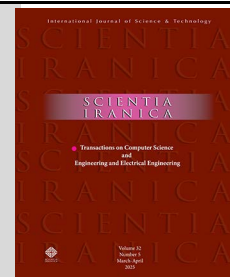




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Scientia Iranica*Transactions on Computer Science & Engineering and Electrical Engineering*<https://scientiairanica.sharif.edu>

A megastable oscillator with two types of attractors

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Received 19 June 2022; received in revised form 3 September 2022; accepted 19 December 2022

Keywords

Megastable oscillator;
Forced system;
Chaos;
Bifurcation analysis;
Multistability.

Abstract

In this paper, a megastable system is designed with particular formation of attractors. It has two formations of attractors: the inner ones with a smaller amplitude, and the outer ones with the Eye of God nebula shape and larger amplitude. To the best of our knowledge, such a megastable oscillator with this special formation of attractors has not been studied before. Afterward, the oscillator is forced, and its attractors are discussed. Different dynamics of this new oscillator are investigated using tools such as bifurcation and Lyapunov exponent diagram, and basins for each attractor.

1. Introduction

Presenting novel chaotic oscillators with unique features has been a controversial subject [1-3] in the field of nonlinear dynamics and chaos. Such oscillators have many applications, such as secure communications [4], and image encryption [5-7]. Regarding the special features, oscillators with different types of equilibria has been considerably investigated [8], such as oscillators without any equilibria [9] and oscillators with a single stable equilibrium [10]. Many of such oscillators are systems with hidden attractors [11]. An existing categorization of dynamical systems is dividing them to self-excited and hidden attractors [12-14]. Another

critical characteristic of chaotic oscillators is the topology of their strange attractor [15,16]. As an example one can mention new chaotic oscillators with different types of symmetry [17], and with multi-scroll attractors [18].

A multistable oscillator is a system that has coexisting attractors regarding the Initial Conditions (ICs) [19,20]. The state of a multistable oscillator can be different for various ICs according to the attractors' basin of attraction. The multistable phenomenon is widespread in the nature and real-world systems [21] such as the catalysis of linear arrays of three cells in chemistry [22], lasers [23], and optics [24].

To cite this article:

B. Ramakrishnan, Z. Faghani, H. Natiq, N. Pal, F. Nazarimehr, S. Jafari "A megastable oscillator with two types of attractors", *Scientia Iranica* (2025); 32(5): 6909. <https://doi.org/10.24200/sci.2022.60637.6909>

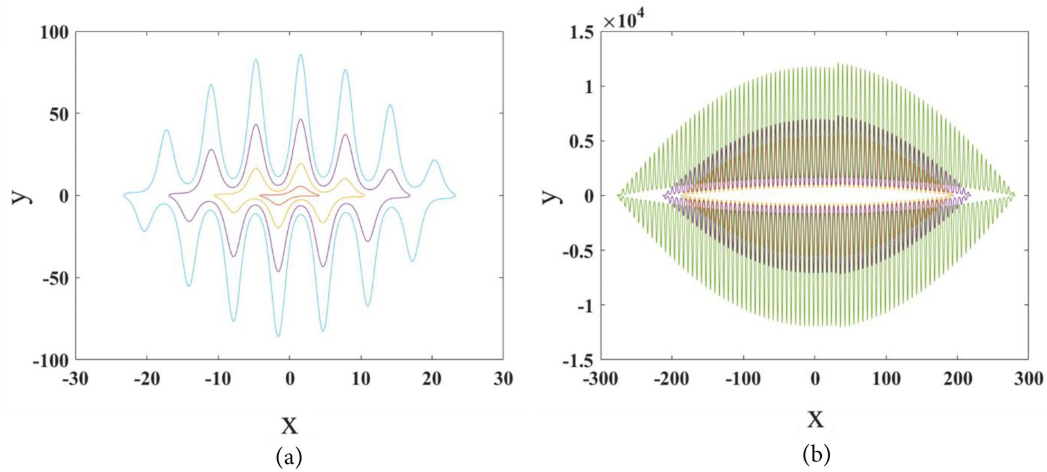


Figure 1. Seven limit cycles for the 14 ICs to show the coexisting limit cycles of Eq. (1); (a) Four inner limit cycles with ICs in $x_0 \in [0, 8\pi], y_0 = 0$ and run-time 4000; (b) Three limit cycles with ICs in $x_0 \in [8.5\pi, 90\pi], y_0 = 0$ and run-time 60000; Specifically, part (a) of the figure shows the limit cycles of System (1) by nine ICs as $[x_0, 0]$ where $x_0 = 0.8889\pi, 1.7778\pi, 2.6667\pi, 3.5556\pi, 4.4444\pi, 5.3333\pi, 6.2222\pi, 7.1111\pi, 8\pi$; for part (b) of the figure three of outer and larger attractors are plotted; the ICs of these attractors are $[x_0, 0]$ where $x_0 = 8.5\pi, 28.875\pi, 49.25\pi, 69.6250\pi, 90\pi$; Some of the ICs result in the same attractors;

Table 1. A comparison of the previously proposed megastable oscillator with the oscillator provided in this study.

#	Reference	# of trigonometric terms	Dynamics
1	[31]	2	Oyster-like
2	[32]	4	Carpet-like
3	[35]	1	Jagged-like
4	[36]	5	-
5	This work	1	Eye of God nebula

Also, multistability has been seen in neuronal models which mimic the behavior of real neurons [25], as well as different disciplines of physics, like semiconductors [26]. Various types of multistability have been investigated, such as extreme multistable oscillators [27] and megastable oscillators [28,29]. Various megastable oscillators have been studied recently. A simple megastable oscillator with a triangular term was discussed in [30]. In [31], a megastable oscillator with oyster-like dynamics was proposed. In [32], a carpet-like megastable oscillator with only sinusoidal terms was investigated. It can be seen that the formation of attractors in megastable oscillators is a core in such researches. Therefore, this paper focuses on proposing a megastable oscillator with a unique attractors' formation. A comparison of the proposed oscillator with the previous ones is shown in Table 1.

Here, a megastable two-dimensional oscillator is designed. By forcing the oscillator, its chaotic dynamics and their bifurcations are discussed. The infinite attractors include cyclic attractors and strange attractors in various parameters. The oscillator has two types of attractors, the inner attractors with smaller amplitudes and the outer ones with larger amplitudes, like the Eye of God nebula. The 2D oscillator is proposed in Section 2, and its attractors are discussed. Then, applying a forcing term, the new attractors are studied. In Section 3, the forced oscillator is analyzed

through its bifurcations and Lyapunov exponents regarding different coexisting attractors. Eventually, the conclusion is stated in Section 4.

2. Proposed oscillator

Here, a two-dimensional megastable oscillator is proposed as follows:

$$\begin{aligned} \dot{x} &= \tanh(y), \\ \dot{y} &= -0.09x + y \cos(x) + x e^{\frac{(x-30)^2}{5}} \tanh(x). \end{aligned} \quad (1)$$

The system is designed based on a simple megastable system [33]. The new system is constructed by varying and adding some terms to the oscillator and using a vast search to find the megastable dynamics. The proposed oscillator has an equilibrium point at $(0,0)$. By using the stability analysis method, the Jacobian matrix of Eq. (1) at the fixed point is:

$$Jac = \begin{pmatrix} 0 & 1 \\ -0.09 & 1 \end{pmatrix} \quad (2)$$

The characteristic equations are:

$$\begin{aligned} |\lambda I - J| &= 0 \rightarrow \begin{vmatrix} \lambda & -1 \\ 0.09 & \lambda - 1 \end{vmatrix} = 0 \\ \rightarrow \lambda^2 - \lambda + 0.09 &= 0. \end{aligned} \quad (3)$$

The eigenvalues are $\lambda_1 = 0.9$, and $\lambda_2 = 0.1$. Since both eigenvalues are positive, the equilibrium point is unstable. So, it repels all the nearby trajectories. Figure 1 shows the coexisting limit cycles for various ICs. The oscillator has two groups of coexisting limit cycles. The first group is the four inner attractors, plotted in part (a) of Figure 1. They are plotted by nine ICs as $[x_0, 0]$ where $x_0 = 0.8889\pi, 1.7778\pi, 2.6667\pi, 3.5556\pi, 4.4444\pi, 5.3333\pi, 6.2222\pi, 7.1111\pi, 8\pi$. The second group is the outer and larger attractors. Three of these attractors are plotted in part (b) of Figure 1.

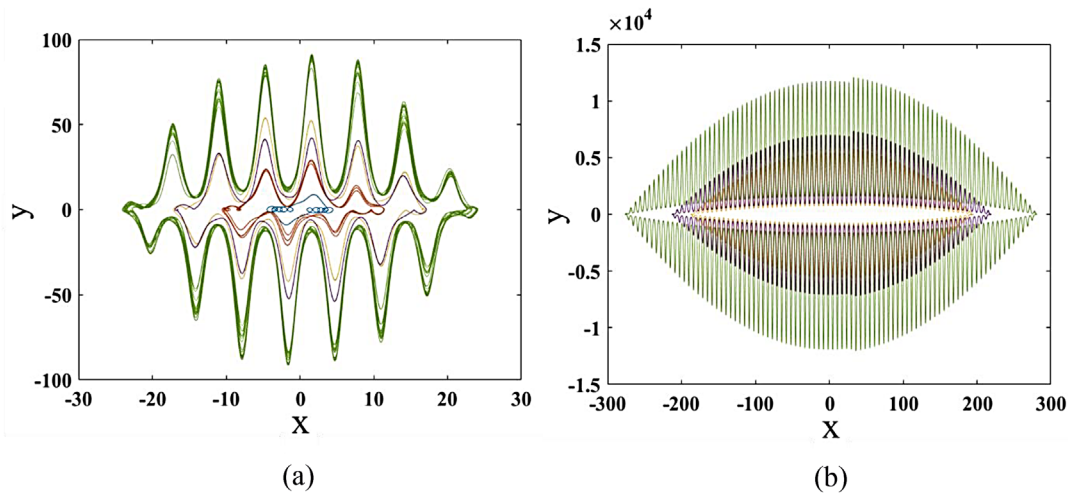


Figure 2. Seven attractors for the 14 ICs to present the coexistence of more complex dynamics like chaotic and limit cycle attractors of Eq. (4) for $A=2.7$; (a) four inner dynamics with ICs in $x_0 \in [0, 8\pi]$, $y_0 = 0$; (b) three outer dynamics with ICs in $x_0 \in [8.5\pi, 90\pi]$, $y_0 = 0$; Specifically, part (a) of the figure shows the dynamics by nine ICs as $[x_0, 0]$ where $x_0 = 0.8889\pi, 1.7778\pi, 2.6667\pi, 3.5556\pi, 4.4444\pi, 5.3333\pi, 6.2222\pi, 7.1111\pi, 8\pi$; for part (b) of the figure three of outer and larger attractors are plotted; the ICs of these attractors are $[x_0, 0]$ where $x_0 = 8.5\pi, 28.875\pi, 49.25\pi, 69.6250\pi, 90\pi$.

The ICs of these attractors are $[x_0, 0]$ where $x_0 = 8.5\pi, 28.875\pi, 49.25\pi, 69.6250\pi, 90\pi$.

The second group limit cycles have oscillations with larger amplitude, and they are like the Eye of God's nebula. They need more run time, and more transient time should be removed. The first group limit cycles are plotted with run time 4000, and the transient half of it is neglected. The second group limit cycles are calculated by run time 60000, and 0.8 of the time series are removed as transient time.

Then, by adding periodically forced term $A \sin(\sqrt{3}t)$ to the System (1), we have:

$$\begin{aligned} \dot{x} &= \tanh(y), \\ \dot{y} &= -0.09x + y \cos(x) + x e^{\frac{-(x-30)^2}{5}} \tanh(x) + A \sin \sqrt{3}t. \end{aligned} \quad (4)$$

Eq. (4) can show more complex dynamics by adding the periodically forcing term. Figure 2 shows the dynamics of Eq. (4) with ICs the same as Figure 1 and $A = 2.7$. Figure 2(a) shows the attractors with ICs in the interval $x_0 \in [0, 8\pi]$, $y_0 = 0$. The figure shows that some of the four inner attractors became chaotic with $A = 2.7$. Figure 2(b) presents the outer attractors with ICs in the interval $x_0 \in [8.5\pi, 90\pi]$, $y_0 = 0$. It seems that the three attractors are not chaotic in $A = 2.7$.

3. The forced system's dynamics

To study the dynamics of Eq. (4), the evolution of the attractors is investigated using the bifurcation diagrams. A is considered a bifurcation parameter. The bifurcation diagram and Lyapunov spectrums

are presented in Figure 3. Figure 3(a) and (b) show the bifurcation and Lyapunov of the largest attractor in the inner group, with the constant ICs $[-7.5\pi, 0]$. Figure 3(a) shows the bifurcation plot for the variable y of Eq. (4). By changing the bifurcation parameter A , the system shows different dynamics like quasiperiodic, chaotic, and periodic attractors regarding the bifurcation diagram in Figure 3(a). The largest Lyapunov exponent is plotted in green in Figure 3(b). The positive largest Lyapunov means that the system presents chaotic dynamics. Using Wolf's method, the Lyapunov exponents are approximated [34] with the run-time 20000.

Figure 4 demonstrates the bifurcation plot and Lyapunov exponents of Eq. (4) by changing the IC of x in the interval $x_0 \in [0, 40]$. The parameter A is set constant to 2.6. Figure 4(a) presents the dynamics of the logarithm of the y 's maxima by varying x_0 . The set of ICs is considered as $[x_0, 0]$. Figure 4(b) shows the largest Lyapunov exponent. The figure illustrates the basin of attractions of different attractors in the x_0 domain. The oscillator shows different strange attractors for the interval $x_0 \in [3.87, 25.88]$. To study Eq. (4) dynamics in different ranges, seven attractors with the selected ICs are plotted in Figure 5.

To investigate the basin of different attractors in 2D, Figure 6 is plotted. The basin of the four inner attractors is plotted in four colors (cyan, yellow, purple, and black). The white color shows the uninvestigated region of the basin of attraction.

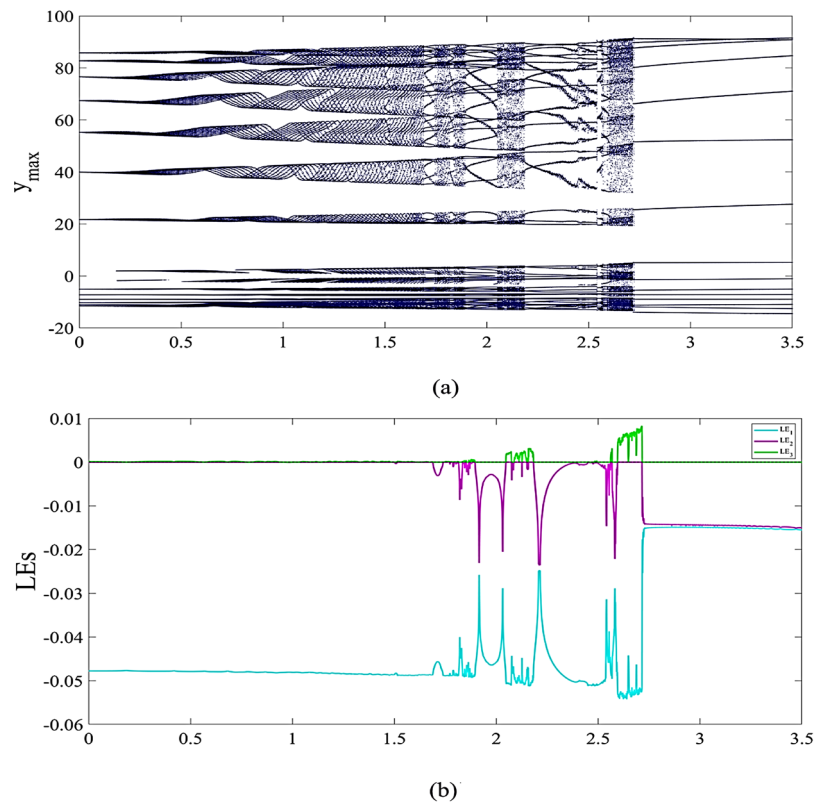


Figure 3. (a) The bifurcation plot of Eq. (4) by varying bifurcation parameter A ; (b) the three Lyapunov Exponents (LEs) in which the largest is plotted in green; where the largest LE is positive, the oscillator has the chaotic dynamics; the diagrams are plotted with the constant ICs $[-7.5\pi, 0]$.

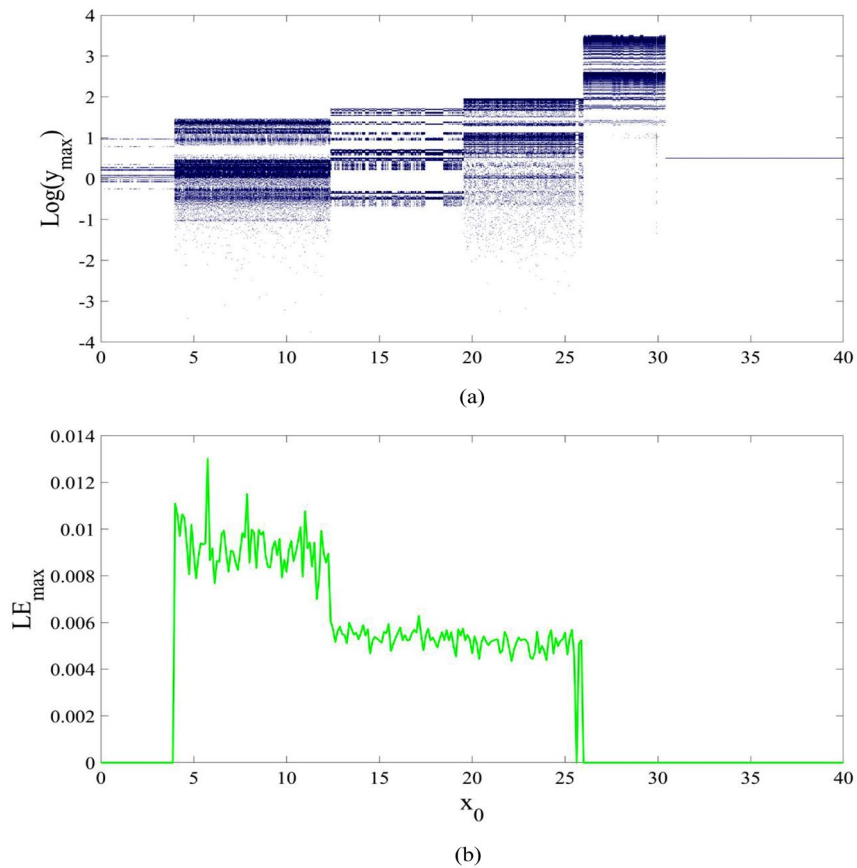


Figure 4. (a) The bifurcation plot of y of Eq. (4) as a function of the value of IC of variable x to investigate the range of chaotic attractors for the values of ICs; (b) The largest Lyapunov Exponent (LE_{\max}); The parameter $A=2.6$ is considered constant during the simulation.

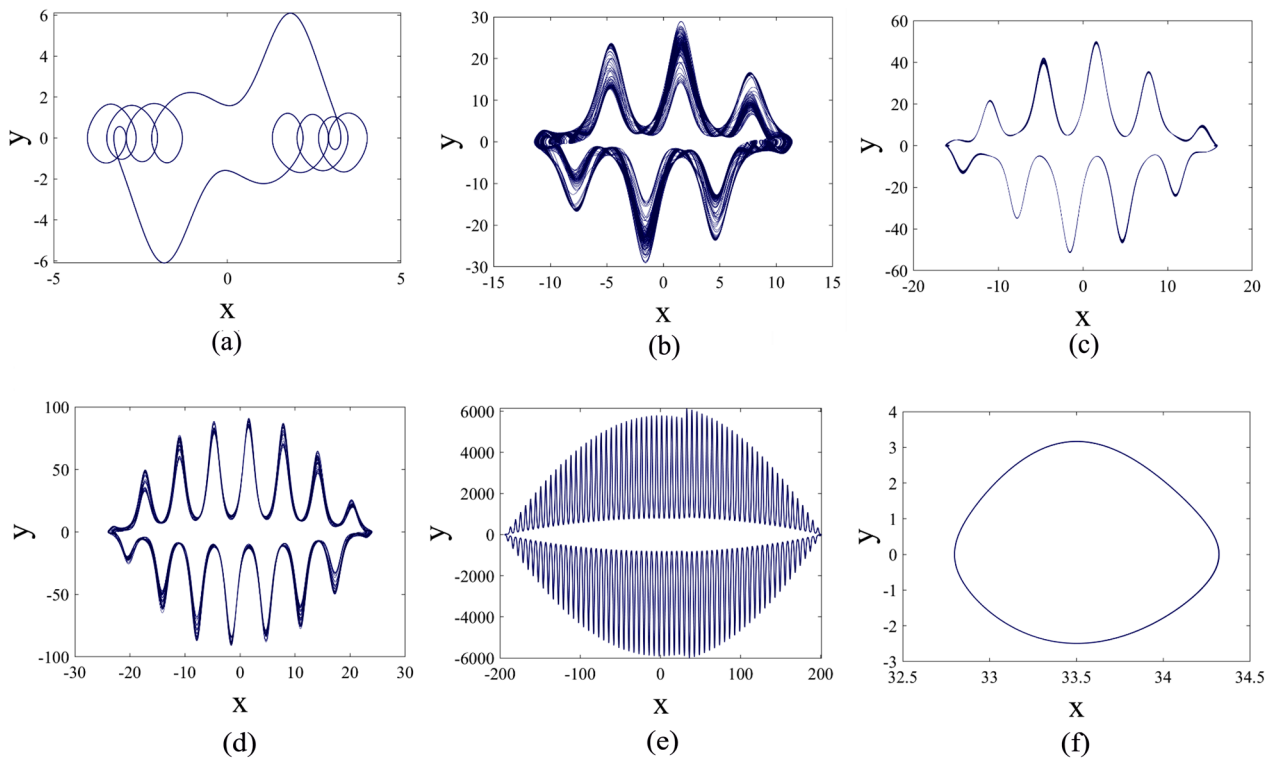


Figure 5. Attractors of Eq. (4) with different ICs corresponding to the different parts of bifurcation of Figure 4(a); (a) with ICs [1, 0]; (b) with ICs [7, 0]; (c) with ICs [15, 0]; (d) with ICs [23, 0]; (e) with ICs [28, 0]; (f) with ICs [35, 0]; Various dynamics of the oscillator by changing ICs of the x variable are presented.

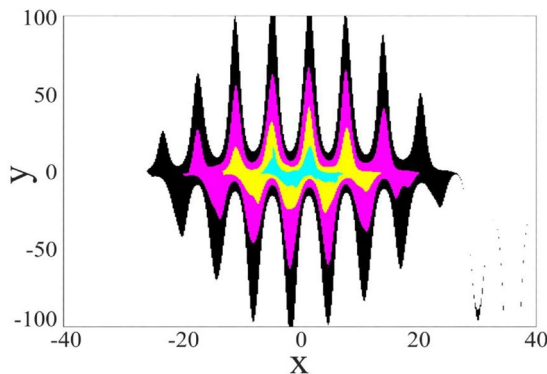


Figure 6. Basin of attraction of Eq. (4) shows the different attractors for the different values of ICs in $A=2.6$; The basin of attraction of four different attractors is plotted in different colors; the white color shows the uninvestigated region.

4. Conclusion

A new megastable oscillator was introduced. Various dynamics of the oscillator were discussed. In addition, the oscillator was studied by adding a periodically forcing term. Various periodic and chaotic attractors of the oscillator were analyzed through its attractors plot, bifurcation plot and Lyapunov exponent diagrams. The evolution of one of the attractors of the oscillator was investigated as a function of parameter A . Another bifurcation diagram was studied as a function of the initial values for the variable x . In this case, the basin of attractions of some of the attractors was discussed. The 2D basin of attraction diagram was also studied to investigate the

variation of basins for two variables' ICs. The results revealed the exciting dynamics of the oscillator.

Funding

This work is funded by the Center for Nonlinear Systems, Chennai Institute of Technology, India, vide funding number CIT/CNS/2022/RD/006.

Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Authors contribution statement

Frist author

Balamurali Ramakrishnan: Conceptualization; Investigation; Writing-original draft

Second author

Zahra Faghani: Investigation; Methodology; Writing- original draft.

Third author

Hayder Natiq: Methodology; Software; Writing-original draft.

Fourth author

Nikhil Pal: Methodology; Validation; Writing-review and editing.

Fifth author

Fahimeh Nazarimehr: Software; Supervision; Writing-review and editing.

Sixth author

Sajad Jafari: Supervision; Validation; Writing-review and editing.

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