On Renewable Energy Source Selection Methodologies Utilizing Picture Fuzzy Hypersoft Information with Choice and Value Matrices

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Abstract

In a decision-making problem, the uncertainty component of refusal and abstain along with the sub-parametrization features in the information are not catered by intuitionistic fuzzy/Pythagorean fuzzy sets. In the present communication, we first introduce the novel notion of picture fuzzy hypersoft matrix along with various important binary operations and properties. The inclusion of the novel notion of picture fuzzy hypersoft matrices allows the decision makers to define their preference in a more general sub-parameterized linguistic form for the evaluation of available alternatives. The proposition concentrates on presenting a robust decision-making framework for identifying the optimal and most suitable renewable energy source. In this regard, the revised definition of picture fuzzy hypersoft choice matrix/weighted choice matrix (PFHSCM/PFHSWCM), value matrix, and total score matrix have been presented. Further, two algorithms of decision-making for the selection of the best renewable energy sources have been provided along with appropriate illustrations and ranking descriptions.

Keywords: Renewable Energy Sources, Picture Fuzzy Set, Hypersoft Set, Decision Making, Choice/Score Matrix

1 Introduction

The role of energy in the field of economic globalization and industrial growth is exceptionally important and inevitable. A significant development in the field of renewable energy source selection procedure under fuzzy decision-making has taken place in recent years. As per the current situation, around 80 percent of the worlds energy production has been channelizing through conventional fossil fuels such as coal, oil and natural gases. Consequently, there has been a significant rise in environmental concerns as well as the crisis in the reserves. In view of the gradual enhancement of the energy crisis and high energy prices, the agencies are striving and searching for the development in the field of renewable energy options such as solar energy, wind energy, water energy, biomass energy, geothermal energy, marine energy etc. Nowadays, various countries have brought forward different energy policies related to the development of renewable energy sources based on different requirements and task-oriented goals [1], [2].

In recent decades, different researchers have been working on almost zero-energy compounds and infrastructures using the integration of renewable energies [3], [4]. However, various interrelated factors, such as technical, economical, socio-economical, environmental constraints may affect the problem of renewable energy source selection and subsequently the development strategies, policies and standards should be chosen carefully in the process of ranking of renewable energy. In view of the limited investment resources [5], it becomes utmost essential to offer optimal performance efficiency under such uncertain and imprecise decision factors. In order to handle such incompleteness and inexactness found in the decision factors, we must go for utilizing advanced kind of fuzzy sets, such as picture fuzzy hypersoft matrices and decision-making algorithms based on these. Thus, an effective and robust decision-making framework is certainly required to identify the best suitable alternative from the available renewable energy sources.

2 Renewable Energy Source Descriptions and Literature Review

In a study [6], it has been highlighted that insufficient public awareness has been a major barrier in the social acceptance of renewable energy sources. Also, on the basis of tweet analysis, it has been suggested that the energy crisis can be overcome with the development of renewable energy technologies. As per the available resources, the prevailing situation of various types of renewable energy alternatives in the existing society can be explained with the help of Figure 1.

The process of renewable energy source selection is supposed to be a complex multi-criteria decision-making (MCDM) problem that certainly involves multiple conflicting criteria, multiple alternatives, and a significant amount of uncertainty. In general, based on the available information on effective indicators, the process comprises six level tasks - structuring of alternatives, selection of criteria, normalization of data, assessment of weights, scoring of alternatives and obtaining result validity. This process has been explained with the help of various effective parameters and criteria detailed in Figure 2 ([7]-[16]).

In literature, a VIKOR approach-based methodology utilizing the Pythagorean fuzzy sets has been proposed by Rani et al.[17] for selecting and evaluating the various different criteria of renewable energy sources/technologies in India. Further, [18] presented a hybrid model using fuzzy AHP and SWOT for a strategic assessment of renewable energy technologies in Pakistan with four prime and seventeen subprime effective indicators. Also, a novel decision-making approach was introduced by Riaz et al.[19] with the incorporation of linear diophantine fuzzy sets. In the joint study, it was observed that socio-political and economic criteria were the crucial indicators as an immediate consequence. In view of sustainability, Wu et al. [20] provided a novel approach by modeling AHP with interval type-2 fuzzy weighted averaging set for assessing the major effectivity of renewable energy sources. In a group decision-making problem, Wang et al. [21] considered the information given by the decision-makers in the form of an interval type-2 fuzzy decision matrix where the information about the alternatives weights is partially known. Further, Wang et al. [22] implemented projection-based VIKOR technique for the risk-oriented infrastructure projects utilizing the picture fuzzy set.

Yuan et. al. [23] studied a novel fuzzy decision-making approach for the selection of RE alternatives in China by using the improved Choquet integral and linguistic hesitant fuzzy set. Dincer and Yuksel [24] presented the selection process of renewable energy alternatives with the help of DEMATEL and TOPSIS technique where interval type 2 hesitant fuzzy sets have been utilized. Further, Jeong and Ramirez [25] provided a novel hybrid approach with fuzzy DEMATEL & GIS (Geographic Information System) for identification of the best location of biomass energy generating plants and put forward important results related to the criteria. Ghenai et al. [26] presented a decision-making approach using SWARA and ARAS methods for the evaluation and selection of renewable energy sources indicating the optimality of the land-based wind energy source.

The literature extension in this field of research can be observed with the help of Figure 3 ([27], [28], [29]-[30], [31], [32], [33]).

On the basis of the above literature review and the lineage (represented through Figure 3), it is being observed that the existing extensions do not incorporate the degree of refusal in the sub-attributes parametrization of objects. There are also some other extensions in the field of soft set theory as Bipolar valued soft sets(BVSSs) given by Mahmood [34] which are the extension of the Bipolar valued fuzzy sets given by Zhang [35]. Also, in order to solve the multi-attribute decision-making problems, Ullah [36] presented the notion of picture fuzzy Maclaurin symmetric mean operators. Further, Liu et al. [37] developed the similarity measures for inter-valued picture fuzzy sets to solve decision-making problems. Javed et al. [38] devised the novel neutrality aggregation operators which are very useful to solve the MCDM problems. In order to deal with such circumstances, we propose to introduce a new concept of picture fuzzy hypersoft matrices (PFHSM) which would certainly improve the enumerating power and variability of the available information. The main objective behind the proposed notion would be to devise a novel structure where the decision-makers/experts would gain ample flexibility and freedom. The proposed notion of PFHSM gives more flexibility to the decision-maker with the inclusion of the degree of refusal and degree of abstain. The novelty of the present manuscript comes from the advantageous feature to deal with any kind of sub-attribute family of parameters in decision-making problems.

In order to illustrate the effectiveness of the proposed picture fuzzy hypersoft matrices, we utilize them in the process of renewable energy source selection with the help of the revised score and value matrices. The strength of the proposed study can be enumerated as follows:

- Proposing a novel notion of picture fuzzy hypersoft information and picture fuzzy hypersoft matrices with various important binary operations and properties.

- Extending the content of the information applicability using the sub-parametrization feature of the picture fuzzy hypersoft environment.

- Proposing a decision-making algorithm using the revised notion of picture fuzzy hypersoft choice/weighted choice matrix.

- Finally, preparing the prioritization table and make comparative analysis based on the computations carried out with the proposed methodologies.

As per our current study in this field, there is no methodology on the optimized selection of renewable energy sources utilizing the concept of picture hypersoft matrices, choice matrix and value matrix which gives more exhaustiveness for the decision-makers.

The present paper has been structured as follows: Section 3 briefly presents very important preliminary definitions and fundamental notions which are available in the literature. In Section 4, the notion of picture fuzzy hypersoft matrix (PFHSM) has been introduced along with various binary operations, prepositions and important matrix-theoretic properties. Two decision-making algorithms illustrating the application of PFHSM in the field of renewable energy source (RES) selection have been duly presented in Section 5. In this regard, numerical illustration and computation based on RES selection have been carried out in Section 6 for a better understanding of the proposed algorithms. The necessary comparative analysis has been accomplished and presented in Section 7 in view of the prioritization table and the existing techniques in the literature. Finally, the paper has been concluded in Section 8.

3 Preliminary Concepts

The notion of Picture fuzzy set [39], soft set [28], hypersoft set/fuzzy hypersoft set/Intuitionistic Fuzzy Hypersoft Set(IFHSS) [31] are available in the literature for ready reference. However, some of the basic preliminaries and notions in connection with introducing the notion of picture fuzzy hypersoft matrices are being outlined in this section.

Definition 1 ***Picture Fuzzy Soft Set(PFSS)[39].*** *"Let V be an initial universe and K be a set of parameters. A pair (R,K) is called a picture fuzzy soft set over V, where R is a mapping given $R: K \to \mathrm{PFS}(V)$ for every $k \in K$, R(k) is a picture fuzzy soft set of V and is called* ***Picture Fuzzy Value*** *for the set of parameter k. Here, $\mathrm{PFS}(V)$ is the set of all picture fuzzy subsets of V and*

$$R(k) = \left\{v, \rho_{R(k)}(v), \tau_{R(k)}(v), \omega_{R(k)}(v) | v \in V\right\}$$

where $\rho_{R(k)}(v), \tau_{R(k)}(v), \omega_{R(k)}(v)$ are the degrees of positive membership, neutral membership and negative membership respectively, with the constraint

$$\rho_{R(k)}(v) + \tau_{R(k)}(v) + \omega_{R(k)}(v) \leq 1,$$

and the degree of refusal is given by

$$\beth_{R(k)}(v) = (1 - (\rho_{R(k)}(v) + \tau_{R(k)}(v) + \omega_{R(k)}(v)) \ (\forall\ v \in V)."$$

Definition 2 ***Hypersoft Set(HSS)[31].*** *"Let V be the universal set and $P(V)$ be the power set of V. Consider $k_1, k_2, \ldots . k_n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attribute values are the sets $K_1, K_2, \ldots, K_n$ with $K_i \cap K_j = \varphi$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(R, K_1 \times K_2 \times \ldots \times K_n)$ is said to be Hypersoft Set over V where $R : K_1 \times K_2 \times \cdots \times K_n \to P(V)$. In other words, Hypersoft Set is a multi-parameterized family of subsets of the set V."*

Definition 3 ***Fuzzy Hypersoft Set(FHSS)[31].*** *"Let V be the universal set and $F(V)$ be the set all Fuzzy subsets of V. Consider $k_1, k_2, \ldots . k_n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attribute values are the sets $K_1, K_2, \ldots, K_n$ with $K_i \cap K_j = \varphi$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(R, K_1 \times K_2 \times \ldots \times K_n)$ is said to be Fuzzy Hypersoft Set over V where $R : K_1 \times K_2 \times \cdots \times K_n \to F(V)$ and, $R(k) = \{v, R(k)(v) | v \in V\}$; $k \in K_1 \times K_2 \times \ldots \times K_n$."*

Definition 4 ***Pythagorean Fuzzy Hypersoft Set (PyFHSS)[40].*** *"Let V be the universal set and PyFS(V) be the set of all Pythagorean fuzzy subsets of V.*
Consider $k_1, k_2, \ldots, k_n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attribute values are the sets $K_1, K_2, \ldots, K_n$ with $K_i \cap K_j = \varphi$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$. Let B_i be the non-empty subsets of K_i for each $i = 1, 2, \ldots, n$. A Pythagorean Fuzzy Hypersoft Set is defined as the pair, $(R, B_1 \times B_2 \times \cdots \times B_n)$, where $R : K_1 \times K_2 \times \cdots \times K_n \to \mathrm{PyFS}(V)$ and

$$R(B_1 \times B_2 \times \cdots \times B_n) = \left\{ < \vartheta, \left(\frac{v}{\rho_{R(\vartheta)}(v), \omega_{R(\vartheta)}(v)} \right) > \mid v \, \epsilon \, V \right\};$$

where $\vartheta \in B_1 \times B_2 \times \cdots \times B_n \subseteq K_1 \times K_2 \times \cdots \times K_n$. It may be noted that ρ and ω represent membership and non-membership degrees respectively, and satisfies the condition

$$\rho^2_{R(\vartheta)}(v) + \omega^2_{R(\vartheta)}(v) \leq 1\}; \text{ where } \rho_{R(\vartheta)}(v), \ \omega_{R(\vartheta)} \in [0, 1];$$

and, $\complement_{R(\vartheta)}(v) = \sqrt{1 - \rho^2_{R(\vartheta)}(v) - \omega^2_{R(\vartheta)}(v)}$ *is called the degree of indeterminacy."*

In the area of soft sets and soft matrix theory, Naim & Serdar [41] first defined the multiplication of two soft matrices and also the multiplication of two fuzzy soft matrices along with various theoretical propositions. In subsequent studies, Zulqarnain [42] and Jafar et al. [33] studied and developed the notion of the neutrosophic hypersoft matrix in a more generalized form with some basic operations, score function and properties. Also, Jafar et al. [32] proposed the notion of Intuitionistic Fuzzy Hypersoft matrices ($IFHSM$) and devised a new algorithm for MCDM problem with the revised score function.

It may be noted that there are some more similar operations on IFHSMs which can also be studied if necessary. In the next sections, we introduce the novel notion of picture fuzzy hypersoft matrices along with different binary operations and applications.

4 Picture Fuzzy Hypersoft Matrices & Operations

In this section, on the basis of the proposed notion of a picture fuzzy hypersoft set, we are also presenting the concept of a new type of hypersoft matrix termed a Picture Fuzzy Hypersoft Matrix (PFHSM) along with various binary operations and important properties.

Picture Fuzzy Hypersoft Matrix. Let $V = \{v^1, v^2, \ldots, v^n\}$ be the universe of discourse and PFS(V) be the collection of all picture fuzzy subsets of V. Suppose $K_1, K_2, \ldots, K_m$ for $m \geq 1$ be m well-defined attributes, whose respective attribute values are the sets $K_1^a, K_2^b, \ldots, K_m^z$ with the relation $K_1^a \times K_2^b \times \cdots \times K_m^z$ where $a, b, c, \ldots, z = 1, 2, \ldots, n$.

The pair $\left(R, K_1^a \times K_2^b \times \cdots \times K_m^z\right)$ is called a picture fuzzy hypersoft set over V where $R : K_1^a \times K_2^b \times \cdots \times K_m^z \to \text{PFS}(V)$ defined by

$$R\left(K_1^a \times K_2^b \times \cdots \times K_m^z\right) = \left\{< v, \rho_\vartheta(v), \tau_\vartheta(v), \omega_\vartheta(v) > \mid v \,\epsilon\, V,\ \vartheta \,\epsilon\, K_1^a \times K_2^b \times \cdots \times K_m^z \right\}.$$

Here, ρ , τ and ω represents the positive membership, neutral membership and negative membership degrees respectively.

Let $Z_v = K_1^a \times K_2^b \times \cdots \times K_m^z$ be the relation with its characteristic function is $\chi_{Z_v} : K_1^a \times K_2^b \times \cdots \times K_m^z \to \text{PFS}(V)$ given by

$$\chi_{Z_v} = \left\{< v, \rho_\vartheta(v), \tau_\vartheta(v), \omega_\vartheta(v) > \mid v \,\epsilon\, V,\ \vartheta \,\epsilon\, K_1^a \times K_2^b \times \cdots \times K_m^z \right\}.$$

The tabular representation of Z_v is given in Table 1

If $B_{ij} = \chi_{Z_v}\left(v^i, K_j^s\right)$ where $i = 1, 2, .., n,\ j = 1, 2, ..., m$ and $s = a, b, c, ..., z$. Then a matrix is defined as

$$[B_{ij}]_{n\times m} = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1m} \\ B_{21} & B_{22} & \cdots & B_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n1} & B_{n2} & \cdots & B_{nm} \end{bmatrix}$$

which is called **Picture Fuzzy Hypersoft Matrix** of order $n \times m$, where

$$B_{ij} = \left(\rho_{K_j^s}(v_i), \tau_{K_j^s}(v_i), \omega_{K_j^s}(v_i), v_i \epsilon V, \left(K_j^s \epsilon\ K_1^a \times K_2^b \times \cdots \times K_m^z\right)\right) = \left(\rho_{ijs}^B, \tau_{ijs}^B, \omega_{ijs}^B\right).$$

Hence, it may be noted that any picture fuzzy hypersoft set can be represented in terms of the picture fuzzy hypersoft matrix. Throughout the paper, we will denote the collection of all picture fuzzy hypersoft matrices by $PFHSM_{n\times m}$.

Example 1: Suppose a need arises for a School to hire a Mathematics teacher for 10th class. A total of five candidates have applied to fill up the void space. The Human Resource cell of the school appoints an expert/decision-maker for this selection process. Let $V = \{v^1, v^2, v^3, v^4, v^5\}$ be the set of all five candidates with their set of attributes as K_1 = Qualification, K_2 = Experience, K_3 = Age, K_4 = Gender. Further, their respective sub-attributes are

$$K_1 = \text{Qualification} = \{\text{BS Hons., MS, M.Phil., Ph.D.}\}$$

$$K_2 = \text{Experience} \; = \; \{3\text{yr}, \; 5\text{yr}, \; 7\text{yr}, \; 10\text{yr}\}$$

$$K_3 = \text{Age} \; = \{\text{Less than twenty five}, \; \text{Great than twenty five}\}$$

$$K_4 = \text{Gender} = \; \{\text{Male}, \; \text{Female}\}\,.$$

Let the function be $R : K_1^a \times K_2^b \times \cdots \times K_m^z \to \text{PFS}\,(V)$. Based on some empirical-hypothetical assumptions and the decision maker's opinion, we present the computed values with respect to each attribute and with their further sub-attributes in the form of tables, Table 2 - Table 5.

Now, let us consider

$$R\left(K_1^a \times K_2^b \times K_3^c \times K_4^d\right) = R\,(\text{MS, 7yr, Greater than twenty five, Male}) = \left(v^1, v^2, v^3, v^5\right).$$

For the above relational expression, the picture fuzzy hypersoft set can be expressed as

$$\begin{aligned}
&R\left(K_1^a \times K_2^b \times K_3^c \times K_4^d\right) = \\
&\quad \{< v^1, (MS(0.1, 0.2, 0.4), 7yr(0.1, 0.7, 0.1), \text{Greater than twenty five}(0.5, 0.3, 0.1), Male(0.2, 0.1, 0.2)) > \\
&\quad < v^2, (MS(0.4, 0.2, 0.3), 7yr(0.4, 0.3, 0.2), \text{Greater than twenty five}(0.4, 0.3, 0.1), Male(0.3, 0.2, 0.3)) > \\
&\quad < v^3, (MS(0.3, 0.5, 0.1), 7yr(0.1, 0.3, 0.5), \text{Greater than twenty five}(0.2, 0.4, 0.3), Male(0.1, 0.2, 0.6)) > \\
&\quad < v^5, (MS(0.1, 0.3, 0.4), 7yr(0.2, 0.5, 0.2), \text{Greater than twenty five}(0.4, 0.3, 0.1), Male(0.5, 0.2, 0.1)) >\}
\end{aligned}$$

The above example of picture fuzzy hypersoft set relational expression can be written in Table 6.

Also, the matrix form of the above representation can be written as

$$[B]_{4\times 4} = \begin{bmatrix} (MS(0.1, 0.2, 0.4)) & (7yr(0.1, 0.7, 0.1)) & (\text{Greater than twenty five}(0.5{,}0.3{,}0.1)) & (Male(0.2, 0.1, 0.2)) \\ (MS(0.4, 0.2, 0.3)) & (7yr(0.4, 0.3, 0.2)) & (\text{Greater than twenty five}(0.4{,}0.3{,}0.1)) & (Male(0.3, 0.2, 0.3)) \\ (MS(0.3, 0.5, 0.1)) & (7yr(0.1, 0.3, 0.5)) & (\text{Greater than twenty five}(0.2{,}0.4{,}0.3)) & (Male(0.1, 0.2, 0.6)) \\ (MS(0.1, 0.3, 0.4)) & (7yr(0.2, 0.5, 0.2)) & (\text{Greater than twenty five}(0.4{,}0.3{,}0.1)) & (Male(0.5, 0.2, 0.1)) \end{bmatrix}.$$

Various Types of Picture Fuzzy Hypersoft Matrices:

Let $B = [B_{ij}]$ be a picture fuzzy hypersoft matrix of order $n \times m$; where $B_{ij} = \left(\rho_{ijs}^B, \tau_{ijs}^B, \omega_{ijs}^B\right)$; then various kinds of important matrices can be presented as below:

- "**Picture fuzzy hypersoft zero matrix** if $\rho_{ijs}^B = 0, \; \tau_{ijs}^B = 0 \;\&\; \omega_{ijs}^B = 0; \forall i, j, s$ and the matrix is denoted by $0 = [0, 0, 0]$."
- "**Picture fuzzy hypersoft square matrix** if $n = m$."
- "**Picture fuzzy hypersoft row matrix** if $m = 1$."
- "**Picture fuzzy hypersoft column matrix** if $n = 1$."
- "**Picture fuzzy hypersoft diagonal matrix** if all its non-diagonal entries are zero $\forall \; i, j, s$."
- "**Picture fuzzy hypersoft ρ-universal matrix** if $\rho_{ijs}^B = 1$, $\tau_{ijs}^B = 0 \;\&\; \omega_{ijs}^B = 0$; $\forall \; i$, j & s, denoted by $\wp_\rho$."

- "**Picture fuzzy hypersoft τ-universal matrix** if $\rho^B_{ijs} = 0$, $\tau^B_{ijs} = 1$ & $\omega^B_{ijs} = 0$; $\forall$ i , j & s, denoted by $\wp_\tau$."

- "**Picture fuzzy hypersoft ω-universal matrix** if $\rho^B_{ijs} = 0$, $\tau^B_{ijs} = 0$ & $\omega^B_{ijs} = 1$; $\forall$ i & j & s, denoted by $\wp_\omega$."

- "**Picture fuzzy hypersoft Scalar multiplication:** for any scalar m, we define $mB = [(m\rho^B_{ijs}, m\tau^B_{ijs}, m\omega^B_{ijs})]$, $\forall$ i , j & s."

- "**Picture fuzzy hypersoft Symmetric Matrix:** if $\left(\rho^B_{ijs}, \tau^B_{ijs}, \omega^B_{ijs}\right) = \left(\rho^B_{jsi}, \tau^B_{jsi}, \omega^B_{jsi}\right)$ i.e.$B^t = B$.

Further, we propose some set-theoretic relations for two given picture fuzzy hypersoft matrices, say, $B = [(\rho^B_{ijs}, \tau^B_{ijs}, \omega^B_{ijs})]$ and $C = [(\rho^C_{ijs}, \tau^C_{ijs}, \omega^C_{ijs})] \in PFHSM_{n\times m}$.

- "**Subsethood:** $B \subseteq C$ if $\rho^B_{ijs} \leq \rho^C_{ijs}$, $\tau^B_{ijs} \geq \tau^C_{ijs}$ & $\nu^B_{ijs} \geq \nu^C_{ijs}$; $\forall$ i, j & s."

- "**Containment:** $B \supseteq C$ if $\rho^B_{ijs} \geq \rho^C_{ijs}$, $\tau^B_{ijs} \leq \tau^C_{ijs}$ & $\omega^B_{ijs} \leq \nu^C_{ijs}$; $\forall$ i, j & s."

- "**Equality:** $B = C$ if $\rho^B_{ijs} = \rho^C_{ijs}$, $\tau^B_{ijs} = \tau^C_{ijs}$ & $\omega^B_{ijs} = \omega^C_{ijs}$; $\forall$ i, j & s."

- "**Max Min Product:**
  Let $B = [B_{ij}] = [(\rho^B_{ijs}, \tau^B_{ijs}, \omega^B_{ijs})] \in PFHSM_{n\times m}$ & $C = [C_{jt}] = [(\rho^C_{jst}, \tau^C_{jst}, \omega^C_{jst})] \in PFHSM_{m\times p}$ be two Picture fuzzy hyoersoft matrices then
  $$B * C = [d_{it}]_{m\times p} = \left[\left(\max(\min_{js}(\rho^B_{ijs}, \rho^C_{jst})), \min(\min_{js}(\tau^B_{ijs}, \tau^C_{jst})), \min(\max_{js}(\omega^B_{ijs}, \omega^C_{jst}))\right)\right];$$
  $\forall$ i, j, s & t"

- "**Average Max Min Product:**
  Let $B = [B_{ij}] = [(\rho^B_{ijs}, \tau^B_{ijs}, \omega^B_{ijs})] \in PFHSM_{n\times m}$ & $C = [C_{jt}] = [(\rho^C_{jst}, \tau^C_{jst}, \omega^C_{jst})] \in PFHSM_{m\times p}$ be two Picture fuzzy hypersoft matrices then
  $$B *_A C = [d_{it}]_{n\times p} = \left[\left(\max_j s(\frac{\rho^B_{ijs}+\rho^C_{jst}}{2}), \min_j s(\frac{\tau^B_{ijs}+\tau^C_{jst}}{2}), \min_j s(\frac{\omega^B_{ijs}, \omega^C_{jst}}{2})\right)\right];$$
  $\forall$ i, j, s & t."

Some Fundamental Binary Operations for Picture fuzzy hypersoft matrices:
Consider two Picture fuzzy hypersoft matrices $B_1 = [(\rho^{B_1}_{ijs}, \tau^{B_1}_{ijs}, \omega^{B_1}_{ijs})]$ and $B_2 = [(\rho^{B_2}_{ijs}, \tau^{B_2}_{ijs}, \omega^{B_2}_{ijs})]$ $\in PFHSM_{n\times m}$. Some of the basic binary operations on these matrices can be presented as follows:

- $B_1^c = \left[\left(\omega^{B_1}_{ijs}, \tau^{B_1}_{ijs}, \rho^{B_1}_{ijs}\right)\right]$; $\forall$ i, j and s.

- $B_1 \cup B_2 = \left[\left(\max(\rho^{B_1}_{ijs}, \rho^{B_2}_{ijs}), \min(\tau^{B_1}_{ijs}, \tau^{B_2}_{ijs}), \min(\omega^{B_1}_{ijs}, \omega^{B_2}_{ijs})\right)\right]$; $\forall$ i, j and s.

- $B_1 \cap B_2 = \left[\left(\min(\rho^{B_1}_{ijs}, \rho^{B_2}_{ijs}), \min(\tau^{B_1}_{ijs}, \tau^{B_2}_{ijs}), \max(\omega^{B_1}_{ijs}, \omega^{B_2}_{ijs})\right)\right]$ $\forall$; i and j.

- $B_1 \otimes B_2 = \left[\left(\rho^{B_1}_{ijs} \cdot \rho^{B_2}_{ijs}, \tau^{B_1}_{ijs} \cdot \tau^{B_2}_{ijs}, \sqrt[n]{(\omega^{B_1}_{ijs})^2 + (\omega^{B_2}_{ijs})^2 - (\omega^{B_1}_{ijs})^2 \cdot (\omega^{B_2}_{ijs})^2}\right)\right]$; $\forall$ i, j amd s.

- $B_1 \oplus B_2 = \left[\left(\sqrt[n]{(\rho_{ijs}^{B_1})^2 + (\rho_{ijs}^{B_2})^2 - (\rho_{ijs}^{B_1})^2 \cdot (\rho_{ijs}^{B_2})^2}, \tau_{ijs}^{B_1} \cdot \tau_{ijs}^{B_2}, \omega_{ijs}^{B_1} \cdot \omega_{ijs}^{B_2}\right)\right]$; $\forall$ i, j and s.

- $B_1 @ B_2 = \left[\left(\frac{\rho_{ijs}^{B_1}+\rho_{ijs}^{B_2}}{2}, \frac{\tau_{ijs}^{B_1}+\tau_{ijs}^{B_2}}{2}, \frac{\omega_{ijs}^{B_1}+\omega_{ijs}^{B_2}}{2}\right)\right]$; $\forall$ i, j and s.

- $B_1 @_w B_2 = \left[\left(\frac{w_1\rho_{ijs}^{B_1}+w_2\rho_{ijs}^{B_2}}{w_1+w_2}, \frac{w_1\tau_{ijs}^{B_1}+w_2\tau_{ijs}^{B_2}}{w_1+w_2}, \frac{w_1\omega_{ijs}^{B_1}+w_2\omega_{ijs}^{B_2}}{w_1+w_2}\right)\right]$; $\forall$ i, j and s ; where $w_1, w_2 > 0$ are the weights.

- $B_1 \$ B_2 = \left[\left(\sqrt[n]{\rho_{ijs}^{B_1} \cdot \rho_{ijs}^{B_2}}, \sqrt[n]{\tau_{ijs}^{B_1} \cdot \tau_{ijs}^{B_2}}, \sqrt[n]{\omega_{ijs}^{B_1} \cdot \omega_{ijs}^{B_2}}\right)\right]$; $\forall$ i, j and s.

- $B_1 \$_w B_2 = \left[\left(((\rho_{ijs}^{B_1})^{w_1} \cdot (\rho_{ijs}^{B_2})^{w_2})^{\frac{1}{w_1+w_2}}, ((\tau_{ijs}^{B_1})^{w_1} \cdot (\tau_{ijs}^{B_2})^{w_2})^{\frac{1}{w_1+w_2}}, ((\omega_{ijs}^{B_1})^{w_1} \cdot (\omega_{ijs}^{B_2})^{w_2})^{\frac{1}{w_1+w_2}}\right)\right]$; $\forall$ i, j and s, where $w_1, w_2 > 0$ are the weights.

- $B_1 \bowtie B_2 = \left[\left(2 \cdot \frac{\rho_{ijs}^{B_1} \cdot \rho_{ijs}^{B_2}}{\rho_{ijs}^{B_1}+\rho_{ijs}^{B_2}}, 2 \cdot \frac{\tau_{ijs}^{B_1} \cdot \tau_{ijs}^{B_2}}{\tau_{ijs}^{B_1}+\tau_{ijs}^{B_2}}, 2 \cdot \frac{\omega_{ijs}^{B_1} \cdot \omega_{ijs}^{B_2}}{\omega_{ijs}^{B_1}+\omega_{ijs}^{B_2}}\right)\right]$; $\forall$ i, j and s.

- $B_1 \bowtie_w B_2 = \left[\left(\frac{w_1+w_2}{\frac{w_1}{\rho_{ijs}^{B_1}}+\frac{w_2}{\rho_{ijs}^{B_2}}}, \frac{w_1+w_2}{\frac{w_1}{\tau_{ijs}^{B_1}}+\frac{w_2}{\tau_{ijs}^{B_2}}}, \frac{w_1+w_2}{\frac{w_1}{\omega_{ijs}^{B_1}}+\frac{w_2}{\omega_{ijs}^{B_2}}}\right)\right]$; $\forall$ i, j and s ; where $w_1, w_2 > 0$ are the weights.

Proposition 1 *Let B_1 and $B_2 \in PFHSM_{n\times m}$ then the following laws hold:*

(i) $B_1 \cup B_2 = B_2 \cup B_1$

(ii) $B_1 \cap B_2 = B_2 \cap B_1$

(iii) $(B_1 \cup B_2)^c = B_1^c \cap B_2^c$

(iv) $(B_1 \cap B_2)^c = B_1^c \cup B_2^c$

(v) $(B_1^c \cap B_2^c)^c = B_1 \cup B_2$

(vi) $(B_1^c \cup B_2^c)^c = B_1 \cap B_2$.

Proof : The proof can be established with the help of proposed operations.

Proposition 2 *Let $B_1 = [(\rho_{ijs}^{B_1}, \tau_{ijs}^{B_1}, \omega_{ijs}^{B_1})] \in PFHSM_{n\times m}$. On the basis of the proposed definitions, the following laws hold:*

(i) $(B_1^c)^c = B_1$

(ii) $(\wp_\rho)^c = \wp_\omega$

(iii) $(\wp_\tau)^c = \wp_\tau$

(iv) $(\wp_\omega)^c = \wp_\rho$

(v) $B_1 \cup B_1 = B_1$

(vi) $B_1 \cup \wp_\rho = \wp_\rho$

(vii) $B_1 \cap \wp_\nu = B_1$

(viii) $B_1 \cap B_1 = B_1$

(ix) $B_1 \cap \wp_\rho = B_1$

(x) $B_1 \cap \wp_\omega = \wp_\omega$.

Proposition 3 *Let B_1 and $B_2 \in PFHSM_{n\times m}$. In view of the weighted form, the following laws hold:*

(i) $(B_1^c @_w B_2^c)^c = B_1 @_w B_2$

(ii) $(B_1^c \$_w B_2^c)^c = B_1 \$_w B_2$

(iii) $(B_1^c \bowtie_w B_2^c)^c = B_1 \bowtie_w B_2$

(iv) $B_1 @_w B_2 = B_2 @_w B_1$

(v) $B_1 \$_w B_2 = B_2 \$_w B_1$

(vi) $B_1 \bowtie_w B_2 = B_2 \bowtie_w B_1$.

Proof : The proof can be carried out with the help of the proposed definitions.

Proposition 4 *For B_1, B_2 and $B_3 \in PFHSM_{n\times m}$, the following associative laws hold:*

(i) $(B_1 \cup B_2) \cup B_3 = B_1 \cup (B_2 \cup B_3)$

(ii) $(B_1 \cap B_2) \cap B_3 = B_1 \cap (B_2 \cap B_3)$

(iii) $(B_1 @ B_2) @ B_3 = B_1 @ (B_2 @ B_3)$

(iv) $(B_1 \$ B_2) \$ B_3 = B_1 \$ (B_2 \$ B_3)$

(v) $(B_1 \bowtie B_2) \bowtie B_3 = B_1 \bowtie (B_2 \bowtie B_3)$.

Proof: The proof can be carried out with the help of the proposed definitions.

Proposition 5 *For B_1, B_2 and $B_3 \in PFHSM_{n\times m}$, the following distributive laws hold:*

(i) $B_1 \cap (B_2 \cup B_3) = (B_1 \cap B_2) \cup (B_1 \cap B_3)$

(ii) $(B_1 \cap B_2) \cup B_3 = (B_1 \cup B_3) \cap (B_2 \cup B_3)$

(iii) $B_1 \cup (B_2 \cap B_3) = (B_1 \cup B_2) \cap (B_1 \cup B_3)$

(iv) $(B_1 \cup B_2) \cap B_3 = (B_1 \cap B_3) \cup (B_2 \cap B_3)$

(v) $(B_1 \cap B_2) @ B_3 = (B_1 @ B_3) \cap (B_2 @ B_3)$

(vi) $(B_1 \cap B_2) \bowtie B_3 = (B_1 \bowtie B_3) \cap (B_2 \bowtie B_3)$

(vii) $B_1 \cup (B_2 @ B_3) = (B_1 \cup B_2) @ (B_1 \cup B_3)$

(viii) $(B_1 \cup B_2) \bowtie B_3 = (B_1 \bowtie B_3) \cup (B_2 \bowtie B_3)$

(ix) $B_1 @ (B_2 \cup B_3) = (B_1 @ B_2) \cup (B_1 @ B_3)$

(x) $B_1 @ (B_2 \cap B_3) = (B_1 @ B_2) \cap (B_2 @ B_3)$

(xi) $B_1 \$ (B_2 \cup B_3) = (B_1 \$ B_2) \cup (B_1 \$ B_3)$

(xii) $(B_1 \cup B_2) \$ B_3 = (B_1 \$ B_3) \cup (B_2 \$ B_3)$

(xiii) $B_1 \cup (B_2 \bowtie B_3) = (B_1 \cup B_2) \bowtie (B_1 \cup B_3)$

(xiv) $B_1 \bowtie (B_2 \cup B_3) = (B_1 \bowtie B_2) \cup (B_1 \bowtie B_3)$

(xv) $B_1 \$ (B_2 \cap B_3) = (B_1 \$ B_2) \cap (B_2 \$ B_3)$

(xvi) $(B_1 \cap B_2) \$ B_3 = (B_1 \$ B_3) \cap (B_2 \$ B_3)$.

Proof :

(i)

$$\begin{aligned} B_1 \cap (B_2 \cup B_3) &= \Bigg[\Bigg(\big[(\rho_{ijs}^{B_1}, \tau_{ijs}^{B_1}, \omega_{ijs}^{B_1})\big] \cap \big[\big(\max\{\rho_{ijs}^{B_2}, \rho_{ijs}^{B_3}\}, \min\{\tau_{ijs}^{B_2}, \tau_{ijs}^{B_3}\}, \min\{\omega_{ijs}^{B_2}, \omega_{ijs}^{B_3}\}\big)\big]\Bigg)\Bigg] \\ &= \Bigg[\Bigg(\min\{\rho_{ijs}^{B_1}, \max\{\rho_{ijs}^{B_2}, \rho_{ijs}^{B_3}\}\}, \min\{\tau_{ijs}^{B_1}, \min\{\tau_{ijs}^{B_2}, \rho_{ijs}^{B_3}\}\}, \\ &\qquad \max\{\omega_{ijs}^{B_1}, \min\{\omega_{ijs}^{B_2}, \omega_{ijs}^{B_3}\}\}\Bigg)\Bigg]. \end{aligned}$$

Now,

$$
\begin{aligned}
(B_1 \cap B_2) \cup (B_1 \cap B_3) &= \Big[\big(\min\{\rho_{ijs}^{B_1}, \rho_{ijs}^{B_2}\}, \min\{\tau_{ijs}^{B_1}, \tau_{ijs}^{B_2}\}, \max\{\omega_{ijs}^{B_1}, \omega_{ijs}^{B_2}\}\big)\Big] \cup \Big[\big(\min\{\rho_{ijs}^{B_1}, \rho_{ijs}^{B_3}\}, \\
&\qquad \min\{\tau_{ijs}^{B_1}, \tau_{ijs}^{B_3}\}, \max\{\omega_{ijs}^{B_1}, \omega_{ijs}^{B_3}\}\big)\Big] \\
&= \Big[\big(\max(\min\{\rho_{ijs}^{B_1}, \rho_{ijs}^{B_2}\}, \min\{\rho_{ijs}^{B_1}, \rho_{ijs}^{B_3}\}),\ \min(\min\{\tau_{ijs}^{B_1}, \tau_{ijs}^{B_2}\}, \min\{\tau_{ijs}^{B_1}, \tau_{ijs}^{B_3}\}), \\
&\qquad \min(\max\{\omega_{ijs}^{B_1}, \omega_{ijs}^{B_2}\}, \max\{\omega_{ijs}^{B_1}, \omega_{ijs}^{B_3}\})\big)\Big] \\
&= \Big[\big(\max(\rho_{ijs}^{B_1}, \min\{\rho_{ijs}^{B_2}, \rho_{ijs}^{B_3}\}), \min(\tau_{ijs}^{B_1}, \min\{\tau_{ijs}^{B_2}, \tau_{ijs}^{B_3}\}), \\
&\qquad \min(\omega_{ijs}^{B_1}, \max\{\omega_{ijs}^{B_2}, \omega_{ijs}^{B_3}\})\big)\Big] \\
&= \Big[\big(\min(\rho_{ijs}^{B_1}, \max\{\rho_{ijs}^{B_2}, \rho_{ijs}^{B_3}\}), \min(\rho_{ijs}^{B_1}, \min\{\rho_{ijs}^{B_2}, \rho_{ijs}^{B_3}\}), \max(\omega_{ijs}^{B_1}, \\
&\qquad \min\{\omega_{ijs}^{B_2}, \omega_{ijs}^{B_3}\})\big)\Big] = B_1 \cap (B_2 \cup B_3).
\end{aligned}
$$

Hence, $B_1 \cap (B_2 \cup B_3) = (B_1 \cap B_2) \cup (B_1 \cap B_3)$ holds.

(ii)

$$
\begin{aligned}
(B_1 \cap B_2) \cup B_3 &= \big[\big(\min\{\rho_{ijs}^{B_1}, \rho_{ijs}^{B_2}\}, \min\{\tau_{ijs}^{B_1}, \tau_{ijs}^{B_2}\}, \max\{\omega_{ijs}^{B_1}, \omega_{ijs}^{B_2}\}\big)\big] \cup \big[\big(\rho_{ijs}^{B_3}, \tau_{ijs}^{B_3}, \omega_{ijs}^{B_3}\big)\big] \\
&= \Big[\big(\max(\min\{\rho_{ijs}^{B_1}, \rho_{ijs}^{B_2}\}, \rho_{ijs}^{B_3}), \min(\min\{\tau_{ijs}^{B_1}, \tau_{ijs}^{B_2}\}, \rho_{ijs}^{B_3}), \\
&\qquad \min(\max\{\omega_{ijs}^{B_1}, \omega_{ijs}^{B_2}\}, \omega_{ijs}^{B_3})\big)\Big].
\end{aligned}
$$

Now,

$$
\begin{aligned}
(B_1 \cup B_3) \cap (B_2 \cup B_3) &= \big[\big(\max\{\rho_{ijs}^{B_1}, \rho_{ijs}^{B_3}\}, \min\{\tau_{ijs}^{B_1}, \tau_{ijs}^{B_3}\}, \min\{\omega_{ijs}^{B_1}, \omega_{ijs}^{B_3}\}\big)\big] \cap \big[\big(\max\{\rho_{ijs}^{B_2}, \rho_{ijs}^{B_3}\}, \\
&\qquad \min\{\tau_{ijs}^{B_2}, \tau_{ijs}^{B_3}\}, \min\{\omega_{ijs}^{B_2}, \omega_{ijs}^{B_3}\}\big)\big] \\
&= \Big[\big(\min(\max\{\rho_{ijs}^{B_1}, \rho_{ijs}^{B_3}\}, \max\{\rho_{ijs}^{B_2}, \rho_{ijs}^{B_3}\}), \min(\min\{\tau_{ijs}^{B_1}, \tau_{ijs}^{B_3}\}, \min\{\tau_{ijs}^{B_2}, \tau_{ijs}^{B_3}\}), \\
&\qquad \max(\min\{\omega_{ijs}^{B_1}, \omega_{ijs}^{B_3}\}, \min\{\omega_{ijs}^{B_2}, \omega_{ijs}^{B_3}\})\big)\Big] \\
&= \Big[\big(\min(\max\{\rho_{ijs}^{B_1}, \rho_{ijs}^{B_2}\}, \rho_{ijs}^{B_3}), \min(\min\{\tau_{ijs}^{B_1}, \tau_{ijs}^{B_2}\}, \tau_{ijs}^{B_3}), \\
&\qquad\quad \max(\min\{\omega_{ijs}^{B_1}, \omega_{ijs}^{B_2}\}, \omega_{ijs}^{B_3})\big)\Big] \\
&= \Big[\big(\max(\min\{\rho_{ijs}^{B_1}, \rho_{ijs}^{B_2}\}, \rho_{ijs}^{B_3}), \min(\min\{\tau_{ijs}^{B_1}, \tau_{ijs}^{B_2}\}, \tau_{ijs}^{B_3}), \\
&\qquad\quad \min(\max\{\omega_{ijs}^{B_1}, \omega_{ijs}^{B_2}\}, \omega_{ijs}^{B_3})\big)\Big] = (B_1 \cap B_2) \cup B_3
\end{aligned}
$$

Hence, $(B_1 \cap B_2) \cup B_3 = (B_1 \cup B_3) \cap (B_2 \cup B_3)$.

On a similar pattern, the rest of the laws can be proved accordingly.

5 Application of PFHSM in Renewable Energy Source Selection

In this section, we consider a basic framework of the renewable energy source selection problem where the formulation of the problem has been considered to be in the form of a picture fuzzy hypersoft matrix & proposed some revised definitions keeping the necessity of the problem into account.

Problem Statement (Renewable Energy Source Selection):
Suppose we have a set of m renewable energy resources $X = \{x_1, x_2, \ldots, x_m\}$ which are to be evaluated against n parameters (criteria) $Z = \{z_1, z_2, \ldots, z_n\}$ having further a set of k sub-attribute's parameters $Q = \{q_1, q_2, \ldots, q_k\}$. For the sake of the best possible selection of the available renewable energy sources, suppose that a committee gets constituted, say, with two experts (decision-makers) having adequate knowledge of the field of engineering, economics, management, government services and national energy policies etc. The computation and the procedure of the decision-making should yield the best suitable source of renewable energy in view of all the interrelated parameters and sub-parameters. In case if we take up a very formal selection process structure where the nature of information is accounted as a picture fuzzy hypersoft matrix and then we need to propose some notions in a revised format that are important and essential for solving such MCDM problems. In view of the widely utilized structure of a decision-making problem and taking the proposed notion of picture fuzzy hypersoft matrices into consideration, we express the following revised definition of choice matrix and weighted choice matrix:

Definition 5 *If $B_1 = [(\rho_{ijs}^{B_1}, \tau_{ijs}^{B_1}, \omega_{ijs}^{B_1})] \in PFHSM_{n\times m}$, then the* ***choice matrix*** *of PFHSM (**PFHSCM**) B_1, in case the weights are same, is defined as*

$$C(B_1) = \left[\left(\frac{\sum\limits_{js=1}^{n}(\rho_{ijs}^{B_1})^q}{n}, \frac{\sum\limits_{js=1}^{n}(\tau_{ijs}^{B_1})^q}{n}, \frac{\sum\limits_{js=1}^{n}(\omega_{ijs}^{B_1})^q}{n}\right)\right]_{n\times 1} \quad ; \ \forall\ i.$$

Definition 6 *If $B_1 = [(\rho_{ijs}^{B_1}, \tau_{ijs}^{B_1}, \omega_{ijs}^{B_1})] \in PFHSM_{n\times m}$, then the* ***weighted choice matrix*** *of PFHSM (**PFHSWCM**) B_1, where $w_{js} > 0$ are weights, is defined by*

$$C_w(B_1) = \left[\left(\frac{\sum\limits_{js=1}^{n} w_{js}(\rho_{ijs}^{B_1})^q}{\sum w_{js}}, \frac{\sum\limits_{js=1}^{n} w_{js}(\tau_{ijs}^{B_1})^q}{\sum w_{js}}, \frac{\sum\limits_{js=1}^{n} w_{js}(\omega_{ijs}^{B_1})^q}{\sum w_{js}}\right)\right]_{n\times 1} \quad \forall\ i.$$

By making use of the revised choice matrix/weighted choice matrix, we present a new technique (Algorithm I) to handle the MCDM problem which is being presented with the help of Figure 4.

Remark: In case of any tie, we select the alternative with the highest membership value and the lowest non-membership value.

Further, in addition to the above methodology for MCDM, we propose an alternative technique (Algorithm II) where the notion of Value matrix and score matrix in the form of picture fuzzy hypersoft information is utilized which is found to be more suitable and consistent.

Definition 7 *Let $B = [B_{ij}]$ be the PFHSM of order $n \times m$, where $B_{ij} = (\rho^B_{ijs}, \tau^B_{ijs}, \omega^B_{ijs})$ then the **value matrix** of B (**PFHSVM**) is denoted by $\delta(B)$ and is defined by $\delta(B) = [B^B_{ij}]$ of order $n \times m$, where $B^B_{ij} = \rho^B_{ijs} - \tau^B_{ijs} - \omega^B_{ijs}$.*

Definition 8 *Let $B = [B_{ij}]$ and $C = [C_{ij}]$ be two picture fuzzy hypersoft matrices of order $n \times m$ then the **score matrix** of B and C is given by $\Gamma(B, C) = \delta(B) + \delta(C)$ and $\Gamma(B, C) = [\Gamma_{ij}]$ where $\Gamma_{ij} = \delta^B_{ij} + \delta^C_{ij}$. The total score of every member is given by $|\sum_{j=1}^{n} \Gamma_{ij}|$.*

Based on the above definitions of value matrix and score matrix, we outline Algorithm II for solving the MCDM problem as given in Figure 5.

Remark: In case while computation of the MCDM problem, if there is a tie in the membership values for determining the objective, then we would select the alternative with the highest membership value and the lowest non-membership value.

6 Numerical Illustration of RES Selection Problem

Suppose $X = \{x_1, x_2, x_3, x_4, x_5\}$ be a set of renewable energy sources (alternatives), where x_1, x_2, x_3, x_4, x_5 represents solar energy, wind energy, geothermal energy, hydropower and biomass energy, respectively. These renewable energy sources are to be examined against the criteria given by $Z = \{z_1, z_2, z_3, z_4, z_5, z_6\}$ and $z_1, z_2, z_3, z_4, z_5, z_6$ represents cost, environmental friendly, yields, maintenance, reliability and less number of peoples are effected from this project. A committee consists of two experts having knowledge of the field of engineering, economics, management, government services and policy-making for the best possible selection of the available resource. In order to formulate the problem into Picture fuzzy hypersoft information let us consider the further sub-attributes of the above attributes given by

- Cost $= z_1 = \{z_{11} = average,\ z_{12} = moderate\}$,
- Environmental Friendly $= z_2$,
- Yields $= z_3$,
- Maintenance $= z_4 = \{z_{41} = \text{predictive}, z_{42} = \text{preventive}, \}$,
- Reliability $= z_5 = \{z_{51} = \text{internal}, z_{52} = \text{external}, \}$,
- People affected from project $= z_6$.

Let $Z' = z_1 \times z_2 \times z_3 \times z_4 \times z_5 \times z_6$ be a set of sub-attributes which is explicitly given by

$$= \left\{ \begin{pmatrix} (z_{11}, z_2, z_3, z_{41}, z_{51}, z_6),\ (z_{11}, z_2, z_3, z_{41}, z_{52}, z_6), (z_{11}, z_2, z_3, z_{42}, z_{51}, z_6), (z_{11}, z_2, z_3, z_{42}, z_{52}, z_6) \\ (z_{12}, z_2, z_3, z_{41}, z_{51}, z_6),\ (z_{12}, z_2, z_3, z_{41}, z_{52}, z_6), (z_{12}, z_2, z_3, z_{42}, z_{51}, z_6), (z_{12}, z_2, z_3, z_{42}, z_{52}, z_6) \end{pmatrix} \right\}$$

For the calculation purpose set of all sub-attributes can be restated as

$$\mathbb{Z}' = \left\{ z_1', \ z_2', z_3', z_4', z_5', z_6', z_7', z_8' \right\}$$

Next, we illustrate the implementation of the proposed algorithms (Algorithm I and Algorithm II) by taking a numerical example existing in literature which has been studied by Feng at al. [43] and Khan et al. [44], [45].

Algorithm I (MCDM Using Choice and Weighted Choice Picture Fuzzy Hypersoft Matrices)

Step 1: The situations are examined by the experts in terms of PFHSMs given by Table 7 and Table 8.

Note: In the picture fuzzy hypersoft matrix (Table 7), the first element $(0.1, 0.1, 0.8)$ defines the degree to which the criterion z_1' is satisfied by the alternative x_1 is 0.1 whereas the degree to which the criterion z_1' is not satisfied by the alternative x_1 is 0.8 and the degree of neutral membership is 0.1.

Note: Similarly, in the picture fuzzy hypersoft matrix (Table 8), the first element $(0.8, 0.1, 0.1)$ defines the degree to which the criterion z_1' is satisfied by the alternative x_1 is 0.8 whereas the degree to which the criterion z_1' is not satisfied by the alternative x_1 is 0.1 and the degree of neutral membership is 0.1.

Step 2: On the basis of the PFHSMs, we construct the respective Choice matrices for the PFHSMs given by both the experts one by one.
Expert - 1
Case 1: (Equal Weights) Here, we assume the equal preference for all the criteria/subcriteria and we calculate the picture fuzzy hypersoft choice matrix as follows:

$$\begin{bmatrix} (0.2463, 0.02, 0.1538) \\ (0.2, 0.0325, 0.1386) \\ (0.2850, 0.0463, 0.1300) \\ (0.3250, 0.12, 0.1175) \\ (0.2462, 0.0163, 0.1375) \end{bmatrix}$$

Case 2: (Unequal weights) Based on the decision-maker's opinion, if different weights $0.1, 0.1, 0.1, 0.1, 0.2, 0.1, 0.15, 0.15$ have been assigned for the set of all sub-attributes

$$\mathbb{Z}' = \left\{ z_1', \ z_2', z_3', z_4', z_5', z_6', z_7', z_8' \right\}$$

respectively, then the picture fuzzy hypersoft weighted choice matrix is being obtained as follows:

$$\begin{bmatrix} (0.238, 0.024, 0.1365) \\ (0.1985, 0.0335, 0.131) \\ (0.2405, 0.0555, 0.138) \\ (0.325, 0.021, 0.102) \\ (0.263, 0.0165, 0.116) \end{bmatrix}$$

Expert - 2
Similarly, we carry out the computation based on the opinion of expert 2 as follows:
Case 1: (Equal Weights) Assuming the equal preference to all the criteria/subcriteria, based on the opinion of expert 2, we calculate the picture fuzzy hypersoft choice matrix as follows:

$$\begin{bmatrix} (0.3438, 0.0225, 0.0563) \\ (0.2738, 0.0338, 0.065) \\ (0.1738, 0.0538, 0.2413) \\ (0.3038, 0.0288, 0.1388) \\ (0.2563, 0.0163, 0.1275) \end{bmatrix}$$

Case 2: (Unequal Weights) Based on the decision-maker's opinion, if different weights $0.1, 0.1, 0.1, 0.1, 0.2, 0.1, 0.15, 0.15$ have been assigned for the set of all sub-attributes

$$\mathbb{Z}' = \left\{ z_1^{'},\ z_2^{'}, z_3^{'}, z_4^{'}, z_5^{'}, z_6^{'}, z_7^{'}, z_8^{'} \right\}$$

respectively, then the picture fuzzy hypersoft weighted choice matrix is being obtained as follows:

$$\begin{bmatrix} (0.3235, 0.023, 0.051) \\ (0.271, 0.0375, 0.0585) \\ (0.173, 0.0665, 0.2055) \\ (0.308, 0.028, 0.119) \\ (0.259, 0.1255, 0.12) \end{bmatrix}$$

Step 3: Analysis by Expert 1
Case 1: Equal Weights As per Step 2, if the equal preferences are given to all sub-attributes then from the choice matrix obtained above the highest membership value is **0.3250**, which is of renewable energy source x_4, i.e., Hydropower energy. Hence, the most suitable renewable energy source would be Hydropower energy.
Case 2: Unequal Weights However, if the preferences are not equal, i.e., if the sub-attribute $z_5^{'}$ is preferred more than other sub-attributes then from the choice matrix obtained above, the highest membership value is **0.325** which is of renewable energy source x_4, i.e., Hydropower energy. Hence, again the most suited renewable energy source would be Hydropower energy.
Analysis by Expert 2
Case 1: Equal Weights As per Step 2, if the equal preferences are given to all sub-attributes then from the choice matrix obtained above the highest membership value is **0.3438**, which is of renewable energy source x_1, i.e., Solar energy. Hence, the most suitable renewable energy source would be Solar energy.
Case 2: Unequal Weights However, if the preferences are not equal, i.e., if the sub-attribute $z_5^{'}$ is preferred more than other sub-attributes then from the choice matrix obtained above, the highest membership value is **0.3235** which is of renewable energy source x_1, i.e., Solar energy. Hence, again the most suited renewable energy source would be Solar energy.

Results and Discussion: On the basis of the opinions of Expert 1 and Expert 2 along with their corresponding choice matrix and weighted choice matrix, it has been observed that there is a one-step variation in the ranking of the renewable energy sources which can be taken as a flip mode. This variation may be eliminated by taking the count of other experts and their majority in opinion. Due to inter-related uncertainty and inexactness in the imprecise information, sometimes it becomes inevitable to tolerate and compromise the precision. The ranking based on the membership values has been pictorially presented in Figure 6. However, to overcome this limitation, in the next algorithm II, the expert's opinions have been duly merged together to obtain a joint decision that would be openly acceptable.

Algorithm II (Using Value & Score Picture Fuzzy Hypersoft Matrices)
Step 1: Same as step 1 of the Algorithm I.
Step 2: Next, we construct the value matrices from the provided picture fuzzy hypersoft matrices obtained in Step 1.

$$\delta(B) = \begin{bmatrix} -0.8 & 0.3 & -0.1 & 0.4 & 0.3 & 0.4 & -0.3 & -0.4 \\ -0.5 & 0.5 & -0.2 & 0.1 & 0.3 & 0.1 & -0.5 & -0.5 \\ -0.4 & 0.6 & 0.2 & 0.5 & -0.5 & 0.5 & -0.1 & -0.4 \\ -0.8 & 0.8 & 0.3 & 0.1 & 0.4 & 0.3 & 0.1 & -0.7 \\ -0.1 & 0.2 & 0.4 & -0.5 & 0.5 & -0.4 & 0.1 & 0.0 \end{bmatrix}$$

$$\delta(C) = \begin{bmatrix} 0.6 & 0.2 & -0.2 & 0.5 & 0.4 & 0.4 & -0.1 & 0.0 \\ 0.3 & 0.4 & -0.1 & 0.2 & 0.2 & 0.0 & 0.1 & -0.1 \\ -0.4 & -0.7 & 0.2 & 0.4 & 0.0 & -0.6 & -0.3 & -0.3 \\ 0.6 & -0.8 & 0.2 & 0.0 & 0.5 & 0.2 & -0.1 & 0.1 \\ -0.2 & 0.3 & 0.4 & 0.2 & 0.5 & -0.3 & -0.3 & -0.4 \end{bmatrix}$$

Step 3: Further, we calculate the score matrices by the above two value matrices:

$$\Gamma(B, C) = \begin{bmatrix} -0.2 & 0.5 & -0.3 & 0.9 & 0.7 & 0.8 & -0.4 & -0.4 \\ -0.2 & 0.9 & -0.3 & 0.3 & 0.5 & 0.1 & -0.4 & -0.6 \\ -0.8 & -0.1 & 0.4 & 0.9 & -0.5 & -0.1 & -0.4 & -1.0 \\ -0.2 & 0.0 & 0.5 & 0.1 & 0.9 & 0.5 & 0.0 & 0.0 \\ -0.3 & 0.5 & 0.8 & -0.3 & 1.0 & -0.7 & -0.2 & -0.4 \end{bmatrix}$$

Step 4: The total score of the above score matrix is given by

$$\begin{bmatrix} 1.6 \\ 0.3 \\ 1.2 \\ 1.8 \\ 1.1 \end{bmatrix}$$

Step 5: Now on the basis of the above total score values, the maximum value comes out to be **1.8** which is corresponding to the alternative x_4, i.e., Hydropower energy. Hence, the best suitable renewable energy source on the basis of the total score value obtained by the proposed algorithm will be hydropower energy. The comparative score values and their ranking can be observed in Figure 7.

Results and Discussion: In order to overcome the limitations of the algorithm I, we deploy algorithm II by merging the expert's opinion to have an optimal renewable energy source. Hence, the best suitable renewable energy source on the basis of the total score value obtained by the proposed algorithm will be hydropower energy. The comparative score values and their ranking can be observed in Figure 7.

7 Comparative Analysis and Advantages of Proposed Methods

In view of the literature available on various types of generalizations of fuzzy sets and information, we tabulate the comparative study in terms of advantages, applicability, and flexibility in Table 9 as follows:

In view of the numerical example under consideration and the results obtained through the existing techniques by various researchers, we present the Table 10 stating the ranking of the alternatives for the decision-making problem:

Important Remarks and Advantages:

- Finally, we are able to state that the proposed notion of picture fuzzy hypersoft matrix (PFHSM) is a novel concept and a valid extension of fuzzy set/hypersoft set theories. The PFHSM has the added advantage to deal with the wider sense of applicability in uncertain situations with the incorporation of the degree of refusal and abstain.
- The existing types of hypersoft sets - intuitionistic fuzzy hypersoft set [31], Pythagorean fuzzy hypersoft set [40], Neutrosophic hypersoft set [31] have their own limitations because of the exclusion of refusal and abstain component.
- The methodology implementing the proposed picture fuzzy hypersoft choice/weighted choice matrix, value matrix and score matrix can be well utilized for various group strategic MCDM models in a generalized framework effectively and consistently.
- For the sake of an overall critical aspect, we observe that eventually with the picture fuzzy information, it won't be possible to suitably address those membership values (given by the decision-makers/experts) whose sum exceeds one. Such restrictions in respect of decision-maker's opinion can be eradicated with the notion of T-spherical fuzzy information.

8 Conclusions & Scope for Future Work

The proposed novel notion of picture fuzzy hypersoft matrix has the capability to model the wider coverage of the inexact/imprecise information. Further, the proposed decision-making algorithms involve the choice matrix, weighted choice matrix, followed by value and score matrix which span the variability of the problem more mathematically. The main purpose of the presented manuscript lies in proposing new fuzzy decision-making methods for evaluating and ranking the available renewable energy sources based on different criteria. Consequently, we successfully illustrated and implemented the formal procedure for solving the problem of renewable energy source selection by utilizing PFHSCM, PFHSWCM, PFHSVM and PFHSTSM. Since the real world is full of uncertainty with various parameters and sub-parameters, the proposed methodologies exhibit the capability to simultaneously span a wider coverage of information in terms of multi-sub attribute features and comprehensiveness of the expert's opinion.

In the future, the proposed methods can further be given effective extension with suitable applications utilizing spherical/T-spherical [46], complex picture fuzzy sets/N-soft sets [47][48] and complex spherical fuzzy/N-soft sets [49] [50]. Further, on the basis of the proposed picture fuzzy hypersoft matrix, new kind of Dombi/Bonferroni aggregation operators may be discussed along with applications in the field of decision sciences [51] and performance evaluation of solar energy cells [52].

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Figure Captions

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- Figure 3: Extensions and Generalizations of Fuzzy Set
- Figure 4: MCDM with Choice/Weighted Choice Picture Fuzzy Hypersoft Matrix
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Table Captions

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- Table 2: Decision maker's opinion for Qualification
- Table 3: Decision maker's opinion for Experience
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Table 1: Tabular form of Z_v

	K_1^a	K_2^b	$\dots$	K_m^z
v^1	$\chi_{Z_v}(v^1, K_1^a)$	$\chi_{Z_v}\left(v^1, K_2^b\right)$	$\dots$	$\chi_{Z_v}(v^1, K_m^z)$
v^2	$\chi_{Z_v}(v^2, K_1^a)$	$\chi_{Z_v}\left(v^2, K_2^b\right)$	$\dots$	$\chi_{Z_v}(v^2, K_m^z)$
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
v^n	$\chi_{Z_v}(v^n, K_1^a)$	$\chi_{Z_v}\left(v^n, K_2^b\right)$	$\dots$	$\chi_{Z_v}(v^n, K_m^z)$

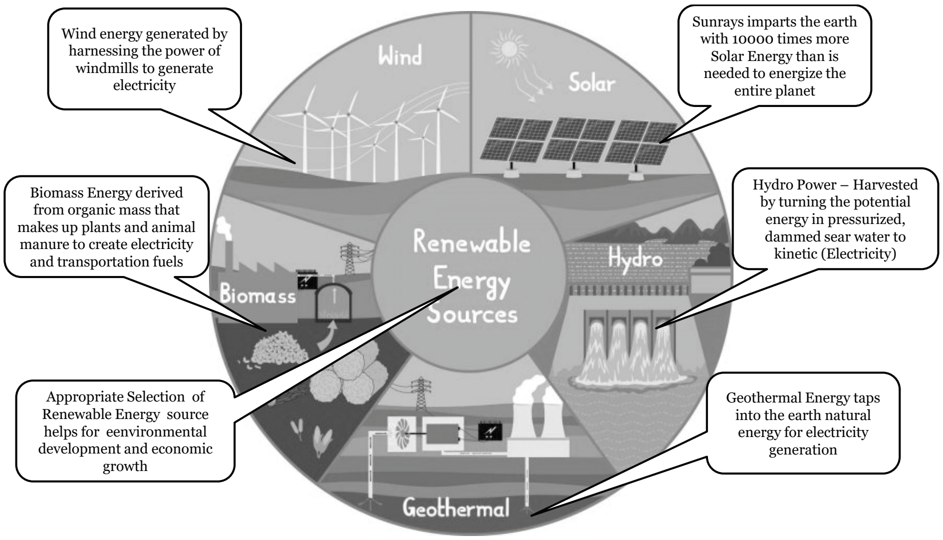

Figure 1: Renewable Energy Sources

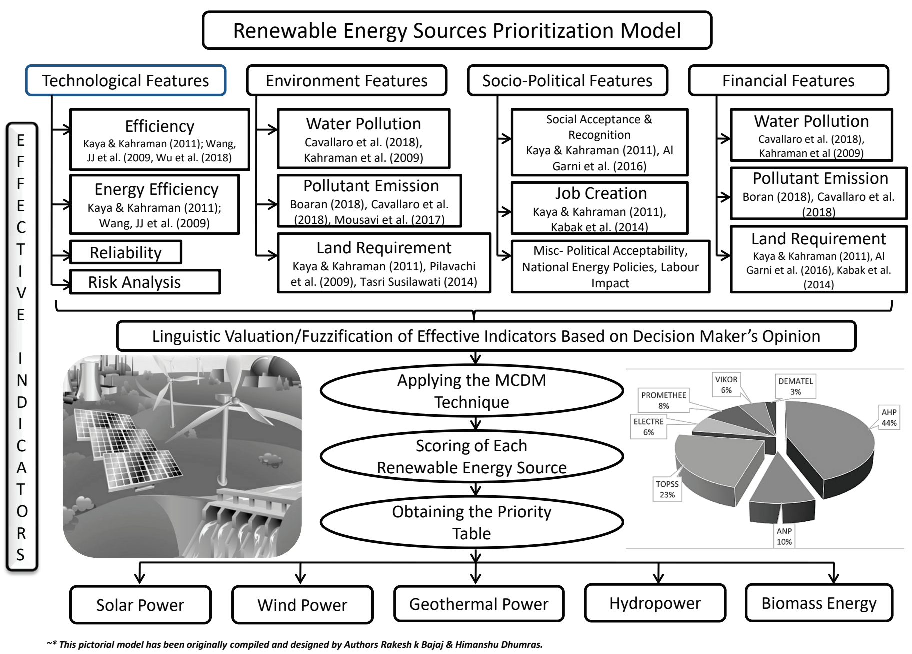

Figure 2: Role of Effective Indicators in RESs Prioritization Model

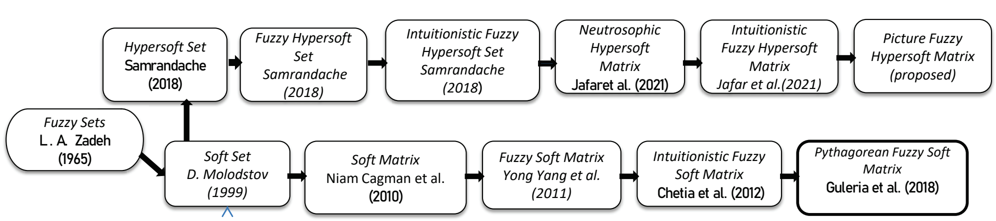

Figure 3: Extensions and Generalizations of Fuzzy Set

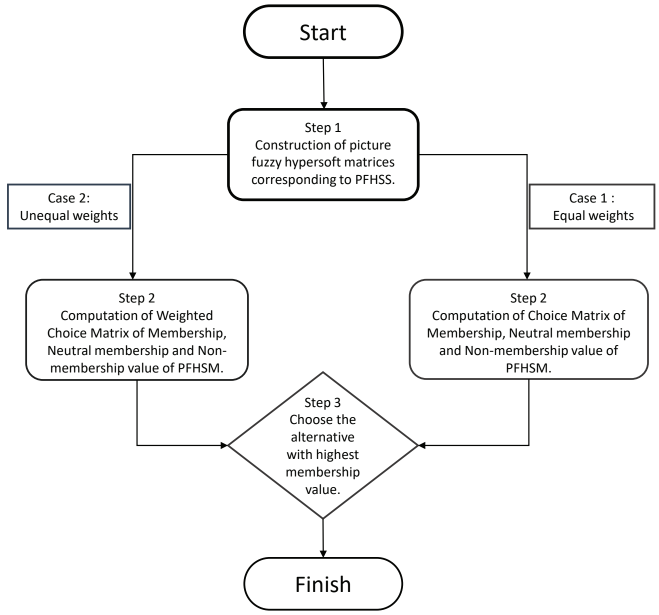

Figure 4: MCDM with Choice/Weighted Choice Picture Fuzzy Hypersoft Matrix

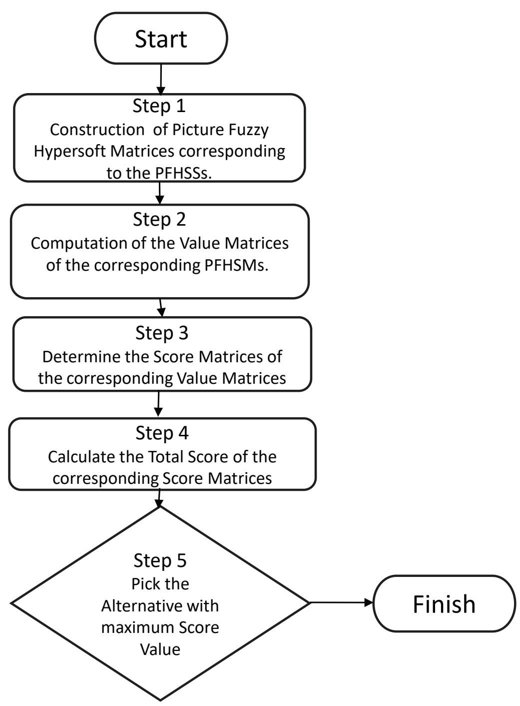

Figure 5: MCDM with Value/Score Picture Fuzzy Hypersoft Matrix

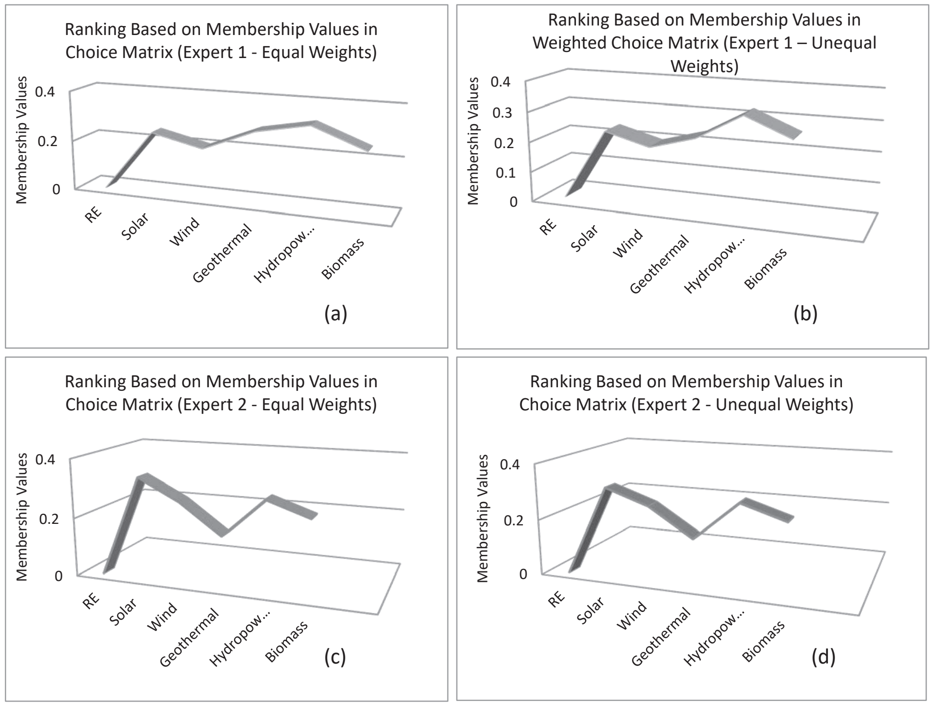

Figure 6: Ranking of Renewable Energy Sources Based on Choice Matrix

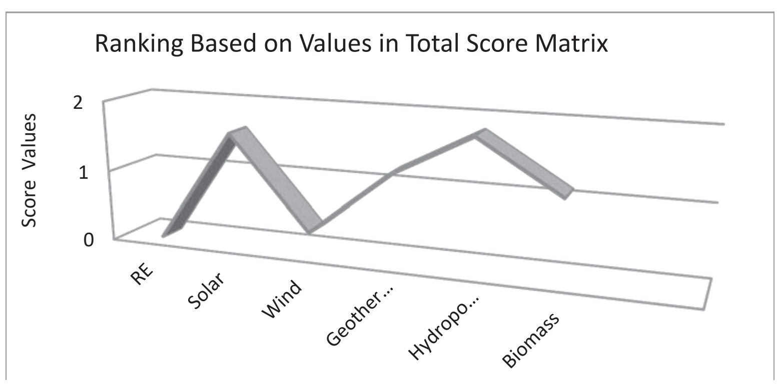

Figure 7: Ranking of Renewable Energy Sources Based on Score Matrix

Table 2: Decision maker's opinion for Qualification

K_1^a (**Qualification**)	v^1	v^2	v^3	v^4	v^5
BS Hons.	$(0.2, 0.3, 0.4)$	$(0.1, 0.3, 0.5)$	$(0.3, 0.5, 0.1)$	$(0.3, 0.2, 0.1)$	$(0.4, 0.1, 0.2)$
MS	$(0.1, 0.2, 0.4)$	$(0.4, 0.2, 0.3)$	$(0.3, 0.5, 0.1)$	$(0.1, 0.1, 0.5)$	$(0.1, 0.3, 0.4)$
M.Phil.	$(0.1, 0.3, 0.5)$	$(0.1, 0.3, 0.5)$	$(0.1, 0.3, 0.5)$	$(0.1, 0.3, 0.5)$	$(0.1, 0.3, 0.5)$
Ph.D.	$(0.3, 0.1, 0.5)$	$(0.4, 0.3, 0.1)$	$(0.2, 0.5, 0.1)$	$(0.2, 0.1, 0.5)$	$(0.3, 0.1, 0.5)$

Table 3: Decision maker's opinion for Experience

K_2^b (**Experience**)	v^1	v^2	v^3	v^4	v^5
$3yr$	$(0.4, 0.3, 0.1)$	$(0.2, 0.3, 0.5)$	$(0.2, 0.2, 0.3)$	$(0.5, 0.1, 0.2)$	$(0.2, 0.3, 0.5)$
$5yr$	$(0.1, 0.3, 0.5)$	$(0.3, 0.3, 0.3)$	$(0.3, 0.3, 0.3)$	$(0.2, 0.4, 0.2)$	$(0.7, 0.1, 0.1)$
$7yr$	$(0.1, 0.7, 0.1)$	$(0.4, 0.3, 0.2)$	$(0.1, 0.3, 0.5)$	$(0.2, 0.2, 0.5)$	$(0.2, 0.5, 0.2)$
$10yr$	$(0.3, 0.5, 0.1)$	$(0.2, 0.4, 0.3)$	$(0.3, 0.3, 0.2)$	$(0.1, 0.3, 0.6)$	$(0.6, 0.3, 0.1)$

Table 4: Decision maker's opinion for Age

K_3^c (**Age**)	v^1	v^2	v^3	v^4	v^5
$Less\ than\ twentyfive$	$(0.2, 0.1, 0.5)$	$(0.3, 0.3, 0.3)$	$(0.5, 0.2, 0.1)$	$(0.6, 0.2, 0.1)$	$(0.2, 0.3, 0.4)$
$Greater\ than\ twentyfive$	$(0.5, 0.3, 0.1)$	$(0.4, 0.3, 0.1)$	$(0.2, 0.4, 0.3)$	$(0.5, 0.2, 0.2)$	$(0.4, 0.3, 0.1)$

Table 5: Decision maker's opinion for Gender

K_4^d (**Gender**)	v^1	v^2	v^3	v^4	v^5
$Male$	$(0.2, 0.1, 0.2)$	$(0.3, 0.2, 0.3)$	$(0.1, 0.2, 0.6)$	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.1)$
$Female$	$(0.2, 0.1, 0.5)$	$(0.4, 0.3, 0.1)$	$(0.3, 0.5, 0.1)$	$(0.2, 0.2, 0.2)$	$(0.4, 0.3, 0.1)$

Table 6: Tabular form of the above relation

	K_1^a	K_2^b	K_3^c	K_4^d
v^1	$(MS(0.1, 0.2, 0.4))$	$(7yr(0.1, 0.7, 0.1))$	(Greater than twenty five(0.5,0.3,0.1))	$(Male(0.2, 0.1, 0.2))$
v^2	$(MS(0.4, 0.2, 0.3))$	$(7yr(0.4, 0.3, 0.2))$	(Greater than twenty five(0.4,0.3,0.1))	$(Male(0.3, 0.2, 0.3))$
v^3	$(MS(0.3, 0.5, 0.1))$	$(7yr(0.1, 0.3, 0.5))$	(Greater than twenty five(0.2,0.4,0.3))	$(Male(0.1, 0.2, 0.6))$
v^5	$(MS(0.1, 0.3, 0.4))$	$(7yr(0.2, 0.5, 0.2))$	(Greater than twenty five(0.4,0.3,0.1))	$(Male(0.5, 0.2, 0.1))$

Table 7: Decision Matrix given by First Expert

	$z_1^{'}$	$z_2^{'}$	$z_3^{'}$	$z_4^{'}$	$z_5^{'}$	$z_6^{'}$	$z_7^{'}$	$z_8^{'}$
x_1	(0.1,0.1,0.8)	(0.6,0.1,0.2)	(0.4,0.1,0.4)	(0.7,0.1,0.2)	(0.6,0.2,0.1)	(0.7,0.0,0.3)	(0.3,0.2,0.4)	(0.1,0.2,0.3)
x_2	(0.2,0.1,0.6)	(0.7,0.0,0.2)	(0.4,0.1,0.5)	(0.5,0.3,0.1)	(0.6,0.1,0.2)	(0.5,0.1,0.3)	(0.1,0.2,0.4)	(0.2,0.3,0.4)
x_3	(0.3,0.0,0.7)	(0.8,0.1,0.1)	(0.6,0.1,0.3)	(0.7,0.1,0.1)	(0.2,0.2,0.5)	(0.7,0.1,0.1)	(0.4,0.2,0.3)	(0.1,0.5,0.3)
x_4	(0.1,0.1,0.8)	(0.9,0.0,0.1)	(0.6,0.1,0.2)	(0.5,0.1,0.3)	(0.7,0.1,0.2)	(0.6,0.1,0.2)	(0.4,0.1,0.2)	(0.4,0.3,0.2)
x_5	(0.4,0.0,0.5)	(0.6,0.1,0.3)	(0.7,0.1,0.2)	(0.2,0.1,0.6)	(0.7,0.1,0.1)	(0.3,0.2,0.5)	(0.5,0.1,0.3)	(0.3,0.2,0.1)

Table 8: Decision Matrix given by Second Expert

	$z_1^{'}$	$z_2^{'}$	$z_3^{'}$	$z_4^{'}$	$z_5^{'}$	$z_6^{'}$	$z_7^{'}$	$z_8^{'}$
x_1	(0.8,0.1,0.1)	(0.6,0.2,0.2)	(0.4,0.2,0.4)	(0.7,0.0,0.2)	(0.6,0.1,0.1)	(0.7,0.0,0.3)	(0.4,0.2,0.3)	(0.3,0.2,0.1)
x_2	(0.6,0.1,0.2)	(0.7,0.1,0.2)	(0.4,0.0,0.5)	(0.5,0.2,0.1)	(0.6,0.2,0.2)	(0.5,0.2,0.3)	(0.4,0.2,0.1)	(0.4,0.3,0.2)
x_3	(0.3,0.0,0.7)	(0.1,0.0,0.8)	(0.6,0.1,0.3)	(0.7,0.2,0.1)	(0.5,0.3,0.2)	(0.1,0.0,0.7)	(0.3,0.2,0.4)	(0.3,0.5,0.1)
x_4	(0.8,0.1,0.1)	(0.1,0.0,0.9)	(0.6,0.2,0.2)	(0.5,0.2,0.3)	(0.7,0.0,0.2)	(0.6,0.2,0.2)	(0.4,0.3,0.2)	(0.4,0.1.0.2)
x_5	(0.4,0.1,0.5)	(0.6,0.0,0.3)	(0.7,0.1,0.2)	(0.6,0.2,0.2)	(0.7,0.1,0.1)	(0.3,0.1,0.5)	(0.3,0.1,0.5)	(0.1,0.2,0.3)

Table 9: Comparison of PFHSM with Some Existing Notions

	Zadeh[27]	Maji et al. [53]	Coung[39]	Coung[39]	Proposed Approach (PFHSM)
Categorization of Uncertainty Component	Fuzzy Set	Fuzzy Soft Set	Picture Fuzzy Set	Picture Fuzzy Soft Set	Proposed Picture Fuzzy Hypersoft Set/Matrices
Membership	✓	✓	✓	✓	✓
Non-membership	×	×	✓	✓	✓
Attributes	✓	✓	✓	✓	✓
Sub-Attributes	×	×	×	×	✓
Loss of Information	×	×	×	×	×
Parametrization	×	✓	×	✓	✓
Abstain & Refusal	×	×	✓	✓	✓
Advantages in terms of dealing factors of uncertainty	Uses Fuzzy Interval	Uses Fuzzy Soft Intervals	Utilize Membership, Non-membership, refusal and degree of abstain	Uses Parametrization of Attributes	Utilizing Parametrization of Multi sub-attributes with the Inclusion of Degree of Refusal and Abstain

Table 10: Comparative Analysis

Method	**Operators/Method Used**	**Developed Ranking**
Feng et al.[43]	Extended Intersection,IFWA	$x_4 > x_1 > x_3 > x_2 > x_5$
Khan et al.[44]	Soft Discernibility Matrix	$x_4 > x_3 > x_1 > x_2 > x_5$
Garg[54]	PFEWA Operator	$x_4 > x_3 > x_1 > x_2 > x_5$
Yager[55]	PFWA Operator	$x_4 > x_3 > x_1 > x_2 > x_5$
Yager[55]	PFWG Operator	$x_4 > x_1 > x_3 > x_2 > x_5$
Khan et al.[45]	VIKOR I	$x_4 > x_2 > x_3 > x_1 > x_5$
Khan et al.[45]	VIKOR II	x_4 , $x_2 > x_3 > x_1 > x_5$
Khan et al.[45]	VIKOR III	$x_4 > x_1 > x_3 > x_2 > x_5$
Khan et al.[45]	VIKOR IV	$x_4 > x_2 > x_3 > x_5 > x_1$
Proposed	PFHSCM (I Expert)	$x_4 > x_3 > x_1 > x_5 > x_2$
Proposed	PFHSWCM (I Expert)	$x_4 > x_5 > x_3 > x_1 > x_2$
Proposed	PFHSCM (II Expert)	$x_1 > x_4 > x_2 > x_5 > x_3$
Proposed	PFHSWCM (II Expert)	$x_1 > x_4 > x_2 > x_5 > x_3$
Proposed	PFHSVM & PFHSTSM	$x_4 > x_1 > x_3 > x_5 > x_2$