Scientia Iranica D (2023) 30(6), 2143–2161

Sharif University of Technology
**Scientia Iranica**
*Transactions D: Computer Science & Engineering and Electrical Engineering*
http://scientiairanica.sharif.edu

# Cost effective indoor HVAC energy efficiency monitoring based on intelligent decision support system under fermatean fuzzy framework

**M. Riaz[a], H.M.A. Farid[a,*], H.M. Shakeel[b], and D. Arif[c]**

a. *Department of Mathematics, University of the Punjab, Lahore, Pakistan.*
b. *Institute of Energy & Environmental Engineering, University of the Punjab, Lahore, Pakistan.*
c. *Department of Electrical Engineering, University of Cape Town, South Africa.*

Received 5 October 2021; received in revised form 20 December 2021; accepted 23 May 2022

**KEYWORDS**
Fermatean fuzzy numbers;
Aggregation operators;
Video processing;
Convolutional neural network;
Human recognition and detection.

**Abstract.** The Heating, Ventilation, and Air Conditioning (HVAC) control system is in charge of the building's energy efficiency. Indoor energy consumption trends can be intelligently monitored and minimized. Occupancy data is essential for saving a significant amount of energy. This energy footprint can play an important part in modern smart buildings to improve indoor green environments while lowering costs. Traditional energy monitoring and control systems can be enhanced by installing an occupancy monitoring system, which consists of a network of sensors and cameras. In this paper, we offer a novel and innovative Convolutional Neural Network (CNN) based on real-time camera occupancy detection and recognition algorithms across various types of sensors, which provides realistic low-cost energy-saving solutions with robust Graphical Processing Units (GPUs). Decision-making tools can be used to select the appropriate occupancy detection and recognition alternative for indoor environment and energy monitoring and management. In this research work, we develop the "Fermatean Fuzzy Prioritized Weighted Average (FFPWA) operator and Fermatean Fuzzy Prioritized Weighted Geometric (FFPWG) operator". In the end, we give an algorithm for an Intelligent Decision Support System (IDSS) using proposed Aggregation Operators (AOs) to compare our CNN based method with other existing sensors techniques.



## 1. Introduction

The concept of smart buildings opens up possibilities for achieving the goals of an integrated power management network and, possibly, an interior sustainable environment system. Because of technological advancements, per capita energy consumption has more than doubled. The output gap has narrowed, influencing the country's lifestyle and economic progress. Energy generation research focuses on energy conservation and management efficiency enhancement. The energy and monitoring and management system can save energy while maintaining the quality of the green environment [1]. Energy is the basic component of household, industry, agriculture, transport, information, infrastructure, and technology development for any nation in the world. The growth of the economy and the

*. *Corresponding author.*
*E-mail addresses: mriaz.math@pu.edu.pk (M. Riaz); hmatharfarid@gmail.com (H.M.A. Farid); hmshakeel.567@gmail.com (H.M. Shakeel); danish.ee@pu.edu.pk (D. Arif)*

doi: 10.24200/sci.2022.59197.6106

improved standard of living are dependent on energy [2]. Therefore, per capita energy consumption should be the maximum for national evolution. There is a direct relationship between energy use and economic growth of industrialized nations. The energy need is rising day by day consistently. Due to the nature of demand, energy availability is getting insecure for the whole world, including industrialized countries [3]. The energy use of domestic and industrial building appliances has gone up dramatically. But on the other hand, there was a decrease in the share of the electric power industry. Energy conservation has developed with two strategies: green energy and green environment. Renewable energy and a green climate are the main global tasks in the current situation. In order to improve people's living conditions, a large amount of money has been expended on renewable energy and the green climate. Fossil fuels have played a significant role in deteriorating both the Earth's ecosystem and weather patterns. After the industrial revolution, we have depleted fossil fuel supplies. 85% of power generation has been shown to be from fossil fuel resources. This critical share of fossil fuel is causing global warming; therefore, renewable energy and energy conservation strategies are getting special attention to explore and to get a solution at the national level [4]. The building energy automation system can be a viable solution to save energy from Heating, Ventilation, and Air Conditioning (HVAC) systems. Intelligent energy monitoring is a balanced approach to energy conservation and management against any energy user. It assesses the exact energy inputs, penetration, and consumption at every energy facility. An automatic energy monitoring system is also able to calculate, categorize, and highlight energy consumption behavior and cost-saving measures for energy use.

Convolutional Neural Networks (CNNs) are a highly efficient model for classifying image information, offering state-of-the-art image segmentation, recognition, identification, and recovery results. The CNN approach achieves almost real-time high detection efficiency on the embedded device. A high-resolution camera can classify the object and count the number of individuals and the precise location of the objects. Real-time object detection plays a crucial role in the energy management of buildings. An automatic intelligent system works with the information of a sensing algorithm. It operates with an HVAC control system and provides multiple predictive outcomes because it integrates sensing technology with optimization and computation strategies, conserving the performance of energy appliances. Occupancy detection and estimation can provide real-time information that can be used in HVAC control systems, limiting energy consumption. It can be a more affordable and efficient solution. It can also save cost. Therefore, an efficient and economical intelligent HVAC monitoring and control system can be a viable and sophisticated solution by implementing occupancy detection technology.

From a technical perspective, existing research studies have confirmed that the continuation of the decision-making support structure further than the classical optimization approach of a single objective function is widely accepted, as demonstrated in a collection of effective solutions [5]. This module makes it easier to address multi-criteria challenges by focusing on finding a solution that meets many, sometimes contradictory, goals. The presence of numerous criteria, the ambiguity of criteria, and the decision-making problem's complicated, subjective, and poorly articulated character provide a model for multi-criteria decision assistance [6]. It is also on these foundational principles that a range of Intelligent Decision Support System (IDSS) have been introduced [7]. The present incarnation in this field indicates the realistic and efficient handling of the problem referred to above by the IDSS strategies. The literature review demonstrates the significance of using IDSS approaches in energy decision-making. The IDSS strategies are used as realistic and useful techniques to tackle energy policy decision-making problems or to pick and test Renewable Energy Sources (RES) technologies [8]. This can easily be verified by overview and meta-analysis papers such as Martín-Gamboa et al. [9], L$\phi$ken [10], Arce et al. [11], and Wang et al. [12], which show the multitude of functional issues in the area of energy policy and IDSS techniques used as a formal context for the models created. The literature review also demonstrated the widespread application of fuzzy evolution (following the notion of Fuzzy Sets (FSs)) of IDSS approaches in climate issues [13]. This ideology is significant because it is founded on the notion that fuzzy number theory models express the uncertainty of individual viewpoints in order to create more sensitive, concrete, and practical modeling outcomes. The main objectives of this article are:

- Fermatean Fuzzy Numbers (FFNs) are suitable to address the complexity issues practically with a Membership Degree (MSD) and a Non-Membership Degree (N-MSD). Taking advantage of FFNs, new Aggregation Operators (AOs) are proposed;
- We develop the "Fermatean Fuzzy Prioritized Weighted Average (FFPWA) operator" and "Fermatean Fuzzy Prioritized Weighted Geometric (FF-PWG) operator";
- These AOs are a new way of modeling decision-making difficulties;
- Finally, we present an algorithm for an IDSS based on the proposed AOs, and we compare our CNN-

based method to other existing sensor methodologies.

The rest of this paper is organized as follows. Section 2 has a basic definition regarding the Fermatean Fuzzy Set (FFS), and Section 3 consists of proposed AOs. We give an algorithm in Section 4 to solve complex decision problems based on proposed AOs. In Section 5, we present our intelligent energy management system. In Section 6, the application of our proposed AOs is given, and in Section 7, we give concluding remarks on this research.

### *1.1. MCDM-based uncertain data modeling*

IDSS is a major component of decision science, with the primary purpose of determining the optimal options given a set of different attributes. In it, an expert or group of specialists evaluates each choice based on a variety of criteria, and their conclusions are expressed as either a crisp value or a linguistic value. However, in today's complicated and tough world, ambiguity is highly common and plays a crucial part in almost every decision-making process. As a result, inconsistencies in the analysis must be accounted for. Uncertainties play a crucial part when dealing with real-world problems such as clustering and classification, decision-making, supplier assessment, and so on, and it is impossible for DMs to obtain accurate results without dealing with imprecise, unclear, or uncertain data. According to Zadeh [14], the perception of a FS is a generalization of a classical set. The membership function computes the degrees of membership of elements in a FS. The literature discusses Zadeh's FS from various angles concerning Multi-Criteria Decision-Making (MCDM). Atanassov [15] extended FS by introducing the idea of the Intuitionistic Fuzzy Set (IFS), which is defined as having a MSD and a N-MSD that is less than or equal to 1. This IFS characteristic is important for many practitioners who deal with real-life issues in an IFS framework. IFS is not beneficial for solving complex decision-making problems since the sum of MSD and N-MSD must be equal to unity. Yager's Pythagorean Fuzzy Set (PFS) [16,17] is a significant generalization of IFS in which the sum of squares of MSD and N-MSD is bounded by 1. Furthermore, it has a wide range of possible applications in a variety of fields, including decision-making and so on. However, the PFS technique could not be acknowledged in several cases. Consider the following scenario: an expert team was called to provide feedback on training institutes, and they were divided into two groups. The degree of MSD is expressed by the first group of experts as 0.92, whereas the degree of N-MSD is expressed by the second group of experts as 0.84. As can be observed, the square sum of MSD and N-MSD is more than one. The IFS and PFS were unable to depict this circumstance. Senapati and Yager [18] developed the concept of FFSs as a logical extension of the IFSs and PFSs to address this complication. In an FFS, the cubic sum of an object's MSD and N-MSD values is bounded by 1. FFS theory is currently playing an important role in a variety of disciplines since it is a solid notion for dealing with ambiguous and imprecise information in a Fermatean fuzzy context. Senapati and Yager [19] also proposed score, accuracy function, and Fermatean Fuzzy Weighted Product Model (FF-WPM).

Several studies have been conducted to date that are only based on FFSs. When there are numerous options for a particular problem, the concept of AOs plays a critical role in determining the best solution. Mesiar and Pap [20] proposed the concept of aggregation of infinite sequences. Senapati and Yager [21] suggested fundamental AOs (geometric and averaging) for the FFS and their application to MCDM. Rani and Mishra [22] introduced the MULTIMOORA technique for selecting electric car charging stations using Fermatean fuzzy Einstein AOs. Jeevaraj presented an interval-valued FFS idea as well as a novel total ordering theory for interval-valued FFS [23]. Garg et al. [24] proposed Yager AOs with application in COVID-19 using FFS. The concept of Hamacher interactive hybrid averaging AOs for FFS was proposed by Shahzadi et al. [25]. Riaz et al. [26] introduced the novel concept of bipolar picture fuzzy. Jana et al. [27] proposed bipolar fuzzy Dombi AOs for MCDM. Sitara et al. [28] introduced q-rung picture fuzzy graph structures. Riaz and Hashmi [29] considerably tested the restrictions related to MSDs and N-MSDs in the structures of FS, IFS, PFS, and FFS in 2019, and those barriers were provided quantitatively. To overcome these challenges, they developed the Linear Diophantine Fuzzy Set (LDFS) by incorporating reference parameters to the nature of IFS. They claim that the concept of LDFS will eliminate the limits in present approaches for other sets and allow the free selection of data in practice. Iampan et al. [30] introduced linear Diophantine fuzzy Einstein AOs and their medical application. Riaz et al. [31] developed prioritized AOs for Linear Diophantine Fuzzy Numbers (LDFNs) with application to third-party logistic provider selection. Ashraf and Abdullah [32] proposed emergency decision support modeling for COVID-19 and also introduced spherical AOs and their application in MCDM [33]. Saha et al. [34] introduced new hybrid hesitant fuzzy weighted AOs depending on Archimedean and Dombi operations. Farid and Riaz [35] also introduced Einstein interactive geometric AOs with application to MCDM for q-ROFNs. Jana et al. introduced trapezoidal neutrosophic AOs [36], bipolar fuzzy Dombi prioritized [37], neutrosophic Dombi power AOs [38], and trapezoidal neutrosophic normal AOs [39]. Yang et al. [40] presented continuous ordered weighted averaging AOs for interval-valued q-ROF information and their use in quality assessment of smartwatch aesthetic design. Chen et al. [41] introduced the notion of enhanced ordered weighted averaging AOs and their application towards MCDM. Chen et al. [42] also proposed power-average AOs for a proportional hesitant fuzzy linguistic term set, with an implementation of an online product recommender system for consumer decision-making. Jana and Pal proposed a dynamical hybrid method to design a decision-making process based on the GRA approach [43].

Yager [44] developed several prioritized AOs. According to Yager, in situations where we choose a child's bike based on safety and cost qualities, we should not allow the cost-related benefit to compensate for the lack of safety. Then, we have a priority hierarchy between these two traits, with security being more important. This condition is known as an aggregation problem since the attributes have a priority relationship. Because we wish to take into account the satisfaction of higher priority criteria, such as safety, in the above example, it is no longer practicable for the AOs in question, such as the average AO and the geometric AO. Yager offered the prioritized AOs in this situation by modeling attribute prioritizing in terms of the weights associated with the attributes based on the satisfaction of the higher priority criteria.

## 2. Preliminaries

We briefly add a few concepts involved in the remaining work to construct this work self-sufficiently.

**Definition 1 [18].** A FFS $\widetilde{\mathscr{O}}$ in $\ddot{\mathscr{D}}$ is given as:

$$\widetilde{\mathscr{O}} = \left\{ \langle \breve{\mathscr{P}}, {\mu^{\gimel}}_{\widetilde{\mathscr{O}}}(\breve{\mathscr{P}}), {\nu^{\gimel}}_{\widetilde{\mathscr{O}}}(\breve{\mathscr{P}}) \rangle : \breve{\mathscr{P}} \in \ddot{\mathscr{D}} \right\},$$

where ${\mu^{\gimel}}_{\widetilde{\mathscr{O}}}, {\nu^{\gimel}}_{\widetilde{\mathscr{O}}} : \ddot{\mathscr{D}} \rightarrow [0,1]$ defines the MSD and N-MSD of the alternative $\breve{\mathscr{P}} \in \ddot{\mathscr{D}}$ and for every $\breve{\mathscr{P}}$ we have:

$$0 \leq {\mu^{\gimel}}^{3}_{\widetilde{\mathscr{O}}}(\breve{\mathscr{P}}) + {\nu^{\gimel}}^{3}_{\widetilde{\mathscr{O}}}(\breve{\mathscr{P}}) \leq 1.$$

In addition, $\pi_{\widetilde{\mathscr{O}}}(\breve{\mathscr{P}}) = \sqrt[3]{1 - {\mu^{\gimel}}^{3}_{\widetilde{\mathscr{O}}}(\breve{\mathscr{P}}) - {\nu^{\gimel}}^{3}_{\widetilde{\mathscr{O}}}(\breve{\mathscr{P}})}$ is given as the indeterminacy degree of $\breve{\mathscr{P}}$ to $\widetilde{\mathscr{O}}$.

**Definition 2 [18].** Let $\overline{\Upsilon}_1 = \langle {\mu^{\gimel}}_1, {\nu^{\gimel}}_1 \rangle$ and $\overline{\Upsilon}_2 = \langle {\mu^{\gimel}}_2, {\nu^{\gimel}}_2 \rangle$ be FFNs. Then:

(1) $\overline{\overline{\Upsilon}}_1 = \langle {\nu^{\gimel}}_1, {\mu^{\gimel}}_1 \rangle$ ,

(2) $\overline{\Upsilon}_1 \vee \overline{\Upsilon}_2 = \langle \max\{{\mu^{\gimel}}_1, {\nu^{\gimel}}_1\}, \min\{{\mu^{\gimel}}_2, {\nu^{\gimel}}_2\} \rangle$,

(3) $\overline{\Upsilon}_1 \wedge \overline{\Upsilon}_2 = \langle \min\{{\mu^{\gimel}}_1, {\nu^{\gimel}}_1\}, \max\{{\mu^{\gimel}}_2, {\nu^{\gimel}}_2\} \rangle$,

(4) $\overline{\Upsilon}_1 \oplus \overline{\Upsilon}_2 = \langle \sqrt[3]{{\mu^{\gimel}}^3_1 + {\mu^{\gimel}}^3_2 - {\mu^{\gimel}}^3_1 {\mu^{\gimel}}^3_2}, {\nu^{\gimel}}_1 {\nu^{\gimel}}_2 \rangle$,

(5) $\overline{\Upsilon}_1 \otimes \overline{\Upsilon}_2 = \langle {\mu^{\gimel}}_1 {\mu^{\gimel}}_2, \sqrt[3]{{\nu^{\gimel}}^3_1 + {\nu^{\gimel}}^3_2 - {\nu^{\gimel}}^3_1 {\nu^{\gimel}}^3_2} \rangle$,

(6) $\sigma \overline{\Upsilon}_1 = \langle \sqrt[3]{1 - (1 - {\mu^{\gimel}}^3_1)^{\sigma}}, {\nu^{\gimel}}^{\sigma}_1 \rangle$,

(7) $\overline{\Upsilon}^{\sigma}_1 = \langle {\mu^{\gimel}}^{\sigma}_1, \sqrt[3]{1 - (1 - {\nu^{\gimel}}^3_1)^{\sigma}} \rangle$.

**Definition 3 [19].** Suppose $\overline{\Upsilon} = \langle \mu^{\gimel}, \nu^{\gimel} \rangle$ is a FFN, then a score function $\mathfrak{E}$ and accuracy function $\mathfrak{R}$ of $\overline{\Upsilon}$ are defined as respectively:

$$\mathfrak{E}(\overline{\Upsilon}) = {\mu^{\gimel}}^3 - {\nu^{\gimel}}^3, \qquad \mathfrak{E}(\overline{\Upsilon}) \in [-1, 1],$$

$$\mathfrak{R}(\overline{\Upsilon}) = {\mu^{\gimel}}^3 + {\nu^{\gimel}}^3, \qquad \mathfrak{R}(\overline{\Upsilon}) \in [0, 1].$$

**Definition 4 [19].** Let $\overline{\Upsilon}_1 = \langle {\mu^{\gimel}}_{\overline{\Upsilon}_1}, {\nu^{\gimel}}_{\overline{\Upsilon}_1} \rangle$ and $\overline{\Upsilon}_2 = \langle {\mu^{\gimel}}_{\overline{\Upsilon}_2}, {\nu^{\gimel}}_{\overline{\Upsilon}_2} \rangle$ be any two FFNs, and $\mathfrak{E}(\overline{\Upsilon}_1)$ and $\mathfrak{E}(\overline{\Upsilon}_2)$ are the score function of $\overline{\Upsilon}_1$, $\overline{\Upsilon}_2$, $\mathfrak{R}(\overline{\Upsilon}_1)$, and $\mathfrak{R}(\overline{\Upsilon}_2)$ are the accuracy function of $\overline{\Upsilon}_1$ and $\overline{\Upsilon}_2$, respectively, then:

(1) If $\mathfrak{E}(\overline{\Upsilon}_1) > \mathfrak{E}(\overline{\Upsilon}_2)$, then $\overline{\Upsilon}_1 > \overline{\Upsilon}_2$;

(2) If $\mathfrak{E}(\overline{\Upsilon}_1) = \mathfrak{E}(\overline{\Upsilon}_2)$, then:

if $\mathfrak{R}(\overline{\Upsilon}_1) > \mathfrak{R}(\overline{\Upsilon}_2)$ then $\overline{\Upsilon}_1 > \overline{\Upsilon}_2$,
if $\mathfrak{R}(\overline{\Upsilon}_1) = \mathfrak{R}(\overline{\Upsilon}_2)$, then $\overline{\Upsilon}_1 = \overline{\Upsilon}_2$.

It must always be remembered that the score function's value is between −1 and 1. To help the following study, we add a further score function, $\bar{\mho}(\overline{\Upsilon}) = \frac{1 + {\mu^{\gimel}}^3_{\overline{\Upsilon}} - {\nu^{\gimel}}^3_{\overline{\Upsilon}}}{2}$. We can see that $0 \leq \bar{\mho}(\overline{\Upsilon}) \leq 1$.

## 3. Fermatean fuzzy prioritized aggregation operators

Here, we present the notion of the FFPWA operator and FFPWG operators. Then, we discussed other properties of proposed operators like boundary, monotonicity, and idempotency.

### *3.1. FFPWA operator*

**Definition 5.** Assume that $\overline{\Upsilon}_j = \langle {\mu^{\gimel}}_j, {\nu^{\gimel}}_j \rangle$ is the collection of FFNs, and FFPWA : $\Lambda^n \rightarrow \Lambda$, is a mapping. if:

$$\text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = \frac{{\kappa^{\gimel}}_1}{\sum_{j=1}^{n} {\kappa^{\gimel}}_j} \overline{\Upsilon}_1 \oplus \frac{{\kappa^{\gimel}}_2}{\sum_{j=1}^{n} {\kappa^{\gimel}}_j} \overline{\Upsilon}_2 \oplus, \ldots, \oplus \frac{{\kappa^{\gimel}}_n}{\sum_{j=1}^{n} {\kappa^{\gimel}}_j} \overline{\Upsilon}_n, \tag{1}$$

then the mapping FFPWA is called FFPWA operator, where ${\kappa^{\gimel}}_j = \prod_{k=1}^{j-1} \bar{\mho}(\overline{\Upsilon}_k)$ $(j = 2, \ldots, n)$, ${\kappa^{\gimel}}_1 = 1$ and $\bar{\mho}(\overline{\Upsilon}_k)$ present the score of $k^{th}$ FFN.

We may also take into account FFPWA by the following theorem depending on FFN's operational rules.

**Theorem 1.** Assume that $\overline{\Upsilon}_j = \langle {\mu^{\gimel}}_j, {\nu^{\gimel}}_j \rangle$ is the collection of FFNs, we can evaluate FFPWA by:

$$\text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) =$$

$$\left\langle \sqrt[q]{(1-\overline{\prod}_{j=1}^{n}(1-\mu^{\daleth}{}_j^3)^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}}}, \overline{\prod}_{j=1}^{n}\nu^{\daleth}{}_j^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}} \right\rangle. \quad (2)$$

***Proof.***

$$\text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = \left(\frac{\kappa^{\daleth}{}_1}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon}_1 \oplus \frac{\kappa^{\daleth}{}_2}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon}_2 \oplus, \ldots, \frac{\kappa^{\daleth}{}_n}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon}_n\right)$$

$$= \left\langle \sqrt[3]{1-\overline{\prod}_{j=1}^{n}(1-\mu^{\daleth}{}_j^3)^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}}}, \overline{\prod}_{j=1}^{n}\nu^{\daleth}{}_j^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}} \right\rangle.$$

We utilize mathematical induction to proof this theorem.
For $n = 2$:

$$\frac{\kappa^{\daleth}{}_1}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon}_1 = \left\langle \sqrt[3]{1-(1-\mu^{\daleth}{}_1^3)^{\frac{\kappa^{\daleth}{}_1}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}}}, \nu^{\daleth}{}_1^{\frac{\kappa^{\daleth}{}_1}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}} \right\rangle,$$

$$\frac{\kappa^{\daleth}{}_2}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon}_2 = \left\langle \sqrt[3]{1-(1-\mu^{\daleth}{}_2^3)^{\frac{\kappa^{\daleth}{}_1}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}}}, \nu^{\daleth}{}_2^{\frac{\kappa^{\daleth}{}_1}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}} \right\rangle.$$

Then we obtained an equation is shown in Box I.

This shows that Eq. (2) is valid for $n = 2$. Now suppose Eq. (2) holds for $n = k$, i.e.,

$$\text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_k) =$$

$$\left\langle \sqrt[3]{1-\overline{\prod}_{j=1}^{k}(1-\mu^{\daleth}{}_j^3)^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}}}, \overline{\prod}_{j=1}^{k}\nu^{\daleth}{}_j^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}} \right\rangle.$$

For $n = k + 1$, we have equation as shown in Box II.

This shows that for $n = k + 1$, Eq. (2) holds. Then:

$$\text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) =$$

$$\left\langle \sqrt[3]{(1-\overline{\prod}_{j=1}^{n}(1-\mu^{\daleth}{}_j^3)^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}}}, \overline{\prod}_{j=1}^{n}\nu^{\daleth}{}_j^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}} \right\rangle. \quad \square$$

**Theorem 2.** (Idempotency) Assume that $\overline{\Upsilon}_j = \langle \mu^{\daleth}{}_j, \nu^{\daleth}{}_j \rangle$ is the assemblage of FFNs, where $\kappa^{\daleth}{}_j = \overline{\prod}_{k=1}^{j-1}\bar{\mho}(\overline{\Upsilon}_k)$ $(j = 2, \ldots, n)$, $\kappa^{\daleth}{}_1 = 1$ and $\bar{\mho}(\overline{\Upsilon}_k)$ is the score of $k$th FFN. If all $\overline{\Upsilon}_j$s are equal, i.e,. $\overline{\Upsilon}_j = \overline{\Upsilon}$ for all $j$, then:

$$\text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = \overline{\Upsilon}.$$

***Proof.*** From Definition 5, we have:

$$\text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = \frac{\widetilde{\kappa^{\daleth}}_1}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon}_1 \oplus \frac{\kappa^{\daleth}{}_2}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon}_2 \oplus, \ldots, \oplus \frac{\kappa^{\daleth}{}_n}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon}_n$$

$$= \frac{\kappa^{\daleth}{}_1}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon} \oplus \frac{\kappa^{\daleth}{}_2}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon} \oplus, \ldots, \oplus \frac{\kappa^{\daleth}{}_n}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon} = \frac{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon} = \overline{\Upsilon}. \quad \square$$

**Corollary 1.** If $\overline{\Upsilon}_j = \langle \mu^{\daleth}{}_j, \nu^{\daleth}{}_j \rangle$ is the assemblage of largest FFNs, i.e,. $\overline{\Upsilon}_j = (1, 0)$ for all $j$, then:

$$\text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = (1, 0).$$

***Proof.*** One can easily prove this corollary in a similar way to the Theorem 2. $\square$

**Corollary 2. (Non-compensatory).** If $\overline{\Upsilon}_1 = \langle \mu^{\daleth}{}_1, \nu^{\daleth}{}_1 \rangle$ is the smallest FFN, i.e,. $\overline{\Upsilon}_1 = (0, 1)$, then:

$$\text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = (0, 1).$$

***Proof.*** Here, $\overline{\Upsilon}_1 = (0, 1)$, then by definition of the score function, we have:

$$\bar{\mho}(\overline{\Upsilon}_1) = 0,$$

since,

$$\kappa^{\daleth}{}_j = \overline{\prod}_{k=1}^{j-1}\bar{\mho}(\overline{\Upsilon}_k) \qquad (j = 2, \ldots, n), \quad \text{and} \quad \kappa^{\daleth}{}_1 = 1,$$

$\bar{\mho}(\overline{\Upsilon}_k)$ is the score of $k$th FFN. We have:

$$\kappa^{\daleth}{}_j = \overline{\prod}_{k=1}^{j-1}\bar{\mho}(\overline{\Upsilon}_k) = \bar{\mho}(\overline{\Upsilon}_1) \times \bar{\mho}(\overline{\Upsilon}_2) \times \ldots \times \bar{\mho}(\overline{\Upsilon}_{j-1}) = 0 \times \bar{\mho}(\overline{\Upsilon}_2) \times \ldots \times \bar{\mho}(\overline{\Upsilon}_{j-1}) \quad (j = 2, \ldots, n),$$

$$\overline{\prod}_{k=1}^{j}\kappa^{\daleth}{}_j = 1.$$

From Definition 5, we have:

$$\text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = \frac{\kappa^{\daleth}{}_1}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon}_1 \oplus \frac{\kappa^{\daleth}{}_2}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon}_2 \oplus, \ldots, \oplus \frac{\kappa^{\daleth}{}_n}{\sum_{j=1}^{n}\kappa^{\daleth}{}_j}\overline{\Upsilon}_n$$

$$= \frac{1}{1}\overline{\Upsilon}_1 \oplus \frac{0}{1}\overline{\Upsilon}_2 \oplus, \ldots, \frac{0}{1}\overline{\Upsilon}_n$$

$$= \overline{\Upsilon}_1 = (0, 1). \quad \square$$

Corollary 2 implied that incentives would not be

$$\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}\overline{\Upsilon}_1 \oplus \frac{\kappa^{\urcorner}_2}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}\overline{\Upsilon}_2$$

$$= \left\langle \sqrt[3]{1-(1-\mu^{\urcorner 3}_1)^{\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}}, \nu^{\urcorner \frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_1 \right\rangle \oplus \left\langle \sqrt[3]{1-(1-\mu^{\urcorner 3}_2)^{\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}}, \nu^{\urcorner \frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_2 \right\rangle$$

$$= \left\langle \sqrt[3]{(1-(1-\mu^{\urcorner 3}_1)^{\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}} + 1-(1-\mu^{\urcorner 3}_2)^{\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}} - \left((1-(1-\mu^{\urcorner 3}_1)^{\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}\right)\left((1-(1-\mu^{\urcorner 3}_2)^{\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}\right)}, \right.$$

$$\left. \nu^{\urcorner \frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_1 . \nu^{\urcorner \frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_2 \right\rangle$$

$$= \left\langle \sqrt[3]{1-(1-\mu^{\urcorner 3}_1)^{\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}} + 1-(1-\mu^{\urcorner 3}_2)^{\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}} - \left(1-(1-\mu^{\urcorner 3}_2)^{\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}} - (1-\mu^{\urcorner 3}_1)^{\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}} + \right.} \right.$$

$$\left. \overline{\left. (1-\mu^{\urcorner 3}_2)^{\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}(1-\mu^{\urcorner 3}_1)^{\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}\right)}, \nu^{\urcorner \frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_1 . \nu^{\urcorner \frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_2 \right\rangle$$

$$= \left\langle \sqrt[3]{1-(1-\mu^{\urcorner 3}_1)^{\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}(1-\mu^{\urcorner 3}_2)^{\frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}}, \nu^{\urcorner \frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_1 . \nu^{\urcorner \frac{\kappa^{\urcorner}_1}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_2 \right\rangle$$

$$= \left\langle \sqrt[3]{1-\overline{\prod}_{j=1}^{2}(1-\mu^{\urcorner 3}_j)^{\frac{\kappa^{\urcorner}_j}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}}, \overline{\prod}_{j=1}^{2}\nu^{\urcorner \frac{\kappa^{\urcorner}_j}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_j \right\rangle.$$

Box I

$$\mathrm{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_{k+1}) = \mathrm{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_k) \oplus \overline{\Upsilon}_{k+1}$$

$$= \left\langle \sqrt[3]{1-\overline{\prod}_{j=1}^{k}(1-\mu^{\urcorner 3}_j)^{\frac{\kappa^{\urcorner}_j}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}}, \overline{\prod}_{j=1}^{k}\nu^{\urcorner \frac{\kappa^{\urcorner}_j}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_j \right\rangle \oplus \left\langle \sqrt[3]{1-(1-\mu^{\urcorner 3}_{k+1})^{\frac{\kappa^{\urcorner}_{k+1}}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}}, \nu^{\urcorner \frac{\kappa^{\urcorner}_{k+1}}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_{k+1} \right\rangle$$

$$= \left\langle \sqrt[3]{1-\overline{\prod}_{j=1}^{k}(1-\mu^{\urcorner 3}_k)^{\frac{\kappa^{\urcorner}_j}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}} + 1-(1-\mu^{\urcorner 3}_{k+1})^{\frac{\kappa^{\urcorner}_{k+1}}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}} - \left(1-\overline{\prod}_{j=1}^{k}(1-\mu^{\urcorner 3}_k)^{\frac{\kappa^{\urcorner}_j}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}\right)\left(1-(1-\mu^{\urcorner 3}_{k+1})^{\frac{\kappa^{\urcorner}_{k+1}}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}\right)}, \right.$$

$$\left. \overline{\prod}_{j=1}^{k}\nu^{\urcorner \frac{\kappa^{\urcorner}_j}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_k . \nu^{\urcorner \frac{\kappa^{\urcorner}_{k+1}}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_{k=1} \right\rangle$$

$$= \left\langle \sqrt[3]{1-\overline{\prod}_{j=1}^{k}(1-\mu^{\urcorner 3}_k)^{\frac{\kappa^{\urcorner}_j}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}(1-\mu^{\urcorner 3}_{k+1})^{\frac{\kappa^{\urcorner}_{k+1}}{\sum_{j=1}^{n}\kappa^{\urcorner}_{k+1}}}}, \overline{\prod}_{j=1}^{k}\nu^{\urcorner \frac{\kappa^{\urcorner}_j}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_k . \nu^{\urcorner \frac{\kappa^{\urcorner}_{k+1}}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_{k+1} \right\rangle$$

$$= \left\langle \sqrt[3]{1-\overline{\prod}_{j=1}^{k+1}(1-\mu^{\urcorner 3}_j)^{\frac{\kappa^{\urcorner}_j}{\sum_{j=1}^{k+1}\kappa^{\urcorner}_j}}}, \overline{\prod}_{j=1}^{k+1}\nu^{\urcorner \frac{\kappa^{\urcorner}_j}{\sum_{j=1}^{n}\kappa^{\urcorner}_j}}_j \right\rangle.$$

Box II

obtained by anyone, although the higher importance conditions had been satisfied by the smallest FFN.

**Theorem 3. (Monotonicity).** Assume that $\overline{\Upsilon}_j = \langle \mu^{\gimel}{}_j, \nu^{\gimel}{}_j \rangle$ and $\overline{\Upsilon}^*_j = \langle \mu^{\gimel *}_j, \nu^{\gimel *}_j \rangle$ are the assemblages of FFNs, where $\kappa^{\gimel}{}_j = \overline{\prod}_{k=1}^{j-1} \bar{\mho}(\overline{\Upsilon}_k)$, $\kappa^{\gimel *}_j = \overline{\prod}_{k=1}^{j-1} \bar{\mho}(\overline{\Upsilon}^*_k)$ $(j = 2, \ldots, n)$, $\kappa^{\gimel}{}_1 = 1$, $\kappa^{\gimel *}_1 = 1$, $\bar{\mho}(\overline{\Upsilon}_k)$ is the score of $\overline{\Upsilon}_k$ FFN, and $\bar{\mho}(\overline{\Upsilon}^*_k)$ is the score of $\overline{\Upsilon}^*_k$ FFN. If $\mu^{\gimel *}_j \geq \mu^{\gimel}{}_j$ and $\nu^{\gimel *}_j \leq \nu^{\gimel}{}_j$ for all $j$, then:

$$\text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) \leq \text{FFPWA}(\overline{\Upsilon}^*_1, \overline{\Upsilon}^*_2, \ldots, \overline{\Upsilon}^*_n).$$

***Proof.*** Here, $\mu^{\gimel *}_j \geq \mu^{\gimel}{}_j$ and $\nu^{\gimel *}_j \leq \nu^{\gimel}{}_j$ for all $j$, If $\mu^{\gimel *}_j \geq \mu^{\gimel}{}_j$.

$$\Leftrightarrow (\mu^{\gimel *}_j)^3 \geq (\mu^{\gimel}{}_j)^3 \Leftrightarrow \sqrt[3]{(\mu^{\gimel *}_j)^3} \geq \sqrt[3]{(\mu^{\gimel}{}_j)^3}$$

$$\Leftrightarrow \sqrt[3]{1-(\mu^{\gimel *}_j)^3} \leq \sqrt[3]{1-(\mu^{\gimel}{}_j)^3}$$

$$\Leftrightarrow \sqrt[3]{(1-(\mu^{\gimel *}_j)^3)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}} \leq \sqrt[3]{(1-(\mu^{\gimel}{}_j)^3)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}}$$

$$\Leftrightarrow \sqrt[3]{\overline{\prod}_{j=1}^n (1-(\mu^{\gimel *}_j)^3)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}}$$

$$\leq \sqrt[3]{\overline{\prod}_{j=1}^n (1-(\mu^{\gimel}{}_j)^3)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}}$$

$$\Leftrightarrow \sqrt[3]{1-\overline{\prod}_{j=1}^n (1-(\mu^{\gimel}{}_j)^3)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}}$$

$$\leq \sqrt[3]{1-\overline{\prod}_{j=1}^n (1-(\mu^{\gimel *}_j)^3)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}}.$$

Now,

$$\nu^{\gimel *}_j \leq \nu^{\gimel}{}_j.$$

$$\Leftrightarrow (\nu^{\gimel *}_j)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}} \leq (\nu^{\gimel}{}_j)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}$$

$$\Leftrightarrow \overline{\prod}_{j=1}^n (\nu^{\gimel *}_j)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}} \leq \overline{\prod}_{j=1}^n (\nu^{\gimel}{}_j)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}.$$

Let:

$$\overline{\overline{\Upsilon}} = \text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n),$$

and

$$\overline{\overline{\Upsilon}^*} = \text{FFPWA}(\overline{\Upsilon}^*_1, \overline{\Upsilon}^*_2, \ldots, \overline{\Upsilon}^*_n),$$

we get that: $\overline{\overline{\Upsilon}^*} \geq \overline{\overline{\Upsilon}}$, so,

$$\text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) \leq \text{FFPWA}(\overline{\Upsilon}^*_1, \overline{\Upsilon}^*_2, \ldots, \overline{\Upsilon}^*_n). \square$$

**Theorem 4 (Boundary).** *Assume that* $\overline{\Upsilon}_j = \langle \mu^{\gimel}{}_j, \nu^{\gimel}{}_j \rangle$ *is the assemblage of FFNs, and:*

$$\overline{\Upsilon}^- = (\min_j (\mu^{\gimel}{}_j), \max_j (\nu^{\gimel}{}_j)) \quad \text{and}$$

$$\overline{\Upsilon}^+ = (\max_j (\mu^{\gimel}{}_j), \min_j (\nu^{\gimel}{}_j)).$$

Then:

$$\overline{\Upsilon}^- \leq \text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) \leq \overline{\Upsilon}^+,$$

where $\kappa^{\gimel}{}_j = \overline{\prod}_{k=1}^{j-1} \bar{\mho}(\overline{\Upsilon}_k)$ $(j = 2, \ldots, n)$, $\kappa^{\gimel}{}_1 = 1$ and $\bar{\mho}(\overline{\Upsilon}_k)$ is the score of $k$th FFN.

***Proof.*** Since,

$$\min_j (\mu^{\gimel}{}_j) \leq \mu^{\gimel}{}_j \leq \max_j (\mu^{\gimel}{}_j), \tag{3}$$

and:

$$\min_j (\nu^{\gimel}{}_j) \leq \nu^{\gimel}{}_j \leq \max_j (\nu^{\gimel}{}_j). \tag{4}$$

From Eq. (3) we can obtained formula is shown in Box III. From Eq. (4) we have:

$$\min_j (\nu^{\gimel}{}_j) \leq \nu^{\gimel}{}_j \leq \max_j (\nu^{\gimel}{}_j) \Leftrightarrow \min_j (\nu^{\gimel}{}_j)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}$$

$$\leq (\nu^{\gimel}{}_j)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}} \leq \max_j (\nu^{\gimel}{}_j)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}$$

$$\Leftrightarrow \overline{\prod}_{j=1}^n \min_j (\nu^{\gimel}{}_j)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}$$

$$\leq \overline{\prod}_{j=1}^n (\nu^{\gimel}{}_j)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}$$

$$\leq \overline{\prod}_{j=1}^n \max_j (\nu^{\gimel}{}_j)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}$$

$$\Leftrightarrow \min_j (\nu^{\gimel}{}_j)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}} \leq \overline{\prod}_{j=1}^n (\nu^{\gimel}{}_j)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}$$

$$\leq \max_j (\nu^{\gimel}{}_j)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}.$$

Let:

$$\text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = \overline{\Upsilon} = (\mu^{\gimel}, \nu^{\gimel});$$

then $\bar{\mho}(\overline{\Upsilon}) = {\mu^{\gimel}}^3 - {\nu^{\gimel}}^3 \leq \max_j (\mu^{\gimel})^3 - \min_j (\nu^{\gimel})^3 = \bar{\mho}(\overline{\Upsilon}_{\max})$. So, $\bar{\mho}(\overline{\Upsilon}) \leq \bar{\mho}(\overline{\Upsilon}_{\max})$.

Again, $\bar{\mho}(\overline{\Upsilon}) = {\mu^{\gimel}}^3 - {\nu^{\gimel}}^3 \geq \min_j (\mu^{\gimel})^3 - \max_j (\nu^{\gimel})^3 = \bar{\mho}(\overline{\Upsilon}_{\min})$. So, $\bar{\mho}(\overline{\Upsilon}) \geq \bar{\mho}(\overline{\Upsilon}_{\min})$.

If, $\bar{\mho}(\overline{\Upsilon}) \leq \bar{\mho}(\overline{\Upsilon}_{\max})$ and $\bar{\mho}(\overline{\Upsilon}) \geq \bar{\mho}(\overline{\Upsilon}_{\min})$, then:

$$\min_j(\mu^{\gimel}{}_j) \leq \mu^{\gimel}{}_j \leq \max_j(\mu^{\gimel}{}_j)$$

$$\Leftrightarrow \sqrt[3]{\min_j(\mu^{\gimel}{}_j)^3} \leq \sqrt[3]{(\mu^{\gimel}{}_j)^3} \leq \sqrt[3]{\max_j(\mu^{\gimel}{}_j)^3}$$

$$\Leftrightarrow \sqrt[3]{1-\max_j(\mu^{\gimel}{}_j)^3} \leq \sqrt[3]{1-(\mu^{\gimel}{}_j)^3} \leq \sqrt[3]{1-\min_j(\mu^{\gimel}{}_j)^3}$$

$$\Leftrightarrow \sqrt[3]{\left(1-\max_j(\mu^{\gimel}{}_j)^3\right)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}} \leq \sqrt[3]{\left(1-(\mu^{\gimel}{}_j)^3\right)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}} \leq \sqrt[3]{\left(1-\min_j(\mu^{\gimel}{}_j)^3\right)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}}$$

$$\Leftrightarrow \sqrt[3]{\prod_{j=1}^n\left(1-\max_j(\mu^{\gimel}{}_j)^3\right)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}} \leq \sqrt[3]{\prod_{j=1}^n\left(1-(\mu^{\gimel}{}_j)^3\right)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}} \leq \sqrt[3]{\prod_{j=1}^n\left(1-\min_j(\mu^{\gimel}{}_j)^3\right)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}}$$

$$\Leftrightarrow \sqrt[3]{1-\max_j(\mu^{\gimel}{}_j)^3} \leq \sqrt[3]{\prod_{j=1}^n\left(1-(\mu^{\gimel}{}_j)^3\right)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}} \leq \sqrt[3]{1-\min_j(\mu^{\gimel}{}_j)^3}$$

$$\Leftrightarrow \sqrt[3]{-1+\min_j(\mu^{\gimel}{}_j)^3} \leq \sqrt[3]{-\prod_{j=1}^n\left(1-(\mu^{\gimel}{}_j)^3\right)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}} \leq \sqrt[3]{-1+\max_j(\mu^{\gimel}{}_j)^3}$$

$$\Leftrightarrow \sqrt[3]{1-1+\min_j(\mu^{\gimel}{}_j)^3} \leq \sqrt[3]{1-\prod_{j=1}^n\left(1-(\mu^{\gimel}{}_j)^3\right)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}} \leq \sqrt[3]{1-1+\max_j(\mu^{\gimel}{}_j)^3}$$

$$\Leftrightarrow \sqrt[3]{\min_j(\mu^{\gimel}{}_j)^3} \leq \sqrt[3]{1-\prod_{j=1}^n\left(1-(\mu^{\gimel}{}_j)^3\right)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}} \leq \sqrt[3]{\max_j(\mu^{\gimel}{}_j)^3}$$

$$\Leftrightarrow \min_j(\mu^{\gimel}{}_j)^3 \leq \sqrt[3]{1-\prod_{j=1}^n\left(1-(\mu^{\gimel}{}_j)^3\right)^{\frac{\kappa^{\gimel}{}_j}{\sum_{j=1}^n \kappa^{\gimel}{}_j}}} \leq \max_j(\mu^{\gimel}{}_j)^3.$$

Box III

$$\overline{\Upsilon}_{\min} \leq \mathrm{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) \leq \overline{\Upsilon}_{\max}. \quad (5)$$

If $\bar{\mho}(\overline{\Upsilon}) = \bar{\mho}(\overline{\Upsilon}_{\max})$, then $\mu^{\gimel 3} - \nu^{\gimel 3} = \max_j(\mu^{\gimel})^3 - \min_j(\nu^{\gimel})^3$,

$$\Leftrightarrow \mu^{\gimel 3} - \nu^{\gimel 3} = \max_j(\mu^{\gimel})^3 - \min_j(\nu^{\gimel})^3,$$

$$\Leftrightarrow \mu^{\gimel 3} = \max_j(\mu^{\gimel})^3, \quad \nu^{\gimel 3} = \min_j(\nu^{\gimel})^3,$$

$$\Leftrightarrow \mu^{\gimel} = \max_j \mu^{\gimel}, \quad \nu^{\gimel} = \min_j \nu^{\gimel}.$$

Now, $H(\overline{\Upsilon}) = \mu^{\gimel 3} + \nu^{\gimel 3} = \max_j(\mu^{\gimel})^3 + \min_j(\nu^{\gimel})^3 = H(\overline{\Upsilon}_{\max})$,

$$\mathrm{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots \overline{\Upsilon}_n) = \overline{\Upsilon}_{\max}. \quad (6)$$

If, $\bar{\mho}(\overline{\Upsilon}) = \bar{\mho}(\overline{\Upsilon}_{\min})$, then $\mu^{\gimel 3} - \nu^{\gimel 3} = \min_j(\mu^{\gimel})^3 - \max_j(\nu^{\gimel})^3$,

$$\Leftrightarrow \mu^{\gimel 3} - \nu^{\gimel 3} = \min_j(\mu^{\gimel})^3 - \max_j(\nu^{\gimel})^3,$$

$$\Leftrightarrow \mu^{\gimel 3} = \min_j(\mu^{\gimel})^3, \quad \nu^{\gimel 3} = \max_j(\nu^{\gimel})^3,$$

$$\Leftrightarrow \mu^{\gimel} = \min_j \mu^{\gimel}, \quad \nu^{\gimel} = \max_j \nu^{\gimel},$$

Now, $H(\overline{\Upsilon}) = \mu^{\gimel 3} + \nu^{\gimel 3} = \min_j(\mu^{\gimel})^3 + \max_j(\nu^{\gimel})^3 = H(\overline{\Upsilon}_{\max})$,

$$\mathrm{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = \overline{\Upsilon}_{\min}. \quad (7)$$

Thus, from Eqs. (5), (6) and (7), we get:

$$\overline{\Upsilon}^{-} \leq \text{FFPWA}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) \leq \overline{\Upsilon}^{+}. \qquad \square$$

### *3.2. FFPWG operator*

**Definition 6.** Assume that $\overline{\Upsilon}_j = \langle \mu^{\daleth}{}_j, \nu^{\daleth}{}_j \rangle$ is the collection of FFNs, and FFPWG: $\Lambda^n \to \Lambda$, is a mapping. If:

$$\text{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = \overline{\Upsilon}_1^{\frac{\kappa^{gimel}{}_1}{\sum_{j=1}^n \kappa^{\daleth}{}_j}} \otimes \overline{\Upsilon}_2^{\frac{\kappa^{\daleth}{}_2}{\sum_{j=1}^n \kappa^{\daleth}{}_j}} \otimes, \ldots, \otimes \overline{\Upsilon}_n^{\frac{\kappa^{\daleth}{}_n}{\sum_{j=1}^n \kappa^{\daleth}{}_j}}, \tag{8}$$

then this mapping FFPWG is called FFPWG operator, where $\kappa^{\daleth}{}_j = \overline{\prod}_{k=1}^{j-1} \bar{\mho}(\overline{\Upsilon}_k)$ $(j = 2, \ldots, n)$, $\kappa^{\daleth}{}_1 = 1$ and $\bar{\mho}(\overline{\Upsilon}_k)$ is the score of $k$th FFN.

**Theorem 5.** Assume that $\overline{\Upsilon}_j = \langle \mu^{\daleth}{}_j, \nu^{\daleth}{}_j \rangle$ is the collection of FFNs, we can find FFPWG by:

$$\text{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = \left\langle \overline{\prod}_{j=1}^{n} \mu^{\daleth}{}_j^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^n \kappa^{\daleth}{}_j}}, \sqrt[3]{(1 - \overline{\prod}_{j=1}^{n} (1 - \nu^{\daleth}{}_j^3)^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^n \kappa^{\daleth}{}_j}}} \right\rangle. \tag{9}$$

***Proof.***

$$\text{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = \overline{\Upsilon}_1^{\frac{\widetilde{\kappa^{\daleth}}_1}{\sum_{j=1}^n \kappa^{\daleth}{}_j}} \otimes \overline{\Upsilon}_2^{\frac{\kappa^{\daleth}{}_2}{\sum_{j=1}^n \kappa^{\daleth}{}_j}} \otimes, \ldots, \otimes \overline{\Upsilon}_n^{\frac{\kappa^{\daleth}{}_n}{\sum_{j=1}^n \kappa^{\daleth}{}_j}}$$

$$= \left\langle \overline{\prod}_{j=1}^{n} \mu^{\daleth}{}_j^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^n \kappa^{\daleth}{}_j}}, \sqrt[3]{(1 - \overline{\prod}_{j=1}^{n} (1 - \nu^{\daleth}{}_j^3)^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^n \kappa^{\daleth}{}_j}}} \right\rangle.$$

We utilize mathematical induction to proof this theorem.
For $n = 2$:

$$\overline{\Upsilon}_1^{\frac{\kappa^{\daleth}{}_1}{\sum_{j=1}^n \kappa^{\daleth}{}_j}} = \left\langle \mu^{\daleth}{}_1^{\frac{\kappa^{\daleth}{}_1}{\sum_{j=1}^n \kappa^{\daleth}{}_j}} \right\rangle, \sqrt[3]{1 - (1 - \nu^{\daleth}{}_1^3)^{\frac{\kappa^{\daleth}{}_1}{\sum_{j=1}^n \kappa^{\daleth}{}_j}}}$$

$$\overline{\Upsilon}_2^{\frac{\kappa^{\daleth}{}_2}{\sum_{j=1}^n \kappa^{\daleth}{}_j}} = \left\langle \mu^{\daleth}{}_2^{\frac{\kappa^{\daleth}{}_1}{\sum_{j=1}^n \kappa^{\daleth}{}_j}}, \sqrt[3]{1 - (1 - \nu^{\daleth}{}_2^3)^{\frac{\kappa^{\daleth}{}_1}{\sum_{j=1}^n \kappa^{\daleth}{}_j}}} \right\rangle.$$

Then we have an equation is shown in Box IV.

This shows that Eq. (9) is valid for $n = 2$, now assume that Eq. (9) holds for $n = k$, i.e.,

$$\text{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_k) = \left\langle \overline{\prod}_{j=1}^{k} \mu^{\daleth}{}_j^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^n \kappa^{\daleth}{}_j}}, \sqrt[3]{1 - \overline{\prod}_{j=1}^{k} (1 - \nu^{\daleth}{}_j^3)^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^n \kappa^{\daleth}{}_j}}} \right\rangle.$$

Now for $n = k + 1$, we can obtained FEPWG as shown in Box V.

This shows that for $n = k + 1$, Eq. (2) holds. Then,

$$\text{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = \left\langle \overline{\prod}_{j=1}^{n} \mu^{\daleth}{}_j^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^n \kappa^{\daleth}{}_j}}, \sqrt[3]{(1 - \overline{\prod}_{j=1}^{n} (1 - \nu^{\daleth}{}_j^3)^{\frac{\kappa^{\daleth}{}_j}{\sum_{j=1}^n \kappa^{\daleth}{}_j}}} \right\rangle. \square$$

Below we define some of FFPWG operator's appealing properties.

**Theorem 6 (Idempotency).** Assume that $\overline{\Upsilon}_j = \langle \mu^{\daleth}{}_j, \nu^{\daleth}{}_j \rangle$ is the assemblage of FFNs, where $\kappa^{\daleth}{}_j = \overline{\prod}_{k=1}^{j-1} \bar{\mho}(\overline{\Upsilon}_k)$ $(j = 2, \ldots, n)$, $\kappa^{\daleth}{}_1 = 1$ and $\bar{\mho}(\overline{\Upsilon}_k)$ is the score of $k$th FFN. If all $\overline{\Upsilon}_j$s are equal, i.e,. $\overline{\Upsilon}_j = \overline{\Upsilon}$ for all $j$, then:

$$\text{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = \overline{\Upsilon}.$$

***Proof.*** From Definition 5, we have:

$$\text{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = \overline{\Upsilon}_1^{\frac{\widetilde{\kappa^{\daleth}}_1}{\sum_{j=1}^n \kappa^{\daleth}{}_j}} \otimes \overline{\Upsilon}_2^{\frac{\kappa^{\daleth}{}_2}{\sum_{j=1}^n \kappa^{\daleth}{}_j}} \otimes, \ldots, \otimes \overline{\Upsilon}_n^{\frac{\kappa^{\daleth}{}_n}{\sum_{j=1}^n \kappa^{\daleth}{}_j}} = \overline{\Upsilon}^{\frac{\widetilde{\kappa^{\daleth}}_1}{\sum_{j=1}^n \kappa^{\daleth}{}_j}} \otimes \overline{\Upsilon}^{\frac{\kappa^{\daleth}{}_2}{\sum_{j=1}^n \kappa^{\daleth}{}_j}} \otimes, \ldots, \otimes \overline{\Upsilon}^{\frac{\kappa^{\daleth}{}_n}{\sum_{j=1}^n \kappa^{\daleth}{}_j}} = \overline{\Upsilon}^{\frac{\sum_{j=1}^n \kappa^{\daleth}{}_j}{\sum_{j=1}^n \kappa^{\daleth}{}_j}} = \overline{\Upsilon}. \qquad \square$$

**Corollary 7.** If $\overline{\Upsilon}_j = \langle \mu^{\daleth}{}_j, \nu^{\daleth}{}_j \rangle$, $j = (1, 2, \ldots, n)$ is the assemblage of largest FFNs, i.e,. $\overline{\Upsilon}_j = (1, 0)$ for all $j$, then:

$$\text{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = (1, 0).$$

***Proof.*** We can easily obtain this corollary similar to the Theorem 2. □

**Corollary 8 (Non-compensatory).** If $\overline{\Upsilon}_1 = \langle \mu^{\daleth}{}_1, \nu^{\daleth}{}_1 \rangle$ is the smallest FFN, i.e., $\overline{\Upsilon}_1 = (0, 1)$, then:

$$\text{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = (0, 1).$$

***Proof.*** Here, $\overline{\Upsilon}_1 = (0, 1)$, and by definition of the score function, we have:

$$\bar{\mho}(\overline{\Upsilon}_1) = 0,$$

$$\overline{\Upsilon}_1^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}} \otimes \overline{\Upsilon}_2^{\frac{\kappa\gimel_2}{\sum_{j=1}^n \kappa\gimel_j}}$$

$$= \left\langle \mu\gimel_1^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}} \sqrt[3]{1-(1-\nu\gimel_1^3)^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}}} \right\rangle \otimes \left\langle \mu\gimel_2^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}}, \sqrt[3]{1-(1-\nu\gimel_2^3)^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}}} \right\rangle$$

$$= \left\langle \mu\gimel_1^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}} .\mu\gimel_2^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}}, \sqrt[3]{(1-(1-\nu\gimel_1^3)^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}} + 1-(1-\nu\gimel_2^3)^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}} - \left((1-(1-\nu\gimel_1^3)^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}}\right)\left((1-(1-\nu\gimel_2^3)^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}}\right)} \right\rangle$$

$$= \left\langle \mu\gimel_1^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}} .\mu\gimel_2^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}}, \sqrt[3]{1-(1-\nu\gimel_1^3)^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}} + 1-(1-\nu\gimel_2^3)^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}} - \left(1-(1-\nu\gimel_2^3)^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}} - (1-\nu\gimel_1^3)^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}} + (1-\nu\gimel_2^3)^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}}(1-\nu\gimel_1^3)^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}}\right)} \right\rangle$$

$$= \left\langle \mu\gimel_1^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}} .\mu\gimel_2^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}}, \sqrt[3]{1-(1-\nu\gimel_1^3)^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}}(1-\nu\gimel_2^3)^{\frac{\kappa\gimel_1}{\sum_{j=1}^n \kappa\gimel_j}}} \right\rangle$$

$$= \left\langle \overline{\prod}_{j=1}^{2} \mu\gimel_j^{\frac{\kappa\gimel_j}{\sum_{j=1}^n \kappa\gimel_j}}, \sqrt[3]{1-\overline{\prod}_{j=1}^{2}(1-\nu\gimel_j^3)^{\frac{\kappa\gimel_j}{\sum_{j=1}^n \kappa\gimel_j}}} \right\rangle.$$

Box IV

$$\mathrm{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_{k+1}) = \mathrm{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_{k}) \otimes \overline{\Upsilon}_{k+1}$$

$$= \left\langle \overline{\prod}_{j=1}^{k} \mu\gimel_j^{\frac{\kappa\gimel_j}{\sum_{j=1}^n \kappa\gimel_j}}, \sqrt[3]{1-\overline{\prod}_{j=1}^{k}(1-\nu\gimel_j^3)^{\frac{\kappa\gimel_j}{\sum_{j=1}^n \kappa\gimel_j}}} \right\rangle \otimes \left\langle \mu\gimel_{k+1}^{\frac{\kappa\gimel_{k+1}}{\sum_{j=1}^n \kappa\gimel_j}}, \sqrt[3]{1-(1-\nu\gimel_{k+1}^3)^{\frac{\kappa\gimel_{k+1}}{\sum_{j=1}^n \kappa\gimel_j}}} \right\rangle$$

$$= \left\langle \overline{\prod}_{j=1}^{k} \mu\gimel_k^{\frac{\kappa\gimel_j}{\sum_{j=1}^n \kappa\gimel_j}} .\mu\gimel_{k=1}^{\frac{\kappa\gimel_{k+1}}{\sum_{j=1}^n \kappa\gimel_j}}, \sqrt[3]{1-\overline{\prod}_{j=1}^{k}(1-\nu\gimel_k^3)^{\frac{\kappa\gimel_j}{\sum_{j=1}^n \kappa\gimel_j}} + 1-(1-\nu\gimel_{k+1}^3)^{\frac{\kappa\gimel_{k+1}}{\sum_{j=1}^n \kappa\gimel_j}} - \left(1-\overline{\prod}_{j=1}^{k}(1-\nu\gimel_k^3)^{\frac{\kappa\gimel_j}{\sum_{j=1}^n \kappa\gimel_j}}\right)\left(1-(1-\nu\gimel_{k+1}^3)^{\frac{\kappa\gimel_{k+1}}{\sum_{j=1}^n \kappa\gimel_j}}\right)} \right\rangle$$

$$= \left\langle \overline{\prod}_{j=1}^{k} \mu\gimel_k^{\frac{\kappa\gimel_j}{\sum_{j=1}^n \kappa\gimel_j}} .\mu\gimel_{k+1}^{\frac{\kappa\gimel_{k+1}}{\sum_{j=1}^n \kappa\gimel_j}}, \sqrt[3]{1-\overline{\prod}_{j=1}^{k}(1-\nu\gimel_k^3)^{\frac{\kappa\gimel_j}{\sum_{j=1}^n \kappa\gimel_j}}(1-\nu\gimel_{k+1}^3)^{\frac{\kappa\gimel_{k+1}}{\sum_{j=1}^n \kappa\gimel_{k+1}}}} \right\rangle$$

$$= \left\langle \overline{\prod}_{j=1}^{k+1} \mu\gimel_j^{\frac{\kappa\gimel_j}{\sum_{j=1}^n \kappa\gimel_j}}, \sqrt[3]{1-\overline{\prod}_{j=1}^{k+1}(1-\nu\gimel_j^3)^{\frac{\kappa\gimel_j}{\sum_{j=1}^{k+1} \kappa\gimel_j}}} \right\rangle.$$

Box V

since:

$$\kappa^{\gimel}{}_j = \overline{\prod}_{k=1}^{j-1} \bar{\mho}(\overline{\Upsilon}_k) \quad (j = 2, \ldots, n), \quad \text{and} \quad \kappa^{\gimel}{}_1 = 1.$$

$\bar{\mho}(\overline{\Upsilon}_k)$ is the score of $k$th FFN. We have:

$$\kappa^{\gimel}{}_j = \overline{\prod}_{k=1}^{j-1} \bar{\mho}(\overline{\Upsilon}_k) = \bar{\mho}(\overline{\Upsilon}_1) \times \bar{\mho}(\overline{\Upsilon}_2) \times \ldots$$

$$\times \bar{\mho}(\overline{\Upsilon}_{j-1}) = 0 \times \bar{\mho}(\overline{\Upsilon}_2) \times \ldots \times \bar{\mho}(\overline{\Upsilon}_{j-1})$$

$$(j = 2, \ldots, n),$$

$$\overline{\prod}_{k=1}^{j} \kappa^{\gimel}{}_j = 1.$$

From Definition 5, we have:

$$\text{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) = \overline{\Upsilon}_1^{\frac{\kappa^{\gimel}{}_1}{\Sigma_{j=1}^n \kappa^{\gimel}{}_j}} \otimes \overline{\Upsilon}_2^{\frac{\kappa^{\gimel}{}_2}{\Sigma_{j=1}^n \kappa^{\gimel}{}_j}}$$

$$\otimes, \ldots, \otimes \overline{\Upsilon}_n^{\frac{\kappa^{\gimel}{}_n}{\Sigma_{j=1}^n \kappa^{\gimel}{}_j}} = \overline{\Upsilon}_1^{\frac{1}{1}} \otimes \overline{\Upsilon}_2^{\frac{0}{1}} \otimes \ldots \overline{\Upsilon}_n^{\frac{0}{1}}$$

$$= \overline{\Upsilon}_1 = (0, 1). \qquad \square$$

Corollary 8 implied that incentives would not be obtained by anyone, although the higher importance conditions had been satisfied by the smallest FFN.

**Theorem 7 (Monotonicity).** Assume that $\overline{\Upsilon}_j = \langle \mu^{\gimel}{}_j, \nu^{\gimel}{}_j \rangle$ and $\overline{\Upsilon}_j^* = \langle \mu^{\gimel *}_j, \nu^{\gimel *}_j \rangle$ are the assemblages of FFNs, where $\kappa^{\gimel}{}_j = \overline{\prod}_{k=1}^{j-1} \bar{\mho}(\overline{\Upsilon}_k)$, $\kappa^{\gimel *}_j = \overline{\prod}_{k=1}^{j-1} \bar{\mho}(\overline{\Upsilon}_k^*)$ $(j = 2, \ldots, n)$, $\kappa^{\gimel}{}_1 = 1$, $\kappa^{\gimel *}_1 = 1$, $\bar{\mho}(\overline{\Upsilon}_k)$ is the score of $\overline{\Upsilon}_k$ FFN, and $\bar{\mho}(\overline{\Upsilon}_k^*)$ is the score of $\overline{\Upsilon}_k^*$ FFN. If $\mu^{\gimel *}_j \geq \mu^{\gimel}{}_j$ and $\nu^{\gimel *}_j \leq \nu^{\gimel}{}_j$ for all $j$, then:

$$\text{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) \leq \text{FFPWG}(\overline{\Upsilon}_1^*, \overline{\Upsilon}_2^*, \ldots, \overline{\Upsilon}_n^*).$$

***Proof.*** Here, $\nu^{\gimel *}_j \geq \nu^{\gimel}{}_j$ and $\mu^{\gimel *}_j \leq \mu^{\gimel}{}_j$ for all $j$, If $\nu^{\gimel *}_j \geq \nu^{\gimel}{}_j$.

$$\Leftrightarrow (\nu^{\gimel *}_j)^3 \geq (\nu^{\gimel}{}_j)^3 \Leftrightarrow \sqrt[3]{(\nu^{\gimel *}_j)^3} \geq \sqrt[3]{(\nu^{\gimel}{}_j)^3}$$

$$\Leftrightarrow \sqrt[3]{1 - (\nu^{\gimel *}_j)^3} \leq \sqrt[3]{1 - (\nu^{\gimel}{}_j)^3},$$

$$\Leftrightarrow \sqrt[3]{(1 - (\nu^{\gimel *}_j)^3)^{\frac{\kappa^{\gimel}{}_j}{\Sigma_{j=1}^n \kappa^{\gimel}{}_j}}} \leq \sqrt[3]{(1 - (\nu^{\gimel}{}_j)^3)^{\frac{\kappa^{\gimel}{}_j}{\Sigma_{j=1}^n \kappa^{\gimel}{}_j}}},$$

$$\Leftrightarrow \sqrt[3]{\overline{\prod}_{j=1}^{n} (1 - (\nu^{\gimel *}_j)^3)^{\frac{\kappa^{\gimel}{}_j}{\Sigma_{j=1}^n \kappa^{\gimel}{}_j}}}$$

$$\leq \sqrt[3]{\overline{\prod}_{j=1}^{n} (1 - (\nu^{\gimel}{}_j)^3)^{\frac{\kappa^{\gimel}{}_j}{\Sigma_{j=1}^n \kappa^{\gimel}{}_j}}},$$

$$\Leftrightarrow \sqrt[3]{1 - \overline{\prod}_{j=1}^{n} (1 - (\nu^{\gimel}{}_j)^3)^{\frac{\kappa^{\gimel}{}_j}{\Sigma_{j=1}^n \kappa^{\gimel}{}_j}}}$$

$$\leq \sqrt[3]{1 - \overline{\prod}_{j=1}^{n} (1 - (\nu^{\gimel *}_j)^3)^{\frac{\kappa^{\gimel}{}_j}{\Sigma_{j=1}^n \kappa^{\gimel}{}_j}}}.$$

Now,

$$\mu^{\gimel *}_j \leq \mu^{\gimel}{}_j,$$

$$\Leftrightarrow (\mu^{\gimel *}_j)^{\frac{\kappa^{\gimel}{}_j}{\Sigma_{j=1}^n \kappa^{\gimel}{}_j}} \leq (\mu^{\gimel}{}_j)^{\frac{\kappa^{\gimel}{}_j}{\Sigma_{j=1}^n \kappa^{\gimel}{}_j}},$$

$$\Leftrightarrow \overline{\prod}_{j=1}^{n} (\mu^{\gimel *}_j)^{\frac{\kappa^{\gimel}{}_j}{\Sigma_{j=1}^n \kappa^{\gimel}{}_j}} \leq \overline{\prod}_{j=1}^{n} (\mu^{\gimel}{}_j)^{\frac{\kappa^{\gimel}{}_j}{\Sigma_{j=1}^n \kappa^{\gimel}{}_j}}.$$

Let:

$$\overline{\overline{\Upsilon}} = \text{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n),$$

and:

$$\overline{\overline{\Upsilon^*}} = \text{FFPWG}(\overline{\Upsilon}_1^*, \overline{\Upsilon}_2^*, \ldots, \overline{\Upsilon}_n^*).$$

We get that: $\overline{\overline{\Upsilon^*}} \geq \overline{\overline{\Upsilon}}$. So,

$$\text{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) \leq$$

$$\text{FFPWG}(\overline{\Upsilon}_1^*, \overline{\Upsilon}_2^*, \ldots, \overline{\Upsilon}_n^*). \qquad \square$$

**Theorem 8 (Boundary).** Assume that $\overline{\Upsilon}_j = \langle \mu^{\gimel}{}_j, \nu^{\gimel}{}_j \rangle$ is the assemblage of FFNs, and:

$$\overline{\Upsilon}^- = (\min_j (\mu^{\gimel}{}_j), \max_j (\nu^{\gimel}{}_j)) \quad \text{and}$$

$$\overline{\Upsilon}^+ = (\max_j (\mu^{\gimel}{}_j), \min_j (\nu^{\gimel}{}_j)).$$

Then:

$$\overline{\Upsilon}^- \leq \text{FFPWG}(\overline{\Upsilon}_1, \overline{\Upsilon}_2, \ldots, \overline{\Upsilon}_n) \leq \overline{\Upsilon}^+,$$

where: $\kappa^{\gimel}{}_j = \overline{\prod}_{k=1}^{j-1} \bar{\mho}(\overline{\Upsilon}_k)$ $(j = 2, \ldots, n)$, $\kappa^{\gimel}{}_1 = 1$ and $\bar{\mho}(\overline{\Upsilon}_k)$ is the score of $k$th FFN.

***Proof.*** The proof of this theorem is the same as Theorem 4. $\square$

## 4. Methodology for IDSS using proposed AOs

Let $\overline{\Pi} = \{\overline{\Pi}_1, \overline{\Pi}_2, \ldots, \overline{\Pi}_m\}$ be the collection of alternatives and $\eta^{\zeta} = \{\eta^{\zeta}{}_1, \eta^{\zeta}{}_2, \ldots, \eta^{\zeta}{}_n\}$ is the assemblage of criteria, priorities are given here between the criteria presented by the linear pattern. $\eta^{\zeta}{}_1 \succ \eta^{\zeta}{}_2 \succ \eta^{\zeta}{}_3, \ldots, \eta^{\zeta}{}_n$ indicates criterion $\eta^{\zeta}{}_J$ has a higher priority than $\eta^{\zeta}{}_i$ if $j > i$. $\mathfrak{K} = \{\mathfrak{K}_1, \mathfrak{K}_2, \ldots, \mathfrak{K}_p\}$ is a team of Decision-Makers (DMs), and DMs are not given the same priority. Prioritization is provided by a linear pattern between the DMs given as, $\overline{\Upsilon}_1 \succ \overline{\Upsilon}_2 \succ \overline{\Upsilon}_3, \ldots, \overline{\Upsilon}_p$ indicates DM $\overline{\Upsilon}_\zeta$ has a higher priority than $\overline{\Upsilon}_\varrho$ if $\zeta > \varrho$. DMs provide a matrix of their own opinion $D^{(p)} = (\mathscr{B}_{ij}^{(p)})_{m\times n}$, where $\mathscr{B}_{ij}^{(p)}$ is given for the alternatives $\overline{\amalg}_i \in \overline{\amalg}$ with respect to the criterion ${\eta^{\zeta}}_j \in \eta^{\zeta}$ by $\overline{\Upsilon}_p$ DM in the form of FFNs. If all criteria are the same types, there is no need for normalization, but there are two types of criteria (benefit type attributes $\tau_b$ and cost type attributes $\tau_c$) in MCGDM, in this case using the normalization formula the matrix $D^{(p)}$ has been changed into normalize matrix $Y^{(p)} = (\mathscr{P}_{ij}^{(p)})_{m\times n}$,

$$(\mathscr{P}_{ij}^{(p)})_{m\times n} = \begin{cases} (\mathscr{B}_{ij}^{(p)})^c; & j \in \tau_c \\ \mathscr{B}_{ij}^{(p)}; & j \in \tau_b. \end{cases} \tag{10}$$

where $(\mathscr{B}_{ij}^{(p)})^c$ shows the compliment of $\mathscr{B}_{ij}^{(p)}$.

The suggested operators will be implemented in the MCGDM, requiring the preceding steps.

**Algorithm**

**Step 1:** Obtain the decision matrix $D^{(p)} = (\mathscr{B}_{ij}^{(p)})_{m\times n}$ in the format of FFNs from DMs.

$$\begin{array}{cc} & \\ \overline{\Upsilon}_1 & \overline{\amalg}_1 \\ & \overline{\amalg}_2 \\ & \\ & \overline{\amalg}_m \\ \overline{\Upsilon}_2 & \overline{\amalg}_1 \\ \vdots & \overline{\amalg}_2 \\ & \\ & \overline{\amalg}_m \\ \overline{\Upsilon}_p & \overline{\amalg}_1 \\ & \overline{\amalg}_2 \\ & \\ & \overline{\amalg}_m \end{array} \begin{array}{c} \begin{array}{cccc} {\eta^{\zeta}}_1 & {\eta^{\zeta}}_2 & & {\eta^{\zeta}}_n \end{array} \\ \begin{bmatrix} ({\mu^{\gimel}}^1_{11}, {\nu^{\gimel}}^1_{11}) & ({\mu^{\gimel}}^1_{12}, {\nu^{\gimel}}^1_{12}) & \cdots\cdots & ({\mu^{\gimel}}^1_{1n}, {\nu^{\gimel}}^1_{1n}) \\ ({\mu^{\gimel}}^1_{21}, {\nu^{\gimel}}^1_{21}) & ({\mu^{\gimel}}^1_{22}, {\nu^{\gimel}}^1_{22}) & \cdots\cdots & ({\mu^{\gimel}}^1_{2n}, {\nu^{\gimel}}^1_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ ({\mu^{\gimel}}^1_{m1}, {\nu^{\gimel}}^1_{m1}) & ({\mu^{\gimel}}^1_{m2}, {\nu^{\gimel}}^1_{m2}) & \cdots\cdots & ({\mu^{\gimel}}^1_{mn}, {\nu^{\gimel}}^1_{mn}) \\ ({\mu^{\gimel}}^2_{11}, {\nu^{\gimel}}^2_{11}) & ({\mu^{\gimel}}^2_{12}, {\nu^{\gimel}}^2_{12}) & \cdots\cdots & ({\mu^{\gimel}}^2_{1n}, {\nu^{\gimel}}^2_{1n}) \\ ({\mu^{\gimel}}^2_{21}, {\nu^{\gimel}}^2_{21}) & ({\mu^{\gimel}}^2_{22}, {\nu^{\gimel}}^2_{22}) & \cdots\cdots & ({\mu^{\gimel}}^2_{2n}, {\nu^{\gimel}}^2_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ ({\mu^{\gimel}}^2_{m1}, {\nu^{\gimel}}^2_{m1}) & ({\mu^{\gimel}}^2_{m2}, {\nu^{\gimel}}^2_{m2}) & \cdots\cdots & ({\mu^{\gimel}}^2_{mn}, {\nu^{\gimel}}^2_{mn}) \\ ({\mu^{\gimel}}^p_{11}, {\nu^{\gimel}}^p_{11}) & ({\mu^{\gimel}}^p_{12}, {\nu^{\gimel}}^p_{12}) & \cdots\cdots & ({\mu^{\gimel}}^p_{1n}, {\nu^{\gimel}}^p_{1n}) \\ ({\mu^{\gimel}}^p_{21}, {\nu^{\gimel}}^p_{21}) & ({\mu^{\gimel}}^p_{22}, {\nu^{\gimel}}^p_{22}) & \cdots\cdots & ({\mu^{\gimel}}^p_{2n}, {\nu^{\gimel}}^p_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ ({\mu^{\gimel}}^p_{m1}, {\nu^{\gimel}}^p_{m1}) & ({\mu^{\gimel}}^p_{m2}, {\nu^{\gimel}}^p_{m2}) & \cdots\cdots & ({\mu^{\gimel}}^p_{mn}, {\nu^{\gimel}}^p_{mn}) \end{bmatrix} \end{array}$$

**Step 2:** The decision matrix has two sorts of criterion: ($tauc$) cost type performance indicators and ($taub$) benefit type performance indicators. There is no need for normalization if all performance indicators are of the identical type; nevertheless, in MCDM, there may be two types of performance indicators. In this scenario, the matrix was modified to the transforming response matrix $Y^{(p)} = (\mathscr{P}_{ij}^{(p)})_{m\times n}$ using the normalization formula Eq. (10).

**Step 3:** Evaluate the values of ${\kappa^{\gimel}}_{ij}^{(p)}$ by given formula as follows:

$${\kappa^{\gimel}}_{ij}^{(p)} = \overline{\prod}_{k=1}^{p-1} \bar{\mho}(\mathscr{P}_{ij}^{(k)}) \quad (p = 2, \ldots, n),$$

$${\kappa^{\gimel}}_{ij}^{(1)} = 1. \tag{11}$$

**Step 4:** Using one of provided AOs to combine all of the independent Farmatian decision matrices $Y^{(p)} = (\mathscr{P}_{ij}^{(p)})_{m\times n}$ into one combined evaluation matrix of the alternatives $W^{(p)} = (\chi^{\gimel}_{ij})_{m\times n}$.

$$\begin{aligned} {\chi^{\gimel}}_{ij} &= \text{FFPWA}(\mathscr{P}_{ij}^{(1)}, \mathscr{P}_{ij}^{(2)}, \ldots, \mathscr{P}_{ij}^{(p)}) \\ &= \Bigg\langle \sqrt[3]{1 - \overline{\prod}_{z=1}^{p} 1 - (({\mu^{\gimel}}^3_{ij})^z)^{\frac{{\kappa^{\gimel}}_j^z}{\sum_{j=1}^n {\kappa^{\gimel}}_j^z}}}, \\ &\quad \overline{\prod}_{z=1}^{p} (({\nu^{\gimel}}^3_{ij})^z)^{\frac{{\kappa^{\gimel}}_j^z}{\sum_{j=1}^n {\kappa^{\gimel}}_j^z}} \Bigg\rangle, \end{aligned} \tag{12}$$

or

$$\begin{aligned} {\chi^{\gimel}}_{ij} &= \text{FFPWA}(\mathscr{P}_{ij}^{(1)}, \mathscr{P}_{ij}^{(2)}, \ldots, \mathscr{P}_{ij}^{(p)}) \\ &= \Bigg\langle \overline{\prod}_{z=1}^{p} (({\mu^{\gimel}}^3_{ij})^z)^{\frac{{\kappa^{\gimel}}_j^z}{\sum_{j=1}^n {\kappa^{\gimel}}_j^z}}, \\ &\quad \sqrt[3]{1 - \overline{\prod}_{z=1}^{p} 1 - (({\nu^{\gimel}}^3_{ij})^z)^{\frac{{\kappa^{\gimel}}_j^z}{\sum_{j=1}^n {\kappa^{\gimel}}_j^z}}} \Bigg\rangle. \end{aligned} \tag{13}$$

**Step 5:** Calculate the values of ${\kappa^{\gimel}}_{ij}$ by following formula:

$${\kappa^{\gimel}}_{ij} = \overline{\prod}_{k=1}^{j-1} \bar{\mho}({\chi^{\gimel}}_{ik}) \quad (j = 2, \ldots, n),$$

$${\kappa^{\gimel}}_{i1} = 1. \tag{14}$$

**Step 6:** Aggregate the q-ROF values ${\chi^{\gimel}}_i$ for each alternative $\overline{\amalg}_i$ by the FFPWA (or FFPWG) operator:

$$\begin{aligned} {\chi^{\gimel}}_{i} &= \text{FFPWA}(\mathscr{P}_{i1}, \mathscr{P}_{i2}, \ldots, \mathscr{P}_{in}) \\ &= \Bigg\langle \sqrt[3]{1 - \overline{\prod}_{j=1}^{n} 1 - ({\mu^{\gimel}}^3_{ij})^{\frac{{\kappa^{\gimel}}_j}{\sum_{j=1}^n {\kappa^{\gimel}}_j}}}, \\ &\quad \overline{\prod}_{j=1}^{n} ({\nu^{\gimel}}^3_{ij})^{\frac{{\kappa^{\gimel}}_j}{\sum_{j=1}^n {\kappa^{\gimel}}_j}} \Bigg\rangle, \end{aligned} \tag{15}$$

or

$$\begin{aligned} {\chi^{\gimel}}_{i} &= \text{FFPWG}(\mathscr{P}_{i1}, \mathscr{P}_{i2}, \ldots, \mathscr{P}_{in}) \\ &= \Bigg\langle \overline{\prod}_{j=1}^{n} ({\mu^{\gimel}}^3_{ij})^{\frac{{\kappa^{\gimel}}_j}{\sum_{j=1}^n {\kappa^{\gimel}}_j}}, \\ &\quad \sqrt[3]{1 - \overline{\prod}_{j=1}^{n} 1 - ({\nu^{\gimel}}^3_{ij})^{\frac{{\kappa^{\gimel}}_j}{\sum_{j=1}^n {\kappa^{\gimel}}_j}}} \Bigg\rangle. \end{aligned} \tag{16}$$

**Step 7:** Analyze the score for all cumulative alternative assessments.

**Step 8:** The alternatives were classified by the score function, and eventually, the most suitable alternative was selected.

## 5. Convolutional neural network object detection model

Here, the theoretical literature is discussed to understand the related techniques of machine learning, sensor networks, deep learning, neural networks, and automatic energy management systems. The algorithm of various conventional techniques is also argued.

### *5.1. Machine learning*

The methodology of our proposed model explains all the stages of human recognition and detection, as shown in Figure 1.

*5.1.1. Video recording and processing*

In our proposed model, a camera was installed to monitor and record the video of human activities, and this video was uploaded to create frames of images. So, it was used to interpret 2D images in the frames. The operation of the real-time object recognition system was based on the sampling video interval of 10 minutes. The duration of each small sample represents a 1-minute video clip to study the behavior of real-time objects.

*5.1.2. Single Shot Detector (SSD) model*

This model is used for real-time object detection and to eliminate the region of application to enhance the speed of the overall process. This model uses 59 Frames of images Per Second (FPS) to recognize and detect an object in real-time. It computes multiple classes for the extraction of features with small convolution filters. It uses multi-scale features and boxes to recuperate the drop-in accuracy [45]. The step-by-step features extraction for this model is shown in Figure 2.

*5.1.3. Object recognition*

In this proposed research, a novel CNN technique is used for object recognition. It has used an image of 190 $\times$ 123 pixel (height $\times$ width) as an input for object recognition. It observes noises and distortion due to a higher image height, creating problems for object recognition. Therefore, the size normalization technique is used for the adjustment of size. Our proposed model encompasses occupancy recognition, activity level classification, and validation of the recognition system [46].

*5.1.4. Feature extraction and classification*

Our proposed research used five convolutional layers and three fully connected layers in CNN architecture [47]. In the convolutional layer, the output feature of an image can be calculated as:

$$x(out) = \frac{x(in) - k + 2p}{s} + 1, \tag{17}$$

where $x(in)$ is the input image feature map (height or width); $p$ the padding size; $k$ the filter size; $s$ the stride size; $x(out)$ the output feature map of an image (height or width).

Therefore, feature map size in 1st convolutional layer is 93 $\times$ 60 $\times$ 96, and the output of the 1st convolutional layer is calculated as; $x(out) = [(190 + 2(0) - 13)/(2)] + 1 = 90$. It has used the maximum size of the filter to avoid invalid noise inside features. The CNN-based algorithm works on neuron activity. Therefore, the neuron's outcome is known as function $f$. The function of input image $x$ is derived as:

$$f(x) = \tanh(x), \tag{18}$$

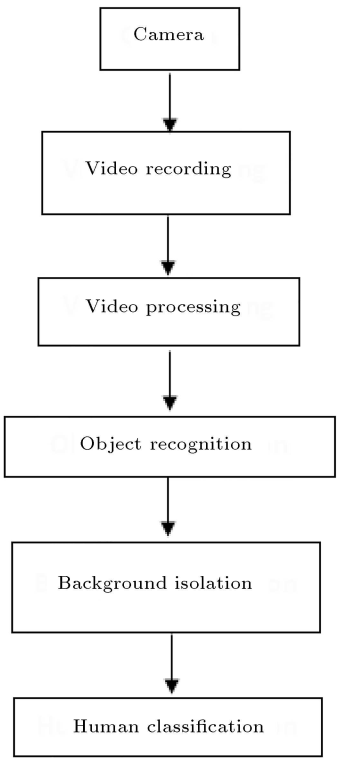


**Figure 1.** Video-based human detection model.

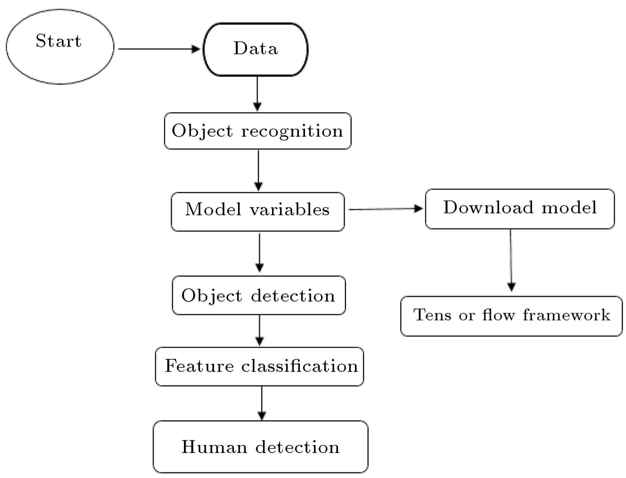


**Figure 2.** Flowchart of SSD model.

or

$$f(x) = (1 + e^{-x})^{-1}. \tag{19}$$

The non-saturating nonlinearity works faster in training than the saturating nonlinearities.

$$f(x) = \max(0, x). \tag{20}$$

The training process of information is faster in Rectified Linear Unit (ReLU) than in tanh. ReLU is also used for the removal of the vanishing gradient. Therefore, it increases the processing speed. After that, learning is initiated after the ReLU-based activity. Generalization is processed with a local normalization scheme, and it is expressed in the form of the following equation:

$$b_{x,y}^{k} = \frac{a_{x,y}^{k}}{i + a\left(\sum_{j=\max(0,k-n/2)}^{N-1,k+n/2} (a_{x,y}^{j})^2\right)^{\beta}}. \tag{21}$$

In the above-mentioned equation, $a_{x,y}^{k}$ is the neuron activity by kernel $k$ at the position level $(x, y)$, and after that, it is applied the ReLU non-linearity. $n$ is the sum of kernel maps at the exact spatial location, and $N$ denotes the total number of kernels used in the convolutional layer. The other constants $i, a$, and $\beta$ are the hyper-parameters. The feature map is attained through the local response normalization channel for the max pooling layer and is used to design the robust and noiseless CNN approach. After the first convolutional layer, the second layer is advanced 128 filters spending the size of $7 \times 7 \times 96$, the stride of $1 \times 1$ pixel, and the padding of $2 \times 2$ pixels in the horizontal and vertical directions. In the structure of CNN, the application of the first two layers is to focus on weak-level images for feature extraction in the form of blobs or edges. The next three layers are applied for the advanced-level extraction of features. Therefore, the convolution of five layers attains the results of feature maps and pixels in the form of 128 feature maps taking the size of $11 \times 7$ pixels. After that, these are fed to the next additional fully connected layers respectively. This approach is finally achieved by human detection and subtracting the background. In the end, the softmax function is applied in the last fully connected layer, which is called the output layer. The function of softmax is given as:

$$\sigma(s)_j = \frac{e^{sj}}{\sum_{m=1}^{k} s^{sn}}. \tag{22}$$

Therefore, the CNN-based human detection system may have the concern of over-fitting. However, it can be decreased by implementing augmentation and dropout technique. The proposed model works on the algorithm for real-time human recognition and detection. The detection of occupancy is done with video processing, which works on a per-frame optimization technique. The accuracy level of the algorithm has achieved better results. The real-time human recognition and detection system SSD model is used for the extraction of feature map and has applied convolutional layers for object detection in video frames. Human detection is achieved with the isolation of backgrounds. Humans can be identified and tracked in a video surveillance system. This system may have multiple applications because it also has the capacity to implement a biometric verification process with human face identification. This model has achieved better results in classification as compared to conventional methods. It works at a very low cost. CNN model shows better recognition and detection results.

## 6. IDSS to energy consumption pattern

This section compares our proposed CNN model with other existing models under the Farmatian fuzzy environment. Other models are energy consumption pattern with Passive Infra-Red (PIR) sensors, energy consumption pattern with ultrasonic sensor, energy consumption pattern with sensors fusion, energy consumption pattern with Radio Frequency (RF) signals, and energy consumption pattern with WLAN, WiFi, and Bluetooth. Planned indoor energy monitoring and management systems are assessed according to the criteria which are given as:

- ${\eta^\zeta}_1$: Environmental protection;
- ${\eta^\zeta}_2$: Cost;
- ${\eta^\zeta}_3$: Total occupancy count (number of people);
- ${\eta^\zeta}_4$: Heating, ventilation, and air condition (Watts);
- ${\eta^\zeta}_5$: Profit;
- ${\eta^\zeta}_6$: Organizational practices that are socially positive.

Consider a set of alternatives $\overline{\text{II}} = \{\overline{\text{II}}_1, \overline{\text{II}}_2, \overline{\text{II}}_3, \overline{\text{II}}_4, \overline{\text{II}}_5\}$, where $\overline{\text{II}}_1$ is the energy consumption pattern with camera based CNN technique, $\overline{\text{II}}_2$ the energy consumption pattern with PIR sensors, $\overline{\text{II}}_3$ the energy consumption pattern with ultrasonic sensor, $\overline{\text{II}}_4$ the energy consumption pattern with RF signals, $\overline{\text{II}}_5$ the energy consumption pattern with WLAN, WiFi, and Bluetooth and $\eta^\zeta = \{{\eta^\zeta}_1, {\eta^\zeta}_2, {\eta^\zeta}_3, {\eta^\zeta}_4, {\eta^\zeta}_5, {\eta^\zeta}_6\}$ is the collection of criteria given above. Priorities shall be defined between all the criteria set out in the linear pattern as ${\eta^\zeta}_1 \succ {\eta^\zeta}_2 \succ {\eta^\zeta}_3, \ldots, {\eta^\zeta}_6$, indicating criterion ${\eta^\zeta}_j$ has a higher priority than ${\eta^\zeta}_i$ if $j > i$. $\overline{\Upsilon} = \{\overline{\Upsilon}_1, \overline{\Upsilon}_2, \overline{\Upsilon}_3\}$ is a team of DMs, and DMs are not given the same priority. Prioritization is provided by a linear pattern between the DMs given as $\overline{\Upsilon}_1 \succ \overline{\Upsilon}_2 \succ \overline{\Upsilon}_3$, indicating DM $\overline{\Upsilon}_\zeta$ has a higher priority than $\overline{\Upsilon}_\varrho$ if $\zeta > \varrho$. DMs are providing a matrix about their own judgment $D^{(p)} = (\mathscr{B}_{ij}^{(p)})_{m \times n}$, where $\mathscr{B}_{ij}^{(p)}$ is given for

**Table 1.** Farmatian fuzzy decision matrix from $\overline{\Upsilon}_1$.

| | $\eta^{\zeta}{}_1$ | $\eta^{\zeta}{}_2$ | $\eta^{\zeta}{}_3$ | $\eta^{\zeta}{}_4$ | $\eta^{\zeta}{}_5$ | $\eta^{\zeta}{}_6$ |
|---|---|---|---|---|---|---|
| $\overline{\text{II}}_1$ | (0.90,0.00) | (0.65,0.35) | (0.75,0.15) | (0.95,0.15) | (0.75,0.00) | (0.45,0.25) |
| $\overline{\text{II}}_2$ | (0.95,0.25) | (0.80,0.30) | (0.55,0.25) | (0.75,0.15) | (0.45,0.45) | (0.35,0.15) |
| $\overline{\text{II}}_3$ | (0.85,0.15) | (0.35,0.55) | (0.75,0.25) | (0.55,0.00) | (0.65,0.35) | (0.45,0.00) |
| $\overline{\text{II}}_4$ | (0.75,0.35) | (0.81,0.25) | (0.65,0.15) | (0.35,0.25) | (0.75,0.25) | (0.35,0.75) |
| $\overline{\text{II}}_5$ | (0.80,0.25) | (0.60,0.00) | (0.25,0.15) | (0.15,0.65) | (0.65,0.15) | (0.25,0.65) |

**Table 2.** Farmatian fuzzy decision matrix from $\overline{\Upsilon}_2$.

| | $\eta^{\zeta}{}_1$ | $\eta^{\zeta}{}_2$ | $\eta^{\zeta}{}_3$ | $\eta^{\zeta}{}_4$ | $\eta^{\zeta}{}_5$ | $\eta^{\zeta}{}_6$ |
|---|---|---|---|---|---|---|
| $\overline{\text{II}}_1$ | (0.75,0.25) | (0.55,0.30) | (0.85,0.15) | (0.95,0.15) | (0.80,0.25) | (0.90,0.15) |
| $\overline{\text{II}}_2$ | (0.55,0.15) | (0.60,0.35) | (0.45,0.15) | (0.75,0.35) | (0.65,0.30) | (0.75,0.00) |
| $\overline{\text{II}}_3$ | (0.90,0.60) | (0.65,0.20) | (0.25,0.55) | (0.65,0.55) | (0.15,0.25) | (0.70,0.30) |
| $\overline{\text{II}}_4$ | (0.50,0.00) | (0.55,0.40) | (0.15,0.10) | (0.50,0.60) | (0.10,0.15) | (0.60,0.35) |
| $\overline{\text{II}}_5$ | (0.85,0.35) | (0.70,0.30) | (0.65,0.55) | (0.25,0.50) | (0.50,0.30) | (0.50,0.25) |

**Table 3.** Farmatian fuzzy decision matrix from $\overline{\Upsilon}_3$.

| | $\eta^{\zeta}{}_1$ | $\eta^{\zeta}{}_2$ | $\eta^{\zeta}{}_3$ | $\eta^{\zeta}{}_4$ | $\eta^{\zeta}{}_5$ | $\eta^{\zeta}{}_6$ |
|---|---|---|---|---|---|---|
| $\overline{\text{II}}_1$ | (0.90,0.15) | (0.85,0.25) | (0.80,0.00) | (0.70,0.35) | (0.80,0.20) | (0.70,0.30) |
| $\overline{\text{II}}_2$ | (0.80,0.25) | (0.55,0.15) | (0.60,0.25) | (0.50,0.30) | (0.60,0.30) | (0.60,0.30) |
| $\overline{\text{II}}_3$ | (0.75,0.15) | (0.65,0.25) | (0.35,0.00) | (0.50,0.35) | (0.75,0.30) | (0.35,0.25) |
| $\overline{\text{II}}_4$ | (0.35,0.35) | (0.50,0.35) | (0.45,0.25) | (0.55,0.45) | (0.25,0.25) | (0.65,0.00) |
| $\overline{\text{II}}_5$ | (0.65,0.25) | (0.65,0.25) | (0.60,0.15) | (0.65,0.25) | (0.65,0.55) | (0.45,0.40) |

**Table 4.** Normalized Farmatian fuzzy decision matrix from $\overline{\Upsilon}_1$.

| | $\eta^{\zeta}{}_1$ | $\eta^{\zeta}{}_2$ | $\eta^{\zeta}{}_3$ | $\eta^{\zeta}{}_4$ | $\eta^{\zeta}{}_5$ | $\eta^{\zeta}{}_6$ |
|---|---|---|---|---|---|---|
| $\overline{\text{II}}_1$ | (0.90,0.00) | (0.35,0.65) | (0.75,0.15) | (0.95,0.15) | (0.75,0.00) | (0.45,0.25) |
| $\overline{\text{II}}_2$ | (0.95,0.25) | (0.30,0.80) | (0.55,0.25) | (0.75,0.15) | (0.45,0.45) | (0.35,0.15) |
| $\overline{\text{II}}_3$ | (0.85,0.15) | (0.55,0.35) | (0.75,0.25) | (0.55,0.00) | (0.65,0.35) | (0.45,0.00) |
| $\overline{\text{II}}_4$ | (0.75,0.35) | (0.25,0.81) | (0.65,0.15) | (0.35,0.25) | (0.75,0.25) | (0.35,0.75) |
| $\overline{\text{II}}_5$ | (0.80,0.25) | (0.00,0.60) | (0.25,0.15) | (0.15,0.65) | (0.65,0.15) | (0.25,0.65) |

**Table 5.** Normalized Farmatian fuzzy decision matrix from $\overline{\Upsilon}_2$.

| | $\eta^{\zeta}{}_1$ | $\eta^{\zeta}{}_2$ | $\eta^{\zeta}{}_3$ | $\eta^{\zeta}{}_4$ | $\eta^{\zeta}{}_5$ | $\eta^{\zeta}{}_6$ |
|---|---|---|---|---|---|---|
| $\overline{\text{II}}_1$ | (0.75,0.25) | (0.30,0.55) | (0.85,0.15) | (0.95,0.15) | (0.80,0.25) | (0.90,0.15) |
| $\overline{\text{II}}_2$ | (0.55,0.15) | (0.35,0.60) | (0.45,0.15) | (0.75,0.35) | (0.65,0.30) | (0.75,0.00) |
| $\overline{\text{II}}_3$ | (0.90,0.60) | (0.20,0.65) | (0.25,0.55) | (0.65,0.55) | (0.15,0.25) | (0.70,0.30) |
| $\overline{\text{II}}_4$ | (0.50,0.00) | (0.40,0.55) | (0.15,0.10) | (0.50,0.60) | (0.10,0.15) | (0.60,0.35) |
| $\overline{\text{II}}_5$ | (0.85,0.35) | (0.30,0.70) | (0.65,0.55) | (0.25,0.50) | (0.50,0.30) | (0.50,0.25) |

the alternatives $\overline{\text{II}}_i \in \overline{\text{II}}$ with respect to the criterion $\eta^{\zeta}{}_j \in \eta^{\zeta}$ by $\overline{\Upsilon}_p$ DM in the form of FFNs.

By applying proposed method, we can solve this problem by following steps:

**Step 1:** Obtain the decision matrix $D^{(p)} = (\mathscr{B}_{ij}^{(p)})_{m\times n}$ in the format of FFNs by the DMs, given in Tables 1–3.

**Step 2:** Using Eq. (10), normalize decision matrixes gained by DMs. $\eta^{\zeta}{}_2$ is the cost type criterion; others are benefit type criteria. Normalized Farmatian fuzzy decision matrices are given in Tables 4–6.

**Step 3:** Determine the $\kappa^{\daleth}{}_{ij}^{(p)}$ values using Eq. (11) to obtained formulas are shown in Box VI.

**Step 4:** Use FFPWA to aggregate all individual

$$\kappa^{\gimel}{}_{ij}^{(1)} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\kappa^{\gimel}{}_{ij}^{(2)} = \begin{pmatrix} 0.8645 & 0.3841 & 0.7093 & 0.9270 & 0.7109 & 0.5378 \\ 0.9209 & 0.2575 & 0.5754 & 0.7093 & 0.5000 & 0.5198 \\ 0.8054 & 0.5618 & 0.7031 & 0.5832 & 0.6159 & 0.5456 \\ 0.6895 & 0.2421 & 0.6356 & 0.5136 & 0.7031 & 0.3105 \\ 0.7482 & 0.3920 & 0.5061 & 0.3644 & 0.6356 & 0.3705 \end{pmatrix}$$

$$\kappa^{\gimel}{}_{ij}^{(3)} = \begin{pmatrix} 0.6078 & 0.1653 & 0.5629 & 0.8593 & 0.5319 & 0.4640 \\ 0.5355 & 0.1065 & 0.3062 & 0.4891 & 0.3119 & 0.3695 \\ 0.6092 & 0.2060 & 0.2985 & 0.3232 & 0.3042 & 0.3590 \\ 0.3878 & 0.1087 & 0.3186 & 0.2334 & 0.3507 & 0.1821 \\ 0.5878 & 0.1341 & 0.2804 & 0.1623 & 0.3489 & 0.3526 \end{pmatrix}$$

Box VI

**Table 6.** Normalized Farmatian fuzzy decision matrix from $\overline{\Upsilon}_3$.

| | $\boldsymbol{\eta^{\zeta}{}_1}$ | $\boldsymbol{\eta^{\zeta}{}_2}$ | $\boldsymbol{\eta^{\zeta}{}_3}$ | $\boldsymbol{\eta^{\zeta}{}_4}$ | $\boldsymbol{\eta^{\zeta}{}_5}$ | $\boldsymbol{\eta^{\zeta}{}_6}$ |
|---|---|---|---|---|---|---|
| $\overline{\amalg}_1$ | (0.90,0.15) | (0.25,0.85) | (0.80,0.00) | (0.70,0.35) | (0.80,0.20) | (0.70,0.30) |
| $\overline{\amalg}_2$ | (0.80,0.25) | (0.15,0.55) | (0.60,0.25) | (0.50,0.30) | (0.60,0.30) | (0.60,0.30) |
| $\overline{\amalg}_3$ | (0.75,0.15) | (0.25,0.65) | (0.35,0.00) | (0.50,0.35) | (0.75,0.30) | (0.35,0.25) |
| $\overline{\amalg}_4$ | (0.35,0.35) | (0.35,0.50) | (0.45,0.25) | (0.55,0.45) | (0.25,0.25) | (0.65,0.00) |
| $\overline{\amalg}_5$ | (0.65,0.25) | (0.25,0.65) | (0.60,0.15) | (0.65,0.25) | (0.65,0.55) | (0.45,0.40) |

**Table 7.** Collective Farmatian fuzzy decision matrix.

| | $\boldsymbol{\eta^{\zeta}{}_1}$ | $\boldsymbol{\eta^{\zeta}{}_2}$ | $\boldsymbol{\eta^{\zeta}{}_3}$ | $\boldsymbol{\eta^{\zeta}{}_4}$ | $\boldsymbol{\eta^{\zeta}{}_5}$ | $\boldsymbol{\eta^{\zeta}{}_6}$ |
|---|---|---|---|---|---|---|
| $\overline{\amalg}_1$ | (0.8648,0.0000) | (0.3304,0.6417) | (0.7997,0.0000) | (0.9171,0.1948) | (0.7796,0.0000) | (0.7312,0.2273) |
| $\overline{\amalg}_2$ | (0.8611,0.2064) | (0.3044,0.7359) | (0.5343,0.2138) | (0.7153,0.2300) | (0.5512,0.3752) | (0.5835,0.0000) |
| $\overline{\amalg}_3$ | (0.8600,0.2382) | (0.3048,0.7192) | (0.6288,0.0000) | (0.5794,0.0000) | (0.6081,0.3474) | (0.5470,0.0000) |
| $\overline{\amalg}_3$ | (0.6472,0.0000) | (0.2988,0.7269) | (0.5491,0.1429) | (0.4394,0.3498) | (0.6184,0.2099) | (0.4825,0.0000) |
| $\overline{\amalg}_4$ | (0.7936,0.2784) | (0.2091,0.6309) | (0.5051,0.2167) | (0.3400,0.5516) | (0.6124,0.2353) | (0.3821,0.4793) |

Farmatian fuzzy decision matrices $Y^{(p)} = (\mathscr{P}_{ij}^{(p)})_{m\times n}$ into one cumulative assessments matrix of the alternatives $W^{(p)} = (\chi^{\gimel}{}_{ij})_{m\times n}$ using Eq. (12) given in Table 7.

**Step 5:** Determine the values of $\kappa^{\gimel}{}_{ij}$ by using Eq. (14).

$$\kappa^{\gimel}{}_{ij} = \begin{pmatrix} 1 & 0.8234 & 0.3178 & 0.2402 & 0.2118 & 0.1561 \\ 1 & 0.8149 & 0.2565 & 0.1466 & 0.0992 & 0.0553 \\ 1 & 0.8113 & 0.2663 & 0.1663 & 0.0993 & 0.0587 \\ 1 & 0.6356 & 0.2042 & 0.1187 & 0.0618 & 0.0379 \\ 1 & 0.7391 & 0.2801 & 0.1567 & 0.0683 & 0.0415 \end{pmatrix}$$

**Step 6:** Aggregate the Farmatian fuzzy values $\chi^{\gimel}{}_{ij}$ for each alternative $\overline{\amalg}_i$ by the FFPWA operator using Eq. (15) given in Table 8.

**Table 8.** Farmatian fuzzy aggregated values $\chi^{\gimel}_i$.

| | |
|---|---|
| $\chi^{\gimel}{}_1$ | (0.7899,0.0000) |
| $\chi^{\gimel}{}_2$ | (0.7319,0.0000) |
| $\chi^{\gimel}{}_3$ | (0.7299,0.0000) |
| $\chi^{\gimel}{}_4$ | (0.5614,0.0000) |
| $\chi^{\gimel}{}_5$ | (0.6576,0.3704) |

**Step 7:** Compute the score for all Farmatian fuzzy aggregated values $\chi^{\gimel}{}_i$.

$\bar{\mho}(\chi^{\gimel}{}_1) = 0.7464,$ $\bar{\mho}(\chi^{\gimel}{}_2) = 0.6960,$

$\bar{\mho}(\chi^{\gimel}{}_3) = 0.6944,$ $\bar{\mho}(\chi^{\gimel}{}_4) = 0.5885,$

$\bar{\mho}(\chi^{\gimel}{}_5) = 0.6168.$

**Step 8:** Rank according to score values.

$$\chi^{⅃}{}_1 \succ \chi^{⅃}{}_2 \succ \chi^{⅃}{}_3 \succ \chi^{⅃}{}_5 \succ \chi^{⅃}{}_4,$$

so,

$$\overline{\amalg}_1 \succ \overline{\amalg}_2 \succ \overline{\amalg}_3 \succ \overline{\amalg}_5 \succ \overline{\amalg}_4.$$

$\overline{\amalg}_1$ indicates that our proposed model is the best among all the existing models.

### *6.1. Comparison with fermatean fuzzy WPM*

The Weighted Product Model (WPM) is a well-known and often used MCDM method for evaluating a set of possible choices in terms of a collection of preferences. By multiplying a number of ratios, one for each chosen criterion, each decision alternative is compared to the others. Each ratio is raised to the power of the related criterion's relative weight. Fermatean fuzzy WPM is proposed by Senapati and Yager [19] to get the optimum alternative in MCDM problems. We solve our given problem by the Fermatean fuzzy WPM and get the same optimal decision. The ranking obtained by the FF-WPM is $\overline{\amalg}_1 \succ \overline{\amalg}_3 \succ \overline{\amalg}_2 \succ \overline{\amalg}_4 \succ \overline{\amalg}_5$. By this, one can see the feasibility, authenticity, and superiority of the proposed method.

## 7. Conclusion

This study focused on occupancy detection models and indoor building energy management algorithms using camera and sensor data. For energy monitoring and management within buildings, we used a new convolutional neural network-based human recognition system based on a camera. For human detection, the camera employs high-infrared and visible lights with greater accuracy. Traditional structures, on the other hand, feature sensors-based occupancy monitoring, which delivers less or restricted information. Sensors are limited in detecting, recognizing, and counting humans. In longer-range applications, sensors are unable to identify and recognize humans. We concluded that camera-based (CNN) person detection can save more Heating, Ventilation, and Air Conditioning (HVAC) energy and costs while promoting a clean atmosphere inside buildings. We offered some additional prioritized Aggregation Operators (AOs) under the Fermatean fuzzy framework to test the applicability of our notion. Some ISSD problem results may be influenced by a failure to consider the links of qualities to the uncertain environment. The Fermatean Fuzzy Numbers (FFNs) are used to express the assessment of decision-makers, the vagueness and incompleteness of the information are effectively addressed. Meanwhile, we developed prioritized AOs named "Fermatean Fuzzy Prioritized Weighted Average (FFPWA) operator and Fermatean Fuzzy Prioritized Weighted Geometric (FFPWG) operator". Various desirable properties of these AOs, as well as their interconnections, have been thoroughly investigated. In further research, considering the superiority of new FFNs, one can extend them to some AOs, such as power mean AOs, Dombi's aggregation operators, Bonferroni mean operators, Heronian mean operators, and so on. Furthermore, we intend to apply the suggested AOs and technique to a wide range of practical applications, including image processing, game theory, pattern recognition, cluster analysis, and uncertain programming.

## References


1. Zuo, J. and Zhao, Z.Y. "Green building research-current status and future agenda: A review", *Renewable & Sustainable Energy Reviews*, **30**, pp. 271–281 (2014).
2. Dwaikat, L.N. and Ali, K.N. "The economic benefits of a green building-evidence from Malaysia", *Journal of Building Engineering*, **18**, pp. 448–453 (2018).
3. Pérez-Lombard, L., Ortiz, J., and Pout, C. "A review on buildings energy consumption information", *Energy and Buildings*, **40**, pp. 394–398 (2018).
4. Deshmukh, S.C. and Patil, V.A. "Energy conservation and audit", *International Journal of Scientific and Research Publications*, **3**(8), pp. 1–5 (2013).
5. Roy, B. "Paradigms and challenges, in multiple criteria decision analysis: state of the art surveys", Springer: New York, NY, USA, pp. 3–24 (2005).
6. Corrente, S., Figueira, J.R., Greco, S., et al. "A robust ranking method extending ELECTRE III to hierarchy of interacting criteria, imprecise weights and stochastic analysis", *Omega*, **73**, pp. 1–17 (2017).
7. Watróbski, J., Jankowski, J., Ziemba, P., et al. "Generalised framework for multi-criteria method selection", *Omega*, **86**, pp. 107–124 (2019).
8. Strantzali, E. and Aravossis, K. "Decision making in renewable energy investments: A review", *Renewable & Sustainable Energy Reviews*, **55**, pp. 885–898 (2016).
9. Martín-Gamboa, M., Iribarren, D., García-Gusano, D., et al. "A review of life-cycle approaches coupled with data envelopment analysis within multi-criteria decision analysis for sustainability assessment of energy systems", *Journal of Cleaner Production*, **150**, pp. 164–174 (2017).
10. L$\phi$ken, E. "Use of multicriteria decision analysis methods for energy planning problems", *Renewable & Sustainable Energy Reviews*, **11**, pp. 1584–1595 (2007).
11. Arce, M.E., Saavedra, A., Míguez, J.L., et al. "The use of grey-based methods in multi-criteria decision analysis for the evaluation of sustainable energy systems: A review", *Renewable & Sustainable Energy Reviews*, **47**, pp. 924–932 (2015).
12. Wang, J.J., Jing, Y.Y., Zhang, C.F., et al. "Review on multi-criteria decision analysis aid in sustainable energy decision making", *Renewable & Sustainable Energy Reviews*, **13**, pp. 2263–2278 (2009).

13. Kaya, I., Çolak, M., and Terzi, F. "A comprehensive review of fuzzy multi criteria decision making methodologies for energy policy making", *Energy Strategy Reviews*, **24**, pp. 207–228 (2019).

14. Zadeh, L.A. "Fuzzy sets", *Information and Control*, **8**, pp. 338–353 (1965).

15. Atanassov, K.T. "Intuitionistic fuzzy sets", *Fuzzy Sets and Systems*, **20**(1), pp. 87–96 (1986).

16. Yager, R.R. and Abbasov, A.M. "Pythagorean membership grades, complex numbers, and decision making", *International Journal of Intelligent Systems*, **28**, pp. 436–452 (2013).

17. Yager, R.R. "Pythagorean fuzzy subsets", *IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS)*, 2013 Joint, Edmonton, Canada, IEEE, pp. 57–61 (2013).

18. Senapati, T. and Yager, R.R. "Fermatean fuzzy sets", *Journal of Ambient Intelligence and Humanized Computing*, **11**, pp. 663–674 (2020).

19. Senapati, T. and Yager, R.R. "Some new operations over fermatean fuzzy numbers and application of fermatean fuzzy WPM in multiple criteria decision making", *Informatica*, **30**(2), pp. 391–412 (2019).

20. Mesiar, R. and Pap, E. "Aggregation of infinite sequences", *Information Sciences*, **178**(18), pp. 3557–3564 (2018).

21. Senapati, T. and Yager, R.R. "Fermatean fuzzy weighted averaging/geometric operators and its application in multi-criteria decision-making methods", *Engineering Applications of Artificial Intelligence*, **85**, pp. 112–121 (2019).

22. Rani, P. and Mishra, A.R. "Fermatean fuzzy Einstein aggregation operators-based MULTIMOORA method for electric vehicle charging station selection", *Expert Systems with Applications*, **182**, 115267 (2021).

23. Jeevaraj, S. "Ordering of interval-valued Fermatean fuzzy sets and its applications", *Expert Systems with Applications*, **185**, 115613 (2021).

24. Garg, H., Shahzadi, G., and Akram, M. "Decision-making analysis based on fermatean fuzzy yager aggregation operators with application in COVID-19 testing facility", *Mathematical Problems in Engineering*, **2020**, 7279027 (2020).

25. Shahzadi, G., Muhiuddin, G., Butt, M.A., et al. "Hamacher interactive hybrid weighted averaging operators under fermatean fuzzy numbers", *Journal of Mathematics*, **2021**, 5556017 (2021).

26. Riaz, M., Garg, H., Farid, H.M.A., et al. "Multi-criteria decision making based on bipolar picture fuzzy operators and new distance measures", *Computer Modeling in Engineering & Sciences*, **127**(2), pp. 771–800 (2021).

27. Jana, C., Pal, M., and Wang, J.Q. "Bipolar fuzzy Dombi aggregation operators and its application in multiple-attribute decision-making process", *Journal of Ambient Intelligence and Humanized Computing*, **10**, pp. 3533–3549 (2019).

28. Sitara, M., Akram, M., and Riaz, M. "Decision-making analysis based on q-rung picture fuzzy graph structures", *Journal of Applied Mathematics and Computing*, **67**, pp 541–577 (2021).

29. Riaz, M. and Hashmi, M.R. "Linear Diophantine fuzzy set and its applications towards multi-attribute decision-making problems", *Journal of Intelligent & Fuzzy Systems*, **37**(4), pp. 5417–5439 (2019).

30. Iampan, A., Garcia, G.S., Riaz, M., et al. "Linear Diophantine fuzzy einstein aggregation operators for multi-criteria decision-making problems", *Journal of Mathematics*, **2021**, 5548033 (2021).

31. Riaz, M., Farid, H.M.A., Aslam, M., et al. "Novel approach for third-party reverse logistic provider selection process under linear Diophantine fuzzy prioritized aggregation operators", *Symmetry*, **13**(7), 1152 (2021).

32. Ashraf, S. and Abdullah, S. "Emergency decision support modeling for COVID-19 based on spherical fuzzy information", *International Journal of Intelligent Systems*, **35**(11), pp. 1–45 (2020).

33. Ashraf, S. and Abdullah, S. "Spherical aggregation operators and their application in multi-attribute group decision-making", *International Journal of Intelligent Systems*, **34**(3), pp. 493–523 (2019).

34. Saha, A., Dutta, D., and Kar, S. "Some new hybrid hesitant fuzzy weighted aggregation operators based on Archimedean and Dombi operations for multi-attribute decision making", *Neural Computing & Application*, **33**(14), pp. 8753–8776 (2021).

35. Farid, H.M.A. and Riaz, M. "Some generalized q-rung orthopair fuzzy Einstein interactive geometric aggregation operators with improved operational laws", *International Journal of Intelligent Systems*, **36**, pp. 7239–7273 (2021).

36. Jana, C., Pal, M., Karaaslan, F., et al. "Trapezoidal neutrosophic aggregation operators and their application to the multi-attribute decision-making process", *Scientia Iranica*, Transaction E, Industrial Engineering, **27**(3), pp. 1655–1673 (2020).

37. Jana, C., Pal, M., and Wang, J.Q. "Bipolar fuzzy Dombi prioritized aggregation operators in multiple attribute decision making", *Soft Computing*, **24**(5), pp. 3631–3646 (2020).

38. Jana, C. and Pal, M. "Multi-criteria decision making process based on some single-valued neutrosophic Dombi power aggregation operators", *Soft Computing*, **25**(7), pp. 5055–5072 (2021).

39. Jana, C., Muhiuddin, G., and Pal, M. "Multi-criteria decision making approach based on SVTrN Dombi aggregation functions", *Artificial Intelligence Review*, **54**(5), pp. 3685–3723 (2021).

40. Yang, Y., Chen, Z.S., Rodriguez, R.M., et al. "Novel fusion strategies for continuous interval-valued q-rung orthopair fuzzy information: a case study in quality assessment of SmartWatch appearance design", *International Journal of Machine Learning and Cybernetics*, **13**, pp. 609–632 (2022). https://doi.org/10.1007/s13042-020-01269-2

41. Chen, Z.S., Yu, C., Chin, K.S., et al. "An enhanced ordered weighted averaging operators generation algorithm with applications for multicriteria decision making", *Applied Mathematical Modelling*, **71**, pp. 467–490 (2019).

42. Chen, Z.S., Yang, L.L., Rodriguez, R.M., et al. "Power-average-operator-based hybrid multiattribute online product recommendation model for consumer decision-making", *International Journal of Intelligent Systems*, **36**, pp. 5272–2617 (2021).

43. Jana, C. and Pal, M. "A dynamical hybrid method to design decision making process based on GRA approach for multiple attributes problem", *Engineering Applications of Artificial Intelligence*, **100**, 104203 (2021).

44. Yager, R.R. "Prioritized aggregation operators", *International Journal of Approximate Reasoning*, **48**, pp. 263–274 (2008).

45. Rastegari, M., Ordonez, V., Redmon, J., et al. "XNOR-net: Imagenet classification using binary convolutional neural networks", *Lecture Notes in Computer Science*, **9908**, pp. 525–542 (2016).

46. Aamir, M., Rahman, Z., Abro, W.A., et al. "An optimized architecture of image classification using convolutional neural network", *International Journal of Image, Graphics and Signal Processing*, **11**(10), pp. 30–39 (2019).

47. Simonyan, K. and Zisserman, A. "Very deep convolutional networks for largescale image recognition", *International Conference on Learning Representations*, pp. 1–14 (2015).

## Biographies

**Muhammad Riaz** has received MSc, MPhil, and PhD degrees in mathematics from the Department of Mathematics, University of the Punjab, Lahore. His research interests include pure mathematics, fuzzy mathematics, topology, algebra, fuzzy sets theory, soft set theory, rough set theory with applications in decision-making problems, medical diagnosis, artificial intelligence, computational intelligence, information measures, image processing, network topology, and pattern recognition. He has 23+ years of regular teaching and research experience. He has published 90+ research articles in international peer-reviewed SCIE and ESCI journals with 1,500+ citations. He has supervised four PhD students and 18 MPhil students. Currently, he is supervising five MPhil and three PhD students. He has been a reviewer for 40+ SCI journals.

**Hafiz Muhammad Athar Farid** received his BS degree in mathematics from the University of Education, Lahore, in 2018. He received an MPhil degree in mathematics from the University of Punjab, Lahore, in 2020. He is currently a PhD scholar at the Department of Mathematics, University of the Punjab, Lahore, Pakistan. He is the author of 20+ SCI research papers. He is also a reviewer of international journals. His research interests include q-Rung orthopair fuzzy sets, neutrosophic sets, linear diophantine fuzzy sets, multi-criteria decision-making problems, aggregation operators, information measures, information fusion, fuzzy modeling, and fuzzy topology.

**Hafiz Muhammad Shakeel** received his BSc degree in Electrical Engineering from the University of the Punjab, Lahore, Pakistan, in 2014. He received his MSc degree in Energy & Environmental Engineering from the same university in 2019. He is a PhD scholar at the University of Huddersfield, United Kingdom. His research interests include artificial intelligence, machine learning, computer vision, data visualization, smart buildings, smart cities, and related applications.

**Danish Arif** is a member of IEEE since 2006. He completed his BSc Engineering in Electrical Engineering from the University of the Punjab, Pakistan, in 2009. He received his MSc degree in Total Quality Management from the same university in 2013. He is a PhD Scholar at the Department of Electrical Engineering, University of Cape Town, South Africa, having research area of Artificial Intelligence and Machine Vision. Between 2009 and 2012, he worked in the defense and telecom industry in Pakistan prior to becoming a lecturer at the Department of Electrical Engineering at the University of the Punjab. He teaches undergraduate and postgraduate subjects in Electrical Engineering. His research areas include systems design, high-performance computing platforms, and artificial intelligence.