problem was evaluated using CODAS, EDAS, WASPAS, and MOORA approaches. Simić et al. [16] extended the WASPAS approach to the picture fuzzy environment for solving the last-mile delivery mode selection problem. Tus and Adal [17] proposed a novel integrated

MCDM method using criteria importance through inter criteria correlation (CRITIC) and WASPAS methods for appraising the time and attendance software selection problem of the private hospital. Urošević et al. [18] presented an integrated approach to select personnel for the position of sales manager in the tourism sector through the SWARA and WASPAS approaches. Prajapati et al. [19] applied a modified SWARA and WASPAS-based methodology to rank the solutions of barriers to reverse logistics application to Indian electrical manufacturing industry. A synthetic method based on MOORA, SWARA and WASPAS approaches was proposed by Jayant et al. [20] for selection of third-party logistics (3PL) in which the SWARA method was applied to specify the criteria weights and the other methods like MOORA and WASPAS were used to attain the rankings of options and opt the best option. Peng and Dai [21] suggested the MABAC, WASPAS and complex proportional assessment (COPRAS) methods to evaluate hesitant fuzzy soft decision-making problem. Keshavarz-Ghorabaee et al. [22] suggested a novel synthetic approach including the IT2F CRITIC and WASPAS methods to assess 3PL providers. In the proposed approach, objective weights were determined using the CRITIC method merged with subjective weights to obtain more real weights. Stojić et al. [23] chosen of suppliers in a company producing polyvinyl chloride (PVC) carpentry through a novel synthetic MCDM in which the criteria weights were attained using the rough analytic hierarchy process (AHP) approach and the evaluation of suppliers was carried out using the new rough WASPAS approach. Deveci et al. [24] introduced WASPAS-based IT2F TOPSIS to assess the best location for solving the car-sharing station problem.

Most researches done on hatch covers are related to lightening weight, type of consuming materials, risk assessment, watertight integrity, strength, etc. Based on our studies, there is only one paper regarding the evaluation of hatch covers. For example, Tawfik et al. [25] applied composites material for mading steel hatch covers to decrease the weight and augment the solidity.

Previous studies mostly emphasized the physical properties and safety situation of hatch covers. Based on our studies, one paper exists regarding the solution of HCEP using the MCDM techniques. Recently, Soner et al. [26] offered an integrated IT2F AHP and VIKOR approach for evaluating the HCEP. Unfortunately, when investigating their VIKOR method, the authors found that the VIKOR method described in their paper has some drawbacks (as shown later in Section 6) such that application of it for the MCDM problems will result in the incorrect calculations. There are many limitations including indices, mathematical operations of weighted type-2 fuzzy decisions, definitions of ideal solutions, and expressions of the worst group scores such that the wrong ranked results have obtained using the used techniques. Therefore, it can be said that the novelty of the present work primarily includes the novel topic, extracting a model based on expert views through a fuzzy approach, and considering objective and subjective weights to extract the proposed MCDM approach.

3. Preliminaries

3.1. Hybrid Shannon entropy

The Shannon entropy technique is a desirable approach when DM standpoints regarding criteria weights are not available. Entropy idea can be effectively employed in the decision-making procedure since it is capable to determine conflicts between collections of data and make clear the DM’s inherent data.

Suppose that the crisp evaluation matrix be as $E=[x_{rt}]_{R\times T}$ in which R alternatives $AL_r\,(r=1,\ldots,R)$ should be appraised with respect to T criteria $C_t\,(t=1,\ldots,T)$ such that x_{rt} shows the appraisal measure of alternative r with respect to criterion t. In addition, assume that $[w_t^o]_{1\times T}$, $[w_t^s]_{1\times T}$, and $[w_t^*]_{1\times T}$ be the objective, subjective, and hybrid (total) criteria weights, respectively. The decision-making process described above can be also represented as below arranged:

$$E=\left[x_{rt}\right]_{R\times T}\;\begin{array}{c} \\ \\ AL_1\\ AL_2\\ AL_3\\ \vdots\\ AL_R\end{array}\begin{array}{c}\begin{array}{ccccc} w_1^* & w_2^* & w_3^* & & w_T^*\\ C_1 & C_2 & C_3 & \ldots & C_T\end{array}\\ \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1T}\\ x_{21} & x_{22} & x_{23} & \cdots & x_{2T}\\ x_{31} & x_{32} & x_{33} & \cdots & x_{3T}\\ \vdots & \vdots & \vdots & \cdots & \vdots\\ x_{R1} & x_{R2} & x_{R3} & \cdots & x_{RT}\end{bmatrix}\end{array}. \tag{1}$$

The below stages show the objective and subjective weights-based hybrid Shannon entropy:

1. Normalize the performance measures of the above matrix to obtain the project outcomes as follows:

$$p_{rt} = \frac{x_{rt}}{\sum_{r=1}^{R} x_{rt}}, \qquad r = 1,\ldots,R,\ t = 1,\ldots,T. \tag{2}$$

2. Compute the entropy measures of project outcomes as below:

$$E_t = -k\sum_{r=1}^{R} p_{rt} \ln p_{rt}, \qquad t = 1,\ldots,T, \tag{3}$$

where $k = \dfrac{1}{\ln(R)}$.

3. Determine the objective weights as follows:

$$w_t^o = \frac{1 - E_t}{\sum_{t=1}^{T}(1 - E_t)}, \qquad t = 1,\ldots,T. \tag{4}$$

4. Obtain the w_t^* based on w_t^o and w_t^s as below formula:

$$w_t^* = \frac{w_t^o w_t^s}{\sum_{t=1}^{T} w_t^o w_t^s}, \qquad t = 1,\ldots,T. \tag{5}$$

3.2. The WASPAS method

The summary explanation of WASPAS is as follows:

Step 1: Structure the crisp evaluation matrix, $E = [x_{rt}]_{R\times T}$, as matrix (1).

Step 2: Normalize the measures of x_{rt} by using the following equation and then construct the normalized decision-making matrix $\hat{N} = [\hat{x}_{rt}]_{R\times T}$:

$$\hat{x}_{rt} = \begin{cases} \dfrac{x_{rt}}{\max\limits_{r} x_{rt}}, & if\ t \in BC \\ \dfrac{\min\limits_{r} x_{rt}}{x_{rt}}, & if\ t \in CC \end{cases}, \tag{6}$$

where benefit criteria (*BC*) and benefit criteria (*CC*).

Step 3: Determine the weighted normalized measures of $\bar{x}_{rt}$ ($r = 1,\ldots,R; t = 1,\ldots,T$) for WSM and $\bar{\bar{x}}_{ij}$ ($r = 1,\ldots,R; t = 1,\ldots,T$) for WPM as follows:

$$\bar{x}_{rt} = \hat{x}_{rt} w_t, \qquad r = 1,\ldots,R,\ t = 1,\ldots,T, \tag{7}$$

$$\bar{\bar{x}}_{rt} = \hat{x}_{rt}^{\ w_t}, \qquad r = 1,\ldots,R,\ t = 1,\ldots,T. \tag{8}$$

Step 4: Calculate the measures of the optimality function regarding each alternative $AL_r\,(r=1,\ldots,R)$ for WSM ($Q_r^{(1)}$) and WPM ($Q_r^{(2)}$), respectively, by using the following equations:

$$Q_r^{(1)}=\sum_{t=1}^{T}\bar{x}_{rt},\qquad r=1,\ldots,R, \tag{9}$$

$$Q_r^{(2)}=\prod_{t=1}^{T}\bar{\bar{x}}_{rt},\qquad r=1,\ldots,R. \tag{10}$$

Step 5: Compute the aggregated optimality function score of WASPAS for each alternative $AL_r\,(r=1,\ldots,R)$ as:

$$Q_r=\lambda Q_r^{(1)}+(1-\lambda)Q_r^{(2)},\qquad r=1,\ldots,R, \tag{11}$$

where λ plays the parameter role of the WASPAS approach by varying at the interval [0, 1]. When $\lambda=1$, the WASPAS approach is converted into WSM and $\lambda=0$ results in WPM.

Step 6: Sort options based on Q_r (the bigger the concession of Q_r, the elder the precedence measure).

3.3. T2FSs and their arithmetic calculations

Table 1 shows the different concepts of T2FSs from which are used to extend the type-2 fuzzy WASPAS approach.

<Take in **Table 1.** >

<Take in **Figure 1.** >

Definition 3.3.1. Let $\tilde{\tilde{B}}^{i(i=L,U)}$ be two non-negative trapezoidal fuzzy numbers (L and U are the lower and upper MFs) [27]. Moreover, assume that $H_{\tilde{B}}^{i(i=L,U)}$ be the heights of $\tilde{\tilde{B}}^{i(i=L,U)}$. On the other hand, let $\left(a_h^L, a_h^U;\ h=1,2,3,4\right)$ be non-negative real values. A trapezoidal interval type-2 fuzzy number (TraIT2FN) is defined as below shown:

$$\tilde{\tilde{A}}=[\tilde{A}^L,\tilde{A}^U]=\left[\left(a_n^L;H_{\tilde{A}}^L\right),\left(a_n^U;H_{\tilde{A}}^U\right),n=1,\ldots,4\right] \tag{12}$$

In addition, suppose that $\tilde{\tilde{A}}_1=[\tilde{A}_1^L,\tilde{A}_1^U]=\left[\left(a_{1n}^L;H_{\tilde{A}_1}^L\right),\left(a_{1n}^U;H_{\tilde{A}_1}^U\right),n=1,2,3,4\right]$ and $\tilde{\tilde{A}}_2=[\tilde{A}_2^L,\tilde{A}_2^U]=\left[\left(a_{2n}^L;H_{\tilde{A}_2}^L\right),\left(a_{2n}^U;H_{\tilde{A}_2}^U\right),n=1,2,3,4\right]$ be two non-negative TraIT2FNs. The interested reader can refer to Kirac and Akan [27] in order to calculate arithmetic operations between them.

Definition 3.3.2. Suppose that $\tilde{\tilde{G}}_1=[\tilde{G}_1^L,\tilde{G}_1^U]=\left[\left(\mu_1^L;\sigma_1^L;H_{\tilde{G}_1}^L\right),\left(\mu_1^U;\sigma_1^U;H_{\tilde{G}_1}^U\right)\right]$ and $\tilde{\tilde{G}}_2=[\tilde{\tilde{G}}_2^L,\tilde{\tilde{G}}_2^U]=\left[\left(\mu_2^L;\sigma_2^L;H_{\tilde{G}_2}^L\right),\left(\mu_2^U;\sigma_2^U;H_{\tilde{G}_2}^U\right)\right]$ be two non-negative normal GIT2FNs ($H_{\tilde{G}_1}^L=H_{\tilde{G}_2}^L=H_{\tilde{G}_1}^U=H_{\tilde{G}_2}^U$). On the other hand, let alpha cut of normal GIT2FN t is presented as follows:

$$\hat{G}_{t\alpha}=\left[\left[\bar{g}_{1t\alpha}^l,\underline{g}_{2t\alpha}^l\right],\mu_t,\left[\underline{g}_{1t\alpha}^r,\bar{g}_{2t\alpha}^r\right]\right], \tag{13}$$

where l and r show the left and right MFs of $\tilde{\tilde{G}}$, respectively. Based on Figure 1, $[\bar{g}_1^l, \underline{g}_2^l]_\alpha$ and $[\underline{g}_1^r, \bar{g}_2^r]_\alpha$ are alpha cut of $\mu^l_{\tilde{\tilde{G}}}(x,u)$ and $\mu^r_{\tilde{\tilde{G}}}(x,u)$. Therefore, some of arithmetic calculations of GIT2FNs are calculated by using the concept of alpha cut for $\alpha = \alpha_1,\ldots,\alpha_P$ (P is the number of alpha cuts) by using the following formulas:

$$\begin{aligned}\hat{G}_{1\alpha} \oplus \hat{G}_{2\alpha} = &\left[\left(\underline{g}^l_{21\alpha} + \underline{g}^l_{22\alpha}, \mu_1 + \mu_2, \underline{g}^r_{11\alpha} + \underline{g}^r_{12\alpha}; \min\left\{H^L_{\tilde{\tilde{G}}_1}, H^L_{\tilde{\tilde{G}}_2}\right\}\right)\right],\\ &\left[\left(\bar{g}^l_{11\alpha} + \bar{g}^l_{12\alpha}, \mu_1 + \mu_2, \bar{g}^r_{21\alpha} + \bar{g}^r_{22\alpha}; \min\left\{H^U_{\tilde{\tilde{G}}_1}, H^U_{\tilde{\tilde{G}}_2}\right\}\right)\right]\end{aligned} \tag{14}$$

$$\begin{aligned}\hat{G}_{1\alpha} \Theta \hat{G}_{2\alpha} = &\left[\left(\underline{g}^l_{21\alpha} - \underline{g}^r_{12\alpha}, \mu_1 - \mu_2, \underline{g}^r_{11\alpha} - \underline{g}^l_{22\alpha}; \min\left\{H^L_{\tilde{\tilde{G}}_1}, H^L_{\tilde{\tilde{G}}_2}\right\}\right)\right],\\ &\left[\left(\bar{g}^l_{11\alpha} - \bar{g}^r_{22\alpha}, \mu_1 - \mu_2, \bar{g}^r_{21\alpha} - \bar{g}^l_{12\alpha}; \min\left\{H^U_{\tilde{\tilde{G}}_1}, H^U_{\tilde{\tilde{G}}_2}\right\}\right)\right]\end{aligned} \tag{15}$$

$$\begin{aligned}\hat{G}_{1\alpha} \otimes \hat{G}_{2\alpha} = &\left[\left(\underline{g}^l_{21\alpha} \times \underline{g}^l_{22\alpha}, \mu_1 \times \mu_2, \underline{g}^r_{11\alpha} \times \underline{g}^r_{12\alpha}; \min\left\{H^L_{\tilde{\tilde{G}}_1}, H^L_{\tilde{\tilde{G}}_2}\right\}\right)\right],\\ &\left[\left(\bar{g}^l_{11\alpha} \times \bar{g}^l_{12\alpha}, \mu_1 \times \mu_2, \bar{g}^r_{21\alpha} \times \bar{g}^r_{22\alpha}; \min\left\{H^U_{\tilde{\tilde{G}}_1}, H^U_{\tilde{\tilde{G}}_2}\right\}\right)\right]\end{aligned} \tag{16}$$

$$\begin{aligned}\hat{G}_{1\alpha} \varnothing \hat{G}_{2\alpha} = &\left[\left(\frac{\underline{g}^l_{21\alpha}}{\underline{g}^r_{12\alpha}}, \frac{\mu_1}{\mu_2}, \frac{\underline{g}^r_{11\alpha}}{\underline{g}^l_{22\alpha}}; \min\left\{H^L_{\tilde{\tilde{G}}_1}, H^L_{\tilde{\tilde{G}}_2}\right\}\right)\right],\\ &\left[\left(\frac{\bar{g}^l_{11\alpha}}{\bar{g}^r_{22\alpha}}, \frac{\mu_1}{\mu_2}, \frac{\bar{g}^r_{21\alpha}}{\bar{g}^l_{12\alpha}}; \min\left\{H^U_{\tilde{\tilde{G}}_1}, H^U_{\tilde{\tilde{G}}_2}\right\}\right)\right],\end{aligned} \tag{17}$$

$$\hat{G}_{1\alpha}.b = b.\hat{G}_{1\alpha} = \begin{cases}\left[\left(b.\underline{g}^l_{21\alpha}, b.\mu_1, b.\underline{g}^r_{11\alpha}; H^L_{\tilde{\tilde{G}}_1}\right), \left(b.\bar{g}^l_{11\alpha}, b.\mu_1, b.\bar{g}^r_{21\alpha}; H^U_{\tilde{\tilde{G}}_1}\right)\right] \ if\ b \geq 0,\\ \left[\left(b.\underline{g}^r_{11\alpha}, b.\mu_1, b.\underline{g}^l_{21\alpha}; H^L_{\tilde{\tilde{G}}_1}\right), \left(b.\bar{g}^r_{21\alpha}, b.\mu_1, b.\bar{g}^l_{11\alpha}; H^U_{\tilde{\tilde{G}}_1}\right)\right] \ if\ b \leq 0,\end{cases} \tag{18}$$

$$\begin{aligned}\hat{G}_{1\alpha}^{\hat{G}_{2\alpha}} = &\left[\left({\underline{g}^l_{21\alpha}}^{\underline{g}^r_{12\alpha}}, {\mu_1}^{\mu_2}, {\underline{g}^r_{11\alpha}}^{\underline{g}^l_{22\alpha}}; \min\left\{H^L_{\tilde{\tilde{G}}_1}, H^L_{\tilde{\tilde{G}}_2}\right\}\right)\right],\\ &\left[\left({\bar{g}^l_{11\alpha}}^{\bar{g}^r_{22\alpha}}, {\mu_1}^{\mu_2}, {\bar{g}^r_{21\alpha}}^{\bar{g}^l_{12\alpha}}; \min\left\{H^U_{\tilde{\tilde{G}}_1}, H^U_{\tilde{\tilde{G}}_2}\right\}\right)\right].\end{aligned} \tag{19}$$

Definition 3.3.3. Suppose that there are Z non-negative normal GIT2FNs, $\tilde{\tilde{G}}_z = [\tilde{G}^L_z, \tilde{G}^U_z] = \left[\left(\mu^L_z; \sigma^L_z; H^L_{\tilde{G}_z}\right), \left(\mu^U_z; \sigma^U_z; H^U_{\tilde{G}_z}\right)\right]$ such that $H^L_{\tilde{G}_z} = H^U_{\tilde{G}_z}$ $(z = 1,\ldots,Z)$. Mean operations, namely $\bar{\hat{G}}_\alpha = (\bar{\bar{g}}^l_{1\alpha}, \underline{\bar{g}}^l_{2\alpha}, \bar{\mu}_\alpha, \underline{\bar{g}}^r_{1\alpha}, \bar{\bar{g}}^r_{2\alpha})$ ($\alpha = \alpha_1,\ldots,\alpha_P$) are calculated as below formulas:

$$\bar{\bar{g}}^l_{1\alpha} = \left(\sum_{z=1}^{Z} \bar{g}^l_{1z\alpha}\right)\Big/Z, \underline{\bar{g}}^l_{2\alpha} = \left(\sum_{z=1}^{Z} \underline{g}^l_{2z\alpha}\right)\Big/Z, \bar{\mu}_\alpha = \left(\sum_{z=1}^{Z} \mu_{z\alpha}\right)\Big/Z, \underline{\bar{g}}^r_{1\alpha} = \left(\sum_{z=1}^{Z} \underline{g}^r_{1z\alpha}\right)\Big/Z, \bar{\bar{g}}^r_{2\alpha} = \left(\sum_{z=1}^{Z} \bar{g}^r_{2z\alpha}\right)\Big/Z, \tag{20}$$

where $\left[\bar{\bar{g}}^l_{1t\alpha}, \underline{\bar{g}}^l_{2t\alpha}\right]$ and $\left[\underline{\bar{g}}^r_{1t\alpha}, \bar{\bar{g}}^r_{2t\alpha}\right]$ are the alpha cut average of $\mu^l_{\tilde{\tilde{G}}}(x,u)$ and $\mu^r_{\tilde{\tilde{G}}}(x,u)$, respectively.

4. The *LDM* approach

Here, the *LDM* approach is explained based on the arithmetic calculations offered in Section 3. Let $\mu_{\tilde{\tilde{G}}}(x,u)$ is divided into $\mu^l_{\tilde{\tilde{G}}}(x,u)$ and $\mu^r_{\tilde{\tilde{G}}}(x,u)$ (the left and right MFs). Based on Figure 1, $\mu^{\min}_{\tilde{\tilde{G}}}(x,u)$ and $\mu^{\max}_{\tilde{\tilde{G}}}(x,u)$ are equal to minimum and maximum bounds. On the other hand, let $[\bar{g}_1^{\min}, \underline{g}_2^{\min}]_\alpha$, $[\underline{g}_1^{\max}, \bar{g}_2^{\max}]_\alpha$, $[\bar{g}_1^l, \underline{g}_2^l]_\alpha$, and $[\underline{g}_1^r, \bar{g}_2^r]_\alpha$ be alpha cut of the minimum, maximum, left, and right MFs, respectively. To this end, positive ideal (*PI*) and negative ideal (*NI*) *LDM*s with respect to *CC* and *BC* are calculated by using the following expressions:

$$LDM_{PI,CC}(\tilde{\tilde{A}}) = \frac{\left(\sum_{\alpha=0.1}^{1}\left|(\bar{g}_1^{l} - \underline{g}_2^{\min})_\alpha + (\underline{g}_2^{l} - \bar{g}_1^{\min})_\alpha\right|\right)}{\left(\sum_{\alpha=0.1}^{1}\left|(\bar{g}_1^{l} - \underline{g}_2^{\min})_\alpha + (\underline{g}_2^{l} - \bar{g}_1^{\min})_\alpha\right|\right) + \left(\sum_{\alpha=0.1}^{1}\left|(\underline{g}_1^{r} - \bar{g}_2^{\max})_\alpha + (\bar{g}_2^{r} - \underline{g}_1^{\max})_\alpha\right|\right)}, \quad (21)$$

$$LDM_{PI,BC}(\tilde{\tilde{A}}) = \frac{\left(\sum_{\alpha=0.1}^{1}\left|(\underline{g}_1^{r} - \bar{g}_2^{\max})_\alpha + (\bar{g}_2^{r} - \underline{g}_1^{\max})_\alpha\right|\right)}{\left(\sum_{\alpha=0.1}^{1}\left|(\underline{g}_1^{r} - \bar{g}_{\max 2})_\alpha + (\bar{g}_2^{r} - \underline{g}_1^{\max})_\alpha\right|\right) + \left(\sum_{\alpha=0.1}^{1}\left|(\bar{g}_1^{l} - \underline{g}_2^{\min})_\alpha + (\underline{g}_2^{l} - \bar{g}_1^{\min})_\alpha\right|\right)}, \quad (22)$$

$$LDM_{NI,CC}(\tilde{\tilde{A}}) = \frac{\left(\sum_{\alpha=0.1}^{1}\left|(\bar{g}_1^{l} - \bar{g}_2^{\max})_\alpha + (\underline{g}_2^{l} - \underline{g}_1^{\max})_\alpha\right|\right)}{\left(\sum_{\alpha=0.1}^{1}\left|(\bar{g}_1^{l} - \bar{g}_2^{\max})_\alpha + (\underline{g}_2^{l} - \underline{g}_1^{\max})_\alpha\right|\right) + \left(\sum_{\alpha=0.1}^{1}\left|(\underline{g}_1^{r} - \underline{g}_2^{\min})_\alpha + (\bar{g}_2^{r} - \bar{g}_1^{\min})_\alpha\right|\right)}, \quad (23)$$

$$LDM_{NI,BC}(\tilde{\tilde{A}}) = \frac{\left(\sum_{\alpha=0.1}^{1}\left|(\underline{g}_1^{r} - \underline{g}_2^{\min})_\alpha + (\bar{g}_2^{r} - \bar{g}_1^{\min})_\alpha\right|\right)}{\left(\sum_{\alpha=0.1}^{1}\left|(\underline{g}_1^{r} - \underline{g}_2^{\min})_\alpha + (\bar{g}_2^{r} - \bar{g}_1^{\min})_\alpha\right|\right) + \left(\sum_{\alpha=0.1}^{1}\left|(\bar{g}_1^{l} - \bar{g}_2^{\max})_\alpha + (\underline{g}_2^{l} - \underline{g}_1^{\max})_\alpha\right|\right)}. \quad (24)$$

5. The entropy-based IT2F WASPAS approach

Figure 2 shows the stages of the methodology applied by this paper. The alternatives (hatch covers) and criteria are determined according to DMs' point of views in the first stage. In the second phase, the hybrid criteria weights (integration of objective and subjective weight) are expressed based on IT2F Shannon entropy and linguistic variables opted by DMs. Afterwards, the alpha cuts-based WSM and WPM techniques are handled to calculate the aggregated optimality function score from the IT2F assessments of each hatch cover with respect to criteria and eventually the most important hatch cover is evaluated through the WASPAS approach.

The summary explanation of the integrated entropy-based IT2F WASPAS technique for prioritizing hatch covers is defined as below shown:

Step 1: Let *K* DMs want to evaluate *R* hatch covers H_r ($r = 1,\ldots,R$) with respect to *T* criteria C_t ($t = 1,\ldots,T$).

Step 2: Introduce two kinds of verbal expressions for the HCEP. The first kind is used to appraise hatch covers with respect to criteria and the next type is adopted to determine criteria weights.

Step 3: Apply the data of second type to weight criteria as the following vector:

$$W = (w_1, w_2, \ldots, w_T). \quad (25)$$

Step 4: Structure the IT2F MCDM (IT2FMCDM) matrix (see Table 2):

<Take in **Table 2.** >

where $\tilde{\tilde{x}}_{rt}^{l}$ is a IT2FN opted by *l*th DM assessing hatch cover *r* ($r = 1,\ldots,R$) with respect to criterion C_t ($t = 1,\ldots,T$).

<Take in **Figure 2.** >

Step 5: Integrate the type-2 fuzzy appraisals $\tilde{\tilde{x}}_{rt}^{l}$ (for all $l = 1,\ldots,L$) with the synthetic type-2 fuzzy appraisal $\tilde{\tilde{x}}_{rt}$ as below defined:

$$\tilde{\tilde{x}}_{rt} = (1/L) \otimes (\tilde{\tilde{x}}_{rt}^{1} \oplus \tilde{\tilde{x}}_{rt}^{2} \oplus \ldots \oplus \tilde{\tilde{x}}_{rt}^{L}), \qquad r = 1,\ldots,R,\ t = 1,\ldots,T, \tag{26}$$

clearly, the above equation is the mean of the type-2 fuzzy appraisals opted by L DMs. Similarly, the type-2 fuzzy weights (T2FW), $\tilde{\tilde{w}}_{t}^{l}$, (for all $l = 1,\ldots,L$) for each criterion are integrated with the synthetic T2FW, $\tilde{\tilde{w}}_{t}$, by:

$$\tilde{\tilde{w}}_{t} = (1/L) \otimes (\tilde{\tilde{w}}_{t}^{1} \oplus \tilde{\tilde{w}}_{t}^{2} \oplus \ldots \oplus \tilde{\tilde{w}}_{t}^{L}), \qquad t = 1,\ldots,T, \tag{27}$$

$$W = (w_1, w_2, \ldots, w_T). \tag{28}$$

The alpha cut representation of GIT2FNs, $\tilde{\tilde{x}}_{rt}$, is as the below formula (see Definition 3.3.2):

$$\hat{x}_{rt\alpha} = (\overline{x}_{1rt_\alpha}^{l}, \underline{x}_{2rt_\alpha}^{l}, \mu_{rt_\alpha}, \underline{x}_{1rt_\alpha}^{r}, \overline{x}_{2rt_\alpha}^{r}), \qquad r = 1,\ldots,R,\ t = 1,\ldots,T, \tag{29}$$

where μ_{rt_α} is equal to the mean of GIT2FN when hatch cover r ($r = 1,\ldots,R$) is appraised with respect to criterion C_t ($t = 1,\ldots,T$). If appraisals be TraIT2FN or triangular interval type-2 fuzzy number (TriIT2FN), μ_{rt_α} can be eliminated from the above expressions.

Similarly, GIT2FN, $\hat{x}_{rt\alpha}^{l}$, opted by lth DM at level α can be given by:

$$\hat{x}_{rt\alpha}^{l} = (\overline{x}_{1rt_\alpha}^{l\,l}, \underline{x}_{2rt_\alpha}^{l\,l}, \mu_{rt_\alpha}, \underline{x}_{1rt_\alpha}^{r\,l}, \overline{x}_{2rt_\alpha}^{r\,l}), \qquad r = 1,\ldots,R,\ t = 1,\ldots,T; l = 1,\ldots,L, \tag{30}$$

obviously, $\overline{\hat{x}}_{rt\alpha} = (\overline{\overline{x}}_{1rt_\alpha}^{l}, \overline{\underline{x}}_{2rt_\alpha}^{l}, \mu_{rt_\alpha}, \overline{\underline{x}}_{1rt_\alpha}^{r}, \overline{\overline{x}}_{2rt_\alpha}^{r})$ is the alpha cut mean of GIT2FNs opted by L DMs that is calculated using the generalization of Equation 20.

Similarly, the average of reference points for subjective weights of criteria opted by L DMs at each alpha cut, namely $\overline{\hat{w}}_{t_\alpha}^{S} = (\overline{\overline{w}}_{1t_\alpha}^{sl}, \overline{\underline{w}}_{2t_\alpha}^{sl}, \mu_{t_\alpha}^{s}, \overline{\underline{w}}_{1t_\alpha}^{sr}, \overline{\overline{w}}_{2t_\alpha}^{sr})$ is calculated through the generalization of Equation 20.

Again, if appraisals and weights are as TraIT2FN or TriIT2FN, eliminate μ_{rt_α} and $\mu_{t_\alpha}^{s}$ from the above equations.

Step 6: Normalize the performance measures in the Table 2. Let $\tilde{\tilde{X}} = [\tilde{X}^{L}, \tilde{X}^{U}] = \left[\left(x_1^L, x_2^L, x_3^L, x_4^L; H_{\tilde{A}}^{L}\right), \left(x_1^U, x_2^U, x_3^U, x_4^U; H_{\tilde{A}}^{U}\right)\right]$ be a TraIT2FN. The normalized performance measures can be calculated for BC and CC, respectively, using the following expressions:

$$\tilde{\tilde{n}}_{rt} = \left[\left(\frac{x_{1rt}^{L}}{x_{4t}^{+}}, \frac{x_{2rt}^{L}}{x_{4t}^{+}}, \frac{x_{3rt}^{L}}{x_{4t}^{+}}, \frac{x_{4rt}^{L}}{x_{4t}^{+}}; H_{\tilde{\tilde{x}}_{rt}}^{L}\right), \left(\frac{x_{1rt}^{U}}{x_{4t}^{+}}, \frac{x_{2rt}^{U}}{x_{4t}^{+}}, \frac{x_{3rt}^{U}}{x_{4t}^{+}}, \frac{x_{4rt}^{U}}{x_{4t}^{+}}; H_{\tilde{\tilde{x}}_{rt}}^{U}\right)\right], \quad \text{for } r = 1,\ldots,R; x_{4t}^{+} = \max_{r} x_{4rt}^{U} \text{ where } t \in BC, \tag{31}$$

and

$$\tilde{\tilde{n}}_{rt} = \left[\left(\frac{x_{1t}^{-}}{x_{4rt}^{L}}, \frac{x_{1t}^{-}}{x_{3rt}^{L}}, \frac{x_{1t}^{-}}{x_{2rt}^{L}}, \frac{x_{1t}^{-}}{x_{1rt}^{L}}; H_{\tilde{\tilde{x}}_{rt}}^{L}\right), \left(\frac{x_{1t}^{-}}{x_{4rt}^{U}}, \frac{x_{1t}^{-}}{x_{3rt}^{U}}, \frac{x_{1t}^{-}}{x_{2rt}^{U}}, \frac{x_{1t}^{-}}{x_{1rt}^{U}}; H_{\tilde{\tilde{x}}_{rt}}^{U}\right)\right], \quad \text{for } r = 1,\ldots,R; x_{1t}^{-} = \min_{r} x_{1rt}^{U} \text{ where } t \in CC. \tag{32}$$

Create the normalized decision matrix, $\widehat{N}_\alpha$, for alpha cuts of $\tilde{\tilde{G}}$ for $r = 1,\ldots,R$ and $t = 1,\ldots,T$ through generalization of the approach defined above as follows:

$$\widehat{n}_{rt\alpha} = \left\{\left[\frac{\overline{x}_{1rt\alpha}^{l}}{x_t^{+}}, \frac{\underline{x}_{2rt\alpha}^{l}}{x_t^{+}}\right], \left[\frac{\underline{x}_{1rt\alpha}^{r}}{x_t^{+}}, \frac{\overline{x}_{2rt\alpha}^{r}}{x_t^{+}}\right]\right\}, \quad \text{for } r = 1,\ldots,R;\ \alpha = \alpha_1,\ldots,\alpha_P; x_t^{+} = \max_{r} \overline{x}_{2rt\alpha}^{r} \text{ where } t \in BC, \tag{33}$$

and

$$\widehat{n}_{rt\alpha} = \left\{ \left[\frac{x_t^-}{\overline{x}_{2rt\alpha}^r}, \frac{x_t^-}{\underline{x}_{1rt\alpha}^r} \right], \left[\frac{x_t^-}{\underline{x}_{2rt\alpha}^l}, \frac{x_t^-}{\overline{x}_{1rt\alpha}^l} \right] \right\}, \text{ for } r = 1,\ldots,R,\ \alpha = \alpha_1,\ldots,\alpha_P;\ x_t^- = \min_r \overline{x}_{1rt\alpha}^l \text{ where } t \in CC, \tag{34}$$

Obviously, measure of $-\sum_{r=1}^{R} \widehat{n}_{rt\alpha} \ln \widehat{n}_{rt\alpha}$ for $\widehat{n}_{rt\alpha} \le 0.4$ and $\widehat{n}_{rt\alpha} \ge 0.4$ is as ascending and descending, respectively. On the other hand, since $-k\sum_{r=1}^{R}\left(\frac{\overline{x}_{2rt\alpha}^r}{x_t^+}\right)\ln(\frac{\overline{x}_{2rt\alpha}^r}{x_t^+})$ and $-k\sum_{r=1}^{R}\left(\frac{x_t^-}{\overline{x}_{1rt\alpha}^l}\right)\ln(\frac{x_t^-}{\overline{x}_{1rt\alpha}^l})$ may be larger than one, thus, the following expressions (for $R \ge 3$) can be used to the final normalization of $t \in BC$ and $t \in CC$, respectively:

$$\widehat{n}'_{rt\alpha} = 0.4 * \frac{\left\{ \left[\frac{\overline{x}_{1rt\alpha}^l}{x_t^+}, \frac{\underline{x}_{2rt\alpha}^l}{x_t^+} \right], \left[\frac{\underline{x}_{1rt\alpha}^r}{x_t^+}, \frac{\overline{x}_{2rt\alpha}^r}{x_t^+} \right] \right\}}{2}, \text{ for } r = 1,\ldots,R;\ \alpha = \alpha_1,\ldots,\alpha_P;\ x_t^+ = \max_r \overline{x}_{2rt\alpha}^r \text{ where } t \in BC, \tag{35}$$

and

$$\widehat{n}'_{rt\alpha} = 0.4 * \frac{\left\{ \left[\frac{x_t^-}{\overline{x}_{2rt\alpha}^r}, \frac{x_t^-}{\underline{x}_{1rt\alpha}^r} \right], \left[\frac{x_t^-}{\underline{x}_{2rt\alpha}^l}, \frac{x_t^-}{\overline{x}_{1rt\alpha}^l} \right] \right\}}{2}, \text{ for } r = 1,\ldots,R,\ \alpha = \alpha_1,\ldots,\alpha_P;\ x_t^- = \min_r \overline{x}_{1rt\alpha}^l \text{ where } t \in CC. \tag{36}$$

Step 7: Compute the entropy measures of project outcomes for each level α ($\alpha = \alpha_1,\ldots,\alpha_P$) using the following equation:

$$\hat{E}_{t\alpha} = -k\sum_{r=1}^{R} \widehat{n}'_{rt\alpha} \ln \widehat{n}'_{rt\alpha}, \qquad t = 1,\ldots,T;\ \alpha = \alpha_1,\ldots,\alpha_P, \tag{37}$$

where $k = \dfrac{1}{\ln(R)}$.

Step 8: Determine the objective weights for each level α ($\alpha = \alpha_1,\ldots,\alpha_P$) as:

$$\hat{w}_{t\alpha}^o = \frac{1-\hat{E}_{t\alpha}}{\sum_{t=1}^{T}(1-\hat{E}_{t\alpha})}, \qquad t = 1,\ldots,T;\ \alpha = \alpha_1,\ldots,\alpha_P, \tag{38}$$

where $\hat{w}_{t\alpha}^O = (\overline{w}_{1t_\alpha}^{ol}, \underline{w}_{2t_\alpha}^{ol}, \underline{w}_{1t_\alpha}^{or}, \overline{w}_{2t_\alpha}^{or})$.

Step 9: Obtain the hybrid entropy weight ($\hat{w}_{t\alpha}^*$) based on the objective ($\hat{w}_{t\alpha}^o$) and subjective weight ($\hat{w}_{t\alpha}^s$) for each level α ($\alpha = \alpha_1,\ldots,\alpha_P$) as below formula:

$$\hat{w}_{t\alpha}^* = \frac{\hat{w}_{t\alpha}^o \hat{w}_{t\alpha}^s}{\sum_{t=1}^{T}\hat{w}_{t\alpha}^o \hat{w}_{t\alpha}^s}, \qquad t = 1,\ldots,T;\ \alpha = \alpha_1,\ldots,\alpha_P, \tag{39}$$

where $\hat{w}_{t\alpha}^S = (\overline{w}_{1t_\alpha}^{sl}, \underline{w}_{2t_\alpha}^{sl}, \underline{w}_{1t_\alpha}^{sr}, \overline{w}_{2t_\alpha}^{sr})$ ($\alpha = \alpha_1,\ldots,\alpha_P$) is determined by using Equation 20.

Step 10: Normalize the measures of $\hat{x}_{rt\alpha}$ for each level α ($\alpha = \alpha_1,\ldots,\alpha_P$) by using Eqs. (33) and (34) and then construct the normalized decision-making matrix $\hat{N}_\alpha = [\hat{x}_{rt\alpha}]_{R\times T}$:

Step 11: Determine the weighted normalized measures, $\hat{\hat{x}}_{ij\alpha}$ ($r = 1,\ldots,R; t = 1,\ldots,T$; $\alpha = \alpha_1,\ldots,\alpha_P$), for WSM and $\hat{\hat{\hat{x}}}_{rt\alpha}$ ($r = 1,\ldots,R; t = 1,\ldots,T$, $\alpha = \alpha_1,\ldots,\alpha_P$) for WPM as follows:

$$\hat{\hat{x}}_{rt\alpha} = \hat{x}_{rt\alpha} w^*_{t\alpha}, \qquad r = 1,\ldots,R,\ t = 1,\ldots,T, \alpha = \alpha_1,\ldots,\alpha_P, \tag{40}$$

$$\hat{\hat{\hat{x}}}_{rt\alpha} = \hat{x}_{rt\alpha}{}^{w^*_{t\alpha}}, \qquad r = 1,\ldots,R,\ t = 1,\ldots,T, \alpha = \alpha_1,\ldots,\alpha_P. \tag{41}$$

Step 12: Calculate the measures of the optimality function for each hatch cover $H_r (r = 1,\ldots,R)$ for WSM ($\hat{Q}_r^{(1)}$) and WPM ($\hat{Q}_r^{(2)}$), respectively:

$$\hat{Q}_{r\alpha}^{(1)} = \sum_{t=1}^{T} \hat{\hat{x}}_{rt\alpha}, \qquad r = 1,\ldots,R, \alpha = \alpha_1,\ldots,\alpha_P, \tag{42}$$

$$\hat{Q}_{r\alpha}^{(2)} = \prod_{t=1}^{T} \hat{\hat{\hat{x}}}_{rt\alpha}, \qquad r = 1,\ldots,R, \alpha = \alpha_1,\ldots,\alpha_P. \tag{43}$$

Step 13: Calculate the aggregated optimality function score of the WASPAS method for each hatch cover $H_r (r = 1,\ldots,M)$ as:

$$\hat{Q}_{r\alpha} = \lambda \hat{Q}_{r\alpha}^{(1)} + (1-\lambda)\hat{Q}_{r\alpha}^{(2)}, \qquad r = 1,\ldots,R, \alpha = \alpha_1,\ldots,\alpha_P, \tag{44}$$

where $\hat{Q}_{r\alpha} = (\overline{Q}^l_{1r_\alpha}, \underline{Q}^l_{2r_\alpha}, \underline{Q}^r_{1r_\alpha}, \overline{Q}^r_{2r_\alpha})$ and λ plays the parameter role of the WASPAS technique such that it can change at the interval [0,1]. When $\lambda = 0$ and $\lambda = 1$, the WASPAS method is transformed with WSM and WPM models, respectively.

Step 14: Compute $LDM(Q_r)$ based on Equation 24 and rank hatch covers according to the descending order $LDM(Q_r)$.

6. Application

6.1. An illustrative example

In order to present the reliability of the corrected VIKOR method and the comparison of the original approach with the others, the authors apply it to the illustrative example applied by Soner et al. [26], where five types of hatch covers (folding (H_1), side-rolling (H_2), lifting (H_3), Piggy-back (H_4), and Roll stowing (H_5)) should be evaluated with respect to nine criteria (water tightness (C_1), physical durability (C_2), installation cost (C_3), opening or closing duration (C_4), flexibility (C_5), maintenance cost (C_6), user-friendly (C_7), operation mechanism (C_8), and repairing time (C_9)). It is worth that C_1, C_2, C_5, C_7, and C_8 are of benefit type and C_3, C_4, C_6, and C_9 are of cost type.

There are a collection of limitations including indices, mathematical operations on weighted type-2 fuzzy decisions, definitions of ideal solutions, and expressions of the worst group scores in the VIKOR method used by Soner et al. [26]. In the following, we explain these drawbacks for each step.

Let HCEP includes n hatch covers $H_j (j = 1,\ldots,n)$ with respect to m criteria $C_i (i = 1,\ldots,m)$ based on standpoints of l DMs $S_k (k = 1,\ldots,l)$ [26]. In real, these notations result in some drawbacks in the steps of IT2F VIKOR defined below:

Step 1: The mean IT2F performance appraisals of hatch covers with respect to criteria are calculated by:

$$E_c = (\tilde{\tilde{e}}_{ij}^k)_{m\times n} = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_m \end{array} \begin{array}{c} \begin{array}{ccccc} H_1 & H_2 & H_3 & \dots & H_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{e}}_{11}^k & \tilde{\tilde{e}}_{12}^k & \tilde{\tilde{e}}_{13}^k & \cdots & \tilde{\tilde{e}}_{1n}^k \\ \tilde{\tilde{e}}_{21}^k & \tilde{\tilde{e}}_{22}^k & \tilde{\tilde{e}}_{23}^k & \cdots & \tilde{\tilde{e}}_{2n}^k \\ \tilde{\tilde{e}}_{31}^k & \tilde{\tilde{e}}_{32}^k & \tilde{\tilde{e}}_{33}^k & \cdots & \tilde{\tilde{e}}_{3n}^k \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \tilde{\tilde{e}}_{m1}^k & \tilde{\tilde{e}}_{m2}^k & \tilde{\tilde{e}}_{m3}^k & \cdots & \tilde{\tilde{e}}_{mn}^k \end{bmatrix} \end{array}, \tag{45}$$

where

$$\tilde{\tilde{e}}_{ij} = \left(\frac{\tilde{\tilde{e}}_{ij}^1 + \tilde{\tilde{e}}_{ij}^2 + \cdots + \tilde{\tilde{e}}_{ij}^l}{l} \right), \qquad i = 1,\ldots,m;\ j = 1,\ldots,n, \tag{46}$$

$\tilde{\tilde{e}}_{ij}$ is an IT2FS and $1 \le i \le m$, $1 \le j \le n$, $1 \le k \le l$ such that index l shows the DMs. Equation 45 represents the performance appraisals of hatch covers with respect to criteria by *k*th DM.

Step 2: The weighted type-2 fuzzy evaluation decision format (matrix) is computed as follows:

$$\tilde{\tilde{V}} = \left[\tilde{\tilde{v}}_{ij}\right]_{m\times n}, \tag{47}$$

where

$$\tilde{\tilde{v}}_{ij} = \tilde{\tilde{w}}_i \otimes \tilde{\tilde{e}}_{ij} = ((f_{i1}^U, f_{i2}^U, f_{i3}^U, f_{i4}^U; H_1(\tilde{F}_i^U), H_2(\tilde{F}_i^U)), ((f_{i1}^L, f_{i2}^L, f_{i3}^L, f_{i4}^L; H_1(\tilde{F}_i^L), H_2(\tilde{F}_i^L)), \tag{48}$$

and $\tilde{\tilde{w}}_i$ is the weight of criterion i ($i = 1,\ldots,m$). The drawback of this stage is that all the reference points in the weighted type-2 fuzzy decisions ($\tilde{\tilde{v}}_{ij}$) include only index *j*. It should include both of indices *i* and *j*.

Step 3: The *PI* solution (P^{e*}, P^{v*}) and *NI* solution (N^{e-}) for upper and lower reference points of IT2FNs are given by:

$$P^{e*} = \left\{\tilde{\tilde{e}}_{ij}^*, \tilde{\tilde{e}}_{ij}^*, \ldots, \tilde{\tilde{e}}_{ij}^*\right\} = \left\{\max_i \tilde{\tilde{e}}_{ij} \,\middle|\, i \in Benefit\right\} \tag{49}$$

$$P^{e*} = ((e_{i1}^{U*}, e_{i2}^{U*}, e_{i3}^{U*}, e_{i4}^{U*}; \max H_1(\tilde{E}_i^U), \max H_2(\tilde{E}_i^U)), ((e_{i1}^{L*}, e_{i2}^{L*}, e_{i3}^{L*}, e_{i4}^{L*}; \max H_1(\tilde{E}_i^L), \max H_2(\tilde{E}_i^L)), \tag{50}$$

$$P^{v*} = \left\{\tilde{\tilde{v}}_{ij}^*, \tilde{\tilde{v}}_{ij}^*, \ldots, \tilde{\tilde{v}}_{ij}^*\right\} = \left\{\max_i \tilde{\tilde{v}}_{ij} \,\middle|\, i \in Benefit\right\} \tag{51}$$

$$P^{v*} = ((f_{i1}^{U*}, f_{i2}^{U*}, f_{i3}^{U*}, f_{i4}^{U*}; \max H_1(\tilde{F}_i^U), \max H_2(\tilde{F}_i^U)), ((f_{i1}^{L*}, f_{i2}^{L*}, f_{i3}^{L*}, f_{i4}^{L*}; \max H_1(\tilde{F}_i^L), \max H_2(\tilde{F}_i^L)), \tag{52}$$

$$N^{e-} = \left\{\tilde{\tilde{e}}_{ij}, \tilde{\tilde{e}}_{ij}, \ldots, \tilde{\tilde{e}}_{ij}\right\} = \left\{\min_j \tilde{\tilde{e}}_{ij} \,\middle|\, i \in Benefit\right\}, \tag{53}$$

$$N^{e-} = ((e_{i1}^{U-}, e_{i2}^{U-}, e_{i3}^{U-}, e_{i4}^{U-}; \min\ H_1(\tilde{E}_i^U), \min H_2(\tilde{E}_i^U)), ((e_{i1}^{L-}, e_{i2}^{L-}, e_{i3}^{L-}, e_{i4}^{L-}; \min H_1(\tilde{E}_i^L), \min H_2(\tilde{E}_i^L)). \tag{54}$$

Unfortunately, the above relations have some drawbacks. According to Soner et al. [26], the number of expressions $\tilde{\tilde{e}}_{ij}$ in $P^{e*}=\{\tilde{\tilde{e}}^{*}_{ij},\tilde{\tilde{e}}^{*}_{ij},\ldots,\tilde{\tilde{e}}^{*}_{ij}\}$ (Equation 49), $P^{v*}=\{\tilde{\tilde{v}}^{*}_{ij},\tilde{\tilde{v}}^{*}_{ij},\ldots,\tilde{\tilde{v}}^{*}_{ij}\}$ (Equation 51), and $N^{e-}=\{\tilde{\tilde{e}}_{ij},\tilde{\tilde{e}}_{ij},\ldots,\tilde{\tilde{e}}_{ij}\}$ (Equation 53) is $m\times n$. However, it should be the number of criteria, i.e., *m*. Also, $\max_j \tilde{\tilde{e}}_{ij}$ and $\max_j \tilde{\tilde{v}}_{ij}$ should be used to *PI* solutions, i.e., $P^{e*}=\{\tilde{\tilde{e}}^{*}_{ij},\tilde{\tilde{e}}^{*}_{ij},\ldots,\tilde{\tilde{e}}^{*}_{ij}\}=\{\max_i \tilde{\tilde{e}}_{ij}\,|\,i\in Benefit\}$ and $P^{v*}=\{\tilde{\tilde{v}}^{*}_{ij},\tilde{\tilde{v}}^{*}_{ij},\ldots,\tilde{\tilde{v}}^{*}_{ij}\}=\{\max_i \tilde{\tilde{v}}_{ij}\,|\,i\in Benefit\}$, respectively. In other words, $\max \tilde{\tilde{e}}_{ij}$ and $\max \tilde{\tilde{v}}_{ij}$ should be determined for hatch covers instead of criteria. Moreover, the negative sign should be taken into account in $N^{e-}=\{\tilde{\tilde{e}}_{ij},\tilde{\tilde{e}}_{ij},\ldots,\tilde{\tilde{e}}_{ij}\}=\left\{\min_j \tilde{\tilde{e}}_{ij}\,|\,i\in Benefit\right\}$ for $\tilde{\tilde{e}}_{ij}$. On the other hand, the above expressions are related to *BC*. There are the different MCDM problems in which the *CC* are used for appraisals.
Then, the S_j (average) and R_j (worst) group scores are determined as below defined:

$$S_j=\sum_{i=1}^{m}\frac{1}{2}(S^U_{ij}+S^L_{ij}),\qquad \forall j=1,\ldots,n, \tag{55}$$

$$R_j=\max_i\left(\frac{1}{2}\left(S^U_{ij}+S^L_{ij}\right)\right),\qquad \forall j=1,\ldots,n, \tag{56}$$

where

$$S^U_{ij}=\sum_j\frac{\sqrt{\frac{1}{4}\sum_{k=1}^{4}\left[(f^{U*}_{i1}-f^{U}_{i4})+(f^{U*}_{i2}-f^{U}_{i3})+(f^{U*}_{i3}-f^{U}_{i2})+(f^{U*}_{i4}-f^{U}_{i1})\right]}}{\sqrt{\frac{1}{4}\sum_{k=1}^{4}\left[(e^{U*}_{i1}-e^{U-}_{i4})+(e^{U*}_{i2}-e^{U-}_{i3})+(e^{U*}_{i3}-e^{U-}_{i2})+(e^{U*}_{i4}-e^{U-}_{i1})\right]}},\qquad \forall j=1,\ldots,n, \tag{57}$$

$$S^L_{ij}=\sum_j\frac{\sqrt{\frac{1}{4}\sum_{k=1}^{4}\left[(f^{L*}_{i1}-f^{L}_{i4})+(f^{L*}_{i2}-f^{L}_{i3})+(f^{L*}_{i3}-f^{L}_{i2})+(f^{L*}_{i4}-f^{L}_{i1})\right]}}{\sqrt{\frac{1}{4}\sum_{k=1}^{4}\left[(e^{L*}_{i1}-e^{L-}_{i4})+(e^{L*}_{i2}-e^{L-}_{i3})+(e^{L*}_{i3}-e^{L-}_{i2})+(e^{L*}_{i4}-e^{L-}_{i1})\right]}},\qquad \forall j=1,\ldots,n. \tag{58}$$

Since S_j and R_j are on index *j* (hatch covers), it is better to select S^U_{ji} and S^L_{ji} in Eqs. (57) to (58). In Eqs. (57) to (58), summation $\sum_{k=1}^{4}$ is not necessary. If it is related to summation of the performance appraisals of DMs, it is unnecessary to use $\sum_{k=1}^{4}$ (measures of f^{U*}_{ir}, f^{U}_{ir}, f^{L*}_{ir}, f^{L}_{ir}, e^{U*}_{ir}, e^{U-}_{ir} e^{L*}_{ir}, e^{L-}_{ir} (*r*=1,2,3,4) are the average of IT2F performance appraisals themselves). On the other hand, reference points with identical numbers should be used at Euclidean distances. This issue has not been satisfied in Eqs. (57) to (58). In addition, $\sum_i$ instead of $\sum_j$ should be used in Eqs. (57) to (58). In other words, the sum of criteria should be used in the Eqs. (57) to (58). Moreover, the expressions in Eqs. (57) to (58) are related to *BC* and the expressions of *CC* should be added to them.

Step 4: Measures of Q_j is computed based on S_j and R_j by Equation 59:

$$Q_j=\nu\frac{(S_j-S^*)}{(S^--S^*)}+(1-\nu)\frac{(R_j-R^*)}{(R^--R^*)},\qquad \forall j=1,\ldots,n, \tag{59}$$

where $S^*=\min_j S_j, S^-=\max_j S_j, R^*=\min_j R_j, R^-=\max_j R_j, \nu\in[0,1]$.

6.2. The application of corrected VIKOR method to an illustrative example

Let there are N criteria $C_j\,(j=1,\ldots,N)$, M hatch covers, $H_i(A_i)(i=1,\ldots,M)$, and L DMs $S_l\,(l=1,\ldots,L)$, as represented in Equation 60. The following steps are the corrected VIKOR method:

Step 1: The mean IT2F performance appraisals of hatch covers with respect to criteria are calculated by Equation 61.

$$E_c = (\tilde{\tilde{e}}_{ij}^{l})_{M\times N} = \begin{array}{c} \\ H_1 \\ H_2 \\ H_3 \\ \vdots \\ H_M \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & \cdots & C_N \end{array} \\ \begin{bmatrix} \tilde{\tilde{e}}_{11}^{l} & \tilde{\tilde{e}}_{12}^{l} & \tilde{\tilde{e}}_{13}^{l} & \cdots & \tilde{\tilde{e}}_{1N}^{l} \\ \tilde{\tilde{e}}_{21}^{l} & \tilde{\tilde{e}}_{22}^{l} & \tilde{\tilde{e}}_{23}^{l} & \cdots & \tilde{\tilde{e}}_{2N}^{l} \\ \tilde{\tilde{e}}_{31}^{l} & \tilde{\tilde{e}}_{32}^{l} & \tilde{\tilde{e}}_{33}^{l} & \cdots & \tilde{\tilde{e}}_{3N}^{l} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \tilde{\tilde{e}}_{M1}^{l} & \tilde{\tilde{e}}_{M2}^{l} & \tilde{\tilde{e}}_{M3}^{l} & \cdots & \tilde{\tilde{e}}_{MN}^{l} \end{bmatrix} \end{array}, \tag{60}$$

where

$$\tilde{\tilde{e}}_{ij} = \left(\frac{\tilde{\tilde{e}}_{ij}^{1} + \tilde{\tilde{e}}_{ij}^{2} + \cdots + \tilde{\tilde{e}}_{ij}^{L}}{L} \right), \qquad 1 \le i \le M,\ 1 \le j \le N,\ 1 \le l \le L. \tag{61}$$

Tables 3 and 4 show the integrated IT2F weights and IT2F performance appraisals of hatch covers with respect to different criteria, respectively, based on the illustrative example applied by Soner et al. [26] in which five hatch covers are to be evaluated by nine criteria.

<Take in **Table 3.** >

<Take in **Table 4.** >

Step 2: The weighted IT2F evaluation matrix is calculated as follows:

$$\tilde{\tilde{V}} = \left[\tilde{\tilde{v}}_{ij}\right]_{M\times N} \tag{62}$$

where

$$\tilde{\tilde{v}}_{ij} = \tilde{\tilde{w}}_j \otimes \tilde{\tilde{e}}_{ij} = ((f_{ij1}^{U}, f_{ij2}^{U}, f_{ij3}^{U}, f_{ij4}^{U}; H_1(\tilde{F}_{ij}^{U}), H_2(\tilde{F}_{ij}^{U})), ((f_{ij1}^{L}, f_{ij2}^{L}, f_{ij3}^{L}, f_{ij4}^{L}; H_1(\tilde{F}_{ij}^{L}), H_2(\tilde{F}_{ij}^{L})). \tag{63}$$

Step 3: The *PI* (P^{e*}, P^{v*}) and *NI* solutions (N^{e-}) of the IT2FNs (both *BC* and *CC*) are given by:

$$P^{e*} = \left\{\tilde{\tilde{e}}_1^{*}, \tilde{\tilde{e}}_2^{*}, \ldots, \tilde{\tilde{e}}_N^{*}\right\} = \left\{ \max_i \tilde{\tilde{e}}_{ij} \,\middle|\, j \in BC, \min_i \tilde{\tilde{e}}_{ij} \,\middle|\, j \in CC, \right\}, \tag{64}$$

$$\begin{aligned} P^{e*} = {} & ((e_{j1}^{U*}, e_{j2}^{U*}, e_{j3}^{U*}, e_{j4}^{U*}; \max_i H_1(\tilde{E}_{ij}^{U}), \max_i H_2(\tilde{E}_{ij}^{U})), \\ & ((e_{j1}^{L*}, e_{j2}^{L*}, e_{j3}^{L*}, e_{j4}^{L*}; \max_i H_1(\tilde{E}_{ij}^{L}), \max_i H_2(\tilde{E}_{ij}^{L})), \qquad \forall j = 1,\ldots,N;\ j \in BC, \end{aligned} \tag{65}$$

$$\begin{aligned} P^{e*} = {} & ((e_{j1}^{U*}, e_{j2}^{U*}, e_{j3}^{U*}, e_{j4}^{U*}; \min_i H_1(\tilde{E}_{ij}^{U}), \min_i H_2(\tilde{E}_{ij}^{U})), \\ & ((e_{j1}^{L*}, e_{j2}^{L*}, e_{j3}^{L*}, e_{j4}^{L*}; \min_i H_1(\tilde{E}_{ij}^{L}), \min_i H_2(\tilde{E}_{ij}^{L})), \qquad \forall j = 1,\ldots,N;\ j \in CC, \end{aligned} \tag{66}$$

$$P^{v*} = \{\tilde{\tilde{v}}_1^*, \tilde{\tilde{v}}_2^*, \ldots, \tilde{\tilde{v}}_N^*\} = \left\{\max_i \tilde{\tilde{v}}_{ij} \,\middle|\, j \in BC, \min_i \tilde{\tilde{v}}_{ij} \,\middle|\, j \in CC\right\}, \tag{67}$$

$$\begin{aligned} P^{v*} = {} & ((f_{j1}^{U*}, f_{j2}^{U*}, f_{j3}^{U*}, f_{j4}^{U*}; \max_i H_1(\tilde{F}_{ij}^U), \max_i H_2(\tilde{F}_{ij}^U)), \\ & ((f_{j1}^{L*}, f_{j2}^{L*}, f_{j3}^{L*}, f_{j4}^{L*}; \max_i H_1(\tilde{F}_{ij}^L), \max_i H_2(\tilde{F}_{ij}^L)), \qquad \forall j = 1, \ldots, N;\ j \in BC, \end{aligned} \tag{68}$$

$$\begin{aligned} P^{v*} = {} & ((f_{j1}^{U*}, f_{j2}^{U*}, f_{j3}^{U*}, f_{j4}^{U*}; \min_i H_1(\tilde{F}_{ij}^U), \min_i H_2(\tilde{F}_{ij}^U)), \\ & ((f_{j1}^{L*}, f_{j2}^{L*}, f_{j3}^{L*}, f_{j4}^{L*}; \min_i H_1(\tilde{F}_{ij}^L), \min_i H_2(\tilde{F}_{ij}^L)), \qquad \forall j = 1, \ldots, N;\ j \in CC, \end{aligned} \tag{69}$$

$$N^{e-} = \{\tilde{\tilde{e}}_1^-, \tilde{\tilde{e}}_2^-, \ldots, \tilde{\tilde{e}}_N^-\} = \left\{\min_i \tilde{\tilde{e}}_{ij}^- \,\middle|\, j \in BC, \max_i \tilde{\tilde{e}}_{ij}^- \,\middle|\, j \in CC\right\}, \tag{70}$$

$$\begin{aligned} N^{e-} = {} & ((e_{j1}^{U-}, e_{j2}^{U-}, e_{j3}^{U-}, e_{j4}^{U-}; \min_i H_1(\tilde{E}_{ij}^U), \min_i H_2(\tilde{E}_{ij}^U)), \\ & ((e_{j1}^{L-}, e_{j2}^{L-}, e_{j3}^{L-}, e_{j4}^{L-}; \min_i H_1(\tilde{E}_{ij}^L), \min_i H_2(\tilde{E}_{ij}^L)), \qquad \forall j = 1, \ldots, N;\ j \in BC, \end{aligned} \tag{71}$$

$$\begin{aligned} N^{e-} = {} & ((e_{j1}^{U-}, e_{j2}^{U-}, e_{j3}^{U-}, e_{j4}^{U-}; \max_i H_1(\tilde{E}_{ij}^U), \max_i H_2(\tilde{E}_{ij}^U)), \\ & ((e_{j1}^{L-}, e_{j2}^{L-}, e_{j3}^{L-}, e_{j4}^{L-}; \max_i H_1(\tilde{E}_{ij}^L), \max_i H_2(\tilde{E}_{ij}^L)), \qquad \forall j = 1, \ldots, N;\ j \in CC, \end{aligned} \tag{72}$$

Based on Soner et al. [26], C_1, C_2, C_5, C_7, and C_8 are of benefit type and C_3, C_4, C_6, and C_9 are of cost type. Tables 5 and 6 represent the *PI* solution (P^{e*}, P^{v*}) based on Eqs. (49) to (52) and Eqs. (64) to (69), respectively. The *PI* solutions (P^{e*}, P^{v*}) obtain regarding maximum of criteria, as shown in Table 5. Obviously, the use of these measures can be resulted in the wrong calculations of the average (S_i) and the worst (R_i) group scores.

<Take in **Table 5.** >

<Take in **Table 6.** >

Table 7 presents the *NI* solution (N^{e-}) based on Eqs. (70) to (72).

<Take in **Table 7.** >

Next, the average (S_i) and the worst (R_i) group scores for each hatch cover are computed as follows:

$$S_i = \sum_{j=1}^{N} \frac{1}{2}(S_{ij}^U + S_{ij}^L), \qquad \forall i = 1, \ldots, M, \tag{73}$$

$$R_i = \max_j \left(\frac{1}{2}\left(S_{ij}^U + S_{ij}^L\right)\right), \qquad \forall i = 1, \ldots, M, \tag{74}$$

where

$$S_{ij}^{U} = \sum_{j\in Benefit} \frac{\sqrt{\frac{1}{4}\left[(f_{j1}^{U*}-f_{ij1}^{U})^2+(f_{j2}^{U*}-f_{ij2}^{U})^2+(f_{j3}^{U*}-f_{ij3}^{U})^2+(f_{j4}^{U*}-f_{ij4}^{U})^2\right]}}{\sqrt{\frac{1}{4}\left[(e_{j1}^{U*}-e_{j1}^{U-})^2+(e_{j2}^{U*}-e_{j2}^{U-})^2+(e_{j3}^{U*}-e_{j3}^{U-})^2+(e_{j4}^{U*}-e_{j4}^{U-})^2\right]}} \qquad \forall i=1,\ldots,M,$$

$$+\sum_{j\in Cost} \frac{\sqrt{\frac{1}{4}\left[(f_{ij1}^{U}-f_{j1}^{U*})^2+(f_{ij2}^{U}-f_{j2}^{U*})^2+(f_{ij3}^{U}-f_{j3}^{U*})^2+(f_{ij4}^{U}-f_{j4}^{U*})^2\right]}}{\sqrt{\frac{1}{4}\left[(e_{j1}^{U-}-e_{j1}^{U*})^2+(e_{j2}^{U-}-e_{j2}^{U*})^2+(e_{j3}^{U-}-e_{j3}^{U*})^2+(e_{j4}^{U-}-e_{j4}^{U*})^2\right]}} \tag{75}$$

$$S_{ij}^{L} = \sum_{j\in Benefit} \frac{\sqrt{\frac{1}{4}\left[(f_{j1}^{L*}-f_{ij1}^{L})^2+(f_{j2}^{L*}-f_{ij2}^{L})^2+(f_{j3}^{L*}-f_{ij3}^{L})^2+(f_{j4}^{L*}-f_{ij4}^{L})^2\right]}}{\sqrt{\frac{1}{4}\left[(e_{j1}^{L*}-e_{j1}^{L-})^2+(e_{j2}^{L*}-e_{j2}^{L-})^2+(e_{j3}^{L*}-e_{j3}^{L-})^2+(e_{j4}^{L*}-e_{j4}^{L-})^2\right]}} \qquad \forall i=1,\ldots,M.$$

$$+\sum_{j\in Cost} \frac{\sqrt{\frac{1}{4}\left[(f_{ij1}^{L}-f_{j1}^{L*})^2+(f_{ij2}^{L}-f_{j2}^{L*})^2+(f_{ij3}^{L}-f_{j3}^{L*})^2+(f_{ij4}^{L}-f_{j4}^{L*})^2\right]}}{\sqrt{\frac{1}{4}\left[(e_{j1}^{L-}-e_{j1}^{L*})^2+(e_{j2}^{L-}-e_{j2}^{L*})^2+(e_{j3}^{L-}-e_{j3}^{L*})^2+(e_{j4}^{L-}-e_{j4}^{L*})^2\right]}} \tag{76}$$

Tables 8 and 9 show the measures of S_{ij}^{U} and S_{ij}^{L} for hatch covers with respect to criteria based on the original and corrected versions, respectively. Obviously, the measures presented in Table 9 are different from calculations obtained by Soner et al. [26].

<Take in **Table 8.** >

<Take in **Table 9.** >

Step 4: The Q_i is calculated based on S_i and R_i by using Equation 77:

$$Q_i = \nu \frac{(S_i - S^*)}{(S^- - S^*)} + (1-\nu)\frac{(R_i - R^*)}{(R^- - R^*)}, \qquad \forall i=1,\ldots,M, \tag{77}$$

where $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, $R^- = \max_i R_i$, $\nu \in [0,1]$.

The final rankings based on indices S_i , R_i , and Q_i are represented in Tables 10 and 11 by using Eqs. (55) to (59) (the original approach) and Eqs. (73) to (77) (the corrected approach). Based on data of these tables and surveying the conditions C_1 and C_2 , the ranking order of the hatch covers regarding the original and corrected approaches is as $H_1 \succ H_2 \succ H_4 \succ H_3 \succ H_5$ (for $\nu = 0.5$). As represented in these tables, there are the obvious differences between two approaches.

On the other hand, values of Q_j have been showed in Tables 12 and 13 based on the original and corrected versions, respectively (for $\nu = 0, 0.1, 0.2, 0.3, 0,4, 0.5, 0.6, 0.7, 0.8, 0.9, \text{and} 1.00$). In addition, Table 14 shows the ranked orders obtained by the various techniques.

<Take in **Table 10.** >

<Take in **Table 11.** >

<Take in **Table 12.** >

<Take in **Table 13.** >

<Take in **Table 14.** >

As represented in Table 14, there are the important differences between the ranking results of original and corrected approaches.

6.3. Implementation of entropy-based IT2F WASPAS approach in the illustrative example

Now, in order to show effectiveness and compare results obtained by entropy-based IT2F WASPAS method with the others, it is implemented in the above illustrative example. By depicting alpha cuts for w_t^s and the matrix of IT2F performance measures (Table 4) and applying Eqs. (35) to (39), Table 15 represents w_t^* based on w_t^o and w_t^s .

<Take in **Table 15.** >

Using Table 15 and Eqs. (42) to (43) (by calculating the measures of $Q_r^{(1)}$ and $Q_r^{(2)}$), the aggregated optimality function score ($\hat{Q}_{r\alpha}$) of the WASPAS method for each hatch cover $H_r (r = 1,\ldots,5)$ is calculated and the results are then presented in Tables 16-18 (for $\lambda = 0, 0.5, 1.0$).

<Take in **Table 16.** >

<Take in **Table 17.** >

<Take in **Table 18.** >

Tables 19 and 20 show measures of *LDM*s (by using Equation 24) and the ranking order of hatch covers (for $\lambda = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, \text{and} 1.0$), respectively.

<Take in **Table 19.** >

<Take in **Table 20.** >

As shown in Table 20, the ranking result is as $H_2 > H_3 > H_1 > H_5 > H_4$ (for $\lambda = 0.5$). Thus, the hatch cover 2 is opted as the optimal hatch cover.

6.4. Accommodation with other techniques

To better explain the reliability of suggested approach, the orders obtained from entropy- based IT2F WASPAS method are compared with several studies. As shown in Table 21, the ranking order is as $H_2 > H_3 > H_1 > H_5 > H_4$ and $H_2 > H_3 > H_1 > H_4 > H_5$ when the entropy-based IT2F WASPAS and IT2F TOPSIS methods [27] were utilized, respectively. Obviously, there are the similar orders for hatch covers 2, 3, and 1 such that these results verify the efficiency of our ranking method. Also, there are the similar situations for hatch covers 2 and 3 as $A_2 > A_3$ in the corrected VIKOR approach ($\lambda = 0.5$). Thus, hatch cover 2 is opted as the best hatch cover. Unfortunately, the authors acquired the different interpretation regarding the interval type 2 VIKOR method [26]. The orders acquired by indices S_i , R_i , and Q_i are as $S_1 < S_2 < S_4 < S_3 < S_5$, $R_1 < R_4 < R_2, R_3 < R_5$, and $Q_1 < Q_2 < Q_4, Q_3 < Q_5$, respectively, that are different from the corrected interval type-2 VIKOR method (see Tables 10 and 11). As shown in Figure 3, the interval type-2 VIKOR method [26] has a different behavior than the others. Based on Figure 3, hatch covers 2 and 3 have the similar priorities according to the last three methods.

<Take in **Table 21.** >

<Take in **Figure 3.** >

6.5. Sensitivity analysis

In the WASPAS method, the parameter λ plays important role in the constancy of the ranking results. In order to show the reliability of the ranking results, different values of the parameter λ should be checked. To this end, the authors considered eleven measures for the parameter λ in order to interpret the different measures of parameters in our approach. As shown in Figure 4, the ranking results are almost reliable for $\lambda \leq 0.7$. In other words, when the aggregated optimality function score of the WASPAS method behaves like WSM, the reliability of the method is reduced. In order to survey the resemblance of the orders for all parameters λ , Spearman correlation coefficient was calculated. Table 22 shows the correlation coefficients. Coefficients measures are bigger than 0.8 for $\lambda \leq 0.7$, as presented in this table. This issue shows that the suggested approach has acceptable stability for $\lambda \leq 0.7$. Moreover, the coefficients measures are less than 0.8 for $\lambda \geq 0.8$, i.e., when the WASPAS model is transformed with WSM. It deduces that the ranking priorities are steady and authentic based on our approach. This argument is consistent with the above conclusion.

<Take in **Figure 4.** >

<Take in **Table 22.** >

7. Conclusions

The T2FSs can greatly reduce hesitancy when solving decision-making problems. This research presents *LDM*s to prioritize the IT2FNs and appraise the HCEP. In order to present the reliability of our approach, it is also implemented in an illustrative example where a synthetic collective decision-making method was used to appraise the HCEP where preferences of DMs are stated by IT2FNs. In real, the DMs have decided to prioritize one type of maritime transportation equipment (hatch cover) with respect to a collection of criteria where all criteria weights are defined as IT2FNs. In our approach, the integrated T2FWs and appraisals are first calculated. Then, the IT2F Shannon entropy approach is applied to merge the subjective and objective weights. Finally, the hatch covers are ranked by WASPAS. Lastly, the results show that the hatch cover 2 is the best hatch cover.

The authors also have expressed the VIKOR method's drawbacks done by Soner et al. [26]. As shown in Tables 5-14, there are many differences in the calculations results between the existence and corrected VIKOR method. For example, the order of indices S_i, R_i, and Q_i are as $S_1 < S_2 < S_4 < S_3 < S_5$, $R_1 < R_4 < R_2, R_3 < R_5$, and $Q_1 < Q_2 < Q_4 < Q_3 < Q_5$ based on the original version and $S_2 < S_3 < S_1 < S_4 < S_5$, $R_2 < R_3 < R_4 < R_1 < R_5$, and $Q_2 < Q_3 < Q_4 < Q_1 < Q_5$ regarding the corrected version. Therefore, the ranking order is not similar for both approaches. On the other hand, the reference points of linguistic variables used for appraisals are decimals. If one applies integers to the reference points, more differences may be obtained in calculations of ideal solutions and Euclidean distances such that the different ranking results will attain for other decision-making problems. In addition, the ranking order obtained from the original version has many differences with the others in Table 21. Based on the obtained results, the corrected version, IT2F TOPSIS [27], and entropy-based IT2F WASPAS approaches have the closer results. Moreover, the ranking results attained by entropy-based IT2F WASPAS approach have high correlation coefficients for $\lambda \leq 0.7$. In other words, when entropy-based IT2F WASPAS approach tends to WSM (for $\lambda \geq 0.8$), it has less reliability. Finally, hatch cover 2 is opted as the best hatch cover using the DMs' point of views based on all methods.

Our methodology was handled for hatch cover. Nonetheless, it can be applied to the other maritime equipment like propeller, lashing bar, etc. In addition, other criteria can be offered for appraising hatch covers. In the proposed *LDM*s, the measure of variations between alpha cuts was opted 0.1. DMs can select the minor values (for example, 0.03). The framework of suggested MCDM is effectiveness for different options and criteria. Moreover, DMs can use it to other decision-making branches of knowledge such as VIKOR, AHP, etc.

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List of figures captions:

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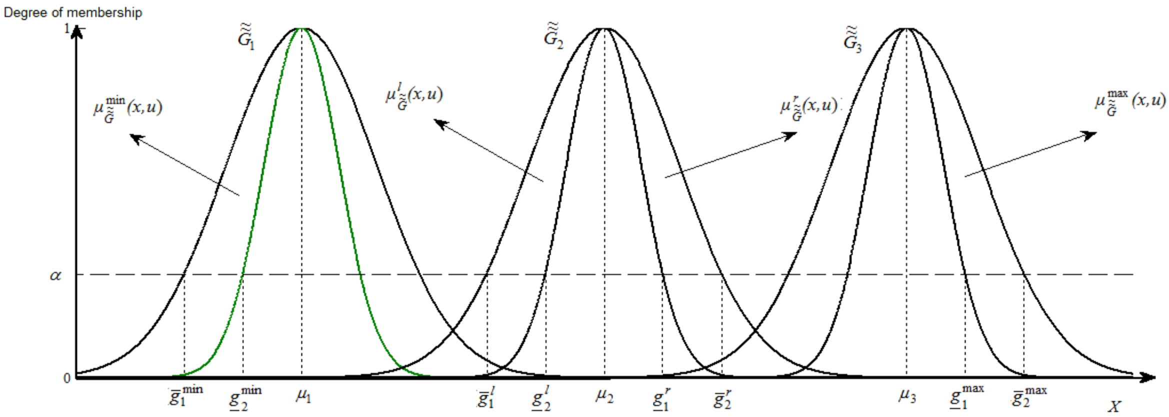

Figure 1. The representation of reference bounds for GIT2FNs.

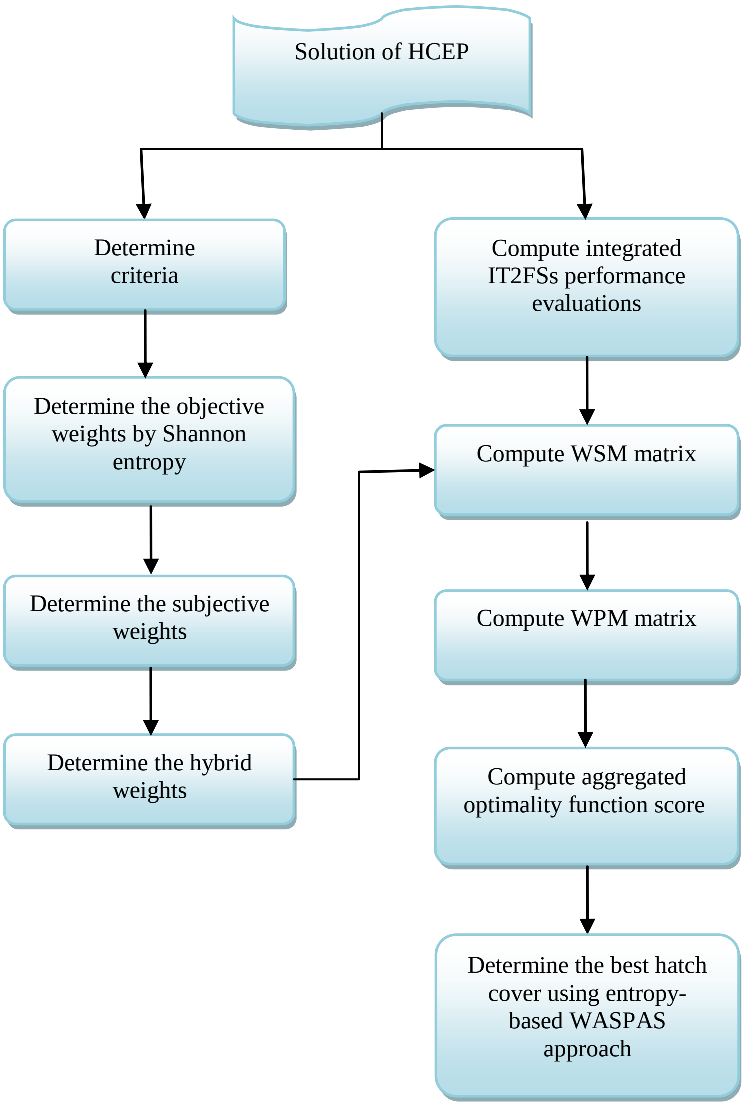

Figure 2. The framework of the proposed methodology.

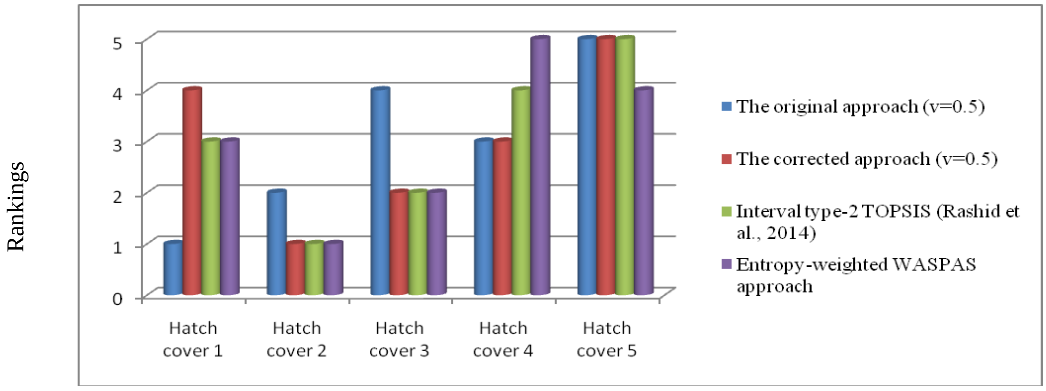

Figure 3. Rankings of hatch cover.

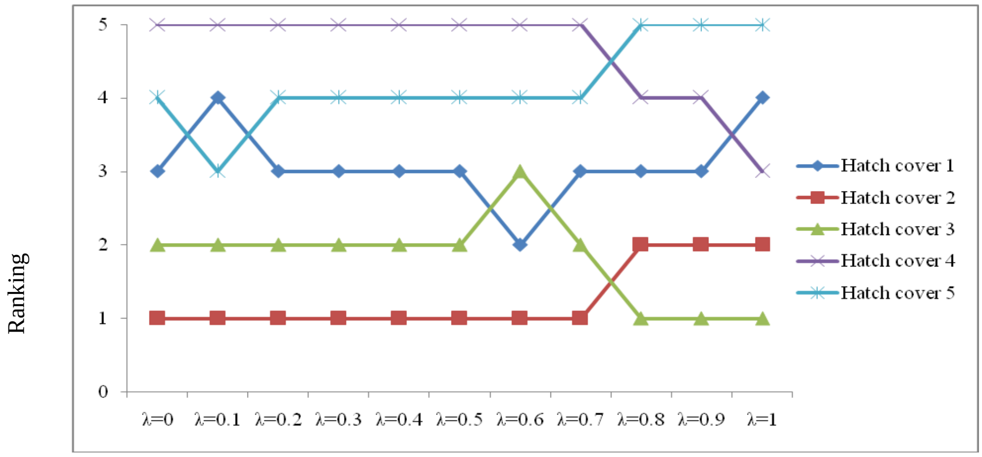

Figure 4. Ranking of hatch covers for different measures λ.

Table 1. The different concepts of T2FSs.

T2FSs [27]
Footprint of uncertainty (FOU) [28]
Normal Gaussian interval type-2 fuzzy numbers (GIT2FNs) [29]
Symmetric GIT2FN represented by Figure 1 [29]
α -cut of T2FSs [30]
Arithmetic calculations of T2FSs [27]

Table 2. The IT2FMCDM matrix for the HCEP.

Hatch covers	Criteria										
		C_1		$\cdots$		C_t		$\cdots$		C_T	
H_1	$\tilde{\tilde{x}}_{11}^{1}$	$\cdots$	$\tilde{\tilde{x}}_{11}^{L}$	$\cdots$	$\tilde{\tilde{x}}_{1t}^{1}$	$\cdots$	$\tilde{\tilde{x}}_{1t}^{L}$	$\cdots$	$\tilde{\tilde{x}}_{1T}^{1}$	$\cdots$	$\tilde{\tilde{x}}_{1T}^{L}$
$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\cdots$	$\vdots$
H_r	$\tilde{\tilde{x}}_{r1}^{1}$	$\cdots$	$\tilde{\tilde{x}}_{r1}^{L}$	$\cdots$	$\tilde{\tilde{x}}_{rt}^{1}$	$\cdots$	$\tilde{\tilde{x}}_{rt}^{L}$	$\cdots$	$\tilde{\tilde{x}}_{rT}^{1}$	$\cdots$	$\tilde{\tilde{x}}_{rT}^{L}$
$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\cdots$	$\vdots$
H_R	$\tilde{\tilde{x}}_{R1}^{1}$	$\cdots$	$\tilde{\tilde{x}}_{R1}^{L}$	$\cdots$	$\tilde{\tilde{x}}_{Rt}^{1}$	$\cdots$	$\tilde{\tilde{x}}_{Rt}^{L}$	$\cdots$	$\tilde{\tilde{x}}_{RT}^{1}$	$\cdots$	$\tilde{\tilde{x}}_{RT}^{L}$

Table 3. The IT2F weights [26].

Criteria	IT2FSs
C_1	((0.204;0.253;0.253;0.3;1;1),(0.229;0.253;0.253;0.277;0.9;0.9))
C_2	((0.085;0.112;0.112;0.141;1;1),(0.098;0.112;0.112;0.126;0.9;0.9))
C_3	((0.117;0.155;0.155;0.197;1;1),(0.136;0.155;0.155;0.174;0.9;0.9))
C_4	((0.044;0.058;0.058;0.078;1;1),(0.051;0.058;0.058;0.067;0.9;0.9))
C_5	((0.055;0.073;0.073;0.093;1;1),(0.064;0.073;0.073;0.082;0.9;0.9))
C_6	((0.043;0.057;0.057;0.077;1;1),(0.05;0.057;0.057;0.066;0.9;0.9))
C_7	((0.019;0.025;0.025;0.035;1;1),(0.022;0.025;0.025;0.029;0.9;0.9))
C_8	((0.017;0.022;0.022;0.03;1;1),(0.019;0.022;0.022;0.025;0.9;0.9))
C_9	((0.027;0.036;0.036;0.049;1;1),(0.031;0.036;0.036;0.041;0.9;0.9))

Table 4. The synthetic IT2F performance measures.

	H_1	H_2	H_3
C_1	((0.770,0.930,0.930,1.000;1), (0.850,0.930,0.930,0.970;0.9))	((0.700,0.870,0.870,0.970;1), (0.780,0.870,0.870,0.920;0.9))	((0.630,0.830,0.830,0.970;1), (0.730,0.830,0.830,0.900;0.9))
C_2	((0.630,0.800,0.800,0.900;1), (0.720,0.800,0.800,0.850;0.9))	((0.630,0.830,0.830,0.970;1), (0.730,0.830,0.830,0.900;0.9))	((0.400,0.570,0.570,0.730;1), (0.480, 0.570,0.570,0.650;0.9))
C_3	((0.570,0.770,0.770,0.900;1), (0.670,0.770,0.770,0.830;0.9))	((0.130,0.300,0.300,0.500;1), (0.220,0.300,0.300,0.400;0.9))	((0.130,0.300,0.300,0.500;1), (0.220,0.300,0.300,0.400;0.9))
C_4	((0.300,0.500,0.500,0.700;1), (0.400,0.500,0.500,0.600;0.9))	((0.430,0.630,0.630,0.800;1), (0.530,0.630,0.630,0.720;0.9))	((0.100,0.200,0.230,0.370;1), (0.150,0.200,0.200,0.280;0.9))
C_5	((0.430,0.630,0.630,0.830;1), (0.530,0.630,0.630,0.730;0.9))	((0.470,0.630,0.630,0.770;1), (0.550,0.630,0.630,0.770;0.9))	((0.070,0.230,0.230,0.430;1), (0.150,0.230,0.230,0.330;0.9))
C_6	((0.400,0.570,0.570,0.730;1), (0.480,0.570,0.570,0.650;0.9))	((0.300,0.500,0.500,0.700;1), (0.400,0.500,0.500,0.600;0.9))	((0.100,0.200,0.230,0.370;1), (0.150,0.200,0.200,0.280;0.9))
C_7	((0.400,0.570,0.570,0.730;1), (0.480,0.570,0.570,0.650;0.9))	((0.530,0.670,0.670,0.770;1), (0.600,0.670,0.670,0.720;0.9))	((0.130,0.300,0.300,0.500;1), (0.220,0.300,0.300,0.400;0.9))
C_8	((0.130,0.300,0.300,0.500;1), (0.220,0.300,0.300,0.400;0.9))	((0.430,0.630,0.630,0.800;1), (0.530,0.630,0.630,0.720;0.9))	((0.630,0.830,0.830,0.970;1), (0.730,0.830,0.830,0.900;0.9))
C_9	((0.030,0.170,0.170,0.370;1), (0.100,0.170,0.170,0.270;0.9))	((0.270,0.430,0.430,0.600;1), (0.350,0.430,0.430,0.520;0.9))	((0.630,0.800,0.800,0.900;1), (0.720,0.800,0.800,0.850;0.9))
	H_4	H_5	
C_1	((0.630,0.830,0.830,0.970;1), (0.730,0.830,0.830,0.900;0.9))	((0.400,0.570,0.570,0.730;1), (0.480,0.570,0.570,0.650;0.9))	
C_2	((0.270,0.400,0.430,0.570;1), (0.330,0.400,0.400,0.480;0.9))	((0.130,0.300,0.300,0.500;1), (0.220,0.300,0.300,0.400;0.9))	
C_3	((0.300,0.500,0.500,0.700;1), (0.400,0.500,0.500,0.600;0.9))	((0.270,0.430,0.430,0.630;1), (0.350,0.430,0.430,0.530;0.9))	
C_4	((0.100,0.200,0.230,0.370;1), (0.150,0.200,0.200,0.280;0.9))	((0.630,0.830,0.830,0.970;1), (0.730,0.830,0.830,0.900;0.9))	
C_5	((0.130,0.300,0.300,0.500;1), (0.220,0.300,0.300,0.400;0.9))	((0.300,0.500,0.500,0.700;1), (0.400,0.500,0.500,0.600;0.9))	
C_6	((0.130,0.300,0.300,0.500;1), (0.220,0.300,0.300,0.400;0.9))	((0.630,0.800,0.800,0.900;1), (0.720,0.800,0.800,0.850;0.9))	
C_7	((0.200,0.370,0.370,0.570;1), (0.280,0.370,0.370,0.470;0.9))	((0.100,0.200,0.230,0.370;1), (0.150,0.200,0.200,0.280;0.9))	
C_8	((0.570,0.770,0.770,0.900;1), (0.670,0.770,0.770,0.830;0.9))	((0.330,0.500,0.500,0.670;1), (0.420,0.500,0.500,0.580;0.9))	
C_9	((0.270,0.430,0.430,0.630;1), (0.350,0.430,0.430,0.530;0.9))	((0.500,0.670,0.670,0.800;1), (0.580,0.670,0.670,0.730;0.9))	

Table 5. The *PI* solutions (P^{e*}, P^{v*}) based on Eqs. (49) to (52).

	P^{e*}	P^{v*}
H_1	((0.770,0.930,0.930,1.000;1), (0.850,0.800,0.800,0.900;0.9))	((0.157,0.235,0.235,0.300;1), (0.194,0.202,0.202,0.249;0.9))
H_2	((0.700,0.870,0.870,0.970;1), (0.730,0.870,0.870,0.920;0.9))	((0.142,0.220,0.220,0.291;1), (0.160,0.220,0.220,0.254;0.9))
H_3	((0.630,0.830,0.830,0.970;1), (0.730,0.830,0.830,0.900;0.9))	((0.128,0.209,0.209,0.291;1), (0.167,0.209,0.209,0.249;0.9))
H_4	((0.630,0.830,0.830,0.970;1), (0.730,0.830,0.830,0.900;0.9))	((0.128,0.209,0.209,0.291;1), (0.167,0.209,0.209,0.249;0.9))
H_5	((0.630,0.830,0.830,0.970;1), (0.730,0.830,0.830,0.900;0.9))	((0.081,0.144,0.144,0.219;1), (0.109,0.144,0.144,0.180;0.9))

Table 6. The *PI* solutions (P^{e*} , P^{v*}) based on Eqs. (64) to (69).

	P^{e*}	P^{v*}
C_1	((0.770,0.930,0.930,1.000;1), (0.850,0.870,0.870,0.920;0.9))	((0.157,0.235,0.235,0.300;1), (0.194,0.220,0.220,0.254;0.9))
C_2	((0.630,0.830,0.830,0.970;1), (0.730,0.830,0.830,0.900;0.9))	((0.053,0.092,0.092,0.136;1), (0.071,0.092,0.092,0.113;0.9))
C_3	((0.130,0.300,0.300,0.500;1), (0.220,0.300,0.300,0.400;0.9))	((0.066,0.119,0.119,0.177;1), (0.015,0.046,0.046,0.098;0.9))
C_4	((0.100,0.200,0.230,0.370;1), (0.150,0.200,0.200,0.280;0.9))	((0.004,0.011,0.013,0.028;1), (0.007,0.011,0.011,0.018;0.9))
C_5	((0.470,0.630,0.630,0.830;1), (0.550,0.630,0.630,0.730;0.9))	((0.025,0.045,0.045,0.077;1), (0.035,0.045,0.045,0.059;0.9))
C_6	((0.100,0.200,0.230,0.370;1), (0.150,0.200,0.200,0.280;0.9))	((0.004,0.011,0.013,0.028;1), (0.007,0.011,0.011,0.018;0.9))
C_7	((0.530,0.670,0.670,0.770;1), (0.600,0.670,0.670,0.720;0.9))	((0.010,0.016,0.016,0.026;1), (0.013,0.016,0.016,0.020;0.9))
C_8	((0.630,0.830,0.830,0.970;1), (0.730,0.830,0.830,0.900;0.9))	((0.010,0.018,0.018,0.029;1), (0.013,0.018,0.018,0.022;0.9))
C_9	((0.030,0.170,0.170,0.370;1), (0.100,0.170,0.170,0.270;0.9))	((0.000,0.006,0.006,0.018;1), (0.003,0.006,0.006,0.011;0.9))

Table 7. The *NI* solutions (N^{e-}) based on Eqs. (70) to (72).

	N^{e-}
C_1	((0.400,0.570,0.570,0.730;1), (0.480,0.570,0.570,0.650;0.9))
C_2	((0.130,0.300,0.300,0.500;1), (0.220,0.300,0.300,0.400;0.9))
C_3	((0.570,0.770,0.770,0.900;1), (0.670,0.770,0.770,0.830;0.9))
C_4	((0.630,0.830,0.830,0.970;1), (0.730,0.830,0.830,0.900;0.9))
C_5	((0.070,0.230,0.230,0.430;1), (0.150,0.230,0.230,0.330;0.9))
C_6	((0.630,0.800,0.800,0.900;1), (0.720,0.800,0.800,0.850;0.9))
C_7	((0.100,0.200,0.230,0.370;1), (0.150,0.200,0.200,0.280;0.9))
C_8	((0.130,0.300,0.300,0.500;1), (0.220,0.300,0.300,0.400;0.9))
C_9	((0.630,0.800,0.800,0.900;1), (0.720,0.800,0.800,0.850;0.9))

Table 8. The upper (S_{ij}^{U}) and lower (S_{ij}^{L}) group scores based on the original approach.

	C_1		C_2		C_3		C_4		C_5	
	S_{ij}^{U}	S_{ij}^{L}	S_{ij}^{U}	S_{ij}^{L}	S_{ij}^{U}	S_{ij}^{L}	S_{ij}^{U}	S_{ij}^{L}	S_{ij}^{U}	S_{ij}^{L}
H_1	0.1400	0.0700	0.1100	0.0700	0.1200	0.0600	0.0800	0.0600	0.0500	0.0300
H_2	0.1400	0.0700	0.1100	0.0700	0.1200	0.0600	0.0800	0.0600	0.0500	0.0300
H_3	0.1500	0.0800	0.1300	0.1000	0.1500	0.1200	0.1000	0.0900	0.0900	0.0700
H_4	0.1500	0.0800	0.1500	0.1200	0.1300	0.0800	0.1000	0.0900	0.0800	0.0600
H_5	0.1800	0.1400	0.1600	0.1400	0.1400	0.0900	0.0600	0.0400	0.0600	0.0500

(continued)

	C_6		C_7		C_8		C_9	
	S_{ij}^{U}	S_{ij}^{L}	S_{ij}^{U}	S_{ij}^{L}	S_{ij}^{U}	S_{ij}^{L}	S_{ij}^{U}	S_{ij}^{L}
H_1	0.0600	0.0500	0.0700	0.0600	0.0800	0.0700	0.0900	0.0700
H_2	0.0700	0.0500	0.0700	0.0600	0.0800	0.0600	0.0800	0.0600
H_3	0.0900	0.0800	0.0800	0.0700	0.0700	0.0500	0.0600	0.0500
H_4	0.0800	0.0700	0.0800	0.0600	0.0700	0.0600	0.0700	0.0600
H_5	0.0500	0.0400	0.0800	0.0700	0.0800	0.0600	0.0700	0.0500

Table 9. The upper (S_{ij}^{U}) and lower (S_{ij}^{L}) group scores based on the corrected approach.

	C_1		C_2		C_3		C_4		C_5	
	S_{ij}^{U}	S_{ij}^{L}	S_{ij}^{U}	S_{ij}^{L}	S_{ij}^{U}	S_{ij}^{L}	S_{ij}^{U}	S_{ij}^{L}	S_{ij}^{U}	S_{ij}^{L}
H_1	0.0000	0.0410	0.0106	0.0076	0.1564	0.1551	0.0303	0.0284	0.0027	0.0016
H_2	0.0398	0.0550	0.0001	0.0000	0.0000	0.0000	0.0422	0.0405	0.0069	0.0030
H_3	0.0681	0.0504	0.0557	0.0553	0.0000	0.0000	0.0000	0.0000	0.0747	0.0732
H_4	0.0681	0.0504	0.0903	0.0914	0.0697	0.0670	0.0000	0.0000	0.0619	0.0604
H_5	0.2480	0.2496	0.1131	0.1122	0.0467	0.0444	0.0614	0.0589	0.0254	0.0245

(continued)

	C_6		C_7		C_8		C_9	
	S_{ij}^{U}	S_{ij}^{L}	S_{ij}^{U}	S_{ij}^{L}	S_{ij}^{U}	S_{ij}^{L}	S_{ij}^{U}	S_{ij}^{L}
H_1	0.0375	0.0358	0.0052	0.0053	0.0229	0.0220	0.0000	0.0000
H_2	0.0316	0.0294	0.0000	0.0000	0.0085	0.0082	0.0154	0.0149
H_3	0.0000	0.0000	0.0204	0.0197	0.0000	0.0000	0.0370	0.0360
H_4	0.0109	0.0101	0.0162	0.0160	0.0029	0.0027	0.0162	0.0151
H_5	0.0594	0.0577	0.0262	0.0253	0.0143	0.013799	0.0295	0.0284

Table 10. The final rankings of hatch covers based on Eqs. (55) to (59).

	H_1	H_2	H_3	H_4	H_5
S_i	0.6760	0.7000	0.8130	0.8020	0.7740
R_i	0.1050	0.1360	0.1360	0.1330	0.1590
Q_i $(\nu = 0.5)$	0.0000	0.3800	0.7900	0.7220	0.8580

Table 11. The final rankings of hatch covers based on Eqs. (73) to (77).

	H_1	H_2	H_3	H_4	H_5
S_i	0.2815	0.1481	0.2454	0.3249	0.6197
R_i	0.1558	0.0474	0.0739	0.0908	0.2488
Q_i $(\nu = 0.5)$	0.4106	0.0001	0.1692	0.2954	1.0003

Table 12. The values of Q_j of the different maximum group utilities based on the original approach.

	$v=0$	$v=0.1$	$v=0.2$	$v=0.3$	$v=0.4$	$v=0.5$	$v=0.6$	$v=0.7$	$v=0.8$	$v=0.9$	$v=1.0$
H_1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
H_2	0.5800	0.5400	0.5000	0.4600	0.4200	0.3800	0.3390	0.2990	0.2590	0.2190	0.1790
H_3	0.5800	0.6220	0.6640	0.7060	0.7480	0.7900	0.8320	0.8740	0.9160	0.9580	1.0000
H_4	0.5250	0.5650	0.6040	0.6430	0.6830	0.7220	0.7620	0.8010	0.8400	0.8800	0.9190
H_5	1.0000	0.9720	0.9430	0.9150	0.8870	0.8580	0.8300	0.8020	0.7740	0.7450	0.7170

Table 13. The values of Q_i of the different maximum group utilities based on the corrected approach.

	$v=0$	$v=0.1$	$v=0.2$	$v=0.3$	$v=0.4$	$v=0.5$	$v=0.6$	$v=0.7$	$v=0.8$	$v=0.9$	$v=1.0$
H_1	0.5385	0.5129	0.4873	0.4618	0.4362	0.4106	0.3850	0.3595	0.3339	0.3083	0.2827
H_2	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
H_3	0.1321	0.1395	0.1469	0.1543	0.1617	0.1692	0.1766	0.1840	0.1914	0.1988	0.2062
H_4	0.2159	0.2318	0.2477	0.2636	0.2795	0.2954	0.3113	0.3272	0.3431	0.3590	0.3749
H_5	1.0006	1.0006	1.0005	1.0005	1.0004	1.0003	1.0003	1.0002	1.0002	1.0001	1.0001

Table 14. The comparison of obtained ranking results with other approaches.

Hatch covers	The original approach (v=0.5)	The corrected approach (v=0.5)
H_1	1	4
H_2	2	1
H_3	4	2
H_4	3	3
H_5	5	5

Table 15. w_t^* with respect to criteria for $\alpha = 0.2, 0.4, 0.6, \text{and}\ 0.8$.

	Criteria							
	C_1				C_2			
α_p	$\overline{w}^{*l}_{1t_\alpha}$	$\underline{w}^{*l}_{2t_\alpha}$	$\underline{w}^{*r}_{1t_\alpha}$	$\overline{w}^{*r}_{2t_\alpha}$	$\overline{w}^{*l}_{1t_\alpha}$	$\underline{w}^{*l}_{2t_\alpha}$	$\underline{w}^{*r}_{1t_\alpha}$	$\overline{w}^{*r}_{2t_\alpha}$
0.2	0.0056	0.0195	0.1678	0.5311	0.0088	0.0248	0.1684	0.4960
0.4	0.0102	0.0255	0.1270	0.2941	0.0146	0.0318	0.1316	0.2850
0.6	0.0177	0.0332	0.0963	0.1692	0.0236	0.0407	0.1033	0.1718
0.8	0.0297	0.0431	0.0728	0.0988	0.0377	0.0521	0.0814	0.1066
	C_3				C_4			
0.2	0.0189	0.0590	0.3673	1.0000	0.0105	0.0280	0.1628	0.4526
0.4	0.0342	0.0754	0.2924	0.6014	0.0173	0.0349	0.1295	0.2725
0.6	0.0574	0.0956	0.2335	0.3776	0.0273	0.0434	0.1034	0.1716
0.8	0.0924	0.1207	0.1866	0.2422	0.0421	0.0539	0.0825	0.1107
	C_5				C_6			
0.2	0.0050	0.0162	0.1248	0.3897	0.0109	0.0293	0.1662	0.4549
0.4	0.0091	0.0211	0.0965	0.2187	0.0180	0.0364	0.1327	0.2747
0.6	0.0157	0.0275	0.0749	0.1287	0.0284	0.0452	0.1063	0.1735
0.8	0.0265	0.0357	0.0584	0.0778	0.0438	0.0560	0.0852	0.1123
	C_7				C_8			
0.2	0.0019	0.0061	0.0454	0.1444	0.0015	0.0038	0.0312	0.0999
0.4	0.0034	0.0079	0.0352	0.0810	0.0026	0.0051	0.0240	0.0558
0.6	0.0058	0.0102	0.0274	0.0475	0.0044	0.0067	0.0185	0.0327
0.8	0.0095	0.0131	0.0213	0.0286	0.0072	0.0088	0.0144	0.0196
	C_9							
0.2	0.0113	0.0276	0.1407	0.3754				
0.4	0.0178	0.0339	0.1140	0.2299				
0.6	0.0272	0.0416	0.0927	0.1475				
0.8	0.0406	0.0510	0.0757	0.0972				

Table 16. The aggregated optimality function score ($\hat{Q}_{r\alpha}$) for $\lambda = 0$ (WPM).

	Criteria							
	H_1				H_2			
α_p	$\overline{Q}^l_{1r_\alpha}$	$\underline{Q}^l_{2r_\alpha}$	$\underline{Q}^r_{1r_\alpha}$	$\overline{Q}^r_{2r_\alpha}$	$\overline{Q}^l_{1r_\alpha}$	$\underline{Q}^l_{2r_\alpha}$	$\underline{Q}^r_{1r_\alpha}$	$\overline{Q}^r_{2r_\alpha}$
0.2	0.0156	0.2684	0.8556	0.9603	0.0263	0.3251	0.8779	0.9647
0.4	0.0969	0.3635	0.8113	0.9196	0.1327	0.4245	0.8416	0.9329
0.6	0.2547	0.4583	0.7579	0.8570	0.3110	0.5196	0.7971	0.8820
0.8	0.4415	0.5480	0.6944	0.7650	0.5029	0.6067	0.7432	0.8044
	H_3				H_4			
0.2	0.0110	0.2721	0.8720	0.9647	0.0001	0.0078	0.1403	0.4338
0.4	0.0884	0.3749	0.8322	0.9313	0.0017	0.0132	0.1064	0.2515
0.6	0.2543	0.4764	0.7828	0.8766	0.0073	0.0206	0.0797	0.1487
0.8	0.4538	0.5711	0.7225	0.7914	0.0194	0.0304	0.0587	0.0865
	H_5							
0.2	0.0035	0.1720	0.7987	0.9314				
0.4	0.0434	0.2584	0.7461	0.8807				
0.6	0.1604	0.3516	0.6838	0.8047				
0.8	0.3337	0.4453	0.6110	0.6955				

Table 17. The aggregated optimality function score ($\hat{Q}_{r\alpha}$) for $\lambda = 0.5$.

	Criteria							
	H_1				H_2			
α_p	$\overline{Q}^{l}_{1r_\alpha}$	$\underline{Q}^{l}_{2r_\alpha}$	$\underline{Q}^{r}_{1r_\alpha}$	$\overline{Q}^{r}_{2r_\alpha}$	$\overline{Q}^{l}_{1r_\alpha}$	$\underline{Q}^{l}_{2r_\alpha}$	$\underline{Q}^{r}_{1r_\alpha}$	$\overline{Q}^{r}_{2r_\alpha}$
0.2	0.0200	0.1753	0.7902	1.7266	0.0268	0.2095	0.8628	1.9575
0.4	0.0708	0.2361	0.6791	1.1177	0.0917	0.2746	0.7410	1.2496
0.6	0.1666	0.3007	0.5864	0.7990	0.2006	0.3424	0.6417	0.8808
0.8	0.2881	0.3680	0.5050	0.5981	0.3297	0.4125	0.5567	0.6570
	H_3				H_4			
0.2	0.0180	0.1810	0.8748	2.0810	0.0109	0.0418	0.4301	1.4950
0.4	0.0677	0.2476	0.7444	1.2960	0.0211	0.0571	0.3232	0.8010
0.6	0.1695	0.3186	0.6383	0.8956	0.0397	0.0775	0.2433	0.4504
0.8	0.3012	0.3930	0.5474	0.6559	0.0725	0.1045	0.1829	0.2588
	H_5							
0.2	0.0098	0.1146	0.6658	1.3763				
0.4	0.0370	0.1673	0.5737	0.9293				
0.6	0.1078	0.2264	0.4935	0.6770				
0.8	0.2150	0.2896	0.4202	0.5057				

Table 18. The aggregated optimality function score ($\hat{Q}_{r\alpha}$) for $\lambda = 1.0$ (WSM).

	Criteria							
	H_1				H_2			
α_p	$\overline{Q}^l_{1r_\alpha}$	$\underline{Q}^l_{2r_\alpha}$	$\underline{Q}^r_{1r_\alpha}$	$\overline{Q}^r_{2r_\alpha}$	$\overline{Q}^l_{1r_\alpha}$	$\underline{Q}^l_{2r_\alpha}$	$\underline{Q}^r_{1r_\alpha}$	$\overline{Q}^r_{2r_\alpha}$
0.2	0.0245	0.0823	0.7248	2.4929	0.0274	0.0938	0.8476	2.9503
0.4	0.0448	0.1087	0.5469	1.3158	0.0508	0.1247	0.6404	1.5664
0.6	0.0786	0.1430	0.4149	0.7411	0.0902	0.1651	0.4861	0.8796
0.8	0.1348	0.1880	0.3156	0.4312	0.1565	0.2184	0.3702	0.5096
	H_3				H_4			
0.2	0.0251	0.0898	0.8776	3.1974	0.0216	0.0758	0.7199	2.5563
0.4	0.0471	0.1204	0.6567	1.6608	0.0404	0.1011	0.5400	1.3504
0.6	0.0846	0.1609	0.4939	0.9146	0.0721	0.1345	0.4068	0.7521
0.8	0.1487	0.2149	0.3724	0.5204	0.1257	0.1786	0.3071	0.4312
	H_5							
0.2	0.0162	0.0572	0.5328	1.8212				
0.4	0.0307	0.0763	0.4012	0.9778				
0.6	0.0552	0.1012	0.3032	0.5493				
0.8	0.0963	0.1340	0.2294	0.3159				

Table 19. The measure of LDMs for the different hatch covers.

Hatch covers	λ										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
H_1	0.597	0.578	0.560	0.542	0.526	0.510	0.495	0.480	0.467	0.454	0.448
H_2	0.627	0.609	0.593	0.577	0.562	0.548	0.535	0.528	0.512	0.501	0.497
H_3	0.605	0.594	0.579	0.568	0.557	0.547	0.447	0.523	0.520	0.512	0.510
H_4	0.151	0.200	0.242	0.278	0.310	0.339	0.364	0.387	0.407	0.426	0.449
H_5	0.543	0.592	0.519	0.481	0.461	0.442	0.423	0.404	0.386	0.368	0.359

Table 20. The ranking order of hatch covers.

Hatch covers	λ										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
H_1	3	4	3	3	3	3	2	3	3	3	4
H_2	1	1	1	1	1	1	1	1	2	2	2
H_3	2	2	2	2	2	2	3	2	1	1	1
H_4	5	5	5	5	5	5	5	5	4	4	3
H_5	4	3	4	4	4	4	4	4	5	5	5

Table 21. The ranking comparison of obtained results with other approaches.

Hatch covers	The original approach (v=0.5)	The corrected approach (v=0.5)	IT2F TOPSIS [27]	Entropy-based IT2F WASPAS approach
H_1	1	4	3	3
H_2	2	1	1	1
H_3	4	2	2	2
H_4	3	3	4	5
H_5	5	5	5	4

Table 22. The correlation coefficients between the different measures λ.

	λ										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	-	0.9902	0.9990	0.9878	0.9702	0.9353	0.8260	0.7620	0.5945	0.3874	0.1750
0.1		-	0.9860	0.9594	0.9315	0.8835	0.7659	0.4917	0.4917	0.2723	0.3572
0.2			-	0.9929	0.9786	0.9482	0.8296	0.7862	0.6252	0.4229	0.2131
0.3				-	0.9960	0.9789	0.8561	0.8531	0.7123	0.5261	0.3260
0.4					-	0.9931	0.8638	0.8957	0.7716	0.5992	0.4083
0.5						-	0.8635	0.9410	0.8405	0.6885	0.5120
0.6							-	0.8205	0.6981	0.5580	0.3991
0.7								-	0.9719	0.8901	0.7687
0.8									-	0.9716	0.8956
0.9										-	0.9754
1.0											-

Biographies

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