Sustainable Supply Chain Evaluation with Shared Resources and Shared Feedbacks: A Common Set of Weights Slack Based Model

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Abstract

Data envelopment analysis (DEA) has been recently employed for performance and efficiency evaluation of decision-making units (DMUs) with multiple inputs and outputs. Complex internal relations within a DMU required more accurate DEA models than existing classic DEA. In real world applications, shared resources, among stages of a supply chain, is of great interest for managers and decision makers. Unfair allotment of shared factors can render the assessment invalid. In this study, an innovative DEA model is formulated for efficiency evaluation of a supply chain with shared factors. In the first step, a linear DEA model is presented in the multiplier form, based on the slack-based measure for efficiency assessment, in order to obtain the optimal proportion of the shared resources and shared feedbacks which are dual-role factors are presented. Then, the aggregate efficiency is decomposed into its stage efficiencies. Next, the presented model is extended for deriving the Common Set of Weights (CSW) for efficiency evaluation of the entire chain and correspondence stages. The case of 20 sustainable supply chains in the oil industry is considered with the developed DEA approach.

Keywords: *DEA; Sustainable Supply Chain; Share resources, Shared Feedbacks, Slack Based Measure of efficiency, CSW.*

1. Introduction

DEA is a mathematical method used extensively in the subject literature for performance evaluation of a set of DMUs. After developing the first DEA models by Charnes, Cooper, and Rhodes [1], which deals with the context of multiple inputs and outputs, several modifications are performed on classic DEA models. Afterward, using DEA models, it is possible to have multiple indicators that consider the effect of different indicators on performance. The key feature of the non-radial measure is to consider the effects of non-radial slacks in the efficiency, but slacks are neglected in radial DEA models which gets the efficiency simply and directly, Tone [2].

Usually, DMUs have network structures with intermediate products. Thus, classic DEA models consider black-box evaluation. Moreover, a more complicated case is where some inputs of the first stage should be split up and used by the second stage. This is a critical issue, as it can understate the efficiency when DEA fails to consider that some of the inputs generate other second-stage outputs. Shared resources are studied in the DEA technique from different aspects. Cook and Zhu [3] presented a paper to extend the usual DEA structure for determining the best resource split to optimize the aggregate efficiency score. Amirteimoori and Shafiei [4] provided a DEA method for omitting the shared resources from all DMUs due to abrupt incidents. Chen et al. [5], and Amirteimoori [6] presented DEA models for the performance assessment of a network consisting of two stages with non-splitable shared inputs. An et al. [7] developed a network DEA model where shared resources are considered with cooperative and non-cooperative gaming. Also, An et al. [8] introduced network processes with shared extra intermediate resources in DEA. Moreover, An et al. [9] introduced profit inefficiency decomposition with resource sharing in DEA. Moghaddas, Mohhamadpour Tosarkani, and Yousefi [10] extended inverse DEA models in network structures. Ma [11] studied partial impact between inputs and outputs for network production systems. Izadikhah et al. [12] and Puri, Yadav, and Garg. [13] considered DEA models with shared resources and measured interval efficiencies. Stefaniec et al. [14] and Zhou and Hu [15] introduced a network with shared resources that incorporates undesirable output. Chai and Zhao [16], Chao, Yu, and Hsieh [17] and Phung et al. [18] respectively examine a parallel production system, a dynamic network DEA, and a mixed-network structure to allocate shared resources for efficiency measurement. Álvarez-Rodríguez et al. [19] proposed a dynamic Network model for sustainability-oriented efficiency evaluation. Jiang et al. [20] suggested allocating shared resources to satisfy conditions for an optimizable operation. Azadi Et al. [21] developed a novel network range directional measure (RDM) approach for evaluating the sustainability of a set of DMUs. Li and Cui [22] conducted a study on different stages in airline departments with shared inputs. Wang et al. [23] obtained overall and individual efficiency scores with altering weight in the model. Avilés-Sacoto et al. [24] and Avilés-Sacoto et al. [25] studied DMUs grouped according to multiple attributes and with multiple shared inputs. Zhang, Wang, and Zhu [26] presented a two-stage network DEA model with shared inputs and fuzzy set qualitative comparative analysis. Wang, Wu, and Chen [27] provided the decomposition of weights in a network DEA model with shared resources. Phung et al. [18] developed a new DEA model to solve a mixed network structure. According to Shi et al. [28], dual-role factors play both input and output roles simultaneously.

Supply chain management is widely affected by measuring the sustainability performance of supply chains, according to Gupta and Palsule-Desai [29]. Thus, production and business practices can be improved as stated by Min and Kim [30]. As a variety of systems extensively benefit from sustainable supply chains, sustainable supply chains are frequently utilized. Also, DEA is applied for the performance and efficiency assessment of supply chains in various aspects. Roy et al. [31] suggested strategies for environmental sustainability by introducing a methodology based on fuzzy cognitive map and DEA. Tavassoli, Ketabi, and Ghandehari [32] and Kalantary et al. [33] evaluated the sustainability of networks. Wang et al. [23] proposed a multi-region input-output DEA model. Álvarez-Rodríguez, Martín-Gamboa, and Iribarren [19] employed DEA modeling for the operational performance. Tang et al. [34] introduced a model for eco-efficiency evaluation. Gilani, Sahebi, and Oliveira [35] studied the fuzzy integrated method for selecting the suitable supply potential points.

Various models have been introduced in the DEA technique to evaluate DMUs with a network structure, yet the SBM model which evaluates efficiency more accurately has been employed less frequently. It is essentials to consider the feedbacks in the applications of network models, which has attracted scanty attention in the field of shared sources. Also, the models have not addressed fair evaluation of efficiency using the common set of weights model, which aims to maximize the efficiency of all units at the same time, that have so far allocated shared resources in network DEA. Therefore, it seems necessary to evaluate the efficiency and determine the optimal shares of network stages from shared resources with the common set of weights method that examines performance evaluation more fairly through the SBM model in a DMU with network structure that includes feedbacks (returned production). Determining optimal shares is an important issue since an inadequate portion of shared factors may cause the system to collapse.

This paper aims to indicate that a DMU as a black box cannot reveal inherent relations among the stages while optimal shares of resources are investigated. Notably, unfair allocation of shared resources can lead to inaccurate assessment of the aggregate and stage efficiencies. In almost every real application, there are complex internal relations, such as shared resources among different stages and importantly shared feedbacks, in networks that significantly affect the efficiency evaluation. The present study aims to fit the multiplier form of the SBM model on a decision-making unit with a network structure consisting of three stages with intermediate inputs/outputs, independent output/inputs, and feedbacks which are dual-role factors. Then, shared resources and feedbacks are considered/investigated in mathematical modeling for evaluating performance and determining the optimal shares of each stage of the network from assets they have in common. Moreover, the present model incorporates the decomposition of aggregate efficiency into its stage efficiencies. Afterward, using the common set of weights method makes it possible to evaluate DMUs fairly. It means maximizing the performance of all DMUs simultaneously.

The contribution of this study can be summarized as follows:

- For the first time, shared resources are obtained from the SBM-Network DEA model.
- An innovative idea is considering shared feedback between the stages of the supply chain.
- With an innovative idea, the presented model extended to find the common set of weights for fairly analyzing the efficiency scores for both the chain and its stages.
- The proposed common set of weights model is a multi-objective model converted into a single objective model via goal programming.
- A linear slack-based network model is presented to evaluate the efficiency scores of the entire supply chain and the stages.
- A case study of 20 sustainable supply chains is presented.

The rest of the paper is organized as follows: in section 2, some DEA preliminaries are briefly reviewed. A new model with shared inputs, feedbacks, and outputs in a three-stage supply chain is introduced in section 3. Section 4 presents the case study for a set of sustainable supply chains. Finally, in section 5, managerial implications are presented and discussed, and section 6 concludes the paper.

2. Research Gap

2.1 literature review

Consider Table 1 in which the literature review for several recent articles, relevant to this research, is mentioned.

2.2 Research Gap

In network systems, resources can be shared by several stages of the network. The use of shared resources and feedbacks and determining the share of each stage from the common resources is the concern of managers and decision makers. In this respect, articles have mainly focused on finding the optimal share of each stage from common sources, and their goal is to maximize the efficiency of each decision-making unit. Note that, feedbacks as dual-role factors play both input and output roles. Also, the multiplier form of the SBM model, which can measure efficiency more accurately due to taking input and output vector components into analysis, has received less attention. Another feature of the model is to evaluate the performance of the network structural DMUs and determine optimal share of each stage of the network from common resources, to find an optimal common weight that can be used to maximize the efficiency of all DMUs simultaneously.

3. DEA Preliminaries

DEA is now extensively used for the performance evaluation of DMUs in production systems and activities. Multi-attribute decision-making methods have been introduced and improved (Liu and Liu [36]) but they have not the ability of deriving relative efficiency score like DEA method. As an early model, the CCR, named after Charnes, Cooper, and Rhodes [1], was quickly introduced and developed. The first non-radial model for efficiency evaluation was introduced by Tone [2], named as Slack Based Measure of efficiency (SBM).

Let X and Y be the input and output matrixes, and x and y be the input and output vectors. Consider λ as the vector of intensifier variables, and s^- and s^+ as the vectors of input slacks and output surpluses. Also, ρ is defined as the free variable for objective function of SBM model (1) which shows the efficiency score of the DMU_o which is under evaluation.

$$\text{Min } \rho = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{ro}}} \quad (1)$$

$$
\begin{aligned}
& s.t. \\
& X\lambda + s^- = x_o, \\
& Y\lambda - s^+ = y_o, \\
& \lambda \geq 0, s^- \geq 0, s^+ \geq 0.
\end{aligned}
$$

Tone (2001) [2] proved $0 < \rho^* \leq 1$, an inefficient DMU_o can improve its inputs and outputs according to $\left(x_o - s^{-*}, y_o + s^{+*}\right)$.

4. Slack-based DEA Model in a Sustainable Supply Chain with Shared Resources

Consider a three-stage supply chain consisting of the supplier, manufacturer, and distributer as in Figure 1. X is the input for the supplier, and Q and Z are corresponding outputs. For the manufacturer, Z and L are the inputs, and Y is the output. Y is an input of the distributer and D is its output.

Enveloping form of the SBM model for the specified supply chain in figure 1 is formulated as model (2). λ^1, λ^2, and λ^3 are non-negative intensifier variables each used for the supplier, the manufacturer, and the distributor, respectively. X is the input matrix of the supplier, and Z is the matrix of intermediate product that is the output for the supplier. Also, Q is a matrix of the independent output for the supplier. The manufacturer has two inputs, L is the matrix of independent input, and Z is the matrix of the intermediate product. Y is the matrix of intermediate product that is the output of the manufacturer and input of the distributer. Also, D is a matrix of desirable outputs for the distributer. Assume s^- to be the slack variable of the input X. Also, assume $\tilde{s}^+$ and $\bar{s}^+$ respectively as the surplus variables for outputs Z and Q for the supplier. Assume $\hat{s}^-$ and s^- respectively as the slack variables of inputs L and Z. Assume s^+ as the surplus variable of output Y for the manufacturer. Let $\dot{s}^-$ be the slack variable of input Y and $\dot{s}^+$ as the surplus variable of output D for the distributer. Consider DMU_o as the DMU under assessment.

$$Min \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{\bar{s}_i}{x_{io}} - \frac{1}{e}\sum_{c=1}^{e} \frac{\hat{s}_c^-}{l_{co}}}{1 + \frac{1}{a}\sum_{c=1}^{a} \frac{\dot{s}_c^+}{d_{co}} + \frac{1}{b}\sum_{p=1}^{b} \frac{\tilde{s}_p^+}{q_{po}}} \quad \text{(a)}$$

(2)

$$s.t.$$

$$X\lambda^1 + s^- = x_o, \quad \text{(b)}$$

$$Z\lambda^1 - \bar{s}^+ = z_o \quad \text{(c)}$$

$$Q\lambda^1 - \tilde{s}^+ = q_o \quad \text{(d)}$$
$$L\lambda^2 + \hat{s}^- = l_o \quad \text{(e)}$$
$$Y\lambda^2 - s^+ = y_o. \quad \text{(f)}$$
$$Y\lambda^3 + \dot{s}^- = y_o \quad \text{(g)}$$
$$D\lambda^3 - \dot{s}^+ = d_o \quad \text{(h)}$$
$$\lambda^1 \ge 0, \lambda^2 \ge 0, \lambda^3 \ge 0 \quad \text{(i)}$$
$$s^- \ge 0, s^+ \ge 0, \hat{s}^- \ge 0,$$
$$\tilde{s}^+ \ge 0, \overline{s}^+ \ge 0, \dot{s}^+ \ge 0, \dot{s}^- \ge 0$$

Model (2) maximizes the efficiency of the entire chain while considering all the links among the stages. Constraints (a), (b), and (c) show the inputs and outputs of the supplier. Constraints (e), (f), and (g) indicate the inputs and outputs of the manufacturer. The same is for constraints (h) and (i) that show the input and output of the distributor. The rest of the constraints show non-negative variables. Using variable transformations introduced in expression (3), nonlinear objective function of model (2) is converted into its linear counterpart. Thus, linear model (4) is obtained.

$$\frac{1}{1 + \frac{1}{a}\sum_{c=1}^{a}\frac{\dot{s}_c^+}{d_{co}} + \frac{1}{b}\sum_{p=1}^{b}\frac{\tilde{s}_p^+}{q_{po}}} = t \quad (3)$$

With reference to (3), it is clear that $t > 0$.

Therefore, the linear counterpart of model (2) is formulated as follows:

$$Min \quad t - \frac{1}{m}\sum_{i=1}^{m}\frac{\overline{s}_i}{x_{io}} - \frac{1}{e}\sum_{c=1}^{e}\frac{\hat{s}_c^-}{l_{co}}$$

$$s.t.$$

$$t + \frac{1}{c}\sum_{c=1}^{a}\frac{\dot{s}_c^+}{d_{co}} + + \frac{1}{b}\sum_{p=1}^{b}\frac{\tilde{s}_p^+}{q_{po}} = 1 \quad \text{(a)}$$
$$X\lambda^1 + s^- = x_o, \quad \text{(b)} \qquad (4)$$
$$Z\lambda^1 - \overline{s}^+ = tz_o \quad \text{(c)}$$
$$Q\lambda^1 - \tilde{s}^+ = tq_o \quad \text{(d)}$$
$$L\lambda^2 + \hat{s}^- = tl_o \quad \text{(e)}$$
$$Y\lambda^2 - s^+ = ty_o \quad \text{(f)}$$

$Z\lambda^2 + s^- = tz_o$ (g)

$Y\lambda^3 + \dot{s}^- = ty_o$ (h)

$D\lambda^3 - \dot{s}^+ = \mathrm{t}d_o$ (i)

$\lambda^1 \geq 0, \lambda^2 \geq 0, \lambda^3 \geq 0$

$s^- \geq 0, s^+ \geq 0, \hat{s}^- \geq 0, \dot{s}^- \geq 0,$

$\tilde{s}^+ \geq 0, \bar{s}^+ \geq 0, \dot{s}^+ \geq 0$, $t > 0.$

Consider variable transformations as $t\lambda^1 = \lambda^1$, $t\lambda^2 = \lambda^2$, $t\lambda^3 = \lambda^3$, $ts^- = s^-$, $t\bar{s}^+ = \bar{s}^+, t\tilde{s}^+ = \tilde{s}^+$, $t\hat{s}^- = \hat{s}^-, ts^+ = s^+, ts^- = s^-, t\dot{s}^- = \dot{s}^-$, and $t\dot{s}^+ = \dot{s}^+$. For simplicity, the notations are considered the same for variables before the transformation. After variable transformation, all new variables are non-negative, because $t > 0$ and the fact that all initial variables were non-negative.

Considering constraints (c) and (g), Z is an intermediate product which is the output of stage 1 and the inputs of stage 2. A combination of these two constraints can be used as follows.

$Z\lambda^1 = tz_o + \bar{s}^+, Z\lambda^2 = tz_o - s^-$, $s^- \geq 0, \bar{s}^+ \geq 0 \rightarrow Z\lambda^2 = tz_o - s^- < tz_o + \bar{s}^+ = Z\lambda^1$

$\rightarrow Z\lambda^2 < Z\lambda^1$

(5)

Y is an intermediate product between manufacturer and distributor, so the same argument holds for constraints (f) and (h). Therefore,

$Y\lambda^3 < Y\lambda^2$ (6)

Expressions (5) and (6) are consistent with the network axiom. It implies that the amount of input used in the next stage, supplied by intermediate production, must be less than the outputs that are produced from the previous stage.

The dual model corresponds to model (4) is formulated as in model (7). Consider the intermediate products z and y in model (4). For the intermediate products, different variables in the dual model (7) are considered. When z is the output of stage 1, the corresponding variable is set to be w^1. Also, when z is the input of stage 2, the corresponding variable is set to be w^2. Similarly, when y is the output of stage 2, the corresponding variable is set to u^1 and for the inputs of stage 3, the corresponding variable is set to be .

$$
\begin{aligned}
& Max \quad \xi \\
& s.t. \\
& \xi - \sigma d_o \ + u^2 y_o - u^1 y_o + kl_c o + w^2 z_o - hq_o - w^1 z_o + vx_o = 1 \\
& \sigma D + u^2 Y + u^1 Y - kL - w^2 Z + hQ + w^1 Z - vX \le 0, \\
& -\sigma \ + \frac{\xi}{a}\left[\frac{1}{d_o}\right] \le 0, \\
& -k \le -\frac{1}{e}\left[\frac{1}{l_o}\right], \\
& -h + \frac{\xi}{b}\left[\frac{1}{q_O}\right] \le 0, \\
& -v \le -\frac{1}{m}\left[\frac{1}{x_o}\right], \\
& -u^2 \le 0, -u^1 \le 0,
\end{aligned} \tag{7}
$$

With regards to expression (5), we should consider a similar dual variable for constraints (c) and (g), which means $w^1 = w^1 = w$. The same holds for constraints (f) and (h), according to expression (6), thus $u^1 = u^2 = u$, as stated in Chen et al. [5]. Therefore, Model (7) will be written as follows:

$$
\begin{aligned}
& Max \quad \xi \\
& s.t. \\
& \xi - \sigma \ d_o + kl_{co} - hq_o + vx_o = 1 \\
& \sigma \ D \ - kL + hQ - vX \le 0, \\
& -\sigma \ + \frac{\xi}{a}\left[\frac{1}{d_o}\right] \le 0, \\
& -k \le -\frac{1}{e}\left[\frac{1}{l_o}\right], \\
& -h + \frac{\xi}{b}\left[\frac{1}{q_O}\right] \le 0, \\
& -v \le -\frac{1}{m}\left[\frac{1}{x_o}\right],
\end{aligned} \tag{8}
$$

According to model (8), $\xi = 1 + hq_o + \sigma \ d_o - kl_{co} - vx_o$ shows the efficiency of the chain. Model (8) can be simplified using the following relation:

$$\xi = 1 + \sigma \ d_o - kl_{co} + hq_o - vx_o \tag{9}$$

Thus, model (10) will be obtained.

$$
\begin{aligned}
&1+\left(Max \ \ \sigma\ d_o - kl_o + hq_o - vx_o\right)\\
&s.t.\\
&\sigma\ D\ \ -kL + hQ - vX\ \le 0,\\
&\sigma\ \ \ge \frac{1+\sigma\ d_o - kl_o + hq_o - vx_o}{a}\left[\frac{1}{d_o}\right],\\
&k \ge \frac{1}{e}\left[\frac{1}{l_o}\right],\\
&h \ge \frac{1+\sigma^1 d_o^1 - \sigma^2 d_o^2 - kl_o + hq_o - vx_o}{b}\left[\frac{1}{q_o}\right],\\
&v \ge \frac{1}{m}\left[\frac{1}{x_o}\right],
\end{aligned}
\tag{10}
$$

Figure 1shows the three-stage supply chain. Having optimal solution (weights) of model (10) efficiency scores of each stage can be calculated via relations (11).

$$
\begin{aligned}
&\text{Efficiency of Stage1} = E.S_1 = \frac{h^* q_o + w^* z_o}{v^* x_o},\\
&\text{Efficiency of Stage2} = E.S_2 = \frac{u^* y_o}{h^* l_o + w^* z_o}\\
&\text{Efficiency of Stage3} = E.S_3 = \frac{\sigma^* d_o}{u^* y_o + \sigma^{*2} d_o^2}
\end{aligned}
\tag{11}
$$

The objective function of the model (10) can be discussed from another viewpoint. The objective function of the model (10) is to maximize the weighted sum of stage-efficiency scores. With regards to the weights defined in (12), equivalent to the objective function of the model (10) is the relation (13). For each stage, weight is defined as a ratio of the sum of weighted inputs of each stage to the sum of weighted inputs of all stages.

$$
\begin{aligned}
&\omega_1 = \frac{vx_o}{vx_o + w\ z_o + kl_o + u\ \ y_o}\\
&\omega_2 = \frac{kl_o + w\ z_o}{vx_o + w\ z_o + kl_o + u\ \ y_o}\\
&\omega_3 = \frac{\sigma\ d_o}{vx_o + w\ z_o + kl_o + u\ \ y_o}
\end{aligned}
\tag{12}
$$

Then, considering relations (11) and (12), the objective function of the model (10) will be defined as the sum of weighted stage efficiency scores (13).

$$\text{Aggregate Efficiency}\left(\text{A.}E\right)=\omega_1 e_1+\omega_2 e_2+\omega_3 e_3=\frac{\sigma\ d_o+uy_o+\text{w}z_o+hq_o}{vx_o+\text{w}z_o+kl_o+\text{u}y_o} \tag{13}$$

For maximizing the ratio in the objective function, the nominator of fraction (13) is maximized, and the denominator is minimized. The linear equivalent to relation (13) is the relation (14).

$$(\sigma\ d_o+\text{u}y_o+\text{w}z_o+hq_o)-\left(vx_o+\text{w}z_o+kl_o+\text{u}y_o\right)=\sigma\ d_o-kl_{co}+hq_o-vx_o \tag{14}$$

Notably, equations (13) and (14) are equivalent in identifying efficient units. In maximizing expression (14), the first term is maximized and the second minimized. After simplification, as can be seen, expression (14) is similar to the objective function of the model (10). Instead of maximizing aggregate efficiency, $\sigma\ d_o+hq_o-vx_o-kl_o$ can be maximizing. Thus, the objective function of the model (10) can be considered as expression (14). Moreover, for deriving the efficiency score of each stage, corresponding relations (b), (c), and (d) are also added in the model (15).

$$
\begin{aligned}
&1+\left(\text{Max}\quad \sigma\ d_o+hq_o-vx_o-kl_o\right) && \text{(a)}\\
&s.t.\\
&hQ+wZ-vX\le 0 && \text{(b)}\\
&u\ \ Y-kL-wZ\le 0 && \text{(c)}\\
&\sigma\ D\ -uY\le 0 && \text{(d)}\\
&\sigma\ D\ -kL+hQ-vX\le 0 && \text{(e)}\\
&\sigma\ \ge\frac{1+\sigma\ d_o-kl_o+hq_o-vx_o}{a_1}\left[\frac{1}{d_o}\right], && \text{(f)}\\
&h\ge\frac{1+\sigma\ d_o-kl_o+hq_o-vx_o}{b}\left[\frac{1}{q_o}\right], && \text{(g)}\\
&k\ge\frac{1}{e}\left[\frac{1}{l_o}\right], && \text{(h)}\\
&v\ge\frac{1}{m}\left[\frac{1}{x_o}\right], && \text{(i)}\\
&u\ge 0,\ w\ge 0,
\end{aligned}
\tag{15}
$$

Constraints (b), (c), and (d) respectively confine the stage efficiency scores in a way that $e_1\le 1, e_2\le 1,$ and $e_3\le 1$. Constraint (e) confines the efficiency of the chain in a way that $E\le 1$. As evident in Figure 2, shared inputs exist in the chain. Input X has two parts of βX, and $\left(1-\beta\right)X$. Clearly, $\beta X+\left(1-\beta\right)X=X$. Note that β is a variable for which $0\le\beta\le 1$. Now, consider βX as the input for the supplier and $\left(1-\beta\right)X$ as the input for the manufacturer.

Now consider model (16) for analyzing the supply chain as demonstrated in Figure 2.

$$
\begin{aligned}
& Max 1+\sigma \ d_o + hq_o - v\beta x_o - v(1-\beta) x_o - kl_o \\
& s.t. \\
& hQ + wZ - v\beta X \le 0 && \text{(a)} \\
& u \ \ Y - kL - wZ - v(1-\beta) X \le 0 && \text{(b)} \\
& \sigma \ D \ \ - uY \le 0, && \text{(c)} \\
& \sigma \ D \ \ - kL - v(1-\beta) X + hQ - v\beta X \le 0, && \text{(d)} \\
& \sigma \ge \frac{1+\sigma \ d_o - kl_o - v(1-\beta) x_o + hq_o - v\beta x_o}{a}\left[\frac{1}{d_o}\right], && \text{(e)} \\
& h \ge \frac{1+\sigma \ d_o - kl_o - v(1-\beta) x_o + hq_o - v\beta x_o}{b}\left[\frac{1}{q_o}\right], && \text{(f)} \\
& k \ge \frac{1}{e}\left[\frac{1}{l_o}\right], && \text{(g)} \\
& v \ge \frac{1}{m\left(\beta x_o + (1-\beta) x_o\right)} = \frac{1}{m}\left[\frac{1}{x_o}\right], && \text{(h)} \\
& w \ \ \ge 0, u \ \ \ge 0, 0 \le \beta \le 1. && \text{(i)}
\end{aligned}
\qquad (16)
$$

Model (16) is non-linear, thus with the following variable transformation (17), the linear counterparts of constraints of the model (12) can be formulated.

$$
v\beta + v(1-\beta) = v \quad \rightarrow \quad v\beta = v - v(1-\beta) \quad \Rightarrow \quad \begin{cases} v(1-\beta) = \hat{v} \\ v\beta = v - \hat{v} \end{cases} \qquad (17)
$$

According to $\beta \ge 0$ and $v > 0$, and using (17), $\hat{v} \ge 0$ and $v - \hat{v} \ge 0$ are obtained. Afterwards, according to the variable transformation introduced in (17), consider model (18) as the following.

$$
\begin{aligned}
& Max 1+\sigma \ d_o + hq_o - \hat{v} x_o - (v-\hat{v}) x_o - kl_o && \text{(a)} \\
& s.t. \\
& hQ + wZ - vX + \hat{v} X \le 0 && \text{(b)} \\
& uY - kL - wZ - \hat{v} X \le 0 && \text{(c)} \\
& \sigma \ D \ \ - uY \le 0 && \text{(d)} \\
& \sigma \ D \ \ - kL + hQ - (v-\hat{v}) X - \hat{v} X \le 0 && \text{(e)} \\
& \sigma \ \ \ge \frac{1+\sigma \ d_o - kl_o - (v-\hat{v}) x_o + hq_o - \hat{v} x_o}{a}\left[\frac{1}{d_o}\right] && \text{(f)} \\
& h \ge \frac{1+\sigma \ d_o - kl_o - (v-\hat{v}) x_o + hq_o - \hat{v} x_o}{b}\left[\frac{1}{q_o}\right], && \text{(g)} \\
& k \ge \frac{1}{e}\left[\frac{1}{l_o}\right] && \text{(h)}
\end{aligned}
\qquad (18)
$$

$$v \geq \frac{1}{m}\left[\frac{1}{x_o}\right] \qquad \text{(i)}$$

$w \geq 0, \quad u \geq 0,$

$v - \hat{v} \geq 0, \quad \hat{v} \geq 0,$

In model (18), expression $\hat{v}x_o + (v - \hat{v})x_o$ can be simplified and then replaced with vx_o.

Definition: Having the optimal solution of model (18), the aggregate efficiency (A.E) and stage efficiencies (S.E) for DMU_o under evaluation are defined as follows:

$$\begin{aligned}
&\text{Agreggate Efficiency} = \text{A.}E = \frac{\sigma^* d_o + u^* y_o + w^* z_o + h^* q_o}{v^* x_o + w^* z_o + k^* l_o + u^* y_o} \\
&\text{Efficiency of Stage 1} = E.S_1 = \frac{h^* Q_o + w^* z_o}{v^* x_o + \hat{v}^* x_o} \\
&\text{Efficiency of Stage 2} = E.S_2 = \frac{u^* Y_o}{k^* L_o + w^* Z_o + \hat{v}^* X_o} \\
&\text{Efficiency of Stage 3} = E.S_3 = \frac{\sigma^* d_o}{u^* Y_o}
\end{aligned} \qquad (19)$$

An important feature of model (18) is that it is based on the SBM model, and that all possible inefficiencies might exist in each element of the input, intermediate, and output vectors. Now consider more complicated sustainable supply chains as shown in Figure 3. Consider the chain demonstrated in Figure 3. $\boldsymbol{\alpha}$ and $\boldsymbol{\delta}$ are variables introduced to recognize shares of $\boldsymbol{Y}$ and $\boldsymbol{L}$, respectively. Note that F is the manufacturer's output entirely consumed by the distributer and $(1-\boldsymbol{\alpha})\boldsymbol{Y}$ is a part of the manufacturer's output that enters the distributor. Therefore, the remaining part (αY) is the feedback of the manufacturer that enters the supplier. Also, let $(1-\boldsymbol{\delta})\boldsymbol{L}$ be the part of the manufacturer's input and the remaining part $(\boldsymbol{\delta L})$ to be the input of the distributer that enters the chain. Consider the expression (20) for introducing the shares of $\boldsymbol{Y}$ and $\boldsymbol{L}$.

$$u\alpha + \mathrm{u}(1-\alpha) = \mathrm{u} \quad \rightarrow \quad \mathrm{u}\alpha = \mathrm{u} - \mathrm{u}(1-\alpha) \quad \Rightarrow \quad \begin{cases} \mathrm{u}(1-\alpha) = \hat{u} \\ \mathrm{u}\alpha = \mathrm{u} - \hat{u} \end{cases} \qquad (20)$$

According to $0 \leq \alpha \leq 1$, $u \geq 0$, $\alpha u \geq 0$, and relation (20), $\hat{u} \geq 0$ and $u - \hat{u} \geq 0$ are obtained

$$k\delta + k(1-\delta) = \delta \quad \rightarrow \quad k\delta = \delta - k(1-\delta) \quad \Rightarrow \quad \begin{cases} k(1-\delta) = \hat{k} \\ k\delta = k - \hat{k} \end{cases} \qquad (21)$$

Similarly, considering $0 \le \delta \le 1$, $k > 0$, $k\delta \ge 0$, and relation (21), $\hat{k} \ge 0$ and $k - \hat{k} \ge 0$ are acquired.

Notice model (22) for efficiency evaluation of a three-stage supply chain as shown in Figure 3. According to the defined variable transformations in model (18), $\alpha\, uY + (1-\alpha)\, \mathrm{uY} = \mathrm{u}\, Y$, $v\beta X + v(1-\beta) X = vX$ and $\delta kL + (1-\delta) kL = kL$ are replaced for simplicity of notations in constraints.

$$
\begin{aligned}
& Max\ 1+\sigma\ d_o + hq_o - vx_o - kl_o \\
& s.t. \\
& h\ Q + \mathrm{wz} - vX + \hat{v}X - \alpha Y \le 0, && \text{(b)} \\
& pF + uY - \delta kL - wZ - \hat{v}X \le 0, && \text{(c)} \\
& \sigma\ D\ -(1-\alpha)Y - (1-\delta)\ L - pF \le 0, && \text{(d)} \\
& \sigma\ D\ -kL + hQ - vX \le 0, && \text{(e)} \\
& \sigma\ \ge \frac{1+\sigma\ d_o + hq_o - vx_o - kl_o}{a}\left[\frac{1}{d_o}\right], && \text{(f)} \\
& h \ge \frac{1+\sigma\ d_o + hq_o - vx_o - kl_o}{b}\left[\frac{1}{q_o}\right], && \text{(g)} \\
& k \ge \frac{1}{e}\left[\frac{1}{l_o}\right], && \text{(h)} \\
& v \ge \frac{1}{m}\left[\frac{1}{x_o}\right], && \text{(i)} \\
& w \ge 0,\ u \ge 0,\ \mathrm{v} \ge 0, p \ge 0, \\
& k \ge 0, u \ge 0,
\end{aligned}
\qquad (22)
$$

The entire Y is the output of stage 2, but only αY is used as an input of stage 3. The other portion, $(1-\alpha)\mathrm{Y}$, is returned to the supplier.

Now according to relations (20) and (21), model (23) will be converted to its linear counterpart as follows:

$$
\begin{aligned}
& Max\ 1+\sigma\ d_o + hq_o - vx_o - kl_o \\
& s.t. \\
& h\ Q + \mathrm{wz} - vX + \hat{v}X - uY + \hat{u}Y \le 0, && \text{(b)} \\
& pF + uY - kL + \hat{k}L - wZ - \hat{v}X \le 0, && \text{(c)} \\
& \sigma\ D\ -\hat{u}Y - \hat{k}\ L - pF \le 0, && \text{(d)} \\
& \sigma\ D\ -k\ L + h\ \ Q - vX \le 0, && \text{(e)}
\end{aligned}
\qquad (23)
$$

$$\sigma \geq \frac{1+\sigma\ d_o + hq_o - vx_o - kl_o}{a}\left[\frac{1}{d_o}\right], \quad \text{(f)}$$

$$h \geq \frac{1+\sigma\ d_o + hq_o - vx_o - kl_o}{b}\left[\frac{1}{q_o}\right], \quad \text{(g)}$$

$$k \geq \frac{1}{e}\left[\frac{1}{l_o}\right], \quad \text{(h)}$$

$$v \geq \frac{1}{m}\left[\frac{1}{x_o}\right], \quad \text{(i)}$$

$$w \geq 0, \ \ \mathrm{u} \geq 0,$$

$$0 \leq \beta v \leq v, 0 \leq \delta k \leq k, 0 \leq \alpha u \ \ \leq u \ ,$$

$$v - \hat{v} \geq 0, \ \ u \ - \hat{u} \ \ \geq 0, \ \ k - \hat{k} \geq 0,$$

Definition 2: Having the optimal solution of model (23), the aggregate efficiency (A.E) and stage efficiencies (S.E) for DMU_O under evaluation are as follows. Moreover, according to this decomposition of stage efficiencies, corresponding weights can be obtained.

$$\text{Aggregate Efficiency} = \mathrm{A.}E = \frac{\sigma^* d_o + u^* y_o + w^* z_o + h^* q_o}{v^* x_o + w^* z_o + k^* l_o + u^* y_o - \hat{u}^* y_o}$$

$$\text{Efficiency of Stage } 1 = E.S_1 = \frac{h^* q_o + w^* z_o}{v^* x_o + \hat{v}^* x_o}$$

$$\text{Efficiency of Stage } 2 = E.S_2 = \frac{u^* Y_o + p^* F_o}{k^* l_o - \hat{k}^* l_o + w^* z_o + \hat{v}^* x_o} \quad (24)$$

$$\text{Efficiency of Stage } 3 = E.S_3 = \frac{\sigma^* d_o}{u^* y_o - \hat{u}^* y_o + \hat{k}^* \ l_o + p^* F_o}$$

According to definition 2, weights that are used in objective function of model (23) are as follows:

$$\omega_1 = \frac{v^* x_o + \hat{v}^* x_o}{v^* x_o + w^* z_o + k^* l_o + u^* y_o - \hat{u}^* y_o}$$

$$\omega_2 = \frac{k^* l_o - \hat{k}^* l_o + w^* z_o + \hat{v}^* x_o}{v^* x_o + w^* z_o + k^* l_o + u^* y_o - \hat{u}^* y_o} \quad (25)$$

$$\omega_3 = \frac{u^* y_o - \hat{u}^* y_o + \hat{k}^* \ l_o + p^* F_o}{v^* x_o + w^* z_o + k^* l_o + u^* y_o - \hat{u}^* y_o}$$

For more efficient investigation among DMUs, model (26) is presented. The innovative idea behind the model (26) is that it searches for the common set of weights based on the SBM model to evaluate the DMUs. This technique in DEA is introduced for fair evaluation, according to Jahanshahloo et al.

[37]. The multi-objective optimization model (26) aims to maximize the efficiency of all DMUs, simultaneously. Note that model (23) considers DMU_o and tries to find optimal weights to maximize the efficiency of DMU_o.

$$\begin{aligned}
&Max\ \{1+\sigma\ d_j+u\ y_j+hq_j-vx_j-kl_j,\ \ldots,1+\sigma\ d_n+u\ y_n+hq_n-vx_n-kl_n\ \}\\
&\text{s.t.}\\
&h\ Q+\mathrm{w}z-vX+\hat{v}X-uY+\hat{u}Y\le 0,\\
&pF+uY-kL+\hat{k}L-wZ-\hat{v}X\le 0, && \text{(b)}\\
&\sigma\ D\ -\hat{u}Y-\hat{k}\ L-pF\le 0, && \text{(c)}\\
&\sigma\ D\ -k\ L+h\ \ Q-vX\le 0, && \text{(d)}\\
&\sigma\ \ge\frac{1+\sigma\ d_j+hq_j-vx_j-kl_j}{a}\left[\frac{1}{d_o}\right],\quad \mathrm{j}=1,\ \ldots,\ \mathrm{n}, && \text{(e)}\\
&h\ge\frac{1+\sigma\ d_j+hq_j-vx_j-kl_j}{b}\left[\frac{1}{q_o}\right],\quad \mathrm{j}=1,\ \ldots,\ \mathrm{n} && \text{(f)} \qquad (26)\\
&k\ge\frac{1}{e}\left[\frac{1}{l_j}\right],\quad \mathrm{j}=1,\ \ldots,\ \mathrm{n} && \text{(g)}\\
&v\ge\frac{1}{m}\left[\frac{1}{x_j}\right],\quad \mathrm{j}=1,\ \ldots,\mathrm{n}, && \text{(h)}\\
&w\ge 0,\ \ \mathrm{u}\ge 0, && \text{(i)}\\
&0\le\beta v\le v,0\le\delta k\le k,0\le\alpha u\ \ \le u\ ,\\
&v-\hat{v}\ge 0,\ \ u\ -\hat{u}\ \ \ge 0,\ \ k-\hat{k}\ge 0,
\end{aligned}$$

Model (26) has “n” objective functions. Considering Goal Programming, Multiple Criteria Decision Making (MCDM) technique, and constraint (e). Let $e_j\ (j=1,\ldots,n)$ be the positive deviation variable. Thus, instead of maximizing the “n” objective function, it is possible to minimize the sum of deviational variables equivalently. The linear counterpart of the model (26) is as follows.

$$\begin{aligned}
&Min\ \sum_{j=1}^{n}e_j\\
&s.t.\\
&h\ Q+\mathrm{w}z-vX+\hat{v}X-uY+\hat{u}Y\le 0, && \text{(b)}\\
&pF+uY-kL+\hat{k}L-wZ-\hat{v}X\le 0, && \text{(c)} \qquad (27)\\
&\sigma\ D\ -\hat{u}Y-\hat{k}\ L-pF\le 0, && \text{(d)}\\
&\sigma\ D\ -k\ L+h\ Q-vX+e_j=0, && \text{(e)}\\
&\sigma\ \ge\frac{1+\sigma\ d_j+hq_j-vx_j-kl_j}{a}\left[\frac{1}{d_o}\right],\quad \mathrm{j}=1,\ \ldots,\ \mathrm{n}, && \text{(f)}
\end{aligned}$$

$$h \geq \frac{1+\sigma\ d_j + hq_j - vx_j - kl_j}{b}\left[\frac{1}{q_o}\right], \quad \text{j=1, ..., n} \tag{g}$$

$$k \geq \frac{1}{e}\left[\frac{1}{l_j}\right], \quad \text{j} = 1, \ldots, \text{ n} \tag{h}$$

$$v \geq \frac{1}{m}\left[\frac{1}{x_j}\right], \quad \text{j} = 1, \ldots, \text{n}, \tag{i}$$

$$w \geq 0, \quad \text{u} \geq 0,$$

$$0 \leq \beta v \leq v, 0 \leq \delta k \leq k, 0 \leq \alpha u \ \leq u \ ,$$

$$v - \hat{v} \geq 0, \quad u \ - \hat{u} \ \geq 0, \quad k - \hat{k} \geq 0, e \geq 0.$$

Based on the optimal weights obtained from model (27), using expression (28), the efficiency score for any $DMU_j \ (j = 1, \ldots, n)$ can be calculated.

Definition 3: The aggregate efficiency (A.E) and stage efficiencies (E.S) for $DMU_j \ (j = 1, \ \ldots, \ n)$ are evaluated with the optimal common set of weights obtained from the model (27).

$$\text{Agreggate Efficiency DMU}_j = (\text{A.}E)_j = \frac{\sigma^* d_j + u^{1*} y_j + w^* z_j + h^* q_j}{v^* x_j + w^* z_j + k^* l_j + u^* y_j - \ \hat{u}^* y_j} \quad j = 1, \ldots, n$$

$$\text{Efficiency Stage 1 for DMU}_j = (E.S_1)_j = \frac{h^* q_j + w^* z_j}{v^* x_j + \hat{v}^* x_j} \quad j = 1, \ldots, n$$

$$\text{Efficiency Stage 2 for DMU}_j = (E.S_2)_j = \frac{u^* Y_j + p^* F_o}{k^* l_j - \hat{k}^* l_j + w^* z_j + \hat{v}^* x_j} \quad j = 1, \ldots, n \tag{28}$$

$$\text{Efficiency Stage 3 or DMU}_j = (E.S_3)_j = \frac{\sigma^* d_j}{u^{2*} y_j - \ \hat{u}^{2*} y_j + \hat{k}^* \ l_j + p^* F_o} \quad j = 1, \ldots, n$$

4. Case Study

Consider the data of a three-stage sustainable supply chain as mentioned in Tables 2 and 3. The mentioned indexes in Table 2 show the influence of sustainability. The data incorporates 20 oil factories from Iranian Stock Exchange Market1. The cost of trained personnel in the field of safety and health is the burden and cost of training personnel for possible occupational accidents and diseases. Materials cost is known as raw material costs. Labor cost per project is a figure obtained from adding up the various rates each worker is paid on a project. Warehousing cost shows the costs involved in storing goods in a warehouse. Supplied Materials means all the materials necessary to produce a product. Transportation cost shows all the expenses related to the transportation of raw materials, final products, and employees. Supplier revenue shows the payments received by the supplier. Necessary tools signify essential and required tools. Revenue is the total amount of income generated by the sale of goods and services. Cost of trained personnel on safety and health issues costs and materials cost are shared inputs between the supplier and manufacturer. Labor cost per project and warehousing cost are shared inputs between the manufacturer and distributor. Supplied material is the output of manufacturer and shared inputs between the supplier and distributor.

The provided model (23) and corresponding aggregate and stage efficiencies were acquired and discussed in the presented case study. The main concern about this model and the findings is efficiency evaluation while optimal shares for resources are determined. Model (23) benefits from the SBM model that is more accurate in efficiency evaluation than radial models. Moreover, converting the final model into a linear model (23) is the other main achievement of this study.
Figure 4shows three stages of a chain with inputs, outputs, intermediate products, and feedback.

Table 4 shows the aggregate and stage efficiencies obtained from expressions (24). According to the stage efficiency, DMUs scores 3, 4, 7, 8, 11, 12, 13, 17, and 19 performed efficiently in supplier and/or manufacturer and/or distributer stages. Also, DMU3 is an aggregate efficient unit and performs efficiently in supplier, manufacturer, and distributer.

Only DMU3 is aggregate efficient. Efficiency scores of DMU12 and DMU17 are 0.96611 and 0.99772, respectively. Among inefficient DMUs, DMU1 and DMU18 have the first and second ranks with 0.33737 and 0.41693, respectively. Consider Figure 5, in which stage and aggregate efficiency scores of DMUs from model (23) are listed.

Table 5 shows the optimal weights for y, obtained from model (23). As observed in Table 5, DMUs 2 and 14 are the only ones that returned a portion of y to the supplier. Thus, the rest of DMUs used y as the input of the distributor. Note that αu shows the share of y for the supplier and $(1-\alpha)u$ share of y for the distributor.

Table 6 shows the optimal weights for x_1 and x_2 that are the independent input of the system and obtained from model (23). It is a feature of the presented model that finds the optimal share of each input element separately. Note that βv_1 shows the share of x_1 for the supplier and $(1-\beta)v_1$ share of

x_1 for the manufacturer. Similarly, βv_2 shows the share of x_2 for the supplier, and the $(1-\beta)v_2$ share of x_2 for the manufacturer.

DMUs 3, 6, 7, 8, 9, 10, 12, 15, and 17 used their shared first input in supplier and the rest DMUs used it the manufacturer. For x_2, DMUs 6, 7, 9, 10, 12, 13, 15, 16, and 17 used their shares from x_1 in the supplier. The remaining DMUs used it in the manufacturer. As seen, DMUS 6, 7, 9, 10, 12, 15, and 17 have all used their shares from x_1 and x_2 in the suppliers.

Tables 7 shows the optimal value for the independent input l_1 and l_2. DMUs 4, 6, and 7 used their shares from independent input l_2 just in the distributor. The rest of the DMUs used their shares for the manufacturer. DMUs 4 and 7 are the only two DMUs that used their shares of independent input l_1 and l_2 for both stages, the manufacturer and distributor.

Joint evaluation is as important as self-assessment. Table 8 summarizes the common set of weights obtained from model (27). Consider the optimal shares of weights obtained from the classic model (23) and the common set of weights model (27).

According to optimal weights of Table 9 and expression (28), aggregate and stage efficiencies are obtained and listed in Table 9. Model (23) finds the minimum sum of distances of DMUs from the efficient frontier, implying that this model has an optimistic viewpoint.

In Table 9, with the common set of weights, none of the units is aggregate efficient. An important point is that the aggregate efficiency score is greater than the minimum efficiency core among the three stages for each DMU, while in Table 5, only one aggregate efficient unit is found. The reason is joint evaluation. In the common set of weights, the goal is to maximize performances of DMUs simultaneously, not just to maximize the performance of a DMU under evaluation. Remarkably, joint evaluation assesses all the units under the same conditions. In self-assessment, the decision makers' perspective is optimistic in favor of the DMU being evaluated. It means that there is a tendency of the efficiency of the DMU under to be maximized.

Note that performance assessment with the common weights aims to maximize the efficiency of all decision-making units. Therefore, each efficient DMU in model (27) is efficient in the classical model (23). In model (27), the problem-solving viewpoint aims to maximize efficiency scores of all DMUs simultaneously. However, if a DMU is efficient under these conditions, it will certainly be efficient in model (23) that aims to evaluate each unit separately.

In model (23), there is no parameter on which sensitivity analysis can be directly applied, because the input and output parameters are the "observed values" of the decision-making units. Therefore, to analyze the results, "efficiency interval" is taken into account. We consider the maximum sum of distances of the DMUs from the efficient frontier. Consequently, all DMUs reach their minimum efficiency, simultaneously. In this case, an efficiency interval is constructed, the upper bound represents optimistic evaluation, and the lower bound represents pessimistic evaluation. Therefore, the efficiency scores calculated from multiple optimal solutions for the decision-making unit certainly will occur in this interval. The results in Table 10 showed the efficiency obtained from the pessimistic perspective of the common set of weight models (27). In the common set of weights model, in an optimistic viewpoint, we sought the minimum sum of distances of DMUs from the efficient frontier.

According to the results of Tables 8 and 9, the efficiency interval, $[E^L, E^U]$, for each unit can be considered as introduced in Jahanshahloo et al. [38].

5. Conclusion

Internal linking and shared resources exist in a variety of networks, which affect the efficiency score. Thus, unfair allocation of shared resources can result in invalid efficiency scores. This study presents a DEA model for the efficiency assessment of networks with shared resources and feedback. This model is on the slack-based measure of efficiency. For further analysis, decomposition of the aggregate efficiency score into stage efficiency scores is applied. The presented model is modified to obtain the common set of weights for a fair assessment. This model tries to maximize the efficiency of all DMUs at the same time. Therefore, the optimal solution of the proposed model shows the best proportion of the shared resources and feedbacks, and enhances the efficiency scores. The case of 20 sustainable supply chains in the oil industry with three stages, supplier, manufacture, and distributer, is considered with the newly developed DEA approach. Then, the aggregate and stage efficiency scores are obtained. Furthermore, optimal shares of resources and feedbacks are obtained. Further analysis shows that the efficiency scores obtained from the classic model and common set of weights model are comparable. According to the results, the efficiency scores in the classic and common set of weights models are different. Note that the variance of efficiency scores obtained from the common set of weights model is less than that of the classic model. Therefore, DMUs acquired optimal fair shares of resources. The common set of weights model aims to maximize the efficiency of DMUs simultaneously. Importantly, alternative optimal weights require pose several limitations for defining diverse strategies based on alternative optimal solutions.

For further research, non-discretionary factors or weak disposability assumptions can be considered in the newly presented model. For further research in this subject, consider the case where the stage's operation has the priority over each other in order to be optimized, thus it is suggested to use the leader-follower method, Vaezi et al. [39].

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References

1. Charnes, A., Cooper, W.W. and Rhodes, E. "Measuring the efficiency of decision making units" *European journal of operational research*, **2**(6), pp. 429-444 (1978).

2. Tone, K. "A slacks-based measure of efficiency in data envelopment analysis" *European journal of operational research*, **130**(3), pp. 498-509 (2001).

3. Cook, W.D. and Zhu, J. "Classifying inputs and outputs in data envelopment analysis." *European Journal of Operational Research*, **180**(2), pp. 692-699 (2007).

4. Amirteimoori, A. and Shafiei, M. "Characterizing an equitable omission of shared resources: A DEA-based approach" *Applied Mathematics and Computation*, **177**(1), pp. 18-23 (2006).

5. Chen, Y., Du, J., Sherman, H.D. and Zhu, J. "DEA model with shared resources and efficiency decomposition" *European Journal of Operational Research*, **207**(1), pp. 339-349 (2010).

6. Amirteimoori, A. "A DEA two-stage decision processes with shared resources" *Cent Eur J Oper Res* **21**, pp.141–151 (2013).

7. An, Q., Wen, Y., Ding, T. and Li, Y. "Resource sharing and payoff allocation in a three-stage system: Integrating network DEA with the Shapley value method" *Omega*, **85**, pp. 16-25 (2019).

8. An, Q., Liu, X. and Ding, S. "Efficiency measurement for mixed two-stage nonhomogeneous network processes with shared extra intermediate resources" *Frontiers of Engineering Management*, **7**(2), pp. 259-274 (2020).

9. An, Q., Wen, Y., Ding, T., Li, Y. "Profit inefficiency decomposition in a serial structure system with resource sharing*" Journal of the Operational Research Society*, 15 Pages, (2018).

10. Moghaddas, Z., Mohhamadpour Tosarkani, B. and Yousefi, S. "Resource reallocation for improving sustainable supply chain performance: An inverse data envelopment analysis" *International Journal of Production Economics*, **252**, pp. 108560 (2022).

11. MA, J. "A two-stage DEA model considering shared inputs and free intermediate measures" *Expert Systems with Applications*, **42**(9), pp. 4339-4347, (2015).

12. Izadikhah, M., Tavana, M., Di Caprio, D. and Santos-Arteaga, F.J. "A novel two-stage DEA production model with freely distributed initial inputs and shared intermediate outputs" *Expert Systems with Applications*, **99**, pp. 213-230 (2018).

13. Puri, J., Yadav, S.P. and Garg, H. "A new multi-component DEA approach using common set of weights methodology and imprecise data: an application to public sector banks in India with undesirable and shared resources" *Annals of Operations Research,* **259**, pp. 351–388 (2017).

14. Stefaniec, A., Hosseini, K., Xie, J. and Li, Y. “Sustainability assessment of inland transportation in China: A triple bottom line-based network DEA approach” *Transportation Research Part D: Transport and Environment*, **80**, p. 102258 (2020).

15. Zhou, H. and Hu, H. “Sustainability evaluation of railways in China using a two-stage network DEA model with undesirable outputs and shared resources” *Sustainability*, **9**(1), p. 150 (2017).

16. Chai, L. and Zhao, D. “Efficiency evaluation and methods of resource allocation of parallel production system based on DEA” *Journal of University of Science and Technology of China*, p. 11 (2016).

17. Chao, S.L., Yu, M.M. and Hsieh, W.F. “Evaluating the efficiency of major container shipping companies: A framework of dynamic network DEA with shared inputs” *Transportation Research Part A: Policy and Practice*, **117**, pp. 44-57 (2018).

18. Phung, M.T., Cheng, C.P., Guo, C. and Kao, C.Y. “Mixed Network DEA with Shared Resources: A Case of Measuring Performance for Banking Industry” *Operations Research Perspectives*, **7**, p. 100173 (2020).

19. Álvarez-Rodríguez, C., Martín-Gamboa, M. and Iribarren, D. “Sustainability-oriented efficiency of retail supply chains: a combination of life cycle assessment and dynamic network data envelopment analysis” *Science of the Total Environment*, **705**, p. 135977 (2020).

20. Jiang, Y., Xu, Q. and Chen, Y. “Developing a joint supply chain plan for the coal industry considering conflict resolution strategies” *Scientia Iranica*, **28**(2), pp. 877-891 (2021).

21. Azadi, M., Moghaddas, Z., Saen, R.F., Gunasekaran, A., Mangla, S.K., Ishizaka, I. “Using network data envelopment analysis to assess the sustainability and resilience of healthcare supply chains in response to the COVID-19 pandemic “*Annals of Operations Research*, 1-14 (2022).

22. Li, Y. and Cui, Q. “Airline efficiency with optimal employee allocation: an input-shared network range adjusted measure” *Journal of Air Transport Management*, **73**, pp. 150-162 (2018).

23. Wang, H., Pan, C., Wang, Q. and Zhou, P. “Assessing sustainability performance of global supply chains: An input-output modeling approach” *European journal of operational research*, **285**(1), pp. 393-404 (2020).

24. Avilés-Sacoto, S.V., Avilés-Sacoto, E.C., Cook, W.D. and Güemes-Castorena, D. “Modelling efficiency in the presence of shared inputs within groups of DMUs” *Journal of the Operational Research Society*, pp. 1-17 (2021).

25. Avilés-Sacoto, S.V., Cook, W.D., Güemes-Castorena, D. and Zhu, J. “Modelling efficiency in regional innovation systems: a two-stage data envelopment analysis problem with shared outputs within groups of decision-making units” *European Journal of Operational Research*, **287**(2), pp. 572-582 (2020).

26. Zhang, Z., Wang, Z. and Zhu, Y. “Optimal path selection of innovation resource allocation in China’s regions with shared inputs” *Economic Research-Ekonomska Istraživanja*, pp. 1-24 (2021).

27. Wang, Q., Wu, Z. and Chen, X. “Decomposition weights and overall efficiency in a two-stage DEA model with shared resources” *Computers & Industrial Engineering*, **136**, pp. 135-148 (2019).

28. Shi, Y., Yu, A., Higgins, H.N. and Zhu, J. “Shared and unsplittable performance links in network DEA” *Annals of Operations Research*, **303**(1), pp. 507-528 (2021).

29. Gupta, S. and Palsule-Desai, O.D. "Sustainable supply chain management: Review and research opportunities" *IIMB Management review*, **23**(4), pp. 234-245 (2011).

30.Min, H. and Kim, I. "Green supply chain research: past, present, and future" *Logistics Research*, **4**(1), pp. 39-47 (2012).

31. Roy, S., Das, M., Ali, S.M., Raihan, A.S., Paul, S.K. and Kabir, G. "Evaluating strategies for environmental sustainability in a supply chain of an emerging economy" *Journal of Cleaner Production*, **262**, p. 121389 (2020).

32. Tavassoli, M., Ketabi, S. and Ghandehari, M. "Developing a network DEA model for sustainability analysis of Iran's electricity distribution network" *International Journal of Electrical Power & Energy Systems*, **122**, p. 106187 (2020).

33. Kalantary, M., Farzipoor Saen, R. and Toloie Eshlaghy, A. "Sustainability assessment of supply chains by inverse network dynamic data envelopment analysis" *Scientia Iranica*, **25**(6), pp. 3723-3743 (2018).

34. Tang, M., Hong, J., Wang, X. and He, R. "Sustainability accounting of neighborhood metabolism and its applications for urban renewal based on emergy analysis and SBM-DEA" *Journal of Environmental Management*, **275**, p. 111177 (2020).

35. Gilani, H., Sahebi, H. and Oliveira, F. "Sustainable sugarcane-to-bioethanol supply chain network design: A robust possibilistic programming model" *Applied Energy*, **278**, p. 115653 (2020).

36. Liu, P. and Liu, X. "A multi-attribute decision-making method based on the third-generation prospect theory and grey correlation degree" *Scientia Iranica*, **28**(2), pp. 1001-1013 (2021).

37. Jahanshahloo, G.R., Memariani, A., Lotfi, F.H. and Rezai, H.Z. "A note on some of DEA models and finding efficiency and complete ranking using common set of weights" *Applied mathematics and computation*, **166**(2), pp. 265-281 (2005).

38. Jahanshahloo, G.R., Lotfi, F.H., Rezaie, V. and Khanmohammadi, M. "Ranking DMUs by ideal points with interval data in DEA" Applied Mathematical Modelling, **35**(1), pp. 218-229 (2011).

39. Vaezi, E., Najafi, S.E., Hajimolana, S.M., Hosseinzadeh Lotfi, F. and Ahadzadeh Namin, M. "Efficiency evaluation of a three-stage leader-follower model by data envelopment analysis with double-frontier viewpoint" *Scientia Iranica*, **28**(1), pp. 492-515 (2021).

40. Ding, J., Feng, C., Bi, G., Liang, L. and Khan, M.R. "Cone ratio models with shared resources and nontransparent allocation parameters in network DEA" *Journal of Productivity Analysis*, **44**(2), pp. 137-155 (2015).

41. Ma, J., Qi, L. and Deng, L. "Additive centralized and Stackelberg DEA models for two-stage system with shared resources" *International Transactions in Operational Research*, **27**(4), pp. 2211-2229 (2020).

42. Ding, J., Dong, W., Liang, L. and Zhu, J. "Goal congruence analysis in multi-Division Organizations with shared resources based on data envelopment analysis" *European Journal of Operational Research*, **263**(3), pp. 961-973 (2017).

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Figure 1. A supply chain

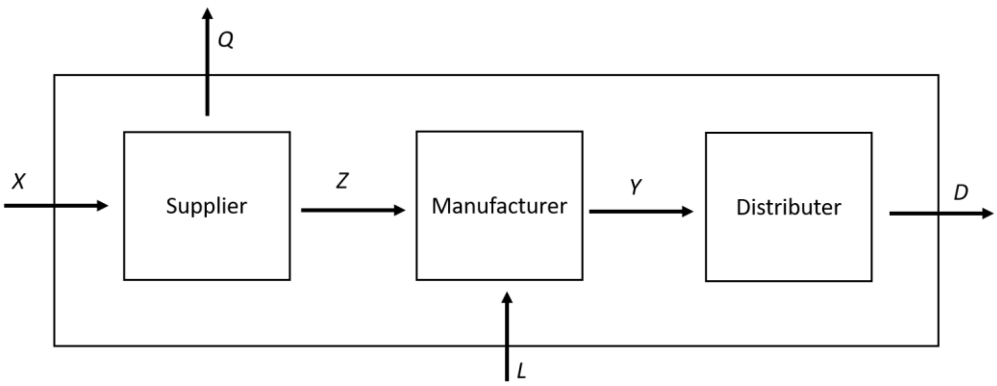

Figure 2. A three-stage supply chain with shared resources

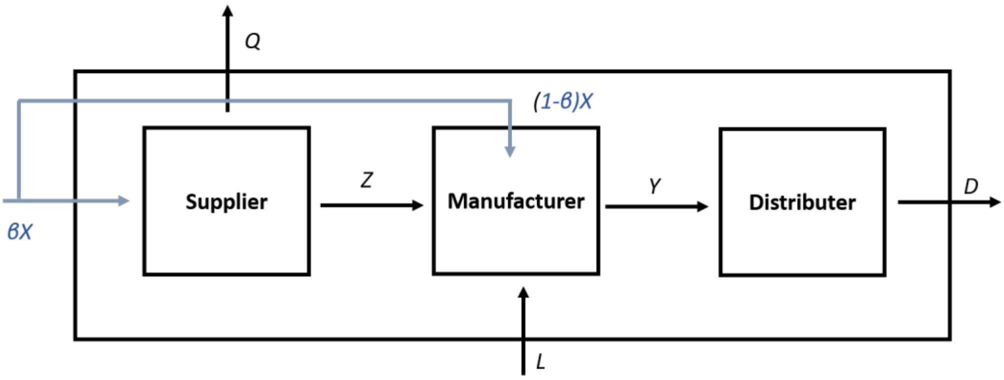

Figure 3.A three-stage supply chain with shared feedbacks

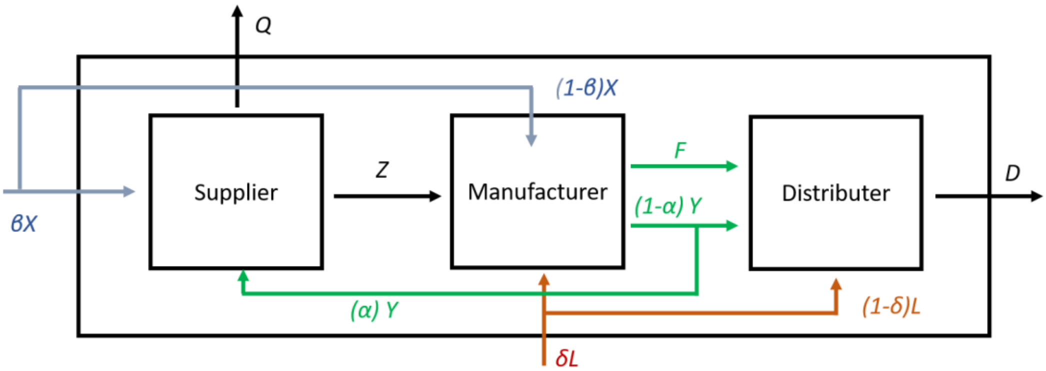

Figure 4.A three-stage supply chain in oil factories

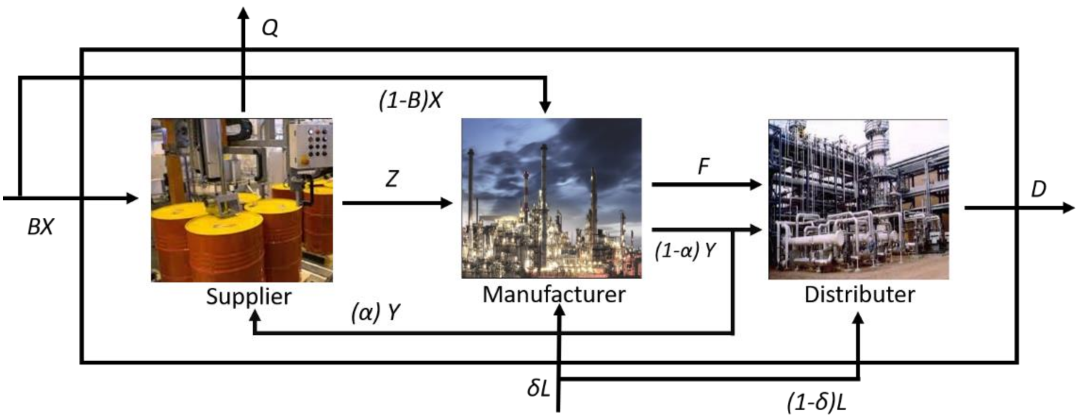

Figure 5. Stage and aggregate efficiency scores from model (23)

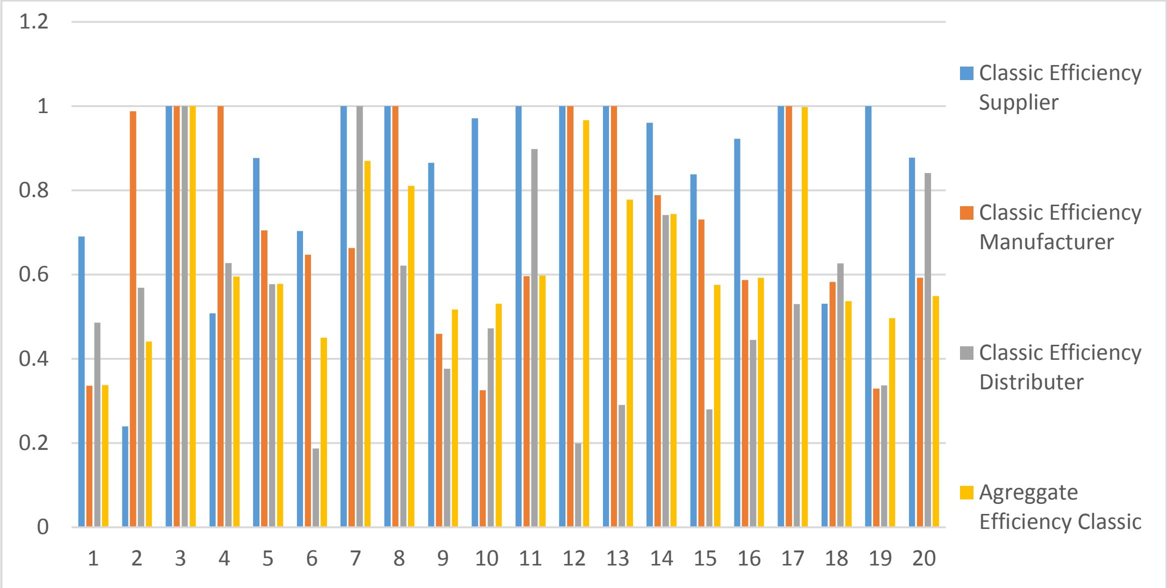

Table 1. Comparison of proposed method with previous methods

Studies	Shared resources	Shared feedbacks	Non-radial Model	CSW model	Sustainable Factors	Shared intermediate
Cook et al. [3]	√	×	×	×	×	×
Amirteimoori and Shafiei [2]	√	×	×	×	×	×
Ma [11]	√	×	×	×	×	√
Álvarez-Rodríguez et al.	√	×	×	×	√	×

[38]						
Chao et al. [17]	√	×	×	×	×	√
Tang et al. [34]	√	×	√	×	×	×
Li and Cui [22]	√	×	√	×	×	×
Zhu and Hu. [15]	√	×	×	×	√	×
Ding et al. [40]	√	×	×	×	×	×
Izadikhah et al. [12]	√	×	×	×	×	×
Phung et al. [18]	√	×	×	×	×	×
An, Liu, and Ding [8]	√	×	×	×	×	√
Ma, Qu, and Deng [41]	√	×	×	×	×	×
Wang et al. [27]	√	×	×	×	√	×
Avilés-Sacoto et al. [25]	√	×	×	×	×	×
Zhang et al. [26]	√	×	×	×	√	√
Chai and Zhao [16]	√	×	×	×	×	×
Puri et al. [13]	√	×	×	×	√	×
Ding et al. [42]	√	×	×	×	×	×
Shi et al. [28]	√	×	×	×	×	√
An et al. [7]	√	×	×	×	×	×
An et al. [9]	√	×	×	×	×	√
Avilés-Sacoto et al. [24]	√	×	×	×	×	×
Present Study	√	√	√	√	√	√

Table 2. Description of indices

Index	Definition		Sustainability	Type of data
x_1	Cost of trained personnel on safety and health issues costs	Shared between stage 1 and 2	Social	10000$
x_2	Additive materials cost	Shared between stage 1 and 2	Economic	1000$
q_1	Supplier revenue	-	Economic	1000T
z_1	Necessary tools	-	Economic	1000$
z_2	Processed material	-	Environmental	1000T
l_1	Labor cost per project	Shared between stage 2 and 3	Social	1000$
l_2	Warehousing cost	Shared between stage 2 and 3	Economic	1000$
y	Supplied material	-	Economic	10000T

f	Transportation cost	Shared Feedback from stage 2 to stage 1	Environment	1000T
d_1	Revenue	-	Economic	10000$

Table 3. Data set

DMUs	f	y	l_1	l_2	z_1	z_2	q	i_1	i_2	D
1	564	231	849	337	53	147	383	7	739	401
2	512	423	1195	222	28	125	193	3	963	323
3	765	125	300	130	57	23	232	2	925	124
4	812	543	684	99	86	109	437	12	889	653
5	876	376	1083	318	73	169	598	7	773	720
6	761	276	601	281	61	43	704	15	1020	385
7	592	217	472	91	93	112	864	14	958	827
8	738	428	720	127	14	65	914	9	568	711
9	827	381	1194	262	61	136	819	12	858	528
10	792	268	1418	273	51	227	885	15	717	432
11	672	241	1064	301	9	170	654	8	644	651
12	778	349	1303	249	5	124	960	12	582	166
13	619	269	973	112	86	159	354	2	706	171
14	729	257	1159	158	94	122	431	4	851	344
15	698	321	671	196	15	201	852	13	733	167
16	734	372	1322	192	19	137	991	11	852	431
17	772	287	1163	335	23	28	791	6	1008	569
18	693	265	1511	194	8	238	442	11	941	767
19	792	275	1467	334	56	89	810	10	846	238
20	827	354	1377	249	47	133	927	14	909	480

Table 4. Aggregate efficiency and stage efficiencies

Classic Efficiency				
DMUs	**Supplier**	**Manufacturer**	**Distributer**	**Aggregate**
1	0.69057	0.33618	0.48536	0.33737
2	0.23943	0.98744	0.56826	0.44108
3	1.00000	1.00000	1.00000	1.00000
4	0.50735	1.00000	0.62708	0.59542
5	0.87631	0.70504	0.57714	0.57763
6	0.70339	0.64687	0.18645	0.44974
7	1.00000	0.66254	1.00000	0.86967
8	1.00000	1.00000	0.62122	0.81063
9	0.86549	0.45892	0.37597	0.51684
10	0.97104	0.32524	0.47233	0.53042
11	1.00000	0.59607	0.89820	0.59741
12	1.00000	1.00000	0.19910	0.96611
13	1.00000	1.00000	0.28996	0.77792

14	0.96033	0.78849	0.74141	0.74341
15	0.83792	0.73017	0.27966	0.57533
16	0.92190	0.58667	0.44453	0.59238
17	1.00000	1.00000	0.52984	0.99772
18	0.53074	0.58258	0.62649	0.53693
19	1.00000	0.32917	0.33650	0.49623
20	0.87692	0.59175	0.84046	0.54870

Table 5. Optimal weights

DMU	$\boldsymbol{\alpha u}$	$\boldsymbol{(1-\alpha)u}$	$\boldsymbol{u}$
1	0.0001000	0.0014740	0.0015740
2	0.0021120	0.0010190	0.0031310
3	0.0001000	0.0043450	0.0044450
4	0.0001000	0.0002460	0.0003460
5	0.0001000	0.0007770	0.0008770
6	0.0001000	0.0012000	0.0013000
7	0.0001000	0.0010180	0.0011180
8	0.0001000	0.0006170	0.0007170
9	0.0001000	0.0009950	0.0010950
10	0.0001000	0.0008960	0.0009960
11	0.0001000	0.0006130	0.0007130
12	0.0001000	0.0008290	0.0009290
13	0.0001000	0.0047370	0.0048370
14	0.0019710	0.0004870	0.0024580
15	0.0001000	0.0010210	0.0011210
16	0.0001000	0.0007630	0.0008630
17	0.0001000	0.0032500	0.0033500
18	0.0001000	0.0002940	0.0003940
19	0.0001000	0.0010630	0.0011630
20	0.0001000	0.0003540	0.0004540

Table 6. Optimal weights of inputs

DMUs	βv_1	βv_2	$(1-\beta)v_1$	$(1-\beta)v_2$	v_1	v_2
1	0.06672	0.00054	0.00472	0.00013	0.07144	0.00068
2	0.01599	0.00010	0.15069	0.00042	0.16668	0.00052
3	0.25003	0.00010	0.00000	0.00044	0.25003	0.00054
4	0.00985	0.00010	0.03182	0.00046	0.04167	0.00056
5	0.03466	0.00028	0.03677	0.00037	0.07144	0.00065
6	0.03334	0.00049	0.00000	0.00000	0.03334	0.00049
7	0.03572	0.00052	0.00000	0.00000	0.03572	0.00052
8	0.05556	0.00049	0.00000	0.00039	0.05556	0.00088
9	0.04167	0.00058	0.00000	0.00000	0.04167	0.00058
10	0.03334	0.00070	0.00000	0.00000	0.03334	0.00070
11	0.02715	0.00022	0.03535	0.00056	0.06251	0.00078

12	0.04167	0.03803	0.00000	0.00000	0.04167	0.03803
13	0.49802	0.00071	0.35054	0.00000	0.84856	0.00071
14	0.01943	0.00014	0.10558	0.00044	0.12501	0.00059
15	0.03847	0.00068	0.00000	0.00000	0.03847	0.00068
16	0.03564	0.00059	0.00982	0.00000	0.04546	0.00059
17	21.66271	0.07424	0.00000	0.00000	21.66271	0.07424
18	0.01240	0.00010	0.03306	0.00043	0.04546	0.00053
19	0.04783	0.00039	0.00218	0.00020	0.05001	0.00059
20	0.01520	0.00012	0.02052	0.00043	0.03572	0.00055

Table 7. Optimal weights of outputs

DMUS	$\boldsymbol{\delta k_1}$	$\boldsymbol{\delta k_2}$	$\boldsymbol{(1-\delta)k_1}$	$\boldsymbol{(1-\delta)k_2}$	$\boldsymbol{k_1}$	$\boldsymbol{k_2}$
1	0.000589	0.001484	0.000000	0.000000	0.000589	0.001484
2	0.000418	0.002252	0.000000	0.000000	0.000418	0.002252
3	0.005074	0.003847	0.023410	0.000000	0.028484	0.003847
4	0.000100	0.004849	0.000631	0.000202	0.000731	0.005051
5	0.000462	0.001572	0.000000	0.000000	0.000462	0.001572
6	0.000832	0.000100	0.000000	0.001680	0.000832	0.001780
7	0.001051	0.000100	0.000009	0.005395	0.001059	0.005495
8	0.000695	0.003937	0.000000	0.000000	0.000695	0.003937
9	0.000419	0.001909	0.000000	0.000000	0.000419	0.001909
10	0.000353	0.001832	0.000000	0.000000	0.000353	0.001832
11	0.000470	0.001661	0.000000	0.000000	0.000470	0.001661
12	0.000384	0.002008	0.000000	0.000000	0.000384	0.002008
13	0.000514	0.004465	0.000000	0.000000	0.000514	0.004465
14	0.000431	0.003165	0.000000	0.000000	0.000431	0.003165
15	0.000798	0.002551	0.000000	0.000000	0.000798	0.002551
16	0.000378	0.002604	0.000000	0.000000	0.000378	0.002604
17	0.000430	0.001493	0.000000	0.000000	0.000430	0.001493
18	0.000100	0.002578	0.000231	0.000000	0.000331	0.002578
19	0.000341	0.001497	0.000000	0.000000	0.000341	0.001497
20	0.000363	0.002008	0.000000	0.000000	0.000363	0.002008

Table 8. Optimal common set of weights

Variable	**Optimal value**	**Variable**	**Optimal value**	**Variable**	**Optimal value**	**Variable**	**Optimal value**
βv_1	0.14508	βv_2	0.00088	$(1-\beta)v_1$	0.10492	$(1-\beta)v_2$	0.00000
δk_1	0.00167	δk_2	0.00424	$(1-\delta)k_2$	0.00000	$(1-\delta)k_2$	0.00126
w_1	0.00515	w_2	0.00233	h	0.00355	σ_1	0.00022
p	0.00138	αu	0.00010	$(1-\alpha)u$	0.00431	σ_2	0.00238

Table 9. Optimal common set of weights optimistic viewpoint

Common set of weights efficiency				
DMUs	**Supplier**	**Manufacturer**	**Distributer**	**Aggregate**
1	0.74242	0.38932	0.34989	0.35582
2	0.36093	0.64917	0.63573	0.27858
3	0.71289	0.90765	0.60070	0.61882
4	0.46251	0.96578	0.63980	0.46622
5	0.87274	0.56710	0.40440	0.46651
6	0.68389	0.50016	0.28543	0.40456
7	1.00000	0.50761	1.00000	0.71469
8	0.95110	0.94788	0.58643	0.85289
9	0.85571	0.53044	0.27134	0.46204
10	0.99330	0.36530	0.51680	0.47318
11	1.00000	0.42270	0.72474	0.54859
12	0.99167	0.51097	0.23306	0.53070
13	1.00000	0.62602	0.47166	0.51813
14	0.94359	0.53661	0.72019	0.53778
15	0.91267	0.57961	0.63028	0.62564
16	0.99733	0.54945	0.44185	0.60551
17	1.00000	0.50764	0.43603	0.56301
18	0.60730	0.39844	0.55314	0.39907
19	0.99801	0.39588	0.45822	0.48134
20	0.88269	0.47549	0.72826	0.56995

Table 10. Optimal common set of weights pessimistic viewpoint

Common set of weights efficiency				
DMUs	**Supplier**	**Manufacturer**	**Distributer**	**Aggregate**
1	0.024286	0.034102	0.048536	0.016357
2	0.021656	0.038143	0.046150	0.015824
3	0.023740	0.108971	0.031833	0.025944
4	0.016708	0.107082	0.051642	0.025226
5	0.034561	0.049157	0.053070	0.025590
6	0.017384	0.057771	0.035283	0.017029
7	0.024613	0.088053	1.000000	0.035791
8	0.036109	0.083557	0.059409	0.040019
9	0.027055	0.048873	0.038618	0.020460
10	0.026545	0.039000	0.046786	0.019346
11	0.032452	0.037770	0.076544	0.027527
12	0.031005	0.044373	0.018396	0.017725
13	0.053412	0.055077	0.028741	0.021200
14	0.036990	0.049874	0.048680	0.024533
15	0.027418	0.064156	0.033820	0.021887
16	0.032771	0.046735	0.041241	0.024000
17	0.035270	0.039771	0.052758	0.025704
18	0.019227	0.037639	0.072220	0.019867
19	0.029432	0.035498	0.033650	0.017582

20	0.025743	0.045061	0.053775	0.022842