A Sustainable Production Recovery Model with Responsiveness in Demand and Quality Consideration during COVID-19 Crisis

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Abstract
The COVID-19 pandemic has resulted in disruption of many supply chains, majorly after the announcement of lockdown in numerous countries as a precaution for the novel coronavirus transmission. The decisions regarding inventory policies are greatly affected by the production rates and volumes, hence during this pandemic, flexibility in production rates and volumes becomes a necessity to handle demand uncertainties. Another big challenge that comes upfront is about rebuilding trust cum awareness in consumers about safe production, services, home deliveries through constant investment in advertisements. As quality and safety becomes the need of the hour, the manufacturers cannot afford any chance to compromise with supreme quality standards. Hence, the paper employs two stage rigorous inspection practices for the imperfect products that arise from the production process, the first inspection process being error prone (delivering Type- I and Type-II errors) while the second one being perfect. The developed model maximizes the manufacturer's total profit by optimizing the investment in service and advertisement with flexible production rate. A numerical example and comprehensive sensitivity analysis are illustrated to support the pragmatism of the model.

Keywords: COVID-19, Carbon emission, Service enhancement, Promotional effort, Flexible production rate, Imperfect rework.

1. Introduction and Literature Review

The rapid spread of novel coronavirus COVID-19 pandemic has brought many flourishing businesses to a standstill outwardly viz. apparel, footwear, furniture, automobile, tourism, hospitality businesses to name a few. While for many others it has brought blooming sales viz. businesses selling essential goods like food, groceries, sanitization products, healthcare being on top of the game. But for handling this increased demand of essential goods, especially in the period of lockdown has been an uphill task for the businessmen. Due to the risk of coronavirus transmission, the customers are not willing to compromise with the safety even while buying goods. Hence, the stores which are promising zero touch home deliveries and ensuring safety methods in various production and servicing methods like minimum staffing, regular sanitization, temperature checking of staff, face shielding, cashless transactions, and other government guidelines at workplace are gaining competitive advantage. While this seems achievable, there come various challenges in the implementation of these new norms. The lockdown has not only impacted international supply chains but has also raised problems for domestic manufacturers, retailers etc. Due to panic buying, hoarding of essential goods, shortage gaming, paradigm shift in consumer preferences and behavior, these supply chain players are left perplexed to forecast the future demand patterns accurately. Thus, the present chaotic situation calls for implementing recovery strategies to handle the socio- economic crisis wisely. Since consumer buying behavior is changing too fast along with the greater spread of novel coronavirus COVID-19, the first and foremost challenge in front of manufacturers is to be acknowledged with the exact predictions of demand. Hence, the manufacturers are trying their best to produce in accordance with the estimated demand with the highest level of accuracy even in this dynamic scenario. This is only possible if firms are continuously agile and flexible enough to adjust the pace of production. Thus, responsiveness is the key to succeed in these times of crisis and volume agility helps in achieving that. However, adjustment in production rates does not guarantee sales volume in these chaotic times. To attract more customers, the manufacturers need to put in extra investment in creating awareness of the safety methods that are being employed by them through advertisements in different media. Related to this, on March 11, World Health Organization (WHO) declared COVID-19 as a pandemic with complete guidelines mentioned in WHO [1]. Since then, researchers have been putting immense efforts in exploring the critical aspects of this spread and its harsh impact on the businesses. Ivanov [2] constructed a viable supply chain model to assist firms in re-building of their supply chains after global crisis of COVID -19 pandemic. Recently, Pavlov, A., Ivanov, D., Werner, F., et al. [3] contributed in conceptualizing a new methodological approach for detecting supply chain disruptions. Alizadeh, M., Paydar, M.M., Hosseini, S.M. et al. [4] studied a multi-level supply chain for the flu vaccine during the COVID-19 pandemic. Roggeveen, and Sethuraman [5] gave some theoretical insights

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on the impact of supply chain disruption caused by COVID-19 for the retailers. del Rio-Chanona, R. M., Mealy, P., Pichler, A. et al. [6] discuss about the demand and supply shocks by categorizing industries into essential and non-essential types during the COVID-19 pandemic. Chesbrough [7] threw light on how innovation will be acting as a recovery method from the economic caused by COVID-19 in many industries. Sarkis, J., Cohen, M. J., Dewick, P. et al. [8] have given few ways for production managers to survive in the times of coronavirus spread. Laing [9] discussed the impact of economic crisis caused by COVID -19 particularly for the mining industries. Laato, S., Islam, A. N., Farooq, A. et al. [10] presented the linking mechanism for quarantines period and making unusual purchases during the COVID-19 pandemic. Hall, M. C., Prayag, G., Fieger, P. et al. [11] proposed a study that grocery-spending in New Zealand increased appreciably in mid-March as compared to the same dates of the last year, and this caused panic-buying and stocking. Other contributories in this direction are Choi [12], Currie, C. S., Fowler, J. W., Kotiadis, K. et al. [13], Ivanov [14], Ivanov [15], Breen and Hannibal [16], Koonin [17], Inoue & Todo [18], Stošić-Mihajlović & Trajković [19], Hobbs [20], Nicola, M., Alsafi, Z., Sohrabi, C. et al. [21], Fransoo & Udenio [22], Bose [23], Lepore, D., Micozzi, A., and Spigarelli, F. [24] etc.

Due to COVID-19 lockdown, there has been limited access or say fluctuations in the availability of various resources required for running perfect production processes. Many suppliers are unable to provide top notch quality of raw material, and hence production of defectives is incontrovertible during the production process. Also, in the production houses, there have been irregular maintenance practices required for correct functioning of machines due to limited access to labor and staff. All these unavoidable factors together contribute to compromised or unreliable product outcomes at the manufacturer's end. In view of this, several eminent contributions have been done in the field of imperfect items in the last decade. Rini, K. M. Kamna, and Priyamvada [25] established the production-inventory system under price-sensitive demand, volume-agility with investment in preservation technology. Recently, Priyamvada, Gautam, P., and Khanna, A. [26] propose a model with imperfect production system.

During this pandemic, when already many businesses are dooming, the manufacturer cannot be at rest knowing that sale of defectives might bring bad name to the firm and the fear of losing customers permanently is at its peak. The imperfect quality environment further encourages the manufacturer to invest in rigorous inspection techniques to prevent the defectives from getting delivered to the end customers. Shafiee-Gol, S., Nasiri, M. M., & Taleizadeh, A. A. [27] investigated the problem of defectives in multi-product single machine manufacturing system by implementing rework and multiple delivery. However, there are always chances of human errors during the inspection process and hence the study also incorporates Type- I and Type-II misclassification errors. When a proportion of perfect items get mistakenly classified as defective items, it is termed as Type-I misclassification error, likewise when a portion of defective items get misclassified as perfect items, it is termed as Type-II misclassification error. Recently, Mokhtari and Asadkhani [28] deliberate an Economic Production Quantity (EPQ) model with preventive maintenance and there were two cases considered for the disposal time of imperfect items at the end of each production or sub-production cycle. Nobil, A. H., Afshar Sedigh, A. H., Tiwari, S. et al. [29] developed an inventory model with an imperfect production system linking multiple products.

To survive these difficult times of COVID-19, any running business would not mind going an extra mile to sustain in the competition. Here also, the manufacturer does not leave any chance to raise the sales volume and hence re-inspects the defective lot to fetch some reworkable items, which can be sent for rework process. A rework process does cost the manufacturer an additional amount but pays off tremendously by selling the reworked items at mark-up price, thereby treating the reworked items as good as perfect ones. The left-over items viz. non reworkable ones derived from this inspection practice are added to the count of defective items, derived from the first inspection practices, and declared as scrap to be disposed of later. Amid the challenging times of COVID-19, there is difficulty in applying all the production and rework practices with perfection. Hence, imperfect scenario is also observed in rework process i.e., all the recovered items do not gain perfect quality and are again segregated into the perfect ones and the final scrap items. All the scrap items are accumulated from stage 1 and stage 2 inspection practices and salvaged together at a cheaper price. Evidently, rework process acts as an efficient recovery strategy for increasing the count of perfect items to an extent and thus boosting the sales also. Recently Jaggi and Rini [30] investigated the scenario of imperfect items, imperfect screening process and imperfect rework process on the production policies. Rini, Kishore, A., Cárdenas-Barrón, L. E. et al. [31] studied two-stage credit financing while implementing order overlapping approach. Other pioneer recent contributions in this direction are Tsao, Y. C., Lee, P. L., Liao, L. W. et al. [32], Marchi, B., Zanoni, S., Zavanella, L. E. et al. [33], Cárdenas-Barrón, L. E., Plaza-Makowsky, M. J. L., Sevilla-Roca, M. A. et al. [34], Hsieh and Chiu [35], to name a few.

While the world is still struggling to cope with calamity caused by COVID-19 pandemic, one must keep a close watch on industrial activities so that one crisis does not lead to another. Various human activities have warmed up the planet over the past 50 years resulting in global warming. The carbon dioxide levels have been raised from 280 parts per million to 412 parts per million in the last 150 years. This calls for a check in the production activities like inventory storage, oil-related emissions, transportation, rework practices, disposal of waste etc. that result in higher carbon emissions and harm the environment. So, firms do need to invest in lowering these dreadful emissions from all their processes. Some practical solutions can be the usage of fuel-efficient vehicles that consume less gas to travel same distance, by utilizing cleaner fuels in all industrial processes that reduce emissions by *80 percent* when they are burned, by using electric cars and trucks for inbound and outbound logistics that consume electricity as fuel, thereby, producing fewer emissions than their conventional counterparts. These can act as some transforming solutions to lower carbon emissions from the industries. Bonney and Jaber [36] put efforts in developing a responsible inventory model that considers controlling environmental hazard in different aspects of production activities viz. packaging, waste management, and plant location. Hua, G., Cheng, T. C. E., and Wang, S. [37] examined the impact of carbon emissions, carbon trade, carbon price, carbon cap on the optimal order quantity. Bouchery, Y., Ghaffari, A., Jemai, Z. et al. [38] constructed a sustainable order quantity model and then also propose a multi-echelon inventory model to analyze the regulatory policies to control carbon emissions. Digiesi, S., Mossa, G., and Mummolo, G. [39] propose a sustainable order quantity model to strategize the best lot sizing policies and transportation means to minimize logistic and environmental costs. Battini, D., Persona, A., and Sgarbossa, F. [40] consider all the sustainable factors viz. internal and external transportation costs, vendor and supplier location and the different freight vehicle utilization ratio to reach the optimal inventory management decisions. Hammami, R., Nouira, I., and Frein, Y. [41] consider a multi-echelon supply chain with different external suppliers, different manufacturing facilities, and different distribution centers to deal with the case of carbon emission tax and carbon emission cap. Some significant contributories in considering control of carbon content in businesses are those of Xu, J., Chen, Y., and Bai, Q. [42], Kazemi, N., Abdul-Rashid, S. H., Ghazilla, R. A. R et al. [43], Taghikhah, F., Voinov, A., and Shukla, N. [44]. Recently, Kabadurmus and Erdogan [45] and Marchi B., Zanoni, S., and Jaber, M.Y. [46] analyzed a constrained supply chain network design considering sustainability and reliability simultaneously.

Our Contribution

The global crisis of COVID-19 has caused every industry to step back and review current strategies, operations, and processes. As the coronavirus curve rises, it is evident that over the coming months and possibly longer, the big winners will be those supply-chains that will adopt techniques which offer flexibility and agility in their methods. In the new normal, the only “constant” in the supply chain will be change. This calls for thinking of some recovery strategies from the point of view of manufacturer, as a lot of end-to-end inventory visibility depends upon his production practices. The present study considers volume agility, as a necessary adjustment for the production manager to handle the uncertainties of demand in the ongoing pandemic. So, the paper incorporates flexible production rate that is dependent on the staff and tool cost in general as a needful assumption. Moreover, as demand of essential goods is showing larger variability in the current times, supply chains are facing significant challenges to meet up the demand. Also, consumers are not willing to compromise with their safety, so the need for door-to-door deliveries of essential goods becomes yet another feature for survival especially in the lockdown period of this pandemic. Hence, the manufacturers strive to invest in advertisements in different media for building trust in new as well as old customers about the implementation of safety measures taken while delivering quality service to them. So, the study considers a demand function that is dependent on two important factors viz. advertisement and service performance investments, which is quite pragmatic in today’s times. Due to the complete lockdown and sky rocketing shoot in demand of essential products, maintaining quality and quantity together has been an arduous task for the manufacturers. Hence, bottlenecks have appeared in almost all the stages of production processes, rendering compromised quality products. Though the production of some faulty products is unavoidable in these times of crisis, but one cannot simply deliver them to the consumers without any quality check. In view of this, the manufacturer implements a two-way inspection technique to safeguard his image in the market. To make the study more relevant in current times of limited skillful staff, the first inspection is considered as error prone while the second inspection process is error free. The first inspection is purposefully done to segregate the defective and the non-defective lots while the second inspection is done to segregate reworkable and non-reworkable items from the defective lot obtained from the first stage of inspection. To ensure supreme standards of quality in this cut-throat competition, the manufacturer does not settle by vaguely assuming the rework process to be perfect and further screens the reworkable items for ensuring delivery of only perfect products in the mainstream. So, the paper tries to contribute in many ways to give some recovery strategies for the manufacturers of the supply chain.

2. Model notations

Following notations are used to model the stated scenario

Parameter	**Description**

I_0	Initial investment to start home delivery service ($ per time unit)
π	Sensitivity parameter for promotional expenditure; $0<\pi<1$
ω	Sensitivity parameter for service enhancement; $0<\omega<1$
d_0	Demand function intercept
a	Demand sensitivity parameter related to promotional expenditure
b	Demand sensitivity parameter related to service enhancement/ effectuation
u_0	Initial setup cost ($)
y_0	Labor cost per cycle ($)
y_1	Tool cost ($)
R	Rate of rework (units per time unit)
α	Imperfect proportion, a random variable; $0<\alpha<1$
β	Non-reworkable proportion, a random variable; $0<\beta<1$
γ	Failed reworked proportion, a random variable; $0<\gamma<1$
m_1	Type-I misclassification proportion, a random variable; $0<m_1<1$
m_2	Type-II misclassification proportion, a random variable; $0<m_2<1$
r_{id}	Production rate of inspected non-conformable items (units per time unit)
r_s	Production rate of scrap items through rework process (units per time unit)
t_p	Production duration (time units)
t_r	Rework duration (time units)
t_d	Inventory depleting time (time units)
c_i	Cost of inspection ($ per unit)
s	Market selling price ($ per unit)
v	Salvage price ($ per unit); $v<s$
c_d	Cost of disposal ($ per unit)
c_{rw}	Cost of rework ($ per unit)
c_r	Type-I misclassification cost ($ per unit)
c_a	Type-II misclassification cost ($ per unit)
c_e	Cost of carbon emission ($ per unit)
c_{eh}	Cost of carbon emission in storing a non-defective unit ($ per unit)
c_{ehd}	Cost of carbon emission in storing a defective unit ($ per unit)
c_h	Storage cost conformable items in ($ per unit per time unit)
c_{hd}	Storage cost for defective items in ($ per unit per time unit)
I	Inventory level after time t_p (units)
I_r	Inventory level after time t_r (units)
ξ^p	Carbon emission rate per produced unit, a random variable; $0<\xi^p<1$

ξ^r	Carbon emission rate per reworked unit, a random variable; $0<\xi^r<1$
ξ^d	Carbon emission rate per disposed unit, a random variable; $0<\xi^d<1$
Q	Production quantity per cycle (units); a dependent decision variable
T	Cycle length (time units); a dependent decision variable
Decision variables	
M	Promotional expenditure (\$ per time unit)
P	Production rate (units per time unit); $P>D$
S_e	Service expenditure (\$ per time unit)

3. Model assumptions

The model is established while assuming the following

i) Demand rate is dependent on promotional expenditure (M) and service enhancement $I(S_e)$, i.e.,

$$D=D(M,I(S_e))=d_0+aM^{\pi}+bI(S_e);\ \ d_0,a,b>0 \text{ and } 0<\pi<1 \tag{1}$$

and $I(S_e)=\omega\ln(S_e);\ \ 0<\omega<1$ (2)

where S_e denotes the service expenditure for maintaining home delivery service. Figure 1 graphically depicts the relationship between service investment and the corresponding service level enhancement.

ii) Production rate is flexible thereby making per unit production cost a variable.

iii) A single item is being produced and shortages are not permissible. Thus, production rate is greater than the demand rate.

iv) Production process is unreliable and engenders imperfect items.

v) Inspection process is erroneous and rework process too is flawed.

vi) Type I error and Type II error proportions are not correlated with defect proportion.

vii) After screening, only items found conformable with quality standards are used to fulfill the demand.

viii) The time horizon is infinite with negligible lead time.

ix) Carbon emission is the repercussion of production, warehousing, rework and disposal process. The rate of carbon emission during production, rework and disposal is a random variable with known probability distribution function. Further, carbon emission cost for holding a conformable/ non-conformable unit is identified with certainty.

< Insert Figure 1 >

4. Problem description and formulation

This section describes the problem structure and provides the mathematical model under the stated assumptions.

4.1 Problem structure

COVID-19 has disrupted operations and finances on a massive scale, challenging manufacturers to assess the impact and analyze best ways to respond quickly to the market changes. Due to the announcement of lockdown by the government, the pandemic has brought concerns regarding the availability of raw material, labor, limited working hours, transportation struggle in inbound- outbound logistics, etc. Some companies are developing recovery plans while others are trying to cope with their survival plans depending upon the adverse effect on the profitability of the company. The situation calls to examine the monetary situation of these manufacturing firms and develop some recovery strategies to assist the operations manager in reformulating the inventory policies. Thus, the manufacturer decides to initiate the home delivery service along with advertising as a response. With the above stated problems occurring in the management of supply chain, in the present study, the manufacturer is unable to produce 100% faultless products and hence defectives are produced at his end. So, before delivering all the products in the mainstream, he implements a rigorous inspection process to segregate the defective items from the perfect items. Adding to the practicality of the situation, the paper also considers inspection errors, namely Type-I and Type-II in the modelling. However, these challenging times do not let the manufacturer settle with sale of compromised quality and loss of profitability due to these errors. So, he further tries to recover some of the defectives with careful rework process. The inspection process done to re-inspect the defective lot to separate the reworkable and non-reworkable items is error free. The non- reworkable items which cannot be reworked are declared as scrap to be disposed of with the previously accumulated defectives. The reworkable items are treated as good as new ones and sold on the markup price in the mainstream.

With the lockdown being imposed by the government as a precaution to avoid novel coronavirus transmit, the main challenge that lies ahead of the industrialists is the forecast of demand in the ongoing scenario of panic buying, shortage gaming, disinterest in non-essential products, etc. Basically, the manufacturers are experiencing a complete paradigm shift in the consumer preferences with the pandemic effect. Whatever the manufacturers produce needs to be sold to the customers, otherwise that becomes an excess burden on the manufacturer to carry the inventory. Also, the production rate needs to be flexible enough to absorb the uncertainties of demand. Further, the demand also needs to be a variable and not fixed to fit in the current times of COVID-19 pandemic. Consumers' priorities need to be recognized and changes need to be implemented accordingly. The new normal calls for a huge investment in creating the awareness of the safety/sanitization measures taken during various supply chain processes viz. procurement, production, transportation (inbound & outbound), packaging, door-step servicing, etc. Thus, an advertisement and service quality dependent demand seem to be pragmatic and in sync with current challenging times. Nevertheless, one cannot neglect the other crisis situations that may occur if required precautions are not taken from now on, especially regarding mother nature and environment. So, the manufacturer does consider the hazardous carbon emissions that take place in various industrial processes like production, transportation, disposal, etc. and keeps a check on these with required investments at various levels. The overall problem that is considered in the present study is valuable and practical for the ongoing pandemic and calls for some solutions to be given to the manufacturer to help him recover the losses gradually. Figure 2 pictorially represents the general problem structure.

< Insert Figure 2 >

4.2 Mathematical model

In line with the assumptions made, a flexible production system has been considered which produces a total of Q units in one production run. Production initiates at time 0 and continues for a time period of t_p, General flow of items has been portrayed through Figure 3.

< Insert Figure 3 >

Since *Q* units are manufactured at the rate *P* in time t_p

$$t_p = \frac{Q}{P} \tag{3}$$

Full inspection policy is adopted to segregate the defective units from the produced lot, where inspection is flawed and includes two kinds of misclassification errors, viz. Type I misclassification error (proportion m_1) and Type II misclassification error (with proportion m_2) such that $0 < m_1, m_2 < 1$. Out of all the units produced, αQ are defectives whereas $(1-\alpha)Q$ are non-defectives which conform to the quality standards. Since, screening is erroneous a proportion $\alpha m_2 Q$ out of total defectives are mistakenly identified as non-defectives (Type II error). Likewise, a proportion $(1-\alpha)m_1 Q$ of total non-defectives are considered as defectives (Type I error), please refer to Figure 3. Consequently, total inspected defectives turn out to be $(1-\alpha)m_1 Q + \alpha(1-m_2)Q$ and total inspected non-defectives identified is $(1-\alpha)(1-m_1)Q + \alpha m_2 Q.$ Thus, the rate of production for inspected defectives is $r_{id}P = \left((1-m_2)\alpha + (1-\alpha)m_1\right)P.$ Therefore, the inventory of non-defective items accumulate at a rate of $P - r_{id}P - D,$ refer to Figure 4.

< Insert Figure 4 >

Segregated defective units are then screened to determine which units can be reworked, which results in $r_{id}\beta Q$ and $r_{id}(1-\beta)Q$ being the non-reworkable and reworkable proportions respectively. The rework process initiates after time t_p at the rate R and continues for a duration of t_r. Rework process being faulty results in $\gamma(1-\beta)r_{id}Q$ units to fail attainment of perfect quality and thus are scrapped. Thus, $(1-\gamma)(1-\beta)r_{id}Q$ units are successfully reworked to perfect quality.

So, $r_{id}(1-\beta)Q$ units are reworked at rate R in time t_r

$$t_r = \frac{r_{id}Q\left(1-\beta\right)}{R} \tag{4}$$

Accordingly, the rate at which scrap units are accrued during rework process is $r_s = \gamma R$. Therefore, during t_r the inventory level of non-defective units rises with the rate $R - r_s - D$, refer to Figure 4. After the rework process, inventory reduces with demand rate D till the end of cycle.

Cycle length

$$T = \frac{\left[\alpha m_2 + \left(1-\alpha\right)\left(1-m_1\right) + r_{id}\left(1-\beta\right)\left(1-\gamma\right)\right]Q}{D(M, I(S_e))} \tag{5}$$

as the total number of units that are conformable with quality standards and can be used to fulfill the demand is $(1-\alpha)(1-m_1)Q + \alpha m_2 Q + (1-\gamma)(1-\beta)r_{id}Q.$

Thus, expected cycle length

$$E[T] = \frac{\left(E[\alpha m_2] + \left(1-E[\alpha]\right)\left(1-E[m_1]\right) + E[r_{id}]\left(1-E[\beta]\right)\left(1-E[\gamma]\right)\right)Q}{D(M, I(S_e))} \tag{6}$$

$$or\ \mathrm{E[T]} = \frac{YQ}{D(M, I(S_e))} \tag{7}$$

where

$$Y = \left(E[\alpha m_2] + \left(1-E[\alpha]\right)\left(1-E[m_1]\right) + E[r_{id}]\left(1-E[\beta]\right)\left(1-E[\gamma]\right)\right) \tag{8}$$

Further, from Figure 4,

$$t_d = \frac{I_r}{D} \tag{9}$$

$$\begin{aligned} I &= \left(P - r_{id}P - D\right)t_p \\ &= \left(P - r_{id}P - D\right)\frac{Q}{P} \end{aligned} \tag{10}$$

$$\begin{aligned} I_r &= I + \left(R - r_s - D\right)t_r \\ &= \left(P - r_{id}P - D\right)\frac{Q}{P} + \left(R - r_s - D\right)\frac{r_{id}Q\left(1-\beta\right)}{R} \end{aligned} \tag{11}$$

Different costs associated are

i) Setup Cost (SC)

$$SC = A \tag{12}$$

ii) Production Cost (PC)

$$PC = \chi(P)Q$$

$$PC = (u_0 + \frac{y_0}{P} + y_1 P)Q \quad (13)$$

iii) Inspection Cost (IC)

$$IC = c_i Q + c_i r_{id} Q + c_i r_{id}(1-\beta)Q \quad (14)$$

iv) Misclassification Cost, pertaining to Type-I error (MC_I)

$$MC_I = c_r(1-\alpha)m_1 Q \quad (15)$$

v) Misclassification Cost, pertaining to Type-II error (MC_{II})

$$MC_{II} = c_a \alpha m_2 Q \quad (16)$$

vi) Rework Cost (RC)

$$RC = c_{rw} r_{id}\left(1-\beta\right)Q \quad (17)$$

vii) Disposal Cost (DC)

$$DC = c_d r_{id} \beta Q \quad (18)$$

Inventory holding cost for non-defective/ conformable items from Figure 4,

$$H_1 = c_h\left[\frac{It_p}{2} + \frac{\left(I + I_r\right)t_r}{2} + \frac{I_r t_d}{2}\right] \quad (19)$$

Inventory holding cost for non-conformable items from Figure 5,

$$H_2 = c_{hd}\left[\frac{r_{id} P t_p^2}{2} + \frac{Rt_r^2}{2} + t_r r_{id}(1-\beta)\gamma Q\right] \quad (20)$$

< Insert Figure 5 >

viii) Inventory Holding Cost (*IHC*)

$$IHC = H_1 + H_2 = c_h\left[\frac{It_p}{2} + \frac{\left(I + I_r\right)t_r}{2} + \frac{I_r t_d}{2}\right] + c_{hd}\left[\frac{r_{id} P t_p^2}{2} + \frac{Rt_r^2}{2} + t_r r_{id}(1-\beta)\gamma Q\right] \quad (21)$$

Using equations (3), (4) and (9) – (11)

$$IHC = c_h \left[\begin{array}{l} \dfrac{\left(P - r_{id}P - D\right)Q^2}{2P^2} + \left(\left(P - r_{id}P - D\right)\dfrac{r_{id}Q^2(1-\beta)}{PR} + \left(R - r_s - D\right)\dfrac{{r_{id}}^2 Q^2 (1-\beta)^2}{2R^2} \right) \\ + \dfrac{1}{2D}\left(\left(P - r_{id}P - D\right)\dfrac{Q}{P} + \left(R - r_s - D\right)\dfrac{r_{id}Q\left(1-\beta\right)}{R} \right)^2 \end{array} \right] + c_{hd}\left[\dfrac{r_{id}Q^2}{2P} + \dfrac{r_{id}^2 Q^2 (1-\beta)^2}{2R} + \dfrac{r_{id}^2 Q(1-\beta)}{R}(1-\beta)\gamma Q \right] \tag{22}$$

Total amount of carbon emissions during the production, rework and disposal

$$= \xi^p P t_p + \xi^r R t_r + \xi^d r_{id}\beta Q \tag{23}$$

Thus, total cost of carbon emissions in production, rework, disposal and holding

$$= c_e\left(\xi^p P t_p + \xi^r R t_r + \xi^d r_{id}\beta Q\right) + c_{eh}\left(\frac{I t_p}{2} + \frac{\left(I + I_r\right)t_r}{2} + \frac{I_r t_d}{2} \right) + c_{ehd}\left(\frac{r_{id} P t_p^2}{2} + \frac{R t_r^2}{2} + t_r r_{id}(1-\beta)\gamma Q \right) \tag{24}$$

ix) Carbon Emission Cost (*CEC*)

Using equations (3), (4), (9) – (11) in equation (24)

$$CEC = c_e\left(\xi^p Q + \xi^r R\frac{r_{id}Q\left(1-\beta\right)}{R} + \xi^d r_{id}\beta Q \right) + c_{ehd}\left[\begin{array}{l} \dfrac{r_{id}Q^2}{2P} + \dfrac{r_{id}^2 Q^2 (1-\beta)^2}{R} \\ + \dfrac{r_{id}Q(1-\beta)}{R} r_{id}(1-\beta)\gamma Q \end{array} \right] + c_{eh}\left[\begin{array}{l} \dfrac{\left(P - r_{id} - D\right)Q^2}{2P^2} + \left(\begin{array}{l} \left(P - r_{id} - D\right)\dfrac{r_{id}Q^2(1-\beta)}{PR} \\ + \left(R - r_s - D\right)\dfrac{{r_{id}}^2 Q^2 (1-\beta)^2}{2R^2} \end{array} \right) \\ + \dfrac{1}{2D}\left(\left(P - r_{id} - D\right)\dfrac{Q}{P} + \left(R - r_s - D\right)\dfrac{r_{id}Q\left(1-\beta\right)}{R} \right)^2 \end{array} \right] + \frac{c_{ehd}\alpha m_2 Q\left[\alpha m_2 + \left(1-\alpha\right)\left(1-m_1\right) + r_{id}\left(1-\beta\right)\left(1-\gamma\right)\right]Q}{2D} \tag{25}$$

x) Service Cost (*SEC*)

$$SEC = I_0 + \frac{I}{D(M, I(S_e))} \tag{26}$$

xi) Advertisement cost (*ADC*)

$$ADC = MT \tag{27}$$

Hence, Total Cost

$$TC = SC + PC + IC + MC_I + MC_{II} + RC + DC + IHC + CEC + SEC + ADC \tag{28}$$

$$TC = A + (u_0 + y_0 / P + y_1 P)Q + c_i (Q + r_{id}Q + r_{id}Q(1-\beta)) + c_r\left(1-\alpha\right)m_1 Q + c_a \alpha m_2 Q$$
$$+c_{rw} r_{id}\left(1-\beta\right)Q + c_r r_{id}\left(1-\beta\right)Q + c_d \beta r_{id} Q$$
$$+(c_h + c_{eh})\left[\begin{array}{l} \dfrac{\left(P - r_{id}P - D\right)Q^2}{2P^2} + \left(\begin{array}{l}\left(P - r_{id}P - D\right)\dfrac{r_{id}Q^2(1-\beta)}{PR} \\ +\left(R - r_s - D\right)\dfrac{r_{id}{}^2 Q^2 (1-\beta)^2}{2R^2}\end{array}\right) \\ +\dfrac{1}{2D}\left(\left(P - r_{id}P - D\right)\dfrac{Q}{P} + \left(R - r_s - D\right)\dfrac{r_{id}Q\left(1-\beta\right)}{R}\right)^2 \end{array}\right] \tag{29}$$
$$+(c_{hd} + c_{ehd})\left[\frac{r_{id}Q^2}{2P} + \frac{r_{id}^2 Q^2 (1-\beta)^2}{2R} + \frac{r_{id}^2 Q(1-\beta)}{R}(1-\beta)\gamma Q\right]$$
$$+c_e\left(\xi^p Q + \xi^r R\frac{r_{id}Q\left(1-\beta\right)}{R} + \xi^d r_{id}\beta Q\right) + I_0 + \frac{I}{D(M, I(S_e))} + MT$$

Revenue Components

i) Revenue from perfect items (R_1)

$$R_1 = s\left[\alpha m_2 Q + \left(1-\alpha\right)\left(1-m_1\right)Q\right] + s\left(1-\beta\right)\left(1-\gamma\right)r_{id}Q \tag{30}$$

ii) Revenue from salvaging (R_2)

$$R_2 = v\left(1-\beta\right)\gamma r_{id} Q \tag{31}$$

Thus, Total Revenue (*TR*)

$$TR = R_1 + R_2 = s\alpha m_2 Q + s\left(1-\alpha\right)\left(1-m_1\right)Q + s\left(1-\beta\right)\left(1-\gamma\right)r_{id}Q + v\left(1-\beta\right)\gamma r_{id}Q \tag{32}$$

Total Profit (*TP*) = Total expected revenue (*TR*) – Total expected cost (*TC*)

Thus, using equations (29) and (32), total profit is given as

$$TP = s\left[\alpha m_2 Q + (1-\alpha)(1-m_1)Q\right] + s(1-\beta)(1-\gamma) r_{id} Q + v(1-\beta)(1-\gamma) r_{id} Q - A$$
$$-(u_0 + y_0 / P + y_1 P)Q - I_0 - \frac{I}{D(M, I(S_e))} - MT$$
$$-c_i (Q + r_{id} Q + r_{id} Q(1-\beta)) - c_r (1-\alpha) m_1 Q - c_a \alpha m_2 Q$$
$$-c_{rw} r_{id} (1-\beta) Q - c_r r_{id} (1-\beta) Q - c_d \beta r_{id} Q$$
$$-(c_h + c_{eh}) \left[\begin{array}{l} \dfrac{(P - r_{id} P - D) Q^2}{2P^2} + \left(\begin{array}{l} (P - r_{id} P - D) \dfrac{r_{id} Q^2 (1-\beta)}{PR} \\ + (R - r_s - D) \dfrac{{r_{id}}^2 Q^2 (1-\beta)^2}{2R^2} \end{array} \right) \\ + \dfrac{1}{2D} \left((P - r_{id} P - D) \dfrac{Q}{P} + (R - r_s - D) \dfrac{r_{id} Q (1-\beta)}{R} \right)^2 \end{array} \right]$$
$$-(c_{hd} + c_{ehd}) \left[\frac{r_{id} Q^2}{2P} + \frac{r_{id}^2 Q^2 (1-\beta)^2}{2R} + \frac{r_{id}^2 Q (1-\beta)}{R} (1-\beta)\gamma Q \right] \tag{33}$$
$$-c_e \left(\xi^p Q + \xi^r R \frac{r_{id} Q (1-\beta)}{R} + \xi^d r_{id} \beta Q \right)$$

Thus, expected total profit is given as

$$E[TP] = s\left[E[\alpha m_2] Q + (1 - E[\alpha])(1 - E[m_1]) Q\right] + s(1 - E[\beta])(1 - E[\gamma]) E[r_{id}] Q$$
$$+ v(1 - E[\beta])(1 - E[\gamma])$$
$$E[r_{id}] Q - A - (u_0 + y_0 / P + y_1 P) Q - I_0 - \frac{I}{D(M, I(S_e))} - \frac{MYQ}{D}$$
$$-c_i (Q + E[r_{id}] Q + E[r_{id}] Q (1 - E[\beta])) - c_r (1 - E[\alpha]) E[m_1] Q - c_a E[\alpha m_2] Q$$
$$-c_{rw} E[r_{id}] (1 - E[\beta]) Q - c_r E[r_{id}] (1 - E[\beta]) Q - c_d E[\beta r_{id}] Q$$
$$-(c_h + c_{eh}) \left[\begin{array}{l} \dfrac{(P - E[r_{id}] P - D) Q^2}{2P^2} + \left(\begin{array}{l} (P - E[r_{id}] P - D) \dfrac{E[r_{id}] Q^2 (1 - E[\beta])}{PR} \\ + (R - r_s - D) \dfrac{E[{r_{id}}^2] Q^2 (1 - E[\beta])^2}{2R^2} \end{array} \right) \\ + \dfrac{1}{2D} \left((P - E[r_{id}] P - D) \dfrac{Q}{P} + (R - r_s - D) \dfrac{E[r_{id}] Q (1 - E[\beta])}{R} \right)^2 \end{array} \right]$$
$$-(c_{hd} + c_{ehd}) \left[\frac{E[r_{id}] Q^2}{2P} + \frac{E[r_{id}^2] Q^2 (1 - E[\beta])^2}{2R} + \frac{E[r_{id}^2] Q (1 - E[\beta])}{R} (1 - E[\beta]) E[\gamma] Q \right]$$
$$-c_e \left(E[\xi^p] Q + E[\xi^r] R \frac{E[r_{id}] Q (1 - E[\beta])}{R} + E[\xi^d r_{id} \beta] Q \right) \tag{34}$$

Using Renewal reward theorem

$$E[TPU] = \frac{E[TP]}{E[T]} \tag{35}$$

$$E[TPU]=\frac{D}{Y}\left\{\begin{array}{c}
s\left[E[\alpha m_2]+\left(1-E[\alpha]\right)\left(1-E[m_1]\right)\right]+s\left(1-E[\beta]\right)\left(1-E[\gamma]\right)E[r_{id}] \\
+v(1-E[\beta])(1-E[\gamma])E[r_{id}] \\
-\frac{A}{Q}-(u_0+y_0/P+y_1P)-\frac{I_0}{Q}-\frac{I}{DQ(M,I(S_e))}-\frac{MY}{D} \\
-c_i(1+E[r_{id}]+E[r_{id}](1-E[\beta]))-c_r\left(1-E[\alpha]\right)E[m_1]-c_aE[\alpha m_2] \\
-c_{rw}E[r_{id}]\left(1-E[\beta]\right)-c_rE[r_{id}]\left(1-E[\beta]\right)-c_dE[\beta r_{id}] \\
-(c_h+c_{eh})\left[\begin{array}{c}\frac{\left(P-E[r_{id}]P-D\right)Q}{2P^2}+\left(\begin{array}{c}\left(P-E[r_{id}]P-D\right)\frac{E[r_{id}]Q(1-E[\beta])}{PR} \\ +\left(R-r_s-D\right)\frac{E[r_{id}{}^2]Q(1-E[\beta])^2}{2R^2}\end{array}\right) \\ +\frac{1}{2D}\left(\left(P-E[r_{id}]P-D\right)\frac{1}{P}+\left(R-r_s-D\right)\frac{E[r_{id}]\left(1-E[\beta]\right)}{R}\right)^2\end{array}\right] \\
-(c_{hd}+c_{ehd})\left[\frac{E[r_{id}]Q}{2P}+\frac{E[r_{id}^2]Q(1-E[\beta])^2}{2R}+\frac{E[r_{id}^2](1-E[\beta])}{R}(1-E[\beta])E[\gamma]Q\right] \\
-c_e\left(E[\xi^p]+E[\xi^r]R\frac{E[r_{id}]\left(1-E[\beta]\right)}{R}+E[\xi^d r_{id}\beta]\right)
\end{array}\right\} \tag{36}$$

5. Optimality

The main objective of the present study is to maximize the total profit of the model by jointly optimizing the investment in service cost (S_e), investment in the advertisement (M) and the production rate (P). To establish optimality, the necessary and sufficient conditions must be satisfied

$$\frac{\partial TP}{\partial S_e}=0,\ \frac{\partial TP}{\partial M}=0 \text{ and } \frac{\partial TP}{\partial P}=0 \tag{37}$$

For the sufficient conditions of optimality w.r.t S_e, M and P, the following Hessian matrix is needed.

$$\text{H}=\begin{bmatrix}\frac{\partial^2 TP}{\partial S_e^2} & \frac{\partial^2 TP}{\partial S_e\partial M} & \frac{\partial^2 TP}{\partial S_e\partial P} \\ \frac{\partial^2 TP}{\partial M\partial S_e} & \frac{\partial^2 TP}{\partial M^2} & \frac{\partial^2 TP}{\partial M\partial P} \\ \frac{\partial^2 TP}{\partial P\partial S_e} & \frac{\partial^2 TP}{\partial P\partial M} & \frac{\partial^2 TP}{\partial P^2}\end{bmatrix} \tag{38}$$

and the conditions are

$$\frac{\partial^2 TP}{\partial S_e^2}<0,\ \begin{vmatrix}\frac{\partial^2 TP}{\partial S_e^2} & \frac{\partial^2 TP}{\partial S_e\partial M} \\ \frac{\partial^2 TP}{\partial M\partial S_e} & \frac{\partial^2 TP}{\partial M^2}\end{vmatrix}>0 \text{ and } |\text{H}|<0 \tag{39}$$

Due to the complexity of the problem the profit function is highly non-linear, thus it is very difficult to establish necessary and sufficiency criteria mathematically, thus the graphical method is used to establish the concavity. Further, Figure 6, Figure 7 and Figure 8 establish the optimality with the help of Maple software.

< Insert Figure 6 >

< Insert Figure 7 >

< Insert Figure 8 >

6. Numerical Analyses

Example 1: For the scenario considered under the influence of COVID-19, it gets important for a manufacturer to adapt with the new normal and take some measures and changes in the production process. With a lot of uncertainty in demand, making the production rate flexible and initiating zero-contact home deliveries while advertising, is must. Moreover, in such a scenario, in the wake of environmental concerns, considering the carbon emissions from the process to decide on the optimal policy is a must. Considering the parametric values from **Table 1**, the optimal production schedule is recorded in **Table 2**.

< Insert Table 1 >

The rates of carbon emission ξ^p, ξ^r and ξ^d follows Beta distribution $j(\xi)$ with parameters $m, n.$ Thus, the p.d.f of ξ is given as

$$j(\xi) = \begin{cases} \dfrac{\xi^{m-1}(1-\xi)^{n-1}}{\sigma(m,n)}, & 0 \le \xi \le 1 \\ 0 \quad , & \text{otherwise} \end{cases} \tag{40}$$

where $\sigma(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (41)

For $p = 1, r = 2$ and $d = 3,$ we get

$$E[\xi^p] = \frac{m}{m+n},\ E[\xi^r] = \frac{m(m+1)}{(m+n)(m+n+1)},\ E[\xi^d] = \frac{m(m+1)(m+2)}{(m+n+2)(m+n+1)(m+n)} \tag{42}$$

Further, the proportion of random variables α, m_1, m_2, β *and* γ are assumed to follow Uniform distribution with following p.d.fs

$$f(\alpha) = \begin{cases} 10, & 0.05 \le x \le 0.15 \\ 0, & \text{otherwise} \end{cases} \quad f(m_1) = \begin{cases} 50, & 0.01 \le \mathrm{q}_1 \le 0.03 \\ 0, & \text{otherwise} \end{cases} \quad f(\beta) = \begin{cases} 10, & 0.01 \le \theta \le 0.11 \\ 0, & \text{otherwise} \end{cases}$$

$$f(\gamma) = \begin{cases} 20, & 0.01 \le \theta_1 \le 0.07 \\ 0, & \text{otherwise} \end{cases} \quad f(m_2) = \begin{cases} 50, & 0.01 \le \mathrm{q}_2 \le 0.03 \\ 0, & \text{otherwise} \end{cases} \tag{43}$$

< Insert Table 2 >

Thus, the optimal schedule calls for a production run of 44 days producing 8751 units with the production rate of 145851 units per year while investing \$87374 in promotions and \$1442714 spent in expansion of services, consequently earning a profit of \$6519882.

Example 2: If the manufacturer decides to run the operations as explained in Example 1 except ignoring the need of promotional campaigns, the effect on optimal policy has been depicted in **Table 3**.

< Insert Table 3 >

Hence, with no investments in different media for advertising, the total profit undergoes a steep fall as the demand experiences a huge decline.

7. Sensitivity Analyses

The objective of this section is to check the robustness of the formulated model further analyzing the effect of variation in values of key parameters on the optimal production policy.

< Insert Table 4 >

It is evident from **Table 4**, when the value of base demand increases, the profit levels are sure to grow significantly. Also, when the base demand has reached maturity and is on increasing trend, the manufacturer can take it as a sign of brand establishment and sufficient customer awareness about the product. Thus, the manufacturer can choose to withdraw some amount of investment in advertisement in different media. Thus, a declining investment in advertisement is an economical decision. Moreover, with more demand, one has to maintain the service levels to match up the standards of customers. With higher demand comes the need to produces more, especially in the imperfect quality scenario where efforts are put to sell only the perfect items. Hence, an increasing rate of production is also justifiable.

< Insert Table 5 >

As **Table 5** conveys, when the proportionality constant of advertisement increases, it is economically favorable for the manufacturer in many ways. Firstly, by creating more awareness in the market, the manufacturer can attract more new customers and also reassure the old customers about new, safety norms applied at various stages of production during COVID-19. To catch up with the increased demand, the production rate must be adjusted to rise according to the demand so that there are no shortages. Hence, the production quantity is showing increasing trends. The increments in the media advertisements get paid off to the manufacturer by escalating profit margins.

< Insert Table 6 >

When the proportionality constant of service investment increases, it shows positive impact on the profit values, refer to **Table 6**. The reason majorly being the combination of imperfect quality items and the recovery policies that are designed for maintaining the required service standards. The compromise in the service performances may lead to more dissatisfied customers especially in the COVID-19 pandemic times, hence, it is a wise decision to gear up the service investments. The customers are greatly affected by the service quality standards given by any firm (as demand is increasing), so the manufacturer can judiciously decide to lower the investment in different advertisement by relying more on the service performances being delivered to customers.

< Insert Table 7 >

When the initial set up cost per unit increases, it is advisable for the manufacturer to go for longer production runs (**Table 7**). Consequently, the production rate as well as the production quantity decrease. The results are also in tune with this. The manufacturer tries to balance out the high investment in setup cost by reducing investments in advertisement and service performance which further reduce the demand. The overall scenario impacts the profit values negatively.

< Insert Table 8 >

With the constraint of lockdown in COVID-19 pandemic and fewer workers left in the production houses, the labor cost per unit increases. The situation can be made economically viable by keeping less but skillful staffing. With the help of overtime by skilled laborers there is a positive impact on the level of service performance. Further, added investments in advertisements may lead to escalating demand patterns. Resultantly, the production rate also needs to be elevated to capture the increased demand. The overall impact of multiple investments leads to decrement in profit values, refer to **Table 8**.

< Insert Table 9 >

The COVID -19 lockdown has hit various secondary processes of the supply chains e.g. repairs, time-based maintenance, productive maintenance, etc. To survive these difficult times of pandemic, the firms need to invest in arrangements of necessary jigs and fixtures for their machinery and their reliable working conditions. The data in **Table 9** indicate that with rise in investment of per unit tool cost, it is advisable for the manufacturer to lower the production rate so as to keep the machinery durable and reliable for longer period. Consequently, lower units are produced per cycle which calls for decreased investment in media and service. Subsequently, the overall impact results in decreased demand and profit values.

< Insert Table 10 >

In any manufacturing process, production of imperfect items is unavoidable. However, these items bring in lot of challenges and their vulnerabilities attached with them. These not only hit the profit values by loss of sales at markup price but also decrease the demand values substantially. The fear of losing customers by sale of imperfect items in the ongoing times of pandemic is worse than ever. Since the production rate is adjustable, it becomes prudent to lower the production rate with the advent of more imperfect items. To recover some losses related to imperfect items, the manufacturer is advised to lower some investment from advertisement campaigning and service also. Once, the production of imperfect is under control, the manufacturer can again regain the lost sales and further enhance the profit margins, refer to **Table 10**.

< Insert Table 11 >

A decrement in non-reworkable proportion implies a greater percentage of reworkable items. Also, as the reworkable items are treated as good as new ones and thus are sold at mark-up price. As evident from the results, the rework process acts as a successful recovery strategy to increase the profit margins through a little investment in the rework procedures. With more perfect items at hand, the manufacturer can focus on investing in the righteous directions of elevating demand viz. investment in advertisement and service quality. To capture the increased demand, the production rate also requires to be elevated to produce more items (**Table 11**).

< Insert Table 12 >

The COVID-19 pandemic has brought unrest in the minds of manufacturers and they need to be doubly sure of their products before launching them in the market. In view of this, the rework process is considered as imperfect and this causes a second inspection process to rule out the slightest chance of selling defectives in the mainstream. Some items fail to get into perfect shape even after the investment of rework process. With the decrease of such failed reworked items, the percentage of successfully reworked items increases, and that is very profitable for the manufacturer. The results also show that the manufacturer can invest more in the advertisement and service levels so as to ensure greater sales with more perfect items at hand. The production rate also needs to be elevated to assure smooth functioning of all the supply chain processes, refer to **Table 12**.

< Insert Table 13 >

Table 13 suggests that to combat the effect of increasing cost associated with carbon emissions, the manufacturer lowers the rate of production and hence a decline in production quantity is observed. Since the overall increase in total cost needs to be balanced with other significant investments, it becomes viable to reduce a certain amount of investment in advertisement sources in some media and keep the service investment almost uncompromised. This causes marginal decrement in demand, thereby decreasing the profit values.

8. Conclusion

COVID-19 has brought severe supply chain disruptions with it that have hit the manufacturing industries in many ways. The crisis has added new layers of unpredictability to supply chains. Industry experts are having to rethink every aspect of business, review current strategies, operations, and processes. Many will realize that a key factor is end-to-end visibility of critical aspects like demand, inventory, resources viz. labor, energy, etc. It gives manufacturers, distribution centers and retailers the flexibility to respond quickly to the ebbs and flows of businesses. Since it becomes difficult to forecast the demand patterns accurately, an adjustment in production rate is a viable alternative to handle these uncertain times of pandemic wisely. Now-a-days, the customers are mostly prioritizing health and safety before purchasing any product. So, all the businesses need to re-create awareness about the mandatory sanitization and 3-ft distancing measures that are supposed to be taken in various industrial processes, viz. procurement, production, transportation, home deliveries, etc. This has put extra financial burden on them to invest in different media to improve their service performances and hence uplift profit margins significantly. However, the imperfect environment makes the task of retaining customers more challenging, especially in the times of COVID-19. So, the manufacturer invests in the inspection practices of the whole produced lot and further in the rework process as well, to not take any chance of losing customers. The key role of manufacturer lies in being dynamic and responsive by being susceptible to the changes required in these unpredictive times.

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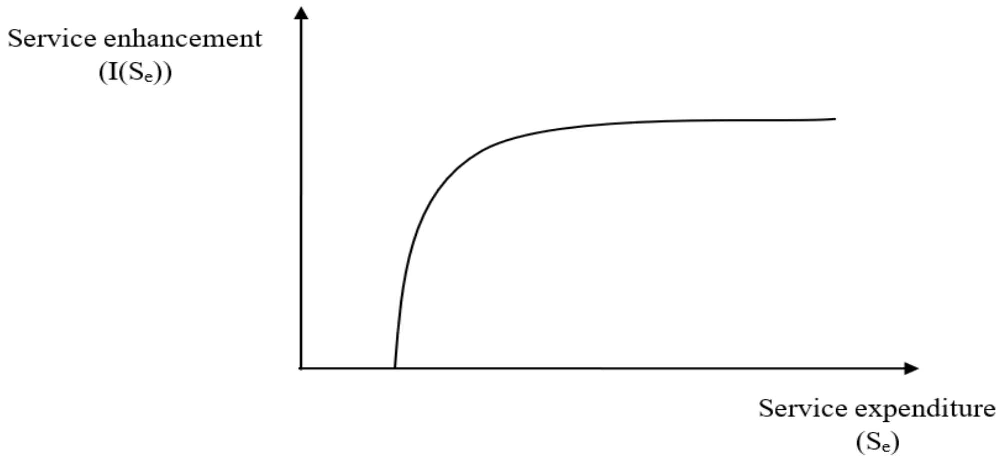

Figure 1. Graphical representation of S_e vs. $I(S_e)$

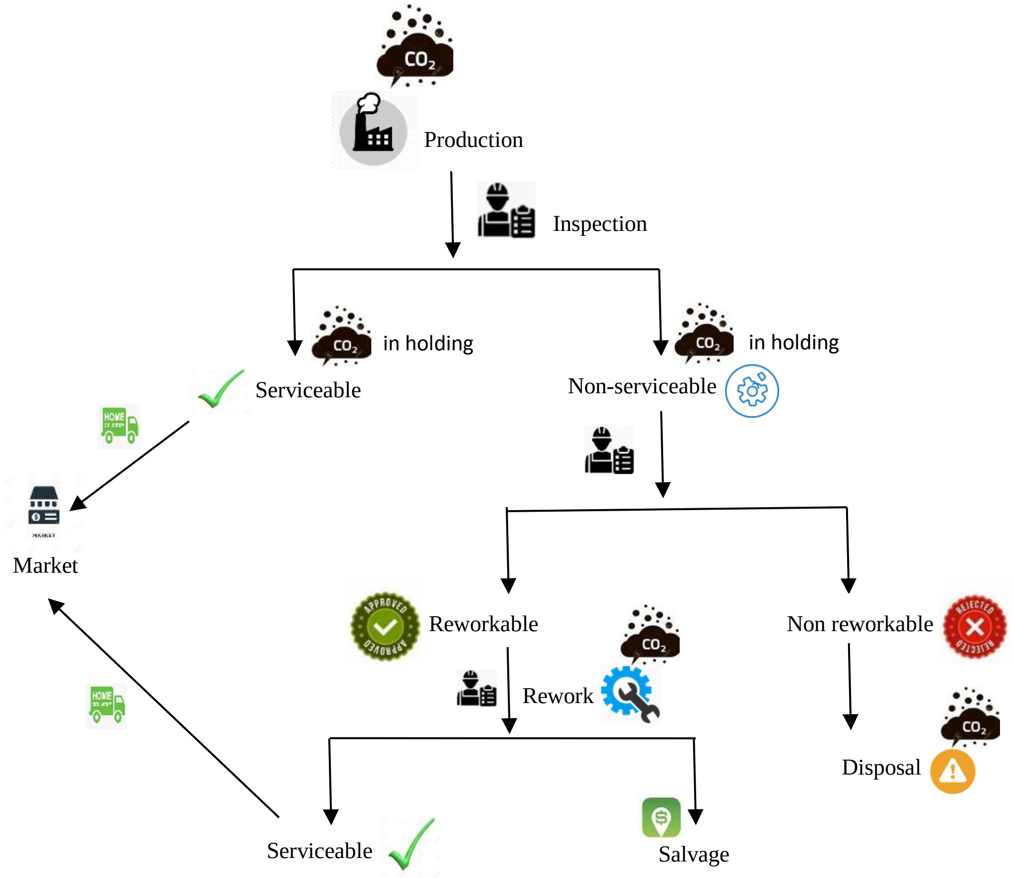

Figure 2. General problem structure

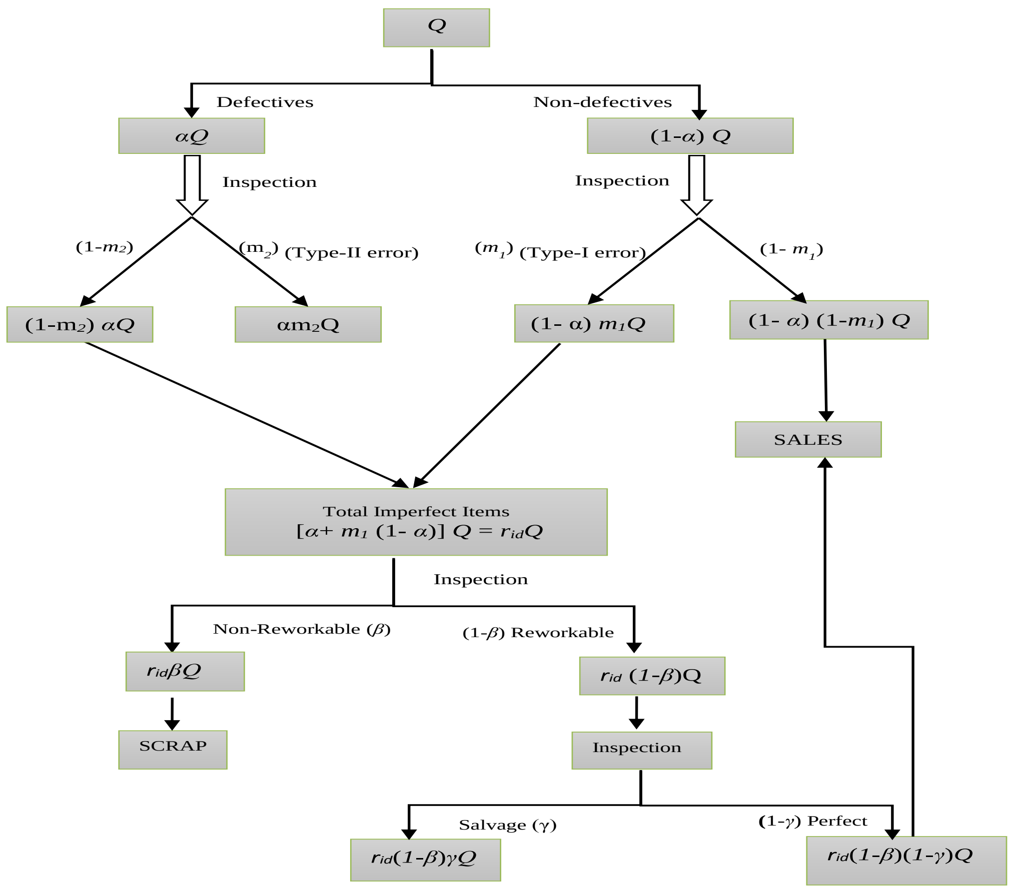

Figure 3. Problem flow

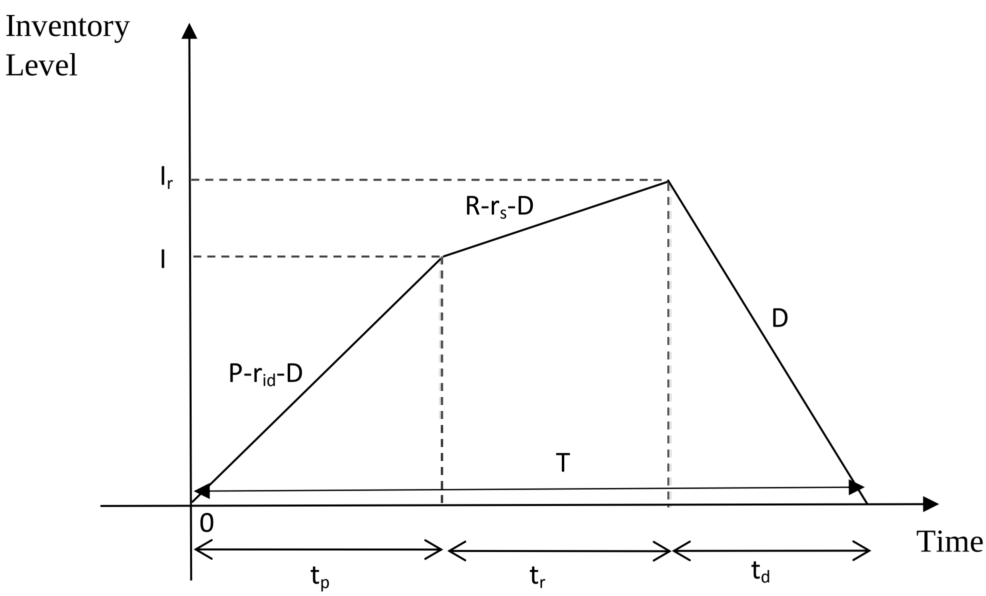

Figure 4. Inventory level of non-defective item

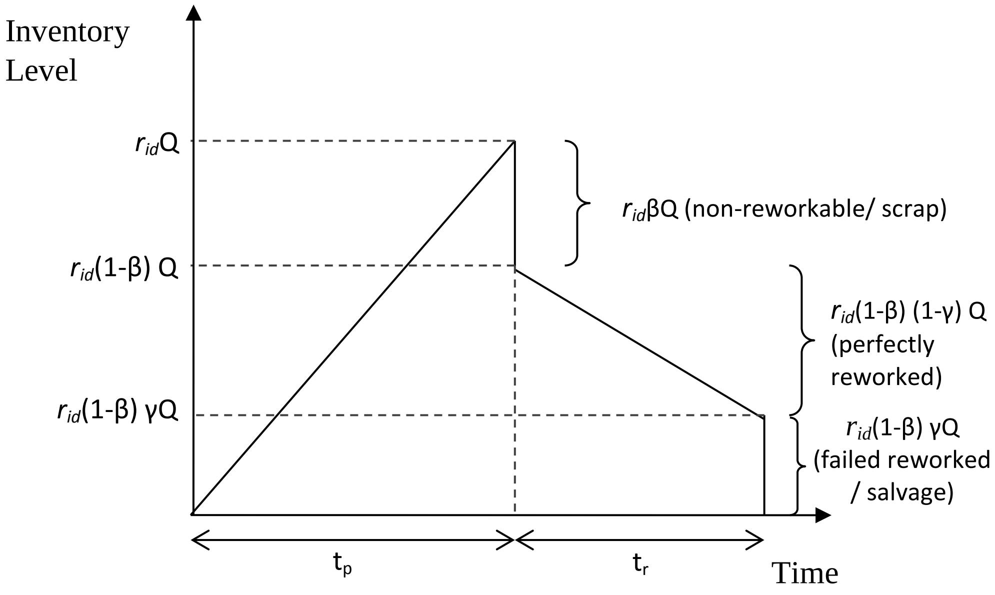

Figure 5. Inventory level of defective items

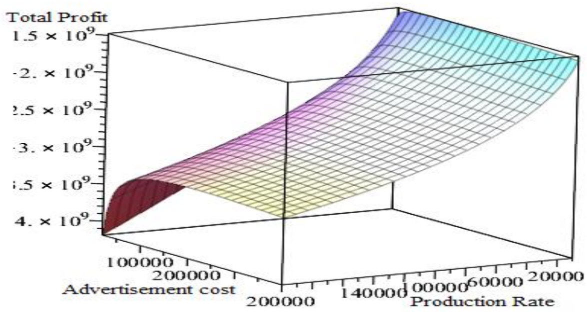

Figure 6. Graphical representation of total profit versus investment in advertisement and production rate

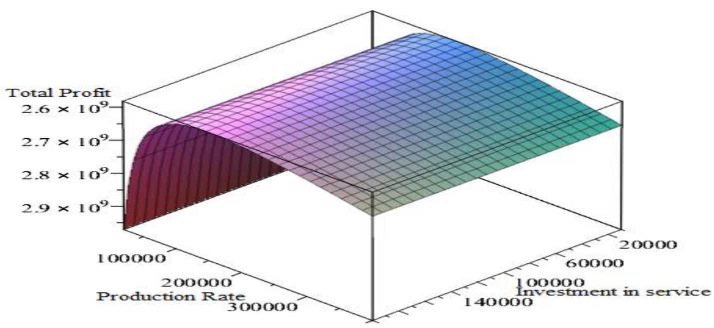

Figure 7. Graphical representation of total profit versus investment in service and production rate

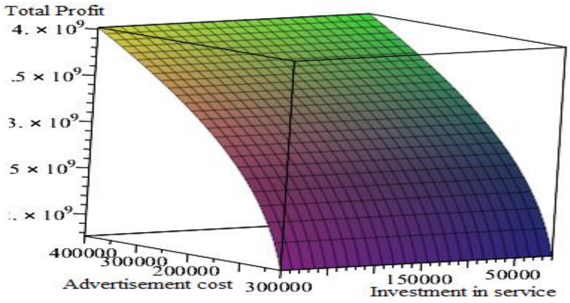

Figure 8. Graphical representation of total profit versus investment in service and advertisement.

Tables

Table 1. Numerical values taken for Example 1

Parameter	Input value	Parameter	Input value	Parameter	Input value
I_0	50000	α	0.15	c_{rw}	40
π	0.25	β	0.06	c_r	500
ω	0.4	γ	0.04	c_a	200
d_0	40000	m_1	0.02	c_e	7.5
a	1800	m_2	0.02	c_{eh}	6.5
b	4.5	t_p	0.06	c_{ehd}	5
u_0	2	c_i	5	h	13.5
y_0	1000	s	150	h_d	10
y_1	0.0001	v	30	m	0.15
R	120000	c_d	8	n	0.35

Table 2. Optimal production schedule for Example 1

Decision variable	Optimal value	Decision variable	Optimal value
P	145851	Q	8751
M	87374	T	44
S_e	1442714	TPU	6519882

Table 3. Optimal production schedule for Example 1 without advertisement investment

Decision variable	Optimal value	Decision variable	Optimal value
P	141768	Q	8506
M	0	T	76
S_e	1402289	TPU	3639855

Table 4. Effect of d_0 on optimal production policy

Parameter	*Parameter Value*	*P*	*M*	S_e	*Q*	*D*	*TPU*
d_0	*70000*	*185403*	*50507*	*1622835*	*11124*	*97010*	*9386603*
	60000	169068	59586	1543215	10144	88148	8416911
	50000	156096	71482	1492765	9366	79458	7460223
	40000	145851	87374	1442715	8751	70972	6519882
	30000	137751	109003	1388795	8265	62732	5600279
	20000	131307	138933	1312245	7878	54777	4707153
	10000	126136	180861	1232665	7568	47146	3847885

Table 5. Effect of *a* on optimal production policy

Parameter	*Parameter Value*	*P*	*M*	S_e	*Q*	*D*	*TPU*
a	3150	170299	133348	1662117	10218	100220	8751307
	2700	163816	120630	1592215	9829	90344	7995685
	2250	155829	105475	1514415	9350	80574	7250170
	1800	145851	87374	1442715	8751	70972	6519882
	1350	133307	65979	1331719	7998	61662	5813847
	900	117794	41726	1262511	7068	52889	5149157
	450	100143	17255	1192400	6009	45183	4561477

Table 6. Effect of *b* on optimal production policy

Parameter	*Parameter Value*	*P*	*M*	S_e	*Q*	*D*	*TPU*
b	7.5	147835	87102	1445728	8950	73987	6521466
	6.5	146141	87173	1444625	8830	72892	6520938
	5.5	145946	87263	1443524	8751	71977	6520410
	4.5	145851	87374	1442715	8751	70972	6519882
	3.5	144856	87484	1441827	8634	69968	6519354
	2.5	143862	87595	1440925	8520	68563	6518826
	1.5	143002	87605	1439823	8452	67858	6518298

Table 7. Effect of u_0 on optimal production policy

Parameter	*Parameter Value*	*P*	*M*	S_e	*Q*	*D*	*TPU*
u_0	3.5	144664	85165	1377913	8680	70775	6411842
	3	145059	85898	1398716	8704	70841	6447821
	2.5	145454	86634	1422722	8727	70907	6483835
	2	145851	87374	1442715	8751	70972	6519882
	1.5	146249	88117	1459915	8775	71038	6555962
	1	146647	88865	1472746	8799	71104	6592075
	0.5	147047	89616	1498264	8823	71169	6628222

Table 8. Effect of y_0 on optimal production policy

Parameter	*Parameter Value*	*P*	*M*	S_e	*Q*	*D*	*TPU*
y_0	1750	145883	87389	1443215	8753	70992	6519511
	1500	145872	87384	1443049	8752	70985	6519634
	1250	145862	87379	1442879	8752	70979	6519758
	1000	145851	87374	1442715	8751	70972	6519882
	750	145841	87369	1442635	8750	70966	6520005
	500	145830	87364	1442470	8750	70957	6520129

	250	145820	87359	1442290	8749	70951	6520252

Table 9. Effect of y_1 on optimal production policy

Parameter	*Parameter Value*	P	M	S_e	Q	D	TPU
y_1	0.000175	95924	50761	844738	5755	67044	5907692
	0.00015	107328	58921	1158264	6440	68069	6081634
	0.000125	122954	70311	1346372	7377	69336	6281807
	0.0001	145851	87374	1442715	8751	70972	6519882
	0.000075	182981	115823	1568263	10979	73232	6817881
	0.00005	254468	172710	1738362	15268	76720	7225606
	0.000025	451844	339674	2028263	27111	83480	7901285

Table 10. Effect of α on optimal production policy

Parameter	*Parameter Value*	P	M	S_e	Q	D	TPU
α	0.2625	141418	78800	1422637	8485	70184	6150013
	0.225	142873	81594	1430019	8572	70448	6273556
	0.1875	144350	84452	1432728	8661	70710	6396844
	0.15	145851	87374	1442715	8751	70972	6519882
	0.1125	147374	90361	1456281	8843	71234	6642672
	0.075	148923	93414	1459915	8935	71494	6765218
	0.0375	150495	96536	1462736	9030	71754	6887524

Table 11. Effect of β on optimal production policy

Parameter	*Parameter Value*	P	M	S_e	Q	D	TPU
β	0.105	145321	86250	1440016	8719	70872	6509849
	0.09	145498	86624	1440719	8730	70906	6513201
	0.075	145674	86998	1441995	8741	70939	6516545
	0.06	145851	87374	1442715	8751	70972	6519882
	0.045	146028	87750	1443245	8762	71006	6523210
	0.03	146205	88127	1443935	8772	71039	6526531
	0.015	146383	88505	1444707	8783	71072	6529844

Table 12. Effect of γ on optimal production policy

Parameter	*Parameter Value*	P	M	S_e	Q	D	TPU
γ	0.07	145479	86589	1440005	8729	70903	6510835
	0.06	145603	86850	1440728	8736	70926	6513856
	0.05	145727	87112	1441736	8744	70949	6516872
	0.04	145851	87374	1442715	8751	70972	6519882
	0.03	145975	87636	1443223	8759	70996	6522886
	0.02	146099	87899	1444708	8766	71019	6525884
	0.01	146223	88162	1445711	8773	71042	6528877

Table 13. Effect of C_e on optimal production policy

Parameter	*Parameter Value*	P	M	S_e	Q	D	TPU
C_e	13.125	144350	84585	1442681	8661	70723	6383233
	11.25	144849	85509	1442695	8691	70806	6428729
	9.375	145349	86438	1442705	8721	70889	6474279
	7.5	145851	87374	1442715	8751	70972	6519882
	5.625	146354	88315	1442729	8781	71055	6565538
	3.75	146859	89263	1442742	8812	71139	6611248
	1.875	147366	90216	1442754	8842	71221	6657011

Biographies

Rini received her Ph.D. degree from the Department of Operational Research, University of Delhi, India. She holds a Master's degree in Operational Research and a Bachelor's degree in Computer Science from University of Delhi, India. Her research areas include Inventory Management, Supply Chain Management and Sustainability. She has published research work in various international journals of repute like Expert Systems with Applications, Rairo-Operations Research, Opsearch etc. She is also UGC-CSIR NET qualified and has been associated with various institutions as an Assistant Professor in the area of Operations.

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