A multi-product multi-layer urban freight distribution problem solved using a hybrid metaheuristic procedure

Cristian Giovanny Gómez-Marin[a*], Conrado Augusto Serna-Urán[a], Julián Andrés Zapata-Córtes[b], Martín Darío Arango-Serna[c]

[a] *Instituto Tecnológico Metropolitano, Medellín, Colombia., cristiangomez@itm.edu.co*

[b] *Institución Universitaria CEIPA, Sabaneta, Colombia.*
[c] *Universidad Nacional de Colombia, Medellín Colombia.*

Abstract

The pick-up and delivery routing problem has received special attention thanks to its application to urban freight distribution processes. However, due to the multiple levels involved in those processes, modeling and analyzing urban distribution networks in urban contexts are complex tasks. As a result, efficient and robust solution methods should be proposed according to the dynamic and uncertain conditions that characterize this type of problems. This article presents a new formulation for the pick-up and delivery problem in a logistics distribution network composed of 3 levels: n: 1: m (n suppliers, 1 urban consolidation center, and m customers). In addition, an algorithm based on a greedy randomized adaptive search procedure (GRASP) heuristic and 2-opt algorithm was implemented here to find solutions to problem, which were compared with the results of the same algorithm for a two-layer vehicle routing problem in several instances. Thus, the proposed procedure achieved a 22% improvement over such algorithm.

Keywords: Urban goods distribution, Urban freight transport, Multi-echelon distribution system, Vehicle routing problem, Mathematical programming, Hybrid metaheuristics.

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[*] Corresponding author. *Calle 73 No. 76A - 354, Vía al Volador.* cristiangomez@itm.edu.co Phone +(574) 440 51 00 opc 9 ext 5290. Celphone +(57) 3006106153. Fax: +(574) 440 51 02.
Calle 73 No. 76A - 354, Vía al Volador. conrado.serna@itm.edu.co Celphone +(57) 3136504960
Calle 77 Sur #No 40–165. julian.zapata@ceipa.edu.co Celphone +(57) 3022891262
Carrera 80 No 65-223 - Campus Robledo. mdarango@unal.edu.co Celphone +(57) 3004817252

1. Introduction

Different stakeholders participate in the process of Urban Goods Distribution (UGD) and are part of the urban network required to perform the required pick-up and/or delivery operations in cities. Their coordination and transport decisions influence the performance of the UGD. Coordination and cooperation among the key actors in the urban supply chain are essential for achieving common objectives, as well as adopting more efficient and less individualistic UGD approaches [1–3].

In general terms, the supply chain structures in urban context are composed of suppliers, producers, distributors, wholesalers, retailers, and customers. Urban supply chain management requires the use of tools to design and evaluate their processes, which includes the coordination of actors to satisfy customers' demands and effectively respond to city limitations and dynamics.

In an urban context, the key stakeholders are customers, suppliers, carriers, and the public administration [4]. The customers can be distributors, wholesalers, and retailers; the freight generators perform as suppliers and producers; the carriers offer transport services between customers and freight generators; and the public administration controls or generates scenarios for these private actors. Generally, all the actors perform as individual decision makers, using large amounts of information, their experience, and their interaction to face the dynamic behaviors of the urban context in order to solve daily operation problems [5].

There are several freight distribution strategies to achieve the objectives of the supply chain, which range from direct deliveries to multi-stop pick-up and delivery routes [6,7]. These strategies are selected based on the characteristics of the distribution context, such as politcies, type of products, accessibility to customers, and quantity of requested orders, among others. The strategies in the urban distribution process seek to satisfy the customers' demands but also have a profitable company operation. This is why they should consider city restrictions and the dynamism of the urban context to promote coordination among actors and improve the performance of the whole distribution process.

Several factors must be taken into account to determine the operational plan for a distribution process, such as travel time, costs, demand changes, and specific delivery policies established by public administrators. Different types of the Vehicle Routing Problem (VRP) (such as the Capacitated Vehicle Routing Problem, CVRP; the Location Routing Problem, LRP; and the Inventory Routing Problem, IRP; among others) are used to stablish those operational plans. From the operational point of view, the urban distribution process should be performed in an effective and efficient way, that is, satisfying the customer's requests using as less resources as possible in the process. In that sense, a complex formulation of a VRP with specific urban characteristics is needed to tackle the complexities of the context and improve the performance of the distribution processes in cities.

Some authors have used the multi-layer strategy for representing the distribution network, the multi-product component for the characteristic and amount of freight, and the pick-up and delivery operation for the flow of goods among the actors. This paper proposes an integration of all these characteristics through a robust model with a modular structure, which is more adjusted to real UGD situations.

The aim of this paper is twofold: (i) to propose a mathematical formulation for an urban freight distribution process involving three important characteristics, i.e., multi-product, multi-layer, and pick-up and delivery operations; and (ii) to develop a solution procedure for the model that can be used in real scenarios.

This paper is structured as follows. Section 2 presents a literature review. Section 3 describes the urban supply chain structure and the Mixed Integer Linear Programming (MILP) model formulation for the UGD. Section 4 details a hybrid metaheuristic solution procedure to solve the model, and its application is analyzed in section 5. Finally, Section 6 draws the conclusions and suggests future research lines.

2. Literature review

Two-tier distribution networks designed for UGD processes have been presented by [8–11]. These networks include one distribution center located on the outskirts and multiple satellite depots inside the city. They use this type of network to design multi-layer routes from the external distribution center to the satellites and from satellites to customers. [12–15] presented a multi-layer distribution structure to coordinate the different flows of goods among the actors located in any layer of the network, which creates tours for the vehicles according to these flows.

Multi-product distribution is another characteristic of the UGD that has been widely studied due to its complexity. Some authors have proposed mathematical formulations. For instance, [16] studied the multi-product cross-dock problem for pick-up and delivery by splitting the pick-up and delivery location nodes that could be visited more than once by one or more vehicles to solve small instances of the problem. Shaabani and Kamalabadi [17] modeled the multi-product problem in a perishable products sector with one manufacturer and multiple customers under the inventory routing problem model. They used multiple distribution strategies to deliver the products and implemented a population-based simulated annealing heuristic to solve the problem. Letchford and Salazar-González, [18] argue that some CVRP problems require the use of additional commodity flow variables, more specifically, the pick-up and delivery multi-product problem.

Only a few articles have incorporated both multi-layer and multi-product characteristics. This integration has been studied by [19], who proposed an integrative three-layer multiproduct distribution network using different routes to integrate the layers into the network. A more complex network with four layers was presented by [20] to solve a pick-up and delivery problem with multiple products and heterogeneous fleets using integrative routes for the multiple layers and considering different numbers of layers in which a vehicle can perform the routes. Boccia, Crainic, Sforza and Sterle [21] proposed an Integer Linear Programming (ILP) formulation, using a flow-intercepting approach, for the location decision of a multi-commodity location routing problem in a three-layer city logistics model. They used a branch and cut algorithm for the solution.

As UGD problems become more complex, some authors have implemented hybrid metaheuristics to solve them. Canales-Bustos et al. [22] presented a case in which these techniques were used. They developed a hybrid particle swarm optimization metaheuristic to solve a three-objective problem model, including the minimization of transport cost, gas emissions, and quality deviation at production plants. Ahkamiraad and Wang [23] proposed a MILP for a distribution system with multiple cross-docks in order to solve a pick-up and delivery CVRP with time windows. For the solution, they used a hybrid genetic algorithm with a Particle Swarm Optimization (PSO) algorithm. Peres et al. (2017) [24] presented a hybrid randomized variable neighborhood descent search to solve an inventory routing problem with transshipment in a retail sector, thus minimizing inventory and transport costs. Pichka et al. [25] proposed a hybrid simulating annealing heuristic to solve a 2-echelon location routing problem in which a third-party logistics company is contracted to perform the routes. In turn, [26] formulated a hybrid multi-population genetic algorithm to solve a multi-depot location routing problem with different delivery options for customers (such as home deliveries or pick-up point deliveries), which represent all the connection decisions between depots, satellites, and customers. This study does not consider direct deliveries, that is, from suppliers to customers without going through depots or satellites.

To sum up, despite the analysis of hundreds of articles of different types of VRP, LRP, IRP, and multi-layer VRP, it appears that the field has not yet explored a three-layer multi-product problem in a three-level urban distribution network n: 1: m (n suppliers, 1 distribution center, and m customers) for pick-up and delivery with three different types of routes.

3. Model structure and formulation

This paper considers an urban distribution network with three layers and three main types of actors: several suppliers, one urban consolidation center (UCC), and multiple customers. The freight is originated by the suppliers in the first layer. Each supplier has unlimited capacity to supply a unique product to fulfill the consolidation center orders. The UCC (i.e., the second layer) takes the customers' orders, consolidates them, and requests the products form the suppliers. The customers, located in the third layer, generate the deterministic demand for different products and share it with the UCC, including information about time windows and quantities.

The UCC consolidates the customers' orders, uses this information to request the products from the suppliers, and allocates the pick-up and delivery routes according to the capacity of a homogeneous fleet of vehicles. The consolidation could be done at the UCC or in the vehicles. The products are distributed by the vehicles using single or interlayer routes. Each vehicle can perform pick-up, delivery, or pick-up + delivery routes, but, once the vehicle is assigned to one type of route, it cannot be changed.

The exchange of products and information has a three-layer structure in which different types of pick-up and delivery routes can be used. The three types of routes allow us to integrate different flows of products between layers and makes the tours more flexible to perform the pick-up and delivery operation. Fig. 1 shows the three types of routes: *R1,* pick-up; *R2,* pick-up and delivery; *R3,* delivery.

R1 (Pick-up): This route is exclusively dedicated to pick up products at supplier locations. According to the demand level, this route can be performed as a direct pick-up from just one supplier or as a tour, visiting several of them. The products are delivered at the UCC.

R2 (Pick-up and delivery): This route is designed for the pick-up and delivery process. First, the products are picked up at the suppliers and then delivered to customers, and the UCC could be a delivery point.

R3 (Delivery): This route contemplates only deliveries to one or more customers from the UCC.

These routes allow the model to propose supply routes from the suppliers to the UCC and direct delivery routes from the UCC to customers, but also routes that combine pick-up and delivery. These routes can be available or not in order to reduce distribution costs.

3.1. Mathematical formulation

This paper presents an urban supply chain model for a multi-layer multi-product pick-up and delivery VRP in which the following assumptions are included:

- Each supplier supplies only one type of product. The demand of all customers for each product should be less than or equal to the supply of the respective supplier.
- Each customer requests at less two different products from the suppliers.
- Homogeneous vehicles are used, and they start and finish their routes at the UCC.

The mathematical formulation of the model for the logistic distribution network proposed here could be described as a direct graph $G=(N,A)$. The set of nodes $N=C\cup F\cup 0$ includes the subset $C:=\{j,\ldots,j'\}$ that represents the customers. The subset $F:=\{i,\ \ldots,i'\}$ represents the suppliers, and node 0 is the UCC. The A set of arcs denotes the links between the nodes. There are complete subgraphs that consist of suppliers $i,\ldots,i'$ and the UCC, as well as customers $j,\ \ldots,j'$ and the UCC. To ensure direct trips between suppliers and customers, G contains the arcs $\{i,j\},\ \ i\in F,\ j\in C$. The homogenous fleet of vehicles is indexed by $k\in K$, and they start and finish their routes $R:\{R1,R2,R3\}$ at the UCC.

To ensure that the UCC acts as a consolidation center, R1 must be executed before R3, which allows the products that arrive to the UCC in R1 to be forwarded to customers. Suppliers are assumed to be able produce enough products to satisfy the demand of all customers. The MILP formulation for the distribution problem (with n suppliers, m customers, and one UCC) is the following:

Parameters:

$K=\{k\}$	$Set\ of\ vehicles$
C_{ij}	$Cost\ of\ travel\ from\ node\ i\ to\ node\ j\ (monetary\ units)$
θ^k	$Capacity\ of\ the\ vehicle\ k\in K\ in\ units\ of\ product\ (200\ units)$
$d_{j,p}$	$Demand\ of\ customer\ j\in C\ for\ product\ p\in P(units\ of\ product)$
$o_{i,p}$	$Quantity\ of\ product\ p\in P\ supplied\ by\ supplier\ i\in F\ (units\ of\ product)$
λ_i^k	$Time\ at\ which\ vehicle\ k\in K\ starts\ its\ service\ to\ customer\ j\in C$
u_j	$Service\ time\ at\ customer\ j\in C\ (minutes)$
$e_j,\ l_j$	$Time\ window\ for\ the\ service\ at\ customer\ j\in C\ (minutes)$
t_{ij}^k	$Travel\ time\ of\ vehicle\ k\in K\ between\ the\ nodes\ i,j\in N\ (minutes)$

Variables:

$q_{i,p}^{k,r}$	$Quantity\ of\ product\ p\in P\ transported\ in\ vehicle\ k\in K\ before\ visiting\ node\ on\ route\ i\in\{F\cup 0\cup C\}$
$\rho_{j,p}^k$	$Quantity\ of\ product\ p\in P\ in\ vehicle\ k\in K\ that\ leaves\ the\ UCC\ and\ must\ be\ delivered\ at\ nodes\ j\in C$
$x_{i,j}^{k,r}$	$Binary\ decision\ variable\ that\ is\ equal\ to\ 1\ if\ vehicle\ k\in K\ uses\ the\ arc\ from\ i\ to\ j\ on\ route\ r\in R;\ otherwise,\ it\ is\ 0$

The Objective Function (OF) seeks to minimize the transportation cost and the number of tours needed to perform the different routes. In this study, the OF was divided into three parts, which correspond to routes *R1*, *R2*, and *R3*. Such OF is given by Eq. (1)

$$
\begin{aligned}
&Min\sum_{k\in K}\left[\sum_{i\in F}C_{0,i}x_{0,i}^{k,r_1}+\sum_{i\in F}\sum_{j\in F,\ i\neq j}C_{i,j}x_{i,j}^{k,r_1}+\sum_{i\in F}C_{i,0}x_{i,0}^{k,r_1}\right]+\\
&\sum_{k=1}^{K}\left[\sum_{i=1}^{F}C_{0,i}x_{0,i}^{k,r_2}+\sum_{i}^{F}\sum_{j,i\neq j}^{F}C_{i,j}x_{i,j}^{k,r_2}+\sum_{i}^{F}\sum_{j}^{C}C_{i,j}x_{i,j}^{k,r_2}+\sum_{i}^{C}\sum_{j,i\neq j}^{C}C_{i,j}x_{i,j}^{k,r_2}+\sum_{i}^{C}C_{i,0}x_{i,0}^{k,r_2}\right]+\\
&\sum_{k=1}^{K}\left[\sum_{j=1}^{C}C_{0,j}x_{0,j}^{k,r_3}+\sum_{i}^{C}\sum_{j,\ i\neq j}^{C}C_{i,j}x_{i,j}^{k,r_3}+\sum_{i=1}^{C}C_{i,0}x_{i,0}^{k,r_3}\right]
\end{aligned}
\quad (1)
$$

The first part, which corresponds to *R1* routes, includes three elements: (a) transport cost from the UCC to the first supplier, (b) transport cost between the suppliers on the route, and (c) cost from the last supplier to the UCC. The second part, for *R2* routes, has five elements: (a) transport cost from the UCC to the supplier, (b) cost between suppliers, (c) cost from the last supplier to the first customer, (d) cost between customers, and (e) cost of returning from the last customer to the UCC. Similarly, the third part of the objective function corresponds to *R3* routes and has three elements: (a) transport cost from the UCC to the customers, (b) cost between customers, and (c) cost from the last customer to the UCC.

The following are the general constraints for all the routes:

$$\sum_{k\in \mathrm{K}}\sum_{r\in \{R\}\setminus r_3}\left[\sum_{i\in F, i\neq j} x_{i,j}^{k,r} + x_{0,i}^{k,r}\right]=1 \qquad \forall\, i\in \mathrm{F} \tag{2}$$

$$\sum_{k\in \mathrm{K}}\left[\sum_{i\in \mathrm{F}\cup \mathrm{C},\ \mathrm{i}\neq \mathrm{j}} x_{i,j}^{k,r_2} + \sum_{i\in \mathrm{C}\cup 0,\ \mathrm{i}\neq \mathrm{j}} x_{i,j}^{k,r_3}\right]=1 \qquad \forall\, j\in \mathrm{C} \tag{3}$$

Each supplier $i \in F$ is visited only once by vehicle $k \in K$ from the UCC or from other suppliers on routes *R1* and *R2*, as stated in Eq. ((2). In the same way, each customer $j \in C$ is visited only once by vehicle $k \in K$ coming from a supplier, from other customer on route *R2*, from the UCC on route *R2*, from other customers on route *R3*, or from the UCC on route *R3*, as established in Eq. ((3).

$$\sum_{i\in F} x_{0,i}^{k,r} \leq 1 \qquad \forall\, k\in K, r\in \{R\}\setminus r_3 \tag{4}$$

$$\sum_{i\in C} x_{0,i}^{k,r_3} \leq 1 \qquad \forall\, k\in K \tag{5}$$

Vehicle $k \in K$ must leave the UCC only once on each route, as stated in Eqs. (4) and (5).

$$\sum_{i\in F} x_{i,0}^{k,r_1} \leq 1 \qquad \forall\, k\in \mathrm{K} \tag{6}$$

$$\sum_{i\in C, i\neq 0} x_{i,0}^{k,r} \leq 1 \qquad \forall\, k\in K, \{R\}\setminus r_1 \tag{7}$$

Additionally, the vehicles must return to the UCC on each route, as in Eq. (6) and Eq. (7).

Constraints at Eq. (8)–(14) consider the flow conservation for each vehicle and each node. That is, for each node *i*, 0, *j* that has a vehicle input, there must be an output:

$$\left(x_{0,i}^{k,r_1} + \sum_{j\in F, i\neq j} x_{j,i}^{k,r_1}\right) - \left(\sum_{j\in F, i\neq j} x_{i,j}^{k,r_1} + x_{i,0}^{k,r_1}\right) = 0 \qquad \forall\, k\in K, i\in F \tag{8}$$

$$\left(x_{0,j}^{k,r_2} + \sum_{i\in F} x_{i,j}^{k,r_2} + \sum_{i\in C, i\neq j} x_{i,j}^{k,r_2}\right) - \left(x_{j,0}^{k,r_2} + \sum_{i\in C, i\neq j} x_{j,i}^{k,r_2}\right) = 0 \qquad \forall\, j\in C, k\in K \tag{9}$$

$$\left(x_{0,j}^{k,r_3} + \sum_{i\in C, i\neq j} x_{i,j}^{k,r_3}\right) - \left(x_{j,0}^{k,r_3} + \sum_{i\in C, i\neq j} x_{j,i}^{k,r_3}\right) = 0 \qquad \forall\, j\in C, k\in K \tag{10}$$

$$\left(x_{0,i}^{k,r_2} + \sum_{j\in F,\ i\neq j} x_{i,j}^{k,r_2}\right) - \left(\sum_{j\in F, i\neq j} x_{j,i}^{k,r_2} + \sum_{j\in C} x_{i,j}^{k,r_2}\right) = 0 \qquad \forall\, k\in K, i\in F \tag{11}$$

$$\sum_{i\in F} x_{0,i}^{k,r_1} - \sum_{i\in F} x_{i,0}^{k,r_1} = 0 \qquad \forall\, k\in K \tag{12}$$

$$\sum_{i\in F} x_{0,i}^{k,r_2} - \sum_{i\in C} x_{i,0}^{k,r_2} = 0 \qquad \forall\, k\in K \tag{13}$$

$$\sum_{i\in C} x_{0,i}^{k,r_3} - \sum_{j\in C} x_{i,0}^{k,r_3} = 0 \qquad \forall\, k\in K \tag{14}$$

The capacity constraints for *R1* routes are presented in Eq. (15) to (18). The vehicles must be empty when they depart from the UCC to the suppliers and return loaded to the UCC, without exceeding their capacity. The production capacity of the supplier o_{ip} is assumed to be enough to meet all customers' demands for product *p*. Constraint formulated by Eq. (15) ensures that the amount of product *p* delivered to the UCC by vehicle *k* is greater than or equal to the quantity of product *p* requested by the UCC. In Eq. (16), the quantity of product *p* transported by each vehicle *k* must be less than or equal to its capacity. Constraint formulated by Eq. (17) establishes that the load of product *p* that is delivered to the UCC by vehicle *k* on arc (i, 0) must be less than or equal to the capacity of the vehicle. The quantity of product *p* transported by each vehicle *k* must be less than or equal to its capacity Eq. (18), including the last arc to the UCC.

$$q_{j,p}^{k,r_1} - o_{i,p} - q_{i,p}^{k,r_1} \leq \theta^k\left(1 - x_{i,j}^{k,r_1}\right) \qquad \forall\ i,j \in F, i \neq j,\ p \in P, k \in K \tag{15}$$

$$q_{0,p}^{k,r_1} - o_{i,p} - q_{i,p}^{k,r_1} \leq \theta^k\left(1 - x_{i,0}^{k,r_1}\right) \qquad \forall\ i \in F,\ p \in P, k \in K \tag{16}$$

$$\sum_{p \in P} q_{i,p}^{k,r_1} \leq \theta^k \qquad \forall\ i \in F \cup 0, k \in K \tag{17}$$

$$q_{0,p}^{k,r_1} \leq \theta^k\left(\sum_{i \in F} x_{i,0}^{k,r_1}\right) \qquad \forall\ k \in K, p \in P \tag{18}$$

The capacity constraint in Eq. (19) should also be applied to routes *R2* and *R3*.

$$\sum_{p \in P} q_{i,p}^{k,r} \leq \theta^k \qquad \forall\ i \in F \cup C,\ k \in K,\ r \in R \backslash \{r_1\} \tag{19}$$

The amount of product $p \in P$ that is loaded into vehicle $k \in K$ at the UCC to perform *R3* routes should be less than or equal to the amount of product $p \in P$ received by the UCC from *R1*. The set of constraints from Eq. (20) to (21) applies to delivery only and pick-up plus delivery routes.

$$q_{i,p}^{k',r_3} \leq \sum_{k \in K} q_{0,p}^{k,r_1} + \theta^k\left(1 - x_{0,i}^{k',r_3}\right) \qquad \forall\ i \in C,\ p \in P, k' \in K \tag{20}$$

Eq. (21) to Eq. (23) limit the quantity of products that can be loaded into vehicles when a route *R2* is performed, that is, the nodes could be either pick-up or delivery nodes. This also applies to the third route at Eq. (24).

$$q_{j,p}^{k,r_2} - o_{i,p} - q_{i,p}^{k,r_2} \leq \theta^k\left(1 - x_{i,j}^{k,r_2}\right) \qquad \forall\ i,j \in F, i \neq j,\ p \in P, k \in K \tag{21}$$

$$q_{j,p}^{k,r_2} - o_{i,p} - q_{i,p}^{k,r_2} \leq \theta^k\left(1 - x_{i,j}^{k,r_2}\right) \qquad \forall\ i \in F, j \in C,\ p \in P, k \in K \tag{22}$$

$$q_{j,p}^{k,r_2} - q_{i,p}^{k,r_2} + d_{i,p} - \leq \theta^k\left(1 - x_{i,j}^{k,r_2}\right) \qquad \forall\ i,j \in C, i \neq j,\ p \in P, k \in K \tag{23}$$

$$q_{j,p}^{k,r_3} - q_{i,p}^{k,r_3} + d_{i,p} - \leq \theta^k\left(1 - x_{i,j}^{k,r_3}\right) \qquad \forall\ i,j \in C, i \neq j,\ p \in P, k \in K \tag{24}$$

In turn, constraints formulated by Eq. (25) to Eq. (27) limit the quantities of products that can be loaded into the vehicles on delivery routes *R2* and *R3* after leaving the UCC .

$$\rho_{i,p}^k - d_{j,p} \leq \rho_{j,p}^k + \theta^k\left(1 - x_{i,j}^{k,r_2}\right) \qquad \forall\ i,j \in C, i \neq j,\ k \in K,\ p \in P \tag{25}$$

$$\rho_{0,p}^k + \theta^k\left(1 - x_{0,j}^{k,r_2}\right) \geq \rho_{j,p}^k \qquad \forall\ j \in C,\ k \in K,\ p \in P \tag{26}$$

$$\rho_{0,p}^k + \theta^k\left(1 - x_{0,j}^{k,r_3}\right) \geq \rho_{j,p}^k \qquad \forall\ j \in C,\ k \in K,\ p \in P \tag{27}$$

In Eq (25), the quantity of product *p* in vehicle *k* arriving at node *j* must be less than or equal to the quantity of product that arrived at the previous node (i) less the demand of customer *j*, while respecting the capacity of the vehicle. According to Eq. (26) and (27), the amount of product that must be loaded into the vehicle must be greater than or equal to the amount of product that must be delivered to customers, which applies to *R2* and *R3* routes.

Eq. (28) to Eq. (30) ensure that customer demand is satisfied. In turn, Equation (32) ensures that vehicle capacities are respected.

$$\rho^{k}_{j,p} \geq d_{j,p} \qquad \forall\ j \in C,\ k \in K,\ p \in P \tag{28}$$

$$\rho^{k}_{0,p} + \theta^{k}\left(1 - x^{k,r_3}_{0,j}\right) \geq \rho^{k}_{j,p} \qquad \forall\ j \in C,\ k \in K,\ p \in P \tag{29}$$

$$\sum_{p \in P} \rho^{k}_{j,p} \leq\ \theta^{k} \qquad \forall\, j \in C \cup 0,\ k \in K \tag{30}$$

In Eq. (28), the quantity of product *p* loaded into vehicle *k* that goes from the UCC to node *j* must be greater than or equal to the quantity of product *p* requested by the customers. In Eq. (29), the amount of product that must be loaded into vehicle *k* at the UCC must be greater than or equal to the amount of product that must be delivered to customers on *R3* routes. Constraint in Eq. (30) ensures that the capacity of the vehicles is not exceeded.

Additionally, Eq. (31) and Eq. (32) guarantee a loading balance on *R2* and *R1* routes.

$$\rho^{k}_{j,p} \leq q^{k,r_2}_{j,p} + \theta^{k}\left(1 - \sum_{i \in FU0} x^{k,r_2}_{i,j}\right) \qquad \forall\, j \in C,\ p \in P, k \in K \tag{31}$$

$$\sum_{k \in K} \rho^{k}_{o,p} = \sum_{k \in K} q^{k,r_1}_{0,p} \qquad \forall\, p \in P \tag{32}$$

Constraint in Eq. (33) ensures the input and output loading balance on *R3* routes. Eq. (34) guarantees the return of the vehicles to the UCC.

$$\rho^{k}_{o,p} \geq \rho^{k}_{j,p} + d_{j.p} - \theta^{k}\left(1 - x^{k,r_3}_{j,0}\right) \qquad \forall\, j \in C,\ p \in P,\ k \in K \tag{33}$$

$$q^{k,r}_{j,p} \leq \theta^{k}\left(\ 1 - x^{k,r}_{j,0}\right) \qquad \forall\, j \in C,\ p \in P, k \in K, r \in R \backslash \{r_1\} \tag{34}$$

The constraints of the time windows are ensured by Eqs. (35) and (36).

$$e_i \leq\ \lambda^{k}_{i}\ \leq\ l_i \qquad \forall\ \ i \in N, k \in K \tag{35}$$

$$x^{k,r}_{i,j}\left(\lambda^{k}_{i} + t^{k}_{i,j} + u_i - \lambda^{k}_{j}\right) \leq 0 \qquad \forall\, i,\, j \in N, k \in K, r \in R \backslash \{r_1\} \tag{36}$$

4. Hybrid greedy randomized solution procedure

The basic VRP with one OF for delivery routes and a common set of few constraints is already a NP-Hard optimization problem [27,28]. In this study, the MILP formulation was tested using GAMS ® (General Algebraic Modeling System) language and the CPLEX Solver in the NEOS server [29]. Nevertheless, a small instance with 15 nodes used up all the computational resources, as shown in Section 4. Due to the complexity of the model, the number of restrictions, and different indexes in the decision variables, an exact solution is not feasible in a short computational time; therefore, a more advanced solution technique should be implemented. This article presents a metaheuristic that uses a procedure that includes the following four steps to solve the problem, as shown in the IDF0 diagram in Fig. 2:

1) Step one: Assign *R2* routes to visit all the suppliers, load the vehicles, and assign the customers according to the number and type of loaded products.
2) Step two: Map routes and improve *R2* routes.

3) Step three: Assign customers and suppliers to *R1* and *R3* routes according to the actors that are not included in *R2* routes.

4) Step four: Design and improve *R1* and *R3* routes.

The solution algorithm for this specific UGD model is based on the cluster first-route second heuristic and a well-known metaheuristic such as the greedy randomized adaptive search procedure (GRASP) [8,30] with a local search heuristic such as the 2-opt optimal operator [31,32].

To perform the routing process, the two heuristics are used based on the cluster first-route second strategy because there are three different types of routes in the distribution model and the nodes must be assigned to each type of route.

The assignment starts by constructing the *R2* routes since they are more complex (pick-up plus delivery) than the other types of routes. First, supplier and customer nodes are classified, and then, according to the vehicle capacity, the suppliers are randomly assigned until the maximum capacity of the vehicle is reached, which generates a wider computational search area to explore. Afterward, customers' demand and the combination of products requested by them are found in order to fulfill the demand, visiting the customer only once. This procedure is repeated until all the pairing possibilities between suppliers and customers have been established.

Once the nodes have been assigned to *R2*, a GRASP algorithm is executed to find an initial solution to the routing problem and, subsequently, a 2-opt algorithm is run on the same route to improve the initial fitness value of the solution.

The GRASP algorithm begins by searching two nodes with the minimum distance from the depot and randomly selecting one of them to start the tour form the UCC to such selected node. This procedure is repeated to select two candidates with the shortest distance from the current node until the tour is completed. After this procedure is carried out, the nodes on the same route are rearranged (2-opt) to improve the solution, and the entire algorithm is executed 10000 times to select the best solution for the tour of *R2* routes.

The nodes that have not been assigned in the previous procedure are selected as supplier nodes for *R1* or as customer nodes for *R3*. For each set of nodes, the GRASP algorithm is executed to find the route solution for those nodes. If the capacity of the vehicle is exceeded, a new route must be created. After all the routes have been mapped, the 2-opt algorithm is run to obtain an improved route.

5. *Results and discussion*

Several instances were tested to study the model and solution algorithm presented here. The values of the parameters for each instance and the results are detailed below:

5.1. Test instances

Initially, the model was formulated in GAMS and solved by the CPLEX Solver in the Neos server. Five small and six medium and large instances were randomly generated considering a maximum of seven suppliers to test the model. The network configuration for the small instances and the cost obtained with the model are presented in Table 1. In bigger instances, no solution was obtained before the limit of the computational resources of the server was reached (8 hours and 3 GB).

Since exact solutions for instances with more than 14 nodes could not be found in a reasonable amount of time, we used the proposed heuristic procedure presented in Section 4 to solve the medium-sized and large instances.

Six instances were used to test the model with complex configurations, including up to 208 nodes, and their solutions were obtained with the hybrid greedy randomized procedure proposed in this article. These instances are based on the case of a real-life food retail company, but the locations of facilities and customers were modified due to a confidentiality agreement. Therefore, the facility locations are represented in a cartesian plane ranging from -200 to 200. The demand of each customer corresponds to real data, and the homogeneus capacity of the vehicles 200 units of product. Table 2 presents the information about each instance. All the tests were run on a laptop computer with a 2.4-Ghz Intel Core i5 processor, 4 GB of RAM, and a 64-bit operating system.

5.2. Results

To solve the model, the algorithms were programmed in Java, and the hole metaheuristic was run 10000 times for all the instances. The distribution costs, as stated in the objective function in Eq. (1), of all the instances are presented in Table 3. This table also shows a comparison between the distribution costs obtained using the proposed model and those calculated implementing the same hybrid greedy randomized solution procedure to solve the two layer VRP between customers and suppliers. In the latter, the routes from suppliers to the UCC and from the UCC to customers were calculated independently.

Table 3 indicates that the proposed model and solution procedure generates better distribution costs than the classic VRP model. Also, this table shows that, when the number of customers is 50 and the suppliers are 5 or 7, the model produces a noticeable improvement in distribution costs compared with the VRP. Similarly, the number of vehicles needed to perform the operation is lower with the proposed model than with the VRP in every instance. However, when the number of customers increases, the model still produces an improvement, but it does not show a relation between the number of customers and the improvement level.

Another interesting finding in this model is that the average vehicle load factor in each instance is generally higher in the proposed model than in the VRP. Only instances 1 and 4 have the same load factor in both models, which is due to the fact that the number of goods loaded and delivered by the only vehicle required in these instances is the same in both models. However, in the VRP, this number of products is downloaded and loaded again at the UCC for the further delivery, creating an additional route, which is an unnecessary operation that only increases the distribution cost.

When there are more suppliers, there is a higher number of product types and products demanded by customers. This generates more diverse routes, in which the rate of products that can be picked up and delivered by the same vehicle is lower. As a result, the quantity of *R2* routes is limited, and, therefore, the products should be delivered to the UCC. Afterward, the products in the UCC are delivered to customers in an increased number of *R3* routes, which makes the total distribution cost of the proposed model similar to that of the VRP, as shown in Table 3.

Table 4 presents the route configurations generated by the proposed model for every instance, where we can see that *R1* routes were not used in the distribution plans generated by the model. The maximum number of *R2* routes in instance 6 is five, and the maximum number of *R3* routes in instance 6 is three. This is because the higher the number of customers and suppliers, the larger the required routes.

Fig. 3 shows the different *R2* and *R3* routes in instance 6, which has the highest number of routes. In this figure, we can observe that the algorithm created 4 *R2* routes to deliver products to customers and also to the UCC. The products delivered to the UCC by vehicles on *R2* routes generate 2 other *R3* routes on which the products are delivered to customers that were not visited on *R2* routes.

Each instance was run 10000 times. With the aim of analyzing the stability of the solution procedure, Fig. 4 presents a distribution plot of all the runs of the six instances, which shows a small variation in every instance. This is expected due to the random behavior of metaheuristic techniques, which do not ensure that an optimal solution to the problem is found. This figure also exhibits the variability between the fitness solution in the tested instances. However, they do not affect the quality of the best solution since the proposed metaheuristic saves all the solutions of the 10000 runs without taking into account if they are good or bad.

6. *Conclusions*

This article presented a model to solve a multi-product, multi-layer pick-up and delivery VRP in which several suppliers, one UCC, and several customers are included. The model proposes a distribution strategy in which the products do not need to go through the UCC; instead, they can be transported directly from suppliers to customers. Using different types of routes designed in the model (*R1*, *R2*, and *R3*) in a single distribution plan allows the integration of distribution strategies and product flows among the three different layers. Furthermore, the proposed model can reduce costs in the distribution process compared to other traditional methodologies such as the single VRP model.

The proposed model was tested using five small instances and six medium-sized and large instances. In the small instances, the solution procedure was a MILP. However, when the number of nodes was higher than 14, said procedure was unable to find an exact solution in a reasonable computation time. In the bigger instances, the model was solved using the hybrid greedy randomized procedure, and the solutions were compared with those of a traditional VRP model. In all the tested instances, the results produced by the proposed model and solution procedure were better than those obtained using the VRP. This difference allows us to conclude that our model can improve distribution networks that have the features included in the problem studied here.

The proposed solution procedure includes a combination of a GRASP and a 2-opt optimal operator. The model and its procedure solution were successfully used to solve a real-life multi-product, multi-layer pick-up and delivery VRP, generating savings of up to 22%, which can be achieved depending on the structure of the distribution network.

Future research in this field should take into account the dynamic context of real UGD processes, in which some parameters are variables (e.g., travel time, service time, and the customers' demands). Additionally, further studies may include other characteristics in the model, such as vehicles with different capacities, time windows, split deliveries, and dynamic behaviors, which are also common in urban distribution contexts.

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Figure and Table list

Fig 1. Type of routes

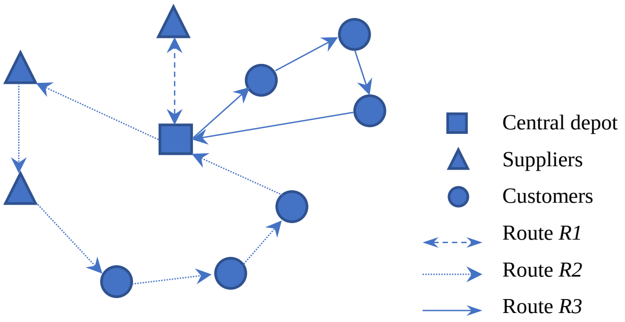


Fig 2. Hybrid greedy randomized procedure

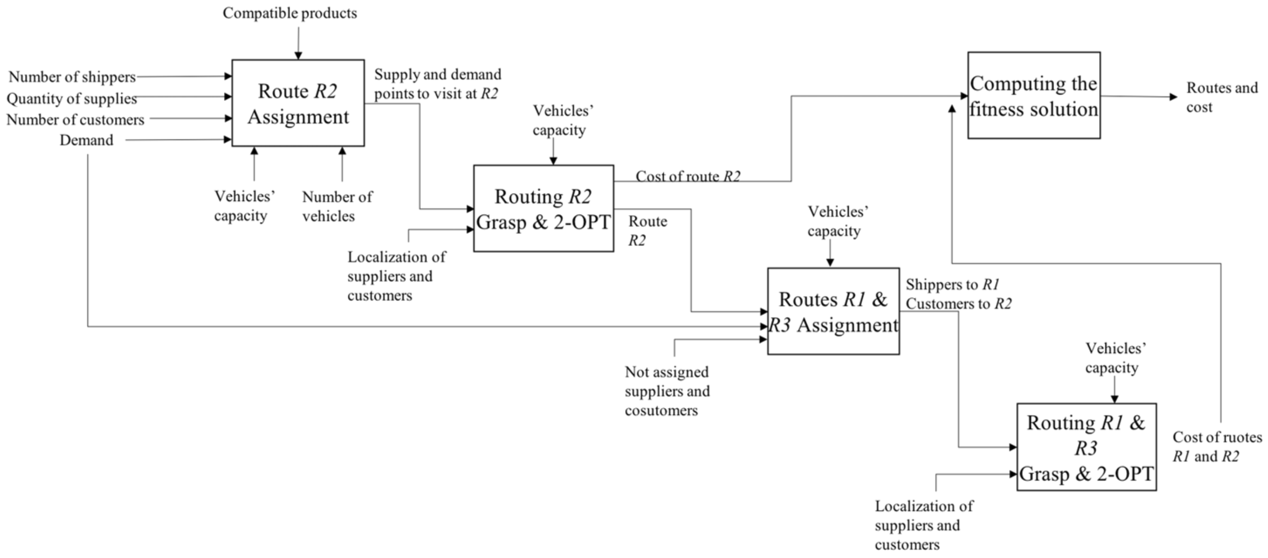


Fig 3. *R2* and *R3* ruotes in instance 6

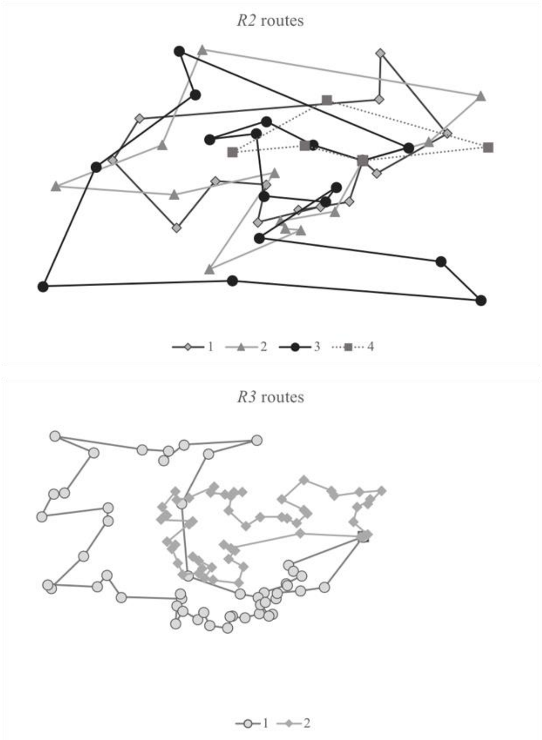
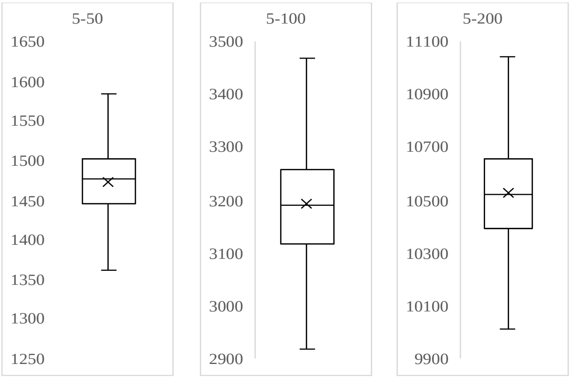
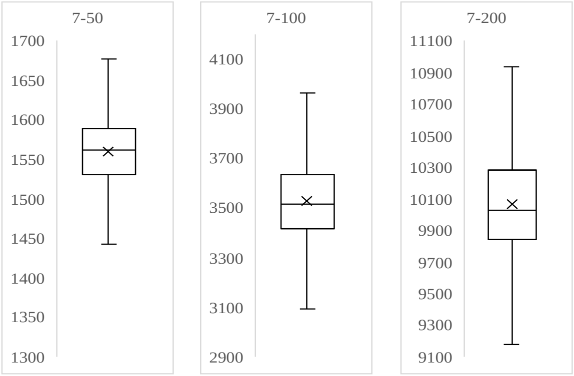


Fig 4. Boxplots of the 10000 runs of the six instances



Instances with 5 suppliers



Instances with 7 suppliers

Table 1. Small test instances for exact solutions

Instance number	Configuration (*F* – *0* – *C*)	Number of nodes	Cost
1	2– 1 – 3	6	244
3	3 – 1 – 6	10	470
2	2 – 1 – 5	8	318
4	3 – 1 – 8	12	493
5	3 – 1 – 10	14	494

F: Shipper, *0*: Urban Consolidation Center (UCC), *C*: final customer

Table 2. Test instances.

Instance Number	Configuration (*F* – *0* – *C*)	Number of nodes
1	5 – 1 – 50	56
2	5 – 1 – 100	106
3	5 – 1 – 200	206
4	7 – 1 – 50	58
5	7 – 1 – 100	108
6	7 – 1 – 200	208

F: Shipper, *0*: Urban Consolidation Center (UCC), *C*: Final customer

Table 3. Comparison of results obtained with the proposed model and the VRP solution.

Instance	Proposed model			Two Layer VRP			Improvement	
	Cost	Number of routes	Average load factor	Cost	Number of routes	Average load factor	Cost	Eliminated routes
1	1297	1	74%	1601	2	74%	23.40%	1
2	2918	3	66%	3112.1	4	51%	6.60%	1
3	9825	5	86%	10231	6	59%	4.10%	1
4	1379	1	98%	1824.5	2	98%	32.30%	1
5	2953	5	66%	2960.6	5	58%	0.26%	0
6	9181	8	82%	10239	8	55%	11.52%	0

Table 4. Freight loaded on each route in different instances.

Instance	*R1*	*R2*					*R3*		
		1	2	3	4	5	1	2	3
1	-	279	--	--	--	--	--	--	--
2	-	247	158	--	--	--	200	--	--
3	-	299	152	156	189	--	298	190	--
4	-	294	--	--	--	--	--	--	--
5	-	243	247	73	--	--	299	129	--
6	-	299	270	189	151	156	300	300	300

Biographies

Cristian G Gómez-Marín received his bachelor degree in industrial engineering, in M.Sc. in administration and a PhD in Industrial engineering from the Universidad Nacional de Colombia in 2002, 2006 and 2020 respectively. Dr. Gómez-Marin is full time Professor with the Instituto Tecnológico Metropolitano. His research interest includes transport and logistics systems design optimization, urban logistics, supply chain management, microsimulation, and multi-agent systems.

Conrado A Serna-Urán received the bachelor degree in industrial engineering, M.S. degree in management engineering and PhD degree in industry and organization engineering from Universidad Nacional de Colombia in 2002, 2009 and 2017 respectively. He is currently the director of the Quality and Production Engineering Department at Instituto Tecnológico Metropolitano. His main research interests include logistics and transportation system design, supply chain management, operations research and multi-agent systems.

Julian A Zapata-Córtes, graduated as Chemical Engineer in 2006, MSc. In Administrative Engineering in 2011 and PhD in Engineering in 2017 at the National University of Colombia, Medellín Campus. Currently he is full time professor at the Institución Universitaria CEIPA. The topics of interest of professor Zapata-Cortes are: transport networks and inventory optimization, Supply chain Management, information and communication technologies applied to the supply chain, business process management, among others.

Martín D Arango-Serna, received the B.S. degree in industrial Engineer from the Universidad Autónoma Latinoamericana, Colombia, in 1991, the M.S. in Systems Engineering in 1997 from the Universidad Nacional de Colombia, and the PhD degree in Industrial Engineering from the Polytechnic University of Valencia, Spain in 2001. He is a full professor at the Department of Organization Engineering at the Faculty of Mines, Universidad Nacional de Colombia, Medellin Campus. The topics in which professor Arango- Serna works are related to logistics processes in the supply chain, operations research, plant design, industrial-organizational optimization techniques.