Evaluation of the functionality of mobile wireless sensor networks using stochastic reward nets

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Received 31 May 2021; received in revised form 21 June 2022; accepted 1 August 2022

KEYWORDS: Wireless Sensor Networks (WSNs); Mobile nodes; Energy harvesting; Analytical modeling; Stochastic Reward Nets (SRNs)

\begin{abstract}
In this paper, the functionality of Mobile Wireless Sensor Networks (MWSNs) is analytically modeled and evaluated using Stochastic Reward Nets (SRNs). In MWSNs, mobile nodes can move around to collect data from the environment and send them to the sink. These nodes use a limited battery as the power source which can be charged according to environmental conditions. If the battery does not have enough power, the mobile node is disabled and, therefore, it cannot move around to collect/send data. The data collected by the nodes will expire if they are not received by the sink in a timely manner. In order to avoid data expiration, mobile nodes can send their data to other nodes around themselves, but this also increases the power consumption because of further communication. Furthermore, moving faster in the environment, the power consumption of the nodes increases. Therefore, environmental and movement conditions as well as communication between nodes can have a major impact on the functionality of the MWSNs. These challenges are considered in the paper and the proposed models analyze the impact of different conditions on the functionality of MWSNs. The results obtained from the comparison of different scenarios demonstrate that the environmental and movement conditions have a greater impact on the system functionality than the communication conditions.
\end{abstract}

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1. Introduction

Various hardware and software factors can affect the functionality of Mobile Wireless Sensor Networks (MWSNs). One of the most important factors affecting the power and performance of these systems is environmental and movement conditions. In the MWSNs, mobile nodes use rechargeable batteries as the power source. These batteries can be charged according to environmental conditions. If the environmental conditions are not suitable, the battery of mobile nodes will discharge after a while, causing the nodes to be disabled. This degrades the performance of the nodes in doing their job, i.e., delivering data to the sink node. In order to prevent degradation of performance, mobile nodes can send their data to other nodes instead of delivering it to the sink, but this also increases the power consumption because they need to establish further communication. On the other hand, if the nodes want to move faster to collect more data and deliver them to the sink, to increase the performance, the power consumed by the nodes will increase. Therefore, analyzing the tradeoff among environmental, commu-
nication, and movement conditions is more important when evaluating the performance of mobile nodes in MWSNs.

Both the power consumption and performance of a system depend on the system functionality; hence, the unified power and performance modeling and evaluation can help us understand how various factors affect system functionality. In order to model the MWSNs and evaluate the impact of different environmental, communication, and movement conditions on the system functionality, the Stochastic Reward Nets (SRNs) [1] are used in this paper. SRNs are stochastic extensions of Petri Nets (PNs) that provide a powerful way for modeling complex systems. Various extensions of PNs have been used to model and evaluate different measures in various distributed computing systems [2-6]. The main contribution of this paper can be summarized in the five following categories:

- The SRN formalism is used to model environmental conditions, mobile and sink nodes, and communication between nodes in MWSNs;
- The movement, battery, and energy harvesting are considered in the models proposed for mobile nodes;
- The impact of different environmental, communication, and movement conditions on the system functionality is analyzed using the proposed SRN models;
- The proposed models are used to evaluate and compare different possible scenarios;
- It is proven that by comparing the results, the environmental and movement conditions have a greater impact on system functionality than the communication conditions.

The rest of this paper is organized as follows. Section 2 presents background information on the MWSNs systems, the SRN formalism, and its supporting tool, and SPNP [7,8]. Section 3 introduces the related state of the art in system formal modeling and evaluation. Section 4 presents the proposed SRN models and figures of merit, which can be computed by applying the proposed models. Section 5 compares the results obtained from different environmental, communication, and movement conditions by analytically solving the proposed SRN models. Finally, Section 6 concludes the paper and gives ideas for future work.

2. Background information

2.1. Description of the MWSNs and our assumptions

Wireless Sensor Networks (WSNs) consist of a large number of sensor nodes deployed to observe or measure a certain phenomenon or detect the occurrence of an event. Sensor nodes send their observations to a base station, such as sink node for further processing [9]. In MWSNs, mobile nodes are equipped with locomotive platforms that allow movement after initial deployment [9]. The movement speed of mobile nodes can affect their power consumption and performance. In accordance with the definitions provided by [9,10], Figure 1 represents the structure of an MWSN considered in this paper, which consists of a sink node and several mobile sensor nodes in different layers. We assume that the geographic environment consists of L layers and each layer, except the layer zero, has data that should be collected by mobile nodes. The sink node is in the layer zero and receives data collected by mobile nodes. Mobile nodes move in up and down directions and collect data from the geographic environment in different layers. They can send their data to the sink only when they are in the layer zero. When a mobile node reaches zero and L layers, its direction changes to up and down, respectively.

We assume that each mobile node has a battery that can be in three statuses: low, medium, and high. The status of the battery affects the node functionality. In case of battery low status, the mobile node is disabled and cannot move around and send data. The batteries can be charged at different rates depending on environmental conditions. We assume that environmental conditions can be at three levels, such as suitable, moderate, and unsuitable. For example, if the batteries are charged based on sunlight, sunny and cloudy days are suitable and moderate environmental conditions, respectively. Moreover, nights are unsuitable environmental conditions. Since data collected by mobile nodes and data received by the sink are expired after a specified time period, mobile nodes can send their data to other nodes around themselves. For example, if the i-th node is on the j-th layer, it can send its data to nearby nodes that are on the way to the sink in the same layer. We assume that power consumption of active mobile nodes

![Figure 1. The MWSNs considered in this paper.](image-url)
increases when they try to communicate with other mobile nodes. Moreover, it is assumed that mobile nodes move at different speeds, affecting their power consumption.

2.2. The SRN formalism and SPNP tool

PNs are an important mathematical formalism to model real systems and evaluate their performance [11]. The advantages of this formalism are simple definition, graphical representation, availability of simulation and analytical solutions, and being supported by various computer tools. The places, transitions, and tokens are three basic elements of the PN formalism [11]. Places are represented by circles and used to model the state variables of a system. Each place can contain zero or several tokens indicating the value of the state variable modeled by the place. Transitions are represented by rectangles and used to model the events/actions that change the values of the state variables. The transitions are divided into two types, namely timed and immediate, which are represented by the hollow and solid rectangles, respectively. There is no waiting time associated with immediate transitions and they fire as soon as they are enabled, while the time taken by timed transitions to fire follows an exponential distribution function.

Stochastic Reward Nets (SRNs) are an important extension of PNs providing a powerful modeling environment to analyze the performance, dependability, and performability of systems [1]. In the SRN formalism, the transitions can have a guard function, a Boolean function, to examine whether a transition is enabled or not. An arc can be marking dependent to specify the number of tokens that should be manipulated in the places during firing of its corresponding transition [7]. The Stochastic Petri Net Package (SPNP) is a well-known tool supporting the SRN formalism to compute a variety of transient, steady-state, cumulative, and sensitivity measures [8]. In this paper, the proposed SRN models are constructed and analyzed by the SPNP tool in the steady state. In order to analytically solve the models and compute measures in the steady state, it is assumed that the transition times of the proposed SRNs follow exponential distributions. The same assumption has been made in related works to model various events in real-world systems, e.g. [2-6].

3. Related work

Some approaches have been proposed to investigate energy harvesting in the MWSNs. These approaches introduce various energy sources, technologies, and challenges that must be addressed to develop reliable energy harvesting systems. Furthermore, there have been other proposed approaches that model MWSNs and evaluate the impact of various factors on power and performance measures, but most of them usually ignore the impact of one of the environmental, communication, and movement conditions on power consumption and system functionality. Table 1 compares our proposed approach against the state-of-the-art approaches.

<table>
<thead>
<tr>
<th>Table 1. Comparison between previous methods and the models presented in this paper.</th>
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<tbody>
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<td>Ref.</td>
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<tr>
<td>The proposed models</td>
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</table>
In [12], different types of wireless smart sensors and communication technologies were reviewed. Sensors must consume less power and reduce the noise produced by wireless communication. In [13], a framework was introduced to evaluate efficient, timely, reliable, and ubiquitous inference of information obtained by nodes in intelligent WSNs. In [14], two mathematical models were introduced to maximize the network lifetime through the controlled mobility of a sink with nonzero travel times without considering energy harvesting and environmental conditions. In [15], the sensor deployment problem was modeled to find an optimal sensor relocation to optimize the sensing quality, energy consumption, and connectivity. However, the impact of energy harvesting on it was not been considered. In [16], a probabilistic algorithm to analyze the network connectivity by taking into account the parameters like network probability, detection area, radius of the individual nodes, and whole detection area was proposed to ensure the network connectivity and communication sustainability. This algorithm optimizes the energy consumption at the utmost level. In [17], by using fuzzy PNs, an energy harvesting aware algorithm for selecting cluster heads was introduced to keep the WSN alive for much longer time. However, its impact on the functionality of WSNs was not considered. In [18], a sensor node with energy harvesting capability was modeled with a new generalized stochastic PN to determine performance parameters and to predict the energy consumption of a sensor node. In this model, a sensor node can be in different modes, such as active, listening, and standby, and it analyzes the impact of the harvesting and sleeping rates on the mean battery charge and mean response time. However, this model ignores the impact of movement conditions. In [19], a PN-based modeling framework was presented for the evaluation of sensing, communication, and movement factors that contribute to energy dissipation in mobile wireless sensor nodes. The model provided by this framework ignores the system functionality. In [20], an approach was proposed for evaluating the WSN lifetime using a set of reusable colored PN models that expressed the power consumption of communication protocols. This approach only evaluates the impact of communication protocols on power consumption of WSN nodes and ignores the impact of environmental and movement conditions. In [21], a methodology that consists of a set of power consumption and reliability models was presented to estimate the power consumption of WSN applications. This methodology also ignores the impact of energy harvesting and movement conditions. In [22, 23], the road traffic flows were considered as WSNs, and PNs were used to model the transportation system and the movement of each vehicle. In [24], the ways to increase the network lifetime, while satisfying the full target coverage in the WSNs, consisting of both non-rechargeable and rechargeable sensors, were studied and, then, a lifetime-enhancing method for this purpose was proposed. In [25], the WSNs were modeled using the colored PNs in order to capture the effect of data aggregation on the overall performance.

4. The proposed SRN models

In this section, various parts of an MWSN, such as mobile and sink nodes, environmental conditions, and communication between nodes are modeled first using the SRN formalism. Then, measures of interest on the proposed models are defined.

4.1 Modeling environmental conditions

As mentioned in Section 2.1, we assume three levels for environmental conditions called suitable, moderate, and unsuitable. Figure 2 represents the proposed SRN for different environmental conditions. The number of tokens inside place Weather which can be equal to one, two, and three represents the unsuitable, moderate, and suitable levels for the environment to charge the battery of mobile nodes, respectively.

The timed transitions $T\_U2S$ and $T\_S2U$ model how environmental conditions change from unsuitable to suitable and from suitable to unsuitable, respectively. These timed transitions follow exponential distributions with rates $\alpha$ and $\beta$, and guard functions $G1$ and $G2$ listed in Table 2 are assigned to them, respectively. The guard function $G1$ ($G2$) checks the number of tokens inside place Weather. If it is smaller (greater) than three (one), function $G1$ ($G2$) returns true to make the timed transition $T\_U2S$ ($T\_S2U$) enabled; otherwise, it returns false to prevent the transition from firing. When timed transition $T\_U2S$ ($T\_S2U$) fires, a token is deposited into (removed from) place Weather.

![Figure 2. The SRN model proposed for environmental conditions.](image)

Table 2. Guard functions assigned to transitions of the SRN represented in Figure 2 modeling the environmental conditions.

<table>
<thead>
<tr>
<th>Guard</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G1$</td>
<td>$mark(Weather) &lt; 3$</td>
</tr>
<tr>
<td>$G2$</td>
<td>$mark(Weather) &gt; 1$</td>
</tr>
</tbody>
</table>
4.2. Modeling the sink node of the MWSNs

As mentioned in Section 2.1, the geographic environment consists of L layers, and each layer, except the layer zero, has data that must be collected by mobile nodes. The sink node receives data collected by mobile nodes in the layer zero. Figure 3 shows the proposed SRN model for the sink node. The place SN(i)_Layer(j), 1 ≤ i ≤ N, 1 ≤ j ≤ L represents the existence of the jth data of the ith node in the sink node. If the number of tokens inside place SN(i)_Layer(j) is equal to one, the jth data of the ith node is available at the sink node; otherwise, if it is equal to zero, the jth data of the ith node is not available at the sink node due to failure to receive or expire.

The timed transition T_Expire_SN(i)_L(j), 1 ≤ i ≤ N, 1 ≤ j ≤ L models the expiration of the jth data of the ith node in the sink node, with rate π. If the number of tokens inside place SN(i)_Layer(j) is equal to one, timed transition T_Expire_SN(i)_L(j) is enabled and when it fires, a token is removed from place SN(i)_Layer(j).

The timed transition T_Send_N(i)2Sink, 1 ≤ i ≤ N sends all data of the ith node to the sink node with rate θ. According to Table 3, the guard function G3 assigned to timed transition T_Send_N(i)2Sink checks whether the ith node is in the layer zero or not. If it is in the layer zero, guard G3 returns true to make transition T_Send_N(i)2Sink enabled; otherwise, it returns false to prevent the transition from firing. When timed transition T_Send_N(i)2Sink fires, #SN(i)L(j), 1 ≤ i ≤ N, 1 ≤ j ≤ L token is deposited into places SN(i)_Layer(j). If the jth data of the ith node is available at the sink node and not available at the sink node, the number #SN(i)L(j) is equal to one; otherwise, it is zero.

4.3. Modeling a mobile node of the MWSNs

As mentioned in Section 2.1, mobile nodes use batteries as the power source and they can move in up and down directions and collect data from the geographic environment in different layers except the layer zero. Figure 4 represents the proposed SRN model for a
mobile node. The number of tokens inside place \( Battery_N(i) \) that can be equal to one, two, and three represents the low, medium, and high statuses of the battery of the \( i \)th mobile node, respectively. Furthermore, the number of tokens inside place \( Direction_N(i) \) which can be equal to one and two represents the down and up directions of the \( i \)th mobile node, respectively. The place \( N(i)_{Layer}(j) \), \( 1 \leq i \leq N, 1 \leq j \leq L \) represents the existence of the \( j \)th data in the \( i \)th node. If the number of tokens inside this place is two, it shows that the \( i \)th node is in the \( j \)th layer and the \( j \)th data is being collected by it. If this number is one, the \( j \)th data is available at the \( i \)th node and if it is zero, the \( j \)th data is not available at the \( i \)th node due to expiration.

The timed transitions \( T_{\text{Charge}}N(i) \) and \( T_{\text{Consume}}N(i) \) model how to charge and consume the battery of the \( i \)th mobile node, respectively. These timed transitions follow an exponential distribution with rates shown in Eqs. (1) and (2), respectively. Two guard functions \( G4 \) and \( G5 \), listed in Table 4, are assigned to transitions \( T_{\text{Charge}}N(i) \) and \( T_{\text{Consume}}N(i) \), respectively.

\[
(\text{mark}(\text{Weather}) - 1) \times \omega.
\]  
(1)

\[
\text{Communication done?} (\omega + 1) : \omega.
\]  
(2)

According to Eq. (1), when environmental condition is suitable, i.e., \( \text{mark}(\text{Weather}) = 3 \), the batteries are charged at twice the rate \( \omega \). If environmental conditions are moderate, i.e., \( \text{mark}(\text{Weather}) = 2 \), the batteries are charged with rate \( \omega \) and finally, if environmental conditions are unsuitable, i.e., \( \text{mark}(\text{Weather}) = 1 \), the batteries are not charged. According to Eq. (2), if the mobile nodes communicate with each other, the

**Figure 4.** The SRN model proposed for a mobile node.
Table 4. Functions assigned to transitions and arcs of the SRN represented in Figure 4 modeling the mobile node.

<table>
<thead>
<tr>
<th>Guard/arc</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>G4</td>
<td>mark (Battery_N(i)) &lt; 3 &amp; mark (Weather) &gt; 1</td>
</tr>
<tr>
<td>G5</td>
<td>mark (Battery_N(i)) &gt; 1</td>
</tr>
<tr>
<td>G6</td>
<td>mark (N(i)_Layer (0)) == 2 &amp; mark (Direction_N(i)) == 1</td>
</tr>
<tr>
<td>G7</td>
<td>mark (N(i)_Layer (L)) == 2 &amp; mark (Direction_N(i)) == 2</td>
</tr>
<tr>
<td>G8</td>
<td>mark (Battery_N(i)) &gt; 1 &amp; mark (N(i)_Layer (j)) == 2 &amp; mark (Direction_N(i)) == 2</td>
</tr>
<tr>
<td>G9</td>
<td>mark (Battery_N(i)) &gt; 1 &amp; mark (N(i)_Layer (j)) == 2 &amp; mark (Direction_N(i)) == 1</td>
</tr>
<tr>
<td>G10</td>
<td>mark (N(i)_Layer (j)) == 1</td>
</tr>
</tbody>
</table>

\[
\text{if}(\text{mark}(\text{N}(i)_\text{Layer}(j+1)) == 0) \\
\text{\#N}(i)_L(j)L(j+1) \\
\text{\#N}(i)_L(j)L(j-1) \text{ return } 2 \\
\text{else} \\
\text{\#N}(i)_L(j)L(j+1) \text{ return } 2 \\
\text{else} \\
\text{\#N}(i)_L(j)L(j-1) \text{ return } 2 \\
\]

batteries are consumed with the rate ($\omega + 1$); otherwise, they are consumed with the rate $\omega$. The guard function G4 checks the situation that the battery is not high and environmental conditions are not unsuitable. If it is, function G4 returns true to make timed transition $\text{T-Charge}_N(i)$ enabled; otherwise, it returns false to prevent the transition from firing. When timed transition $\text{T-Charge}_N(i)$ fires, a token is deposited into place $\text{Battery}_N(i)$. The guard function G5 checks if the battery is not low. If it is evaluated to true, timed transition $\text{T-Consum}_N(i)$ is enabled; otherwise, it cannot fire. When timed transition $\text{T-Consum}_N(i)$ fires, a token is removed from place $\text{Battery}_N(i)$.

The immediate transition $\text{I-UpDir}_N(i)$ and $\text{I-DownDir}_N(i)$ model how the direction of the $i$th mobile node changes from down to up and from up to down, respectively. The guard functions G6 and G7 listed in Table 4 are assigned to transitions $\text{I-UpDir}_N(i)$ and $\text{I-DownDir}_N(i)$, respectively. According to guard function G6 (G7), if the mobile node is in the zero (Lth) layer and its direction is down (up), immediate transition $\text{I-UpDir}_N(i)$ ($\text{I-DownDir}_N(i)$) is enabled. When immediate transition $\text{I-UpDir}_N(i)$ ($\text{I-DownDir}_N(i)$) fires, a token is deposited into (removed from) place $\text{Direction}_N(i)$ ($\text{Direction}_N(i)$).

The timed transition $\text{T-Expire}_N(i)_L(j)$, $1 \leq i \leq N$, $1 \leq j \leq L$ expires the $j$th data in the $i$th node with rate $\pi$. According to the guard function G10, if the number of tokens inside place $\text{N}(i)_\text{Layer}(j)$ is equal to one, timed transition $\text{T-Expire}_N(i)_L(j)$ is enabled and when it fires, a token is removed from place $\text{N}(i)_\text{Layer}(j)$.

The timed transitions $\text{T-Move}_N(i)_L(j)2L(j+1)$ and $\text{T-Move}_N(i)_L(j)L(j-1)$, $1 \leq i \leq N$, $1 \leq j \leq L$ model upward and downward movements of the $i$th node in different layers, respectively. These timed transitions follow an exponential distribution with rate $\mu$, and guard functions G8 and G9, listed in Table 4, are assigned to them, respectively. The guard function G8 (G9) checks whether the battery is not low, the $i$th node is in the $j$th layer and the direction of the $i$th node is up (down). If it is, function G8 (G9) returns true to make timed transition $\text{T-Move}_N(i)_L(j)2L(j+1)$ ($\text{T-Move}_N(i)_L(j)L(j-1)$) enabled; otherwise, it returns false to prevent the transition from firing.

When timed transition $\text{T-Move}_N(i)_L(j)2L(j+1)$ fires, a token is removed from place $\text{N}(i)_\text{Layer}(j)$ and $\#\text{N}(i)_L(j)L(j+1)$, $1 \leq i \leq N$, $1 \leq j \leq L$ tokens are deposited into places $\text{N}(i)_\text{Layer}(j+1)$. If the $j + 1$th data is not available at the $i$th node, the number $\#\text{N}(i)_L(j)L(j+1)$ is equal to two, otherwise, it is equal to one. Similarly, when timed transition $\text{T-Move}_N(i)_L(j)L(j-1)$ fires, a token is removed from place $\text{N}(i)_\text{Layer}(j)$ and the $\#\text{N}(i)_L(j)L(j-1)$, $1 \leq i \leq N$, $1 \leq j \leq L$ token is deposited into places...
$N(i)_\text{Layer}(j - 1)$. If the $(j - 1)$th data is not available at the $i$th node, the number $\#N(i)L(j)L(j - 1)$ is equal to two; otherwise, it is equal to one.

4.4 Modeling communication between mobile nodes

As mentioned in Section 2.1, mobile nodes can send their data to other nodes around themselves. For example, if the $i$th node is in the $j$th layer, it can send its data to the $(i + 1)$th and $(i - 1)$th nodes which are on the way to the sink in the same layer. Figure 5 represents the proposed SRN model to ensure a communication between mobile nodes. The places $Data_N(i)N(i - 1)$ and $Data_N(i)N(i + 1)$. $1 \leq i \leq N$ represent the existence of the data of the $(i - 1)$th and $(i + 1)$th nodes in the $i$th node, respectively. If the number of tokens inside place $Data_N(i)N(i - 1)$ $(Data_N(i)N(i + 1))$ is equal to one, all data of the $(i - 1)$th $(i + 1)$th node is available at the $i$th node; otherwise, if it is equal to zero, the data is not available at the $i$th node.

The timed transitions $T_{Send-N}(i)N(i - 1)$ and $T_{Send-N}(i)N(i + 1)$. $1 \leq i \leq N$ send the data of the $i$th node to the $(i - 1)$th and $(i + 1)$th nodes, respectively. These timed transitions follow an exponential distribution with rate $\theta$, and guard functions $G11$ and $G12$ listed in Table 5, are assigned to them, respectively. In case the data of the $i$th node are

<table>
<thead>
<tr>
<th>Guard/arc</th>
<th>Function</th>
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</table>
| $G11$     | $mark(Data_N(i)N(i - 1)) == 0 &$<br>$mark(Battery_N(i)) > 1 & mark(Battery_N(i - 1)) > 1 &$<br>($\exists_{1 \leq j \leq L}$) $mark(N(i)_{Layer}(j)) == mark(N(i - 1)_{Layer}(j)) == 2)$<br>$mark(Data_N(i)N(i + 1)) == 0 &$<br>$mark(Battery_N(i)) > 1 & mark(Battery_N(i + 1)) > 1 &$<br>($\exists_{1 \leq j \leq L}$) $mark(N(i)_{Layer}(j)) == mark(N(i + 1)_{Layer}(j)) == 2)$<br>$mark(Data_N(i)N(i - 1)) > 1 & mark(Battery_N(i - 1)) == 1 &$<br>($\exists_{1 \leq j \leq L}$) $mark(N(i)_{Layer}(j)) == mark(N(i - 1)_{Layer}(j)) == 2)$<br>$mark(Data_N(i)N(i + 1)) > 1 & mark(Battery_N(i + 1)) == 1 &$<br>($\exists_{1 \leq j \leq L}$) $mark(N(i)_{Layer}(j)) == mark(N(i + 1)_{Layer}(j)) == 2)$

$G12$
not available at the \((i - 1)\)th \((i + 1)\)th node, the
battery of the \((i - 1)\)th \((i + 1)\)th and \(i\)th nodes is
not low and the direction of the \((i - 1)\)th \((i + 1)\)th
and \(i\)th nodes is down and up, respectively, and they
will be in the same layer. Function \(G11\) \((G12)\) returns
true to make timed transition \(T_{Send\_N(i)}(i - 1)\)
\((T_{Send\_N(i)}(i + 1))\) enabled; otherwise, it returns
false to prevent the transition from firing. When timed
transition \(T_{Send\_N(i)}(i - 1)\) \((T_{Send\_N(i)}(i + 1))\) fires, a token is deposited into place \(Data\_N(i - 1)\) \((Data\_N(i + 1))\).

The timed transitions \(T_{\_Expire\_N(i)}(i - 1)\) and
\(T_{\_Expire\_N(i)}(i + 1)\), \(1 \leq i \leq N\) expire the data
of the \((i - 1)\)th and \((i + 1)\)th nodes in the \(i\)th node
with rate \(\pi\), respectively. If the number of tokens inside
place \(Data\_N(i)\) \((i - 1)\) is one, then, timed transition
\(T_{\_Expire\_N(i)}(i - 1)\) is enabled and empties this
place upon firing. The same happens for transition
\(T_{\_Expire\_N(i)}(i + 1)\) and place \(Data\_N(i)\) \((i + 1)\).

As mentioned in Section 4.2, the timed transition
\(T_{\_Send\_N(i)\_2Sink}\), \(1 \leq i \leq N\) sends all data of the
\(i\)th node to the sink node with rate \(\theta\). If the mobile
nodes can communicate with each other, this transition also
sends all data of the \((i - 1)\)th and \((i + 1)\)th nodes to the
sink node. Therefore, when timed transition \(T_{\_Send\_N(i)\_2Sink}\) fires, the \#\(SN(i - 1)L(j)\) and
\#\(SN(i + 1)L(j)\), \(1 \leq i \leq N\), \(1 \leq j \leq L\) tokens
are deposited into places \(SN(i - 1)_Layer(j)\) and
\(SN(i + 1)_Layer(j)\), respectively. If the \(j\)th data of the
\((i - 1)\)th \((i + 1)\)th node is available at the \(i\)th
node and is not available at the sink node, the number
\#\(SN(i - 1)L(j)\) \((\#\(SN(i + 1)L(j)\)) is equal to one;
otherwise, it is zero.

4.5. Interesting measures

In order to assess the functionality of the MWSNs,
we define some reward functions in the proposed SRN
models. The expected cumulative rewards can be
computed as \(\sum \rho_i \times \pi_i\), where \(\rho_i\) is a reward assigned to
marking \(i\) of the SRN model and \(\pi_i\) is the probability
of being in marking \(i\) in the steady state [2,3]. The
interesting measures in the proposed SRN models are
as follows:

- **Average status of the battery of each mobile node** \((AB\_Node(i))\). As mentioned earlier, the
battery of each mobile node can be in three statuses,
such as low, medium, and high. Since the number of
tokens inside place \(Battery\_N(i)\) represents three
statuses of the battery of the \(i\)th mobile node,
we compute the measure \(AB\_Node(i)\) by defining
reward function in Eq. (3):

\[
AB\_Node(i) = \sum_{k=1}^{3} \pi(\text{mark}(Battery\_N(i)) == k) \times k. \quad (3)
\]

- **Availability of environment data at each mobile node** \((AD\_j\_Node(i))\). As mentioned
earlier, the geographic environment consists of
the \(L\) layers and each layer except the layer
zero has data that should be collected by mobile
nodes. Furthermore, the data collected by each
mobile node are expired after a while. Since the number
of tokens inside place \(N(i)\_Layer(j)\) represents the
existence of the \(j\)th data in the \(i\)th node, we compute
the measure \(AD\_j\_Node(i)\) by defining
reward function in Eq. (4):

\[
AD\_j\_Node(i) = \pi(\text{mark}(N(i)_Layer(j)) == 1) + \pi(\text{mark}(N(i)_Layer(j)) == 2). \quad (4)
\]

- **Availability of environment data at the sink node** \(AN(i)_D(j)\_Sink\). As mentioned earlier,
the sink node is in the layer zero and receives
data collected by mobile nodes. Furthermore,
the data received by the sink node are expired after
a while. Since the number of tokens inside place
\(SN(i)_Layer(j)\) represents the existence of the \(j\)th
data of the \(i\)th node in the sink node, we compute
the measure \(AN(i)_D(j)\_Sink\) by defining reward
function in Eq. (5):

\[
AN(i)_D(j)\_Sink = \pi(\text{mark}(SN(i)_Layer(j)) == 1). \quad (5)
\]

5. Numerical results

In this section, the results obtained by numerically
solving the proposed SRN models in the steady state
using the SPNP tool [8] are discussed in three parts.
In Section 5.1, we compare the results obtained from
six different environmental conditions regardless of
the communication between mobile nodes. In Section 5.2,
the results of the previous sub-section are compared
against the results that consider communication
between mobile nodes. In Section 5.3, new movement
conditions are defined and the results of the previous
sub-sections, following the same movement conditions,
are compared to the results of new movement conditions.

5.1. Comparison of different environmental conditions

Herein, we define six different environmental conditions
and evaluate their impact on interesting measures.
These conditions are listed below:

- **Suitable**. The environmental conditions are always
suitable;

- **Moderate**. The environmental conditions are always
moderate;

- **Unsuitable**. The environmental conditions are always
unsuitable;
• $\alpha = \beta$. The occurrence rates of suitable and unsuitable environmental conditions are equal;

• $\alpha = 2 \times \beta$. The occurrence rate of suitable environmental conditions is twice that of unsuitable environmental conditions;

• $\alpha = \frac{1}{2} \times \beta$. The occurrence rate of suitable environmental conditions is half that of unsuitable environmental conditions.

Table 6 provides the value of input parameters used in all of the following experiments. Furthermore, Table 7 represents the results obtained from the proposed SRN models in different environmental conditions regardless of the communication between mobile nodes.

As can be seen in Table 7, the environmental conditions can affect the battery status and availability of environmental data at each mobile node. In suitable environmental conditions, the battery of mobile nodes is lower, making them more active. This makes it less likely that the data collected by the mobile nodes will expire, because the nodes arrive at the desired layers in time. Furthermore, the sink node receives more data from mobile nodes in suitable environmental conditions. The opposite of the above is true for unsuitable environmental conditions.

As can be seen in Table 7, the data of middle layers is more up-to-date at mobile nodes, because the nodes reach those layers in a shorter amount of time. However, the data of the first layers are more up-to-date at the sink node, because they are sent to the sink sooner after collection.

5.2. Comparison of different communication conditions

The previous section evaluates the impact of six different environmental conditions on the system functionality without considering the communication between mobile nodes. In this section, the results in the previous section are compared to those that consider communication between mobile nodes. Table 8 represents the results obtained from the proposed SRN models in different environmental conditions considering the communication between mobile nodes.

As can be seen in Tables 7 and 8, since the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of layers ($L$)</td>
<td>5</td>
</tr>
<tr>
<td>Rate of changing conditions from suitable to unsuitable ($\alpha$)</td>
<td>Conditional</td>
</tr>
<tr>
<td>Rate of changing conditions from unsuitable to suitable ($\beta$)</td>
<td>1 event/h</td>
</tr>
<tr>
<td>Rate of changing status of the battery of each mobile node ($\omega$)</td>
<td>4 event/h</td>
</tr>
<tr>
<td>Movement rate of each mobile node ($\mu$)</td>
<td>10 layers/h</td>
</tr>
<tr>
<td>Sending rate of data ($\theta$)</td>
<td>100 data/h</td>
</tr>
<tr>
<td>Expiration rate of data ($\pi$)</td>
<td>2 data/h</td>
</tr>
</tbody>
</table>

**Table 7. Results obtained from different environmental conditions regardless of the communication between mobile nodes.**

<table>
<thead>
<tr>
<th>Node</th>
<th>Environmental conditions</th>
<th>Availability of environmental data</th>
<th>Battery status</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Layer 1</td>
<td>Layer 2</td>
</tr>
<tr>
<td>Mobile</td>
<td>Suitable</td>
<td>0.62</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>Moderate</td>
<td>0.60</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>Unsuitable</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\alpha = \beta$</td>
<td>0.51</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 2 \times \beta$</td>
<td>0.58</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>$\alpha = \frac{1}{2} \times \beta$</td>
<td>0.43</td>
<td>0.48</td>
</tr>
</tbody>
</table>

|      | Suitable                | 0.41      | 0.34      | 0.29      | 0.24      | 0.20      | —              |
|      | Moderate                 | 0.39      | 0.32      | 0.26      | 0.22      | 0.18      | —              |
|      | Unsuitable               | 0         | 0         | 0         | 0         | 0         | —              |
|      | $\alpha = \beta$         | 0.30      | 0.24      | 0.20      | 0.16      | 0.13      | —              |
|      | $\alpha = 2 \times \beta$| 0.37      | 0.30      | 0.25      | 0.21      | 0.17      | —              |
|      | $\alpha = \frac{1}{2} \times \beta$ | 0.22      | 0.17      | 0.13      | 0.10      | 0.08      | —              |
Table 8. Results obtained from different environmental conditions considering the communication between mobile nodes.

<table>
<thead>
<tr>
<th>Node</th>
<th>Environmental conditions</th>
<th>Availability of environmental data</th>
<th>Battery status</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Layer 1</td>
<td>Layer 2</td>
</tr>
<tr>
<td>Mobile</td>
<td>Suitable</td>
<td>0.61</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>Moderate</td>
<td>0.58</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>Unsuitable</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\alpha = \beta$</td>
<td>0.50</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 2 \times \beta$</td>
<td>0.57</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>$\alpha = \frac{1}{2} \times \beta$</td>
<td>0.42</td>
<td>0.46</td>
</tr>
<tr>
<td>Sink</td>
<td>Suitable</td>
<td>0.45</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>Moderate</td>
<td>0.41</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>Unsuitable</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\alpha = \beta$</td>
<td>0.31</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 2 \times \beta$</td>
<td>0.39</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>$\alpha = \frac{1}{2} \times \beta$</td>
<td>0.22</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 9. Results obtained from different environmental and communication conditions in new movement conditions.

<table>
<thead>
<tr>
<th>Node</th>
<th>Environmental conditions</th>
<th>Availability of environmental data</th>
<th>Battery status</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Layer 1</td>
<td>Layer 2</td>
</tr>
<tr>
<td>Mobile</td>
<td>Suitable</td>
<td>0.72</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>Moderate</td>
<td>0.67</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>Unsuitable</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\alpha = \beta$</td>
<td>0.57</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 2 \times \beta$</td>
<td>0.66</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>$\alpha = \frac{1}{2} \times \beta$</td>
<td>0.46</td>
<td>0.45</td>
</tr>
<tr>
<td>Sink</td>
<td>Suitable</td>
<td>0.51</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>Moderate</td>
<td>0.45</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>Unsuitable</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\alpha = \beta$</td>
<td>0.34</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 2 \times \beta$</td>
<td>0.44</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>$\alpha = \frac{1}{2} \times \beta$</td>
<td>0.24</td>
<td>0.26</td>
</tr>
</tbody>
</table>

communication between mobile nodes increases their power consumption, the battery of mobile nodes is not that high, thus making them less active. This makes it more likely that the data collected by the mobile nodes will expire, because the nodes do not arrive at the desired layers in time. However, the data of the sink node in Table 8 is more up to date when compared with those shown in Table 7, because the mobile nodes send their data to others. The results of Tables 7 and 8 demonstrate that compared to the communication conditions, the environmental conditions have a greater impact on system functionality and availability of data, because the communication between nodes increases power consumption.

5.3. Comparison of different movement conditions

In this section, the movement rate ($\mu$) of mobile nodes is doubled and if a mobile node communicates with others, its battery is consumed with the rate ($2 \times \omega + 1$); otherwise, it is consumed with the rate $2 \times \omega$. Table 9 shows the results obtained from different environmental and communication conditions in new movement conditions. In Table 9, the columns labeled
with “No” represent the situations where there is no communication between mobile nodes, and inversely, columns labeled with “Yes” stand for the situations in which mobile nodes communicate to each other.

Although power consumption is doubled by doubling the movement rate, the results of Tables 7, 8, and 9 demonstrate that the availability of environmental data at each mobile node increases with increasing the movement rate in different environmental and communication conditions. Furthermore, the data of the sink node in Table 9 are more up to date than those shown in Tables 7 and 8. For example, in Table 7, where the movement rate is \( \mu \) and there is no communication between nodes, the availability of environmental data of the sink node in Layer 1 in suitable environment conditions is equal to 0.41; however, in Table 8, where the movement rate is \( \mu \) and there is a communication between nodes, it is equal to 0.45. However, in Table 9, where the movement rate is \( 2\mu \), these results are equal to 0.51 and 0.56, respectively.

The results of Tables 7, 8, and 9 show that the environmental and movement conditions, compared to the communication conditions, have a greater impact on system functionality and data availability.

6. Conclusions and future work

This study presented analytical SRN models to analyze the impact of different environmental, communication, and movement conditions on the functionality of mobile wireless sensor networks. In previously done research, the impacts of all of these conditions had not been simultaneously evaluated in the MWSNs. In the proposed models, the mobile nodes move around to collect data and, then, send it to the sink node. They use the battery as the power source that can be charged according to environmental conditions. The data of the sink and mobile nodes expires after a while and needs to be constantly updated. The results obtained from numerically solving the proposed models demonstrated that the environmental and movement conditions had a greater impact on system functionality and availability of data, compared to the communication conditions.

The routing algorithms and communication between nodes were the most important factors influencing the MWSNs. In this context, the communication constraints, such as the malfunction and noise, can be considered in the communication. Furthermore, the random movement of mobile nodes can be considered in these systems. As a future work, one can model and evaluate the impact of these factors on the power consumption, lifetime, and system functionality. Furthermore, the impact of other factors, e.g., heterogeneity of nodes and different levels of failures, can be modeled and analyzed using SRNs in the MWSNs. In other words, the impact of these factors on other aspects of MWSNs like reliability, availability, and dependability can be assessed in the future.

References


**Biographies**

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