Innovative q-Rung Orthopair Fuzzy Prioritized Interactive Aggregation Operators to Evaluate Efficient Autonomous Vehicles for Freight Transportation

Hafiz Muhammad Athar Farid*, Muhammad Riaz †

Abstract

Freight transportation is essential to both the manufacturing industry and daily living. It gives vital supplies for manufacturing and technical activities, as well as supplies for human use. However, the vehicles have a significant environmental effects. It is critical to monitor transportation usage and promote the use of ecologically friendly vehicles, fuels, and technology. This is the most pressing issue confronting all stake holders involved in urban freight transportation. In multi-criteria group decision-making (MCGDM) strategies, the lack of contact between membership degree (MSD) and non-membership degree (NMSD) would be the basic factor for poor results in many MCGDM. To address these drawbacks, we define new aggregation operators (AOs) methods based on generalized membership grades of q-rung orthopair fuzzy (q-ROF) information, in this way, the input evaluation is interpreted in terms of q-rung orthopair fuzzy numbers (q-ROFNs). While interactive operators are well-known for interrelationship between generalized membership grades, prioritized operators are well-suited to exploit prioritized relationships among various criterion. Based on the characteristics of such flexible operators, two novel hybrid aggregation operators are proposed. Several significant features of these AOs are also investigated. The suitability and validity of the suggested operators is discussed for sustainable freight transportation selection. Numerical examples for data analysis is illustrated.

Keywords: Aggregation operators, q-rung orthopair fuzzy numbers, interactive relation, freight transportation, prioritized relation and sustainable.

1 Introduction

Throughout the last two generations of 20th century, several countries around the globe have become increasingly concerned about environmental pollution as well as role of transportation resources in supply chain. Countries are also being pushed to seek alternative sources of energy due to the prospect of a possible reduction in oil supply. Due to the rapid human consumption of fossil fuels, rising prices and the environmental impact of carbon dioxide (CO_2) emissions, the production of highly efficient energy supply systems and the use of renewable energy sources are crucial to the long-term supply of energy [1]. The United States and other governments have raised concerns and some have already passed legislation or introduced legislation to increase the environmental performance of their freight transport networks, with an emphasis on zero and near-zero emission vehicles (ZEVs). Fortunately, these technology's technical characteristics, market readiness, and other related factors differ, and the best choices for different uses and customer demands are not really clear [2]. Previous research has shown that, in addition to vehicle intensity, purchase price and running costs, various types of rewards can be decisive factors in the advancement of their adoption [3]. In addition, a better understanding of the behavior and attitudes of freight stakeholder will help to establish effective programmes and regulations to promote their deployment.As a consequence, the aim of this study is to investigate the factors that influence ZEV prioritization and acceptance in last-mile shipment operations using multi-criteria decision-making (MCDM) techniques. Transport is essential for capital accumulation activity and citizens' daily lives because it promotes the flow of manufactured goods and people, which boosts the economy. Analysis and forecasting of transport planning is one of the most relevant topics in the transport sector. As a result, fundamental and industrial transport planning are paying special attention to this topic [4]. The logistics industry is one of the main daily energy consumers in Europe, resulting in increased greenhouse gas (GHG) and air pollution (AP) emissions. Important regulatory and research and technology efforts are focused on improving attainability, i.e. reducing its effect on energy sources and the environment. At the same time, mobility of goods and people is a cornerstone of today's economy and should be further encouraged as transport becomes more effective. The propulsion technologies that will be promoted in the future and the improvements they will bring along are therefore key factors in terms of whether attainability targets can be achieved. Road freight mass transit, which is traditionally heavily dependent on compression ignition (diesel) engines, is one area where there are few sustainable alternatives. Increased use of bio-fuels and the use of second-generation bio-fuels in conjunction

*Department of Mathematics, University of the Punjab, Lahore, Pakistan. Email: hmatharfarid@gmail.com
†Department of Mathematics, University of the Punjab, Lahore, Pakistan. Email: mriaz.math@pu.edu.pk

with modern combustion concepts (low temperature combustion) will provide important benefits in terms of performance and reduction of AP and GHG emissions. Electric vehicles are able to reduce GHG and AP emissions substantially compared to conventional technologies. However, costs, infrastructure requirements and battery capacity remain major obstacles to widespread adoption. While technology is making rapid progress, there are no definitive statements as to whether and how much cost-efficiency the battery can improve. Furthermore, the availability and cost of materials for large-scale battery and engine production remains uncertain. Hybrid vehicles have some benefits over conventional vehicles in terms of GHG and AP emissions. However, they cannot be considered a long-term option, even with their dependence on fossil fuels.

Consistent with previous research, we can conclude that decision-making challenges in today's world are becoming increasingly complex. It is critical to explain the unknown details more competitively in order to select the best alternative(s) for multi-criteria decision-making (MCDM) difficulties. Furthermore, understanding how to manage the prioritized relationship between several criteria is critical. Using several of these qualities, as well as q-ROFNs, prioritized AOs and interactive AOs are merged to form the notion of prioritized interactive AOs. When it comes to taking prioritized relationships among distinct criterion as well as inter-correlations between NSDs and NMSDs, offered AOs are the best solution. As a result of these considerations, the following are the primary goals of this manuscript.

1. q-ROFNs are flexible and reliable to address vague information practically with generalized membership grades (NSD and NMSD). They provide freedom to the decision makers in selecting these grades in the unit square $[0,1]^2$.

2. The interactive operators are well-known AOs for interrelationship between MSD and NMSD, prioritized operators are well-suited to exploit prioritized relationships among various criteria. Based on the characteristics of these operators, two novel hybrid aggregation operators are proposed named as "q-rung orthopair fuzzy prioritized interactive weighted averaging (q-ROFPIWA) operator and the q-rung orthopair fuzzy prioritized interactive weighted geometric (q-ROFPIWG) operator".

3. Various illustrative examples are given to elaborate the innovative concepts regarding newly prioritized AOs for information fusion. The suggested operators cover drawbacks of existing operators and they provide more reliable and accurate information.

4. A new MCGDM approach is proposed for modeling vagueness in real-life problems. q-ROFNs have the ability to meet various MCGDM challenges.

5. An application of the proposed MCGDM technique is provided. A comparative analysis of the suggested MCGDM with existing methodologies is also offered to debate the practicality and efficiency of the suggested operators and MCGDM approach.

The remainder of the paper is organized as follows. Section 2 is a survey of the literature on sustainable automotive technologies and uncertain data modeling. Section 3 defines the q-rung orthopair fuzzy set (q-ROFS) as well as other important ideas. Section 4 has interactive AOs with q-ROF prioritization. In Section 5, we used the presented AOs to construct an MCGDM method. Section 6 goes into detail about the case study, including numerical examples and a comparison to current AOs. Section 7 outlines the important findings of the study paper.

2 Literature review

In this section we discuss some literature review about sustainable automotive technologies and uncertain data modeling.

2.1 Approaches to selecting sustainable automotive technologies

The process of prioritizing, screening, and selecting a collection of alternatives under often distinct, incompatible, or conflicting conditions is referred to as MCDM. Several literary works have been published that use various tactics and strategies to test the deployment of alternative technology in heavy and medium-duty vehicles. A potential market analysis of hybrid automobiles was conducted, and the elements that contribute to their cost-effectiveness when compared to traditional diesel vehicle models were reviewed [5]. Brownstone *et al.* [6] designed a micro-simulation inventory administration system to generate annual forecasting models including overall modern and reused vehicle trends by type of vehicle and geographical region. Miller *et al.* [7] analyzed California freight data to create a truck purchasing decision-making model that took into account commercial vehicle performance, vehicle expense and maintenance costs, efficiency and performance criteria, and other vital purchase decision factors for various types of trucks and fleets are all important considerations. Zhang *et al.* [8] forecasts choices made by truck fleets using organized surveys by Chinese fleets, taking into account the factors influencing their willingness to purchase alternative fuel vehicles, as well as the timing and purchase rates. Bunch *et al.* [9] looked at the possibility of alternative heavy-duty fuel use in California. Kurani *et al.* [10] the decision to purchase an electric vehicle in view of the household's accumulated vehicle stock, car purchasing practises and travel operation. Jeremy and Richard [11] proposed a life-cycle model that compares the pollution and cost of alternative fuel vehicles (AFVs).

MCDM approaches have also been used to test various vehicle technologies in urban and cargo transportation. Aydin and Kahraman [12] assessed public transportation vehicle selection alternatives. To analyse alternative fuel heavy duty vehicles, Yavuz *et al.* [13] developed a "hierarchical hesitant fuzzy" linguistic paradigm. In addition, Wtróbski *et al.* [14] PROMETHEE II and fuzzy TOPSIS have been used to test electric vans for urban logistics applications. Jaller and Otay [15] used Spherical Fuzzy TOPSIS and AHP to evaluate sustainable vehicle technologies for freight transport. Alkharabsheh *et al.* [16] has invented an integrated MCDM approach to assessing urban public transport networks. For market forecasting models for AFVs, Ma *et al.* [17] combined AHP and logit regression models. Erdem *et al.* [18] used a web-based survey to assess the willingness of Turkish customers to pay for electric vehicles. Maria *et al.* [19] addressed public expectations and demand for AFVs, as well as offering an overview of various methodological approaches. In Denmark, Mabit and Fosgerau [20] looked at the future of AFVs. Mass transit operations are among the most important aspects of economic development and human well-being. Bus transport plays a vital role in public transport. Bus is a very efficient way of travelling that is inexpensive, flexible and, in many cases, tailored to the needs of users, both in terms of ability and speed. Busses are the most viable option for healthy and sustainable urban development from an economical, ecological and social perspective. Behnam *et al.* [21] used two groundbreaking, ambiguous, MCDM methods to address the issue of alternative-fuel bus selection. Zhang *et al.* [22] investigated the understanding of the demand in electric vehicles and the factors that affect it. According to the existing literature, MCDM techniques have been widely used in a variety of applications, and the various techniques offer a balance in solutions in terms of process complexities, technical expert experience, and other operational issues. Koohathongsumrit and Meethom [23] developed an adaptive approach for route selection in multi-functional transportation networks based on a fuzzy risk assessment model and data envelopment analysis. Poudenx [24] discussed how transportation policies affect energy consumption and greenhouse gas emissions from urban passenger transportation. Daryanto *et al.* [25] presented a three-tiered supply chain model that takes into account carbon emissions and item deterioration. In the tabular form of gaining more clarity in comprehending the literature of selecting sustainable automobile technology given in Table 1.

Table 1: Literature of selecting sustainable automobile technology

Authors	Applications
Miller *et al.* [7]	To analyzed California freight data to create a truck purchasing decision-making model
Zhang *et al.* [8]	To forecasts choices made by truck fleets using organized surveys by Chinese fleets
Bunch *et al.* [9]	To looked at the possibility of alternative heavy-duty fuel use in California
Kurani *et al.* [10]	Decision to purchase an electric vehicle in view of the household's accumulated vehicle stock
Jeremy and Richard [11]	To proposed a life-cycle model that compares the pollution and cost of AFVs
Aydin and Kahraman [12]	To assessed public transportation vehicle selection alternatives
Wtróbski *et al.* [14]	To test electric vans for urban logistics applications
Jaller and Otay [15]	To evaluate sustainable vehicle technologies for freight transport
Alkharabsheh *et al.* [16]	To invented an integrated MCDM approach to assessing urban public transport networks
Erdem *et al.* [18]	To used a web-based survey to assess the willingness of Turkish customers to pay for electric vehicles
Maria *et al.* [19]	To addressed public expectations and demand for AFVs, as well as offering an overview of various methodological approaches
Mabit and Fosgerau [20]	To looked at the future of AFVs
Zhang *et al.* [22]	To investigated the understanding of the demand in electric vehicles and the factors that affect it

2.2 MCDM based uncertain data modeling

For decades, unclear and false information has been a major source of concern. Data collecting is critical for making decisions in the corporate, social, organizational, technological, clinical, machine intelligence, and psychological domains. Understanding of the alternative has always been considered as a precise number or linguistic quality. However, due to the data's ambiguity, it cannot be successfully aggregated. MCDM is a common intellectual processing technique whose main purpose is to select among a limited number of alternatives depending on decision-makers (DMs) preferences. However, because it integrates the complexities of human reasoning abilities, the MCDM technique is ambiguous and imprecise, making it difficult for DMs in the assessment phase to provide proper appraisal. It is vital to resolve this issue because, in addition to dealing with inconsistencies, Zadeh [26] gave the notion of "fuzzy set theory". Atanassov [27] developed the concept of "intuitionistic fuzzy set" (IFS). Yager and Abbasov [28] and Yager [29,30] proposed the "Pythagorean fuzzy set" (PFS) as an extension of IFS. Yager established the q-ROFS approach after newly generalizing IFS and PFS. The constraint of q-ROFS is that the sum of qth MSD power and NMSD power will be smaller than or equal to one. Clearly, the higher the rung q, the more orthopair's fulfil the bounding requirement, and hence the larger the universe of fuzzy data that may be defined by q-ROFSs [31]. q-ROFs outperform IFS and PFS in terms of their capacity to deal with both complete lack of transparency and disregarded data.

Liu & Wang [32] presented some basic geometric and averaging AOs related to q-ROFNs. Garg [33] introduced novel

concept of connection number based q-ROFS and their applications towards MCDM. Peng *et al.* [34] gave the novel exponential operational law for q-ROFNs and their AOs. They also defined new score function for the ranking of q-ROFNs. Jana *et al.* [35] developed famous Dombi AOs for q-ROFNs and their applications to MCDM. Wei *et al.* [36] presented the idea of Heronian mean AOs for the aggregation of q-ROFNs.Lin *et al.* [37] initiated the idea of linguistic q-ROFS and linguistic interactional partitioned Heronian mean AOs. Riaz *et al.* [38] introduced the innovative concept of q-ROFS Einstein AOs and their application towards sustainable energy planning decision management. Riaz *et al.* [39] developed the prioritized AOs for q-ROFNs with application in green supply chain management (GSCM). q-ROF prioritized AOs are employed when there is a prioritized relationship exit between two conflicting criteria. Riaz *et al.* [40] also initiated the concept of Einstein prioritized AOs for q-ROFNs which is the hybrid structure of Einstein AOs and prioritized AOs. Farid and Riaz [41] developed innovative Einstein interactive geometric AOs for q-ROFNs. Liu & Liu [42] proposed the Bonferroni mean AOs for q-ROFNs. Joshi & Gegov [43] proposed "q-ROF confidence based AOs". Riaz *et al.* [44, 45] suggested a comprehensive approach to q-ROF group-generalized and generalized and q-ROF interaction AOs for modeling vagueness in information fusion and MCDM problems. Akram *et al.* [46] developed a hybrid complex spherical fuzzy decision-making framework based on complex spherical fuzzy prioritized weighted aggregation operators. Jana *et al.* [47] proposed new multiple attribute dynamic decision making approach with the help of some complex aggregation functions in CQROF setting. Feng *et al.* [48] introduced the notion of Minkowski weighted score functions of intuitionistic fuzzy values. Ashraf *et al.* [49] proposed single valued neutrosophic Sine trigonometric aggregation operators and fuzzy decision support modeling for hydrogen power plant selection. Liu *et al.* [50] introduced "power Maclaurin symmetric mean" AOs for q-ROFNs and their application MCDM. Xing *et al.* [51] presented the novel idea of point weighted AOs for q-ROFNs. Liu & Wang [52] gave the idea of "Archimedean Bonferroni AOs" for q-ROFNs. Liu *et al.* [53] developed heterogeneous relationship among criterion for q-ROFNs. Mahmood & Ali [54] proposed complex q-ROF Hamacher AOs for MCDM. Saha *et al.* [55] gave the idea of fairly AOs for q-ROFSs. Hussain *et al.* [56] proposed AOs for hesitant q-ROFS with their applications in MCDM. Jana *et al.* [57] initiated the concept of AOs for MCDM method using bipolar fuzzy soft set.
There are many AOs related to interactive concept, given in Table 2.

Table 2: Some work related to interactive AOs

Authors	Aggregation Operators
Wang *et al.* [58]	Pythagorean fuzzy interactive Hamacher power AOs
Wang and Garg [59]	Archimedean based Pythagorean fuzzy interactive AOs
Wang and Li [60]	Pythagorean fuzzy interaction power Bonferroni mean AOs
Wei [61]	Pythagorean fuzzy interaction weighted AOs
Wang and Li [62]	Pythagorean fuzzy interaction AOs
Farid and Riaz [41]	q-ROF Einstein interactive geometric AOs
Riaz *et al.* [45]	q-ROF interactive AOs
Garg [63]	Intuitionistic fuzzy Hamacher interactive weighting AOs
Garg [64]	Generalised Pythagorean fuzzy geometric Einstein interactive AOs
Garg and Arora [65]	Prioritized intuitionistic fuzzy soft interactive AOs
Farid and Riaz [66]	Single-valued neutrosophic Einstein interactive AOs
Lin *et al.* [67]	Picture fuzzy interactional partitioned Heronian mean AOs
Luo and Xing [68]	Picture fuzzy interaction partitioned Heronian AOs

However, due to the expanded complexities of present situation challenges, the following MCDM problems should be addressed.

- The AOs devised by Liu & Wang [32] are dependent on algebraic operational rules, which do not allow for interaction betwixt the MSD and NMSDs. Let $\aleph_t = (\mu_t, \Re_t)$, be the assemblage of q-ROFNs. If $\aleph_j = (\mu_j, 0)$ with μ_j is not zero, then by [32], we get $\Re_{\aleph_t \times \aleph_j} = 0$, i.e. NMSD of product of all $\aleph_t$ and $\aleph_j$ is zero if one of the NMSD become zero and other NMSDs is not zero. Moreover, If we consider q-ROFWA$(\aleph_1, \aleph_2, \ldots \aleph_t)$, we get $\Re_{q-ROFWA(\aleph_1,\aleph_2,\ldots\aleph_t)} = 0$, if one of the NMSD of $(\aleph_1, \aleph_2, \ldots \aleph_t)$ is zero but others are non-zero. As a result, the basic operational principles of q-ROFNs must be improved.

- Certain characteristics are constantly associated in such a way that their interrelationships should be examined in many other decision-making problems. We also should paying close attention to aggregation algorithms that may account for a wide range of attribute interdependencies.

He *et al.* [69] appear to be able to tackle the first problem outlined above by introducing interaction operational laws that take into account the interactions betwixt MSD and NMSDs. As according Yager, in circumstances where we choose a child's bike based on both protection and budget factors, we really shouldn't permit the expense edge to prevent the loss of protection. Then there is a type of prioritized connection between these two criteria, with protection getting primacy. Because the attributes have a priority connection, this is referred to as an aggregation problem. The AOs in question, including the prioritized geometric and average AOs, are notable because they allow us to assess higher priority criteria, such as protection in the former example. In this case, Yager [70] provided prioritized AOs by describing attribute prioritization in terms of criterion weights based on the fulfilment of the higher value attributes. Given the aforementioned, we designed hybridized AOs, which are a combination of q-ROF interactive AOs and q-ROF Prioritized AOs.

3 Preliminaries

In this section, we review some fundamentals of q-ROFSs [31] and q-ROFNs [32].
List of symbols used in the paper:
$\breve{\Lambda}$ = q-rung orthopair fuzzy set
μ = membership degree
$\Re$ = non-membership degree
$\aleph$ = q-rung orthopair fuzzy number
$\widehat{\mathscr{F}}$ = score function
$\acute{\mathscr{A}}$ = accuracy function
$\perp$ = weight vector
$\top$ = alternative
γ = criteria
$\pounds$ = decision maker

Definition 3.1. [31] Consider q-ROF $\breve{\Lambda}$ in $\breve{\mho}$ is defined as

$$\breve{\Lambda} = \{\langle \tilde{\hbar}, \mu_{\breve{\Lambda}}(\tilde{\hbar}), \Re_{\breve{\Lambda}}(\tilde{\hbar})\rangle : \tilde{\hbar} \in \breve{\mho}\}$$

where $\mu_{\breve{\Lambda}}, \Re_{\breve{\Lambda}} : \breve{\mho} \to [0,1]$ characterized the MSD and NMSD of the alternative $\tilde{\hbar} \in \breve{\mho}$ and $\forall \tilde{\hbar}$ we have

$$0 \leq (\mu_{\breve{\Lambda}})^q(\tilde{\hbar}) + (\Re_{\breve{\Lambda}})^q(\tilde{\hbar}) \leq 1.$$

Furthermore, Liu and Wang recommended that q-ROFN data be combined with the following operating laws.

Definition 3.2. [32] Let $\aleph_1 = \langle \mu_1, \Re_1 \rangle$ and $\aleph_2 = \langle \mu_2, \Re_2 \rangle$ be q-ROFNs. Then
(1) $\overline{\aleph_1} = \langle \Re_1, \mu_1 \rangle$
(2) $\aleph_1 \vee \aleph_2 = \langle max\{\mu_1, \Re_1\}, min\{\mu_2, \Re_2\}\rangle$
(3) $\aleph_1 \wedge \aleph_2 = \langle min\{\mu_1, \Re_1\}, max\{\mu_2, \Re_2\}\rangle$
(4) $\aleph_1 \oplus \aleph_2 = \langle \sqrt[q]{\mu_1^q + \mu_2^q - \mu_1^q \mu_2^q}, \Re_1 \Re_2 \rangle$
(5) $\aleph_1 \otimes \aleph_2 = \langle \mu_1 \mu_2, \sqrt[q]{\Re_1^q + \Re_2^q - \Re_1^q \Re_2^q} \rangle$
(6) $\sigma \aleph_1 = \langle \sqrt[q]{(1 - (1 - \mu_1^q)^\sigma)}, \Re_1^\sigma \rangle$
(7)$\aleph_1^\sigma = \langle \mu_1^\sigma, \sqrt[q]{(1 - (1 - \Re_1^q)^\sigma)} \rangle$

Definition 3.3. [32] Consider $\aleph = \langle \mu, \Re \rangle$ is the q-ROFN, score function (SF) $\widehat{\mathscr{F}}$ of $\aleph$ is defines as

$$\widehat{\mathscr{F}}(\aleph) = (\mu)^q - (\Re)^q$$

$\widehat{\mathscr{F}}(\aleph) \in [-1, 1]$. The q-ROFN score will define its ranking, with the highest score determining the highest q-ROFN priority. In some circumstances, the SF is detrimental to q-ROFN. As a result, using the SF to evaluate the q-ROFNs is insufficient. We're adding a new function, namely an accuracy function (AF).

Definition 3.4. [32] Consider $\aleph = \langle \mu, \Re \rangle$ is the q-ROFN, then an AF $\acute{\mathscr{A}}$ of $\aleph$ is defines as

$$\acute{\mathscr{A}}(\aleph) = (\mu)^q + (\Re)^q$$

$\acute{\mathscr{A}}(\aleph) \in [0, 1]$.

Definition 3.5. Consider $\aleph = \langle \mu_\aleph, \Re_\aleph \rangle$ and $\breve{\Upsilon} = \langle \mu_{\breve{\Upsilon}}, \Re_{\breve{\Upsilon}} \rangle$ are two q-ROFNs, and $\widehat{\mathscr{F}}(\aleph), \widehat{\mathscr{F}}(\breve{\Upsilon})$ are the SF of $\aleph$ and $\breve{\Upsilon}$, and $\acute{\mathscr{A}}(\aleph), \acute{\mathscr{A}}(\breve{\Upsilon})$ are the AF of $\aleph$ and $\breve{\Upsilon}$, respectively, then
(a) If $\widehat{\mathscr{F}}(\aleph) > \widehat{\mathscr{F}}(\breve{\Upsilon})$, then $\aleph > \breve{\Upsilon}$
(b) If $\widehat{\mathscr{F}}(\aleph) = \widehat{\mathscr{F}}(\breve{\Upsilon})$, then

if $\acute{\mathscr{A}}(\aleph) > \acute{\mathscr{A}}(\breve{\Upsilon})$ then $\aleph > \breve{\Upsilon}$,
if $\acute{\mathscr{A}}(\aleph) = \acute{\mathscr{A}}(\breve{\Upsilon})$, then $\aleph = \breve{\Upsilon}$.

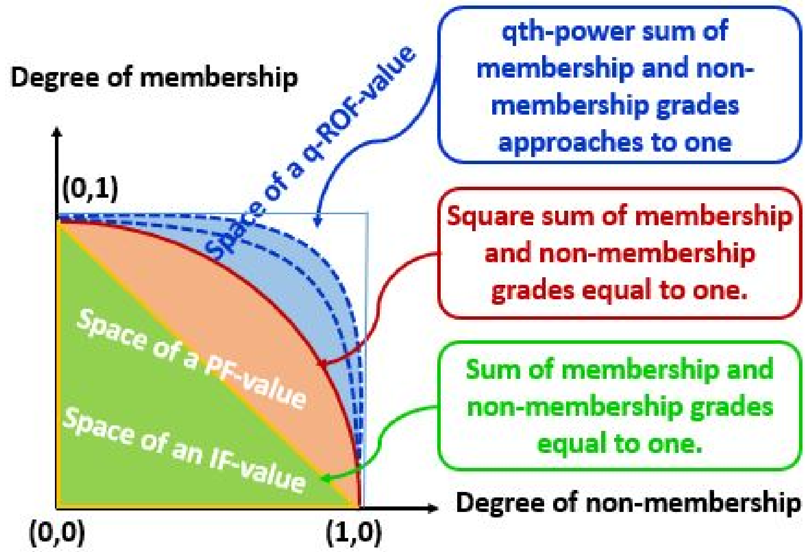

Figure 1: Geometrical representation of q-ROFS

It must always be remembered that the SF's value is betwixt-1 and 1. To help the following study, we add a further SF, $\mathscr{H}(\aleph) = \frac{1+\mu_{\aleph}^q - \Re_{\aleph}^q}{2}$. We can see that $0 \leq \mathscr{H}(\aleph) \leq 1$.

The geometrical depiction of q-ROFS with IFS and PFS is shown in Figure 1.

3.1 q-ROF interactive operations

Definition 3.6. [45] Let $\aleph$, $\aleph_1$ and $\aleph_2$ be the two q-ROFNs, the interactive operations for q-ROF environment is defined as

1. $\aleph_1 \oplus \aleph_2 = \left(\sqrt[q]{(\mu_1)^q + (\mu_2)^q - (\mu_1)^q(\mu_2)^q}, \sqrt[q]{(\Re_1)^q + (\Re_2)^q - (\Re_1)^q(\Re_2)^q - (\Re_1)^q(\mu_2)^q - (\mu_1)^q(\Re_2)^q} \right)$
2. $\aleph_1 \otimes \aleph_2 = \left(\sqrt[q]{(\mu_1)^q + (\mu_2)^q - (\mu_1)^q(\mu_2)^q - (\mu_1)^q(\Re_2)^q - (\Re_1)^q(\mu_2)^q}, \sqrt[q]{(\Re_1)^q + (\Re_2)^q - (\Re_1)^q(\Re_2)^q} \right)$
3. $\lambda\aleph = \left(\sqrt[q]{1-(1-\mu^q)^\lambda}, \sqrt[q]{(1-\mu^q)^\lambda - (1-(\mu^q+\Re^q))^\lambda} \right), \quad \lambda > 0$
4. $\aleph^\lambda = \left(\sqrt[q]{(1-\Re^q)^\lambda - (1-(\Re^q+\mu^q))^\lambda}, \sqrt[q]{1-(1-\Re^q)^\lambda} \right), \quad \lambda > 0$

Definition 3.7. [45] Let $\aleph_q = \langle \mu_q, \Re_q \rangle$ be the assemblage of q-ROFNs, and q-ROFIWA : $\breve{\mathcal{L}}^n \to \breve{\mathcal{L}}$, is a mapping.

$$\text{q-ROFIWA}(\aleph_1, \aleph_2, \dots \aleph_r) = \bigoplus_{g=1}^{r} \perp_g \aleph_g \tag{3.1}$$

then the mapping q-ROFIWA is called "q-rung orthopair fuzzy interactive weighted averaging operator", where $(\perp_1, \perp_2, \dots \perp_r)$ be the WV of considered q-ROFNs with the condition that $\perp_j > 0$, $\perp_g \in [0,1]$ and $\sum_{g=1}^{r} \perp_g = 1$

Theorem 3.8. *[45] Consider* $\aleph_q = \langle \mu_q, \Re_q \rangle$ is the assemblage of q-ROFNs, then

q-ROFIWA$(\aleph_1, \aleph_2, \dots \aleph_r)$

$$\begin{aligned} &= \bigoplus_{g=1}^{r} \perp_g \aleph_g \\ &= \left(\sqrt[q]{1 - \prod_{g=1}^{r} \left(1-(\mu_g)^q\right)^{\perp_g}}, \sqrt[q]{\prod_{g=1}^{r} \left(1-(\mu_g)^q\right)^{\perp_g} - \prod_{g=1}^{r} \left(1-((\mu_g)^q + (\Re_g)^q)\right)^{\perp_g}} \right) \end{aligned} \tag{3.2}$$

Definition 3.9. [45] Consider $\aleph_q = \langle \mu_q, \Re_q \rangle$ is the assemblage of q-ROFNs, and q-ROFIWG : $\breve{\mathcal{L}}^n \rightarrow \breve{\mathcal{L}}$, is a mapping.

$$\text{q-ROFIWG}(\aleph_1, \aleph_2, \ldots \aleph_r) = \bigotimes_{g=1}^{r} (\aleph_g)^{\perp_g} \tag{3.3}$$

then the mapping q-ROFIWG is called "q-rung orthopair fuzzy interactive weighted geometric operator", where $(\perp_1, \perp_2, \ldots \perp_r)$ be the WV of considered q-ROFNs with the condition that $\perp_j > 0$, $\perp_g \in [0,1]$ and $\sum_{g=1}^{r} \perp_g = 1$

Theorem 3.10. *[45]* Consider $\aleph_q = \langle \mu_q, \Re_q \rangle$ is the assemblage of q-ROFNs, then

$$\begin{aligned} &\text{q-ROFIWG}(\aleph_1, \aleph_2, \ldots \aleph_r) \\ &= \bigotimes_{g=1}^{r} (\aleph_g)^{\perp_g} \\ &= \left(\sqrt[q]{\prod_{g=1}^{r} \left(1 - (\Re_g)^q\right)^{\perp_g} - \prod_{g=1}^{r} \left(1 - ((\Re_g)^q + (\mu_g)^q)\right)^{\perp_g}}, \sqrt[q]{1 - \prod_{g=1}^{r} \left(1 - (\Re_g)^q\right)^{\perp_g}} \right) \end{aligned} \tag{3.4}$$

4 q-ROF prioritized interactive AOs

In this section, we present hybrid AOs namely, q-ROF prioritized interactive AOs.

4.1 q-ROF prioritized interactive averaging AOs

Definition 4.1. Consider $\aleph_q = \langle \mu_q, \Re_q \rangle$ is the assemblage of q-ROFNs, and q-ROFPIWA : $\breve{\mathcal{L}}^n \rightarrow \breve{\mathcal{L}}$, is a mapping. if

$$\text{q-ROFPIWA}(\aleph_1, \aleph_2, \ldots \aleph_r) = \frac{\zeta_1}{\sum_{g=1}^{r} \zeta_g} \aleph_1 \oplus \frac{\zeta_2}{\sum_{g=1}^{r} \zeta_g} \aleph_2 \oplus \ldots, \oplus \frac{\zeta_r}{\sum_{g=1}^{r} \zeta_g} \aleph_r \tag{4.1}$$

then the mapping q-ROFPIWA is called "q-rung orthopair fuzzy prioritized interactive weighted averaging (q-ROFPIWA) operator", where $\zeta_j = \prod_{k=1}^{j-1} \widehat{F}(\aleph_k)$ $(j = 2 \ldots, n)$, $\zeta_1 = 1$ and $\widehat{F}(\aleph_k)$ is the score of k^{th} q-ROFN.

Based on q-ROF interactive operations we have the following theorem.

Theorem 4.2. Consider $\aleph_q = \langle \mu_q, \Re_q \rangle$ is the assemblage of q-ROFNs, then

$$\begin{aligned} &\text{q-ROFIPWA}(\aleph_1, \aleph_2, \ldots \aleph_r) \\ &= \left(\sqrt[q]{1 - \prod_{g=1}^{r} \left(1 - (\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r} \zeta_g}}}, \sqrt[q]{\prod_{g=1}^{r} \left(1 - (\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r} \zeta_g}} - \prod_{g=1}^{r} \left(1 - ((\mu_g)^q + (\Re_g)^q)\right)^{\frac{\zeta_g}{\sum_{g=1}^{r} \zeta_g}}} \right) \end{aligned} \tag{4.2}$$

Proof is given in appendix.

Example 4.3. Consider $\aleph_1 = (0.96, 0.00)$, $\aleph_2 = (0.35, 0.35)$ and $\aleph_3 = (0.50, 0.15)$ the q-ROFNs, there are six criterions for these q-ROFNs. Priorities are assigned betwixt the criteria provided by the linear orientation in this case. $\gamma_1 \succ \gamma_2 \succ \gamma_3 \ldots \gamma_6$ indicates criteria γ_J has a high priority than γ_i if $j > i$ and we take $q = 3$.

$$\begin{aligned} \sqrt[q]{1 - \prod_{g=1}^{3} \left(1 - (\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{6} \zeta_g}}} &= 0.8475 \\ \sqrt[q]{\prod_{g=1}^{3} \left(1 - (\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{6} \zeta_g}} - \prod_{g=1}^{3} \left(1 - ((\mu_g)^q + (\Re_g)^q)\right)^{\frac{\zeta_g}{\sum_{g=1}^{6} \zeta_g}}} &= 0.1934 \end{aligned}$$

and we have,

$$\begin{aligned} &\text{q-ROFPIWA}(\aleph_1, \aleph_2, \aleph_3) \\ &= \left(\sqrt[q]{1 - \prod_{g=1}^{3} \left(1 - (\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{6} \zeta_g}}}, \sqrt[q]{\prod_{g=1}^{3} \left(1 - (\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{6} \zeta_g}} - \prod_{g=1}^{3} \left(1 - ((\mu_g)^q + (\Re_g)^q)\right)^{\frac{\zeta_g}{\sum_{g=1}^{6} \zeta_g}}} \right) \\ &= (0.8475, 0.1934) \end{aligned}$$

Some of the extremely attractive characteristics of q-ROFPWA operator are described below.

Theorem 4.4. *(Idempotency) Consider* $\aleph_q = \langle \mu_q, \Re_q \rangle$ *is the assemblage of q-ROFNs, where* $\zeta_q = \prod_{g=1}^{q-1} \widehat{F}(\aleph_g)$ $(q = 2\ldots, n)$, $\zeta_1 = 1$ *and* $\widehat{F}(\aleph_g)$ *is the score of* g^{th} *q-ROFN. If all* $\aleph_q$ *are equal, i.e,.* $\aleph_q = \aleph$ *for all q, then*

$$\textit{q-ROFPIWA}(\aleph_1, \aleph_2, \ldots \aleph_r) = \aleph$$

Proof. From Definition 4.1, we have

$$\begin{aligned}
\text{q-ROFPIWA}(\aleph_1, \aleph_2, \ldots \aleph_r) &= \frac{\zeta_1}{\sum_{g=1}^{r} \zeta_g}\aleph_1 \oplus \frac{\zeta_2}{\sum_{g=1}^{r} \zeta_g}\aleph_2 \oplus \ldots, \oplus \frac{\zeta_r}{\sum_{g=1}^{r} \zeta_g}\aleph_r \\
&= \frac{\zeta_1}{\sum_{g=1}^{r} \zeta_g}\aleph \oplus \frac{\zeta_2}{\sum_{g=1}^{r} \zeta_g}\aleph \oplus \ldots, \oplus \frac{\zeta_r}{\sum_{g=1}^{r} \zeta_g}\aleph \\
&= \frac{\sum_{g=1}^{r} \zeta_g}{\sum_{g=1}^{r} \zeta_g}\aleph \\
&= \aleph
\end{aligned}$$

□

Corollary 4.5. *If* $\aleph_q = \langle \mu_q, \Re_q \rangle$ *is the assemblage of largest q-ROFNs, i.e,.* $\aleph_q = (1, 0)$ *for all* j*, then*

$$\textit{q-ROFPIWA}(\aleph_1, \aleph_2, \ldots \aleph_r) = (1, 0)$$

Proof. We can simply obtain a proof of this corollary by Theorem 4.13. □

Corollary 4.6. *If* $\aleph_1 = \langle \mu_1, \Re_1 \rangle$ *is the smallest q-ROFN, i.e,.* $\aleph_1 = (0, 1)$*, then*

$$\textit{q-ROFPIWA}(\aleph_1, \aleph_2, \ldots \aleph_r) = (0, 1)$$

Proof. Here, $\aleph_1 = (0, 1)$ then by the SF, we have,

$$\widehat{F}(\aleph_1) = 0$$

Since,

$$\zeta_q = \prod_{g=1}^{q-1} \widehat{F}(\aleph_g) \qquad (q = 2\ldots, n), \quad \text{and} \quad \zeta_1 = 1$$

$\widehat{F}(\aleph_g)$ is the score of g^{th} q-ROFN.
We have,
$\zeta_q = \prod_{g=1}^{q-1} \widehat{F}(\aleph_g) = \widehat{F}(\aleph_1) \times \widehat{F}(\aleph_2) \times \ldots \times \widehat{F}(\aleph_{q-1}) = 0 \times \widehat{F}(\aleph_2) \times \ldots \times \widehat{F}(\aleph_{q-1}) \qquad (q = 2\ldots, n)$

$$\prod_{g=1}^{q} \zeta_g = 1$$

From Definition 4.1, we have

$$\begin{aligned}
\text{q-ROFPIWA}(\aleph_1, \aleph_2, \ldots \aleph_r) &= \frac{\zeta_1}{\sum_{g=1}^{r} \zeta_g}\aleph_1 \oplus \frac{\zeta_2}{\sum_{g=1}^{r} \zeta_g}\aleph_2 \oplus \ldots, \oplus \frac{\zeta_r}{\sum_{g=1}^{r} \zeta_g}\aleph_r \\
&= \frac{1}{1}\aleph_1 \oplus \frac{0}{1}\aleph_2 \oplus \ldots \frac{0}{1}\aleph_r \\
&= \aleph_1 = (0, 1)
\end{aligned}$$

□

As a Corollary 4.15, if the higher priority attribute is smallest q-ROFN, rewards will not be received by other criteria even though they are fulfilled.

Theorem 4.7. *(Monotonicity) Consider* $\aleph_q = \langle \mu_q, \Re_q \rangle$ *and* $\aleph_q^* = \langle \mu_q^*, \Re_q^* \rangle$ *are the assemblages of q-ROFNs, where* $\zeta_q = \prod_{g=1}^{q-1} \widehat{F}(\aleph_g)$, $\zeta_q^* = \prod_{g=1}^{q-1} \widehat{F}(\aleph_g^*)$ $(g = 2\ldots, n)$, $\zeta_1 = 1$, $\zeta_1^* = 1$, $\widehat{F}(\aleph_g)$ *is the score of* $\aleph_g$ *q-ROFN, and* $\widehat{F}(\aleph_g^*)$ *is the score of* $\aleph_g^*$ *q-ROFN. If* $\mu_q^* \geq \mu_q$ *and* $\Re_q^* \leq \Re_q$ *for all q, then*

$$\textit{q-ROFPIWA}(\aleph_1, \aleph_2, \ldots \aleph_r) \leq \textit{q-ROFPIWA}(\aleph_1^*, \aleph_2^*, \ldots \aleph_r^*)$$

Proof. Here, $\mu_g^* \geq \mu_g$ and $\Re_g^* \leq \Re_g$, $\forall g$
If $\mu_g^* \geq \mu_g$.

$\Leftrightarrow (\mu_g^*)^q \geq (\mu_g)^q \Leftrightarrow \sqrt[q]{(\mu_g^*)^q} \geq \sqrt[q]{(\mu_g)^q} \Leftrightarrow \sqrt[q]{1-(\mu_g^*)^q} \leq \sqrt[q]{1-(\mu_g)^q}$

$\Leftrightarrow \sqrt[q]{(1-(\mu_g^*)^q)^{\frac{\zeta_g}{\sum_{g=1}^r \zeta_g}}} \leq \sqrt[q]{(1-(\mu_g)^q)^{\frac{\zeta_g}{\sum_{g=1}^r \zeta_g}}}$

$\Leftrightarrow \sqrt[q]{\prod_{g=1}^r (1-(\mu_g^*)^q)^{\frac{\zeta_g}{\sum_{g=1}^r \zeta_g}}} \leq \sqrt[q]{\prod_{g=1}^r (1-(\mu_g)^q)^{\frac{\zeta_g}{\sum_{g=1}^r \zeta_g}}}$

$\Leftrightarrow \sqrt[q]{1-\prod_{g=1}^r (1-(\mu_g)^q)^{\frac{\zeta_g}{\sum_{g=1}^r \zeta_g}}} \leq \sqrt[q]{1-\prod_{g=1}^r (1-(\mu_g^*)^q)^{\frac{\zeta_g}{\sum_{g=1}^r \zeta_g}}}$

Now, again we take,
$\mu_g^* \geq \mu_g$ and $\Re_g^* \leq \Re_g$, $\forall g$
If $\mu_g^* \geq \mu_g$.

$\Leftrightarrow (\mu_g^*)^q \geq (\mu_g)^q \Leftrightarrow \sqrt[q]{(\mu_g^*)^q} \geq \sqrt[q]{(\mu_g)^q} \Leftrightarrow \sqrt[q]{1-(\mu_g^*)^q} \leq \sqrt[q]{1-(\mu_g)^q}$

$\Leftrightarrow \sqrt[q]{(1-(\mu_g^*)^q)^{\frac{\zeta_g}{\sum_{g=1}^r \zeta_g}}} \leq \sqrt[q]{(1-(\mu_g)^q)^{\frac{\zeta_g}{\sum_{g=1}^r \zeta_g}}}$

$\Leftrightarrow \sqrt[q]{\prod_{g=1}^r (1-(\mu_g^*)^q)^{\frac{\zeta_g}{\sum_{g=1}^r \zeta_g}} - \prod_{g=1}^r \left(1-((\mu_g^*)^q+(\Re_g^*)^q)\right)^{\frac{\zeta_g}{\sum_{g=1}^r \zeta_g}}} \leq$
$\sqrt[q]{\prod_{g=1}^r (1-(\mu_g)^q)^{\frac{\zeta_g}{\sum_{g=1}^r \zeta_g}} - \prod_{g=1}^r \left(1-((\mu_g)^q+(\Re_g)^q)\right)^{\frac{\zeta_g}{\sum_{g=1}^r \zeta_g}}}$

Let

$$\overline{\aleph} = \text{q-ROFPIWA}(\aleph_1, \aleph_2, \ldots \aleph_r)$$

and

$$\overline{\aleph^*} = \text{q-ROFPIWA}(\aleph_1^*, \aleph_2^*, \ldots \aleph_r^*)$$

We get that $\overline{\aleph^*} \geq \overline{\aleph}$. So,

$$\text{q-ROFPIWA}(\aleph_1, \aleph_2, \ldots \aleph_r) \leq \text{q-ROFPIWA}(\aleph_1^*, \aleph_2^*, \ldots \aleph_r^*)$$

□

Theorem 4.8. *(Boundary) Consider* $\aleph_q = \langle \mu_q, \Re_q \rangle$ *is the family of q-ROFNs, and*

$$\aleph^- = (min_q\,(\mu_q), max_q\,(\Re_q)) \quad and \quad \aleph^+ = (max_q\,(\mu_q), min_q\,(\Re_q))$$

Then,

$$\aleph^- \leq q\text{-}ROFPIWA(\aleph_1, \aleph_2, \ldots \aleph_r) \leq \aleph^+$$

where $\zeta_g = \prod_{g=1}^{q-1} \widehat{F}(\aleph_g)$ $(q = 2 \ldots, r)$, $\zeta_1 = 1$ *and* $\widehat{F}(\aleph_g)$ *is the score of* g^{th} *q-ROFN.*

Theorem 4.9. *Consider* $\aleph_q = \langle \mu_q, \Re_q \rangle$ *and* $\ddot{\breve{\mathcal{L}}}_q = \langle \sigma_q, \tau_q \rangle$ *are two assemblages of q-ROFNs, where* $\zeta_g = \prod_{g=1}^{r-1} \widehat{F}(\aleph_g)$ $(g = 2 \ldots, r)$, $\zeta_1 = 1$ *and* $\widehat{F}(\aleph_g)$ *is the score of* j^{th} *q-ROFN. If* $R > 0$ *and* $\ddot{\breve{\mathcal{L}}} = \langle \mu_{\ddot{\breve{\mathcal{L}}}}, \Re_{\ddot{\breve{\mathcal{L}}}} \rangle$ *is an q-ROFN, then*

1. *q-ROFPIWA*$(\aleph_1 \oplus \ddot{\breve{\mathcal{L}}}, \aleph_2 \oplus \ddot{\breve{\mathcal{L}}}, \ldots \aleph_r \oplus \ddot{\breve{\mathcal{L}}}) =$ *q-ROFPIWA*$(\aleph_1, \aleph_2, \ldots \aleph_r) \oplus \ddot{\breve{\mathcal{L}}}$

2. *q-ROFPIWA*$(R\aleph_1, R\aleph_2, \ldots R\aleph_r) = R$ *q-ROFPIWA*$(\aleph_1, \aleph_2, \ldots \aleph_r)$

3. *q-ROFPIWA*$(\aleph_1 \oplus \ddot{\breve{\mathcal{L}}}_1, \aleph_2 \oplus \ddot{\breve{\mathcal{L}}}_2, \ldots \aleph_r \oplus \ddot{\breve{\mathcal{L}}}_n) =$ *q-ROFPIWA*$(\aleph_1, \aleph_2, \ldots \aleph_r) \oplus$ *q-ROFPIWA*$(\ddot{\breve{\mathcal{L}}}_1, \ddot{\breve{\mathcal{L}}}_2, \ldots \ddot{\breve{\mathcal{L}}}_r)$

4. *q-ROFPIWA* $R\aleph_1 \oplus \ddot{\breve{\mathcal{L}}}, R\aleph_2 \oplus \ddot{\breve{\mathcal{L}}}, \ldots \oplus R\aleph_r \oplus \ddot{\breve{\mathcal{L}}}) = R$ *q-ROFPIWA*$(\aleph_1, \aleph_2, \ldots \aleph_r) \oplus \ddot{\breve{\mathcal{L}}}$

4.2 q-ROF prioritized interactive geometric AOs

Definition 4.10. Consider $\aleph_q = \langle \mu_q, \Re_q \rangle$ is the assemblage of q-ROFNs, and q-ROFPIWG : $\breve{\mathcal{L}}^n \to \breve{\mathcal{L}}$, is a mapping. if

$$\text{q-ROFPIWG}(\aleph_1, \aleph_2, \ldots \aleph_r) = \aleph_1^{\frac{\zeta_1}{\sum_{g=1}^{r} \zeta_g}} \otimes \aleph_2^{\frac{\zeta_2}{\sum_{g=1}^{r} \zeta_g}} \otimes \ldots, \otimes \aleph_r^{\frac{\zeta_r}{\sum_{g=1}^{r} \zeta_g}} \tag{4.3}$$

then the mapping q-ROFPIWG is called "q-rung orthopair fuzzy prioritized interactive weighted geometric (q-ROFPIWG) operator", where $\zeta_j = \prod_{k=1}^{j-1} \widehat{F}(\aleph_k)$ $(j = 2 \ldots, n)$, $\zeta_1 = 1$ and $\widehat{F}(\aleph_k)$ is the score of k^{th} q-ROFN.

Based on q-ROF interactive operations we have the following theorem.

Theorem 4.11. *Consider $\aleph_q = \langle \mu_q, \Re_q \rangle$ is the assemblage of q-ROFNs, then*

$$\begin{aligned} \textit{q-ROFIPWG}(\aleph_1, \aleph_2, \ldots \aleph_r) &= \left(\sqrt[q]{\prod_{g=1}^{r} \left(1 - (\Re_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r} \zeta_g}} - \prod_{g=1}^{r} \left(1 - ((\Re_g)^q + (\mu_g)^q)\right)^{\frac{\zeta_g}{\sum_{g=1}^{r} \zeta_g}}}, \right. \\ &\left. \sqrt[q]{1 - \prod_{g=1}^{r} \left(1 - (\Re_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r} \zeta_g}}} \right) \end{aligned} \tag{4.4}$$

Example 4.12. Consider $\aleph_1 = (0.96, 0.00)$, $\aleph_2 = (0.35, 0.35)$ and $\aleph_3 = (0.50, 0.15)$ the q-ROFNs, there are six criterions for these q-ROFNs. Priorities are assigned betwixt the criteria provided by the linear orientation in this case. $\gamma_1 \succ \gamma_2 \succ \gamma_3 \ldots \gamma_6$ indicates criteria γ_J has a high priority than γ_i if $j > i$ and we take $q = 3$.

$$\begin{aligned} \sqrt[q]{\prod_{g=1}^{3} \left(1 - (\Re_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{6} \zeta_g}} - \prod_{g=1}^{3} \left(1 - ((\Re_g)^q + (\mu_g)^q)\right)^{\frac{\zeta_g}{\sum_{g=1}^{6} \zeta_g}}} &= 0.8426 \\ \sqrt[q]{1 - \prod_{g=1}^{3} \left(1 - (\Re_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{6} \zeta_g}}} &= 0.2602 \end{aligned}$$

and we have,

$$\begin{aligned} &\text{q-ROFPIWG}(\aleph_1, \aleph_2, \aleph_3) \\ &= \left(\sqrt[q]{\prod_{g=1}^{3} \left(1 - (\Re_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{6} \zeta_g}} - \prod_{g=1}^{3} \left(1 - ((\Re_g)^q + (\mu_g)^q)\right)^{\frac{\zeta_g}{\sum_{g=1}^{6} \zeta_g}}}, \sqrt[q]{1 - \prod_{g=1}^{3} \left(1 - (\Re_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{6} \zeta_g}}} \right) \\ &= (0.8426, 0.2602) \end{aligned}$$

Some of the extremely attractive characteristics of q-ROFPWG operator are described below.

Theorem 4.13. *(Idempotency) Consider $\aleph_q = \langle \mu_q, \Re_q \rangle$ is the assemblage of q-ROFNs, where $\zeta_q = \prod_{g=1}^{q-1} \widehat{F}(\aleph_g)$ $(q = 2 \ldots, n)$, $\zeta_1 = 1$ and $\widehat{F}(\aleph_g)$ is the score of g^{th} q-ROFN. If all $\aleph_q$ are equal, i.e,. $\aleph_q = \aleph$ for all q, then*

$$\textit{q-ROFPIWG}(\aleph_1, \aleph_2, \ldots \aleph_r) = \aleph$$

Proof. From Definition 4.10, we have

$$\begin{aligned} \text{q-ROFPIWG}(\aleph_1, \aleph_2, \ldots \aleph_r) &= \aleph_1^{\frac{\zeta_1}{\sum_{g=1}^{r} \zeta_g}} \otimes \aleph_2^{\frac{\zeta_2}{\sum_{g=1}^{r} \zeta_g}} \otimes \ldots, \otimes \aleph_r^{\frac{\zeta_r}{\sum_{g=1}^{r} \zeta_g}} \\ &= \aleph^{\frac{\zeta_1}{\sum_{g=1}^{r} \zeta_g}} \otimes \aleph^{\frac{\zeta_2}{\sum_{g=1}^{r} \zeta_g}} \otimes \ldots, \otimes \aleph^{\frac{\zeta_r}{\sum_{g=1}^{r} \zeta_g}} \\ &= \aleph^{\frac{\sum_{g=1}^{r} \zeta_g}{\sum_{g=1}^{r} \zeta_g}} \\ &= \aleph \end{aligned}$$

□

Corollary 4.14. *If $\aleph_q = \langle \mu_q, \Re_q \rangle$ $q = (1, 2, \ldots n)$ is the assemblage of largest q-ROFNs, i.e,. $\aleph_q = (1, 0)$ for all j, then*

$$\textit{q-ROFPIWG}(\aleph_1, \aleph_2, \ldots \aleph_r) = (1, 0)$$

Proof. We can easily obtain Corollary similar to the Theorem 4.13. □

Corollary 4.15. *(Non-compensatory) If $\aleph_1 = \langle \mu_1, \Re_1 \rangle$ is the smallest q-ROFN, i.e,. $\aleph_1 = (0, 1)$, then*

$$\text{q-ROFPIWG}(\aleph_1, \aleph_2, \dots \aleph_r) = (0, 1)$$

Proof. Here, $\aleph_1 = (0, 1)$ then by definition of the SF, we have,

$$\widehat{F}(\aleph_1) = 0$$

Since,

$$\zeta_q = \prod_{g=1}^{q-1} \widehat{F}(\aleph_g) \qquad (q = 2 \dots, n), \quad \text{and} \quad \zeta_1 = 1$$

$\widehat{F}(\aleph_g)$ is the score of g^{th} q-ROFN.
We have,
$\zeta_q = \prod_{g=1}^{q-1} \widehat{F}(\aleph_g) = \widehat{F}(\aleph_1) \times \widehat{F}(\aleph_2) \times \dots \times \widehat{F}(\aleph_{q-1}) = 0 \times \widehat{F}(\aleph_2) \times \dots \times \widehat{F}(\aleph_{q-1}) \qquad (q = 2 \dots, n)$

$$\prod_{g=1}^{q} \zeta_g = 1$$

From Definition 4.1, we have

$$\begin{aligned}
\text{q-ROFPIWG}(\aleph_1, \aleph_2, \dots \aleph_r) &= \aleph_1^{\frac{\zeta_1}{\sum_{g=1}^{r} \zeta_g}} \otimes \aleph_2^{\frac{\zeta_2}{\sum_{g=1}^{r} \zeta_g}} \otimes \dots, \otimes \aleph_r^{\frac{\zeta_r}{\sum_{g=1}^{r} \zeta_g}} \\
&= \aleph_1^{\frac{1}{1}} \otimes \aleph_2^{\frac{0}{1}} \otimes \dots \aleph_r^{\frac{0}{1}} \\
&= \aleph_1 = (0, 1)
\end{aligned}$$

□

Corollary 4.15 meant that, if the higher priority criteria are met by the smallest q-ROFN, rewards will not be received by other criteria even though they are fulfilled.

Theorem 4.16. *(Monotonicity) Consider $\aleph_q = \langle \mu_q, \Re_q \rangle$ and $\aleph_q^* = \langle \mu_q^*, \Re_q^* \rangle$ are the assemblages of q-ROFNs, where $\zeta_q = \prod_{g=1}^{q-1} \widehat{F}(\aleph_g)$, $\zeta_q^* = \prod_{g=1}^{q-1} \widehat{F}(\aleph_g^*)$ $(g = 2 \dots, n)$, $\zeta_1 = 1$, $\zeta_1^* = 1$, $\widehat{F}(\aleph_g)$ is the score of $\aleph_g$ q-ROFN, and $\widehat{F}(\aleph_g^*)$ is the score of $\aleph_g^*$ q-ROFN. If $\mu_q^* \geq \mu_q$ and $\Re_q^* \leq \Re_q$ for all q, then*

$$\text{q-ROFPIWG}(\aleph_1, \aleph_2, \dots \aleph_r) \leq \text{q-ROFPIWG}(\aleph_1^*, \aleph_2^*, \dots \aleph_r^*)$$

Proof. Proof of this theorem is same as Theorem 4.7.

□

Theorem 4.17. *Consider $\aleph_q = \langle \mu_q, \Re_q \rangle$ and $\breve{\breve{\pounds}}_q = \langle \sigma_q, \tau_q \rangle$ are two assemblages of q-ROFNs, where $\zeta_g = \prod_{g=1}^{r-1} \widehat{F}(\aleph_g)$ $(g = 2 \dots, r)$, $\zeta_1 = 1$ and $\widehat{F}(\aleph_g)$ is the score of j^{th} q-ROFN. If $R > 0$ and $\breve{\breve{\pounds}} = \langle \mu_{\breve{\breve{\pounds}}}, \Re_{\breve{\breve{\pounds}}} \rangle$ is an q-ROFN, then*

1. *$\text{q-ROFPIWG}(\aleph_1 \otimes \breve{\breve{\pounds}}, \aleph_2 \otimes \breve{\breve{\pounds}}, \dots \aleph_r \otimes \breve{\breve{\pounds}}) = \text{q-ROFPIWG}(\aleph_1, \aleph_2, \dots \aleph_r) \otimes \breve{\breve{\pounds}}$*

2. *$\text{q-ROFPIWG}(R\aleph_1, R\aleph_2, \dots R\aleph_r) = R \ \text{q-ROFPIWG}(\aleph_1, \aleph_2, \dots \aleph_r)$*

3. *$\text{q-ROFPIWG}(\aleph_1 \otimes \breve{\breve{\pounds}}_1, \aleph_2 \otimes \breve{\breve{\pounds}}_2, \dots \aleph_r \otimes \breve{\breve{\pounds}}_n) = \text{q-ROFPIWG}(\aleph_1, \aleph_2, \dots \aleph_r) \otimes \text{q-ROFPIWG}(\breve{\breve{\pounds}}_1, \breve{\breve{\pounds}}_2, \dots \breve{\breve{\pounds}}_r)$*

4. *$\text{q-ROFPIWG} \ R\aleph_1 \otimes \breve{\breve{\pounds}}, R\aleph_2 \otimes \breve{\breve{\pounds}}, \dots \otimes R\aleph_r \otimes \breve{\breve{\pounds}}) = R \ \text{q-ROFPIWG}(\aleph_1, \aleph_2, \dots \aleph_r) \otimes \breve{\breve{\pounds}}$*

5 Methodology for MCGDM using proposed AOs

Let $\top = \{\top_1, \top_2, \dots, \top_m\}$ be the collection of alternatives and $\gamma = \{\gamma_1, \gamma_2, \dots, \gamma_n\}$ is the assemblage of criterions, in this situation, priority are assigned between the criteria offered by the linear orientation. $\gamma_1 \succ \gamma_2 \succ \gamma_3 \dots \gamma_n$ indicates criteria γ_J has a high priority than γ_i if $j > i$. $\pounds = \{\pounds_1, \pounds_2, \dots, \pounds_p\}$ are the DMs. Prioritization betwixt the DMs given as, $\pounds_1 \succ \pounds_2 \succ \pounds_3 \dots \pounds_p$ shows DM $\pounds_\zeta$ has a high importance than $\pounds_\varrho$ if $\zeta > \varrho$. And no need for normalization if all performer parameters are of the same type; however, because MCGDM has two types of assessment criteria (benefit kind features

τ_b and cost kind features τ_c), the matrix $D(p)$ has been changed into a normalize matrix using the normalization formula $Y^{(p)} = (\mathscr{P}_{ij}^{(p)})_{m\times n}$,

$$(\mathscr{P}_{ij}^{(p)})_{m\times n} = \begin{cases} (\mathscr{B}_{ij}^{(p)})^c; & j \in \tau_c \\ \mathscr{B}_{ij}^{(p)}; & j \in \tau_b. \end{cases} \tag{5.1}$$

where $(\mathscr{B}_{ij}^{(p)})^c$ show the compliment of $\mathscr{B}_{ij}^{(p)}$.
The suggested operators will be implemented to the MCGDM, which will require the preceding steps.

Procedural steps

Step 1:
Obtain the decision matrix $D^{(p)} = (\mathscr{B}_{ij}^{(p)})_{m\times n}$ in the format of q-ROFNs from DMs.

$$\begin{array}{cc} & \\ \mathcal{L}_1 & \top_1 \\ & \top_2 \\ & \\ & \top_m \\ \mathcal{L}_2 & \top_1 \\ \vdots & \top_2 \\ & \\ & \top_m \\ \mathcal{L}_p & \top_1 \\ & \top_2 \\ & \\ & \top_m \end{array}
\begin{array}{c} \begin{array}{cccc} \gamma_1 & \gamma_2 & & \gamma_n \end{array} \\
\begin{bmatrix}
(\mu_{11}^1, \Re_{11}^1) & (\mu_{12}^1, \Re_{12}^1) & \cdots\cdots & (\mu_{1n}^1, \Re_{1n}^1) \\
(\mu_{21}^1, \Re_{21}^1) & (\mu_{22}^1, \Re_{22}^1) & \cdots\cdots & (\mu_{2n}^1, \Re_{2n}^1) \\
\vdots & \vdots & \ddots\ddots & \vdots \\
(\mu_{m1}^1, \Re_{m1}^1) & (\mu_{m2}^1, \Re_{m2}^1) & \cdots\cdots & (\mu_{mn}^1, \Re_{mn}^1) \\
(\mu_{11}^2, \Re_{11}^2) & (\mu_{12}^2, \Re_{12}^2) & \cdots\cdots & (\mu_{1n}^2, \Re_{1n}^2) \\
(\mu_{21}^2, \Re_{21}^2) & (\mu_{22}^2, \Re_{22}^2) & \cdots\cdots & (\mu_{2n}^2, \Re_{2n}^2) \\
\vdots & \vdots & \ddots\ddots & \vdots \\
(\mu_{m1}^2, \Re_{m1}^2) & (\mu_{m2}^2, \Re_{m2}^2) & \cdots\cdots & (\mu_{mn}^2, \Re_{mn}^2) \\
(\mu_{11}^p, \Re_{11}^p) & (\mu_{12}^p, \Re_{12}^p) & \cdots\cdots & (\mu_{1n}^p, \Re_{1n}^p) \\
(\mu_{21}^p, \Re_{21}^p) & (\mu_{22}^p, \Re_{22}^p) & \cdots\cdots & (\mu_{2n}^p, \Re_{2n})^p \\
\vdots & \vdots & \ddots\ddots & \vdots \\
(\mu_{m1}^p, \Re_{m1}^p) & (\mu_{m2}^p, \Re_{m2}^p) & \cdots\cdots & (\mu_{mn}^p, \Re_{mn}^p)
\end{bmatrix} \end{array}$$

Step 2:
Find the normalization matrix $Y^{(p)} = (\mathscr{P}_{ij}^{(p)})_{m\times n}$ using the Equation 5.1
Step 3:
Evaluate the values of $\breve{\varpi}_{ij}^{(p)}$ by given formula.

$$\breve{\varpi}_{ij}^{(p)} = \overline{\prod}_{k=1}^{p-1} \bar{\mho}(\mathscr{P}_{ij}^{(k)}) \quad (p = 2\ldots, n), \tag{5.2}$$

$$\breve{\varpi}_{ij}^{(1)} = 1$$

Step 4:
Using one of provided AOs to combine all of the independent q-ROF decision matrices $Y^{(p)} = (\mathscr{P}_{ij}^{(p)})_{m\times n}$ into one combined evaluation matrix of the alternatives $W^{(p)} = (\mathcal{E}_{ij})_{m\times n}$.

$\mathcal{E}_{ij} = \text{q-ROFPIWA}(\mathscr{P}_{ij}^{(1)}, \mathscr{P}_{ij}^{(2)}, \ldots \mathscr{P}_{ij}^{(p)})$

$$= \left(\sqrt[q]{1 - \prod_{z=1}^{p} \left(1 - ((\mu_{ij}^z)^q\right)^{\frac{\breve{\varpi}_j^z}{\sum_{j=1}^n \breve{\varpi}_j^z}}}, \sqrt[q]{\prod_{z=1}^{p} \left(1 - (\mu_{ij}^z)^q\right)^{\frac{\breve{\varpi}_j^z}{\sum_{j=1}^n \breve{\varpi}_j^z}} - \prod_{z=1}^{p} \left(1 - ((\mu_{ij}^z)^q + (\Re_{ij}^z)^q)\right)^{\frac{\breve{\varpi}_j^z}{\sum_{j=1}^n \breve{\varpi}_j^z}}} \right) \tag{5.3}$$

or
$\mathcal{E}_{ij} = \text{q-ROFPIWG}(\mathscr{P}_{ij}^{(1)}, \mathscr{P}_{ij}^{(2)}, \ldots \mathscr{P}_{ij}^{(p)})$

$$= \left(\sqrt[q]{\prod_{z=1}^{p} \left(1 - (\Re_{ij}^z)^q\right)^{\frac{\breve{\varpi}_j^z}{\sum_{j=1}^n \breve{\varpi}_j^z}} - \prod_{z=1}^{p} \left(1 - ((\Re_{ij}^z)^q + (\mu_{ij}^z)^q)\right)^{\frac{\breve{\varpi}_j^z}{\sum_{j=1}^n \breve{\varpi}_j^z}}}, \sqrt[q]{1 - \prod_{z=1}^{p} \left(1 - ((\Re_{ij}^z)^q\right)^{\frac{\breve{\varpi}_j^z}{\sum_{j=1}^n \breve{\varpi}_j^z}}} \right) \tag{5.4}$$

Step 5:
Calculate the values of $\breve{\varpi}_{ij}$ by following formula.

$$\breve{\varpi}_{ij} = \overline{\prod}_{k=1}^{j-1} \bar{\mho}(\mathcal{E}_{ik}) \quad (j = 2 \ldots, n), \tag{5.5}$$

$$\breve{\varpi}_{i1} = 1$$

Step 6:
Aggregate the q-ROF values $\mathcal{E}_{ij}$ for each alternative $\top_i$ by the q-ROFPIWA (or q-ROFPIWG) operator.

$\mathcal{E}_{ij} = \text{q-ROFPIWA}(\mathscr{P}_{i1}, \mathscr{P}_{i2}, \ldots \mathscr{P}_{in})$

$$= \left(\sqrt[q]{1 - \prod_{j=1}^{n} \left(1 - \mu_{ij}^{q}\right)^{\frac{\breve{\varpi}_j}{\sum_{j=1}^{n} \breve{\varpi}_j}}}, \sqrt[q]{\prod_{j=1}^{n} \left(1 - \mu_{ij}^{q}\right)^{\frac{\breve{\varpi}_j}{\sum_{j=1}^{n} \breve{\varpi}_j}} - \prod_{j=1}^{n} \left(1 - (\mu_{ij}^{q} + \Re_{ij}^{q})\right)^{\frac{\breve{\varpi}_j}{\sum_{j=1}^{n} \breve{\varpi}_j}}} \right) \tag{5.6}$$

or
$\mathcal{E}_{ij} = \text{q-ROFPIWG}(\mathscr{P}_{i1}, \mathscr{P}_{i2}, \ldots \mathscr{P}_{in})$

$$= \left(\sqrt[q]{\prod_{j=1}^{n} \left(1 - \Re_{ij}^{q}\right)^{\frac{\breve{\varpi}_j}{\sum_{j=1}^{n} \breve{\varpi}_j}} - \prod_{j=1}^{n} \left(1 - (\Re_{ij}^{q} + \mu_{ij}^{q})\right)^{\frac{\breve{\varpi}_j}{\sum_{j=1}^{n} \breve{\varpi}_j}}}, \sqrt[q]{1 - \prod_{j=1}^{n} \left(1 - \Re_{ij}^{q}\right)^{\frac{\breve{\varpi}_j}{\sum_{j=1}^{n} \breve{\varpi}_j}}} \right) \tag{5.7}$$

Step 7:
Analyze the score for all cumulative alternative assessments.

Step 8:
The alternatives were classified by the SF and, eventually, the most suitable alternative was selected.

6 Case Study

Significant progress in many countries in reducing air pollution appears to have negative effects on natural substantiality, the economy and public health. Road freight transport, in particular, is related to a range of negative externalizes, including risks to human health and the overuse of non-renewable environmental assets. Transportation is a high-priority field of action for long-term substantiality. It plays an important role in the economy due to its omnipresence in the supply chain and at all geographic scales. However, transport is also considered to be the sector with the highest rate of rise in pollution. The fight against anthropogenic climate change is one of the most urgent challenges facing humanity today. The transport sector is a major energy user and a major source of GHG emissions due to its high dependence on fossil fuels. CO_2 and other GHG affect climate change. Sources of air pollution are on-road vehicles and engines and non-road vehicles and engines. Emission reductions, for example, can result in cleaner air and greater health as a solution to vehicle transport air pollution. Catalytic converters, when combined with low sulphur levels and unleaded fuel, greatly cut nitrogen oxide and hydrocarbon emissions. Furthermore, fuel requirements minimize exposure to contaminants like benzene and lead. Renewable fuels help to minimize CO_2 emissions. Computer controls, multi-valve engines,variable valve timing, turbo-charging, and gasoline direct injection are examples of engine technology that improve fuel efficiency and reduce CO_2 emissions. Transmissions with seven or more gears, "dual clutch transmissions" (DCTs), and "continuously variable transmissions" (CVTs) reduce CO_2 emissions and fuel consumption. Diesel filters are used in both off-road and on-road diesel vehicles to remove particulate matter. Alternative automotive systems, such as fuel cells and plug-in hybrids, emit no emissions at all. Better planning for passenger and freight transportation decreases emissions and fuel consumption.

Transport, in addition to the generation of electricity and industrial activity, is a major source of air pollution. In the enlarged European Union, current levels of air pollution are causing serious health effects, resulting in 370,000 premature lives lost each year, increasing hospital visits, additional treatment, and millions of missed working days. Transport contributes greatly to global CO_2 emissions and is one of the key sources of environmental pollution. If the global average temperature increases above the 2C safety level, this could have catastrophic implications for environmental sustainability [71]. Transport accounted for 23 per cent of global carbon emissions from energy consumption in 2014, while road transport accounted for 20 per cent. Road transport accounts for almost 92% of all CO_2 emissions, including 6% of total CO_2 emissions, in the United Kingdom, among others. In some nations, such as India and China, CO_2 emissions have increased and decreased in other countries, such as Europe, following ratification of the Kyoto protocol on climate change. While CO_2 emissions have decreased significantly in many other sectors of the economy, CO_2 emissions in the transportation sector have continued to rise, and transportation pollution mitigation appears to be more expensive than in other sectors due to its reliance on solid resources and conventional supporting structures. As a result, both stakeholder and consumers of road transportation

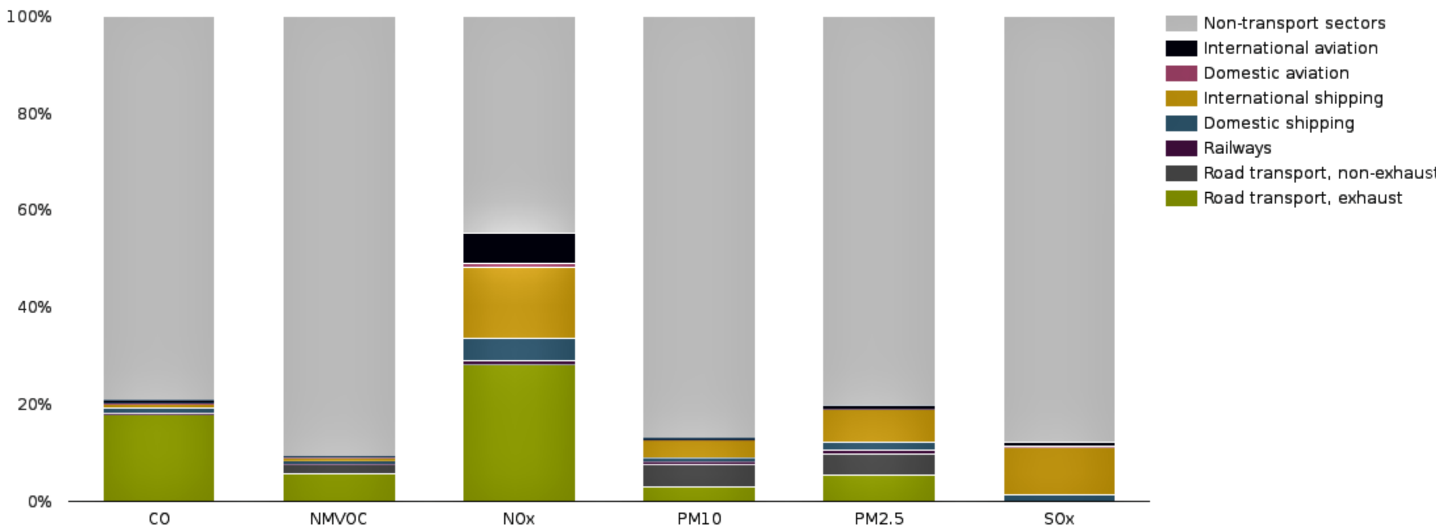

Figure 2: The transportation sector's contribution to overall emissions of the major air pollutants [72]

services have tried to incorporate some creative solutions and clean technology to minimize energy usage by transport and climate change. Cars are a significant cause of air pollution that is detrimental to public health. Vehicle emissions leads to the development of ground-level ozone (smog), which can worsen asthma, minimize lung capacity, and increase susceptibility to respiratory diseases such as bronchitis and pneumonia. Fine particulate matter is often released by motor vehicles, in particular those used for freight. Particulate matter in the air has been linked in many clinical studies to severe health problems such as asthma, chronic bronchitis and heart attacks. Because long-term exposure to diesel particulate matter is likely to cause lung cancer, this is of particular concern.

Between 1990 and 2017, the transportation industry dramatically reduced emissions of "sulphur oxides" (66%), "non-methane volatile organic compounds" and "carbon monoxide (CO)" (both by approximately 87%), and "nitrogen oxides" (40%). Since 2000, particulate matter emissions have decreased. (44 % for PM2.5, and 35% for PM10). Over the previous two decades, emissions from road transportation have fallen less than projected and continue to fall. Emissions were lower in 2017 than the last year: "nitrogen oxide" emissions decreased by 3%, "carbon monoxide" emissions decreased by 3.2 percent, and PM10 and PM2.5 emissions decreased by 1.4 and 3.6 percent, respectively. "Sulphur oxide (SOx)" emissions rose by 2.7 percent in 2017 compared to 2016, but they are still less than one percent of what they were in 1990. Emissions of air pollutants have decreased for all modes of transportation since 1990, with the exception of shipping, which has increased "nitrogen oxide (NOx)" emissions, and aviation, which has increased emissions of all pollutants. Since 1990, significant progress has been made in reducing the emissions of many air contaminants from the transportation sector. Despite a general rise in operation in the industry, emissions from all modes of transportation have decreased since 1990. Figure 2 show transportation sector's contribution to overall emissions of the major air pollutants and Figure 3 define summary trends in transportation-related air pollution emissions. During 1990 to 2017, emissions of NOx from transport decreased by 40% across the EEA-33, SOx decreased by 66%, and CO decreased by 87% [72].

There are four ways for transport authorities and local governments to minimize traffic-related air pollution and improve air quality:

1. Develop safer mobility options by extending mass transit networks, improving the efficiency of public transport and, among other things, developing or improving bicycle and pedestrian facilities.
2. Reduce the distance betwixt the main destinations required to meet daily needs by making land use planning and growth more effective, making walking or cycling more attractive and convenient as a mode of transport.
3. Install or promote the growth of green charging platforms, such as electric vehicle charging stations and hydrogen refueling stations.
4. Regulate the mass transit system to enhance automobile and system performance by implementing anti-idling policies, enhanced incident response, real-time public transportation travel information, and congestion control.

Appropriate policy instruments are urgently required to help reduce and track the negative impact of transport activities. Indicators can be useful policy instruments for assessing and evaluating the efficiency of transport systems. Attributes are often seen as quantitative dimensions that can be used to "simply explain and communicate complex phenomena, including patterns and development over time." The academic group and policy analysts have widely used indicators to quantify attainability issues over the last 20 years. The 1992 United Nations "conference on environment and development" (UNCED) in Rio de Janeiro was the first time that sustainable development indicators have been placed on the political agenda.The UNCED Policy Statement Agenda 21 called on countries to develop indicators at national level, as well as foreign governments and non-governmental stakeholder, to develop indicators at international level in order to improve information on decision-making. Since then, indicators have been considered to be useful tools for evaluating different aspects of attainability,

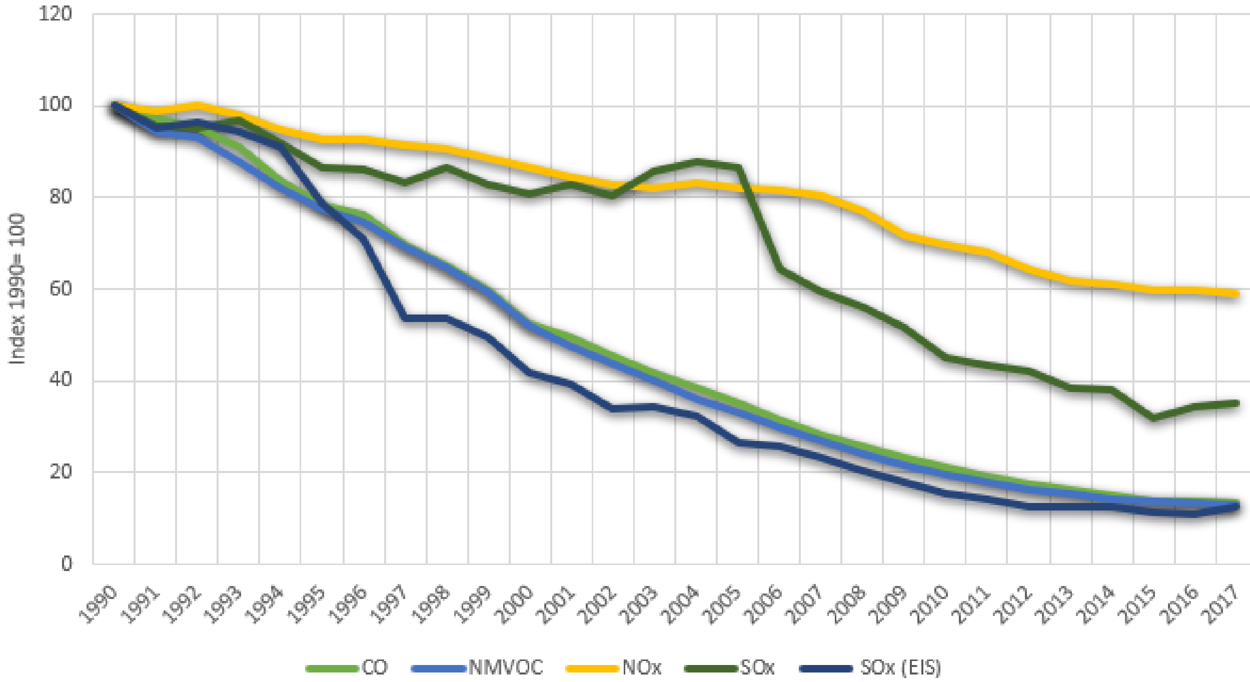

Figure 3: Trends in transportation-related air pollution emissions

including transport related issues. There are main 5 classes of criterions which are economic, social, environmental, technical & operational and institutional. In economic we include, transport infrastructure, demand, intensity, costs and prices. In social criteria accessibility, mobility, health impacts, afford-ability and employment are observed. In environmental criteria we analysis transport emissions (PM_{10} and $PM_{2.5}$ emissions (per capita), SO_x emissions (per capita), O_3 concentration (per capita), CO_2 emissions (per capita), N_2O emissions (per capita), CH_4 emissions (per capita)). Technical & operational criteria evaluated occupancy of transportation and technology status, institutional criteria evaluated measures to Improve Transport sustainability and institutional development [73].

To illustrate the possibilities for Vehicles Freight Transportation problem by means of reasoning based on the q-ROFSs, There are five alternatives $\top_i$ $(i = 1, 2, 3, 4, 5)$, where $\top_1$ = hybrid electric vehicle, $\top_2$ = battery electric, $\top_3$ = CNG/RNG vehicles, $\top_4$ = diesel vehicles and $\top_5$ = Fuel Cell (H_2) vehicles. We consider γ_1 = transport infrastructure, γ_2 = cost, γ_3 = environmental criteria, γ_4 = technical & operational criteria, γ_5 = social criteria and γ_6 = energy efficiency as attributes. Details related to criterion are given in Table 3. Priorities are assigned betwixt the criteria provided by the linear orientation in this case. $\gamma_1 \succ \gamma_2 \succ \gamma_3 \ldots \gamma_6$ indicates criteria γ_J has a high priority than γ_i if $j > i$. In this example we use q-ROFNs as input data for ranking the given alternatives under the given attributes. Here three DMs are involved i.e $\overline{\breve{\pounds}}_1$, $\overline{\breve{\pounds}}_2$ and $\overline{\breve{\pounds}}_3$. DMs are not given the same priority. Prioritization is provided by a linear pattern betwixt the DMs given as, $\pounds_1 \succ \pounds_2 \succ \pounds_3$ shows DM $\pounds_\zeta$ has a high imprtance than $\pounds_\varrho$ if $\zeta > \varrho$.

Table 3: Details of criterion

Indicators	Details
Transport infrastructure (γ_1)	Opportunities to achieve additional benefits with the use of information and communication technologies and needs to develop refuelling or communication systems.
Cost (γ_2)	Research, development and production costs. Environmental and health costs induced by the use of the particular technology.
Environmental criteria (γ_3)	NOx, HC, CO and PM emissions produced by the vehicle's propulsion.
Technical & operational criteria (γ_4)	Identify the essential capabilities, associated requirements, performance measures, and the process or series of actions to be taken in effecting the results
Social criteria (γ_5)	Assessment of the people's response to particular technology.
Energy efficiency (γ_6)	Efficiency of converting the on-board fuel energy content to vehicle displacement.

Using q-ROFPIWA operator

Step 1:

Obtain the decision matrix $D^{(p)} = (\mathscr{B}_{ij}^{(p)})_{m \times n}$ in the format of q-ROFNs by the DMs, given in Table 4, Table 5 and Table 6.

Table 4: q-ROF decision matrix from $\pounds_1$.

	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
$\top_1$	(0.96,0.00)	(0.75,0.23)	(0.85,0.13)	(0.87,0.11)	(0.87,0.00)	(0.66,0.14)
$\top_2$	(0.85,0.15)	(0.75,0.45)	(0.15,0.75)	(0.55,0.25)	(0.35,0.35)	(0.44,0.16)
$\top_3$	(0.74,0.15)	(0.35,0.55)	(0.75,0.25)	(0.55,0.00)	(0.65,0.45)	(0.45,0.00)
$\top_4$	(0.40,0.35)	(0.75,0.45)	(0.55,0.15)	(0.45,0.25)	(0.65,0.35)	(0.25,0.65)
$\top_5$	(0.70,0.35)	(0.65,0.00)	(0.25,0.25)	(0.35,0.55)	(0.45,0.25)	(0.45,0.75)

Table 5: q-ROF decision matrix from $\pounds_2$.

	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
$\top_1$	(0.35,0.35)	(0.40,0.30)	(0.55,0.15)	(0.90,0.15)	(0.80,0.25)	(0.90,0.15)
$\top_2$	(0.75,0.15)	(0.66,0.35)	(0.75,0.15)	(0.75,0.35)	(0.65,0.30)	(0.35,0.00)
$\top_3$	(0.80,0.60)	(0.55,0.20)	(0.35,0.55)	(0.65,0.65)	(0.25,0.25)	(0.70,0.30)
$\top_4$	(0.40,0.00)	(0.53,0.40)	(0.35,0.10)	(0.50,0.45)	(0.50,0.15)	(0.20,0.35)
$\top_5$	(0.35,0.35)	(0.60,0.30)	(0.55,0.55)	(0.25,0.30)	(0.30,0.30)	(0.30,0.25)

Table 6: q-ROF decision matrix from $\pounds_3$.

	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
$\top_1$	(0.50,0.15)	(0.75,0.25)	(0.80,0.00)	(0.50,0.35)	(0.70,0.20)	(0.70,0.30)
$\top_2$	(0.70,0.25)	(0.45,0.15)	(0.50,0.35)	(0.40,0.30)	(0.50,0.30)	(0.50,0.30)
$\top_3$	(0.45,0.15)	(0.55,0.25)	(0.15,0.00)	(0.40,0.35)	(0.35,0.30)	(0.25,0.25)
$\top_4$	(0.65,0.35)	(0.40,0.35)	(0.35,0.35)	(0.35,0.45)	(0.15,0.25)	(0.55,0.00)
$\top_5$	(0.55,0.25)	(0.50,0.15)	(0.45,0.25)	(0.35,0.25)	(0.55,0.55)	(0.35,0.40)

Step 2:
Using Equation 5.1, normalize decision matrixes gained by DMs. γ_2 is cost type criteria and others are benefit type criterions, , given in Table 7, Table 8 and Table 9.

Table 7: Normalized q-ROF decision matrix from $\pounds_1$.

	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
$\top_1$	(0.96,0.00)	(0.23,0.75)	(0.85,0.13)	(0.87,0.11)	(0.87,0.00)	(0.66,0.14)
$\top_2$	(0.85,0.15)	(0.45,0.75)	(0.15,0.75)	(0.55,0.25)	(0.35,0.35)	(0.44,0.16)
$\top_3$	(0.74,0.15)	(0.55,0.35)	(0.75,0.25)	(0.55,0.00)	(0.65,0.45)	(0.45,0.00)
$\top_4$	(0.40,0.35)	(0.45,0.75)	(0.55,0.15)	(0.45,0.25)	(0.65,0.35)	(0.25,0.65)
$\top_5$	(0.70,0.35)	(0.00,0.65)	(0.25,0.25)	(0.35,0.55)	(0.45,0.25)	(0.45,0.75)

Table 8: Normalized q-ROF decision matrix from $\pounds_2$.

	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
$\top_1$	(0.35,0.35)	(0.30,0.40)	(0.55,0.15)	(0.90,0.15)	(0.80,0.25)	(0.90,0.15)
$\top_2$	(0.75,0.15)	(0.35,0.66)	(0.75,0.15)	(0.75,0.35)	(0.65,0.30)	(0.35,0.00)
$\top_3$	(0.80,0.60)	(0.20,0.55)	(0.35,0.55)	(0.65,0.65)	(0.25,0.25)	(0.70,0.30)
$\top_4$	(0.40,0.00)	(0.40,0.53)	(0.35,0.10)	(0.50,0.45)	(0.50,0.15)	(0.20,0.35)
$\top_5$	(0.35,0.35)	(0.30,0.60)	(0.55,0.55)	(0.25,0.30)	(0.30,0.30)	(0.30,0.25)

Table 9: Normalized q-ROF decision matrix from $\pounds_3$.

	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
$\top_1$	(0.50,0.15)	(0.25,0.75)	(0.80,0.00)	(0.50,0.35)	(0.70,0.20)	(0.70,0.30)
$\top_2$	(0.70,0.25)	(0.15,0.45)	(0.50,0.35)	(0.40,0.30)	(0.50,0.30)	(0.50,0.30)
$\top_3$	(0.45,0.15)	(0.25,0.55)	(0.15,0.00)	(0.40,0.35)	(0.35,0.30)	(0.25,0.25)
$\top_4$	(0.65,0.35)	(0.35,0.40)	(0.35,0.35)	(0.35,0.45)	(0.15,0.25)	(0.55,0.00)
$\top_5$	(0.55,0.25)	(0.15,0.50)	(0.45,0.25)	(0.35,0.25)	(0.55,0.55)	(0.35,0.40)

Step 3:
Determine the $\breve{\varpi}_{ij}^{(p)}$ values using the Equation 5.2 .

$$\breve{\varpi}_{ij}^{(1)} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\breve{\varpi}_{ij}^{(2)} = \begin{pmatrix} 0.9423 & 0.2951 & 0.8060 & 0.8286 & 0.8293 & 0.6297 \\ 0.8053 & 0.3346 & 0.2908 & 0.5000 & 0.5000 & 0.5405 \\ 0.7009 & 0.5618 & 0.7031 & 0.5918 & 0.5918 & 0.5456 \\ 0.5106 & 0.3346 & 0.5815 & 0.6159 & 0.6159 & 0.3705 \\ 0.6501 & 0.3627 & 0.4383 & 0.5378 & 0.5378 & 0.3346 \end{pmatrix}$$

$$\breve{\varpi}_{ij}^{(3)} = \begin{pmatrix} 0.4712 & 0.1421 & 0.4687 & 0.7149 & 0.6204 & 0.5433 \\ 0.5712 & 0.1264 & 0.2005 & 0.3448 & 0.3119 & 0.2818 \\ 0.4541 & 0.2364 & 0.3082 & 0.2959 & 0.2959 & 0.3590 \\ 0.2716 & 0.1531 & 0.3029 & 0.3184 & 0.3454 & 0.1802 \\ 0.3250 & 0.1471 & 0.2192 & 0.2658 & 0.2689 & 0.1692 \end{pmatrix}$$

Step 4:
Use q-ROFPIWA to aggregate all individual q-ROF decision matrices $Y^{(p)} = (\mathscr{P}_{ij}^{(p)})_{m\times n}$ into one cumulative assessments matrix of the alternatives $W^{(p)} = (\mathcal{E}_{ij})_{m\times n}$ using Equation 5.3 given in Table 10.

Table 10: Collective q-ROF decision matrix

	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
$\top_1$	(0.8475, 0.1934)	(0.2497, 0.7121)	(0.8129, 0.1287)	(0.8375, 0.2005)	(0.8169, 0.1854)	(0.7741, 0.2035)
$\top_2$	(0.7979, 0.1789)	(0.4178, 0.7207)	(0.4923, 0.6516)	(0.6114, 0.3026)	(0.5026, 0.3277)	(0.4294, 0.1885)
$\top_3$	(0.7307, 0.4752)	(0.4647, 0.4556)	(0.6303, 0.3868)	(0.5721, 0.4901)	(0.5463, 0.4016)	(0.5420, 0.2323)
$\top_4$	(0.4357, 0.2983)	(0.4313, 0.6995)	(0.4799, 0.2057)	(0.4552, 0.3759)	(0.5712, 0.3037)	(0.3254, 0.5768)
$\top_5$	(0.6097, 0.3404)	(0.1902, 0.6274)	(0.4078, 0.3994)	(0.3266, 0.4736)	(0.4385, 0.3573)	(0.4157, 0.6855)

Step 5:
Determine the values of $\breve{\varpi}_{ij}$ by using Equation 5.5.

$$\breve{\varpi}_{ij} = \begin{pmatrix} 1 & 0.8007 & 0.2620 & 0.2011 & 0.1588 & 0.1222 \\ 1 & 0.7511 & 0.2624 & 0.1105 & 0.0663 & 0.0362 \\ 1 & 0.6414 & 0.3226 & 0.1924 & 0.1029 & 0.0565 \\ 1 & 0.5281 & 0.1948 & 0.1073 & 0.0559 & 0.0324 \\ 1 & 0.5936 & 0.2202 & 0.1106 & 0.0514 & 0.0267 \end{pmatrix}$$

Step 6:
Aggregate the q-ROF values $\mathcal{E}_{ij}$ for each alternative $\top_i$ by the q-ROFPIWA operator using Equation 5.6 given in Table 11.

Table 11: q-ROF Aggregated values $\mathcal{E}_i$

$\mathcal{E}_1$	(0.774416, 0.425423)
$\mathcal{E}_2$	(0.679893, 0.518536)
$\mathcal{E}_3$	(0.644932, 0.459714)
$\mathcal{E}_4$	(0.444681, 0.505356)
$\mathcal{E}_5$	(0.511121, 0.475054)

Step 7:
Compute the score for all q-ROF aggregated values $\mathcal{E}_i$.

$$\widehat{F}(\mathcal{E}_1) = 0.693719$$

$$\widehat{F}(\mathcal{E}_2) = 0.58743$$

$$\widehat{F}(\mathcal{E}_3) = 0.585548$$

$$\widehat{F}(\mathcal{E}_4) = 0.479436$$

$$\widehat{F}(\mathcal{E}_5) = 0.513159$$

Step 8:
Ranks according to score values.

$$\mathcal{E}_1 \succ \mathcal{E}_2 \succ \mathcal{E}_3 \succ \mathcal{E}_5 \succ \mathcal{E}_4$$

So,

$$\top_1 \succ \top_2 \succ \top_3 \succ \top_5 \succ \top_4$$

$\top_1$ is best alternative among all other alternatives.

6.1 Comparison analysis

In this section, we compare recommended operators to specific AOs that are currently in use. The fact that they both produce the same result demonstrates the superiority of our proposed AOs. We compare our outcomes and arrive at the same ideal conclusion by resolving the information data with certain current AOs. This demonstrates the robustness and consistency of the paradigm we suggested. We obtain $\top_1 \succ \top_3 \succ \top_2 \succ \top_5 \succ \top_4$ rating by our proposed AOs; to validate our optimal option, We evaluate this issue through other AOs that are already in place. The fact that we arrive at the same optimal decision demonstrates the validity of our proposed AOs. The Table 12 compares the AOs available with certain existing AOs.

Table 12: Comparison of proposed operators with some exiting operators

Authors	AOs	Ranking of alternatives	The optimal alternative
Riaz *et al.* [38]	q-ROFEWA	$\top_1 \succ \top_2 \succ \top_3 \succ \top_5 \succ \top_4$	$\top_1$
	q-ROFEOWA	$\top_1 \succ \top_2 \succ \top_3 \succ \top_5 \succ \top_4$	$\top_1$
Riaz *et al.* [39]	q-ROFPWA	$\top_1 \succ \top_2 \succ \top_5 \succ \top_3 \succ \top_4$	$\top_1$
	q-ROFPWG	$\top_1 \succ \top_2 \succ \top_5 \succ \top_3 \succ \top_4$	$\top_1$
Liu & Wang [32]	q-ROFWA	$\top_1 \succ \top_2 \succ \top_3 \succ \top_5 \succ \top_4$	$\top_1$
	q-ROFWG	$\top_1 \succ \top_3 \succ \top_2 \succ \top_4 \succ \top_5$	$\top_1$
Liu & Liu [42]	q-ROFWBM	$\top_1 \succ \top_2 \succ \top_3 \succ \top_5 \succ \top_4$	$\top_1$
	q-ROFWGBM	$\top_1 \succ \top_3 \succ \top_2 \succ \top_4 \succ \top_5$	$\top_1$
Jana *et al.* [35]	q-ROFDWA	$\top_1 \succ \top_2 \succ \top_3 \succ \top_5 \succ \top_4$	$\top_1$
	q-ROFDWG	$\top_1 \succ \top_2 \succ \top_3 \succ \top_5 \succ \top_4$	$\top_1$
Peng *et al.* [34]	q-ROFEWA	$\top_1 \succ \top_2 \succ \top_3 \succ \top_5 \succ \top_4$	$\top_1$
	q-ROFEWG	$\top_1 \succ \top_2 \succ \top_4 \succ \top_5 \succ \top_3$	$\top_1$
Riaz *et al.* [40]	q-ROFEPWA	$\top_1 \succ \top_2 \succ \top_3 \succ \top_5 \succ \top_4$	$\top_1$
	q-ROFEPWG	$\top_1 \succ \top_2 \succ \top_3 \succ \top_5 \succ \top_4$	$\top_1$
Joshi & Gegov [43]	CQROFWA	$\top_1 \succ \top_2 \succ \top_3 \succ \top_5 \succ \top_4$	$\top_1$
	CQROFWG	$\top_1 \succ \top_2 \succ \top_3 \succ \top_5 \succ \top_4$	$\top_1$
Proposed	q-ROFPIWA	$\top_1 \succ \top_2 \succ \top_3 \succ \top_5 \succ \top_4$	$\top_1$
	q-ROFPIWG	$\top_1 \succ \top_3 \succ \top_2 \succ \top_5 \succ \top_4$	$\top_1$

7 Conclusion

The failure to grasp attribute correlations in an unpredictability scenario can have an impact on the conclusions in a number of MCGDM difficulties. To address these issues, we developed a novel approach for selecting sustainable freight transportation utilizing q-ROF data, in which the q-ROFNs took into account the DM's judgement. The q-ROFNs were used to express DM's assessments, and the uncertainty and inadequacy of the information were successfully resolved. Since AOs serve a significant part in decision-making, therefore in this paper we provided hybrid AOs based on MSD-NMSD interactions of q-ROFNs and prioritized relation betwixt criterions, namely the "q-rung orthopair fuzzy prioritized interactive weighted averaging (q-ROFPIWA) operator and q-rung orthopair fuzzy prioritized interactive weighted geometric (q-ROFPIWG) operator". We discussed some basic properties of the developed operators. Finally, a descriptive example was given concerning the selection of sustainable freight transportation to highlight the possibilities for applying the proposed approach. q-ROFS has been shown to be an effective method to clarify vague and fuzzy aspects of MCDM problems and to provide data on interactions. Overall, due to their ability to incorporate the information conveyed in q-ROFNs, established operators are applicable to complex instances of MCDM. The incorporation of the interaction betwixt the MSD and the NMSD suggested by the DMs themselves increases the effectiveness of the study. If we talk about the limitations of our proposed work, it is not work properly if our information is not q-ROFNs. The proposed model works efficiently when the input is q-ROFNs. However, with some minor changes, the proposed model can be extended to handle other types of the input data.

Future studies will look at how the suggested operators can be used with different types of information and how they work in different areas. A wide range of real-world situations can benefit from the ideas in this paper. They can be used to deal with ambiguity effectively in business, machine intelligence, cognitive science, electoral system, pattern recognition, learning techniques, pyrognostics, trade analysis, forecasts, agricultural estimation, microelectronics and so on.

Declaration of Interests:
The authors has no conflict of interest.

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Appendix

Proof of Theorem 4.2
Definition 4.1 and Theorem 4.2 are easily preceded by the first statement. This is shown in the following parts.

q-ROFPIWA$(\aleph_1, \aleph_2, \ldots \aleph_r)$

$$
\begin{aligned}
&= \frac{\zeta_1}{\sum_{g=1}^{r}\zeta_g}\aleph_1 \oplus \frac{\zeta_2}{\sum_{g=1}^{r}\zeta_g}\aleph_2 \oplus \ldots, \oplus \frac{\zeta_r}{\sum_{g=1}^{r}\zeta_g}\aleph_r \\
&= \left(\sqrt[q]{1 - \prod_{g=1}^{r}\left(1-(\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}}},\ \sqrt[q]{\prod_{g=1}^{r}\left(1-(\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}} - \prod_{g=1}^{r}\left(1-((\mu_g)^q+(\Re_g)^q)\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}}} \right)
\end{aligned}
$$

This theorem is completed by mathematical induction.
when $g = 2$

$$
\text{q-ROFPIWA}(\aleph_1, \aleph_2) = \frac{\zeta_1}{\sum_{g=1}^{r}\zeta_g}\aleph_1 \oplus \frac{\zeta_2}{\sum_{g=1}^{r}\zeta_g}\aleph_2
$$

By interactive laws of q-ROFNs, we have

$$
\begin{aligned}
\frac{\zeta_1}{\sum_{g=1}^{r}\zeta_g}\aleph_1 &= \left(\sqrt[q]{1 - \left(1-\mu_1^q\right)^{\frac{\zeta_1}{\sum_{g=1}^{r}\zeta_g}}},\ \sqrt[q]{\left(1-\mu_1^q\right)^{\frac{\zeta_1}{\sum_{g=1}^{r}\zeta_g}} - \left(1-(\mu_1^q+\Re_1^q)\right)^{\frac{\zeta_1}{\sum_{g=1}^{r}\zeta_g}}} \right) \\
\frac{\zeta_2}{\sum_{g=1}^{r}\zeta_g}\aleph_2 &= \left(\sqrt[q]{1 - \left(1-\mu_2^q\right)^{\frac{\zeta_2}{\sum_{g=1}^{r}\zeta_g}}},\ \sqrt[q]{\left(1-\mu_2^q\right)^{\frac{\zeta_2}{\sum_{g=1}^{r}\zeta_g}} - \left(1-(\mu_2^q+\Re_2^q)\right)^{\frac{\zeta_2}{\sum_{g=1}^{r}\zeta_g}}} \right)
\end{aligned}
$$

Then,
q-ROFPIWA$(\aleph_1, \aleph_2)$

$$= \frac{\zeta_1}{\sum_{g=1}^{r}\zeta_g}\aleph_1 \oplus \frac{\zeta_2}{\sum_{g=1}^{r}\zeta_g}\aleph_2$$

$$= \left(\sqrt[q]{1-\left(1-\mu_1^q\right)^{\frac{\zeta_1}{\sum_{g=1}^{r}\zeta_g}}}, \sqrt[q]{\left(1-\mu_1^q\right)^{\frac{\zeta_1}{\sum_{g=1}^{r}\zeta_g}} - \left(1-(\mu_1^q+\Re_1^q)\right)^{\frac{\zeta_1}{\sum_{g=1}^{r}\zeta_g}}} \right.$$

$$\left. \oplus \sqrt[q]{1-\left(1-\mu_2^q\right)^{\frac{\zeta_2}{\sum_{g=1}^{r}\zeta_g}}}, \sqrt[q]{\left(1-\mu_2^q\right)^{\frac{\zeta_2}{\sum_{g=1}^{r}\zeta_g}} - \left(1-(\mu_2^q+\Re_2^q)\right)^{\frac{\zeta_2}{\sum_{g=1}^{r}\zeta_g}}} \right)$$

$$= \left(\sqrt[q]{1-\left(1-\mu_1^q\right)^{\frac{\zeta_1}{\sum_{g=1}^{r}\zeta_g}}\left(1-\mu_2^q\right)^{\frac{\zeta_2}{\sum_{g=1}^{r}\zeta_g}}}, \right.$$

$$\left. \sqrt[q]{\left(1-\mu_1^q\right)^{\frac{\zeta_1}{\sum_{g=1}^{r}\zeta_g}}\left(1-\mu_2^q\right)^{\frac{\zeta_2}{\sum_{g=1}^{r}\zeta_g}} - \left(1-(\mu_1^q+\Re_1^q)\right)^{\frac{\zeta_1}{\sum_{g=1}^{r}\zeta_g}}\left(1-(\mu_2^q+\Re_2^q)\right)^{\frac{\zeta_2}{\sum_{g=1}^{r}\zeta_g}}} \right)$$

$$= \left(\sqrt[q]{1-\prod_{g=1}^{2}\left(1-(\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}}}, \sqrt[q]{\prod_{g=1}^{2}\left(1-(\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}} - \prod_{g=1}^{2}\left(1-(\mu_g+\Re_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}}} \right)$$

Suppose result holds for $g = d$

q-ROFPIWA$(\aleph_1, \aleph_2, \dots \aleph_d)$

$$= \bigoplus_{g=1}^{d} \frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}\aleph_g$$

$$= \left(\sqrt[q]{1-\prod_{g=1}^{d}\left(1-(\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}}}, \sqrt[q]{\prod_{g=1}^{d}\left(1-(\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}} - \prod_{g=1}^{d}\left(1-((\mu_g)^q+(\Re_g)^q)\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}}} \right)$$

Now, we shall prove it for $g = d+1$

q-ROFPIWA$(\aleph_1, \aleph_2, \dots \aleph_d, \aleph_{d+1})$

$$= \bigoplus_{g=1}^{d} \frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}\aleph_g \oplus \frac{\zeta_{d+1}}{\sum_{g=1}^{r}\zeta_g}\aleph_{d+1}$$

$$= \left(\sqrt[q]{1-\prod_{g=1}^{d}\left(1-(\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}}}, \sqrt[q]{\prod_{g=1}^{d}\left(1-(\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}} - \prod_{g=1}^{d}\left(1-((\mu_g)^q+(\Re_g)^q)\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}}} \right)$$

$$\oplus \left(\sqrt[q]{1-\left(1-\mu_{d+1}^q\right)^{\frac{\zeta_{d+1}}{\sum_{g=1}^{r}\zeta_g}}}, \sqrt[q]{\left(1-\mu_{d+1}^q\right)^{\frac{\zeta_{d+1}}{\sum_{g=1}^{r}\zeta_g}} - \left(1-(\mu_{d+1}^q+\Re_{d+1}^q)\right)^{\frac{\zeta_{d+1}}{\sum_{g=1}^{r}\zeta_g}}} \right)$$

$$= \left(\sqrt[q]{1-\prod_{g=1}^{d+1}\left(1-(\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}}}, \sqrt[q]{\prod_{g=1}^{d+1}\left(1-(\mu_g)^q\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}} - \prod_{g=1}^{d+1}\left(1-((\mu_g)^q+(\Re_g)^q)\right)^{\frac{\zeta_g}{\sum_{g=1}^{r}\zeta_g}}} \right).$$

Biographies

Hafiz Muhammad Athar Farid received his BS degree in mathematics from the University of Education, Lahore, in 2018. He received MPhil degree in mathematics from the University of Punjab, Lahore, in 2020. He is currently PhD scholar at Department of Mathematics, University of the Punjab, Lahore, Pakistan. He is the author of 23+ SCI research papers. He is also a reviewer of international journals. His research interests include q-rung orthopair fuzzy sets, neutrosophic sets, linear diophantine fuzzy sets, multi-criteria decision-making problems, aggregation operators, information measures, information fusion, fuzzy modelling, and fuzzy topology.

Muhammad Riaz has received MSc, MPhil and PhD degrees in mathematics from Department of Mathematics, University of the Punjab, Lahore. His research interests include pure mathematics, fuzzy mathematics, topology, algebra, fuzzy sets theory, soft set theory, rough set theory with applications in decision-making problems, medical diagnosis, artificial intelligence, computational intelligence, information measures, image processing, network topology and pattern recognition. He has 23+ years regular teaching and research experience. He has published 90+ research articles in international peer-reviewed SCIE and ESCI journals with 1,500+ citations. He has been supervised 04 PhD students and 18 MPhil students. Currently, he is supervising 05 MPhil and 03 PhD students. He has been a reviewer for 40+ SCI journals.