Competition between multi-level supply chains with hybrid distribution structures under two service systems

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Abstract
Distribution management is an important part of the business cycle for distributors and wholesalers. It is also important for businesses to have successful distribution management systems in order to remain competitive and keep customers satisfied. In this research, we consider the competition between two multi-echelon supply chains under two different service systems where price and service level are considered as performance measures. For every supply chain, there are two different indirect channels for the sale of the products that have not been taken into account previously. Thus, each supply chain benefits from two rival sale channels with different structures. The different structures of these channels induce magnificent changes in the demand function. Hence, we develop two alternate scenarios which are based on the centralized and decentralized servicing conditions. In the centralized conditions, the manufacturer is authorized to service all the members of the chains. In the decentralized conditions, the sale channels are responsible for decision-making about service level for every member of the chains. Furthermore, the effects of competition and sensitivity analysis on the equilibrium of the scenarios and supply chain profit are discussed in a numerical example.

Keywords: Common & exclusive retailer, Distribution channel, Pricing, Service level, Supply chain management.

1. Introduction
As firms become more specialized, they get increasingly aware of the growing interdependencies and opportunities that exist between organizations, suppliers, and customers. Organizations and

firms have become more conscious of the fact that they cannot compete with other firms in isolation from their suppliers and other units in the supply chain [1].

Distribution channels are of great significance in the delivery of the product to the customer in a supply chain. Supply chain distribution is the term used to refer to the methodology adopted for that purpose. The primary purpose of any channel of distribution is to bridge the gap between the manufacturer of a product and its user. There are basically four types of marketing channel: direct sale, sale through intermediaries, dual distribution, and reverse channels[1].

When a supply chain uses multiple channels of distribution to sell products, the channels will compete to sell more and gain greater profit. Therefore, a competition between supply chains is in fact one between their distribution channels.

The competition in the field of supply chains is broadly divided into two parts: 1) competition among the members of a supply chain and 2) competition among supply chains.

Many researchers have discussed the issue of competition among the members of a supply chain ([2]-[4]). The competition among supply chains is a distinct type of competition. McGuire and Staelin [5] conducted a study on price competition between two suppliers whose products are sold through independent retailers in a duopoly market with two competitive supply chains. The results demonstrated that in a deterministic model with substitutable products and competition on price, the decentralized structure is preferred by the chains over the centralized structure as the degree of substitution between the products rises.

Moorthy et al. [6] linked the concept of strategic interaction with the notion of decentralization value. They investigated why centralized chains are associated with greater profit for the manufacturer and the whole chain, and concluded that the decentralized structure is associated with higher profit for the channel as the degree of competition rises. Boyaci and Gallego [7] studied three competitive scenarios between two supply chains: 1) where both chains are centralized, 2) where both chains are decentralized, and 3) where one chain is centralized while the other is decentralized. They assumed that the two chains had selected similar prices for their products, and that they competed in terms of customer service. They realized that both chains had selected the integrated structure for their dominant strategies. Qian [8] studied the price competition between Parallel Distribution Channels (PDCs), where each channel has a manufacturer and a retailer. This article was, in fact, the first to consider the competition between two supply chains assuming that one chain was the leader. Under the assumption of a definite demand, Qian demonstrated that PDC, moving as the second individual, had the competitive advantage. In addition to the degree of competition, the sequence had a significant impact on the players' decisions. Xiao and Yang [9] investigated the price-and-service competition between two supply chains, each with a risk-neutral supplier and risk-averse retailer. The products of the suppliers were substitutable, and their demand function was nondeterministic, depending on retail price and service. They realized that as the investment efficiency of the retailer increased on a certain service, the optimal price and the competitor's service declined. Wu et al. [10] developed a competitive model with two supply chains through simultaneous decision-making on price and quantity in the competitive model from one to an indefinite number of time periods between the two chains.

They considered joint pricing and quantity decisions and competition under three possible supply chain strategies: Vertical Integration (VI), Manufacturer's Stackelberg (MS), and bargaining the wholesale price. Demand was nondeterministic in their model.

[1]https://courses.lumenlearning.com

Anderson and Bao [11] investigated the competition between two-level supply chains where exclusive retailers competed for their final customers, and the supply chains were either integrated or decentralized. They studied the effect of variation in competition level on the members' profit, and concluded that although industry profit is reduced by intensified horizontal competition, an acceptable level of competition can increase profit if the underlying market shares of the supply chains are not widely scattered. Provided that the market shares are equal as competition increases, the decentralized supply chain will be more profitable. Li et al. [12] investigated contract selection by the manufacturer to establish coordination in the competition between two supply chains. They assumed that the manufacturer had the dominant power, and needed to decide on the wholesale price or discount contract with the retailer. Two types of supply chain structure were taken into account: 1) supply chains with two common retailers and 2) supply chains with exclusive retailers.

Mahmoodi and Eshghi [13] studied the horizontal chain-to-chain competition based on price. The structure of the industry was taken into account in three modes: 1) where the two chains were integrated, 2) where one chain was centralized, and the other was decentralized, and 3) where both chains were decentralized. A single product was used in the model, aiming to determine the retailing and wholesale prices and order. Amin Naseri and Azari Khojasteh [14] investigated a competitive model between leader-follower supply chains, each with a risk-neutral manufacturer and a risk-averse retailer. They demonstrated that in both supply chains, an increase in the risk aversion of the retailers might lead to a decrease in the total profit of the supply chain. Baron et al. [15] extended the work by MacGuire and Staelin [5], and studied the Nash Equilibrium in an industry with two competitive supply chains. They demonstrated that when demand is deterministic, both the Stackelberg manufacturer and the vertical integration strategies represent particular modes of Nash bargaining on the wholesale prices.

The price competition between two supply chains was studied by Zheng et al. [16]. They assumed that one of the supply chains was normal, and the other was reverse. The supply chains might have the same or different structures. The authors further studied the impact of the degree of competition intensity between the two chains and the product return rate of the reverse supply chain on the profit and price equilibria. Hafezalkotob et al. [17] formulated a competitive model in multi-product green supply chains under government supervision to reduce the environmental pollution cost. They provided a novel approach to construct a model that maximized the government tariffs and the profits of the suppliers and manufacturers in all the green supply chains. The results demonstrated that the fiscal policy of the government greatly affected the reduction of environmental pollution costs. Hafezalkotob [18] established production competition models between sets of green and regular supply chains. He considered two schemas of government intervention: direct tariffs (DTs) and tradable permits (TPs), both with and without baselines. The research sought to evaluate how the green supply chains and non-green supply chains responded to the DT or TP schema.

Moradinasab et al. [19] investigated the modeling of a petroleum supply chain with sustainability and pricing issues for the first time, and developed a sustainable competitive petroleum supply chain (SCPSC) model to minimize pollution while maximizing the profits and job creation. The problem involved a two-level model. The first level considered the competition between the supply chains of the government and the private sector, and was modeled by the game theory approach including Nash and Stackelberg equilibria. At the second level, the optimal values of the decision variables in the design of the petroleum supply chain were obtained using Mixed Integer Linear Programming (MILP) under the above three objective functions.

Taleizadeh and Sadeghi [20] considered two competitive reverse supply chains that competed in collection and refurbishment of used products after their useful lives. One of the chains collected eligible obsolete products through traditional and electronic channels, while the competitor used only the traditional channel. It was demonstrated that the e-channel proposed more appropriate rewards to the customer because it was less costly than the traditional channel, so the former channel achieved a more substantial share of the market. Khanlarzade et al. [21] considered the competition between two multi-echelon supply chains on price and service under balanced and imbalanced market power between the chains with different structures. They demonstrated that the chain did not always have the second-mover advantage in the Stackelberg game. Moreover, when the chains made their decisions sequentially, the service and the price jointly played a strategic role in earning profits.

Wang and Liu [22] studied the effects of vertical selection of contracts in two parallel competitive shipping supply chains. They realized that Pareto optimization occurred in the chain which adopted the revenue-sharing contract, and the overall profit of that chain was also maximized.

Moreover, they identified a lose-lose situation; that is, a classic Prisoner's Dilemma occurs when both chains adopt the revenue-sharing contract.

Guan et al. [23] investigated the issue of sharing demand information in two competing supply chains, each consisting of one manufacturer, who provided the consumers with free after-sale service, and one retailer, who had private information about uncertain demand.

The demand was linearly affected by the retail prices and service levels of both supply chains, which captured both price and service competition. The authors examined the impacts of information sharing on price and service decisions, and explored the value of information sharing to each supply chain. The results demonstrated that information sharing enabled the manufacturers to adjust their wholesale prices and service levels responsively to demand.

Wang et al. [24] studied the competitive and sustainable supply chain network design problem, and considered the chain-to-chain competition between two supply chains. The retail price and carbon emission equilibrium at the competition stage were studied. It was found that (1) considering the carbon emission competition in the supply chain lowered both total optimized cost and carbon emission and (2) the carbon emission of the total supply chain network increased with a rise in the probability of capacity satisfaction.

Khanlarzade et al. [25] considered the competition between two multi-echelon supply chains with identical structures on price under two market power structures. They investigated the effects of different relationships between the market sizes of supply chains and the supply chain structures on price and profit along with an analysis of power in the market.

The authors found that the supply chain did not always have the second-mover advantage in the price Stackelberg game. Furthermore, both of the supply chains, with centralized structures, benefitted from the presence of the leader in the market at different combinations of market size.

El Ouardighi et al. [26] investigated the impact of both horizontal and vertical competition, on the one hand, and strategy types (commitment-based versus contingent-based equilibrium strategies), on the other, on the pollution accumulated by two supply chains over time. They considered a two-stage game model where two manufacturers and two retailers were involved in a wholesale price contract in order to supply the demand over a finite time horizon. They sought to identify the combination of market structure and strategy type that led the two supply chains to generate the lowest pollution intensity and the highest abatement intensity.

The works by Ha and Tong [27], Wu [28], Li and Li [29], and Chan et al. [30] can be pointed out in this regard.

Table 1 provides a summary of the reviewed papers in the field and the position of this study among them in terms of the factors of competition, number of levels for each supply chain, and proper sale channels in each chain.

Please Insert Table 1 about here.

1.1. Classification of Distribution Channels

For a better understanding of the problem, techniques considered in the literature for goods distribution (sale channels) are classified below.

- Techniques studied for cases where the competition is among the members of a supply chain

1) One-channel distribution ([2])
2) Two-channel distribution or dual channels (direct and indirect channels) ([31], [32])
3) Two-channel distribution or dual channels (identical indirect channels) ([33], [34]).

Clearly, two different indirect distribution channels have not been considered so far in the supply chain (in the sense that the sale channels are structurally different from each other).

- Techniques studied for cases where the competition is among supply chains

1) One-channel distribution (Each manufacturer uses only a specific retailer, as considered by [11])
2) Two-channel distribution (as considered by [12])

1.2. Contribution

Distribution is part of the supply chain that is aimed at delivering goods/services to the consumers.

Distribution management is critical to a company's financial success and corporate longevity. Its successful implementation requires effective management of the entire distribution process.

Due to the great significance of goods distribution in supply chains, this paper analyzes the price- and service-level competition between supply chains, postulating a hybrid distribution structure for each.

Therefore, the *contribution* of this study can be summarized as follows.

1) Goods distribution process

As stated in the previous subsection, five types of distribution structure have been investigated so far in the literature. On that basis, the distribution structure examined for the first time in this paper is described below.

- In each supply chain, the manufacturer uses two indirect distribution channels that are different in terms of structure. The difference between the two distribution channels is that one sells goods only of the same chain (exclusive retailer), while the other distribution channel is allowed to sell goods of the rival supply chain along with commodities in its own chain (common retailer).

  Therefore, each manufacturer has three distribution channels for selling its product.

- In the competition among the supply chains, each manufacturer simultaneously uses a retailer in common with the other chain and an exclusive retailer.
  Therefore, the competition between the two chains is in fact one between four sale channels (two common and two exclusive retailers).

2) Demand function

Because of the competition between sale channels that are structurally different (There is a competition between four different sale channels in the competition between the two supply chains), a new demand function is defined in this study.

3) Service system

In this study, two centralized and decentralized service systems are analyzed in each supply chain. In the centralized system, the manufacturer determines the level of service provided to the customer. In the decentralized system, however, service level is determined by exclusive and common retailers. These two types of system have not been evaluated and compared so far.

3. Problem Description

This research is conducted based on the activity of two supply chains in the home appliance industry (audiovisual equipment in particular) in Iran (They are not referred to directly here as requested). These two chains are the main rivals in the Iranian market because of their production of substitutable products where they compete on price and service level. Therefore, customers of these types of product are highly sensitive to both price and service level. These chains use decentralized structures to sell their goods. The manufacturers sell the products to the distributor, who then sells them to an exclusive retailer (brand shop); next, the brand shop sells them to a common retailer or multi-shop, where the customer can buy the product of either chain from either of the retailers. The two brands considered in the present study have the same level of background and reputation in the market. Thus, the supply chains make their decisions simultaneously (These chains refer to the upper-level and lower-level sellers as brand shops and multi-shops, respectively. Therefore, the titles *exclusive retailer*, *upper-level seller*, and *brand shop* are regarded as the same in this paper, as are *common retailer*, *lower-level seller*, and *multi-shop*).

Given the properties of the above-mentioned supply chains, we study the effect of the service provided by different levels of each supply chain on the profit of the supply chains.

In this study, each supply chain comprises four levels as follows. There is a manufacturer at the first level. The second level includes a distributor, who buys the goods from the manufacturer, and sells them to the third level, *i.e.* the upper-level seller (exclusive retailer). The upper-level seller performs two types of activity. The first activity comprises sale of the product to a customer in the market, and the second activity includes sale of the product to the fourth level or the lower-level seller (common retailer). In each supply chain, the lower-level seller is permitted to sell the goods of the rival chain as well as those of its own chain. The upper-level seller, however, is monopolized for each chain, and is not allowed to do so. Hence, the structures of the two sale channels are different in each chain. The competition between the two supply chains comprises the competitions between their downstream levels [11]. Besides the competition between the sale channels of the two supply chains, therefore, there is a competition between the upper- and lower-level sellers in each supply chain. Therefore, there is finally a competition between four shops, including two upper-level sellers and two lower-level sellers. The commodities of the two supply chains are substitutable, and the competition between the chains takes place at the price and service levels. Due to the focus on price- and service-level competition, the demand of each retailer is considered as a function of price and service level. As in the reports by Wang et al. [4], Zhao et al. [35], and Tsay and Agrawal [2], service encompasses all efforts made to increase demand. These efforts include after-sale service, service provided before the sale, advertising, in-store promotion, product placement, and the overall quality of the shopping experience. A schema of the structure of the supply chains is given in Fig. 1.

Please Insert Figure 1 about here.

In each supply chain, it is the manufacturer because of its power that first makes a decision. Therefore, we model the follower-leader relationship as a Stackelberg game including one leader and multiple followers. Alternatively, it is assumed that the supply chains make their decisions in the market simultaneously. Thus, a Nash game is played among them. The presented model is analyzed in two general scenarios.

Centralized servicing (CS) scenario. In this scenario, the manufacturer determines the service levels for all sale channels to which its product is sold.

Decentralized servicing (DS) scenario. The service level for each product is determined here by the shop in which the given product is sold to the customer.

The purpose of this study is to analyze the effects of the above two scenarios on profit and the decision variables upon a competition among the four sale channels. We intend to examine how the method of service provision influences the profitability of the members at each level and in the rival chain. Finally, we seek to specify which of the above scenarios would be to the manufacturer's and other members' benefit if selected.

The rest of this paper is organized as follows. Section 4 is dedicated to the demand function and its parameters. In Section 5, the mathematical model is proposed, and the equilibrium solutions are extracted from Game Theory (GT). Section 6 provides a numerical analysis of the obtained results of the model under two servicing scenarios. Section 7 analyzes the sensitivity of the decision variables and the profits of the supply chains in various scenarios and various modes, compared to that of the model parameters. Finally, in Section 8, we summarize the findings, and provide suggestions for future research directions.

4. Demand Function

The manufacturer uses two indirect sale channels in each supply chain to supply the product to the customer. The first sale channel (upper-level seller) is allowed only to sell the product of the same supply chain, while the second sale channel (lower-level seller) can sell the products from both supply chains. Given the above assumption, there are two types of demand function for these sale centers. The symbols that are used are given in Table 2.

Please Insert Table 2 about here.

It is to be noted that $0<\theta_p,\theta_s,\gamma_p,\gamma_s,\rho,\beta<1$, $\eta_{(i)}>0$. Based on Table 2, the demand functions for the multi-shop and the brand shop are as follows:

$$D_{w(i)}^{(j)}=1+\left\{-p_{w(i)}^{(j)}+\left(p_{R(i)}^{(j)}+p_{R(3-i)}^{(j)}\right)\left(\rho\gamma_p-\beta\gamma_p\left(1+\theta_p\right)\right)+\left(p_{R(i)}^{(3-j)}+p_{R(3-i)}^{(3-j)}\right)\left(\rho\gamma_p\theta_p-\beta\gamma_p\theta_p\right)+p_{w(3-i)}^{(3-j)}\left(\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\right\}$$
$$+\left\{s_{w(i)}^{(j)}-\left(s_{R(i)}^{(j)}+s_{R(3-i)}^{(j)}\right)\left(\rho\gamma_s-\beta\gamma_s\left(1+\theta_s\right)\right)-\left(s_{R(i)}^{(3-j)}+s_{R(3-i)}^{(3-j)}\right)\left(\rho\gamma_s\theta_s-\beta\gamma_s\theta_s\right)-s_{w(3-i)}^{(3-j)}\left(\rho\gamma_s\theta_s-2\beta\gamma_s\theta_s\right)\right\}$$

(1)

where $i=j=1,2$;

$$D_{R(i)}^{j}=\begin{cases} if\ i=j;1+\{-p_{R(i)}^{(j)}+p_{R(3-i)}^{(j)}\left(\beta\gamma_p\left(1-\theta_p\right)-\rho\gamma_p\right)+p_{R(i)}^{(3-j)}\left(\beta\theta_p\left(1-\gamma_p\right)-\rho\gamma_p\theta_p\right)+p_{R(3-i)}^{(3-j)}\left(\beta\theta_p\gamma_p-\rho\gamma_p\theta_p\right) \\ \qquad +p_{w(i)}^{(j)}\beta\left(\gamma_p\left(1-\theta_p\right)-\gamma_p\left(1+\theta_p\right)\right)+p_{w(3-i)}^{(3-j)}\left(-\rho\theta_p\gamma_p\right)\} \\ \qquad +\{s_{R(i)}^{(j)}-s_{R(3-i)}^{(j)}\left(\beta\gamma_s\left(1-\theta_s\right)-\rho\gamma_s\right)-s_{R(i)}^{(3-j)}\left(\beta\theta_s\left(1-\gamma_s\right)-\rho\gamma_s\theta_s\right)-s_{R(3-i)}^{(3-j)}\left(\beta\theta_s\gamma_s-\rho\gamma_s\theta_s\right) \\ \qquad -s_{w(i)}^{(j)}\beta\left(\gamma_s\left(1-\theta_s\right)-\gamma_s\left(1+\theta_s\right)\right)-s_{w(3-i)}^{(3-j)}\left(-\rho\gamma_s\theta_s\right)\} \\ if\ i\neq j;1+\{-p_{R(i)}^{(j)}+p_{R(3-i)}^{(j)}\left(\beta\gamma_p\left(1-\theta_p\right)-\rho\gamma_p\right)+p_{R(i)}^{(3-j)}\left(\beta\theta_p\left(1-\gamma_p\right)-\rho\gamma_p\theta_p\right)+p_{R(3-i)}^{(3-j)}\left(\beta\theta_p\gamma_p-\rho\gamma_p\theta_p\right) \\ \qquad +p_{w(3-i)}^{(j)}\beta\left(\gamma_p\left(1-\theta_p\right)-\gamma_p\left(1+\theta_p\right)\right)+p_{w(i)}^{(3-j)}\left(-\rho\theta_p\gamma_p\right)\} \\ \qquad +\{s_{R(i)}^{(j)}-s_{R(3-i)}^{(j)}\left(\left(1-\theta_s\right)-\rho\gamma_s\right)-s_{R(i)}^{(3-j)}\left(\beta\theta_s\left(1-\gamma_s\right)-\rho\gamma_s\theta_s\right)-s_{R(3-i)}^{(3-j)}\left(\beta\theta_s\gamma_s-\rho\gamma_s\theta_s\right) \\ \qquad -s_{w(3-i)}^{(j)}\beta\left(\gamma_s\left(1-\theta_s\right)-\gamma_s\left(1+\theta_s\right)\right)-s_{w(i)}^{(3-j)}\left(-\rho\gamma_s\theta_s\right)\} \end{cases}$$

(2)

where $i=1,2,\ j=1,2$.

A brand shop is an exclusive, expert shop, which is different from the common retailer (or multi-shop) in many respects such as the site allocated for display of goods, method of presentation of commodities and shelf-setting, goods inventory in the shop, and field of activity. A multi-shop is restricted in terms of the arrangement of goods, levels of presentation of various services, and inventory of commodities, where small volumes of any product can in fact be presented for the sake of goods diversity. A brand shop, however, can allocate the entire space of the shop to its products and supply more services to the customer. For example, there is a division in some shops called *training in the shop* (*e.g.* in Apple and Microsoft stores), which can guide the customer on very trivial, unimportant issues until he/she has no question about them, in addition to the type of necessary training provided to him/her. From the viewpoint of a multi-shop, however, it is not favorable to spend this time on a customer because a multi-shop has to sell several other goods as well at the same time. Although a multi-shop sells both products *i* and *j*, it is not considered as a proper market for the sale of these commodities, because the shop sells several products at the same time, as a result of which the tendency of costumers to purchase their needed goods from these markets is gradually reduced. Thus, it is implied by the features of these two types of shop that the brand shop is deemed a powerful rival for the multi-shop. Therefore, we assume in this study that $\rho>\beta$. Now, we consider the demand for the jth product at the i^th^ lower-level shop to describe the demand function statements. The following three events may occur as the price of the commodity rises at the rival lower-level shop.

1) The customers tend to purchase the i^{th} product from the j^{th} lower-level seller;

2) the customers are inclined to purchase rival goods from the i^{th} lower-level seller;

3) the customers tend to purchase the commodity from the upper-level seller in the i^{th} supply chain.

As already mentioned above, however, the customers are more likely to purchase the commodity from the upper-level seller since this seller is considered as a powerful rival for the lower-level seller. Thus, the third event may occur with a higher probability than the first and second. Hence, the expression $p_{R(3-i)}^{(j)}$ appears in the demand function ($D_{R(i)}^{(j)}$) with the following coefficient: $\beta\gamma_p - \beta\gamma_p\theta_p - \rho\gamma_p = \beta\gamma_p\left(1-\theta_p\right) - \rho\gamma_p$.

Let us consider Product 1 in the brand shop of Supply Chain 1. It is sold in the multi-shops of both Chains as well as in the above brand shop, where the variation in the product price affects the demand for Product 1. This product also has a rival (that manufactured in Chain 2), the variation in the price of which is effective on the demand for Product 1. Therefore, the demand for this product increases as the price of the same product in the other stores (the multi-shops of Chains 1 and 2) rises ($+\left(p_{R(1)}^{(1)} + p_{R(2)}^{(1)}\right)\left(\rho\gamma_p - \beta\gamma_p\left(1+\theta_p\right)\right)$). As the price of the product itself increases, the demand for it decreases ($-p_{w(1)}^{(1)}$). With an increase in the price of Product 2 in the multi-shops of Chains 1 and 2, demand increases ($+\left(p_{R(1)}^{(2)} + p_{R(2)}^{(2)}\right)\left(\rho\gamma_p\theta_p - \beta\gamma_p\theta_p\right)$). Finally, demand increases with a rise in the price of Product 2 in the brand shop of Chain 2 ($+p_{w(2)}^{(2)}\left(\rho\gamma_p\theta_p - 2\beta\gamma_p\theta_p\right)$). This is also the case for service level."

5. Solution Procedure

The model is analyzed in two separate scenarios (based on service provision).

Scenario 1) Centralized Servicing (CS)
Due to the service provided by the manufacturer in each supply chain, the objective functions for the common retailer, exclusive retailer, distributor, and manufacturer are as follows, respectively.

$$\Pi_{R(i)} = \left(p_{R(i)}^{(j)} - w_{w(i)}\right)D_{R(i)}^{(j)} + \left(p_{R(i)}^{(3-j)} - w_{w(3-i)}\right)D_{R(i)}^{(3-j)} \tag{3}$$

$$\Pi_{W(i)} = \left(p_{w(i)}^{(j)} - w_{d(i)}\right)D_{w(i)}^{(j)} + \left(w_{w(i)} - w_{d(i)}\right)\left(D_{R(i)}^{(j)} + D_{R(3-i)}^{(j)}\right) \tag{4}$$

$$\Pi_{D(i)} = \left(w_{d(i)} - w_{m(i)}\right)\left(D_{w(i)}^{(j)} + D_{R(i)}^{(j)} + D_{R(3-i)}^{(j)}\right) \tag{5}$$

$$\Pi_{M(i)} = w_{m(i)}\left(D_{w(i)}^{(j)} + D_{R(i)}^{(j)} + D_{R(3-i)}^{(j)}\right) - \frac{1}{2}\eta_{(i)}\left(\left(s_{w(i)}^{(j)}\right)^2 + \left(s_{R(i)}^{(j)}\right)^2 + \left(s_{R(3-i)}^{(j)}\right)^2\right) \tag{6}$$

(for $i = j = 1,2$)

Scenario 2) Decentralized Servicing (DS)

Unlike in the former scenario, service cost is calculated here in the objective functions for the retailers, as follows, because the service is provided by the sale channels in this scenario.

$$\Pi_{R(i)}=\left(p_{R(i)}^{(j)}-w_{w(i)}\right)D_{R(i)}^{(j)}+\left(p_{R(i)}^{(3-j)}-w_{w(3-i)}\right)D_{R(i)}^{(3-j)}-\frac{1}{2}\eta_{(i)}\left(s_{R(i)}^{(j)}\right)^2-\frac{1}{2}\eta_{(3-i)}\left(s_{R(i)}^{(3-j)}\right)^2 \tag{7}$$

$$\Pi_{W(i)}=\left(p_{w(i)}^{(j)}-w_{d(i)}\right)D_{w(i)}^{(j)}+\left(w_{w(i)}-w_{d(i)}\right)\left(D_{R(i)}^{(j)}+D_{R(3-i)}^{(j)}\right)-\frac{1}{2}\eta_{(i)}\left(s_{w(i)}^{(j)}\right)^2 \tag{8}$$

$$\Pi_{D(i)}=\left(w_{d(i)}-w_{m(i)}\right)\left(D_{w(i)}^{(j)}+D_{R(i)}^{(j)}+D_{R(3-i)}^{(j)}\right) \tag{9}$$

$$\Pi_{M(i)}=w_{m(i)}\left(D_{w(i)}^{(j)}+D_{R(i)}^{(j)}+D_{R(3-i)}^{(j)}\right) \tag{10}$$

Using sequential differentiations from the profit function of each supply chain member, one can solve such models to specify the equilibrium of the decision variables as a parametric equation. The following theorems hold in regard to the objective functions at the different levels of the supply chain.

Theorem 1. The common retailer's profit in Eq. (3) is strictly concave.
Proof. See Appendix.
It can be stated based on Theorem 1 that Eq. (3) is strictly concave; consequently, the values obtained for $p_{R(i)}^{(j)}$ and $p_{R(i)}^{(3-j)}$ are optimal and unique.

Theorem 2. The exclusive retailer's profit function is strictly concave.
Proof. See Appendix.
According to Theorem 2, the values obtained for $p_{w(i)}^{(j)}$ and $w_{w(i)}$ are optimal and unique since the objective function of the exclusive retailer is concave.

Theorem 3. The distributor's profit function is strictly concave.
Proof. See Appendix.
According to Theorem 3, $\Pi_{D(i)}$ is strictly concave; consequently, the value obtained for $w_{d(i)}$ is optimal and unique.

Theorem 4. The manufacturer's profit function for the $\{\gamma_p=\gamma_s,\theta_p=\theta_s,\eta_1=\eta_2\}$ condition is strictly concave.
Proof. See Appendix.
Therefore, $\Pi_{M(i)}$ is strictly concave; consequently, the values obtained for $w_{m(i)}$ and the service levels are optimal and unique.

The GT used in this work to solve the mathematical model is similar to those of Mahmoodi and Eshgh [13] and Anderson and Bao [11]. In each chain, the manufacturer is assumed to be the leader and distributor, and the retailers are followers. Following the decision made by the leader (*i.e.* manufacturer), the distributor, exclusive retailer, and common retailer make decisions. Hence, we observe the Stackelberg game in each supply chain. Furthermore, the two chains make their decisions simultaneously; therefore, a Nash game is gone through between them. In the Stackelberg game, the leader makes decisions to maximize its own profit according to the follower's response function.

The problem may be solved using backward induction. Thus, the problem of the follower's response function is solved for each scenario of the Stackelberg game given that the follower has observed the leader's decision (Due to the similarity of the solution procedures of the two scenarios, the solution steps only of Scenario 1 are described).

The consecutive procedures used to obtain the price and service equilibria in different scenarios stepwise are given in Appendix A.

6. Numerical Analysis

For proper analysis, the problem is solved in four different modes. In the first mode, we assume that the intensity of competition is low, *i.e.* $0 < \theta_p, \theta_s, \gamma_p, \gamma_s \le 0.5$.

For the second mode, the intensity of competition is high. In other words, $0.5 < \theta_p, \theta_s, \gamma_p, \gamma_s < 1$.

In the third mode, the intensity of competition is high among the products but low among the stores. That is, $0 < \gamma_p, \gamma_s \le 0.5 \, and \, 0.5 < \theta_p, \theta_s < 1$. In the fourth mode, the intensity of competition is high among the stores but low among the products. That is, $0 < \theta_p, \theta_s \le 0.5 \, and \, 0.5 < \gamma_p, \gamma_s < 1$.

Therefore, the following hold.

Mode 1: $\{\theta_p = 0.2 \,, \theta_s = 0.3 \,, \gamma_p = 0.3 \,, \gamma_s = 0.2 \,, \rho = 0.4 \,, \beta = 0.3 \,, \eta_1 = 1.1 \,, \eta_2 = 1.3\}$

Mode 2: $\{\theta_p = 0.6 \,, \theta_s = 0.8 \,, \gamma_p = 0.9 \,, \gamma_s = 0.7 \,, \rho = 0.4 \,, \beta = 0.3 \,, \eta_1 = 1.1 \,, \eta_2 = 1.3\}$

Mode 3: $\{\theta_p = 0.6 \,, \theta_s = 0.8 \,, \gamma_p = 0.3 \,, \gamma_s = 0.2 \,, \rho = 0.4 \,, \beta = 0.3 \,, \eta_1 = 1.1 \,, \eta_2 = 1.3\}$

Mode 4: $\{\theta_p = 0.2 \,, \theta_s = 0.3 \,, \gamma_p = 0.9 \,, \gamma_s = 0.7 \,, \rho = 0.4 \,, \beta = 0.3 \,, \eta_1 = 1.1 \,, \eta_2 = 1.3\}$

Table 3 presents the profits of the supply chain members and the equilibria for the decision variables in all the four modes for different servicing scenarios.

Please Insert Table 3 about here.

- Analysis of each scenario

- An increase in competition reduces price, and increases service level.
- Similarly, an increase in competition leads to a rise in the members' profits in both supply chains; this leads to an increase in the total profit of the supply chain.
- An increase in price competition causes each supply chain to reduce its price to achieve a higher profit than the competitor. Moreover, an increase in competition at the service level causes each supply chain to consider a higher service level to draw greater customer attention despite its higher cost of service.
- Clearly from Table 3, the supply chain with a lower service cost factor than the competitor provides a higher service level for its product. Likewise, it can be observed in each supply chain that the upper-level sellers not only sell their products at lower prices than the lower-level sellers, but also provide higher-level service. This is because of the exclusive nature of higher-level sellers.

It is worth noting that since the upper-level seller in each chain sells its product at the same price as both of the lower-level sellers, sale price is identical for the two lower-level sellers. Similarly, since the manufacturer in each chain provides the same service cost factor for its product to both of the lower-level sellers, the service level for every product is similar for the two.
- Modes 3 and 4 are mixed modes, and somehow supplement each other. In Mode 3, there is intense competition between the products on price and service, while the intensity of competition between the sale centers is low, and the manufacturer considers a higher level of service for the product in Scenario 1. When the competition between the products is reduced (Mode 4), however, the level of service provided by the manufacturer also decreases. In Mode 4, with a high level of competition between the sale centers, they provide higher levels of service. As the level of competition assumes a decremental trend (Mode 3), however, the level of service provided by the sale channels is also lowered.
If Scenario 2 is selected, the members' highest profits are gained in Mode 4. This will occur in Mode 2 upon selection of Scenario 1.

- Comparison between the scenarios

The price and service levels in the second scenario are lower and higher, respectively, than those in the first scenario in all the four modes. Likewise, the supply chain members in the second scenario acquire higher profit than those in the first scenario. Thus, it can be concluded that the conditions in the second scenario are to the benefit of the chain members in all the four modes. Therefore, if the specifications of the service level are delegated to the sale channels by the manufacturer, all the members will gain greater profits. Hence, when the sale channels are responsible for presentation of these specifications to the end customers, the customers buy the product at lower prices than in the first scenario, in addition to receiving a higher service level. The supply chain members are can acquire greater profit than in the first scenario.
Likewise, it is implied in regard to the demand for products in the sale channels in each scenario that:
- the demand for products in each scenario increases as competition rises;
- the demand for products is greater in the second scenario than in the first in both modes. This means that the customers prefer the structure given in the second scenario.

7. Sensitivity Analysis

In this section, we intend to analyze the effect of variation in the parameters of the problem on the profit of the supply chain members in both scenarios. For this purpose, the results obtained in this section are analyzed in the following three parts. The variation in profit is compared to that in substitution degree for two products and sale channels in the first and second parts, respectively. The third part is dedicated to a sensitivity analysis of profit versus the significance coefficients in exclusive and common retailers.

7.1. Intensity of Competition between Products

To obtain the intensity of competition between products, we set $\theta_s = 0.5$, and then analyze the variation in profit for the members for different values of θ_p in the (0,1) range. Table 4 shows the profits gained by the members for these values.

Please Insert Table 4 about here.

The following quantities are assumed in this part for the other parameters:
$\gamma_p = 0.1$, $\gamma_s = 0.1$, $\rho = 0.4$, $\beta = 0.3$, $\eta_1 = \eta_2 = 1.1$.

A small value of $\frac{\theta_p}{\theta_s}$ indicates the lower sensitivity of customers to the price of the product. Clearly from Table 4, profit increases at all the levels of the supply chain as the above ratio rises. Thus, an increase in the price competition intensity between the products is to the benefit of all the supply chain members. Therefore, price plays a more dominant role than service level in the competition between the products in both scenarios. Thus, the more replaceable the products in terms of price (the higher the value of $\frac{\theta_p}{\theta_s}$), the greater the profit gained by the supply chain members.

7.2. Intensity of Competition among Sale Channels

To obtain the intensity of competition among sale channels, we set $\gamma_s = 0.5$, and then analyze the variation in the members' profit for different values of γ_p in the (0,1) range. Table 5 shows the profits gained by the members for these values. The following quantities are assumed in this part for the other parameters:
$\theta_p = 0.1$, $\theta_s = 0.1$, $\rho = 0.4$, $\beta = 0.3$, $\eta_1 = \eta_2 = 1.1$.
When there is a competition among the sale channels ($\gamma_p, \gamma_s \neq 0$), the profit gained by the supply chain members decreases as $\frac{\gamma_p}{\gamma_s}$ increases (Table 5). In other words, profit decreases for all the members as the substitution degree of the sale channels with respect to price increases. Thus, all the levels will acquire greater profits if $\gamma_s > \gamma_p$ regardless of who is responsible for provision of service to the customer in the supply chain. In this case, the customers tend to purchase from the shops that provide higher service levels. Hence, service level is a more important factor than price in the competition between the sale channels.

Please Insert Table 5 about here.

The computations in 7-1 and 7-2 are based only on the values $\theta_s = 0.5$ and $\gamma_s = 0.5$. Similar computations are made for other quantities of γ_s and θ_s; since they are too long, however, the relevant tables are disregarded. Finally, the following figure is extracted from the given computations.
(In Figure 2, $\theta_p > \theta_s$ ($\gamma_p > \gamma_s$) holds for Area I, $\theta_p = \theta_s$ ($\gamma_p = \gamma_s$) for Area II, and $\theta_p < \theta_s$ ($\gamma_p < \gamma_s$) for Area III.)

Please Insert Figure 2 about here.

The maximum profit in the competition between two products is acquired in Part I of Figure 2. In fact, the profit gained by the members assumes an incremental trend as it moves from Part III to Part I. Therefore, price is the dominant concern. An increase in the intensity of competition on price between products leads to decrease in their prices. Hence, the level of service provided for the products is also decreased proportionally to the decrease in the prices.
When moving from Part I to Part III, profit increases for the members in the competition between the sale channels; therefore, service level increases as γ_s rises, and price increases as a result. Thus, the degree of competition on service in the sale channels can be increased if higher costs are considered for service, and the level of service provided to the customer is enhanced. This leads to an increase in profit at all the levels of the supply chain for both scenarios.
In the competition between products, therefore, price is the dominant concern, while in the competition between sale channels, it is service that is the dominant concern.

For clarification of the behavior of the profit functions for different values of γ_p (as given in Table 5), the profit functions for the supply chain members and the total profit of the supply chain for the first scenario are given in Fig. 3.

Please Insert Figure 3 about here.

It can be found from the above figures that the maximum profit of the supply chain members occurs in the $0 < \gamma_p < 0.3$ range when the parameters are assumed to be constant, and γ_p is variable. It can therefore be stated that maximum supply chain profit is observed for a low level of price competition intensity between sale centers. Of course, the above result is obtained in a case where other parameters are assumed to be constant.

7.3. Significance Coefficients of Sale Channels

Given the following values for the parameters of the problem, we investigate the effect of various values of ρ and β on profit for the supply chain members. Table 6 shows the results obtained in this section ($\theta_p = 0.1$, $\theta_s = 0.1$, $\gamma_p = 0.1$, $\gamma_s = 0.1$, $\eta_1 = \eta_2 = 1.1$).

Please Insert Table 6 about here.

In this table, ρ and β are the significance coefficients for the exclusive and common retailers, respectively. In this study, we assume that $\rho > \beta$. This means that the exclusive retailer is considered as a powerful rival for the common retailer. Profit is reduced for the common retailer as the difference between ρ and β increases, while it rises for the exclusive retailer. This causes the gap of profit between these two levels to increase. Moreover, we observe increase in profit for the manufacturer, distributor, and supply chain.
If $(\rho - \beta) \to 0$, however, the difference in profit between the two levels is reduced as the profit for the common retailer increases, and that for the exclusive retailer decreases.

8. Conclusion and Future Directions

This paper investigated the competition between two multi-level supply chains under two centralized and decentralized service systems with hybrid distribution channels.

The first scenario comprised a CS system in which the manufacturer provided a service level in each supply chain, while the sale channels designated the service level for the customer in the other scenario. The following managerial insights can be proposed based on the obtained results.

- If the manufacturer equips the exclusive retailer excessively, with considerations for even the most trivial customer demands, the profits made by all the supply chain members and total supply chain profit will increase, except for the common retailer.

Because it is only the profit made by the common retailer that is reduced, the manufacturer can satisfy it using incentives (such as discounts or profit participation).

- The following strategies can be adopted by a supply chain to gain greater profit.

1) To establish severer competition between the products on price rather than on service, meaning that supply chain member profit increases as low product prices are set. To intensify competition on service level among sale centers, measures such as the following are taken: 24-hour service, improvement of response to customer request, after-sale service, and assurance of original parts.

2) To establish severer competition between the sale centers on service rather than on price; it is suggested that better service be provided to the customer by the sale centers to gain greater profits (in the decentralized service system).

For example, companies hold sales festivals or offer promotions to intensify price competition between products.

- The customer prefers to have the sale centers (exclusive and common retailers) determine service level. The supply chain members also gain greater profits in this mode than in the centralized service system. Therefore, the manufacturer had better leave service level to be determined by the retailers.

The model actually attempted to analyze the impact of different servicing systems on price, service, and profit in the supply chains. To investigate how competition affected the equilibrium and each member's profit in each scenario, a numerical example was analyzed, and the results revealed the following.

- The DS system performed better than the CS system. Specifically, the DS system increased profit for all the supply chain members, and, finally, the total profit of the supply chain.

- since service was provided by the sale channels in the DS system, the shops presented higher service levels there than in the CS system, and sale price was also lower for the products in the DS system than for those in the CS system. This led to greater satisfaction for the customer.

- According to a sensitivity analysis, service was the dominant concern in the competition between retailers, and price was the dominant concern in the competition at the level of products.

These conclusions were drawn based on the assumptions made for this competitive model. The model can be developed in several ways. Firstly, service and price were taken into account as the competing parameters, while other factors such as product quality and product stability degree can be considered as well. This competitive model can be investigated for complementary products under various conditions of the demand function. Demand uncertainty can be taken into account, for instance. Secondly, we assumed in this paper that all the chain members had symmetric information on the demand, while asymmetric information for all the members can be developed in other models. The model was analyzed based on the Stackelberg manufacturer. Future studies can consider different power structures in the supply chain. Another assumption was that there was only one member at each level. More members can be assumed at each level in future research. Another assumption made in this study was that the exclusive retailers sold their products at the same prices as the common retailers in both supply chains. One can also analyze the effect of this process on profit and the decision variables by relaxing this assumption. Availability of a distributor in the supply chain is effective on the manufacturer and, eventually, on the entire supply chain in a large number of ways. Potential lines of research involve investigation of the effect of distributor performance from different aspects and elimination of the distributor from the supply chain and its impact on the overall performance of the supply chain and its members.

Another issue that can be addressed is the simultaneous specification of service level by both the manufacturer and the sale centers and the analysis of the problem in this case.

Finally, we assumed in this study that all the members in both supply chains are risk-neutral. In future research, risk-averse members can be considered.

Data Availability Statement

The data generated or analyzed during the study are available from the corresponding author upon request.

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Tables List

Table 1. Summary of the literature on supply chain competition

Number of levels	Paper	Factor of competition			Sale channels	
		Price	Service	Other	Exclusive	Common
[5], [6],[8],[11],[13],[14],[15], [22],[26]	2	*			*	
[27],[23],[29],[19],[18]	2			*	*	
[7]	2		*		*	
[9]	2	*	*		*	
[28],[10],[24]	2	*		*	*	
[12]	2	*		*	*	*
[31]	2	*			*	*
[16]	2	*			*	
[17]	2			*	*	
[20]	2			*	*	
[21]	3	*	*		*	
[25]	3	*			*	
This Study	4	*	*		*	*

Table 2. Model parameters and indices

Index/Parameter		Description
i		Supply chain index
j		Product index
θ_p		Degree (intensity) of price competition between products
θ_s		Degree (intensity) of service competition between products
γ_p		Degree (intensity) of price competition among sale channels
γ_s		Degree (intensity) of service competition among sale channels
$\eta_{(i)}$		Manufacturer's service cost factor in the i[th] supply chain
ρ		Significance coefficient of the exclusive retailer (a coefficient to indicate the importance of the exclusive retailer)
β		Significance coefficient of the common retailer (a coefficient to indicate the importance of the common retailer)
$w_{m(i)}$		Manufacturer's price in the i[th] supply chain
$w_{d(i)}$		Distributor's price in the i[th] supply chain
$w_{w(i)}$		Exclusive retailer's price in the i[th] supply chain
$i = j = 1,2$	$D_{w(i)}^{(j)}$	Demand for the j[th] product at the exclusive retailer of the i[th] supply chain
	$p_{w(i)}^{(j)}$	Price of the j[th] product at the exclusive retailer of the i[th] supply chain
	$s_{w(i)}^{(j)}$	Service level of the j[th] product at the exclusive retailer of the i[th] supply chain
$i = 1,2$, $j = 1,2$	$D_{R(i)}^{(j)}$	Demand for the j[th] product at the common retailer of the i[th] supply chain
	$p_{R(i)}^{(j)}$	Price of the j[th] product at the common retailer of the i[th] supply chain
	$s_{R(i)}^{(j)}$	Service level of the j[th] product at the common retailer of the i[th] supply chain

Table 3. Results of solving the model in different scenarios

	Scenario 1			Scenario 2		
		Sc 1	Sc 2		Sc 1	Sc 2
Mode1	Level1	$w_{m(1)} = 0.214$ $\Pi_{M(1)} = 1.1425$	$w_{m(2)} = 0.174$ $\Pi_{M(2)} = 1.1309$	Level1	$w_{m(1)} = 0.1815$ $\Pi_{M(1)} = 1.3069$	$w_{m(2)} = 0.1616$ $\Pi_{M(2)} = 1.2341$
	Level2	$w_{d(1)} = 0.266$ $\Pi_{D(1)} = 1.1131$	$w_{d(2)} = 0.2217$ $\Pi_{D(2)} = 0.9971$	Level2	$w_{d(1)} = 0.2223$ $\Pi_{D(1)} = 1.1938$	$w_{d(2)} = 0.1927$ $\Pi_{D(2)} = 1.1496$
	Level3	$p_{w(1)}^{(1)} = 0.3036$ $w_{w(1)} = 0.2879$ $s_{w(1)}^{(1)} = 0.2136$ $\Pi_{N(1)} = 0.8583$	$p_{w(2)}^{(2)} = 0.2622$ $w_{w(2)} = 0.2352$ $s_{w(2)}^{(2)} = 0.1958$ $\Pi_{N(2)} = 0.8125$	Level3	$p_{w(1)}^{(1)} = 0.2586$ $w_{w(1)} = 0.2398$ $s_{w(1)}^{(1)} = 0.2917$ $\Pi_{N(1)} = 1.0784$	$p_{w(2)}^{(2)} = 0.2262$ $w_{w(2)} = 0.2192$ $s_{w(2)}^{(2)} = 0.2553$ $\Pi_{N(2)} = 1.0593$
	Level4	$p_{R(1)}^{(1)} = 0.3134$ $p_{R(1)}^{(2)} = 0.2716$ $s_{R(1)}^{(1)} = 0.1528$ $s_{R(1)}^{(2)} = 0.1426$ $\Pi_{R(1)} = 0.6693$	$p_{R(2)}^{(1)} = 0.3134$ $p_{R(2)}^{(2)} = 0.2716$ $s_{R(2)}^{(1)} = 0.1528$ $s_{R(2)}^{(2)} = 0.1426$ $\Pi_{R(2)} = 0.6693$	Level4	$p_{R(1)}^{(1)} = 0.2830$ $p_{R(1)}^{(2)} = 0.2673$ $s_{R(1)}^{(1)} = 0.2484$ $s_{R(1)}^{(2)} = 0.2079$ $\Pi_{R(1)} = 0.8423$	$p_{R(2)}^{(1)} = 0.2830$ $p_{R(2)}^{(2)} = 0.2673$ $s_{R(2)}^{(1)} = 0.2484$ $s_{R(2)}^{(2)} = 0.2079$ $\Pi_{R(2)} = 0.8423$
		Sc 1	Sc 2		Sc 1	Sc 2
Mode2	Level1	$w_{m(1)} = 0.1786$ $\Pi_{M(1)} = 1.2636$	$w_{m(2)} = 0.1703$ $\Pi_{M(2)} = 1.2479$	Level1	$w_{m(1)} = 0.1599$ $\Pi_{M(1)} = 1.3658$	$w_{m(2)} = 0.1519$ $\Pi_{M(2)} = 1.3490$
	Level2	$w_{d(1)} = 0.2142$ $\Pi_{D(1)} = 1.1610$	$w_{d(2)} = 0.2060$ $\Pi_{D(2)} = 1.0507$	Level2	$w_{d(1)} = 0.1825$ $\Pi_{D(1)} = 1.2821$	$w_{d(2)} = 0.1770$ $\Pi_{D(2)} = 1.1941$
	Level3	$p_{w(1)}^{(1)} = 0.2416$ $w_{w(1)} = 0.2347$ $s_{w(1)}^{(1)} = 0.2688$ $\Pi_{N(1)} = 0.8887$	$p_{w(2)}^{(2)} = 0.2337$ $w_{w(2)} = 0.2129$ $s_{w(2)}^{(2)} = 0.2423$ $\Pi_{N(2)} = 0.8527$	Level3	$p_{w(1)}^{(1)} = 0.2200$ $w_{w(1)} = 0.2045$ $s_{w(1)}^{(1)} = 0.3499$ $\Pi_{N(1)} = 1.1083$	$p_{w(2)}^{(2)} = 0.2056$ $w_{w(2)} = 0.1979$ $s_{w(2)}^{(2)} = 0.2995$ $\Pi_{N(2)} = 1.0880$
	Level4	$p_{R(1)}^{(1)} = 0.2697$ $p_{R(1)}^{(2)} = 0.2418$ $s_{R(1)}^{(1)} = 0.2045$ $s_{R(1)}^{(2)} = 0.1899$ $\Pi_{R(1)} = 0.7021$	$p_{R(2)}^{(1)} = 0.2697$ $p_{R(2)}^{(2)} = 0.2418$ $s_{R(2)}^{(1)} = 0.2045$ $s_{R(2)}^{(2)} = 0.1899$ $\Pi_{R(2)} = 0.7021$	Level4	$p_{R(1)}^{(1)} = 0.2436$ $p_{R(1)}^{(2)} = 0.2386$ $s_{R(1)}^{(1)} = 0.2789$ $s_{R(1)}^{(2)} = 0.2413$ $\Pi_{R(1)} = 1.0068$	$p_{R(2)}^{(1)} = 0.2436$ $p_{R(2)}^{(2)} = 0.2386$ $s_{R(2)}^{(1)} = 0.2789$ $s_{R(2)}^{(2)} = 0.2413$ $\Pi_{R(2)} = 1.0068$

Table 3. Results of solving the model in different scenarios (continued)

		Scenario 1			Scenario 2	
		Sc 1	Sc 2		Sc 1	Sc 2
Mode 3	Level 1	$w_{m(1)}=0.1866$ $\Pi_{M(1)}=1.2032$	$w_{m(2)}=0.1815$ $\Pi_{M(2)}=1.1871$	Level 1	$w_{m(1)}=0.1704$ $\Pi_{M(1)}=1.3223$	$w_{m(2)}=0.1659$ $\Pi_{M(2)}=1.3202$
	Level 2	$w_{d(1)}=0.2279$ $\Pi_{D(1)}=1.1333$	$w_{d(2)}=0.2197$ $\Pi_{D(2)}=1.1108$	Level 2	$w_{d(1)}=0.2189$ $\Pi_{D(1)}=1.252$	$w_{d(2)}=0.2113$ $\Pi_{D(2)}=1.2463$
	Level 3	$p_{w(1)}^{(1)}=0.2493$ $w_{w(1)}=0.2402$ $s_{w(1)}^{(1)}=0.2589$ $\Pi_{N(1)}=0.8611$	$p_{w(2)}^{(2)}=0.2413$ $w_{w(2)}=0.2367$ $s_{w(2)}^{(2)}=0.2383$ $\Pi_{N(2)}=0.8544$	Level 3	$p_{w(1)}^{(1)}=0.2342$ $w_{w(1)}=0.1957$ $s_{w(1)}^{(1)}=0.2983$ $\Pi_{N(1)}=1.2238$	$p_{w(2)}^{(2)}=0.2278$ $w_{w(2)}=0.1915$ $s_{w(2)}^{(2)}=02763$ $\Pi_{N(2)}=1.1982$
	Level 4	$p_{R(1)}^{(1)}=0.2878$ $p_{R(1)}^{(2)}=0.2599$ $s_{R(1)}^{(1)}=0.1849$ $s_{R(1)}^{(2)}=0.1765$ $\Pi_{R(1)}=0.6981$	$p_{R(2)}^{(1)}=0.2819$ $p_{R(2)}^{(2)}=0.2486$. $s_{R(2)}^{(1)}=0.1694$ $s_{R(2)}^{(2)}=0.1536$ $\Pi_{R(2)}=0.6878$	Level 4	$p_{R(1)}^{(1)}=0.2707$ $p_{R(1)}^{(2)}=0.2599$ $s_{R(1)}^{(1)}=0.2711$ $s_{R(1)}^{(2)}=0.2662$ $\Pi_{R(1)}=1.1884$	$p_{R(2)}^{(1)}=0.2657$ $p_{R(2)}^{(2)}=0.2513$ $s_{R(2)}^{(1)}=0.2532$ $s_{R(2)}^{(2)}=0.2525$ $\Pi_{R(2)}=1.1769$
Mode 4		Sc 1	Sc 2		Sc 1	Sc 2
	Level 1	$w_{m(1)}=0.1802$ $\Pi_{M(1)}=1.1934$	$w_{m(2)}=0.1759$ $\Pi_{M(1)}=1.1808$	Level 1	$w_{m(1)}=0.1767$ $\Pi_{M(1)}=1.334$	$w_{m(2)}=0.1674$ $\Pi_{M(2)}=1.3308$
	Level 2	$w_{d(1)}=0.2111$ $\Pi_{D(1)}=1.13$	$w_{d(2)}=0.2055$ $\Pi_{D(2)}=1.1107$	Level 2	$w_{d(1)}=0.2202$ $\Pi_{D(1)}=1.2541$	$w_{d(2)}=0.1996$. $\Pi_{D(2)}=1.2234$
	Level 3	$p_{w(1)}^{(1)}=0.2386$ $w_{w(1)}=0.2306$ $s_{w(1)}^{(1)}=0.2464$ $\Pi_{N(1)}=0.8546$	$p_{w(2)}^{(2)}=0.2311$ $w_{w(2)}=0.2249$ $s_{w(2)}^{(2)}=0.2342$ $\Pi_{N(2)}=0.8511$	Level 3	$p_{w(1)}^{(1)}=0.2407$ $w_{w(1)}=0.2001$ $s_{w(1)}^{(1)}=0.3212$ $\Pi_{N(1)}=1.2389$	$p_{w(2)}^{(2)}=0.2237$ $w_{w(2)}=0.1914$ $s_{w(2)}^{(2)}=0.308$. $\Pi_{N(2)}=1.2201$
	Level 4	$p_{R(1)}^{(1)}=0.2732$ $p_{R(1)}^{(2)}=0.2437$ $s_{R(1)}^{(1)}=0.1709$ $s_{R(1)}^{(2)}=0.1679$	$p_{R(2)}^{(1)}=0.2779$ $p_{R(2)}^{(2)}=0.2414$ $s_{R(2)}^{(1)}=0.1609$ $s_{R(2)}^{(2)}=0.1443$	Level 4	$p_{R(1)}^{(1)}=0.2633$ $p_{R(1)}^{(2)}=0.2417$ $s_{R(1)}^{(1)}=0.3012$ $s_{R(1)}^{(2)}=0.2914$	$p_{R(2)}^{(1)}=0.2533$ $p_{R(2)}^{(2)}=0.2413$ $s_{R(2)}^{(1)}=0.2864$ $s_{R(2)}^{(2)}=0.2851$

		$\Pi_{R(1)} = 0.6905$	$\Pi_{R(2)} = 0.6606$		$\Pi_{R(1)} = 1.2016$	$\Pi_{R(2)} = 1.1963$

Table 4. Variation in profit vs. intensity of competition between two products

		$\frac{\theta_p}{\theta_s} < 1$				$\frac{\theta_p}{\theta_s} = 1$	$\frac{\theta_p}{\theta_s} > 1$			
	Profit	$\theta_s = 0.5$ $\theta_p = 0.1$	$\theta_s = 0.5$ $\theta_p = 0.2$	$\theta_s = 0.5$ $\theta_p = 0.3$	$\theta_s = 0.5$ $\theta_p = 0.4$	$\theta_s = \theta_p = 0.5$	$\theta_s = 0.5$ $\theta_p = 0.6$	$\theta_s = 0.5$ $\theta_p = 0.7$	$\theta_s = 0.5$ $\theta_p = 0.8$	$\theta_s = 0.5$ $\theta_p = 0.9$
Scenario 1	Π_M Π_D Π_N Π_R	1.1424 1.0573 0.7896 0.6895	1.1466 1.0589 0.7905 0.6907	1.1487 1.0606 0.7915 0.6919	1.1512 1.0632 0.7947 0.6956	1.1533 1.0671 0.8003 0.6998	1.1586 1.0706 0.825 0.7238	1.1601 1.073 0.8411 0.7279	1.1619 1.0745 0.8464 0.7304	1.163 1.0758 0.8499 0.7343
Scenario 2	Π_M Π_D Π_N Π_R	1.3159 1.196 1.0903 0.8558	1.3167 1.1973 1.0911 0.857	1.3176 1.1984 1.092 0.8579	1.3186 1.2001 1.0933 0.8588	1.3205 1.2011 1.0942 0.8603	1.3232 1.2028 1.096 0.8622	1.3247 1.2041 1.0973 0.8636	1.3256 1.2055 1.0984 0.864	1.327 1.2067 1.0992 0.8649

Table 5. Variation in profit vs. intensity of competition between sale channels

		$\frac{\gamma_p}{\gamma_s} < 1$				$\frac{\gamma_p}{\gamma_s} = 1$	$\frac{\gamma_p}{\gamma_s} > 1$			
	Profit	$\gamma_s = 0.5$ $\gamma_p = 0.1$	$\gamma_s = 0.5$ $\gamma_p = 0.2$	$\gamma_s = 0.5$ $\gamma_p = 0.3$	$\gamma_s = 0.5$ $\gamma_p = 0.4$	$\gamma_s = \gamma_p = 0.5$	$\gamma_s = 0.5$ $\gamma_p = 0.6$	$\gamma_s = 0.5$ $\gamma_p = 0.7$	$\gamma_s = 0.5$ $\gamma_p = 0.8$	$\gamma_s = 0.5$ $\gamma_p = 0.9$
Scenario 1	Π_M Π_D Π_N Π_R	1.1482 1.0639 0.7478 0.6067	1.1474 1.063 0.7466 0.6059	1.1464 1.0621 0.7458 0.6046	1.1456 1.0608 0.7444 0.6037	1.144 1.0593 0.742 0.6011	1.1413 1.0571 0.7394 0.5973	1.1397 1.0558 0.7373 0.5955	1.1382 1.0538 0.736 0.594	1.137 1.0523 0.7348 0.5931
Scenario 2	Π_M Π_D Π_N Π_R	1.3085 1.1977 1.0945 0.8628	1.3074 1.1965 1.0933 0.8617	1.3069 1.1955 1.0925 0.8605	1.3055 1.1948 1.0913 0.8589	1.3047 1.1935 1.0904 0.8584	1.3034 1.1927 1.0887 0.8576	1.3024 1.1919 1.0875 0.8566	1.3011 1.1901 1.0869 0.8552	1.3003 1.189 1.0856 0.8543

Table 6. Variation in profit vs. the coefficients of significance in the sale channels

		$1 < \frac{\rho}{\beta} < 2$	$\frac{\rho}{\beta} = 2$	$\frac{\rho}{\beta} > 2$
	Profit	$\rho = 0.4$ $\beta = 0.3$	$\rho = 0.6$ $\beta = 0.3$	$\rho = 0.8$ $\beta = 0.3$
Scenario 1	Π_M Π_D Π_N Π_R Π_{Sc}	1.1475 1.0633 0.7456 0.6048 3.5612	1.1483 1.0639 0.7496 0.6005 3.5623	1.149 1.0645 0.7547 0.5981 3.5663
Scenario 2	Π_M Π_D Π_N	1.3151 1.1986 1.0948 0.8614	1.3157 1.1995 1.0989 0.8584	1.3168 1.2006 1.1049 0.8533

	Π_R Π_{Sc}	4.4699	4.4727	4.4756

Figures List

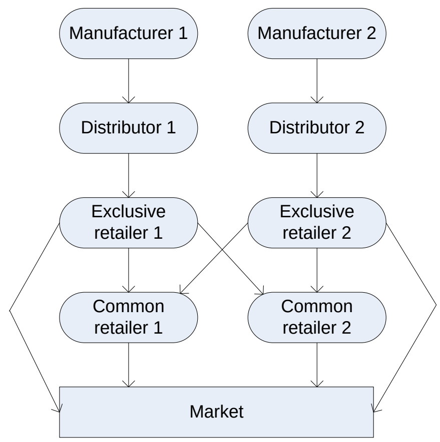


Figure 1. Graphic structure of the supply chains

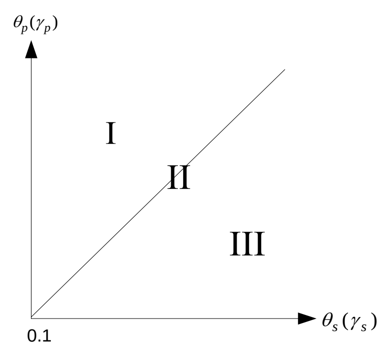
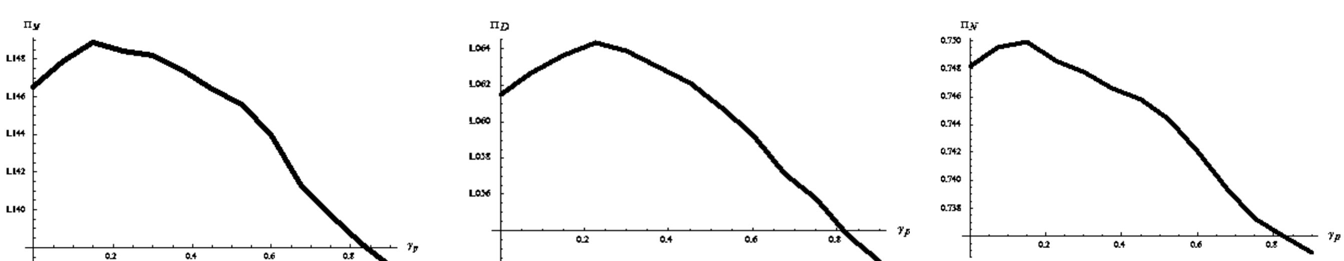


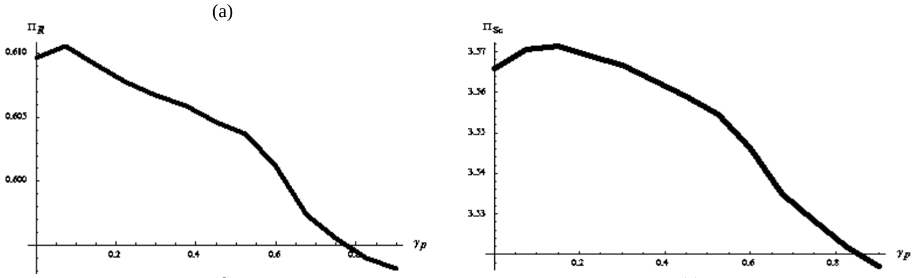
Figure 2. Variation in profit with respect to competition intensity



(b)

(c)

(a)



(d) (e)

Figure 3. Variation in the profit function with respect to γ_p for the (a) manufacturer, (b) distributor, (c) exclusive retailer, (d) common retailer, and (e) supply chain

Appendix A

The solution steps are as follows.

(1) Differentiate function (3) with respect to $p_{R(i)}^{(j)}$ and $p_{R(i)}^{(3-j)}$.

(2) Solve the equations $\dfrac{\partial \Pi_{R(i)}}{\partial p_{R(i)}^{(j)}} = 0$ and $\dfrac{\partial \Pi_{R(i)}}{\partial p_{R(i)}^{(3-j)}} = 0$ simultaneously for $i = j = 1,2$ and specification of the retailer prices as follows.

$$p_{R(1)}^{(1)} = f\left(w_{w(1)}, w_{w(2)}, s_{w(1)}^{(1)}, s_{w(2)}^{(2)}, p_{w(1)}^{(1)}, p_{w(2)}^{(2)}, s_{R(1)}^{(1)}, s_{R(2)}^{(1)}, s_{R(1)}^{(2)}, s_{R(2)}^{(2)}\right) \quad (11)$$

$$p_{R(1)}^{(2)} = f\left(w_{w(1)}, w_{w(2)}, s_{w(1)}^{(1)}, s_{w(2)}^{(2)}, p_{w(1)}^{(1)}, p_{w(2)}^{(2)}, s_{R(1)}^{(1)}, s_{R(2)}^{(1)}, s_{R(1)}^{(2)}, s_{R(2)}^{(2)}\right) \quad (12)$$

$$p_{R(2)}^{(1)} = f\left(w_{w(1)}, w_{w(2)}, s_{w(1)}^{(1)}, s_{w(2)}^{(2)}, p_{w(1)}^{(1)}, p_{w(2)}^{(2)}, s_{R(1)}^{(1)}, s_{R(2)}^{(1)}, s_{R(1)}^{(2)}, s_{R(2)}^{(2)}\right) \quad (13)$$

$$p_{R(2)}^{(2)} = f\left(w_{w(1)}, w_{w(2)}, s_{w(1)}^{(1)}, s_{w(2)}^{(2)}, p_{w(1)}^{(1)}, p_{w(2)}^{(2)}, s_{R(1)}^{(1)}, s_{R(2)}^{(1)}, s_{R(1)}^{(2)}, s_{R(2)}^{(2)}\right) \quad (14)$$

Proof of Theorem 1. For the common retailer's profit, Eq. (3), the Hessian matrix is as follows:

$$Hessian\,Matrix = H = \begin{pmatrix} \dfrac{\partial^2 \Pi_{R(i)}}{\partial \left(p_{R(i)}^{(j)}\right)^2} & \dfrac{\partial^2 \Pi_{R(i)}}{\partial p_{R(i)}^{(j)} \partial p_{R(i)}^{(3-j)}} \\ \dfrac{\partial^2 \Pi_{R(i)}}{\partial p_{R(i)}^{(3-j)} \partial p_{R(i)}^{(j)}} & \dfrac{\partial^2 \Pi_{R(i)}}{\partial \left(p_{R(i)}^{(3-j)}\right)^2} \end{pmatrix}$$

Where $\frac{\partial^2 \Pi_{R(i)}}{\partial\left(p_{R(i)}^{(j)}\right)^2} = -2$, $\frac{\partial^2 \Pi_{R(i)}}{\partial\left(p_{R(i)}^{(3-j)}\right)^2} = -2$,and

$\frac{\partial^2 \Pi_{R(i)}}{\partial p_{R(i)}^{(j)} \partial p_{R(i)}^{(3-j)}} = \frac{\partial^2 \Pi_{R(i)}}{\partial p_{R(i)}^{(3-j)} \partial p_{R(i)}^{(j)}} = 2\beta(1-\gamma\mathrm{p})\theta\mathrm{p} - 2\alpha\gamma\mathrm{p}\theta\mathrm{p} = -2\left(\beta(-1+\gamma\mathrm{p})+\alpha\gamma\mathrm{p}\right)\theta\mathrm{p}$.

The determinant of the Hessian matrix is calculated as follows: $|H| = 4-4\left(\left(\beta(-1+\gamma\mathrm{p})+\alpha\gamma\mathrm{p}\right)\theta\mathrm{p}\right)^2 > 0$.

The above expression is always positive for any value of the problem parameters; thus, the Hessian matrix is negative definite.
Hence, the profit function expressed by Eq. (3) is strictly concave. ■

(3) Replace Eqs. (11)-(14) in the profit function of the exclusive retailer.
(4) Specify the first derivatives of the profit function with respect to $p_{w(i)}^{(j)}$ and $w_{w(i)}$.
(5) Solve the equations $\frac{\partial \Pi_{W(i)}}{\partial p_{w(i)}^{(j)}} = 0$ and $\frac{\partial \Pi_{W(i)}}{\partial w_{w(i)}} = 0$ simultaneously for $i = j = 1,2$. As a result, the following will hold.

$$p_{w(1)}^{(1)} = f\left(w_{d(1)}, w_{d(2)}, s_{w(1)}^{(1)}, s_{w(2)}^{(2)}, s_{R(1)}^{(1)}, s_{R(2)}^{(1)}, s_{R(1)}^{(2)}, s_{R(2)}^{(2)}\right) \quad (15)$$

$$p_{w(2)}^{(2)} = f\left(w_{d(1)}, w_{d(2)}, s_{w(1)}^{(1)}, s_{w(2)}^{(2)}, s_{R(1)}^{(1)}, s_{R(2)}^{(1)}, s_{R(1)}^{(2)}, s_{R(2)}^{(2)}\right) \quad (16)$$

$$w_{w(1)} = f\left(w_{d(1)}, w_{d(2)}, s_{w(1)}^{(1)}, s_{w(2)}^{(2)}, s_{R(1)}^{(1)}, s_{R(2)}^{(1)}, s_{R(1)}^{(2)}, s_{R(2)}^{(2)}\right) \quad (17)$$

$$w_{w(2)} = f\left(w_{d(1)}, w_{d(2)}, s_{w(1)}^{(1)}, s_{w(2)}^{(2)}, s_{R(1)}^{(1)}, s_{R(2)}^{(1)}, s_{R(1)}^{(2)}, s_{R(2)}^{(2)}\right) \quad (18)$$

Proof of Theorem 2. For the exclusive retailer's profit in step (3), the Hessian matrix is as follows:

$$Hessian\,Matrix = H = \begin{pmatrix} \frac{\partial^2 \Pi_{W(i)}}{\partial\left(p_{w(i)}^{(j)}\right)^2} & \frac{\partial^2 \Pi_{W(i)}}{\partial p_{w(i)}^{(j)} \partial w_{w(i)}} \\ \frac{\partial^2 \Pi_{W(i)}}{\partial w_{w(i)} \partial p_{w(i)}^{(j)}} & \frac{\partial^2 \Pi_{W(i)}}{\partial\left(w_{w(i)}\right)^2} \end{pmatrix}$$

where

$$\frac{\partial^2\Pi_{W(i)}}{\partial\left(p_{w(i)}^{(j)}\right)^2}=-\rho,\qquad \frac{\partial^2\Pi_{W(i)}}{\partial\left(w_{w(i)}\right)^2}=-\rho,\qquad \frac{\partial^2\Pi_{W(i)}}{\partial p_{w(i)}^{(j)}\partial w_{w(i)}}=\frac{\partial^2\Pi_{W(i)}}{\partial w_{w(i)}\partial p_{w(i)}^{(j)}}=-\beta\rho .$$

The determinant of the Hessian matrix is calculated as follows:

$$\cdot\ \left|H\right|=\rho^2-\rho\beta\left(\beta\rho\right)$$

Because $\left|H\right|>0$, the Hessian matrix is negative definite.

Thus, the profit function expressed by Eq. (4) is strictly concave. ■

(6) Replace Eqs. (15)-(18) in the profit function of the distributors.
(7) Specify the first derivative of the profit function in step (6) with respect to $w_{d(i)}$.

(8) Solve the equations $\frac{\partial\Pi_{D(i)}}{\partial w_{d(i)}}=0$ simultaneously for $i=1,2$. As a result, the following will hold.

$$w_{d(1)}=f\left(w_{m(1)},w_{m(2)},s_{w(1)}^{(1)},s_{w(2)}^{(2)},s_{R(1)}^{(1)},s_{R(2)}^{(1)},s_{R(1)}^{(2)},s_{R(2)}^{(2)}\right) \tag{19}$$

$$w_{d(2)}=f\left(w_{m(1)},w_{m(2)},s_{w(1)}^{(1)},s_{w(2)}^{(2)},s_{R(1)}^{(1)},s_{R(2)}^{(1)},s_{R(1)}^{(2)},s_{R(2)}^{(2)}\right) \tag{20}$$

Proof of Theorem 3. For the distributor's profit function in step (6), $\frac{\partial^2\Pi_{D(i)}}{\partial\left(w_{d(i)}\right)^2}=-\rho^2\beta\left(1-\rho\right)\eta_1<0$ holds.

Given the above equation, $\frac{\partial^2\Pi_{D(i)}}{\partial\left(w_{d(i)}\right)^2}<0$; therefore, $\Pi_{D(i)}$ is also concave. ■

(9) Replace Eqs. (19)-(20) in the manufacturer's profit function.
(10) Specify the first derivatives of the manufacturer's profit function, in step (9), with respect to $w_{m(i)}$, $s_{w(i)}^{(j)}$, $s_{R(i)}^{(j)}$, and $s_{R(3-i)}^{(j)}$.

(11) Solve the equations $\{\frac{\partial\Pi_{M(i)}}{\partial w_{m(i)}}=0,\frac{\partial\Pi_{M(i)}}{\partial s_{w(i)}^{(j)}}=0,\frac{\partial\Pi_{M(i)}}{\partial s_{R(i)}^{(j)}}=0,\frac{\partial\Pi_{M(i)}}{\partial s_{R(3-i)}^{(j)}}=0\}$ simultaneously for $i=j=1,2$.

Thus, the equilibria of the manufacturer's price and service level in each shop are as follows.

$$w_{m(1)}=\left(\frac{1}{2}\right)\Big(\Big(\beta\left(-1+\gamma_p{}^2\eta_1\right)+\rho\gamma_p\Big)\theta_p-\left(\rho-\beta\right)\gamma_p{}^2\left(\beta\left(-1+\theta_p\right)\right)\theta_p\Big)-\beta\theta_s+\rho\gamma_s\theta_s+\rho\beta^2\gamma_s\theta_s+\rho\gamma_s\theta_s+5\beta\gamma_s\theta_s+\beta^2\gamma_s\theta_s$$

$$-\beta\gamma_s+2\beta\gamma_s{}^2\theta_s+((\beta(-1+\gamma_p)+\rho\gamma_p)\theta_p(2(1+(\rho-\beta)\gamma_s)+(\rho\gamma_p+\beta(-2+3\gamma_p))\theta_p+(1+\beta(-1+\gamma_p)\theta_p+\rho\gamma_p\theta_p)$$
$$(2+(-\rho+\beta)\gamma_p\theta_p)+\gamma_p\theta_p(-\rho-\beta^2-2\rho+\beta+\rho^2\beta-4\beta-\rho^2\gamma_s+2\rho\beta\gamma_s-\beta^2\gamma_s+(\rho-\eta_2\beta)((\rho+2\beta)\gamma_p\eta_1+(\beta-$$
$$(\rho+\beta)\gamma_p{}^2))\theta_p)+(-2\beta+2\rho\gamma_s+2(\rho+\eta_2\beta)\gamma_s+(-\rho\eta_1+\beta)\gamma_p{}^2(-\beta\eta_1+(\rho+\beta)\gamma_s)\theta_p)\theta_s)\eta_2)+(-1-\rho\gamma_s+\beta\gamma_s+\beta\theta_p$$
$$+\beta\theta_p-\rho\gamma_p\theta_p-\rho^2\gamma_p\theta_p+\rho\beta\gamma_p\theta_p-\beta\gamma_p\theta_p-\beta^2\gamma_p\theta_p+2\beta\gamma_p\theta_p+\beta\theta_s-\rho\gamma_s\theta_s-\rho\beta^2\gamma_s\theta_s-11\rho\gamma_s\theta_s-\beta\gamma_s\theta_s-\rho^2\beta\gamma_s\theta_s$$
$$+\rho\beta\gamma_s\theta_s-2\beta\gamma_s\theta_s)/((\frac{\rho\gamma_p\theta_p}{2}-\frac{\beta\gamma_p\theta_p}{2}-\left(\eta_1(-\beta\theta_p+\rho\gamma_p\theta_p+\beta\gamma_p\theta_p)\left(4-(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p)^2\right)\right)/(-2(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p{}^2\theta_p)-(2\beta\theta_p-2\rho\gamma_p\theta_p$$
$$-2\beta\gamma_p\theta_p)(-\rho\gamma_p\theta_p\beta\gamma_p\theta_p))$$

$$-((\frac{\rho\gamma_p}{2}-\frac{\beta\gamma_p}{2}+\frac{\beta\gamma_p{}^2\theta_p}{2}-((-\beta\theta_p+\rho\gamma_p\theta_p+\beta\gamma_p\theta_p)(-2(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\eta_1)+(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p)))$$
$$/(-2\eta_2(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)-(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p)))$$

$$((-2(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)-(2\beta\theta_p-2\rho\gamma_p{}^2\theta_p-2\beta\gamma_p\theta_p)(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p))(-2(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)$$
$$-(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)(-\rho\gamma_p\theta_p+\eta_2\beta\gamma_p\theta_p))-(-(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)-2(-\rho\gamma_p\theta_p+$$
$$\beta\gamma_p\theta_p))\left(4-(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p)^2\right))))$$

(21)

$$\boldsymbol{w}_{m(2)}=(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)\eta_1(-1-\rho\gamma s+\beta\gamma s+\beta\theta_p+\beta\gamma_p\theta_p-\rho\gamma_p\theta_p-\beta^2\rho\gamma_p\theta_p+\rho\gamma_p{}^2\theta_p-\beta\gamma_p\theta_p-\beta\gamma_p\theta_p+2\beta\gamma_p{}^2\theta_p+\beta\theta s$$
$$-\rho\gamma_s\theta_s-\rho\beta^2\gamma_s\theta_s-\rho\gamma_s\theta_s-\beta\gamma_s\theta_s-\beta^2\gamma_s\theta_s+\beta\gamma_s\theta_s-2\beta\gamma_s\theta_s))-(-(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)-2(-\eta_1\rho\gamma_p\theta_p$$
$$+\beta\gamma_p{}^2\theta_p))(-(-\rho\gamma_p{}^2\theta_p+\beta\gamma_p\theta_p)(-\eta_2-\rho\gamma_s+\beta\gamma_s+\beta\theta_p+\beta\theta_p-\rho\gamma_p\theta_p-\rho\gamma_p{}^2\theta_p+\rho\gamma_p\theta_p-\beta\gamma_p\theta_p-\rho\beta\gamma_p\theta_p+2\beta\gamma_p\theta_p+\beta\theta_s-\rho\gamma_s\theta_s$$
$$-\rho^2\gamma_s\theta_s-\rho\gamma_s\theta_s-\beta^2\gamma_s\theta_s-\beta\gamma_s\theta_s+\beta^2\gamma_s\theta_s-2\beta\gamma_s\theta_s)-2\eta_1(-1-\rho\gamma_s+\beta\gamma_s+\beta\theta_p+\beta\theta_p-\rho\gamma_p\theta_p-\rho^2\gamma_p\theta_p+\rho\gamma_p\theta_p-\beta\gamma_p\theta_p-\beta\gamma_p\theta_p$$
$$+2\beta\gamma_p\theta_p+\beta\theta_s-\rho\gamma_s\theta_s-1)/\left(\left(4\eta_1-(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)^2\right)(-2(-\rho\gamma_p{}^2+\beta\gamma_p-\beta\gamma_p\theta_p)-(2\beta\theta_p-2\rho\gamma_p\theta_p-2\eta_2\beta\gamma_p\theta_p)(-\rho\gamma_p\theta_p\right.$$
$$+\beta\gamma_p\theta_p))-(-(2\eta_2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)-2(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p))(-2(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)-(-\rho\gamma_p+\beta\gamma_p$$
$$-\beta\gamma_p\theta_p)(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p)))-\eta_2((\frac{\rho\gamma_p{}^2\theta_p}{2}-\frac{\beta\gamma_p\theta_p}{2}-\left(\eta_1(-\beta\theta_p+\rho\gamma_p\theta_p+\beta\gamma_p\theta_p)\left(4-(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p)^2\right)\right)/(-2(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p{}^2\theta_p)$$
$$-(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)\eta_2(-\rho\gamma_p\theta_p+\beta\gamma_p{}^2\theta_p))))$$

(22)

$$\boldsymbol{s}_{w(1)}^{(1)}=(-((-\eta_2(2\beta\theta_p-2\rho\gamma_p{}^2\theta_p-2\beta\gamma_p\theta_p)(-\rho^2\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)-2(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p))(-2(-\rho\gamma_p+\rho^2\gamma_p-\beta\gamma_p\theta_p)-(2\beta\theta_p-\rho\gamma_p\theta_p$$
$$-2\beta\gamma_p\theta_p)(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p))-\left(4-(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)^2\right)(-2(2\beta\theta_p-2\rho\gamma_p{}^2\theta_p-2\beta\gamma_p\theta_p)-(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)(-\eta_2\rho\gamma_p\theta_p+$$
$$\beta\gamma_p\theta_p)))((-2\eta_1(-\rho^2\gamma_p{}^2+\beta\gamma_p-\beta\gamma_p\theta_p)-(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)(-\rho\gamma_p{}^2\theta_p+\beta\gamma_p\theta_p))(-2(-1-\rho^2\gamma_s+\beta\gamma_s+\beta^2\theta_p+\beta\theta_p-\rho\beta\gamma_p\theta_p$$
$$-\rho\beta^2\gamma_p\theta_p+\rho\gamma_p\theta_p-\beta\gamma_p\theta_p-\beta\gamma_p\theta_p+2\beta\gamma_p\theta_p+\eta_1\beta\theta_s-\rho\beta\gamma_s\theta_s-\rho\gamma_s\theta_s-\rho\beta\gamma_s\theta_s-\beta\gamma_s\theta_s+\gamma_s\theta_s-\beta\gamma_s\theta_s-2\beta^2\gamma_s\theta_s)-\eta_1(-\rho\gamma_p+\beta\gamma_p$$

$$-\beta{\gamma_p}^2\theta_p)(-1-\rho\gamma_s+\beta\gamma_s+\beta\theta_p+8\rho\beta\theta_p-\beta^2\gamma_p\theta_p-\rho^2\gamma_p\theta_p+\rho\gamma_p\theta_p-\beta{\gamma_p}^2\theta_p-\beta\rho\gamma_p\theta_p+2\beta\gamma_p\theta_p+\beta\theta_s-\rho\gamma_s\theta_s-\rho^2\gamma_s\theta_s-\rho\gamma_s\theta_s$$

$$-\beta\gamma_s\theta_s-\beta\rho\gamma_s\theta_s+\beta\gamma_s\theta_s-2\beta\gamma_s\theta_s))/((\left(4-\eta_1\left(-\rho^2\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)^2\right)(-2\left(-\rho{\gamma_p}^2+\beta\gamma_p-\beta\gamma_p\theta_p\right)-\left(2\beta\theta_p-2\rho{\gamma_p}^2\theta_p-2\beta\gamma_p\theta_p\right)$$

$$\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right))-\left(-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho{\gamma_p}^2+\beta\gamma_p-\beta\gamma_p\theta_p\right)-2\eta_2\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\right)(-2\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)$$

$$-\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)))(\left(-2\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho{\gamma_p}^2\theta_p+\beta\gamma_p\theta_p\right)\right)^2-(4$$

$$-\left(2\beta\theta_p-2\rho{\gamma_p}^2\theta_p-2\beta\gamma_p\theta_p\right)^2)\left(4-\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)^2\right))-((-\left(2\beta\theta_p-2\rho^2\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho^2+\beta\gamma_p-\beta\gamma_p\theta_p\right)-2\eta_1(-\rho\gamma_p\theta_p+$$

$$\beta\gamma_p\theta_p))\left(-2\eta_2\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p4-\left(2\beta\theta_p-2\rho\gamma_p\theta_p\eta_1-2\beta\gamma_p\theta_p\right)^2\right)\right)) \tag{23}$$

$$\boldsymbol{s}_{\boldsymbol{w}(2)}^{(2)}=(-\left(-\rho{\gamma_p}^2\theta_p+\beta\gamma_p\theta_p\right)(-\eta_1-\rho\beta^2\gamma_s+\beta\gamma_s+\rho\beta\theta_p+\beta\theta_p-\rho\gamma_p\theta_p-\rho\gamma_p\theta_p+\rho\beta^2\gamma_p\theta_p-\beta\gamma_p\theta_p-\beta^2\gamma_p\theta_p+2\beta\gamma_p\theta_p+\beta^2\theta_s$$

$$-\rho\gamma_s\theta_s-\rho\beta^2\gamma_s\theta_s-\rho\gamma_s\theta_s-\eta_1\beta\gamma_s\theta_s-\beta^2\gamma_s\theta_s+\beta\gamma_s\theta_s-2\beta\gamma_s\theta_s)-2(-1-\rho\gamma_s+\beta^2\gamma_s+\beta{\gamma_p}^2\theta_p+\beta\theta_p-\rho\beta{\gamma_p}^2\theta_p-\eta_2\rho\gamma_p\theta_p+\rho\beta^2\gamma_p\theta_p$$

$$-\beta\gamma_p\theta_p-\beta^2\gamma_p\theta_p+2\beta\gamma_p\theta_p+\rho\beta\theta_s-3\rho\beta\gamma_s\theta_s-\rho^2\gamma_s\theta_s-6\rho\gamma_s\theta_s+\beta\gamma_s\theta_s-\beta\rho\gamma_s\theta_s-\eta_1\beta\gamma_s\theta_s-2\beta\gamma_s\theta_s))/(-2\left(-\rho\eta_2\gamma_p+\beta\gamma_p-2\beta\gamma_p\theta_p\right)$$

$$-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho{\gamma_p}^2\theta_p+3\beta\gamma_p\theta_p\right))-((-2\left(4\beta\theta_p-2\rho{\gamma_p}^2\theta_p-2\beta\gamma_p\theta_p\right)\left(-\left(-\rho\gamma_p+\beta{\gamma_p}^2-\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p+\beta\gamma_p\theta_p\right)\right)$$

$$\left(\left(-2\eta_1\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-\left(2\beta\theta_p-2\rho^2\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho{\gamma_p}^2\theta_p+\beta\gamma_p\theta_p\right)\right)\right)/((-\left(-\rho{\gamma_p}^2+\beta\gamma_p-\beta{\gamma_p}^2\theta_p\right)-(2\beta\eta_1\theta_p-2\rho\gamma_p\theta_p$$

$$-2\beta\gamma_p\theta_p)\left(-\rho\gamma_p\theta_p+\beta\rho^2\gamma_p\theta_p\right))(\left(4-\left(-\rho{\gamma_p}^2+\beta\gamma_p-\beta\gamma_p\theta_p\right)^2\right)(-2\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)(-\rho\gamma_p\theta_p$$

$$+\beta\gamma_p\theta_p))-\left(-\left(2\beta\theta_p\eta_2-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p+\beta{\gamma_p}^2\eta_1-\beta\gamma_p\theta_p\right)-2\eta_2\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\right)(-2\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta{\gamma_p}^2\theta_p\right)-$$

$$\left(-\rho\gamma_p+\beta\rho^2\gamma_p-\beta\gamma_p\theta_p\right)\left(-\rho{\gamma_p}^2\theta_p+\beta\gamma_p\theta_p\right))))\left(-2\eta_2\left(2\beta\rho^2\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)-\left(-\rho\gamma_p\eta_1+\beta\gamma_p-\beta\gamma_p\theta_p\right)\left(-\rho^2\gamma_p\theta_p+\beta{\gamma_p}^2\theta_p\right)\right)) \tag{24}$$

$$\boldsymbol{s}_{\boldsymbol{R}(1)}^{(1)}=\boldsymbol{s}_{\boldsymbol{R}(2)}^{(1)}=(-(\left(-\left(\beta\rho^2\theta_p-2\rho\gamma_p\theta_p-2\eta_2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-2\left(-\rho^2\theta_p+\beta\gamma_p\theta_p\right)\right)(-2\left(-\rho{\gamma_p}^2+\beta\rho^2\gamma_p-\beta\gamma_p\theta_p\right)-(2\beta\theta_p$$

$$-2\gamma_p\theta_p-2\beta\gamma_p\theta_p)\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right))-\left(4-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)^2\right)(-2\left(2\beta\theta_p-2\rho{\gamma_p}^2\theta_p-2\beta\gamma_p\theta_p\right)-\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)$$

$$\left(-\rho\gamma_p\theta_p+\beta\gamma_p\right)))(\left(-2\left(-\rho\gamma_p+\beta\gamma_p-\beta{\gamma_p}^2\theta_p\right)-\left(2\beta\rho\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p+\beta\gamma_p\theta_p\right)\right)(-2(-\eta_1-\rho\gamma_s+\beta\gamma_s+\beta^2\theta_p+\eta_1\beta\theta_p$$

$$-\beta^2\rho\gamma_p\theta_p-\rho\beta{\gamma_p}^2\theta_p+\rho^2\gamma_p\theta_p-\beta\gamma_p\theta_p-\beta{\gamma_p}^2\theta_p+2\beta\gamma_p\theta_p+\beta\theta_s-\rho\gamma_s\theta_s-\rho\gamma_s\theta_s-\rho\beta\gamma_s\theta_s-\beta\gamma_s\theta_s+\beta\rho^2\gamma_s\theta_s-\eta_2\beta\gamma_s\theta_s-2\beta\gamma_s\theta_s)$$

$$-\left(-\rho{\gamma_p}^2+\beta\gamma_p-\beta\gamma_p\theta_p\right)(-1-\rho^2\gamma_s+\beta\gamma_s+\beta\theta_p+\beta\theta_p-\rho^3\gamma_p-\rho\gamma_p\theta_p+\rho{\gamma_p}^2\theta_p-\beta\gamma_p\theta_p-\beta\gamma_p\theta_p+2\beta\gamma_p\theta_p+\beta\theta_s-\rho^2\gamma_s\theta_s-\rho\gamma_s\theta_s-$$

$$\rho\gamma_s\theta_s-\rho^2\beta\gamma_s\theta_s-\beta\gamma_s\theta_s+5\beta\gamma_s\theta_s-2\beta\gamma_s\theta_s))-\left(-\eta_2\left(2\beta\theta_p-\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-2\eta_2\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\right)$$

$$\left(-\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\left(-2-\rho\gamma_s+\beta\gamma_s+\eta_1\beta\theta_p\right)\right)))/(\left(4\rho^2\eta_2-\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)^2\right)(-2\left(-\rho\gamma_p+\beta\gamma_p-\beta\rho^2\gamma_p\theta_p\right)-(2\beta\theta_p-2\rho\gamma_p\theta_p$$

$$-2\beta\gamma_p\theta_p)\left(-\rho+\beta\gamma_p\theta_p\right))-\left(-\left(2\beta\theta_p-2\rho^2\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p+\beta\gamma_p-\beta\rho^2\gamma_p\theta_p\right)-2\left(-\rho^2\gamma_p\theta_p+\beta{\gamma_p}^2\theta_p\right)\right)(-2(2\beta\theta_p-2\rho{\gamma_p}^2\theta_p$$

$$-2\beta\gamma_p\theta_p)-\left(-\rho\gamma_p{}^2+\beta\gamma_p-\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)))(-\left(4-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)^2\right)(-\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)(-1+\rho^2\gamma_s+\beta\gamma_s$$
$$+\beta\theta_p+\beta\theta_p-\eta_1\rho\gamma_p\theta_p-\rho^2\gamma_p\theta_p+\rho\beta\gamma_p\theta_p-\beta\rho^2\gamma_p\theta_p-\beta\gamma_p\theta_p+2\beta\gamma_p\theta_p+\beta\theta_s-\rho\gamma_s\theta_s-\eta_2\rho\gamma_s\theta_s-\rho\gamma_s\theta_s-\beta\gamma_s\theta_s-\beta\gamma_s\theta_s+\beta\gamma_s\theta_s-$$
$$2\beta\gamma_s\theta_s)-2\eta_1(1-\rho\gamma_s+\beta\gamma_s+\beta\theta_p+\beta\rho\theta_p-\rho\gamma_p\theta_p-\rho^2\gamma_p\theta_p+\rho\beta\gamma_p\theta_p-\beta\gamma_p\theta_p-\eta_1\beta\gamma_p\theta_p+2\beta\gamma_p{}^2\theta_p+\beta\theta_s-\rho^2\theta_s-\eta_2\rho\gamma_s\theta_s-\rho\beta\gamma_s\theta_s$$
$$+\beta\rho^2\gamma_s\theta_s-\beta^2\gamma_s\theta_s-3\beta\gamma_s\theta_s-2\beta\gamma_s\theta_s)))$$

(25)

$$\mathbf{s}_{\boldsymbol{R}(1)}^{(2)}=\mathbf{s}_{\boldsymbol{R}(2)}^{(2)}=(-(\left(-\left(\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p{}^2+\beta\gamma_p-\beta\gamma_p\theta_p\right)-2\eta_2\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\right)(-2\left(-\rho\gamma_p+\beta-\beta\gamma_p\theta_p\right)-(2\beta\theta_p$$
$$-2\rho\gamma_p\theta_p-2\beta\gamma_p)\left(-1+\gamma_p\right))-\left(4-\eta_1\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)^2\right)(-2\left(-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)-\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)(-\rho\gamma_p\theta_p$$
$$+\beta\gamma_p\theta_p)))(\left(-2\left(-\rho\gamma_p+\beta-\beta\gamma_p\theta_p\right)-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\right)\right)(-2(-1-\rho\gamma_s+\beta\eta_1\gamma_s+\beta\theta_p+\beta\rho^2\theta_p-\rho\beta\gamma_p\theta_p$$
$$-\rho^2\gamma_p\theta_p+\rho\gamma_p\theta_p-\beta\gamma_p{}^2\theta_p-\beta\gamma_p\theta_p+2\beta\gamma_p{}^2\theta_p+\beta^2\theta_s-\rho\gamma_s\theta_s-\rho\eta_2\gamma_s\theta_s-\beta^2\gamma_s\theta_s+\beta\gamma_s\theta_s-2\beta\rho^2\gamma_s\theta_s)-\left(-\rho+\beta\gamma_p-\beta\gamma_p\theta_p\right)(-1$$
$$-\rho\gamma_s+\beta\gamma_s+\beta^2\theta_p+\beta\theta_p-\rho\gamma_p{}^2\theta_p-\beta\gamma_p\theta_p-\beta\rho^2\gamma_p\theta_p+\beta\theta_s-\rho\gamma_s\theta_s-2\beta\gamma_s))-(-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)$$
$$-2\left(-\rho\gamma_p{}^2\theta_p+\beta\gamma_p\theta_p\right))(-\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)(-1-\rho\gamma_s+\beta\gamma_s+\beta\theta_p+\beta\theta_p-\rho\beta^2\gamma_p\theta_p-\rho\eta_1\gamma_p\theta_p+\rho-\beta^2\gamma_p\theta_p+2\beta\gamma_p\theta_p+\beta\theta_s-\rho\gamma_s\theta_s$$
$$-\beta\gamma_s\theta_s+\beta\gamma_s\theta_s-2\beta\gamma_s\theta_s)-2\eta_1\left(\beta\gamma_s+\beta\theta_p-\rho^2\theta_p-\rho\gamma_p\theta_p+\rho^2\gamma_p-\beta\gamma_p\theta_p+2\beta\gamma_p{}^2\theta_p+\beta\theta_s+\gamma_s\theta_s-2\beta\gamma_s\theta_s\right)))/(((4-\eta_2$$
$$\left(-\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)^2)\left(-2\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\right)\left(-\rho\gamma_p{}^2\theta_p+\beta\gamma_p\theta_p\right)\right)-(-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)$$
$$\left(-\rho\gamma_p{}^2+\gamma_p-\beta\gamma_p\theta_p\right)-2\eta_1\left(-\rho^2\gamma_p\theta_p+\beta\gamma_p\right))\left(-2\left(2\beta\theta_p-\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)-\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\right))(((1$$
$$-\left(-\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)^3)\left(\left(-\rho\gamma_p+\eta_1\gamma_p-\beta\theta_p\right)-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\right)\left(-\rho\gamma_p{}^2\theta_p+\beta\right)\right)-(-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-\beta\gamma_p\theta_p\right)\left(\beta\gamma_p-\beta\gamma_p{}^2\theta_p\right)$$
$$-2\left(-\rho\gamma_p\theta_p+\beta\gamma_p\right))\left(-2\left(\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)-\eta_2\left(-\rho\gamma_p+\gamma_p-\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\right))$$
$$\left(\left(-2\left(-\gamma_p-\beta\gamma_p\theta_p\right)-\left(2\beta^2\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho+\beta\gamma_p\theta_p\right)\right)^2-\left(6-\left(2\beta\theta_p-2\rho\gamma_p{}^2\theta_p-2\gamma_p\theta_p\right)^2\right)\left(1-\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)^2\right)\right)$$
$$-(\left(-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-\gamma_p{}^2\theta_p\right)\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-2\left(-\rho\gamma_p\theta_p+\beta\theta\mathrm{p}\right)\right)(-2\left(-\eta_1-\beta\gamma_p{}^2\theta_p\right)-\left(2\beta\theta_p-\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)(-\eta_1\rho\gamma_p\theta_p$$
$$+\beta\gamma_p\theta_p))-\left(4-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-\gamma_p\theta_p\right)^2\right)\left(-2\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\gamma_p\theta_p\right)-\eta_2\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)\left(-\rho\theta_p+\beta\gamma_p{}^2\right)\right))))$$

(26)

$$\boldsymbol{w}_{\boldsymbol{d}(1)}=(((\left(2\eta_1\beta^2\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right))-(-\left(2\beta\theta_p-\gamma_p\theta_p-2\eta_2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p+\beta\gamma_p{}^2-\beta\gamma_p\theta_p\right)-2(-\rho\gamma_p{}^2\theta_p$$
$$+\beta\gamma_p\theta_p))\left(-2\left(\beta\theta_p-\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)-\left(-\rho\gamma_p-\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\right))$$
$$(\left(-2\left(-\rho\gamma_p{}^2+\beta\gamma_p\right)-\left(2\beta\theta_p-2\rho\gamma_p{}^2\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\eta_2\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\right)^2-\left(3-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-\beta\gamma_p\theta_p\right)^2\right)(4\theta_s\eta_1$$
$$-\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)^2))-(\left(-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-\gamma_p\theta_p\right)\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-2\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\right)(-2\left(\beta\gamma_p-\beta\gamma_p\theta_p\right)-(-2\rho\gamma_p\theta_p$$

$$-2\beta\gamma\mathrm{p})\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right))-\left(4-\left(2\beta\theta_p-2\rho\eta_2\gamma_p\theta_p-2\beta\gamma_p{}^2\theta_p\right)^2\right)(-2\left(\beta\theta_p-2\rho\gamma_p\theta_p-\beta\gamma_p\theta_p\right)-\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)(-\rho\gamma_p{}^2\theta_p$$

$$+\beta\gamma_p\theta_p)))(\left(-2\left(\beta\eta_1\gamma_p-\beta\gamma_p\theta_p\right)-\left(2\beta\theta_p-\rho\gamma_p\theta_p-\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\right)(-2\eta_1\left(2\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)-\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)$$

$$\left(-\rho\gamma_p+\gamma_p\theta_p\right))-\left(-\left(2\eta_2\beta\theta_p-\gamma_p\theta_p-\beta\gamma_p\right)\left(-\rho\gamma_p+\beta\gamma_p-\gamma_p\theta_p\right)-2\left(-\rho\gamma_p{}^2\theta_p+\beta\gamma_p\theta_p\right)\right)\left(4\eta_1-\left(-\gamma_p\theta_p+\beta\gamma_p\theta_p\right)^2\right)))/\eta_2(((-1-\rho\gamma_s$$

$$+\beta\gamma_s+\beta\theta_p+\gamma_p\theta_p-\rho\beta\gamma_p{}^2\theta_p+\beta^2\rho\gamma_p\theta_p-\beta^2\gamma_p\theta_p+2\beta\gamma_p\theta_p+\beta\theta_s-\rho\gamma_s\theta_s-\rho\beta^2\gamma_s\theta_s-\rho\gamma_s\theta_s-\rho\beta\gamma_s\theta_s-\beta\gamma_s+\beta^2\gamma_s\theta_s-2\beta\gamma_s\theta_s)$$

$$-2\left(-\beta\gamma_s+\beta^2\theta_p+\beta\rho^2\theta_p-\rho\gamma_p\theta_p-\beta\gamma_p{}^2-\beta\gamma_p\theta_p+\beta\theta_s-\rho^2\gamma_s\theta_s-\rho\beta\gamma_s-\beta\gamma_s\theta_s+\beta\eta_1\gamma_s\theta_s-\beta\gamma_s\theta_s-2\rho^2\beta\gamma_s\theta_s\right))))$$

(27)

$$\boldsymbol{w}_{\boldsymbol{d}(2)}=2(1-\rho^2\gamma_s+\beta\gamma_s+\beta\rho\theta_p+\beta\theta_p-\rho\eta_1\gamma_p\theta_p-\rho\gamma_p{}^2\theta_p+\rho\beta^2\gamma_p\theta_p-\gamma_p\theta_p-\beta\gamma_p\theta_p+2\beta\gamma_p\theta_p+\beta\beta\theta_s-\rho\gamma_s\theta_s-\rho\gamma_s\theta_s+\eta_2\beta\gamma_s\theta_s$$

$$-\beta\gamma_s\theta_s-\beta\rho^2\gamma_s\theta_s-2\beta\gamma_s\theta_s)))+(\left(4-\left(-\rho\eta_1\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)^2\right)(-2\left(\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)(-\rho\gamma_p\theta_p$$

$$+\beta\gamma_p\theta_p))-\left(-\left(2\beta\theta_p-\rho\gamma_p{}^2-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p{}^2-\beta\gamma_p+\beta\gamma_p\theta_p\right)-2\left(-\rho\gamma_p\theta_p+\beta\theta_p\right)\right)(-2\left(2\beta\theta_p-2\rho\gamma_p\theta_p-\beta^2\gamma_p\theta_p\right)-(-\rho\gamma_p+\beta\gamma_p$$

$$-\beta\gamma_p\theta_p)\left(-\rho\theta_p+\beta\gamma_p\theta_p\right)))(-\left(4\beta^2-\eta_2\left(\beta\theta_p-\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)^2\right)(-\eta_1\eta_2\left(-\beta\gamma_p{}^2\theta_p+\theta_p\right)(-\rho\gamma_s+\beta\theta_p-\rho\gamma_p\theta_p-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p-\beta\gamma_p\theta_p$$

$$-\beta\gamma_p\theta_p+2\beta\rho\gamma_p\theta_p+\beta\theta_s-\gamma_s\theta_s-2\rho\gamma_s\theta_s-\beta\gamma_s\theta_s-\beta\rho^2\gamma_s\theta_s+\beta\gamma_s\theta_s-2\beta\gamma_s\theta_s)-2(-\frac{1}{2}\eta_1-\rho\gamma_s-\beta\gamma_s+\beta\gamma_p\theta_p-\beta\gamma_p\theta_p+2\beta\gamma_p\theta_p+$$

$$\beta\theta_s-3\rho\beta^2\gamma_s\theta_s+\beta\gamma_s\theta_s-\beta\eta_2\gamma_s\theta_s-\beta\gamma_s\theta_s-2\beta\gamma_s\theta_s))/\left(-\rho\gamma_p{}^2+\beta\gamma_p-\beta\gamma_p\theta_p\right)(\beta\theta_p+\beta\theta_p-\rho\gamma_p\theta_p-\rho\gamma_p{}^2\theta_p+\rho\eta_2\gamma_p\theta_p+\beta\gamma_p\theta_p+\beta\theta_s$$

$$-\rho\gamma_s\theta_s-\rho\theta_s-\rho\beta\gamma_s\theta_s-\beta\gamma_s\theta_s-\beta\gamma_s\theta_s+\beta\gamma_s\theta_s-\beta\rho^2\gamma_s\theta_s))-(-\left(\beta\theta_p-\rho\gamma_p\theta_p-\beta\eta_1\gamma_p\right)\left(-\rho\gamma_p+\beta^2\gamma_p-\beta\gamma_p\theta_p\right)-2\eta_2(-\rho\gamma_p\theta_p$$

$$+\beta\gamma_p\theta_p))(-\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)(-\eta_2-\left(\beta\theta_p-\rho\gamma_p-2\beta\gamma_p\theta_p\right)^2+\beta\gamma_s+\beta\gamma_p{}^2\theta_p+\beta\theta_p-\rho\gamma_p\theta_p-\beta\gamma_p\theta_p-\beta\gamma_p{}^2\theta_p+2\beta\gamma_p\theta_p+\rho\beta^2\theta_s$$

$$-\rho\beta\gamma_s\theta_s-\rho^2\gamma_s\theta_s-\rho\gamma_s\theta_s-\beta^2\gamma_s\theta_s-\beta\eta_1\gamma_s\theta_s+\beta\gamma_s\theta_s-2\gamma_s\theta_s)$$

(28)

$$\boldsymbol{p}_{\boldsymbol{w}(1)}^{(1)}=(-(\left(-\left(2\beta\theta_p-2\rho\gamma_s\theta_s-2\beta^2\gamma_p\theta_p\right)\left(-\rho^2\gamma_p+\beta\gamma_p-\beta\gamma_p{}^2\theta_p\right)-2\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\right)$$

$$\left(-3\eta_2\left(-\rho\beta^2\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-\eta_2\left(2\eta_1\beta\theta_p-2\rho\gamma_p{}^2\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\theta_p+\beta\gamma_p\theta_p\right)\right)$$

$$\left(+\left(4-\left(2\beta\theta_p-\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)^2\right)\left(-2\left(2\beta\theta_p-\rho\beta^2\gamma_p\theta_p-\beta\gamma_p\theta_p\right)-\left(-\rho\gamma_p+2\beta^4\gamma_p-\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta^3\gamma_p\theta_p\right)\right)\right)$$

$$(\left(-2\eta_1\left(\rho\gamma_p+\beta\gamma_p-\beta\rho^2\gamma_p\theta_p\right)-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\gamma_p\right)\right)$$

$$((-2\left(\rho\gamma\mathrm{s}+\beta\gamma\mathrm{s}+\beta\eta_1\theta_p+\beta\theta_p-\rho\gamma_p\theta_p-\rho^3\gamma_p\theta_p+\rho\gamma_p{}^2\theta_p-\beta\gamma_p\theta_p-\beta\gamma_p{}^2\theta_p+2\beta\gamma_p\theta_p-\rho^2\beta\gamma_s\theta_s\right)$$

$$+\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)\left(-\eta_1-\rho\gamma_p\theta_p-\rho\beta^2\gamma_p\theta_p+\rho^2\gamma_p\theta_p-\beta\gamma_p\theta_p-\beta^3\gamma_p\theta_p+2\beta\gamma_p{}^2\theta_p+\beta\theta_s-\rho\gamma_s\theta_s-\rho^2\beta\gamma_s\theta_s-\rho\gamma_s\theta_s-\beta\gamma_s\theta_s\right))))$$

$$/\eta_2(\left(\left(2\beta\theta_p-2\rho\gamma_p\theta_p-\beta\gamma_p\theta_p\right)\left(-\rho\theta_p+\beta\gamma_p\theta_p\right)\right)-\left(-\left(\beta\theta_p-2\rho\gamma_p{}^2\theta_p-\gamma_p\theta_p\right)\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-2\left(-\rho\gamma_p{}^2\theta_p+\beta\gamma_p\theta_p\right)\right)$$

$$\left(-2\eta_1\left(2\beta\theta_p-\rho\gamma_p\theta_p-\beta\gamma_p\theta_p\right)-\left(-\rho\eta_1\gamma_p+\beta\gamma_p{}^2-\beta\gamma_p\theta_p\right)\left(-\rho^5\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\right))((\left(4-\left(-\rho\gamma_p{}^2+\beta\gamma_p-\beta\gamma_p\theta_p\right)^2\right)(-2(-\rho\beta^2\theta_s$$

$$+\beta\gamma_p{}^2-\beta\gamma_p{}^2\theta_p)-\left(2\beta\theta_p-\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\gamma_p\right))-(-\eta_2\left(2\beta\theta_p+2\eta_1\rho\gamma_p\theta_p-2\gamma_p{}^2\theta_p\right)\left(-\rho\eta_1\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)$$

$$-2\left(-\eta_2\gamma_p\theta_p+\beta^3\gamma_p\theta_p\right))\left(-2\left(2\beta\theta_p-2\rho\gamma_p\theta_p-\gamma_p\theta_p\right)-\left(-\rho^2\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)\left(-\rho\theta_p+\beta\gamma_p\theta_p\right)\right))$$

$$(\left(-2\eta_2\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-\left(\eta_1\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\right)^2-\left(4\eta_1-\left(2\beta^3\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)^2\right)(4$$

$$-\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)^3))-((\left(-\left(2\beta\theta_p-\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)+2\left(-\rho\eta_2+\beta\gamma_s\theta_s\right)\right)$$

(29)

$$\boldsymbol{p}_{w(2)}^{(2)}=(-\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)(-2\beta\eta_1-\rho\gamma_s+\beta\gamma_s+\beta^2\theta_p+\beta\theta_p-\rho^2\gamma_p\theta_p-\rho\beta\gamma_p\theta_p+\rho\gamma_p{}^2\theta_p-\beta\rho^2\gamma_p\theta_p-\beta\eta_2\gamma_p\theta_p+2\beta\gamma_p\theta_p+\beta\theta_s$$

$$-\rho\gamma_s\theta_s-\beta^2\gamma_s\theta_s+\beta\rho^2\gamma_s\theta_s-2\beta\gamma_s\theta_s)-2(\beta\gamma_s+\beta\theta_p+\beta\rho\theta_p-\rho\gamma_p\theta_p-\rho\gamma_p+\rho\gamma_p\theta_p-\beta\gamma_p\theta_p-\beta\gamma_p\theta_p+2\beta\gamma_p\theta_p+\beta\theta_s-\rho^4\gamma_s\theta_s$$

$$-\beta\gamma_s\theta_s-\beta\gamma_s\theta_s-2\beta\gamma_s\theta_s))+\left(-2\left(-\rho+\beta\gamma_p-\beta\gamma_p\theta_p\right)-\left(2\beta\eta_1\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p{}^4\theta_p+\beta\gamma_p\theta_p\right)\right)(-(2\beta\theta_p-2\rho\gamma_p\theta_p$$

$$-2\beta\eta_2\gamma_p\theta_p)(-1/2+\beta\gamma_s+\beta\theta_p+\beta\theta_p-\rho\gamma_p\theta_p-\rho\theta_p+2\rho\beta\gamma_p\theta_p-\beta\eta_2\gamma_p\theta_p-\beta\gamma_p{}^2\theta_p+2\beta\gamma_p\theta_p+\beta\eta_1\theta_s-\rho\gamma_s\theta_s-\rho\beta\gamma_s\theta_s-\rho\gamma_s\theta_s$$

$$-\beta\gamma_s\theta_s-\gamma_s\theta_s+\beta^3\gamma_s\theta_s-\beta\gamma_s\theta_s)-3(\rho\gamma_p\theta_p-\beta\gamma_p\theta_p-\beta\gamma_p\theta_p+2\beta\gamma_p\theta_p+\beta\theta_s-\rho\gamma_s\theta_s-\rho\theta_s-\rho\gamma_s-\beta\gamma_s\theta_s+\beta\gamma_s\theta_s-\beta\theta_s$$

$$-2\beta\gamma_s\theta_s)))/(\left(4-\left(-\rho\gamma_p+\beta^2\gamma_p-\beta\gamma_p\theta_p\right)^2\right)\left(-2\left(-\rho^2\gamma_p+\beta\gamma_p-\beta\gamma_p{}^2\theta_p\right)-\left(2\beta\eta_2\theta_p-\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p\right)\right)$$

$$-\left(-\left(2\eta_2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\left(-\rho\eta_1\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-2\left(-\rho\theta_p+\beta\alpha^3\gamma_p\theta_p\right)\right)(-2\left(2\eta_1\beta\theta\mathrm{p}-2\rho\gamma_p{}^2\theta_p-2\beta\gamma_p\theta_p\right)-(-\rho\gamma_p+\beta\gamma_p$$

$$-\beta^3\gamma_p\theta_p)\left(-\rho\theta_p+\beta\gamma_p\theta_p\right)))(-\eta_1\left(4-\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\right)^2\right)(-\eta_2\left(-\rho\gamma_p+\beta^2\gamma_p\theta_p\right)(-\beta\gamma_s-\rho\gamma_p\theta_p-\rho\theta_p+\rho\gamma_p-2\beta\gamma_p\theta_p-\beta\gamma_p\theta_p$$

$$+\beta\gamma_p\theta_p+\beta\theta_s-\rho\gamma_s\theta_s-\alpha\gamma_s-\rho\beta^2\gamma_s\theta_s-\beta\gamma_s\theta_s-\beta\gamma_s\theta_s+\beta\gamma_s\theta_s-2\beta\gamma_s\theta_s)-2\eta_1(\left(-2+\theta_p\left(-1+\beta\left(-3+\beta\theta_p\left(-2+\theta_p{}^2\right)\right)\right)\right)\left(1+3\rho^2\theta_s\right)$$

$$+\eta_1\gamma_p\left(2-2\left(-1+\theta_p\right)\eta_2\theta_p+\rho\beta\gamma_p\theta_p-\beta\gamma_p\theta_p-\beta\gamma_p\theta_p+2\beta\gamma_p\theta_p+\beta\theta_s-\rho\gamma_s\theta_s-\rho\gamma_S{}^2\theta_s-\rho\theta_s+\beta\gamma_s\theta_s-\beta^3\gamma_s\theta_s-\beta\gamma_s\theta_s-2\beta\gamma_s\theta_s\right))$$

$$+\left(-2\eta_2\left(-\rho\gamma_p+\beta^2\gamma_p-\beta\gamma_p\theta_p\right)-\left(2\beta\theta_p-2\rho\eta_1\gamma_p-\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\gamma_p{}^3\right)\right)$$

(30)

$$\boldsymbol{w}_{w(1)}=-(8-\rho^4\gamma_p{}^3\left(-1+3\theta_p\right)\left(\gamma_s+\gamma_p\theta_p+3\gamma_s\theta_s\right)+4\eta_1\beta\left(\gamma_p+2\gamma_s\left(-1+\theta_s\right)-2\left(2\theta_p+\theta_s\right)\right)-2\beta^2(\gamma_p{}^2\left(1+\theta_p+7\theta_p{}^2\right)+2\theta_p(\gamma_s+3\theta_p$$

$$+\theta_s-2\gamma_s\theta_s)+\gamma_p\left(\theta_p-13\theta_p{}^2+2\theta_s-4\theta_p\theta\mathrm{s}+2\gamma_s\left(-1+2\theta_p\right)\left(-1+2\theta_s\right)\right))+\beta^4\eta_2(2\theta_p{}^3\left(\gamma_s+2\theta_p+\theta_s-2\gamma_s\theta_s\right)-\gamma_p\theta_p{}^2\left(1+11\theta_p\right)(\gamma_s$$

$$+2\theta_p+\theta_s-2\gamma_s\theta_s)+\gamma_p{}^3\left(\left(1+2\theta_p\right)\left(1+\theta_p\left(-5+2\theta_p\right)\right)\left(2\theta_p+\rho\theta_s\right)+\gamma_s\left(1-2\theta_s+\theta_p\left(-3-2\theta_p\left(4+\theta_p\left(-2+\eta_2\theta_s\right)-6\theta_s\right)+6\theta_s\right)\right)\right)$$

$$+\gamma_p{}^2\theta_p\left(\left(-2+\theta_p\left(13+8\theta_p\right)\right)\left(2\eta_1\theta_p+\theta_s\right)+\eta_2\gamma_s\left(-2+4\theta_s+\theta_p\left(13+8\theta_p-23\eta_1\theta_s-21\theta_p\theta\mathrm{s}\right)\right)\right))-\beta^{3\eta_2}(\gamma_p{}^3\left(-1+\theta_p\right)(-1+4\theta_p$$

$$+6\theta_p{}^2)-4\theta_p{}^2\left(\gamma_s+2\theta_p+\rho-\gamma_s\theta_s\right)+\gamma_p\theta_p\left(6\theta_s+\theta_p\left(11+19\theta_p+10\theta_s\right)+2\gamma_s\left(3+5\theta_p-6\theta_s-4\theta_p\theta_s\right)\right)+\gamma_p{}^2(\theta_p(\theta_p\left(11-43\theta_p-20\theta_s\right)$$

$$+2\eta_1\left(-2+\theta_s\right))-2\theta_s+2\gamma_s\left(-1+\theta_s+\theta_p\left(1-10\theta_p+\theta_s+13\theta_p\theta_s\right)\right))))+\eta_2\rho^2\gamma_p(3\beta^2\gamma_p{}^3\theta_p\left(1+\theta_p\left(-3+\theta_p+8\theta_p{}^2\right)\right)+\gamma_s(-4+\theta_p(-12$$

$$+\beta\left(-6+\theta_p\left(4+\beta\left(-1+6\theta_p\right)\right)\right)))(1+3\theta_s)-\gamma_p(2+4\theta_p\left(-1+3\theta_p\right)-2\beta\eta_1(\theta_p\left(2+5\left(-3\eta_2+\theta_p\right)\theta_p-6\theta_s\right)+\theta_s+\gamma_s(3+5\theta_s+\theta_p(-13$$

$$+4\theta_p-9\theta_s+15\theta_p\theta_s)))+\beta^2\theta_p\left(2\theta_s+\theta_p\eta_2\left(4+\theta_p-12\theta_p{}^2-3\theta_p\theta_s\right)+\gamma_s\left(6+8\theta_s+\theta_p\left(-13+27\theta_p-42\theta_s+96\theta_p\theta_s\right)\right)\right))/(-16$$

$$+\rho^4\gamma_p{}^4\left(-1+\theta_p+7\theta_p{}^2+3\theta_p{}^3-6\theta_p{}^4\right)+\rho^2\gamma_p{}^2(2\beta\eta_1\theta_p\left(3+\left(9+\gamma_p\left(11-14\theta_p\right)\right)\theta_p\right)+4\left(2\eta_1+\theta_p+7\theta_p{}^2\right)+\beta^2(\theta_p{}^2\left(5+2\theta_p-34\theta_p{}^2\right)$$

$$+\gamma\mathrm{p}\theta_p{}^2\left(13+2\theta_p\left(-15\eta_2+56\theta_p\right)\right)-\gamma_p{}^2\left(9+\theta_p\left(-22+\theta_p\left(19+\theta_p+73\theta_p{}^2\right)\right)\right)))-\rho^3\gamma_p{}^3\eta_2(4\theta_p\left(2+5\theta_p\right)+\beta(\theta_p{}^2\left(11+6\left(1-4\theta_p\right)\theta_p\right)$$

$$+\gamma_p\left(-5+\theta_p\left(8+\theta_p\left(4+\theta_p\left(-26+41\theta_p\right)\right)\right)\right)))+\beta^2(16\theta_p{}^2-4\beta^2\theta_p{}^4+24\gamma_p\theta_p{}^2\left(-2+\beta^2\theta_p{}^2\right)+\beta^2\gamma_p{}^4\left(-2+\theta_p\left(9+\theta_p-22\theta_p{}^2+4\theta_p{}^3\right)\right)$$

$$+4\beta\gamma_p{}^3\theta_p{}^2\left(10\left(-1+\theta_p\right)+\beta\left(-4\eta_1+\theta_p\left(9+4\theta_p\right)\right)\right)+\gamma_p{}^2\left(12+\theta_p\left(-24+\theta_p\left(52+\beta\left(28-32\theta_p-3\beta\left(-2+\theta_p\left(5+13\theta_p\right)\right)\right)\right)\right)\right))$$

$$+\rho\beta\gamma_p(20\theta_p{}^2\left(-2+\beta^2\theta_p{}^2\right)+\beta^2\gamma_p{}^3\left(7-\theta_p\eta_1\left(24+\theta_p\left(-15+2\theta_p\left(8+17\theta_p\right)\right)\right)\right)+\beta\gamma_p{}^2\theta_p(8\eta_2+\theta_p(22+4\theta_p+\beta(14+\theta_p(7$$

$$+113\theta_p))))+\gamma_p\left(-20+\theta_p\left(16+44\theta_p-\beta\left(6+\theta_p\left(40-26\theta_p+\beta\left(11+2\theta_p\left(-6+47\theta_p\right)\right)\right)\right)\right)\right))$$

(31)

$$\boldsymbol{w}_{w(2)}=-(\rho^4\gamma_p{}^3\theta_p\left(1+\theta_p\left(-5+2\theta_p\right)\right)\left(\eta_1\gamma_s+\gamma_p\theta_p+3\gamma_s\theta_s\right)+\rho^3\gamma_p{}^2(\theta_p\left(\gamma_p\left(-1+6\theta_p{}^2\right)+2\gamma_s\left(3+2\theta_p\right)\left(1+3\theta_s\right)\right)+\beta(\gamma_p{}^2\theta_p(-2+\theta_p(3$$

$$+\theta_p\left(-5+9\theta_p\right)))-\gamma_s\left(-1+\theta_p\right)\eta_2\theta_p\left(-1+8\theta_p\right)\left(1+3\theta_s\right)+\gamma_p(\gamma_s\left(-2+2\theta_p+7\theta_p{}^3+\left(-6+\theta_p\left(12+\theta_p\left(-23+27\theta_p\right)\right)\right)\theta_s\right)-\eta_1\theta_p(\theta_s$$

$$+\theta_p\left(3-5\theta_s+\theta_p\left(-19\eta_2+12\theta_p+2\theta_s\right)\right)))))+\rho^2\gamma_p(\beta^2\gamma_p{}^3\theta_p\left(4+\theta_p\left(-6+\theta_p\left(3+16\theta_p\right)\right)\right)+2\gamma_s(-2+\theta_p(-6+\beta(-3+\beta\theta_p(-2$$

$$+5\theta_p))))\left(\eta_2+3\theta_s\right)+\gamma_p(2-4\left(-1+\theta_p\right)\theta_p+\eta_2\beta^2\theta_p(26\theta_p{}^3+\theta_s-6\gamma_s\theta_s+\theta_p\left(2-6\gamma_s-9\theta_s+25\gamma_s\theta_s\right)-\theta_p{}^2(22+17\gamma_s-8\theta_s$$

$$+79\gamma_s\theta_s))+\beta\left(\theta_p+2\gamma_s\left(2-3\theta_p\left(2+\theta_p\right)+5\theta_s+\left(-4+\theta_p\right)\theta_p\theta_s\right)-2\theta_p\left(3\theta_s+2\theta_p\left(2+4\theta_p+\theta_s\right)\right)\right))+\beta\gamma_p{}^2(\left(-2+3\theta_p\right)(1+2(-1$$

$$+\theta_p)\theta_p)+\beta(2\theta_s+\theta_p\left(4-3\theta_s+\theta_p\left(-7+\theta_p\left(13-43\theta_p-9\theta_s\right)+5\theta_s\right)\right)+\gamma_s(6\eta_1+8\theta_s+\theta_p(-9-13\theta_s+\eta_1\theta_p(8+7\theta_p+\theta_s$$

$$+48\theta_p\theta_s))))))-\beta\left(2\beta^2\gamma_p{}^3\left(\eta_1-\theta_p\left(1+\theta_p\right)\left(1+2\theta_p\right)+\beta\left(\gamma_s+2\theta_p\right)\left(-1+\theta_p+6\theta_p{}^2\right)\right)\right)/\beta\eta_2(-1+\theta_p+6\theta_p{}^2+\gamma_s(2+\theta_p(-4+\theta_p(-9$$

$$+2\theta_p))))\theta_s)-4\left(-2+\beta^2\theta_p{}^2\right)\left(\theta_p+\theta_s+\beta\theta_p\left(2\theta_p+\theta_s\right)+\gamma_s\left(1+\beta\theta_p-\eta_1\left(2+\beta\theta_p\right)\theta_s\right)\right)+\beta\gamma_p{}^2(2\theta_p\left(1+2\theta_p\right)+\beta^2\theta_p(-\left(\gamma_s+2\theta_p\right)(-2$$

$$+3\theta_p\left(5+4\theta_p\right))+\left(2-3\theta_p\left(5+4\theta_p\right)+\gamma_s\left(-2+\theta_p\left(27+7\theta_p\right)\right)\right)\theta_s)+\beta(-4\theta_s+\theta_p\left(-10+\theta_p\left(19-17\theta_p-16\theta_s\right)+4\eta_1\theta_s\right)+2\gamma_s(-2$$

$$+4\theta_s+\theta_p\left(2-8\theta_p-\theta_s+14\theta_p\theta_s\right))))+2\gamma_p(-2+4\theta_p+\beta(2\theta_s+\theta_p(1+\theta_p\left(-9+\beta\left(4+\theta_p\left(7+4\beta+14\beta\theta_p\right)\right)\right))-4\theta_s+\beta(3+\theta_p(7+2\beta$$

$$+7\beta\theta_p))\theta_s)+\gamma_s\left(2-4\theta_s+\eta_2\theta_p\left(4\left(-1+\theta_s\right)+\beta\left(3-6\theta_s+\theta_p\left(7+2\beta+7\beta\theta_p-2\eta_2\left(7+2\beta+3\beta\theta_p\right)\theta_s\right)\right)\right)\right)))+\rho\eta_1(2\beta^3\gamma_p{}^4\theta_p(-1+\theta_p$$

$$+6\theta_p{}^2)-4\gamma_s\left(1+\beta\theta_p\right)\left(-2+\beta^2\theta_p{}^2\right)\left(1+3\theta_s\right)+\beta\gamma_p{}^2(-2+\beta(-4\theta_s+\theta_p\left(-11+6\theta_s+2\theta_p\left(9-3\theta_p+\theta_s\right)\right))-2\gamma_s(4+\theta_s+\theta_p(-5+7\theta_p$$

$$-3\theta_s\eta_1+27\theta_p\theta_s)))+\beta^2\theta_p(\theta_s+\theta_p(2-3\theta_s+\theta_p(-2+64\theta_p+25\theta_s))+\gamma_s(3+7\theta_s-\theta_p(18-13\theta_p+39\theta_s+52\theta_p\theta_s))))+\beta^2\gamma_p{}^3(4$$
$$+\theta_p(-8+\theta_p(9+\theta_p))+\beta(-4\theta_s-\theta_p(8\eta_1-6\theta_s+\theta_p(-14+3\theta_s+\eta_2\theta_p(21+44\theta_p+16\theta_s)))$$

(32)

$$\boldsymbol{p}_{\boldsymbol{R}(1)}^{(1)}=\eta_1\gamma_s(2(-3+\theta_s)+\theta_p(8-10\theta_s+\theta_p(9-16\theta_p+41\theta_s+13\theta_p\theta_s)))))+2\eta_2\gamma_p(-2(1+\theta_p)+\beta(2\theta_s+\theta_p(1+\theta_p(12+\beta(4+\theta_p(3$$
$$+4\beta-12\beta\theta_p)))+6\theta_s+\beta(3+\beta(2-5\theta_p)\theta_p)\theta_s)+\gamma_s(2(2+\theta_s)+\theta_p(2-20\theta_s+\beta(3\eta_1(2+\theta_s)+\theta_p$$
$$(7+21\theta_s+\beta(4+2\theta_p+2\theta_s+25\theta_p\theta_s))+\rho\beta\gamma_p(20\rho^3\theta_p{}^2(-2+\beta^2\theta_p{}^2)+\eta_2\beta^2\gamma_p{}^3(7-\theta_p(24+\theta_p(-15+2\theta_p(8+17\theta_p))))+$$
$$\beta\gamma_p{}^2\theta_p(8+\theta_p(22+4\theta_p+\beta(14+\theta_p(7+113\theta_p)))))+\gamma_p(-20+\theta_p(16+44\theta_p-\beta(6+\theta_p\eta_1(40-26\theta_p+\beta(11+2\theta_p\eta_1(-6$$
$$+47\theta_p)))/(-6+\rho^4\gamma_p{}^4(-1+\theta_p+7\eta_1\theta_p{}^2+3\theta_p{}^3-6\theta_p{}^4)+\rho^2\gamma_p{}^2(2\beta\theta_p(3+\eta_2(9+\gamma_p(11-14\theta_p))\theta_p)+4(2+\theta_p+7\theta_p{}^2)+\beta^2(\theta_p{}^2(5$$
$$+2\theta_p-34\theta_p{}^2)+\gamma_p\theta_p{}^2(13+2\theta_p(-15+56\theta_p))-\gamma_p{}^2(9+\theta_p(-22\eta_1+\theta_p(19+\theta_p+73\theta_p{}^2)))))-\rho^3\gamma_p{}^3(4\theta_p(2+5\theta_p)+\beta\eta_2(\theta_p{}^2(11$$
$$+6(1-4\theta_p)\theta_p)+\gamma_p(-5+\theta_p(8+\theta_p(4+\theta_p(-26+41\theta_p)))))))+\beta^2(16\theta_p{}^2-4\beta^2\theta_p{}^4+24\gamma_p\theta_p{}^2(-2+\beta^2\theta_p{}^2)+\beta^2\gamma_p{}^4(-2+\theta_p(9\eta_2$$
$$+\theta_p-22\theta_p{}^2+4\theta_p{}^3))+4\beta\gamma_p{}^3\theta_p{}^2(10(-1+\theta_p)+\beta(-4+\theta_p(9+4\theta_p)))+\gamma_p{}^2(12+\theta_p(-24+\theta_p(52+\beta(28-32\theta_p-3\beta(-2\eta_1+\theta_p(5$$
$$+13\theta_p)))))))$$

(33)

$$\boldsymbol{p}_{\boldsymbol{R}(1)}^{(2)}=(-((4-(-\rho^2\gamma_p{}^2+\beta\gamma_p-\beta\gamma_p\theta_p{}^2)(-\rho\gamma_p+2\beta\gamma_p-\beta\gamma_p\theta_p))$$
$$(-2(-\rho\eta_1\gamma_p+\beta^2\gamma_p-2\beta\gamma_p\theta_p)-(\beta\theta_p{}^2-2\rho\gamma_p\theta_p-2\beta\gamma_p{}^2\theta_p)(-\rho\eta_2\gamma_p+\beta\gamma_p\theta_p))$$
$$-(-2\eta_1\eta_2(\beta^2\theta_p-\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)-(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)(-\rho\gamma_p{}^2+\beta\gamma_p\theta_p))$$
$$(-(\beta\theta_p-2\rho^3\gamma_p\theta_p{}^2-2\beta\gamma_p\theta_p)(-\rho\gamma_p+2\beta\gamma_p-\beta\gamma_p\theta_p)-2(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p))$$
$$((-2\eta_2(-\rho\gamma_p+\beta\gamma_p-2\beta\gamma_p\theta_p)-(\beta\theta_p-2\rho^2\gamma_p\theta_p-2\beta\gamma_p\theta_p)(-\rho\gamma_p+\beta\eta_1\gamma_p\theta_p{}^2)).$$
$$(-2(-\rho\gamma_s+\beta^2\gamma_s+2\beta\theta_p-\rho\eta_1\gamma_p\theta_p+\beta\theta_s-3\gamma_s\theta_s-2\beta\gamma_s\theta_s)-(2\beta\theta_p-2\rho\gamma_p{}^2\theta_p-2\beta\gamma_p\theta_p)))$$
$$-(6-(\beta\eta_2\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p{}^2\theta_p)(2\beta^2\theta_p{}^2-2\rho\theta_p-2\beta\gamma_p\theta_p))$$
$$(-\rho\gamma_p\theta_p{}^2+\beta\theta\mathrm{s}-3\eta_2\rho\gamma_s\theta_s-\beta\gamma_s\theta_s)-(8\eta_1-(\beta 2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)(2\beta\theta_p{}^2-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p))$$
$$((-2(\beta\theta_p-\rho\gamma_p{}^2\theta_p-2\beta\gamma_p\theta_p)-(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)(-\rho^2\gamma_p+\beta\gamma_p\theta_p)))).$$

$$/(-2\eta_2(-\rho\gamma_p+\beta\gamma_p-2\beta\eta_1\gamma_p\theta_p^2)-(\beta\theta_p-2\rho\gamma_p\theta_p-2\beta^2\gamma_p\theta_p)(-\rho\gamma_p+\beta\gamma_p\theta_p))(-\eta_1(2\beta^2\theta_p-2\rho^2\gamma_p\theta_p-2\beta\gamma_p\theta_p^2)(-\rho\gamma_p^2$$
$$+\beta\eta_1\gamma_p-\beta\gamma_p\theta_p)-2(-\rho\gamma_p^2\theta_p+\beta\gamma_p\theta_p)))((-2\eta_2(-\rho\gamma_p+\beta\gamma_p-2\beta\gamma_p\theta_p)-(\beta^2\theta_p-2\rho\gamma_p^2\theta_p-2\beta\gamma_p\theta_p)(-\rho\gamma_p$$
$$+\beta\gamma_p\theta_p))(-2(-1-\rho\gamma_s+\beta\gamma_s+2\beta\theta_p-\rho^2\gamma_p\theta_p+\beta\theta_s-3\rho\gamma_s\theta_s-2\beta\gamma_s\theta_s)-\eta_1(-\rho\gamma_p+2\beta\gamma_p-\beta\theta_p^2)(-\eta_2-\rho\gamma_s+\beta^2\gamma_s$$
$$+2\beta\theta_p-\rho\gamma_p\theta_p^2+\beta\theta_s-3\rho^2\gamma_s\theta_s-\beta^2\gamma_s\theta_s))-(-(\beta\theta_p-2\rho\gamma_p\theta_p-2\beta^2\gamma_p)(-\rho\gamma_p+2\beta\gamma_p-\beta\gamma_p\theta_p)-2(-\rho\gamma_p\theta_p^2$$
$$+\beta\gamma_p))(-2\eta_1(-1-\rho\gamma_s+\beta\gamma_s+2\beta\theta_p-\rho\gamma_p^2\theta_p+\beta\eta_2\theta_s-3\rho\gamma_s\theta_s-2\beta\gamma_s\theta_s)-(-\rho^2\gamma_p+\beta\gamma_p^2\theta_p)(-1-\rho\eta_1\gamma_s+\beta\gamma_s+2\beta\theta_p.$$
$$-\rho^2\gamma_p\theta_p+\beta^2\theta_s-3\rho\gamma_s\theta_s))))$$

(34)

$$\boldsymbol{p}_{R(2)}^{(1)}=(-((-(-\rho+\beta\gamma_p-\beta\gamma_s\theta_s)(-\rho\gamma_p+\gamma_p-\beta^2\gamma_p\theta_p))(-2(-\rho\gamma_p+\beta\gamma_p-2\beta\gamma_p\theta_p)-\eta_1(\beta\theta_p-2\rho\gamma_p\theta_p-\beta^2\gamma_p\theta_p)(-\rho\gamma_p+\beta\gamma_p\theta_p))$$
$$-(-2\eta_1(\beta^2\theta_p-\rho\gamma_p\theta_p-\beta\gamma_p^2\theta_p)-(-\rho+\beta\gamma_p-\beta\gamma_p\theta_p)(-\rho\gamma_p+\beta\gamma_s\theta_s))(-(\beta\theta_p-\rho^2\gamma_p\theta_p-2\beta\gamma_p\theta_p)(-\rho\gamma_p+2\beta\gamma_p-\beta\gamma_p^2\theta_p)-2(-\rho$$
$$+\beta\gamma_s\theta_p)))((-2(-\rho\eta_2\gamma_p+\beta^2\gamma_p-2\beta\gamma_p\theta_p)-(\beta\theta_p-2\rho\gamma_p\theta_p-\beta\gamma_s\theta_s)(-\rho\gamma_p+\beta\gamma_p\theta_p))(-2\eta_2(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)-(2\beta\theta_p-2\rho\gamma_p\theta_p$$
$$-2\beta\gamma_s\theta_s)(-\eta_1\rho\gamma_p\theta_p+\beta\theta_p))-(4\eta_2-(\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_s\theta_s)(2\beta\theta_p-2\rho\gamma_p-2\beta^2\gamma_p\theta_p))(4-(-\rho^2\gamma_p+\beta\theta_p)(-\rho\gamma_p\theta_p+\beta\gamma_p^2)))$$
$$+(-(4-(\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\theta_p)(2\beta^2-2\rho^2\gamma_p^2\theta_p-2\beta\gamma_p^2\theta_p))(-2(\beta\theta_p-\rho^2\gamma_p\theta_p-2\beta\gamma_p\theta_p)-(-\rho\gamma_p+\beta-\beta\gamma_p\theta_p)(-\rho\gamma_p+\beta^2\theta_p))$$
$$+(-2(-\rho\gamma_p+\beta\gamma_p-2\beta\theta_p)-(\beta^2\theta_p-2\rho\gamma_s\theta_s-\eta_1\gamma_p\theta_p)(-\rho\gamma_p+\beta\gamma_p\theta_p))(-(2\beta^2\theta_p-2\rho\eta_1\gamma_p\theta_p-\beta\gamma_p\theta_p)(-\rho\gamma_p+\gamma_p-\beta\gamma_p\theta_p)$$
$$-2(-\rho\gamma_s\theta_s+\beta\gamma_p)))((-2\eta_1(-\rho^3\gamma_p+3\beta\gamma_p-\beta\gamma_p\theta_p)-(\beta\theta_p-\rho\gamma_p-\beta\gamma_p\theta_p)(-\rho^2\gamma_p+\beta\gamma_p\theta_p))$$
$$(-2\eta_2(2\beta\theta_p-\gamma_p^2\theta_p-2\beta\gamma_p\theta_p)-(-\rho\gamma_p+2\beta\gamma_p^2-\beta\gamma_p\theta_p)\eta_1(-\rho\gamma_p\theta_p+\beta))+(-(\beta\theta_p-2\rho\gamma_p\theta_p-2\gamma_p\theta_p)(-\rho\gamma_p+4\gamma_p-\beta\gamma_p\theta_p)$$
$$-2(-\rho^2+\beta\gamma_p\theta_p))(4-(-\rho\eta_2\gamma_p+\beta\gamma_p\theta_p)(-\rho\gamma_p\theta_p+\beta\gamma_p\theta_p))))$$
$$/(((-2(-\gamma_p^4+\beta\gamma_p-2\beta\theta_s)-\eta_1\eta_2(\beta^2\theta_s-2\rho\gamma_p\theta_p-2\gamma_p\theta_p)(-\rho\gamma_p+\beta\gamma_p\theta_p))$$
$$(-2(-\rho^2\gamma_p+\beta\gamma_p^2-\beta\gamma_p\theta_p)-(2\beta\theta_s-2\rho\gamma_p^2\theta_p-2\beta\gamma_p\theta_p)(-\rho\gamma_p\theta_p^2+\beta^2\gamma_p\theta_p))$$
$$-(4-(\beta^2\theta_s-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)(2\beta\theta_s-2\rho^2\gamma_p\theta_p-2\beta\gamma_p\theta_p)))$$
$$(-(\beta\theta_s-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)(-\rho\gamma_p+2\beta\gamma_p-\beta\gamma_p\theta_p)-2(-\rho\gamma_p\theta_p+\beta\gamma_p^2\theta_p)))((-2\eta_2(-\rho\gamma_p+\beta\gamma_p^2-2\beta\gamma_p\theta_p)-(\beta\theta_s-2\rho\gamma_p\theta_p$$
$$-2\beta^2\gamma_p)(-\rho\gamma_p+\beta\gamma_p\theta_p))(-2(-\rho^2\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p)-\eta_1(2\beta\theta_s-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)(-\rho\theta_s+\beta\gamma_p\theta_p))-(4\eta_1-(\beta\theta_s-2\rho\gamma_p\theta_p^2$$
$$+2\beta\gamma_p\theta_p)(2\beta^2\theta_s-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p))(4-\eta_2(-\rho\gamma_p+\beta\gamma_p\theta_p)(-\rho^2\gamma_p\theta_p+\beta\gamma_p)))+(-(4-(\beta\theta_s-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)(6\beta\theta_s$$
$$-2\rho\gamma_p\theta_p-\gamma_p^2\theta_p))(-2(\beta\theta_s-\eta_1\gamma_p\theta_p-2\beta\gamma_p\theta_p)-(-\rho(\gamma_p^2+\beta^2\gamma_p-\gamma_p\theta_p)(-\rho\gamma_p+\beta\gamma_p\theta_p))+\ -2(-\rho\gamma_p+\beta\gamma_p^4-2\beta\gamma_p\theta_p)$$
$$-(\beta\eta_1\theta_s-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p)(-\rho^2\gamma_p+\beta\theta_s))(-(2\beta\eta_2\theta_s-2\rho\gamma_p^2\theta_p-2\gamma_p\theta_p)(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p^2)-2(-\rho\gamma_p\theta_p$$
$$+\beta^2)))((-2(-\rho\gamma_p+\beta\gamma_p-2\beta\gamma_p^2\theta_p)-(\beta\theta_s-2\rho\gamma_p\theta_p-2\gamma_p\theta_p^2)(-\rho\gamma_p+\beta\gamma_p\theta_p))(-2(2\beta\theta_s-\eta_1\gamma_p\theta_p-2\beta\gamma_p\theta_p)-(-\rho\gamma_p$$

$$+2\beta\gamma_p^{\,4}-\beta\gamma_p\theta_p)\left(-\rho\eta_1\gamma_p\theta_p+\beta\gamma_p\right))-\left(-\left(\beta\theta_s-2\rho\gamma_p\theta_p-2\beta^2\gamma_p\theta_p\right)\left(-\rho^2\gamma_p+2\beta\gamma_p-\beta\gamma_p\theta_p\right)-2\eta_2\left(-\rho+\beta\gamma_p\theta_p\right)\right)(4-(-\rho\gamma_p^{\,2}$$

$$+\beta\gamma_p\theta_p)\left(-\rho\gamma_p\theta_p+\beta^2\theta_p\right))))$$

(35)

$$p_{R(2)}^{(2)}=\left(4-\left(-\rho^2\gamma_p+\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p\theta_p+\beta\gamma_s\theta_s\right)\right))(-(\left(4\eta_2-\left(-\rho\gamma_p+\beta 2\gamma_p^{\,2}-\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_s\theta_s\right)\right)(-2\left(-\rho\gamma_p+\beta\gamma_p-2\beta\gamma_s\theta_s\right)$$

$$-\left(\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_s\theta_s\right)\left(-\rho^2\gamma_p+\beta\gamma_s\theta_s\right))-\left(-2\left(\beta\theta_p^{\,2}-\rho\gamma_p\theta_p-2\beta(\gamma_p^{\,2}\theta_p\right)-\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p+\beta\gamma_s\theta_s\right)\right)-(\beta\eta_1\theta_p$$

$$-2\rho\gamma_s\theta_s-2\beta\gamma_p\theta_p^{\,2})\left(-\rho\gamma_p+2\beta\gamma_p-\beta\gamma_s\theta_s\right)-2\left(-\rho\gamma_s\theta_s+\beta\gamma_s\theta_s\right)))((-2\left(-\rho\gamma_p+\beta\gamma_p-2\gamma_p\theta_p\right)-\left(\beta\theta_p-\gamma_p\theta_p-2\beta\gamma_s\theta_s\right)(-\rho^2\gamma_p$$

$$+\beta\gamma_s\theta_s))(-2\left(-\rho\gamma_s+\beta\gamma_s+2\beta\theta_p-\rho\gamma_p\theta_p+\beta\theta_s-3\eta_1\rho\gamma_s\theta_s-2\beta\gamma_s\theta_s\right)-\left(4-\left(\beta\theta_p-2\rho\gamma_s\theta_s-\gamma_s\theta_s\right)\left(2\beta\theta_p-2\rho\gamma_s\theta_s-2\beta\theta_p\right)\right)(-2(-1$$

$$-\rho\gamma_s+\beta\gamma_s+2\beta^2\theta_p-\rho 3\gamma_p^{\,2}\theta_p+\beta\theta_s-\rho\gamma_s\theta_s-2\beta\eta_2\gamma_s\theta_s)-\left(-\rho\gamma_p+\beta\gamma_p\theta_p\right)\left(-\eta_1-\rho\gamma_s+2\beta\theta_p-\rho\gamma_p\theta_p+\beta\theta_s-3\rho^2\gamma_s\theta_s-\beta\gamma_s\theta_s\right)))$$

$$+(-\eta_2\left(4-\left(\beta^2\theta_p-2\rho\gamma_s\theta_s-2\beta\gamma_p\theta_p\right)\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_s\theta_s\right)\right)(-2\left(\beta\theta_p-\rho\gamma_p\theta_p-2\beta\gamma_s\theta_s\right)-\left(-\rho(\gamma_p^{\,2}+\beta\gamma_p-\beta\gamma_p\theta_p\right)-\rho\gamma_p$$

$$+\beta\gamma_p\theta_p))+\left(-\rho^2\eta_2\gamma_p+\beta\gamma_p-\beta\gamma_s\theta_s\right)-2\left(-\rho\gamma_p\theta_p+\beta\gamma_p\right)))((-2\eta_2\left(-\rho\gamma_p+\beta\gamma_p-2\beta\gamma_p\theta_p\right)-\left(\beta\theta_p-2\gamma_p\theta_p-2\beta\gamma_s\theta_s\right)(-\rho^2\gamma_p$$

$$+\beta\gamma_p\theta_p))(-2\left(-1+\beta\gamma_s+2\beta\theta_p^{\,2}-\rho\eta_1\gamma_p\theta_p+\beta\theta_s-3\rho\gamma_s\theta_s-2\beta\gamma_s\theta_s\right)-\left(-\rho\gamma_p+2\beta\eta_1\gamma_p-\beta^2\gamma_p\theta_p\right)(-1-\rho^2\gamma_s+\beta\gamma_s+2\beta\eta_1\theta_p-\rho\gamma_p\theta_p$$

$$+\beta\theta_s-3\rho\gamma_s\theta_s-\beta^2\gamma_s\theta_s))-\eta_1\left(-\left(\beta\theta_p-2\rho^2\gamma_p\theta_p-2\beta\gamma_s\theta_s\right)\left(-\rho\eta_2\gamma_p+2\beta 2\gamma_p^{\,2}-\beta\gamma_s\theta_s\right)-\eta_1\left(-\rho\gamma_s\theta_s+\beta\theta_p\right)\right)/((-(4-(\beta^2\theta_p-2\rho\eta_2\theta_p$$

$$-2\beta\gamma_s\theta_s)\eta_2\left(2\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_p\theta_p\right))\left(-2\left(\beta\theta_p-\rho^2\gamma_s\theta_s-2\beta\gamma_p\theta_p\right)-\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_s\theta_s\right)\left(-\rho\gamma_p+\beta(\gamma_p^{\,2}\theta_p\right)\right)+-2(-\rho\gamma_p+\beta^2\gamma_p$$

$$-2\beta\gamma_s\theta_s)-\left(-\rho\gamma_p^{\,2}+\beta\gamma_p\theta_p\right)-\left(-2\left(\beta\theta_p-\rho^2\gamma_s\theta_s-2\beta\gamma_p\theta_p\right)-\left(-\rho\gamma_p+\beta\gamma_p-\beta\eta_1\gamma_p\theta_p\right)\left(-\rho^2\gamma_p+\beta\gamma_p\theta_p\right)\right)(-\eta_1(\beta\theta_p^{\,2}-2\rho\gamma_p\theta_p$$

$$-2\beta\gamma_p\theta_p)\left(-\rho\gamma_p+2\beta\gamma_p-\beta\eta_1\gamma_s\theta_s\right)-2\left(-\rho\gamma_s\theta_s+\beta\right)))((-2\eta_2\left(-\rho^2\gamma_p+\beta\gamma_p-2\beta\gamma_p\theta_p\right)-\left(\beta\theta_p-2\rho\gamma_p\theta_p-2\beta\gamma_s\theta_s\right)(-\rho\gamma_p$$

$$+\beta\gamma_s\theta_s))\left(-2\left(-\rho\gamma_p+\beta^2\gamma_p-\beta\gamma_s\theta_s\right)-\left(2\beta\theta_p-2\gamma_s\theta_s-2\beta\gamma_s\theta_s\right)\left(-\rho+\beta\gamma_p\theta_p\right)\right)-(4-\left(\beta\theta_p^{\,2}-2\rho\gamma_p\theta_p-2\beta\gamma_s\theta_s\right)(2\beta\eta_1\theta_p-2\rho\gamma_s\theta_s$$

$$-2\gamma_s\theta_s))\left(4-\eta_2\left(-\rho\gamma_p+\beta\theta_p\right)\left(-\rho\gamma_s\theta_s+\beta\gamma_p\theta_p^{\,2}\right)\right))+(-\left(4\eta_1-\left(\beta^2\theta_p-\gamma_p\theta_p^{\,2}-\gamma_p\theta_p\right)\left(2\beta\theta_p-2\rho^2\gamma_p\theta_p-2\beta\gamma_p\theta_p\right)\right)(-2(\beta\theta_p-\gamma_p\theta_p$$

$$-2\beta\gamma_p\theta_p)-\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p^{\,2}\right)\left(-\rho\gamma_p+\beta\gamma_p^{\,2}\theta_p\right)+(-2\left(-\rho\gamma_p+\beta\gamma_p-2\beta^2\gamma_s\theta_s\right)-\left(\beta\eta_2\theta_p-\gamma_s\theta_s-2\beta\gamma_p\theta_p\right)(-\rho\gamma_p$$

$$+\beta\gamma_p\theta_p))\left(-\left(2\beta\theta_p-2\rho\gamma_s\theta_s-2\beta\gamma_p\theta_p\right)\left(-\rho\gamma_p+\beta\gamma_p-\beta\gamma_p\theta_p\right)-2\left(-\rho\gamma_s\theta_s+\beta^2\gamma_p\theta_p\right)\right))((-2\left(-\rho\gamma_p+\beta\eta_1\gamma_p-2\beta(\gamma_p^{\,2}\theta_p\right)-\beta\theta_p$$

$$-2\rho\gamma_s\theta_s)\left(-\rho^2+\beta\gamma_s\theta_s\right))\left(-2\left(2\beta\theta_p-2\rho\gamma_s\theta_s-2\beta\gamma_s\theta_s\right)-\left(-\rho\gamma_p+2\beta\gamma_p-\beta\gamma_s\theta_s\right)\left(-\rho\gamma_p\theta_p+\beta^2\theta_p\right)\right)-(-(\beta\theta_p-2\rho\gamma_p\theta_p$$

$$-2\beta\gamma_p\theta_p)\left(-\rho\eta_1\gamma_p+2\beta\gamma_p-\beta\gamma_p\theta_p\right)-2\left(-\rho\gamma_p\theta_p+\beta\gamma_s\theta_s\right))\left(4-\left(-\rho^2\gamma_p+\beta\gamma_p\theta_p\right)\left(-\rho\eta_2\gamma_s\theta_s+\beta\gamma_p\theta_p\right)\right))))$$

(36)

Proof of Theorem 4. For the profit function in step (9), the Hessian matrix is as follows.

$$Hessian\,matrix = H = \begin{pmatrix} \frac{\partial^2 \Pi_{M(i)}}{\partial\left(w_{m(i)}\right)^2} & \frac{\partial^2 \Pi_{M(i)}}{\partial w_{m(i)} s_{w(i)}^{(j)}} & \frac{\partial^2 \Pi_{M(i)}}{\partial w_{m(i)} s_{R(i)}^{(j)}} & \frac{\partial^2 \Pi_{M(i)}}{\partial w_{m(i)} s_{R(3-i)}^{(j)}} \\ \frac{\partial^2 \Pi_{M(i)}}{\partial s_{w(i)}^{(j)} w_{m(i)}} & \frac{\partial^2 \Pi_{M(i)}}{\partial\left(s_{w(i)}^{(j)}\right)^2} & \frac{\partial^2 \Pi_{M(i)}}{\partial s_{w(i)}^{(j)} s_{R(i)}^{(j)}} & \frac{\partial^2 \Pi_{M(i)}}{\partial s_{w(i)}^{(j)} s_{R(3-i)}^{(j)}} \\ \frac{\partial^2 \Pi_{M(i)}}{\partial s_{R(i)}^{(j)} w_{m(i)}} & \frac{\partial^2 \Pi_{M(i)}}{\partial s_{R(i)}^{(j)} s_{w(i)}^{(j)}} & \frac{\partial^2 \Pi_{M(i)}}{\partial\left(s_{R(i)}^{(j)}\right)^2} & \frac{\partial^2 \Pi_{M(i)}}{\partial s_{R(i)}^{(j)} s_{R(3-i)}^{(j)}} \\ \frac{\partial^2 \Pi_{M(i)}}{\partial s_{R(3-i)}^{(j)} w_{m(i)}} & \frac{\partial^2 \Pi_{M(i)}}{\partial s_{R(3-i)}^{(j)} s_{w(i)}^{(j)}} & \frac{\partial^2 \Pi_{M(i)}}{\partial s_{R(3-i)}^{(j)} s_{R(i)}^{(j)}} & \frac{\partial^2 \Pi_{M(i)}}{\partial\left(s_{R(3-i)}^{(j)}\right)^2} \end{pmatrix}$$

To calculate the elements of this matrix, we consider the function $\Pi_{M(1)}$ ($i = j = 1$), for instance, as follows.

$$\Pi_{M(1)} = w_{m(1)}\left(D_{w(1)}^{(1)} + D_{R(1)}^{(1)} + D_{R(2)}^{(1)}\right) - \frac{1}{2}\eta_1\left(\left(s_{w(1)}^{(1)}\right)^2 + \left(s_{R(1)}^{(1)}\right)^2 + \left(s_{R(2)}^{(1)}\right)^2\right)$$

We first replace the expressions $D_{w(1)}^{(1)}$, $D_{R(1)}^{(1)}$, and $D_{R(2)}^{(1)}$ in the above equation, and then replace Equations (21), (23)-(26), (29)-(30), and (33)-(36) in the obtained equation. Then, we calculate the required partial derivatives in the Hessian matrix.

Thus,

$$|H|_{2*2} = \left(\frac{\partial^2 \Pi_{M(i)}}{\partial\left(w_{m(i)}\right)^2}\right)\left(\frac{\partial^2 \Pi_{M(i)}}{\partial\left(s_{w(i)}^{(j)}\right)^2}\right) - \left(\frac{\partial^2 \Pi_{M(i)}}{\partial w_{m(i)} s_{w(i)}^{(j)}}\right)^2 = \beta\gamma_p - \beta\gamma_p\theta_p^{\,3} + \rho\gamma_s - 2\beta\gamma_p\theta_p^{\,2} - \rho\gamma_p = \beta\gamma_p\left(1 - \theta_p^{\,3} - 2\theta_p^{\,2}\right) > 0$$

.

(Since $0 < \theta_p < 1$, $\theta_p^{\,3} - 2\theta_p^{\,2} < 0$. Therefore, $1 - \theta_p^{\,3} - 2\theta_p^{\,2}$ is always positive.)

On the other hand,

$$|H|_{3*3} = \frac{\partial^2 \Pi_{M(i)}}{\partial\left(w_{m(i)}\right)^2}\left(\frac{\partial^2 \Pi_{M(i)}}{\partial\left(s_{w(i)}^{(j)}\right)^2} \times \frac{\partial^2 \Pi_{M(i)}}{\partial\left(s_{R(i)}^{(j)}\right)^2} - \left(\frac{\partial^2 \Pi_{M(i)}}{\partial s_{w(i)}^{(j)} s_{R(i)}^{(j)}}\right)^2\right)$$

$$- \frac{\partial^2 \Pi_{M(i)}}{\partial w_{m(i)} s_{w(i)}^{(j)}}\left(\frac{\partial^2 \Pi_{M(i)}}{\partial s_{w(i)}^{(j)} w_{m(i)}} \times \frac{\partial^2 \Pi_{M(i)}}{\partial\left(s_{R(i)}^{(j)}\right)^2} - \frac{\partial^2 \Pi_{M(i)}}{\partial s_{w(i)}^{(j)} s_{R(i)}^{(j)}} \times \frac{\partial^2 \Pi_{M(i)}}{\partial s_{R(i)}^{(j)} w_{m(i)}}\right)$$

$$+ \frac{\partial^2 \Pi_{M(i)}}{\partial w_{m(i)} s_{R(i)}^{(j)}}\left(\frac{\partial^2 \Pi_{M(i)}}{\partial s_{w(i)}^{(j)} w_{m(i)}} \times \frac{\partial^2 \Pi_{M(i)}}{\partial s_{R(i)}^{(j)} s_{w(i)}^{(j)}} - \frac{\partial^2 \Pi_{M(i)}}{\partial\left(s_{w(i)}^{(j)}\right)^2} \times \frac{\partial^2 \Pi_{M(i)}}{\partial s_{R(i)}^{(j)} w_{m(i)}}\right) = -\gamma_p\beta\eta_2 < 0$$

Moreover, $\left|H\right|_{4*4} = \beta^2\gamma_p - \beta^4\rho^3\gamma_p\theta_p > 0$. Hence, the above Hessian matrix is negative definite. Therefore, $\Pi_{M(i)}$ is also concave. ■

(It should be noted that it was impossible to demonstrate in general that $\Pi_{M(1)}$ was concave due to the high complexity of the derivatives obtained for the Hessian matrix. Therefore, this was carried out using the simplifying assumption that $\{\gamma_p = \gamma_s, \theta_p = \theta_s, \eta_1 = \eta_2\}$. Of course, the concaveness of the manufacturer's profit function was demonstrated numerically for all the specific parameter values in the paper, as exemplified in Figure 3.)