A New Robust Bidding Approach for Wind Power Producers Participating in Competitive Power Markets with Correlated Market Prices

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In this research, a bidding problem for a wind-power plant participating in a day-ahead power market with uncertain correlated market prices is studied. A new robust optimization approach considering correlation among uncertainty on the hourly prices in a day-ahead market is developed. This results in solutions with lower level of over-conservatism. For this purpose a new correlated polyhedral uncertainty set is introduced. To consider the uncertainty of market clearing prices and the value of power produced by wind power producer a bidding algorithm is developed. Results of the study using a robust modelling the bidding problem reveal that the appliance of the proposed model on the bidding problem for a price-taker wind power plant in a day-ahead market with uncertain correlated data leads to solutions with superior performance than that of the conventional polyhedral uncertainty sets.

Keywords: Wind power producer; Bidding problem; Robust optimization; Uncertainty set; Correlated market price

1. Introduction

In a day-ahead market, power producers propose the hourly prices for the following day to the market operator. The market operator covers the demanded power via received bids with the minimum prices while regarding the transaction restrictions. The maximum value of the accepted hourly prices by the market operator is called market clearing price of the hour. As the market clearing prices depend on the prices proposed by market participants, the hourly market clearing prices are uncertain, hence participants of day-ahead markets try to estimate the hourly market clearing prices in order to propose the optimal bid with the maximum profit. For further details on this subject the readers are referred to [1]. Several approaches are applied by researchers to tackle the uncertainty in optimization problems. Robust optimization approach is one of these approaches while assumes that there is no knowledge on the probability distribution function. In the robust optimization approach, an uncertainty set includes all perturbation that obtained solutions that are robust against all of them.

Robust optimization approach is used in solving different problems. Researchers has developed different robust models for self-scheduling and bidding problems considering uncertainties of market clearing prices, demands bid, load variations, power output of the wind-power plants etc. Baringo, Baringo and Arroyo [2] formulated the self-scheduling problem of a VPP based on a stochastic adaptive robust optimization approach. Darvishi, Sheisi and Aghaei [3] studied both price-maker roles and price-taker roles (balancing market). Baringo, Baringo and Arroyo [4] modelled the self-scheduling problem of a VPP via a stochastic adaptive robust optimization approach. Hasanzad and Rastegar [5] modelled the power system SCUC problem under wind power generation uncertainty using an adaptive robust approach and solved it by Benders Decomposition algorithm. Khaloie and Anvari-Moghaddam [6] investigated the problem of determining the optimal generation scheduling of a hybrid thermal-energy storage system. They utilized the robust optimization approach to mitigate the risk of the energy market price uncertainty. Abdalla, Adma and Ahmed[7] considered the correlation existing between the renewable resources uncertainties to determine the generation expansion planning methodology based on a new correlated polyhedral uncertainty set. Zhang et al. [8] established an uncertainty set and proposed a robust dispatching method to cover the uncertainty of the value of the power proposed by wind power producers. Gu et al. [9] proposed a robust model for a network including all of the solar, thermal, wind producers and storages. Dai et al. [10] modelled the bidding problem of a wind power producer trading in an energy market considering the uncertainty of the wind power producer output and the value of the demanded power. Han, Kardakos and Hug [11] proposed a two-stage robust framework to derive the optimal bidding strategies for WPPs while considering the uncertainty of wind power generation.

To study the correlation among successive prices of a power market, the values of MCP for seven days, 336 time intervals, starting from April 1 2019 till April 7 2019 are indicated in Figure1.

(Figure 1)

As shown in this figure, the trend of the prices is seasonal. To apply Autocorrelation function, the collected data are converted to stationary series using Minitab 19.2.0 software. The relevant Autocorrelation function plot is indicated in Figure 2. As shown in this figure, there is strong autocorrelations among hourly market prices.

(Figure 2)

This correlation is not considered in researches that have employed robust optimization approach to model the self-scheduling problem. Hence the solutions obtained are protected against perturbations with low or even zero probability of occurrence that leads to over-conservative solutions. It means that the solutions robustness against perturbations with low probability of occurrence decreases the optimality of solutions. In this regard Jalilvandnejad, Shafaei and Shahriari [12] developed a robust model for the self-scheduling problem for a power generation company based on a couple of polyhedral uncertainty sets that consider the existing correlation among hourly prices.

Daneshvari and Shafaei [13] improved the uncertainty set presented by Jalilvandnejad et al. [14] and introduced a new correlated uncertainty set leading to solutions with superior robustness and over conservatism levels compared with the other methods.

In this research, to avoid over-conservatism of the solutions, the new polyhedral uncertainty set that considers the existing correlation among hourly prices is employed to model the bidding problem for a wind power plant participating in a day-ahead market. To remove void spaces of the uncertainty set corresponding to perturbations with negligible probability of occurrence, the borders of the proposed uncertainty set are changed corresponding to the correlation level. On the other hand the employed uncertainty set includes all probable perturbations that leads to robustness of solutions against occurrence of such perturbations. On the other hand, a new bidding algorithm is developed to consider the uncertainty of market clearing prices and the value of power produced by wind power producer. To this aim, an uncertain bidding problem for a price-taker wind power producer is modelled.

2. Robust approach

Robust optimization, fuzzy programming, and stochastic optimization are used o model the uncertainty existing in real world problems [15]. Robust optimization is applied by researchers in different areas among them includes power engineering [16], production systems [17], supply chain management [18- 20], etc.

Solutions obtained using robust models are protected against all perturbations that are included in the corresponding uncertainty sets, hence the applied uncertainty set is important. Let's consider the linear programming problem presented below:

$$\begin{aligned} & Max C'x \\ & Ax < b \\ & l \leq x \leq u \end{aligned} \tag{1}$$

The uncertain matrix A includes the actual values of uncertain coefficients a_{ij} vary in the range of $\left[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}\right]$ where a_{ij} is the nominal value of $\tilde{a}_{ij}$ and $\hat{a}_{ij}$ is the maximum deviation from the nominal value. In this case, the real value of the uncertain coefficient can be formulized as below:

$$\tilde{a}_{ij} = a_{ij} + \zeta_{ij} \hat{a}_{ij} \tag{2}$$

Where ξ_{ij} is a random variable perturbing in the range of $\left[-1, +1\right]$. In a case when all of the absolute values of ζ_{ij} take values lower than $\varphi(\varphi \leq 1)$, the uncertainty set obtained from the interaction among perturbations is called box uncertainty set that can be formulated as follows:

$$U^{A} = \left\{\tilde{a}_{ij} = a_{ij} + \zeta_{ij} \hat{a}_{ij} \left\| \zeta_{ij} \right| \leq \varphi_{i}; \forall i, j \right\} \tag{3}$$

Soyster [21] took into account the worst value of each coefficient by setting the value of φ equal to 1 that leads to the most conservative solutions. Figure 3. illustrates the box uncertainty set associated with two different values of φ.

(Figure 3)

As the value of φ_i increases the corresponding solutions become more conservative. Corners of box uncertainty set indicate perturbations with the maximum deviation from the nominal value. [22-25] proposed an ellipsoidal uncertainty set to avoid over conservative solutions via omitting corners of the box uncertainty set. The formulation of the ellipsoidal uncertainty set is presented as follows:

$$U^{A}=\left\{\tilde{a}_{ij}=a_{ij}+\zeta_{ij}\hat{a}_{ij}\middle|\sum_{j}\zeta^{2}_{ij}\leq\Omega_{i}^{2};\forall i\right\} \tag{4}$$

Where Ω_i adjusts the robustness level. When a problem contains two different uncertain parameters, the associated ellipsoidal uncertainty set is illustrated in Figure 4.

(Figure4)

The robust models developed under ellipsoidal uncertainty set are convex and has a non-linear structure. Bertsimas and Sim [26] introduced the polyhedral uncertainty set based on the trade -off between the robustness and optimality level of solutions. The proposed polyhedral uncertainty set is formulated as follows:

$$U^{A}=\left\{\tilde{a}_{ij}=a_{ij}+\zeta_{ij}\hat{a}_{ij}\middle|\sum_{j}\left|\zeta_{ij}\right|\leq\Gamma_{i};\forall i\right\} \tag{5}$$

The protection level Γ is calculated as follows. This adjusts the robustness level of obtained solutions:

$$\sum_{t=1}^{24}\zeta_{t} \tag{6}$$

As the level of parameter Γ increases, the over conservatism level of solutions are increased. Figure 5 illustrates the polyhedral uncertainty set for a problem with two uncertain coefficients.

(Figure 5)

Researchers have applied box, ellipsoidal and polyhedral uncertainty sets on the self-scheduling problems. Jabr [27] applied a box uncertainty set to model self-scheduling problem for a thermal unit participating in a day-ahead market. The ellipsoidal uncertainty set is used in self-scheduling models of [28-31]. The studies [16, 32-35] formulated robust self-scheduling models under a polyhedral uncertainty set.

As mentioned before, correlations among data are not considered in the introduced uncertainty sets. It means that applying these uncertainty sets in problems with correlated data leads to solutions that are protected against those perturbations with very low occurrence probability. Areas C in Figure 6 indicate such perturbations while the mentioned uncertainty sets include them.

(Figure 6)

To consider the existing correlation among data, Pachamanova [36] introduced a formulation appropriate for the situations where the covariance matrix among the uncertain coefficients is known. Such perturbations in the formulation of polyhedral uncertainty set, proposed by Pachamanova [36], are assumed to be unbounded. Therefore the solutions obtained are robust against perturbations out of the uncertainty bounds which lead to over conservatism in problems with bounded perturbations. The polyhedral uncertainty set proposed in [36] is formulated as follows:

$$U^A = \left\{ \tilde{A} \,\middle|\, \left\| \boldsymbol{\Sigma}^{-\frac{1}{2}} (vec(\tilde{A}) - vec(\breve{A})) \right\| \le \Gamma \right\} \tag{7}$$

Where $\tilde{A}$ and $\breve{A}$ represent the matrices of actual values and expected values of the uncertain coefficients matrix A, respectively. $vec\left(\tilde{A}\right)$ and $vec\left(\breve{A}\right)$ are obtained by stacking the rows of matrix A on top of one another.

Bertsimas and Sim [26] introduced another uncertainty set that considers the correlation among data. They assumed that all sources of uncertainty and the corresponding relation between the sources of uncertainty and uncertain coefficients are known while in practice it is difficult to identify such information. Jalilvandnejad, Shafaei, and Shahriari [14] develop an uncertainty set that covers probable perturbations and removes those perturbations with low probability of occurrence while considering the correlation among uncertain coefficients. They bended the borders of the polyhedral uncertainty set to focus on covering perturbations around the diagonals. The proposed correlated polyhedral uncertainty set is formulated as follows:

$$U^{A}=\left\{\tilde{a}_{ij}=a_{ij}+\zeta_{ij}\hat{a}_{ij}\left\|\zeta_{ij}\right|+\sum_{k\neq j}\left[(1-(\frac{(n-\Gamma_{i})}{n-1})\left|\rho_{ijk}\right|)\left|\zeta_{ij}\right|\right]\leq\Gamma_{i};\forall i,j\right\} \tag{8}$$

Where the existing correlation among uncertain coefficients a_j and a_k is denoted by ρ_{jk}. This is further illustrated in Figure 7:

(Figure 7)

As shown in Figure 8, areas named α indicate perturbations with low or even zero probability of occurrence that includes the uncertainty set introduced by Jalilvandnejad et al. [14] which leads to over conservative solutions. On the other hand, areas named β indicate perturbations with high probability of occurrence that are not included by the uncertainty set addressed above, hence the obtained solutions are not robust against the occurrence of such perturbations.

(Figure 8)

In this research, the improved correlated polyhedral uncertainty set proposed by Daneshvari and Shafaei [13] as illustrated in Figure 9 is applied to develop a robust modelling of a self-scheduling problem for a price taker wind power producer participating in a day-ahead market.

(Figure 9)

As shown above, perturbations with low probability of occurrence are omitted from the proposed uncertainty set. On the other hand, the proposed uncertainty set includes all probable perturbations that lead to solutions with superior level of robustness.

The formulation of the uncertainty set illustrated in Figure 9 is presented as follows:

$$U^{A}=\left\{\begin{array}{l}\tilde{a}_{ij}=a_{ij}+\zeta_{ij}\hat{a}_{ij}\left|\begin{array}{l}\sum_{k\neq j}\left|\zeta_{ij}\right|(1-\beta\Gamma_{i}+\left|\zeta'_{ik}\right|(\beta-1))\geq\\ \sum_{k\neq j}\left|\zeta_{ij_{\min}}\right|(\left|\zeta_{ik}\right|-\beta\Gamma_{i}-(1-\beta)\left|\zeta'_{ik}\right|);\forall i,j\end{array}\right.\\ \left|\zeta_{ij}\right|+\sum_{k\neq j}\left[(1-(\frac{(n-\Gamma_{i})}{n-1})\left|\rho_{ijk}\right|)\left|\zeta_{ij}\right|\right]\leq\Gamma_{i};\forall i,j\\ \left|\zeta_{ij}\right|\leq 1;\forall i,j\end{array}\right\} \tag{9}$$

Where the decision making parameter β with values variying in the range of $[-1,+1]$ makes a trade-off between the optimality and robustness of the solutions. To set $\beta=1$ maximizes the robustness level of the model but reduces the optimality level of solutions. On the other hand, the optimality level of model is maximized when $\beta=0$ but the robustness level of solutions is set on the minimum level. Having considered the existing regression relationships between the intersection points of the proposed uncertainty set borders with horizontal and vertical axes and correlation coefficient ρ, the formulation of the uncertainty set illustrated in Figure 9 is rewritten as follows:

$$U^{A}=\left\{\begin{array}{l}\tilde{a}_{ij}=a_{ij}+\zeta_{ij}\hat{a}_{ij}\left|\begin{array}{l}\sum_{k\neq j}\left|\zeta_{ij}\right|(1-\beta\Gamma_{i}+(1-\rho_{ijk})(\beta-1))\geq\\ \sum_{k\neq j}(\rho_{ijk})(\left|\zeta_{ik}\right|-\beta\Gamma_{i}-(1-\beta)(1-\left|\rho_{ijk}\right|));\forall i,j\end{array}\right.\\ \left|\zeta_{ij}\right|+\sum_{k\neq j}\left[(1-(\frac{(n-\Gamma_{i})}{n-1})\left|\rho_{ijk}\right|)\left|\zeta_{ij}\right|\right]\leq\Gamma_{i};\forall i,j\\ \left|\zeta_{ij}\right|\leq 1;\forall i,j\end{array}\right\} \tag{10}$$

3. Self-scheduling problem

Power producers can be categorized in two types namely price-maker and price-taker producers. Prices proposed by Price-makers affect the market clearing prices, while a price-taker can't influence the market clearing prices. Power producers participating in day-ahead markets intend to maximize profit by solving the self-scheduling problem of the units. For a generation company, solutions obtained via solving the self-scheduling problem determine the optimal hourly output of the units. The used notations in this paper are as follows:

λ_t :	Market clearing price at t.
q_t :	Power produced by wind power producer at t.
r^+ :	The ratio of the real time market price to the day-ahead market price paid to the wind power producer for its excess of generation
Δ_t^+ :	Positive deviation of injected power at from the offered bid at t.
r^- :	The ratio of the real time market price to the day-ahead market price to be charged for deficit of generation
Δ_t^- :	Negative deviation of injected power at from the offered bid at t
$C_t(q_t)$:	Total cost of production at t.
C^{fix} :	Fix cost
$C_t^{inveest}$:	Investment cost at t.
C_t^{mtn} :	Maintenance cost at t.
$b_{j,t}$:	A binary variable, equals to 1 if the wind power output includes the block j at t.
ρ_{tk} :	Correlation coefficient between t and k.
Γ :	Uncertainty budget
ζ_j :	Perturbation of uncertain coefficient j.
β :	Decision making parameter with values in the range of $[0,1]$

For a price-taker wind power producer, the self-scheduling problem can be formulated as follows:

$$Max \sum_{t=1}^{T} (\tilde{\lambda}_{t,w}^{DA}(q_t + r_{t,w}^{+} \Delta_{t,w}^{+} - r_{t,w}^{-} \Delta_{t,w}^{-}) - (C^{invest} + C^{mtn} + C^{fix})) \tag{11}$$

$$Qb_{j,t} \leq Qb_j^m - Qb_{j-1}^m \qquad \forall t, j = 2, ..., N_j \tag{12}$$

$$Qb_{j,t} \geq Qb_j^m - Qb_{j-1}^m - M(1 - b_{j+1,t}) \qquad \forall t, j = 2, ..., N_j \tag{13}$$

$$Qb_{1,t} \le Qb_1^m - Q^{\min} \qquad \forall t \qquad (14)$$

$$Qb_{1,t} \le Qb_1^m - Q^{\min} - M.(1 - b_{2,t}) \qquad \forall t \qquad (15)$$

$$\varepsilon.Qb_{1,t} \le b_{j,t} \le M.Qb_{1,t} \qquad \forall t \qquad (16)$$

$$q_t = \sum_j Qb_{j,t} + Q^{\min}.u^t \qquad \forall t \qquad (17)$$

$$\Delta_t = \Delta_t^+ - \Delta_t^- \qquad \forall t \qquad (18)$$

$$\Delta_t = W_t - q_t \qquad \forall t \text{ s} \qquad (19)$$

As it written above, in equation (11), the price-taker wind power producer tries to maximize the total revenue minus the corresponding total cost. Constraints (12) to (19) imposes that the wind power producer offers should not be higher than the generation capacity of the installed units and calculate the total energy deviations. These technical constraints specify the range of each bidding block and force each block to be used once the previous bidding block is fully utilised.

As mentioned before, actual market clearing prices for different hours of a day market, i.e. $\tilde{\lambda}_t$, are correlated uncertain variables varying in a range of $\left[\lambda_t - \hat{\lambda}_t, \lambda_t + \hat{\lambda}\right]$. In this case, $\tilde{\lambda}_t$ can be formulated as:

$$\tilde{\lambda}_t = \lambda_t + \zeta_t \hat{\lambda}_t \qquad (20)$$

Where λ_t is the nominal value and $\zeta_t \hat{\lambda}_t$ denotes the deviation term from the actual price. As mentioned in (7), the perturbations considered in the uncertainty set proposed by Pachamanova [36] are assumed to be unbounded so that it leads to over conservative solutions. To decrease the over conservatism level of the solutions, Jalilvandnejad et al. [12] restricted the bounds of the uncertainty set introduced by Pachamanova [36] by adding the constraints of a box uncertainty set assuming $\varphi = 1$ as follows:

$$U^A = \left\{ \tilde{a}_j = a_j + \zeta_j \hat{a}_j \left| \sum_j \left| \sum_k r_{jk} \zeta_k \hat{a}_k \right| \le \Gamma_i ; \left| \zeta_k \right| \le 1 ; \forall j \right. \right\} \qquad (21)$$

They illustrated the robust counterpart of the uncertain self-scheduling problem for a generation company under the uncertainty set presented in (21). The details are presented at below:

$$Max \sum_{t=1}^{N_t} (\lambda_t q_t - C(q_t)) - \underset{U^A = \left\{ \tilde{a}_j = a_j + \zeta_j \hat{a}_j \left| \sum_j \left| \sum_k r_{jk} \zeta_k \hat{a}_k \right| \le \Gamma_i ; \left| \zeta_k \right| \le 1 ; \forall j \right. \right\}}{MAX} \sum_t \left| \zeta_t \right| q_t \hat{\lambda}_t)$$

(22)

$$q_t \in S^{q'}$$

In this relation, $C(q_t)$ indicates the producer's total cost and $S^{q'}$ is a set of all feasible solutions of the problem under the unit's technical constraints. On the other hand, the self-scheduling problem presented in (22) under the uncertainty set introduced by Jalilvandnejad et al. [14] is written as follows:

$$Max \sum_{t=1}^{N_t} (\lambda_t q_t - C(q_t))$$
$$- \underset{U^A = \left\{ \tilde{a}_{ij} = a_{ij} + \zeta_{ij} \hat{a}_{ij} \left| \left| \zeta_{ij} \right| + \sum_{k \ne j} \left[(1 - (\frac{(n - \Gamma_i)}{n-1}) \left| \rho_{ijk} \right|) \left| \zeta_{ij} \right| \right] \le \Gamma_i ; \forall i, j \right. \right\}}{MAX} \sum_t \left| \zeta_t \right| q_t \hat{\lambda}_t)$$

(23)

This model is a nonlinear model and hence, Jalilvandnejad et al. [12] rewrote this model by replacing the deviation term of the objective function with its corresponding dual problem. The model is written as follows:

$$Max \sum_{t=1}^{N_T} (\lambda_t (q_t + r_t^+ \Delta_t^+ - r_t^- \Delta_t^-) - (C^{invest} + C^{mtn} + C^{fix})) -$$
$$\sum_t Z_t \Gamma - \sum_t W_t \qquad (24)$$

$$Z_t + \sum_{k \neq t} \left[(1 - (\frac{(N_T - \Gamma_i)}{N_T - 1}) \left| \rho_{tk} \right|) z_k \right] + W_t \geq \hat{\lambda}_t (q_t + r^+ \Delta_t^+ - r^- \Delta_t^-) \quad \forall t \tag{25}$$

$$Z_t, W_t \geq 0 \quad \forall t \tag{26}$$

$$q_t \in S^{q'} \tag{27}$$

In the next section, the robust self-scheduling problem (11)-(19) is modelled under the improved correlated polyhedral uncertainty set proposed by Daneshvari and Shafaei [13] and the results are compared with those obtained using the model presented in [14].

4. Robust modelling under an improved correlated polyhedral uncertainty set

Here, the robust counterpart of the self-scheduling problem presented in (11)-(19) is modelled using the improved correlated polyhedral uncertainty set proposed by Daneshvari and Shafaei [13]:

$$Max \sum_{t=1}^{N_T} (\lambda_t (q_t + r_t^+ \Delta_t^+ - r_t^- \Delta_t^-) - (C^{invest} + C^{mtn} + C^{fix})) \tag{28}$$

$$- \underset{U^A = \left\{ \begin{array}{l} \tilde{a}_{ij} = a_{ij} + \zeta_{ij} \hat{a}_{ij} \left| \begin{array}{l} \sum_{k \neq j} \left| \zeta_{ij} \right| (1 - \beta \Gamma_i + \left| 1 - \rho_{ijk} \right| (\beta - 1)) \geq \\ \sum_{k \neq j} (\left| \rho_{ijk} \right|)(\left| \zeta_{ik} \right| - \beta \Gamma_i - (1-\beta)(\left| 1 - \rho_{ijk} \right|)); \forall i, j \end{array} \right. \\ \left| \zeta_{ij} \right| + \sum_{k \neq j} \left[(1 - (\frac{(n - \Gamma_i)}{n-1}) \left| \rho_{ijk} \right|) \left| \zeta_{ij} \right| \right] \leq \Gamma_i; \forall i, j \\ \left| \zeta_{ij} \right| \leq 1; \forall i, j \end{array} \right\}}{Max} \sum_t \left| \zeta_t \right| \hat{\lambda}_t (q_t + r_t^+ \Delta_t^+ - r_t^- \Delta_t^-)$$

$$q_t \in S^{q'} \tag{29}$$

In this model, given vector of optimal solution q^*, the deviation term of the objective function is formulated as follows:

$$Max \sum_t \left| \zeta_t \right| \hat{\lambda}_t (q_t^* + r_t^+ \Delta_t^+ - r_t^- \Delta_t^-) \tag{30}$$

$$\begin{aligned}&\sum_{k\neq j}\left|\zeta_{ij}\right|(1-\beta\Gamma_i+(\left|1-\rho_{ijk}\right|)(\beta-1))\geq \\ &\sum_{k\neq j}(\left|\rho_{ijk}\right|)(\left|\zeta_{ik}\right|-\beta\Gamma_i-(1-\beta)(\left|1-\rho_{ijk}\right|))\end{aligned} \qquad \forall i,j \qquad (31)$$

$$\left|\zeta_{ij}\right|+\sum_{k\neq j}\left[(1-(\frac{(n-\Gamma_i)}{n-1})\left|\rho_{ijk}\right|)\left|\zeta_{ij}\right|\right]\leq\Gamma_i \qquad \forall i,j \qquad (32)$$

$$0\leq\left|\zeta_{ij}\right|\leq 1 \qquad \forall i,j \qquad (33)$$

The dual problem of (30)-(33) is also formulated as follows:

$$Min\left[\sum_t M_t\sum_{k\neq t}(\left|\rho_{tk}\right|)(\beta\Gamma+(1-\beta)(\left|1-\rho_{ijk}\right|))+\sum_t \Gamma N_t+\sum_t Z_t\right] \qquad (34)$$

$$\begin{aligned}&\sum_{k\neq t}M_k(\left|\rho_{tk}\right|)+\sum_{k\neq t}M_t(\beta\Gamma-1+(\left|1-\rho_{ijk}\right|)(1-\beta))+N_t+ \\ &\sum_{k\neq t}\left[(1-(\frac{n-\Gamma}{n-1})\left|\rho_{tk}\right|)\left|N_k\right|\right]+Z_t\geq\hat{\lambda}_t\left|q_t^*+r^+\Delta^+-r^-\Delta^-\right|\end{aligned} \qquad \forall t \qquad (35)$$

$$M_t\geq 0 \qquad \forall t \qquad (36)$$

$$N_t\geq 0 \qquad \forall t \qquad (37)$$

$$Z_t\geq 0 \qquad \forall t \qquad (38)$$

$$q_t\in S^{q'} \qquad \forall t \qquad (39)$$

Where M_t, N_t and Z_t are dual variables for constraints (31) , (32) and (33) respectively. If this problem has a feasible and bounded solution, the equivalent model of the non-linear model (28)-(29) can be replaced by a linear model presented at below:

$$\begin{aligned}&Max\sum_t(\lambda_t(q_t+r^+\Delta^+-r^-\Delta^-)-(C^{invest}+C^{mtn}+C^{fix})) \\ &-\sum_t M_t\sum_{k\neq t}(\left|\rho_{tk}\right|)(\beta\Gamma+(1-\beta)(\left|1-\rho_{ijk}\right|))-\sum_t\Gamma N_t-\sum_t Z_t\end{aligned}$$

(40)

$$\sum_{k\neq t} M_k(|\rho_{tk}|) + \sum_{k\neq t} M_t(\beta\Gamma - 1 + (|1-\rho_{ijk}|)(1-\beta)) + N_t + \sum_{k\neq t}\left[(1-(\frac{n-\Gamma}{n-1})|\rho_{tk}|)|N_k|\right] + Z_t \geq \hat{\lambda}_t\left[q_t^* + r^+\Delta^+ - r^-\Delta^-\right] \qquad \forall t$$

(41)

$$M_t \geq 0 \qquad \forall t \qquad (42)$$

$$N_t \geq 0 \qquad \forall t \qquad (43)$$

$$Z_t \geq 0 \qquad \forall t \qquad (44)$$

$$q_t \in S^{q'} \qquad \forall t \qquad (45)$$

As shown above, the self-scheduling problem for a wind power plant is formulized as the linear programming problem (40)-(45) that considers the existing correlation between data. In the next section, the performance of the presented model is compared with that of the other covariance based models.

5. Experimental results

In order to study the performance of the proposed model, its performance is compared with that of the conventional polyhedral uncertainty set, covariance based polyhedral uncertainty set presented by Pachamanova [36], correlated polyhedral uncertainty set presented by Jalilvandnejad et al. [14], for this purpose, the self-scheduling problem for a price-taker wind power producer participating in Iranian day-ahead market is studied under different protection level.

To start with, the nominal prices for six months namely February, April, June, August, October and December 2019 of Iranian day-ahead market are used. As illustrated in [12], there is a strong correlation among hourly prices of Iranian day-ahead market. The data from other countries would also follow the similar pattern. Therefore, it is expected that models containing correlated perturbations lead to solutions with lower level of over conservatism. The technical data applied for a price-taker wind power plant is presented in Table 1:

(Table 1)

Bertsimas and Sim [26] introduced the price of robustness as the change in the value of the objective function initiated by alteration in the protection level that can be formulated as follows:

$$thepriceofrobustness = \left(\frac{F_D - F_R}{F_D} \right) \quad (46)$$

Where F_D and F_R are the values of the objective function in the deterministic and robust problems respectively. The self-scheduling problem is modeled and solved under the mentioned robust models. Figure 10 represents results obtained using the market clearing prices of October 2019.

(Figure 10)

As shown above in terms of the price of robustness, conventional robust model results in the largest value. This is in the line with the assumption that the hourly prices are correlated, but such a correlation is not considered in the uncertainty set defined in the conventional model. On the other hand, as the value of protection level decreases, the performance of the other models have less deviations while by increasing the value of the protection level, the solutions obtained using the model introduced by Pachamanova [36] have smaller price of robustness than those model introduced by Jalilvandnejad et al. [12] and the model proposed when the value of parameter β is close to 1. It is because by increasing the value of protection level, the borders of both correlated polyhedral uncertainty set proposed by Jalilvandnejad et al. [12] and also the proposed correlated polyhedral uncertainty set converge to the borders of the conventional polyhedral uncertainty set. On the other hand, the decision making parameter β used in the proposed model lets the decision makers to set the borders of the improved correlated polyhedral uncertainty set as the obtained uncertainty set which covers just those perturbations with low or high probability of occurrence. As a result, the solutions corresponding to those cases that the value of β is close to 0 have smaller values of the price of robustness. As shown in Figure 10, for a risk seeker wind power producer which chooses small value of protection level to gain more profits, the proposed correlated polyhedral uncertainty set leads to solutions with smaller or equal values of the price of robustness than that of the other models investigated. Figure 11 illustrates the performance of the proposed robust self- scheduling models.

(Figure 11)

As shown above, setting values close to 0 for parameter β leads to improvement in the value of solutions price of robustness. On the other hand, to study the performance of the proposed model, the probability of violation is calculated for each solution. For this purpose the nominal hourly prices for six months i.e. February, April, June, August, October and December 2019 of Iranian day-ahead market are used, therefore there are 6*30*24=4320 hourly prices that are used to run the model and the corresponding probabilities of violation are calculated. The graphical results are presented in Figure 12.

(Figure 12)

In Figure 12, X is the probability of violation, and Y is the value of objective function.

In this figure, each point is obtained based on the value of the protection level, hence as the value of the protection level decreases the probability of violation and also the value of the objective value are increased.

As shown in this figure, since the conventional uncertainty set does not consider the existing correlation among data, leads to results with the highest value of the probability of violation. On the other hand, the results obtained from the models developed based on the proposed uncertainty set have lower values of probability of violation than the results obtained from the other models. The results show that decreasing the value of the decision making parameter β improves the performance of the model. Hence the model developed based on the proposed model with the decision making parameter $\beta = 0$ has the lowest values of the probability of violation. In this case, the borders of the uncertainty set obtained include just correlated data. On the other hand, as the value of the decision making parameter β increases, the performance of the proposed model gets closer to the performance of the model proposed by Jalilvandnejad et al. [14]. The presented results reveal that the proposed model leads to solutions with lower level of over conservatism and outperforms the conventional models in terms of robustness. Thus, it is recommended to apply the proposed robust optimisation approach to model uncertain production and scheduling problems.

6. Wind power uncertainty

As the value of wind power production depends on the wind speed, the wind power uncertainty that leads to uncertainty of wind power units output plays an important role in the self-scheduling and bidding problems of wind power producers. Because of the uncertainty of wind speed, determining the bidding strategy is difficult for WPP's [36]. To consider uncertainty factors existing in WPP's self-scheduling problem, the improved correlated polyhedral uncertainty set proposed by Daneshvari and Shafaei [13] is used as follows. In this relations it is assumed $r^+ = r^- = r$.

$$Max\sum_{t}(\tilde{\lambda}_t(q_t + r\Delta^{+} - r\Delta^{-}) - (C^{invest} + C^{mtn} + C^{fix})) \tag{47}$$

$$Qb_{j,t} \le Qb^{m}_{j,t} - Qb^{m}_{j-1,t} \qquad \forall t, j = 2,..., N_j \tag{48}$$

$$Qb_{j,t} \ge Qb^{m}_{j,t} - Qb^{m}_{j-1,t} - M.(1 - b_{j+1,t}) \qquad \forall t, j = 2,..., N_j \tag{49}$$

$$Qb_{1,t} \le Qb^{m}_{1,t} - Q^{\min} \qquad \forall t \tag{50}$$

$$Qb_{1,t} \ge Qb^{m}_{1,t} - Q^{\min} - M.(1 - b_{2,t}) \qquad \forall t \tag{51}$$

$$\varepsilon.Qb_{1,t} \le b_{j,t} \le M.Qb_{1,t} \qquad \forall t, j \tag{52}$$

$$q_t = \sum_{j} Qb_{j,t} + Q^{\min} \cdot u_t \qquad \forall t, j \tag{53}$$

$$\Delta_t = \Delta^{+}_t - \Delta^{-}_t \qquad \forall t \tag{54}$$

$$\Delta_t = \tilde{W}_t - q_t \qquad \forall t \tag{55}$$

Where $\tilde{\lambda}_t$ and $\tilde{W}_t$ are real values of market clearing price and wind power at time t. the presented model can be reformulated as below by replacing constraints (54) and (55) in (47):

$$Max\sum_{t}(\tilde{\lambda}_t(q_t + r(\tilde{W}_t - q_t) - (C^{invest} + C^{mtn} + C^{fix})) \tag{56}$$

$$Qb_{j,t} \le Qb^{m}_{j,t} - Qb^{m}_{j-1,t} \qquad \forall t, j = 2,..., N_j \tag{57}$$

$$Qb_{j,t} \ge Qb^{m}_{j,t} - Qb^{m}_{j-1,t} - M.(1 - b_{j+1,t}) \qquad \forall t, j = 2,..., N_j \tag{58}$$

$$Qb_{1,t} \le Qb^{m}_{1,t} - Q^{\min} \qquad \forall t \tag{59}$$

$$Qb_{1,t} \ge Qb^{m}_{1,t} - Q^{\min} - M.(1 - b_{2,t}) \qquad \forall t \tag{60}$$

$$\varepsilon.Qb_{1,t} \le b_{j,t} \le M.Qb_{1,t} \qquad \forall t, j \tag{61}$$

$$q_t = \sum_j Qb_{j,t} + Q^{\min} \cdot u_t \qquad \forall t, j \tag{62}$$

As there is no uncertainty in constraints (57) to (62), the model addressed above can be presented as follows:

$$Max \sum_t (\lambda_t (q_t + r(\tilde{W}_t - q_t) - (C^{invest} + C^{mtn} + C^{fix}))$$

$$-\underset{U^A = \left\{ \begin{array}{l} \tilde{\lambda}_{ij} = \lambda_{ij} + \zeta_{ij} \hat{\lambda}_{ij} \left| \begin{array}{l} \sum_{k \neq t} \left| \zeta_t \right| (1 - \beta \Gamma_\zeta + (\left| 1 - \rho_{\zeta tk} \right|)(\beta - 1)) \geq \\ \sum_{k \neq t} \left| \rho_{\zeta tk} \right| (\left| \zeta_k \right| - \beta \Gamma_\zeta - (1-\beta)(\left| 1 - \rho_{\zeta tk} \right|)); \forall t \end{array} \right. \\ \left| \zeta_t \right| + \sum_{k \neq t} \left[(1 - (\frac{(n - \Gamma_\zeta)}{n-1}) \left| \rho_{\zeta tk} \right|) \left| \zeta_k \right| \right] \leq \Gamma_\zeta ; \forall t \\ \left| \zeta_t \right| \leq 1; \forall t \end{array} \right\}}{Max} \sum_t \lambda_t \hat{W}_t \left| \zeta_t \right| r$$

$$-\underset{U^A = \left\{ \begin{array}{l} \tilde{W}_{ij} = W_{ij} + \varepsilon_{ij} \hat{W}_{ij} \left| \begin{array}{l} \sum_{k \neq t} \left| \varepsilon_t \right| (1 - \beta \Gamma_\varepsilon + (\left| 1 - \rho_{\varepsilon tk} \right|)(\beta - 1)) \geq \\ \sum_{k \neq t} \left| \rho_{\varepsilon tk} \right| (\left| \varepsilon_k \right| - \beta \Gamma_\varepsilon - (1-\beta)(\left| 1 - \rho_{\varepsilon tk} \right|)); \forall t \end{array} \right. \\ \left| \varepsilon_t \right| + \sum_{k \neq t} \left[(1 - (\frac{(n - \Gamma_\varepsilon)}{n-1}) \left| \rho_{\varepsilon tk} \right|) \left| \varepsilon_k \right| \right] \leq \Gamma_\varepsilon ; \forall t \\ \left| \varepsilon_t \right| \leq 1; \forall i, j \end{array} \right\}}{Max} \sum_t \left[\hat{\lambda}_t \left| \varepsilon_t \right| (q_t (1 - r) + r W_t) \right]$$

$$-\underset{U^A = \left\{ \begin{array}{l} \tilde{a}_{ij} = a_{ij} + \varepsilon_{ij} \zeta_{ij} \hat{a}_{ij} \left| \begin{array}{l} \sum_{k \neq t} \left| \varepsilon_t \right| \left| \zeta_t \right| (1 - \beta \Gamma_{\varepsilon\zeta} + (\left| 1 - \rho_{\varepsilon\zeta tk} \right|)(\beta - 1)) \geq \\ \sum_{k \neq t} \left| \rho_{\varepsilon\zeta tk} \right| (\left| \varepsilon_k \right| \left| \zeta_k \right| - \beta \Gamma_{\varepsilon\zeta} - (1-\beta)(\left| 1 - \rho_{\varepsilon\zeta tk} \right|)); \forall t \end{array} \right. \\ \left| \varepsilon_t \right| \left| \zeta_t \right| + \sum_{k \neq t} \left[(1 - (\frac{(n - \Gamma_{\varepsilon\zeta})}{n-1}) \left| \rho_{\varepsilon\zeta tk} \right|) \left| \varepsilon_t \right| \left| \zeta_t \right| \right] \leq \Gamma_{\varepsilon\zeta} ; \forall t \\ \left| \varepsilon_t \right| \left| \zeta_t \right| \leq 1; \forall t \end{array} \right\}}{Max} \sum_t \left[\hat{\lambda}_t \left| \varepsilon_t \right| (q_t (1 - r) + r W_t) \right] \tag{63}$$

$$q_t \in S^{q'} \tag{64}$$

As ζ_k and ε_k are correlated perturbations that $0 \leq |\varepsilon_k| \leq 1$ and $0 \leq |\zeta_k| \leq 1$ then $0 \leq |\varepsilon_k \zeta_k| \leq 1$ can be replaced by the slack perturbation γ. Consequently the model (63) - (64) can be written as the following relations:

$$Max\sum_t (\lambda_t(q_t + r(\tilde{W}_t - q_t) - (C^{invest} + C^{mtn} + C^{fix}))$$

$$-\underset{U^A=\left\{\begin{array}{l}\tilde{\lambda}_{ij}=\lambda_{ij}+\zeta_{ij}\hat{\lambda}_{ij}\left|\begin{array}{l}\sum_{k\neq t}|\zeta_t|(1-\beta\Gamma_\zeta+(|1-\rho_{\zeta tk}|)(\beta-1))\geq\\ \sum_{k\neq t}|\rho_{\zeta tk}|(|\zeta_k|-\beta\Gamma_\zeta-(1-\beta)(|1-\rho_{\zeta tk}|));\forall t\end{array}\right.\\ |\zeta_t|+\sum_{k\neq t}\left[(1-(\frac{(n-\Gamma_\zeta)}{n-1})|\rho_{\zeta tk}|)|\zeta_k|\right]\leq\Gamma_\zeta;\forall t\\ |\zeta_t|\leq 1;\forall t\end{array}\right\}}{Max} \sum_t \lambda_t \hat{W}_t |\zeta_t| r$$

$$-\underset{U^A=\left\{\begin{array}{l}\tilde{W}_{ij}=W_{ij}+\varepsilon_{ij}\hat{W}_{ij}\left|\begin{array}{l}\sum_{k\neq t}|\varepsilon_t|(1-\beta\Gamma_\varepsilon+(|1-\rho_{\varepsilon tk}|)(\beta-1))\geq\\ \sum_{k\neq t}|\rho_{\varepsilon tk}|(|\varepsilon_k|-\beta\Gamma_\varepsilon-(1-\beta)(|1-\rho_{\varepsilon tk}|));\forall t\end{array}\right.\\ |\varepsilon_t|+\sum_{k\neq t}\left[(1-(\frac{(n-\Gamma_\varepsilon)}{n-1})|\rho_{\varepsilon tk}|)|\varepsilon_k|\right]\leq\Gamma_\varepsilon;\forall t\\ |\varepsilon_t|\leq 1;\forall i,j\end{array}\right\}}{Max} \sum_t \left[\hat{\lambda}_t |\varepsilon_t| (q_t(1-r) + rW_t)\right]$$

$$-\underset{U^A=\left\{\begin{array}{l}\tilde{a}_{ij}=a_{ij}+\gamma_{ij}\hat{a}_{ij}\left|\begin{array}{l}\sum_{k\neq t}|\gamma_t|(1-\beta\Gamma_\gamma+(|1-\rho_{\gamma tk}|)(\beta-1))\geq\\ \sum_{k\neq t}|\rho_{\gamma tk}|(|\gamma_k|-\beta\Gamma_\gamma-(1-\beta)(|1-\rho_{\gamma tk}|));\forall t\end{array}\right.\\ |\gamma_t|+\sum_{k\neq t}\left[(1-(\frac{(n-\Gamma_\gamma)}{n-1})|\rho_{\gamma tk}|)|\gamma_t|\right]\leq\Gamma_\gamma;\forall t\\ |\gamma_t|\leq 1;\forall t\end{array}\right\}}{Max} \sum_t \left[\hat{\lambda}_t |\varepsilon_t| (q_t(1-r) + rW_t)\right]$$

(65)

$$q_t \in S^{q'} \quad (66)$$

Given the optimal solution q^*, the uncertain part of the mentioned objective function can be reformulated as below:

$$Max\sum_t \left[\lambda_t \hat{W}_t r \ |\zeta_t| + \hat{\lambda}_t |\varepsilon_t| (q^*(1-r) + rW_t) + \hat{\lambda}_t W_t\right] \qquad (67)$$

$$\begin{array}{l}\sum_{k\neq t}|\zeta_t|(1-\beta\Gamma_\zeta+(|1-\rho_{\zeta tk}|)(\beta-1)) \geq \\ \sum_{k\neq t}|\rho_{\zeta tk}|(|\zeta_k|-\beta\Gamma_\zeta-(1-\beta)(|1-\rho_{\zeta tk}|))\end{array} \qquad \forall t \qquad (68)$$

$$\left|\zeta_t\right| + \sum_{k\neq t}\left[(1-(\frac{(n-\Gamma_\zeta)}{n-1})\left|\rho_{\zeta tk}\right|)\left|\zeta_k\right|\right] \leq \Gamma_\zeta \qquad \forall t \qquad (69)$$

$$\sum_{k\neq t}\left|\varepsilon_t\right|(1-\beta\Gamma_\varepsilon+(\left|1-\rho_{\varepsilon tk}\right|)(\beta-1)) \geq \sum_{k\neq t}\left|\rho_{\varepsilon tk}\right|(\left|\varepsilon_k\right|-\beta\Gamma_\varepsilon-(1-\beta)(\left|1-\rho_{\varepsilon tk}\right|)) \qquad \forall t \qquad (70)$$

$$\left|\varepsilon_t\right| + \sum_{k\neq t}\left[(1-(\frac{(n-\Gamma_\varepsilon)}{n-1})\left|\rho_{\varepsilon tk}\right|)\left|\varepsilon_k\right|\right] \leq \Gamma_\varepsilon \qquad \forall t \qquad (71)$$

$$\sum_{k\neq t}\left|\gamma_t\right|(1-\beta\Gamma_\gamma+(\left|1-\rho_{\gamma tk}\right|)(\beta-1)) \geq \sum_{k\neq t}\left|\rho_{\gamma tk}\right|(\left|\gamma_k\right|-\beta\Gamma_\gamma-(1-\beta)(\left|1-\rho_{\gamma tk}\right|)) \qquad \forall t \qquad (72)$$

$$\left|\gamma_t\right| + \sum_{k\neq t}\left[(1-(\frac{(n-\Gamma_\gamma)}{n-1})\left|\rho_{\gamma tk}\right|)\left|\gamma_t\right|\right] \leq \Gamma_\gamma \qquad \forall t \qquad (73)$$

$$\left|\zeta_t\right| \leq 1 \qquad \forall t \qquad (74)$$

$$\left|\varepsilon_t\right| \leq 1 \qquad \forall t \qquad (75)$$

$$\left|\gamma_t\right| \leq 1 \qquad \forall t \qquad (76)$$

The dual problem of model (67) to (76) can be presented as follows:

$$Min\begin{bmatrix} \sum_t M_{\lambda t}\sum_{k\neq t}\left|\rho_{\zeta tk}\right|(\beta\Gamma_{\zeta}+(1-\beta)(\left|1-\rho_{\zeta tk}\right|))+\sum_t N_{\lambda t}\Gamma_{\zeta}+\sum_t Z_{\lambda t} \\ +\sum_t M_{wt}\sum_{k\neq t}\left|\rho_{\varepsilon tk}\right|(\beta\Gamma_{\varepsilon}+(1-\beta)(\left|1-\rho_{\varepsilon tk}\right|))+\sum_t N_{wt}\Gamma_{\varepsilon} \\ +\sum_t Z_{wt}+\sum_t M_{\gamma t}\sum_{k\neq t}\left|\rho_{\gamma tk}\right|(\beta\Gamma_{\gamma}+(1-\beta)(\left|1-\rho_{\gamma tk}\right|)) \\ +\sum_t N_{\gamma}\Gamma_{\zeta}+\sum_t Z_{\gamma t} \end{bmatrix} \quad \forall t \qquad (77)$$

$$\sum_{k\neq t} M_{\lambda t}(\beta\Gamma_{\zeta}-1+(\left|1-\rho_{\zeta tk}\right|)(1-\beta))+\sum_{k\neq t}\left|\rho_{\zeta tk}\right|(M_{\lambda k})+N_{\lambda t}$$
$$+\sum_{k\neq t}\left[(1-(\frac{n-\Gamma_{\zeta}}{n-1})\left|\rho_{\zeta tk}\right|)N_{\lambda k}\right]+Z_{\lambda t}\geq\sum_t \lambda_t\hat{w}_t r \quad \forall t \qquad (78)$$

$$\sum_{k\neq t} M_{wt}(\beta\Gamma_{\varepsilon}-1+(\left|1-\rho_{\varepsilon tk}\right|)(1-\beta))+\sum_{k\neq t}\left|\rho_{\varepsilon tk}\right|(M_{wk})+N_{wt}$$
$$+\sum_{k\neq t}\left[(1-(\frac{n-\Gamma_{\varepsilon}}{n-1})\left|\rho_{\varepsilon tk}\right|)N_{wk}\right]+Z_{wt}\geq\sum_t \hat{\lambda}_t(q^{*}(1-r)+rW_t) \quad \forall t \qquad (79)$$

$$\sum_{k\neq t} M_{\gamma t}(\beta\Gamma_{\gamma}-1+(\left|1-\rho_{\gamma tk}\right|)(1-\beta))+\sum_{k\neq t}\left|\rho_{\gamma tk}\right|(M_{\gamma k})+N_{\gamma t}$$
$$+\sum_{k\neq t}\left[(1-(\frac{n-\Gamma_{\gamma}}{n-1})\left|\rho_{\gamma tk}\right|)N_{\gamma k}\right]+Z_{\gamma t}\geq\sum_t r\hat{\lambda}_t\hat{W}_t \quad \forall t \qquad (80)$$

$$M_{wt}, N_{wt}, Z_{wt}\geq 0 \quad \forall t \qquad (81)$$

$$M_{\lambda t}, N_{\lambda t}, Z_{\lambda t}\geq 0 \quad \forall t \qquad (82)$$

$$M_{\gamma t}, N_{\gamma t}, Z_{\gamma t}\geq 0 \quad \forall t \qquad (83)$$

Where $M_{\gamma t}$, $Z_{\gamma t}$, $N_{\gamma t}$, $M_{\lambda t}$, $Z_{\lambda t}$, $N_{\lambda t}$, M_{wt}, N_{wt} and Z_{wt} are dual variables. If this problem has a feasible and bounded solution, the equivalent model of the non-linear model (65)-(66) can be replaced by a linear model presented at below:

$$Max\sum_t(\lambda_t(q_t+r(W_t-q_t)-(C^{invest}+C^{mtn}+C^{fix})))$$
$$-\sum_t M_{\lambda t}\sum_{k\neq t}\left|\rho_{\zeta tk}\right|(\beta\Gamma_\zeta+(1-\beta)(\left|1-\rho_{\zeta tk}\right|))+\sum_t N_{\lambda t}\Gamma_\zeta$$
$$+\sum_t Z_{\lambda t}+\sum_t M_{wt}\sum_{k\neq t}\left|\rho_{\varepsilon tk}\right|(\beta\Gamma_\varepsilon+(1-\beta)\left|1-\rho_{\varepsilon tk}\right|)$$
$$+\sum_t N_{wt}\Gamma_\varepsilon+\sum_t Z_{wt}+\sum_t M_{\gamma t}\sum_{k\neq t}\left|\rho_{\gamma tk}\right|(\beta\Gamma_\gamma+(1-\beta)\left|1-\rho_{\gamma tk}\right|)$$
$$+\sum_t N_{\gamma t}\Gamma_\zeta+\sum_t Z_{\gamma t} \qquad \forall t \qquad (84)$$

$$\sum_{k\neq t}M_{\lambda t}(\beta\Gamma_\zeta-1+(\left|1-\rho_{\zeta tk}\right|)(1-\beta))+\sum_{k\neq t}\left|\rho_{\zeta tk}\right|(M_{\lambda k})+N_{\lambda t}$$
$$+\sum_{k\neq t}\left[(1-(\frac{n-\Gamma_\zeta}{n-1})\left|\rho_{\zeta tk}\right|)N_{\lambda k}\right]+Z_{\lambda t}\geq\sum_t\lambda_t\hat{w}_t r \qquad \forall t \qquad (85)$$

$$\sum_{k\neq t}M_{wt}(\beta\Gamma_\varepsilon-1+(\left|1-\rho_{\varepsilon tk}\right|)(1-\beta))+\sum_{k\neq t}\left|\rho_{\varepsilon tk}\right|(M_{wk})+N_{wt}$$
$$+\sum_{k\neq t}\left[(1-(\frac{n-\Gamma_\varepsilon}{n-1})\left|\rho_{\varepsilon tk}\right|)N_{wk}\right]+Z_{wt}\geq\sum_t\hat{\lambda}_t(q^*(1-r)+rW_t) \qquad \forall t \qquad (86)$$

$$\sum_{k\neq t}M_{\gamma t}(\beta\Gamma_\gamma-1+(\left|1-\rho_{\gamma tk}\right|)(1-\beta))+\sum_{k\neq t}\left|\rho_{\gamma tk}\right|(M_{\gamma k})+N_{\gamma t}$$
$$+\sum_{k\neq t}\left[(1-(\frac{n-\Gamma_\gamma}{n-1})\left|\rho_{\gamma tk}\right|)N_{\gamma k}\right]+Z_{\gamma t}\geq\sum_t r\hat{\lambda}_t\hat{W}_t \qquad \forall t \qquad (87)$$

$$M_{wt}, N_{wt}, Z_{wt}\geq 0 \qquad \forall t \qquad (88)$$

$$M_{\lambda t}, N_{\lambda t}, Z_{\lambda t}\geq 0 \qquad \forall t \qquad (89)$$

$$M_{\gamma t}, N_{\gamma t}, Z_{\gamma t}\geq 0 \qquad \forall t \qquad (90)$$

$$q_t\in S^{q'} \qquad \forall t \qquad (91)$$

7. Bidding strategy for wind power producer

In this section, the third step of the bidding procedure provided in [37] is studied using a new bidding algorithm. To this aim, the algorithm proposed by Baringo and Conejo [16] is considered. They used a polyhedral uncertainty set to make their bidding to the power market. On this line they divided a range of possible values for uncertain market clearing price to n-1 equal ranges. In the first step, the highest bidding point with the maximum values for (λ_t, q_t) is obtained when the maximum value for market clearing price is used to solve the self-scheduling problem. In the next step, the uncertainty range of market clearing price is reduced to $\left[\lambda_t^{\max} - \left(\frac{\lambda_t^{\max} - \lambda_t^{\min}}{n-1}\right), \lambda_t^{\max}\right]$ and the self-scheduling problem is solved to obtain the second bidding floor. In each iteration of the bidding algorithm proposed in [16] the market clearing price uncertainty range is added by $\frac{\lambda_t^{\max} - \lambda_t^{\min}}{n-1}$. To obtain a bidding diagram with n floors, these steps are replicated n times. As Baringo and Conejo [16] set the value of uncertainty budget to 24, considering the uncertainty range for market clearing prices has no effect on the values of the uncertain coefficients λ_t and the algorithm selects the minimum values for market clearing prices in every replication. For example, although the algorithm considers the uncertainty range $\left[\lambda_t^{\max} - (k-1)\left(\frac{\lambda_t^{\max} - \lambda_t^{\min}}{n-1}\right), \lambda_t^{\max}\right]$ for the market clearing price in the kth replication, the value of the market clearing price is equal to its predefined value $\lambda_t^{\max} - (k-1)\left(\frac{\lambda_t^{\max} - \lambda_t^{\min}}{n-1}\right)$. In other words, the modelled robust self-scheduling problem would be considered as an ineffective.

On the other hand, the uncertainty of wind power is not been considered in the literature. To this aim, here a new bidding algorithm that considers the uncertainty of market clearing prices and the uncertainty of wind power is introduced as shown in Figure 13:

(Figure 13)

8. Bidding algorithm evaluation

To evaluate the introduced bidding algorithm, the nominal market clearing prices of Iranian power market and the wind speed data registered for Manjil weather station June till july 2021 are used. The applied technical data are same as those presented in section 5. The results obtained using the introduced algorithm are presented in Figure 14:

(Figure 14)

The results obtained using the introduced algorithm reveals that the performance of the introduced bidding algorithm is superior that the other method. In addition the results show that the proposed model leads to solutions with lower level of over conservatism and results in higher profit than the other methods.

9. Discussions and conclusions

In this study, a bidding problem for a price-taker wind power plant participating in a day-ahead market with uncertain correlated hourly prices is studied. To prevent over conservatism a new correlated polyhedral uncertainty set is presented. A decision making parameter is used in defining the borders of the proposed correlated polyhedral uncertainty set. This leads decision makers to define the uncertainty set in a way that it only includes the perturbations with the probably occurrence. The results of the study revealed that the appliance of the proposed model on the self-scheduling problem for a price-taker wind power plant in a day-ahead market with uncertain correlated data leads to solutions with smaller values in terms of the price of robustness and lower level of over conservatism.

In this study a new bidding algorithm was introduced that considers an uncertainty on market clearing prices as well as wind power. The results revealed that the performance of the proposed bidding algorithm along with the new self-scheduling model leads to solutions with lower level of over conservatism and higher profit values for a wind power producers participating in power markets.

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Table 1. WPP'S DATA

Figure 1.The values of market clearing prices.

Figure 2.Autocorrelation function plot of market clearing prices.

Figure 3.Box uncertainty set.

Figure 4. Ellipsoidal uncertainty set.

Figure 5. Polyhedral uncertainty set.

Figure 6.Areas with low or zero probability of occurrence included by box, ellipsoidal and polyhedral uncertainty sets.

Figure 7.Correlated polyhedral uncertainty set in terms of different correlations and protection levels [14].

Figure 8.Areas with high probability of occurrence which includes correlated polyhedral uncertainty set and areas with low probability of occurrence covered by polyhedral uncertainty set and areas with $\rho = 0.8$ and $\Gamma = 0.6$.

Figure 9.The proposed borders of the improved correlated polyhedral uncertainty set for different values of β.

Figure 10.A comparison among robust self-scheduling models under different uncertainty sets with respect to the price of robustness.

Figure 11.Performance of the proposed models compared with those of other robust models.

Figure 12.Graphical results of solving the self-scheduling problem under different uncertainty sets.

Figure 13.The introduced bidding algorithm for wind power producers.

Figure 14.The bidding diagram for t=9.

Table 1

C^{mtn} $(^{\$}/_{MWh})^{*}$	C^{invest} $(^{\$}/_{h})^{**}$	$P^{MAX}(MW)$
4	658	30

*Maintenance/operation cost of wind turbine equals to 35 $/kW/year.
**The wind turbine investment cost equals to 1250 $/kW with lifetime of 25 Years.

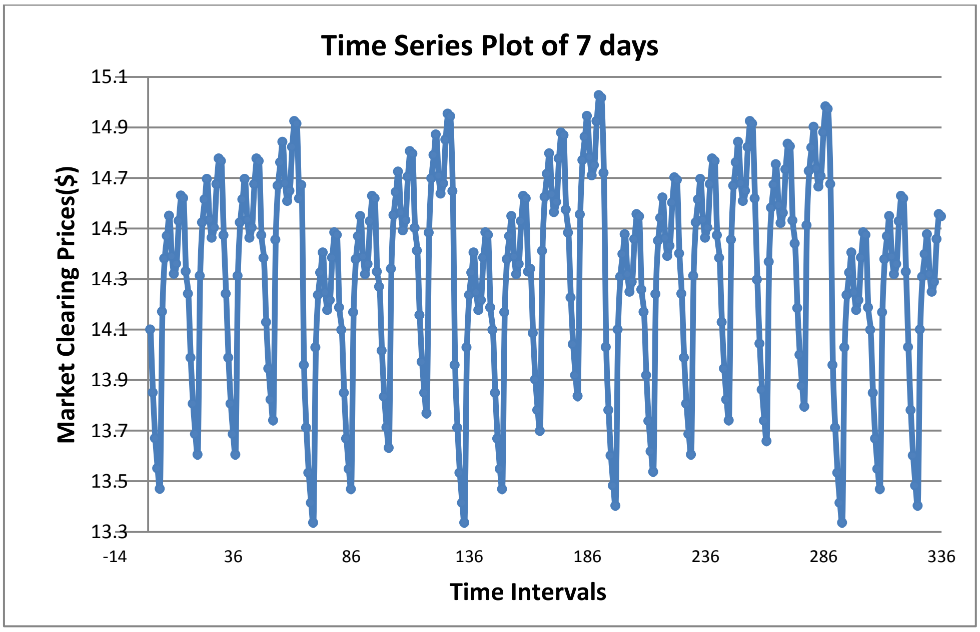


Figure 1

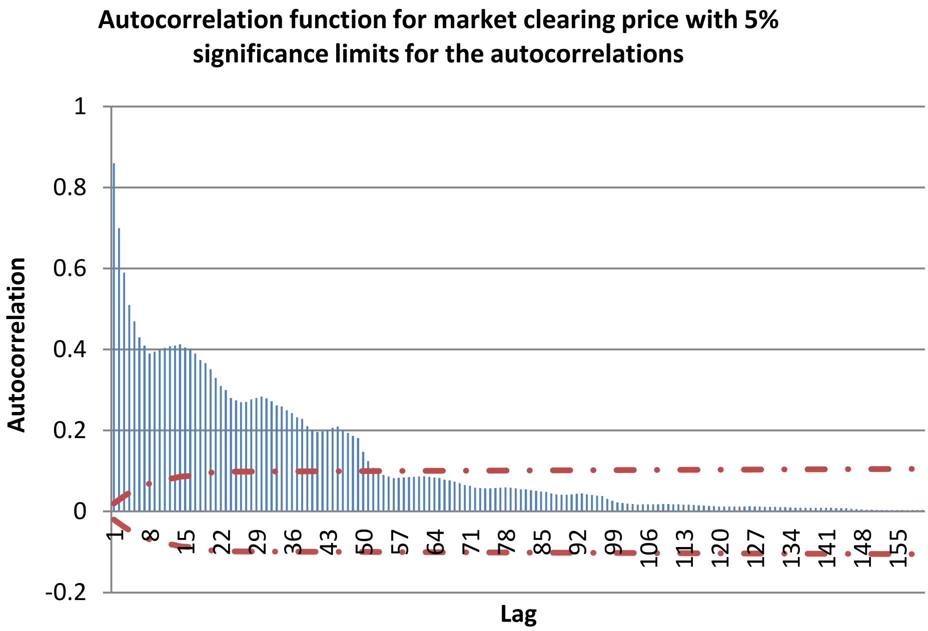


Figure 2

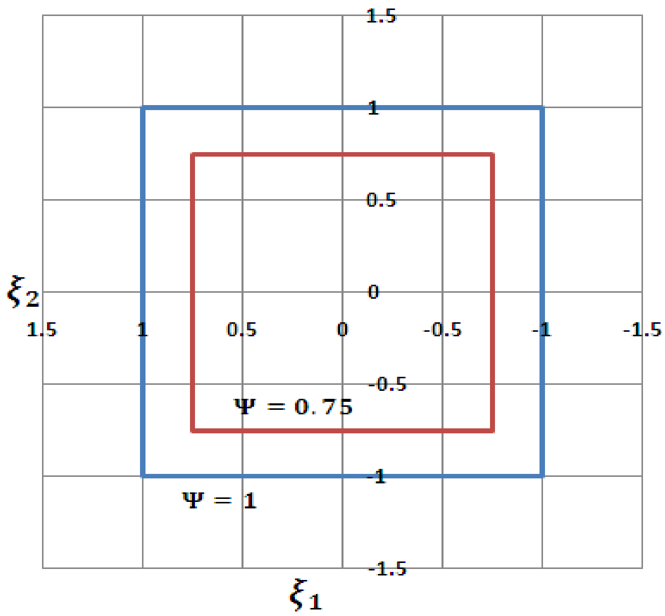
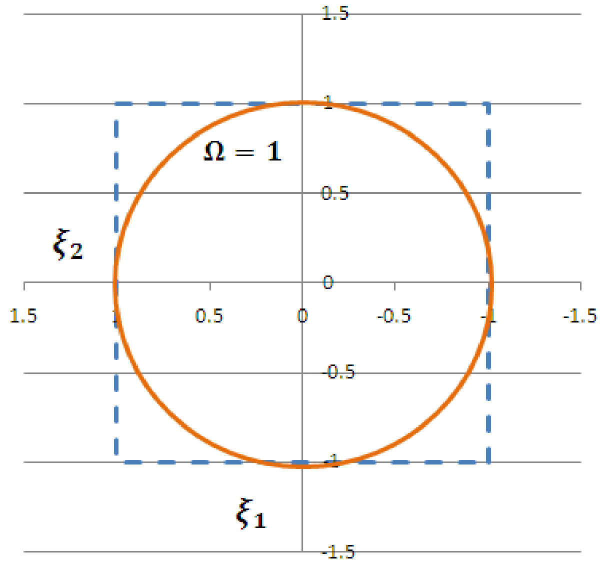


Figure 3



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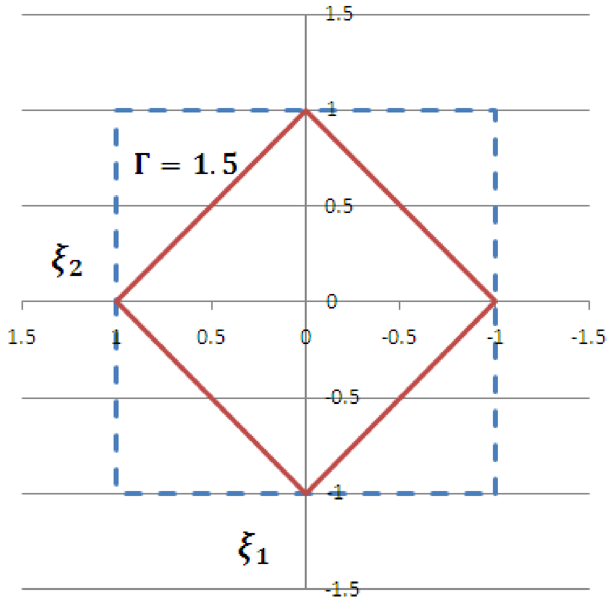

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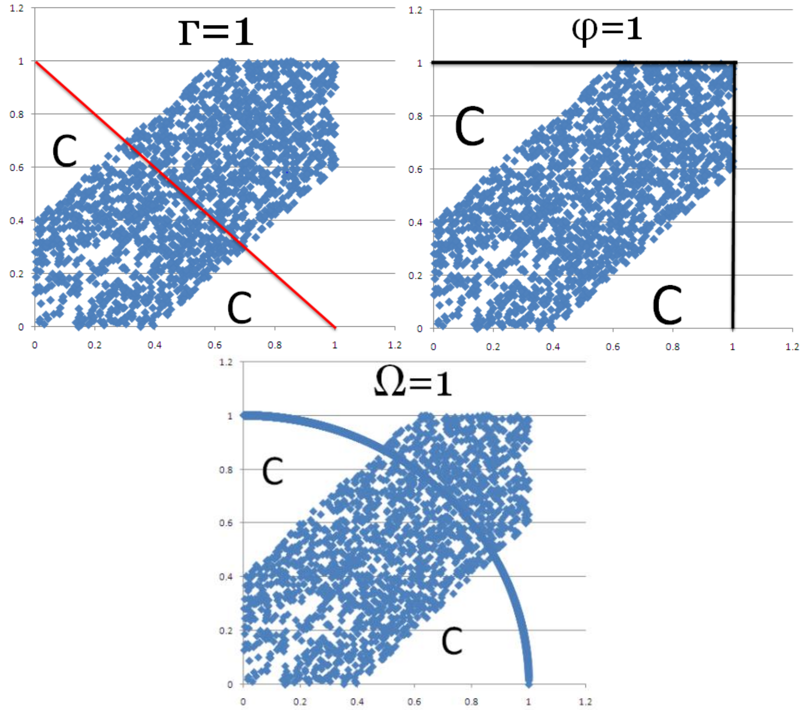

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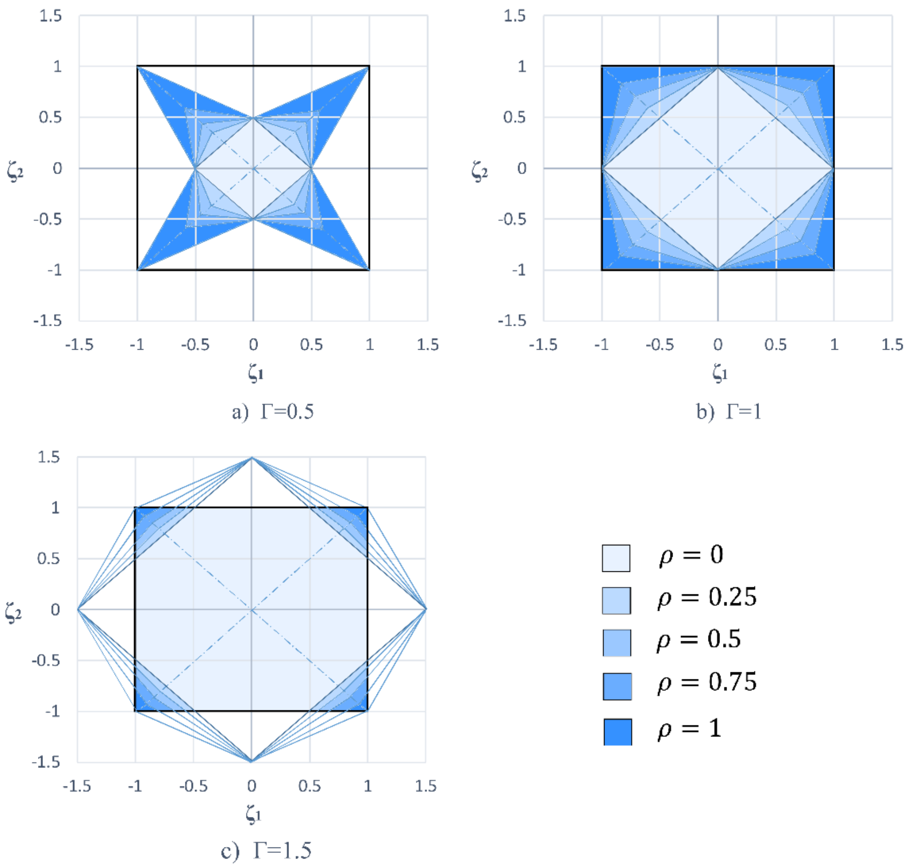

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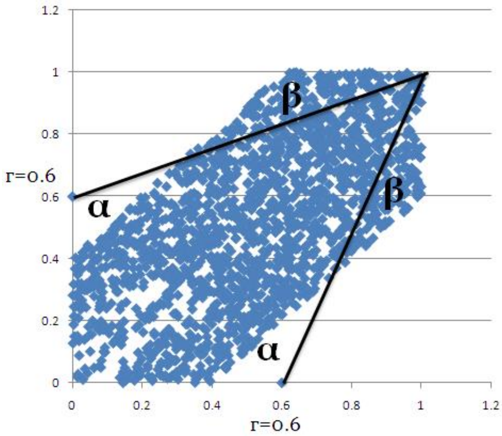

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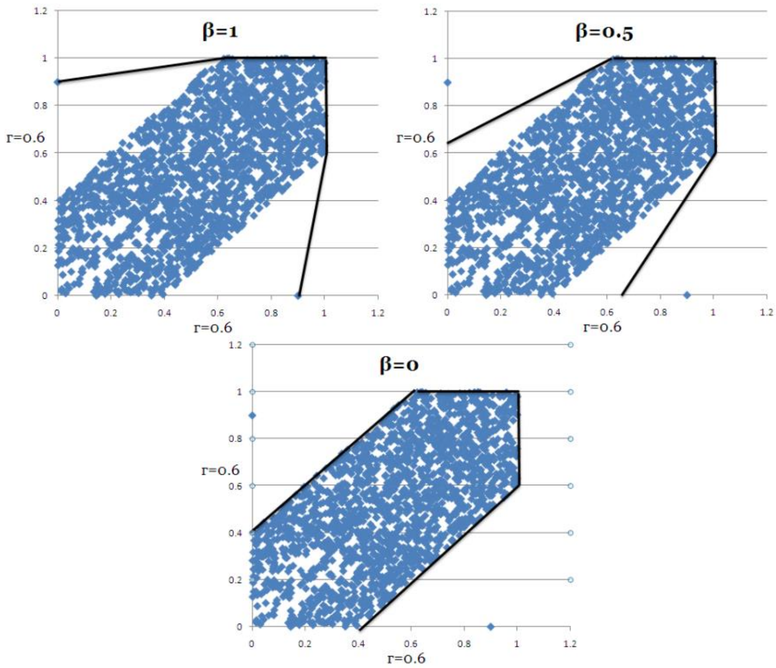


Figure 9

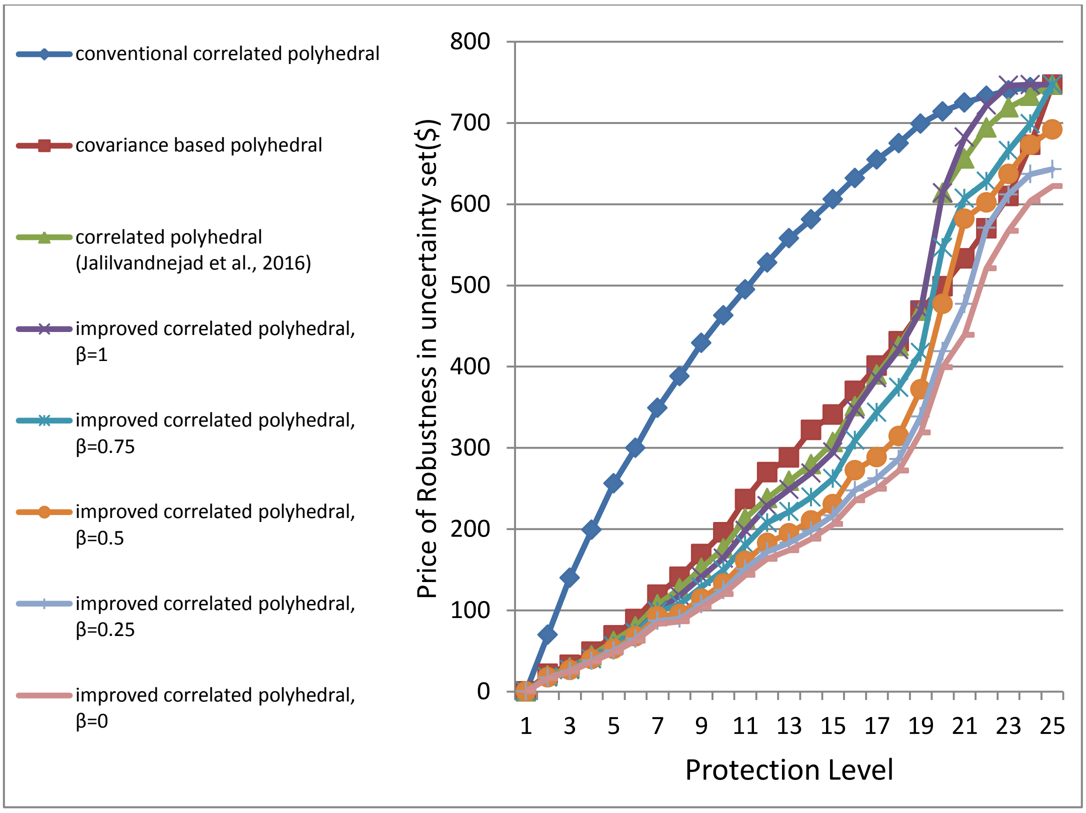


Figure 10

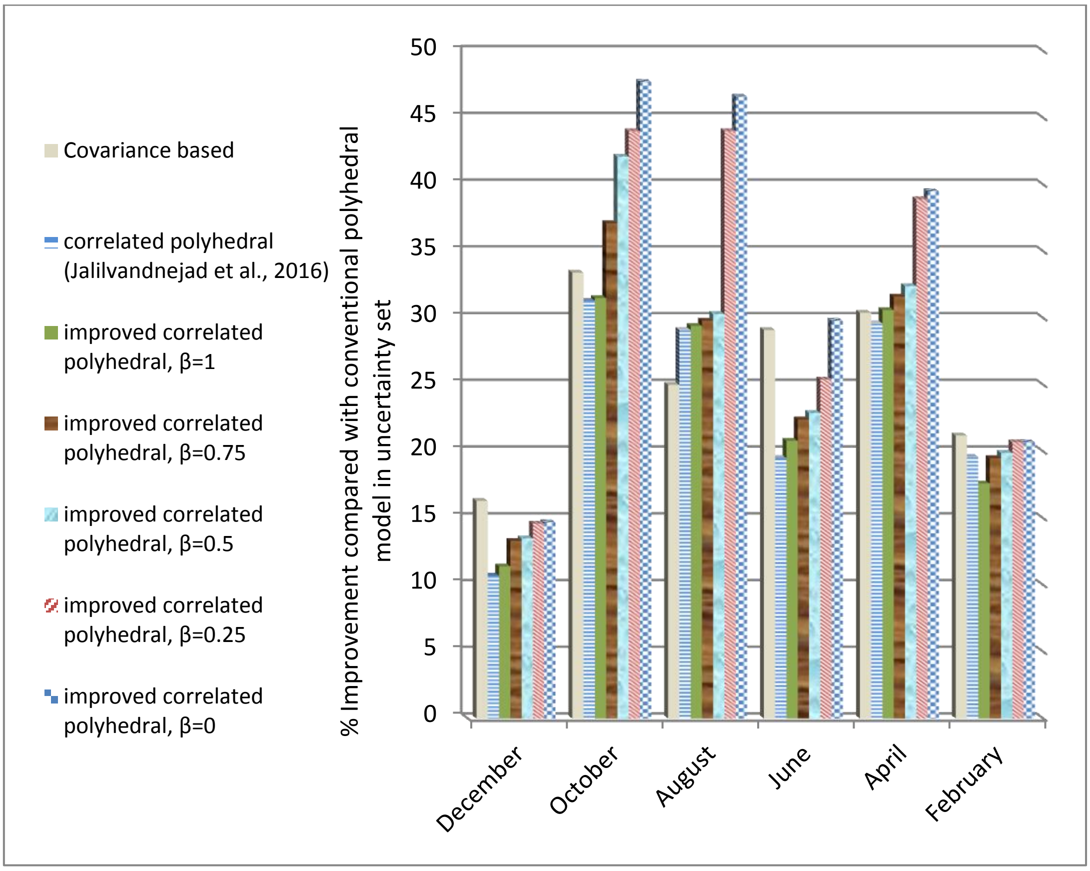
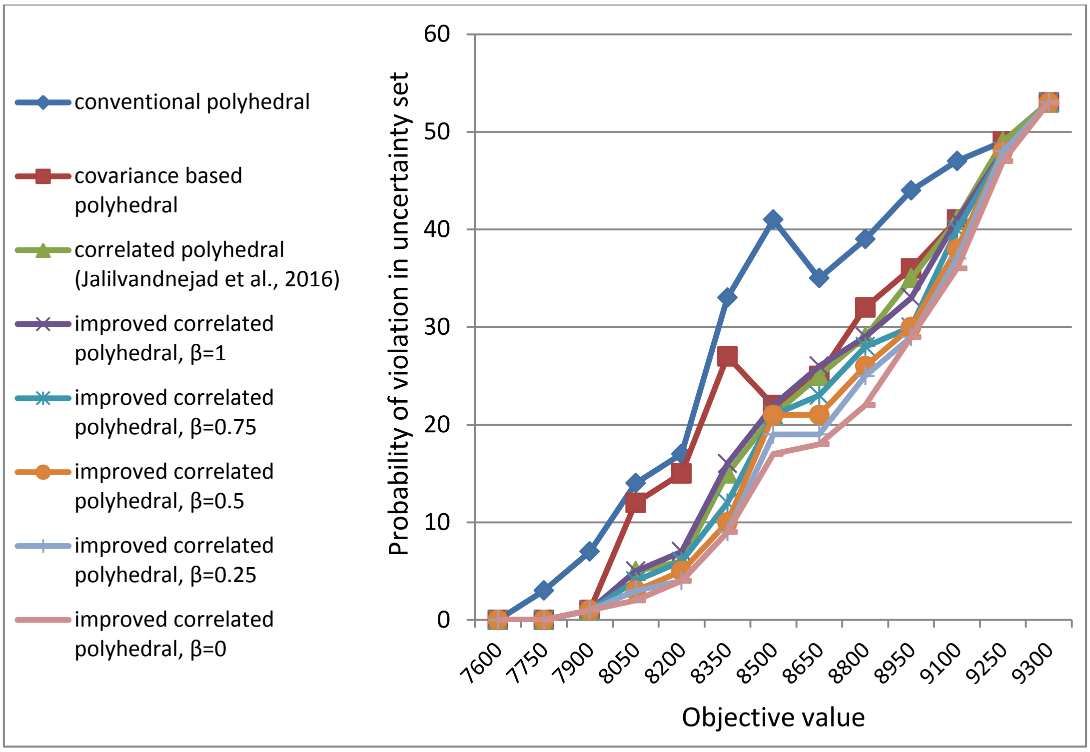


Figure 11



conventional polyhedral
covariance based polyhedral
correlated polyhedral (Jalilvandnejad et al., 2016)
improved correlated polyhedral, β=1
improved correlated polyhedral, β=0.75
improved correlated polyhedral, β=0.5
improved correlated polyhedral, β=0.25
improved correlated polyhedral, β=0
Probability of violation in uncertainty set
60
50
40
30
20
10
0
7600
7750
7900
8050
8200
8350
8500
8650
8800
8950
9100
9250
9300
Objective value

Figure 12

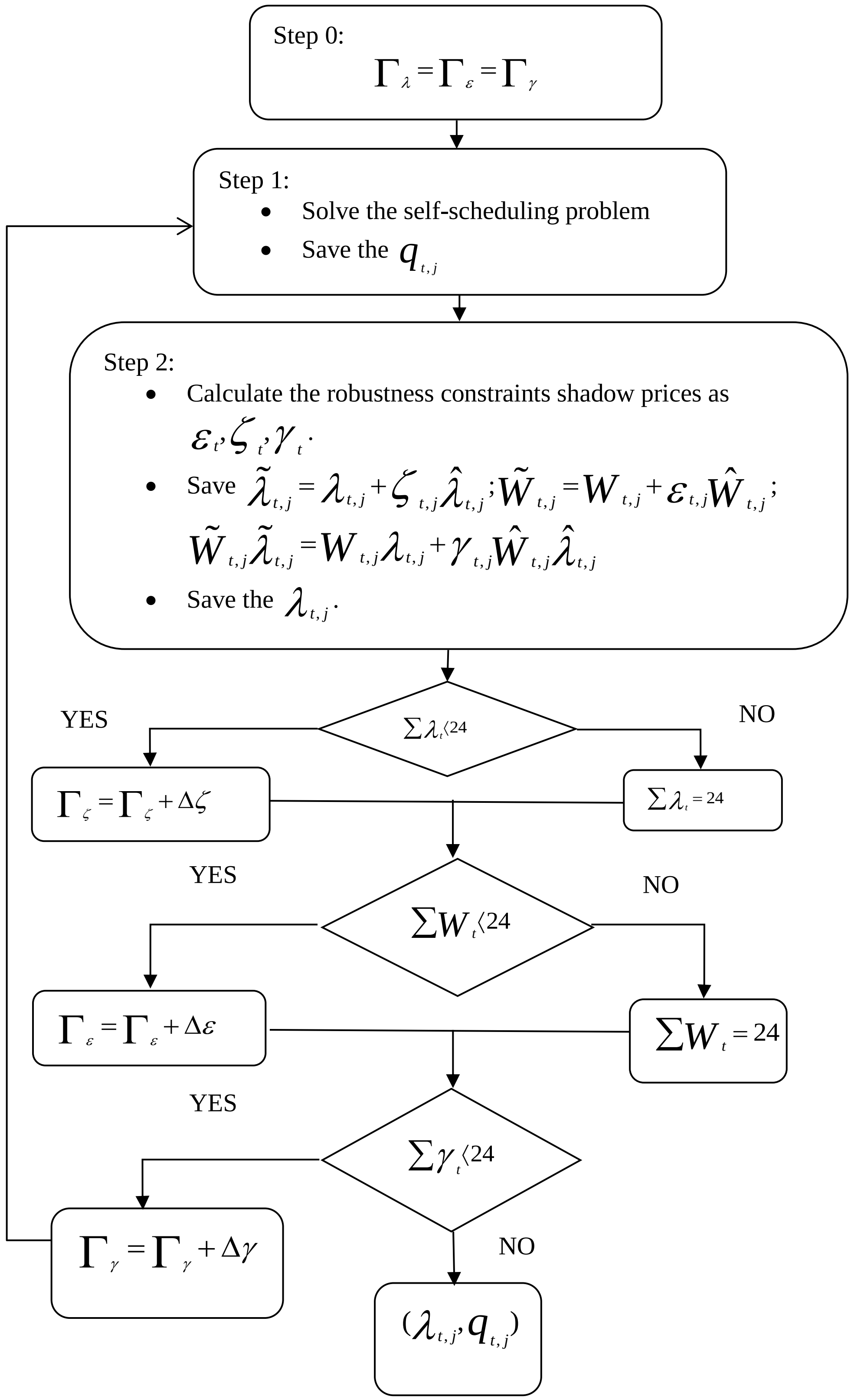
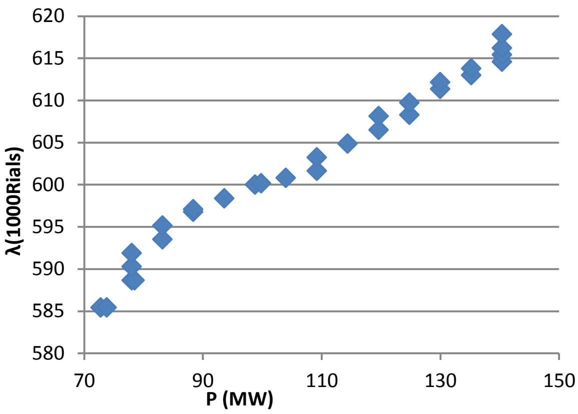
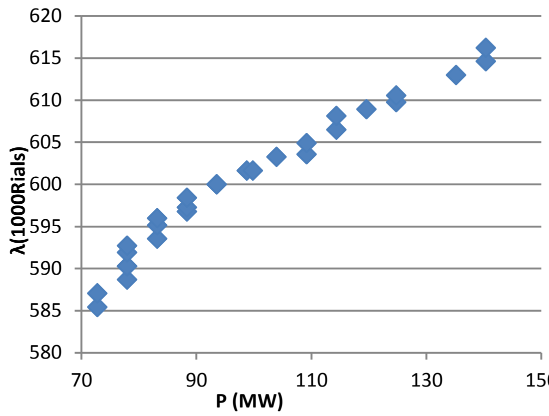

Step 0:
$\Gamma_\lambda = \Gamma_\varepsilon = \Gamma_\gamma$
Step 1:
Solve the self-scheduling problem
Save the $q_{t,j}$
Step 2:
Calculate the robustness constraints shadow prices as $\varepsilon_t, \zeta_t, \gamma_t$.
Save $\tilde{\lambda}_{t,j} = \lambda_{t,j} + \zeta_{t,j}\hat{\lambda}_{t,j}; \tilde{W}_{t,j} = W_{t,j} + \varepsilon_{t,j}\hat{W}_{t,j}$;
$\tilde{W}_{t,j}\tilde{\lambda}_{t,j} = W_{t,j}\lambda_{t,j} + \gamma_{t,j}\hat{W}_{t,j}\hat{\lambda}_{t,j}$
Save the $\lambda_{t,j}$.
YES
$\sum \lambda_t \langle 24$
NO
$\Gamma_\zeta = \Gamma_\zeta + \Delta\zeta$
$\sum \lambda_t = 24$
YES
$\sum W_t \langle 24$
NO
$\Gamma_\varepsilon = \Gamma_\varepsilon + \Delta\varepsilon$
$\sum W_t = 24$
YES
$\sum \gamma_t \langle 24$
$\Gamma_\gamma = \Gamma_\gamma + \Delta\gamma$
NO
$(\lambda_{t,j}, q_{t,j})$

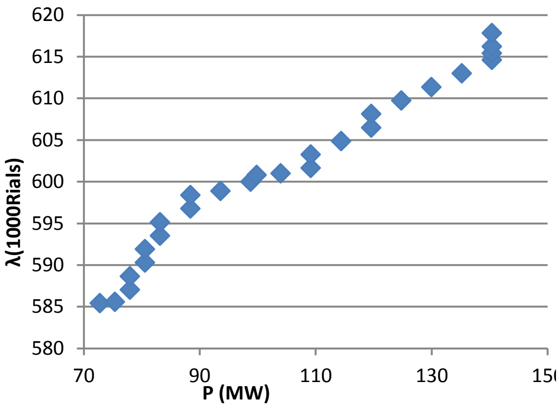
Figure 13



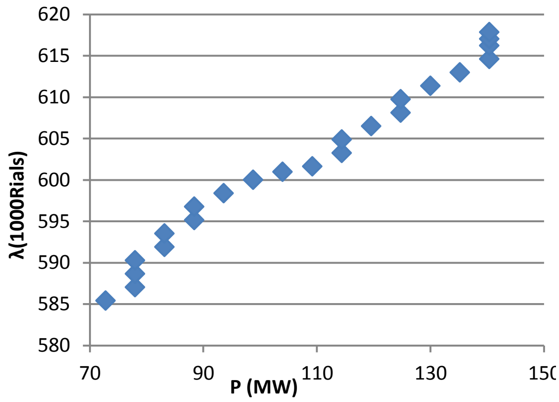
(a) The model based on the covariance based correlated



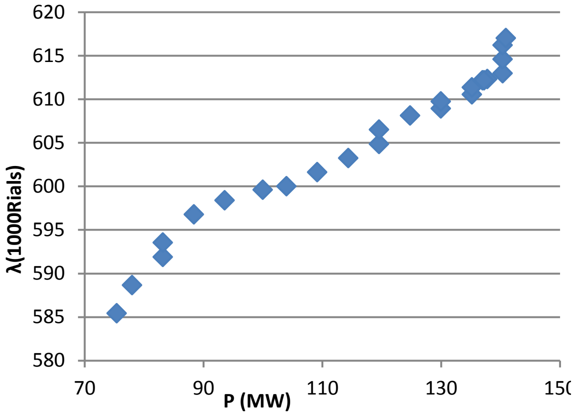
(b) The model based on the correlated polyhedral uncertainty set proposed by Jalilvandnejad et al. (2016)



(c) The model based un the improved correlated polyhedral uncertainty set proposed by Daneshvari and Shafaei (2021), $\beta = 1$.



(d) The model based un the improved correlated polyhedral uncertainty set proposed by Daneshvari and Shafaei (2021), $\beta = 0.5$.



(e) The model based un the improved correlated polyhedral uncertainty set proposed by Daneshvari and Shafaei (2021), $\beta = 0$.

Figure 14

Biographies

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