Group multiple attribute decision making using a modified TOPSIS method in the presence of interval data

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Abstract

TOPSIS is a well-known technique in multiple criteria decision making and has found several applications in recent years. However, as mentioned in literature TOPSIS has several shortcomings. In this paper, we present an extension of TOPSIS method to determine the weight of decision makers (DMs) in group multiple attribute decision making problems with interval information. Our method is based on the concept that the best alternative is closer to the positive ideal solution and far away from the negative ideal solution, simultaneously. The contribution of the proposed method is that while it overcomes the shortcomings of the TOPSIS method it can be used to weight the decision making team and ranking the alternatives, as well. The method is illustrated through three examples.

Keywords: Group multiple attribute decision making; weight of decision makers; TOPSIS; ranking; interval data

1 Introduction

Multiple attribute decision making (MADM) problems are comparing multiple alternatives based on multiple attributes, which are often inconsistent, ranking alternatives and selecting the best one. The MADM models have been proposed in many numerous fields such industry [1], engineering [2], risk assessment society [3], management [4], automobile industry [5] and etc. Moreover, in recent years the attention of many authors is located on it and solved these problems with different methods [6, 7, 8, 9, 10, 11, 12]. An important and easy to use method for solving these problems is the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), that is a famous technique for solving MADM problem which was first introduced by Hwang and Yoon in 1981 [13]. This method is based on this concept that the best alternative is closer to Positive Ideal Solution (PIS) and farther from Negative Ideal Solution (NIS), simultaneously. The PIS and NIS are two virtual alternatives that show the best and worst performances of alternatives based on attributes, respectively. The ranking of alternatives computed on the basis of closeness

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coefficients of them. The closeness coefficients calculated by dividing the distance of each alternative from the NIS to sum of the distances of each alternative from the PIS and NIS. Some researchers extended the TOPSIS method for ranking the alternatives in different situations [14]. Jahanshahloo et al. [15] extended TOPSIS method for ranking the alternatives with interval numbers. They defined the PIS and NIS for each alternative separately by real values. The closeness coefficient of each alternative determined with interval form. Finally with two approaches the intervals of alternatives compared and ranked them. In 2013 Dymova et al. [16] proposed a new approach to interval extension of TOPSIS method. They claimed their method has not any heuristic assumptions like as suggested interval extensions of TOPSIS method which are based on different heuristic approaches to definition of PIS and NIS, which are not attainable in a decision matrix.

Saffarzadeh et al. [17] proposed a method such that being away from NIS and being close to PIS have the same effect in alternatives ranking. In their proposed method, the PIS and NIS are determined as interval numbers and distance of each alternative from PIS and NIS is calculated by extension of Euclidean distance. Then, a compromise index is defined to rank the alternatives.

Sadabadi et al. [18] presented an approach based on linear programming to solve MADM problems. In their methodology two scores are computed for each alternative and then by integrating these two score the final score of alternative is calculated. Fuzzy data in MADM problems studied by some researchers such as [19, 20, 21, 22].

Because of complexity of real-life, the decision making take place in group. The group of Decision Makers (DMs) proposed their opinion about alternatives based on attributes. In recent decades, some researchers suggested the methods based on TOPSIS method for solving Group Multiple Attribute Decision Making (GMADM) problems. For example, Shih et al. [23] studied the effects of normalization and aggregation approaches in GMADM problems. They applied two normalization approach (linear and vector normalization) and two mean (arithmetic mean and geometric mean) for aggregation. In their examples the best and worst alternatives do not changed but other alternative's ranking changed. Anisseh et al. [24] proposed a fuzzy extension of TOPSIS method for GMADM problems under fuzzy environment. They converted the DM's fuzzy decision matrix into an aggregated decision matrix. Then the closeness coefficients computed based on TOPSIS method.

In group decision making environment, DMs have different skills, knowledge, and experiences. In numerous GMADM problems, the difference of knowledge and experiences of DMs (importance or weight of DMs) is not considered in decision making process and all DMs have the same importance and weights. Obviously, this is unreasonable in real environment and causes error and uncertainly in final solution. In recent years, some methods based on TOPSIS method have suggested to determining the weight of DMs. For example, Ataei et al. [25] presented the ordinal priority approach method for calculating the DMs's weight. They first determined the DMs and their priorities. After prioritization of the DMs, attributes are prioritized by each DM. Then, each DM ranked the alternatives based on each attribute. By solving the presented linear programming model of this method, the weights of the attributes, alternatives and DMs obtained simultaneously.

Yue [26] determined the weight of DMs based on TOPSIS method. First he considered

the mean of all decisions as PIS. Then assumed the NIS in two parts. The left and right NIS were minimum and maximum of all decisions, alternatively. Finally by using the closeness coefficient of TOPSIS method, the weight of DMs calculated. Also with these calculated weights, the decisions aggregated and derived a group decision. The values of each alternative in his row, added and obtained the score of that alternative. The ranking of alternatives are performed with these scores. Besides, Yue [27] extended this method for GMADM problems with interval numbers. First normalized the decision matrix with interval numbers in two steps. Then by using the weight of attributes, computed the weighted normalized decision matrix. The PIS defined as the mean of all weighted normalized decision matrix. The minimum of the left values of intervals and maximum of the right values of intervals considered as left and right NIS, respectively. Finally closeness coefficient of each DM computed based on TOPSIS method. The normalized closeness coefficients defined as weight of DMs. The group decision matrix computed as aggregation of weighted normalized decision matrix with computed weights. Each row added and the degree of possibility of intervals calculated. The sum of the degree of possibility of each row is the score of corresponding alternative. In 2012, Yue [28] used the mentioned method but changed the definitions of PIS and NIS to intersection and union of intervals of all DMs. Yue [29] computed the weight of DMs in interval forms. He defined PIS as mean ,left NIS as minimum and right NIS as maximum of all matrix of DMs. Then the left (right) closeness coefficient calculated as minimum (maximum) of closeness coefficients calculated with distances of each DM from PIS and left NIS (PIS and right NIS). The interval weight of DMs computed with normalized intervals with left and right closeness coefficients. Liu et al. [30] computed the weight of attributes with mean and standard deviations. The weight of DMs calculated with TOPSIS method like as Yue [26]. In 2018, Yang et al. [31] for determining the weight of DMs, computed the weighted normalized decision matrix as the Yue [27] method. Then putted the left and right values of intervals in two matrix and called lower and upper decision matrix. For each of these matrix, calculated the group decision matrix. Then performed rough group decision matrix. They computed the lower and upper PIS and NIS based on best and worst performances, respectively. The mean of lower and upper PIS and NIS considered as overall PIS and NIS. The closeness coefficients calculated as TOPSIS method. These closeness coefficients supposed as weight of DMs.
In spite of all advantageous and applications of TOPSIS method, this method has some disadvantageous. One of these disadvantageous is related to normalization. When the normalization method changes, the ranking also changes. Another flaw of TOPSIS method is the way of aggregate the distances of each alternative from PIS and NIS. There are several methods for aggregation, such as, the classic method of TOPSIS, sum of these distances and subtract of two distances. Some researchers introduced two weights as the relative importance, one for benefit attributes and other for cost attributes. Kuo [32] represented the closeness coefficients of TOPSIS method is irrespective of the weights of distances of an alternative from the PIS and NIS. In other words, not important what weights the DM assigns to these two distances, the ranking results would not vary as if DM has no preference for these two distances. For solving this flaw, Kuo reduced the original problem to a new problem with two attributes only, the distances of an alternative from the PIS and NIS as a cost attribute and a benefit attribute, respectively. The new closeness

coefficient suggested with considering two weights corresponded to two new attributes. In his method, the weights changed with respect to DM's opinion and not unique. Diwivedi et al. [33] suggested the weights putted in exponent of distances of an alternative from the PIS and NIS. Opricovic and Tzeng [34] proposed the TOPSIS method and this flaw. They pointed to Lai et al.[35] paper and stated that this issue remained as open question. As mentioned in Kuo's method, the distances of an alternative from the PIS and NIS are two types. first is a cost attribute and second is a benefit attribute. So summing two attributes from two types is unreasonable. Another flaw of TOPSIS method that relate to aggregation method is, it might a DM that is closer to PIS than other DMs, rank worse than others and it is inconsistency of the previous definition of TOPSIS method that the best alternative is closer to PIS and farther from NIS, simultaneously. In this paper, we propose a method to overcome these flaws of TOPSIS method without needing to consider the weights. The structure of this study is as follows:
In section 2, we review the TOPSIS method and extension of this method for interval numbers express in section 3. Section 4 proposes our method. It is illustrated through using some examples in section 5. Section 6 concludes the paper.

2 The TOPSIS method

In this section we review the TOPSIS method for multiple attribute decision making (MADM) problems. TOPSIS is a well-known method for solving the MADM problems that was proposed by Hwang and Yoon at first. The TOPSIS method chooses the best alternative that is closer to PIS and far away from NIS, simultaneously, where the PIS is the best virtual decision and the NIS has the maximum distance from the PIS.
Suppose $A = \{A_1, ..., A_n\}$ be the set of n alternatives and $U = \{u_1, ..., u_m\}$ be the set of m attributes. We have two types of attributes, benefit attributes and cost attributes. We denote the benefit attributes set by U_1 and cost attributes set by U_2 where $U_1 \bigcap U_2 = \phi$ and $U = U_1 \bigcup U_2$. The value of ith alternative based on jth attribute that defined by DM is shown by x_{ij}. The steps of TOPSIS are as follows:
Step 1. Normalize the values x_{ij} to the corresponding normalized values r_{ij} with the following formulation

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}, \quad i = 1, ..., n, \quad j = 1, ..., m \tag{1}$$

Step 2. Calculate the weighted normalized values by the product of each normalized value r_{ij} in its weight w_j

$$v_{ij} = w_j \; r_{ij}, \quad i = 1, ..., n, \quad j = 1, ..., m \tag{2}$$

Step 3. The PIS which is the best value for each attributes compute as follows

$$v_j^+ = \begin{cases} \max\limits_{1 \le i \le n} \{v_{ij}\} & u_j \in U_1 \\ \min\limits_{1 \le i \le n} \{v_{ij}\} & u_j \in U_2 \end{cases} \tag{3}$$

And the NIS which has the most distance from PIS characterize as follows

$$v_j^- = \begin{cases} \min\limits_{1\leq i\leq n} \{v_{ij}\} & u_j \in U_1 \\ \max\limits_{1\leq i\leq n} \{v_{ij}\} & u_j \in U_2 \end{cases} \tag{4}$$

Step 4. Calculate the distance of each alternative from the PIS and NIS

$$\begin{aligned} S_i^+ &= \sqrt{\sum_{j=1}^{m} \left(v_j^+ - v_{ij}\right)^2}, \quad i = 1, ..., n \\ S_i^- &= \sqrt{\sum_{j=1}^{m} \left(v_{ij} - v_j^-\right)^2}, \quad i = 1, ..., n \end{aligned} \tag{5}$$

Step 5. The closeness coefficient of ith alternative compute as

$$RC_i = \frac{S_i^-}{S_i^+ + S_i^-}, \quad i = 1, ..., n \tag{6}$$

Step 6. Rank all the alternatives according to the decreasing order of RC_is. RC_is show closest to PIS and farthest from to NIS, simultaneously.

3 Extended TOPSIS

In this section we review the extended TOPSIS method for GMADM problems with interval numbers that proposed by Yue [26].

Definition 1. A nonnegative interval number a is a set of the form $\{x \mid 0 < a^l \leq x \leq a^u\}$, which denoted by $a = [a^l, a^u]$ ([36]).

With the notations of the previous section and additional assumptions that there is t DMs, $\{DM_1, DM_2, ..., DM_t\}$, where each DM obtained his/her preferences of each alternative based on attributes with a matrix. the X_k $(k = 1, 2, ..., t)$ is decision matrix of the DM_k as follows:

$$X_k = ([x_{ij}^{k(l)}, x_{ij}^{k(u)}])_{n\times m} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & ... & u_m \end{array} \\ \begin{pmatrix} [x_{11}^{k(l)}, x_{11}^{k(u)}] & [x_{12}^{k(l)}, x_{12}^{k(u)}] & ... & [x_{1m}^{k(l)}, x_{1m}^{k(u)}] \\ [x_{21}^{k(l)}, x_{21}^{k(u)}] & [x_{22}^{k(l)}, x_{22}^{k(u)}] & ... & [x_{2m}^{k(l)}, x_{2m}^{k(u)}] \\ \vdots & \vdots & ... & \vdots \\ [x_{n1}^{k(l)}, x_{n1}^{k(u)}] & [x_{n2}^{k(l)}, x_{n2}^{k(u)}] & & [x_{nm}^{k(l)}, x_{nm}^{k(u)}] \end{pmatrix} \end{array} \tag{7}$$

by the following steps the DM's weight construct:

Step1. Compute the normalized decision matrix R_k $(k = 1, 2, ..., t)$ with two steps as follows

$$\begin{cases} y_{ij}^{k(l)} = \dfrac{x_{ij}^{k(l)}}{\sum\limits_{i=1}^{n} x_{ij}^{k(u)}}, & y_{ij}^{k(u)} = \dfrac{x_{ij}^{k(u)}}{\sum\limits_{i=1}^{n} x_{ij}^{k(l)}}, \quad u_j \in U_1 \\ y_{ij}^{k(l)} = \dfrac{1/x_{ij}^{k(u)}}{\sum\limits_{i=1}^{n} 1/x_{ij}^{k(l)}}, & y_{ij}^{k(u)} = \dfrac{1/x_{ij}^{k(l)}}{\sum\limits_{i=1}^{n} 1/x_{ij}^{k(u)}}, \quad u_j \in U_2 \end{cases} \tag{8}$$

And

$$r_{ij}^{k(l)} = \frac{y_{ij}^{k(l)}}{\sqrt{\sum\limits_{i=1}^{n}\left(\left(y_{ij}^{k(l)}\right)^2 + \left(y_{ij}^{k(u)}\right)^2\right)}}, \quad r_{ij}^{k(u)} = \frac{y_{ij}^{k(u)}}{\sqrt{\sum\limits_{i=1}^{n}\left(\left(y_{ij}^{k(l)}\right)^2 + \left(y_{ij}^{k(u)}\right)^2\right)}} \tag{9}$$

Step2. Compute the weighted normalized decision matrix $V_k = ([v_{ij}^{k(l)}, v_{ij}^{k(u)}])_{n\times m}$.
Step3. Define the PIS $A^+ = ([v_{ij}^{+(l)}, v_{ij}^{+(u)}])_{n\times m}$ as

$$v_{ij}^{+(l)} = \frac{1}{t}\sum_{k=1}^{t} v_{ij}^{k(l)}, v_{ij}^{+(u)} = \frac{1}{t}\sum_{k=1}^{t} v_{ij}^{k(u)} \tag{10}$$

Step4. Define the NIS $A^- = ([v_{ij}^{-(l)}, v_{ij}^{-(u)}])_{n\times m}$ as

$$v_{ij}^{-(l)} = \min_{1\le k\le t}\left\{v_{ij}^{k(l)}\right\}, v_{ij}^{-(u)} = \max_{1\le k\le t}\left\{v_{ij}^{k(u)}\right\} \tag{11}$$

Step5. Calculate the distances of DM_k from PIS

$$S_k^+ = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\left(v_{ij}^{k(l)} - v_{ij}^{+(l)}\right)^2 + \left(v_{ij}^{k(u)} - v_{ij}^{+(u)}\right)^2\right)} \tag{12}$$

Step6. Calculate the distances of DM_k from NIS

$$S_k^- = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\left(v_{ij}^{k(l)} - v_{ij}^{-(l)}\right)^2 + \left(v_{ij}^{k(u)} - v_{ij}^{-(u)}\right)^2\right)} \tag{13}$$

Step7. Determine the closeness coefficient of DM_k

$$RC_k = \frac{S_k^-}{S_k^- + S_k^+} \tag{14}$$

Step8. Determine the weight of DM_k as

$$\lambda_k = \frac{RC_k}{\sum\limits_{k=1}^{t} RC_k} \tag{15}$$

4 The proposed method

In this section, first we explain a drawback of extended TOPSIS method (hereafter called ET method), and then proposed our method for solving this flaw.
Suppose that DM_j has the shortest distances from A^+ and A^-, simultaneously and DM_k has the farthest distances from A^+ and A^-, simultaneously. It is clear that both have one positive score (DM_j has the shortest distances from A^+ and DM_k has the farthest distances from A^-) and one negative score (DM_j has the shortest distances from A^- and DM_k has the farthest distances from A^+). So they must have the equal closeness coefficients and ranked similar. But by using ET method, until ended the computations, we do not have a certain ranking and may have different closeness coefficients. Also it may that a DM that is closer to PIS than other DMs, rank worse than others. For clarifies this discussion, suppose that the DM_j rank better than DM_k, then $RC_j > RC_k$ and therefore:

$$RC_j > RC_k \Rightarrow \frac{S_j^-}{S_j^+ + S_j^-} > \frac{S_k^-}{S_k^+ + S_k^-} \Rightarrow S_j^+ < \frac{S_j^- S_k^+}{S_k^-}$$

Let, DM_k has this property that $S_k^+ = S_k^-$. Then all alternatives DM_j with $S_j^+ > S_k^+$ and $S_j^+ < S_j^-$ have the better rank than DM_k, since

$$S_j^+ < S_j^- \Rightarrow S_j^+ + S_j^- < 2S_j^- \Rightarrow \frac{S_j^-}{S_j^+ + S_j^-} > \frac{1}{2} \Rightarrow RC_j > \frac{1}{2}$$

On the other hand,

$$RC_k = \frac{S_k^-}{S_k^+ + S_k^-} = \frac{1}{2}$$

Then $RC_j > RC_k$. But since $S_j^+ > S_k^+$, DM_k has less distance to PIS than DM_j.
For solving this flaw, the following method is proposed.
We compute the values of $\left\{S_1^+, S_2^+, ..., S_t^+\right\}$ by steps (1-6) of the previous section. Then consider them as cost attributes, since small values of S_k^+ are better. Now set

$$S^{+*} = \max_{1\leq k\leq t}\left\{S_k^+\right\}, \quad S_-^+ = \min_{1\leq k\leq t}\left\{S_k^+\right\}$$

And define

$$\tilde{S_k^+} = \frac{S^{+*} - S_k^+}{S^{+*} - S_-^+} \tag{16}$$

It is clear that $\tilde{S_k^+} \in [0, 1]$.
Similarity, the values of $\left\{S_1^-, S_2^-, ..., S_t^-\right\}$ are computed using Steps $(1-6)$ of the previous section and they are considered as benefit attributes (Since big values are better). Now set

$$S^{-*} = \max_{1\leq k\leq t}\left\{S_k^-\right\}, \quad S_-^- = \min_{1\leq k\leq t}\left\{S_k^-\right\}$$

And define

$$\tilde{S_k^-} = \frac{S_k^- - S_-^-}{S^{-*} - S_-^-} \tag{17}$$

It is clear that $\tilde{S_k^-} \in [0,1]$.
Now set

$$\xi_k = \tilde{S_k^+} + \tilde{S_k^-} \tag{18}$$

If $\xi_k = 0$, then $\tilde{S_k^+} = 0$, $\tilde{S_k^-} = 0$. So DM_k has the shortest distance from NIS and the farthest distance from PIS. Therefore this DM is the worst one . Also if $\xi_k = 2$, then $\tilde{S_k^+} = 1$, $\tilde{S_k^-} = 1$. So DM_k has the shortest distance from PIS and the farthest distance from NIS and consequently this DM is the best.

Lemma 1. $0 \le \xi_k \le 2$.

Proof : The proof is clear and hence omitted. ■

Lemma 2. *Suppose that DM_j has the shortest distances from A^+ and A^-, simultaneously and DM_k has the farthest distances from A^+ and A^-, simultaneously. Then, DM_j and DM_k has the same rank.*

Proof : Suppose that DM_j has the shortest distances from A^+ and A^-, simultaneously and DM_k has the farthest distances from A^+ and A^-, simultaneously. Then

$$S^{+*} = S_k^+, \quad S_-^+ = S_j^+, \quad S^{-*} = S_k^-, \quad S_-^- = S_j^-$$

Therefore

$$\begin{cases} \tilde{S_j^+} = \frac{S^{+*}-S_j^+}{S^{+*}-S_-^+} = \frac{S_k^+-S_j^+}{S_k^+-S_j^+} = 1 \\ \\ \tilde{S_j^-} = \frac{S_j^- - S_-^-}{S^{-*}-S_-^-} = \frac{S_j^- - S_j^-}{S_k^- - S_j^-} = 0 \end{cases} \Rightarrow \quad \xi_j = 1 + 0 = 1$$

And

$$\begin{cases} \tilde{S_k^+} = \frac{S^{+*}-S_k^+}{S^{+*}-S_-^+} = \frac{S_k^+-S_k^+}{S_k^+-S_j^+} = 0 \\ \\ \tilde{S_k^-} = \frac{S_k^- - S_-^-}{S^{-*}-S_-^-} = \frac{S_k^- - S_j^-}{S_k^- - S_j^-} = 1 \end{cases} \Rightarrow \quad \xi_k = 0 + 1 = 1$$

Hence, DM_j and DM_k has the same rank. ■

Lemma 3. *If DM_j has the shorter distance from PIS and the farther distance from NIS than DM_k, then DM_j has the better rank than DM_k.*

Proof : Since DM_j has the shorter distance from PIS than DM_k, So

$$S_j^+ < S_k^+ \Rightarrow S^{+*} - S_j^+ > S^{+*} - S_k^+ \Rightarrow \frac{S^{+*} - S_j^+}{S^{+*} - S_-^+} > \frac{S^{+*} - S_k^+}{S^{+*} - S_-^+} \Rightarrow \tilde{S_j^+} > \tilde{S_k^+}$$

And DM_j has the farther distance from NIS than DM_k, So

$$S_j^- > S_k^- \Rightarrow S_j^- - S_-^- > S_k^- - S_-^- \Rightarrow \frac{S_j^- - S_-^-}{S^{-*} - S_-^-} > \frac{S_k^- - S_-^-}{S^{-*} - S_-^-} \Rightarrow \tilde{S_j^-} > \tilde{S_k^-}$$

Therefore
$\tilde{S_j^+} + \tilde{S_j^-} > \tilde{S_k^+} + \tilde{S_k^-} \Rightarrow \xi_j > \xi_k$ ■
In sum, the steps of the proposed method are as follows:
Step 1. Define the decision matrix X_k $(k = 1, 2, ..., t)$.
Step 2. Utilize Eqs.(8-9) to compute the normalized decision matrix R_k $(k = 1, 2, ..., t)$.
Step 3. Compute the weighted normalized decision matrix V_k $(k = 1, 2, ..., t)$ using Eq.(2).
Step 4. Calculate the PIS and NIS by Eq.(10) and Eq.(11), respectively.
Step 5. Utilize Eqs.(12-13) to determine the distances of DM_k from PIS and NIS, respectively.
Step 6. Compute the closeness coefficient of DM_k with Eq.(18).
Step 7. Calculate the weight of DM_k as $\lambda_k = \frac{\xi_k}{\sum_{k=1}^{t} \xi_k}$.

5 Illustrative examples

In this section we illustrate the proposed method using three examples.
Example 1. This example has been taken from Yue [26, 27].
"The Pearl River Delta Regional Air Quality Monitoring Network (the Network) was jointly established by the Guangdong Provincial Environmental Monitoring Center (GDEMC) and the Environmental Protection Department of the Hong Kong Special Administrative Region (HKEPD) from 2003 to 2005. It came into operation on November 30, 2005 and has been providing data for reporting of Regional Air Quality Index to the public since then. The Network comprises 16 automatic air-quality monitoring stations across the Pearl River Delta region. All stations are installed with equipment to measure the ambient concentrations of respirable suspended particulate (PM_{10} or RSP), sulphur dioxide (SO_2) and nitrogen dioxide (NO_2).
In what follows, we will present a comprehensive evaluation of the air quality in Guangzhou for the Novembers of 2006, 2007, and 2008 for the 16th Asian Olympic Games. The air-quality monitoring stations can be considered as DMs. For convenience, we select three air-quality monitoring stations located in Guangzhou from the 16 air-quality monitoring stations across the Pearl River Delta region, i.e., $D = \{DM_1, DM_2, DM_3\} = \{LuhuPark, Wanqingsha, Tianhu\}$. The measured values are shown in Tables $1 - 3$.
The monthly air quality for the Novembers of 2006, 2007 and 2008, respectively, can be considered as alternative. For convenience, let $A = \{A_1, A_2, A_3\}$ be the set of alternatives, $U = \{u_1, u_2, u_3\} = \{SO_2, NO_2, PM_{10}\}$ be the set of attributes."

Table 1. Air quality data derived from Luhu Park monitoring station X_1

Alternative	SO_2	NO_2	PM_{10}
A_1	[0.013, 0.129]	[0.028, 0.144]	[0.021, 0.136]
A_2	[0.013, 0.107]	[0.038, 0.139]	[0.047, 0.155]
A_3	[0.003, 0.042]	[0.018, 0.054]	[0.014, 0.150]

Table 2. Air quality data derived from Wanqingsha monitoring station X_2

Alternative	SO_2	NO_2	PM_{10}
A_1	[0.040, 0.161]	[0.034, 0.093]	[0.047, 0.199]
A_2	[0.047, 0.127]	[0.040, 0.081]	[0.102, 0.206]
A_3	[0.014, 0.113]	[0.016, 0.086]	[0.030, 0.187]

Table 3. Air quality data derived from Tianhu monitoring station X_3

Alternative	SO_2	NO_2	PM_{10}
A_1	[0.006, 0.118]	[0.004, 0.053]	[0.003, 0.174]
A_2	[0.015, 0.046]	[0.001, 0.026]	[0.021, 0.157]
A_3	[0.009, 0.034]	[0.005, 0.019]	[0.011, 0.103]

The normalized decision matrix by Step 2 calculated and shown in Tables (4-6).

Table 4. Normalized air quality data derived from Luhu Park monitoring station R_1

Alternative	SO_2	NO_2	PM_{10}
A_1	[0.0019,0.2194]	[0.0270,0.5007]	[0.0121,0.5383]
A_2	[0.0022,0.2194]	[0.0280,0.3689]	[0.0106,0.2405]
A_3	[0.0057,0.9506]	[0.0721,0.7788]	[0.0110,0.8075]

Table 5. Normalized air quality data derived from Wanqingsha monitoring station R_2

Alternative	SO_2	NO_2	PM_{10}
A_1	[0.0154,0.3178]	[0.0433,0.3991]	[0.0291,0.5215]
A_2	[0.0195,0.2705]	[0.0498,0.3392]	[0.0281,0.2403]
A_3	[0.0219,0.9081]	[0.0469,0.8480]	[0.0310,0.8171]

Table 6. Normalized air quality data derived from Tianhu monitoring station R_3

Alternative	SO_2	NO_2	PM_{10}
A_1	[0.0069,0.7891]	[0.0014,0.2381]	[0.0008,0.9557]
A_2	[0.0178,0.3156]	[0.0028,0.9524]	[0.0008,0.1365]
A_3	[0.0241,0.5261]	[0.0038,0.1905]	[0.0013,0.2607]

For the weight vector $w = (w_1, w_2, w_3) = (0.4, 0.2, 0.4)$ of attributes, the next step is to computing the weighted normalized decision matrix By Step 3, which are show in Tables 7-9.

Table 7. Weighted normalized air quality data derived from Luhu Park monitoring station V_1

Alternative	SO_2	NO_2	PM_{10}
A_1	[0.00074,0.08775]	[0.00541,0.10013]	[0.00485,0.21532]
A_2	[0.00090,0.08775]	[0.00560,0.07378]	[0.00426,0.09621]
A_3	[0.00228,0.38025]	[0.01442,0.15576]	[0.00440,0.32298]

Table 8. Weighted normalized air quality data derived from Wanqingsha monitoring station V_2

Alternative	SO_2	NO_2	PM_{10}
A_1	[0.00615,0.12714]	[0.00867,0.07981]	[0.01165,0.20862]
A_2	[0.00780,0.10820]	[0.00995,0.06784]	[0.01125,0.09613]
A_3	[0.00877,0.36325]	[0.00937,0.16960]	[0.01240,0.32684]

Table 9. Weighted normalized air quality data derived from Tianhu monitoring station V_3

Alternative	SO_2	NO_2	PM_{10}
A_1	[0.00278,0.31564]	[0.00027,0.04762]	[0.00030,0.38229]
A_2	[0.00713,0.12626]	[0.00056,0.19047]	[0.00034,0.05461]
A_3	[0.00964,0.21043]	[0.00076,0.03809]	[0.00052,0.10426]

By Step 4, the PIS and NIS are shown as Table 10 and 11, respectively.

Table 10. Positive ideal solution

Alternative	SO_2	NO_2	PM_{10}
A_1	[0.00323,0.17684]	[0.00478,0.07585]	[0.00560,0.26874]
A_2	[0.00527,0.10740]	[0.00537,0.11070]	[0.00528,0.08232]
A_3	[0.00690,0.31798]	[0.00818,0.12115]	[0.00577,0.25136]

Table 11. Negative ideal solution

Alternative	SO_2	NO_2	PM_{10}
A_1	[0.00074,0.31564]	[0.00027,0.10013]	[0.00030,0.38229]
A_2	[0.00090,0.12626]	[0.00056,0.19047]	[0.00034,0.09621]
A_3	[0.00228,0.38025]	[0.00076,0.16960]	[0.00052,0.32684]

The distances from PIS and NIS, S_k^+ and S_k^- , are calculated by Step 5, which are shown in Table 12.

Table 12. Distances of each air-quality monitoring station from PIS and NIS.

Distances	DM_1	DM_2	DM_3
S_k^+	0.1537	0.1356	0.2841
S_k^-	0.3089	0.2872	0.3166

The closeness coefficients and weights of air-quality monitoring stations are calculated by Steps 6 and 7 of ET method, respectively. These closeness coefficients, weights and their ranking are summarized in Table 13.

Table 13. Closeness coefficients, weights and ranking of air-quality monitoring station with ET method.

Monitoring stations	RC_k	λ_k	Ranking
DM_1	0.6678	0.3563	2
DM_2	0.6793	0.3625	1
DM_3	0.5270	0.2812	3

As we see in Table 13, based on ET method DM_2 has the best rank and DM_3 has the worst rank. But Table 12 shows that DM_2 and DM_3 have one positive score (DM_2 is closest to PIS and DM_3 is farthest from NIS) and one negative score (DM_2 is farthest from PIS and DM_3 is nearest to NIS). So they must have the same rank and DM_1 should be ranked as best. Therefore the ranking order of ET method is not reasonable. Now consider the proposed method, the results are shown in Table 14. As we see, $\xi_2 = \xi_3$ and $\lambda_2 = \lambda_3$ and DM_1 is selected as s the best. Hence, the proposed method provides the reasonable result.

Table 14. Closeness coefficients, weights and ranking of air-quality monitoring station with proposed method.

Monitoring stations	ξ_k	λ_k	Ranking
DM_1	1.6146	0.4467	1
DM_2	1.0000	0.2767	2(3)
DM_3	1.0000	0.2767	3(2)

Example 2. The ET method has another drawback: If a DM is closer to PIS than other DMs, this DM might be ranked worse than others. We show this problem through a simple example:
Suppose that we have 4 DM such that their distances from PIS and NIS are as the second and third column of Table 15. The weights and ranking of DMs with ET and proposed methods are as shown in four last columns of Table 15.

Table 15. Distances, weights and ranking with ET and proposed method.

Decision Makers	S_k^+	S_k^-	λ_k of ET method	Rank	λ_k of proposed method	Rank
DM_1	0.5196	0.5196	0.2291	4	0.2763	2
DM_2	0.6082	0.7810	0.2576	2	0.2430	3
DM_3	0.6000	0.8485	0.2684	1	0.3231	1
DM_4	0.6164	0.7071	0.2448	3	0.1575	4

As we see, DM_1 has the equal distance from PIS and NIS. Also this DM is closer to PIS than other DMs, but its weights with ET method is less than others, and, consequently is ranked worse than others. But by using proposed method the DM_1 has the second rank. This is true, because according to second and third column of Table 15, the DM_3 has one positive score, since this DM has the farthest distance from NIS. The fourth DM has one negative score, because this DM is farthest from PIS. And the DM_1 has one positive score and one negative score, since DM_1 is nearest to PIS and is farthest from NIS. So DM_3 is ranked as first and DM_4 is located at last place. By the proposed method (the last two columns), DM_3 is ranked as first and DM_4 obtains the last rank. So the proposed method constructs the reasonable results.

Example 3. We consider an example where the core enterprise of the virtual enterprise has to select a partner for a sub-project and proposed in Ye and Li [37]. The partner selection decision is made on the basis of five main attributes including Cost, Time, Trust, Risk and Quality. Cost, Time and Risk are cost type, while Trust and Quality are benefit type. There are four partners have been identified as alternatives, and four decision makers are responsible for the partner selection problem. The decision matrix and the vector of corresponding weight of each attribute are given in Table 16.

Table 16. The decision matrix and the vector of corresponding weight of each attribute

DM	Attribute and weight	Cost	Time	Trust	Risk	Quality
DM_1	C_1	[10, 12]	[21, 25]	[80, 84]	[0.95, 0.98]	[0.95, 0.96]
	C_2	[11, 15]	[24, 25]	[84, 85]	[0.92, 0.93]	[0.96, 0.97]
	C_3	[12, 13]	[22, 24]	[87, 89]	[0.88, 0.91]	[0.96, 0.97]
	C_4	[14, 16]	[18, 20]	[91, 93]	[0.89, 0.90]	[0.99, 1.00]
	Weight	0.22	0.17	0.25	0.15	0.21
DM_2	C_1	[9, 13]	[24, 25]	[79, 82]	[0.93, 0.94]	[0.96, 0.98]
	C_2	[11, 12]	[21, 23]	[83, 84]	[0.92, 0.94]	[0.97, 0.98]
	C_3	[10, 12]	[22, 23]	[88, 89]	[0.89, 0.91]	[0.98, 0.99]
	C_4	[15, 16]	[19, 20]	[89, 92]	[0.90, 0.92]	[0.99, 1.00]
	Weight	0.19	0.18	0.22	0.16	0.25
DM_3	C_1	[11, 13]	[19, 22]	[74, 78]	[0.96, 0.97]	[0.93, 0.96]
	C_2	[12, 14]	[18, 25]	[76, 80]	[0.93, 0.96]	[0.94, 0.96]
	C_3	[12, 15]	[21, 22]	[82, 85]	[0.90, 0.92]	[0.95, 0.96]
	C_4	[13, 17]	[18, 23]	[86, 88]	[0.91, 0.94]	[0.97, 0.98]
	Weight	0.21	0.19	0.23	0.17	0.20
DM_4	C_1	[13, 14]	[22, 23]	[76, 78]	[0.95, 0.96]	[0.94, 0.95]
	C_2	[13, 15]	[19, 23]	[81, 82]	[0.94, 0.95]	[0.93, 0.94]
	C_3	[16, 18]	[20, 22]	[84, 86]	[0.89, 0.92]	[0.94, 0.95]
	C_4	[15, 17]	[19, 21]	[87, 88]	[0.88, 0.91]	[0.95, 0.96]
	Weight	0.24	0.18	0.21	0.18	0.19

The weight of DMs and ranking of them with ET and proposed method are shown in Table 17.

Table 17. The weight and ranking of DMs with ET and proposed method.

DMs	λ_k of ET method	Rank	λ_k of proposed method	Rank
DM_1	0.21	4	0.19	4
DM_2	0.28	1	0.30	1
DM_3	0.28	1	0.23	3
DM_4	0.23	3	0.28	2

As we see, with the ET method, DM_2 and DM_3 have equal weights and thus ranking the same. But using proposed method, No two DMs have the same weight. In addition, different ranking has been achieved, so that DM_3 is third and DM_4 has the second rank.

6 Conclusion

One of the most important subject in GMADM problem is determining the importance of DMs or the weight of each DM in decision making process. TOPSIS is a well-known method, that suggested for solving this problem. The basic idea of TOPSIS method is: The chosen decision is closest to PIS and farthest from NIS, simultaneously. But, TOPSIS method has some flaws. One of these flaws is related to aggregation method. TOPSIS aggregated two measures that are in two types, benefit and cost types. Based on this aggregation, TOPSIS may introduce a decision as the best decision; however this decision is only farthest from NIS and not closer to PIS. Also the closeness coefficients of this method is not reasonable enough. In this paper we proposed a method to determine the weight of DMs and overcome to the shortcomings of TOPSIS method. For future research one can do the sensitivity analysis such as done in [38] or extend the proposed approach in fuzzy decision making.

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