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Manufacturer-retailer integrated inventory model with controllable lead time and service level constraint under the effect of learning-forgetting in the setup cost

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Service level constraint.

Abstract. The present study aims to establish a manufacturer-retailer integrated inventory model to jointly compute the optimal values of order quantity, lead time, reorder point, and number of shipments considering the effect of learning-forgetting phenomenon on the setup cost. The fabrication process of the manufacturer is not perfect; therefore, a certain level of product quality can be obtained for an additional cost. Service level constraint is incorporated into the inventory model to evade the backorder, which has negative impact on the company reputation. The lead time is also reduced by crashing cost. The proposed inventory model is illustrated through an example according to which centralized decision is preferred over the decentralized one. In addition, the analysis reveals that players should make a compromise concerning their profit if they wish to enhance the service level and product quality. The profit of the centralized system increases under the effect of learning-forgetting on the setup cost.

1. Introduction

All members of the supply chain should have some preliminary information to meet their consumer demands. In addition, a favorable level of coordination between the decisions of both manufacturers and retailers is required. In this respect, the manufacturer-retailer inventory problem has received considerable attention in recent years. With the growing necessity for establishing an integrated system, decision-makers should have a better understanding of the causal relations that exist in current systems in order to effectively formulate and implement supply chain collaboration strategies. Decision-makers must perfectly understand how to manage stocks throughout the whole supply more efficiently with well coordination to decrease the setup cost by a natural phenomenon called learning-forgetting. Decision-makers are also aware of the fact that decreasing the lead time without diminishing customer service is a significant factor in the acquisition of competitive advantage with the competitors. In addition, they know quite well that defective items are produced in the manufacturing processes. There are four options with regard to defective products:

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reject, repair, rework, or taking back defective ones upon customer request. They incur considerable costs in all of these cases. For this reason, in an imperfect fabrication system, the manager needs to invest capital in the quality improvement programs.

Among different industries, mobile and fashion industries enjoy particular characteristics in common such as short product lifecycle, unpredictable demand, and high diversity of products available in the market. Therefore, an efficient integrated system is required to manage these characteristics. This paper puts its main focus on the customer service level constraint which basically depends on the shorter lead time. Consequently, this research established an integrated inventory model by determining the order quantity, lead time, reorder point, and number of shipments.

As observed in the literature, several researchers and academicians have studied the collaboration among the partners of supply chains. Goyal [1] investigated the joint optimization of the total cost in a single-vendor-single-buyer inventory model. A decade later, Banerjee [2] formulated the Goyal's [1] inventory model by including a finite production rate where the vendor implemented a lot-for-lot strategy. Then, Goyal [3] relaxed the lot-for-lot supposition put forward by Banerjee [2] and derived a more general joint economic lot sizing inventory model. Pan and Yang [4] created an integrated production-inventory problem with controllable lead time. Basically, they developed an algorithm to determine the optimal operation policy. One year later, Goyal [5] revisited the inventory model of Pan and Yang [4] and proposed a simple algorithm to determine the operation policy. In addition, Joint Economic Lot Size (JELS) model was studied and extended in numerous forms. The studies on JELS are classified into diverse categories based on different subjects such as deteriorating items [6], quality [7], setup and order cost reduction [8], controllable lead time [9], multiple buyers [10], and stochastic lead time [11], to name a few. Ben-Daya et al. [12] presented an ample review of the JELS problem. Lin [13] developed an inventory model where both ordering cost and lead time decreased with investment. Ye and Xu [14] derived an asymmetric Nash bargaining model to allocate the cost proportion and share the benefits of both members in a decentralized supply chain with controllable lead time. Later, Li et al. [15] addressed the issue of coordination in a decentralized supply chain comprised of a buyer and a vendor. Since the buyers face service level constraint, they consider additional costs to decrease the lead time. Yadav et al. [16] developed an inventory model considering several decision variables such as the order quantity, lead time, and backorder price discount. While the probability density function of the demand for lead time cannot be determined, the first two moments can be calculated. They employed minimax distribution method to determine the optimal values of decision variables. At the same time, Arkan and Hejazi [17] extended the inventory model for a two-level supply chain composed of a single supplier and a single buyer, considering the credit option. They stated that uncertain demand was normally distributed and lead time was reduced by crashing cost, indicating that lead time was manageable. Afterwards, Song et al. [18] applied Stackelberg game framework to model the interactions between a manufacturer and a retailer, where the lead time demand had free distribution and the mean and variance were uniquely known. Later, Heydari [19] proposed a coordination mechanism based on per-order extra payment with the objective of avoiding aggregation of the lead times of both supplier and retailer. In recent times, Zhu [20] considered a decentralized supply chain with one supplier and one retailer where the demand was simultaneously affected by price and lead time. Zhu [20] employed a Stackelberg game where the supplier (leader) determined the wholesale price and capacity and the retailer (follower) determined the lead time and sale price. Sarkar and Mahapatra [21] presented a periodic review of fuzzy inventory model with decision variables of lead time, reorder point, and cycle length. They considered a logarithmic investment function for lost-sale rate reduction. Heydari et al. [22] proposed a scheme to coordinate an integrated system by means of controlling the lead time. They supposed that lead time could be reduced by spending more and using the fast-shipping mode. They suggested an incentive mechanism to motivate the buyer to participate in the joint decision-making. Ben-Ammar et al. [23] formulated and solved an integrated model with a known dynamic demand. They pointed out that the lead times of the order were independent and discrete random variables with known and bounded probability distributions. They developed Genetic Algorithm (GA) to obtain optimal values for lead times and safety stock. They concluded that under certain assumptions, it is better to optimize the planned lead times rather than to implement safety stocks.

In the real world, estimation of the stockout cost is actually problematic due to ever-existing intangible losses such as damages to the credibility and reputation of firms. Of note, the stockout impacts the customers' satisfaction in a negative form since they fail to purchase the goods required at the time they need, hence the dissatisfaction and low loyalty of customers. Therefore, it is suggested that a service level constraint be included in the objective function instead of the stockout cost. For this reason, several inventory practitioners have included a service level constraint into their proposed inventory models. In this regard, Ouyang and Wu [24] incorporated a service level constraint into the objective function instead of the stockout cost. Jha and Shanker [25] derived a two-echelon supply chain inventory model considering both service level constraint and controllable lead time. Bijvank and Vis [26] studied the optimal replenishment, base-stock, and (R, s, S) policies in an inventory system. Moreover, they derived lower and upper limits on the order up to level and also proposed an effective and efficient algorithm to compute the order up to level. Sarkar et al. [27] carried out a continuous review of an inventory system that could jointly determine the lot size, backorder price discount, reorder point, lead time, and process quality. Basically, they derived two inventory models. While the lead time demand had a normal distribution in the first inventory model, it did not have any particular probability density distribution in the second inventory model. However, the mean and standard deviations were calculated. Shin et al. [28] proposed two different inventory models: In first case, they assume that lead time demand has a normal distribution; in the second case, they suppose that lead time demand does not have a particular probability distribution. However, both mean and standard deviation values were known. They considered the service level constraint in their modeling to prevent backorder cost. Albrecht [29] analyzed the inventory problem considering the service level as the objective or constraint, which was also the key performance indicator in retail environments where customers requested complete deliveries. Gruson et al. [30] evaluated the impact of service level constraints on the inventory policy in the context of capacitated and incapacitated lot sized inventory problems. They assumed the demands as deterministic and considered backlogging over a finite time horizon. They also considered several service level constraints taken from the stochastic inventory literature to evaluate the impact of First In First Out (FIFO) policy and found that it could improve the solutions. Sakulsom and Tharmmaphornphilas [31] designed an integrated inventory model to determine an ordering policy for the two-echelon case to minimize the inventory cost on condition that the expected service level constraint be satisfied. At the end of retailing, the demand was considered seasonal with a short cycle. Recently, Cárdenas-Barrón et al. [32] revisited the inventory model optimization with the objective of understanding service inventories to enhance the overall performance.

Specifically, in their inventory model, Rosenblatt and Lee [33] supposed that defective products could be immediately remanufactured with additional costs. They concluded that the existence of defective items would encourage fabrication of smaller lot sizes. Teng and Thompson [34] derived an inventory model for a new product and established some price and quality policies. Through the maximum principle, they determined the optimal price and quality levels over time on condition that the unit cost decreased due to learning and increased if the quality was made greater. On the contrary, Salameh and Jaber [35] supposed that the defective products were sold as a single lot at a discounted price before the arrival of the subsequent lot. Jaber and Bonney [36] developed an economic lot-sizing inventory model from the manufacturer's perspective and studied the effect of learning and forgetting in setup and product quality. Yang and Pan [37] established an inventory model considering the variable lead time and quality improvement investment when the demand had a normal distribution. In their study, stockout cost was not taken into account and the reorder point was a given parameter. Then, Ouyang et al. [38] revisited and readdressed the inventory model proposed by Yang and Pan [37] by incorporating the stockout cost and optimizing the reorder point. They concluded that the lot size increased upon increase in the proportion of imperfect quality pieces. Chen et al. [39] introduced an inventory model considering the imperfect product process with its shortages. They also evaluated the effect of learning in the unit production time on optimal lot and built an effective algorithm to determine the optimal production quantity and shortages at each cycle to minimize the total inventory cost. Hsu and Yu [40] conducted a study on the inventory model of Salameh and Jaber [35] considering the quality issue. Roy et al. [41] proposed an inventory model that took into account an exponential partial backlogging rate and the lot had imperfect quality pieces. Pal et al. [42] investigated an inventory model with an imperfect fabrication system over two kinds of cycles. In the first cycle, the retailer vends only products of perfect quality at a regular price and in the second cycle, the retailer vends those of imperfect quality with a discount price. Jaber et al. [43] revisited the inventory model of Salameh and Jaber [35], supposing that a distant supplier could provide shipment; therefore, it was not possible to substitute the imperfect pieces with an additional order from the same supplier. To handle this constraint, they presented two inventory models. In the first inventory model, imperfect pieces were sent to a repair shop to change the cost plus a markup margin. Imperfect pieces in the second inventory model were substituted by perfect ones from a local provider at a higher cost. Kumar and Goswami [44] presented an inventory model considering the imperfect product process with partial backlogging and evaluated the effect of the unit production time learning on the optimal lot size in an uncertain environment. They used fuzzy expectation and signed the distance method to defuzzify the fuzzy random cost function into an equivalent crisp function. Giri and Glock [45] examined a single-manufacturer single-retailer supply chain inventory model under the effect of learning and forgetting in production and inspection of returned items. The main objective of the inventory model was to obtain the optimal number of shipments, shipment size, and retail price. They stated that the profit of the closed-loop supply chain was much greater than that of the basic inventory model which ignored worker learning. Jaggi et al. [46] built a two-warehouse inventory model for deteriorating products with imperfect quality when the supplier offered to the buyer a permissible delay in payments. Nobil et al. [47] formulated and solved a multi-machine economic production quantity for products considering scrapped, rework, shortages, and allocation decisions. Other relevant research works were conducted by Gautam and Khanna [48], Guatam et al. [49], Khanna et al. [50], Khanna et al. [51], and Kishore et al. [52]. Dey and Giri [53] developed an integrated inventory model to determine the optimal number of shipments and shipment size of the vendor. They presumed that the vendor's production system was not perfect; hence, buyer would receive the ordered quantity from the vendor in a number of equal-sized shipment and carry out a screening process upon delivery of each batch; however, it was assumed that the screening process was erroneous, hence susceptible to misclassification errors (Types I and II). The effect of learning in the screening process is discussed here.

Table 1 presents a comparison among the contributions of several recent research works.

In the case of the integrated inventory models presented in Table 1, none of these researchers considered the well-known human phenomenon called learning-forgetting in their studies. Thus, it can be concluded that the issue of integrated inventory model to obtain the optimal decisions under service level constraint and controllable lead time, with the assumption that the setup cost is reduced due to learning-forgetting effect, has never been adequately examined. The main contribution of this paper is the development of an integrated inventory model with emphasis on the following issues:

1. To calculate the value of the stockout cost, the constraint of the service level instead of the stockout cost was taken into account to bind the occurrence of the stockout in each cycle;
2. The negative exponential crashing cost and component crashing cost functions were employed to reduce the lead time, i.e., the response time to fill the customer demand;
3. It was supposed that the manufacturing process was not perfect and the defective units were immediately reworked;
4. Two inventory situations were also taken into consideration: in the first situation, the manufacturer and the retailer pursue conflicting objectives and work as different entities. In the second situation, both are coordinated to form a supply chain and work as a single unit;
5. In fact, learning-forgetting is a human phenomenon. Hence, the effect of learning-forgetting in the setup cost should be considered.

The rest of this research work is organized as follows. Section 2 elaborates on the assumptions and notation under which the two different inventory models, i.e., non-coordinated and coordinated supply chain models, were developed. Section 3 illustrates the proposed inventory models using a numerical example. Section 4 gives concluding remarks and suggests future research areas.

Table 1. A comparison of the contributions of some research works.

Author(s)	Supply chain	Lead time reduction		Service level constraint	Learning and forgetting	Quality improvement
		Component wise crashing cost	Exponential crashing cost			
Li et al. [15]	✓	✓		✓		
Jha and Shanker [25]	✓	✓		✓		
Shin et al. [28]		✓		✓		
Ouyang et al. [38]	✓	✓				✓
Ouyang et al. [54]		✓				✓
Hoque [55]	✓	✓				
Sarkar and Majumder [56]	✓	✓				
Jha and Shanker [57]	✓	✓		✓		
Moon et al. [58]			✓	✓		
Sarkar and Moon [59]	✓	✓				✓
Sarkar et al. [60]		✓		✓		✓
Li et al. [61]	✓		✓	✓		
Kim and Sarkar [62]	✓	✓				✓
Sarkar et al. [63]	✓					✓
This paper	✓	✓	✓	✓	✓	✓

2. Formulation of integrated inventory model

The following notation and assumptions are defined.

Notation

Decision's variables

Q	Retailer's order quantity (units)
L	Length of the lead time (unit of time)
m	The number of lots where the manufactured goods are sent from the manufacturer to the retailer in one cycle (an integer number)

Dependent variable

r	Reorder point (units)

Parameters

D	Demand rate (units/ unit of time)
p	Retailer's retail price (\$/unit)
A	Retailer's ordering cost (\$/order)
h_r	Retailer's holding cost (\$/unit/unit of time)
X	Lead time demand (units)
μ	Average lead time demand (units)
$E(X)$	Mathematical expectation of X
X^+	max $\{X, 0\}$
$E(X-r)^+$	Expected shortage amount at the end of the cycle (units)
P	Manufacturer production rate; $P > D$ (units/ unit of time)
$\tilde{S}_i$	Manufacturer's setup cost for the ith cycle (\$/setup)
w	Manufacturer's wholesale price (\$/unit)
c	Manufacturer's fabrication cost (\$/unit)
h_m	Manufacturer's holding cost (\$/unit/ unit of time)
θ_1	Probability of the manufacturing process going out of control
x	Manufacturer's rework cost (\$/unit)
y	Manufacturer's opportunity cost of capital (\$/unit)

Assumptions

1. There is one manufacturer and one retailer in the integrated system;
2. The demand X, through the lead time L, has a normal probability density function (p.d.f) $f(X)$ with the finite mean of μL and standard deviation of $\sigma\sqrt{L}$;
3. The retailer operates with a continuous review inventory policy. In other words, he or she makes replenishment when the stock level reaches the reorder point r. The reorder point r is calculated by the sum of the expected demand during lead time and a safety stock: $r = \mu L + k\sigma\sqrt{L}$ where k is a safety factor;
4. The retailer orders a lot of size Q and the manufacturer fabricates the product in lots of size mQ at a finite production rate of $P(P > D)$ at one setup; however, he sends Q units to the retailer over m times. The retailer incurs an ordering cost for each order and the manufacturer incurs a setup cost for each production run;
5. A negative exponential crashing cost function as in the study of Wu et al. [64] is considered. The total crashing cost is represented as follows:

   $$R(L) = \alpha e^{-\theta L}.$$

   The value of the lead time crashing cost for the value of 1 is utilized to calculate the parameters α and θ. The service level is higher than 50% and this constraint is imposed to reduce the cost associated with the distribution-free continuous-review inventory model;
6. When the manufacturer is manufacturing a lot, the fabrication process may go out of control with a probability θ_1; another piece is manufactured each time. The manufacturing process is supposed to be under control at the beginning of the fabrication process. In case the manufacturing process becomes out of control, the manufacturing process produces defective pieces [65];
7. The manufacturers' capital investment, $W(\theta_1)$, in enhancing the process quality to decrease the out-of-control probability from θ_0 to θ_1 follows a logarithmic function. Mathematical speaking: $W(\theta_1) = b\log(\theta_0/\theta_1)$ for $0 < \theta_1 \leq \theta_0$ where $b = 1/\delta$. Here, δ represents the proportion decrease in θ_1 per dollar increase in investment $W(\theta_1)$ [66].

2.1. Learning-forgetting in setup cost (from Jaber and Bonney [36])

Wright [67] was probably the first researcher who formulated the relations among learning variables in a numerical manner. Wright's [67] learning curve is given by:

$$T_i = \begin{cases} T_1 j^{-e} & \text{if} \quad j < j_s \\ T_{\min} & \text{if} \quad j \geq j_s \end{cases}$$

where T_i is the time (cost) needed to do (manufacture) the jth repetition (unit), T_1 the time (cost) to do (manufacture) the repetition (unit) at the first time, and e the slope of the learning curve. In addition, $e = -(\log(\varphi/100))/\log 2$ where φ is the learning rate given in percentage and j_s the number of repetitions (units) necessary to be done to achieve the minimum time (standard time). Moreover, $T_{\min}$ considers a parallel learning relationship which is related to the setup so that if we consider S_1 the cost of the first setup, the cost of the nth setup is given by:

$$S_n = \begin{cases} S_1 n^{-e} & \text{if} \quad n < n_s \\ S_{\min} & \text{if} \quad n \geq n_s \end{cases} \tag{1}$$

where S_n represents the cost of the nth setup and $S_{\min}$ the minimum setup cost obtained as $n = n_s$.

It is supposed that the function that defines forgetting or knowledge decay in the setup follows the function of Loftus [68] as expressed below:

$$d_i(\delta t) = d_i e^{-\gamma \delta t}, \tag{2}$$

where $d_i(\delta t)$ is the residual of the knowledge assimilation (or the strength of memory) of the ith repetition by lapsed time δt. The lapse time is calculated as $\delta t = t_r - t_e$ where t_e and t_r are the times when the information is encoded and retrieved, respectively. In addition, d_i is the quantity of knowledge assimilated at repetition i and γ the forgetting exponent. By generalizing Eq. (2) for setup i, the strength of memory at the beginning of setup i is written as follows:

$$m_i = \sum_{j=0}^{i-1} d e^{-\gamma(t_{r_i} - t_{e_j})} - 1, \qquad 1 \leq i \leq n. \tag{3}$$

Here, $t_{e_0} = 0$ is the time when the first setup occurs and t_{r_j} the retrieval time of knowledge obtained in i setup. In addition, $d = 1$ and n is the number of setups. Using Eqs. (1) and (3), we have:

$$\tilde{S}_i = \begin{cases} S_1(m_i + 1)^{-e} & \text{if} \quad m_i < n_s \\ S_{\min} & \text{if} \quad m_i \geq n_s \end{cases} \tag{4}$$

where $\tilde{S}_i$ represents the cost of the ith setup resulting from forgetting or knowledge decay.

The setup cost variation in different repetitions of setups is shown in Table 2. Suppose that $d = 1$ unit of information is acquired at Repetition 1 at time 0; thus, let the period of two consecutive setups be 10 units, e.

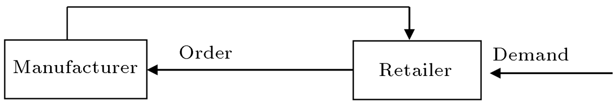

Figure 1. Two-level supply chain model.

According to Table 2, setup cost in the case of forgetting is always higher than that in the case of without forgetting. Of note, when there is no decay in setup knowledge and $i = n$, we have $m_n = n - 1$ based on Eq. (3); hence, Eq. (4) turns into Eq. (1). However, it is necessary to modify Eq. (3) with the purpose of accommodating the supposition of equal lot sizes, which is equivalent to the equal cycle times: $t_{e_i} = Q/D$ where $i = 1, 2, \cdots, m$. Therefore, Eq. (3) is expressed for $i \in [1, m]$ as:

$$m_i = \begin{cases} \sum_{j=0}^{i-1} e^{\frac{-\gamma(i-1-j)Q}{D}} - 1, & \gamma > 0 \\ i - 1, & \gamma = 0 \end{cases}$$

or:

$$m_i = \begin{cases} -\dfrac{e^{\frac{-\gamma(i-1)Q}{D}} - 1}{e^{\frac{\gamma Q}{D}} - 1}, & \gamma > 0 \\ i - 1, & \gamma = 0 \end{cases} \tag{5}$$

This study aimed to develop an integrated inventory model for a two-level supply chain with a single manufacturer and a single retailer (see Figure 1) where the manufacturer fulfills the retailer's demand.

2.2. Non-coordinated supply chain model

First, the optimal policies for the manufacturer and retailer are obtained independently. In this non-coordinated situation, each supply chain participant maximizes its profit individually.

Figure 2 depicts the structure of the supply chain. It consists of a single-setup multi-delivery policy for a single-supplier and single-retailer chain. The order the manufacturer sends to the retailer is in m batches. If the retailer's order quantity is of Q units, the manufacturer fabricates mQ units in one setup at a manufacturing rate of P, when $P > D$, to decrease the setup cost. In order to decrease the inventory cost, the manufacturer sends the lot size Q to the retailer over m times where m is a positive integer. Therefore, the length of each production cycle for the manufacturer is mQ/D and that of each retailer ordering cycle for the retailer is Q/D.

Table 2. Setup cost in different repetitions of setups.

	Second repetition of setup	**Third repetition of setup**	**Fourth repetition of setup**
Setup cost (\$) (with forgetting, $\gamma = 0.04$)	879.63	828.77	801.69
Setup cost (\$) (without forgetting, $\gamma = 0$)	840.89	759.83	707.10

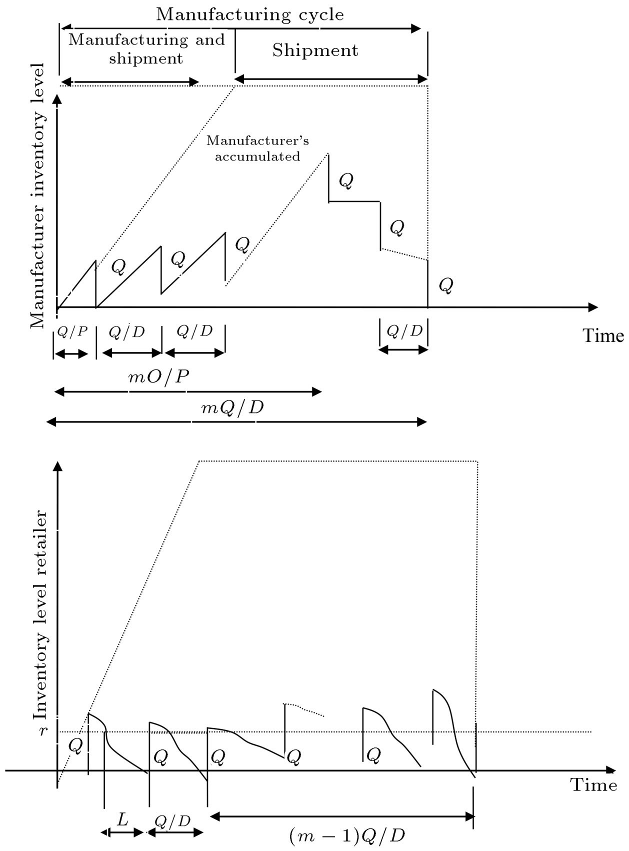

Figure 2. The inventory pattern for manufacturer and retailer.

2.2.1. The retailer's inventory model

The main assumption in this subsection is that the retailer operates a continuous review of inventory policy (Figure 2). In the inventory policies of the deterministic review, the order quantity is frequently computed under the supposition of constant lead time. However, it can be stated from the practical point of view that the lead time is manageable by the crashing cost. To obtain a more realistic inventory situation, the effects of the function of the investment cost for the lead time reduction should be carefully studied. To be specific, the role of the capital investment $R(L)$ in reducing the lead time is a function of the lead time L. The investment needed for shortening the lead time is a convex and strictly decreasing function. The retailer takes into account the usual (Q, r) continuous review of inventory policy with deterministic variable lead time.

The retailer's expected profit is calculated by the difference between the revenue and total inventory costs. Here, the total inventory cost is comprised of purchasing, ordering, holding, and lead time crashing costs. If we consider p as the selling price, w is the purchasing cost and D the demand; then, pD is the retailer's revenue and wD the purchasing cost. The retailer's orders quantity is Q and the expected cycle length is Q/D. Therefore, the ordering cost for the retailer is AD/Q. When the amount of stock reaches the reorder point r, an order of Q units is placed by the retailer. The expected stock level prior to the reception of an order is $r - \mu L$, and the expected amount of the stock after the arrival of the order is $Q + r - \mu L$. The average amount of the stock during a cycle is determined as $\left(\frac{Q}{2} + r - \mu L\right)$, indicating that the retailer's expected holding cost is computed as $h_r\left(\frac{Q}{2} + r - \mu L\right)$. The lead time crashing cost function for the retailer is $\frac{D\alpha e^{-\theta L}}{Q}$. Thus, the retailer's profit function is:

$$(p-w)D - \frac{AD}{Q} - h_r\left(\frac{Q}{2} + r - \mu L\right) - \frac{D}{Q}\alpha e^{-\theta L}. \quad (6)$$

The main objective here is to maximize the retailer's profit subject to an indicated fill rate. The fill rate is defined as the partial demand covered directly from the stock. The fill rate is in fact a service measure represented by β which is determined by Eq. (7) shown in Box I. The fill rate (β) is the fraction of the customers' demands satisfied normally as expressed below:

$$\beta = 1 - \frac{E(X-r)^+}{Q} \Rightarrow E(X-r)^+ = (1-\beta)Q. \quad (8)$$

Let $K = r - \mu L$ where K is the safety stock. Thus, from Eq. (6), we have:

$$TP_R = (p-w)D - AD/Q - h_r\left(\frac{Q}{2} + K\right) - \frac{D}{Q}\alpha e^{-\theta L}. \quad (9)$$

According to Gallego and Moon [69], the following inequality is required to obtain the least favorable distribution in F:

$$E(X-r)^+ \le \frac{\sqrt{\sigma^2 L + (r-\mu L)^2} - (r - \mu L)}{2},$$

for any $F \in \mathcal{F}$.

$$\beta = \frac{\text{Expected demand satisfied per replenishment cycle}}{\text{Expected demand per replenishment cycle}}. \quad (7)$$

Box I

Of note, the upper bound is tight. The value of K_β can be obtained considering K_β as the safety stock with respect to β, as shown in the following:

$$\frac{\sqrt{\sigma^2 L + K_\beta^2} - K_\beta}{2} = (1-\beta)Q$$

$$\Rightarrow K_\beta = \frac{\sigma^2 L}{4(1-\beta)Q} - (1-\beta)Q. \tag{10}$$

According to Eq. (9), $TP_R(Q,L)$ is:

$$TP_R(Q,L) = (p-w)D - \frac{AD}{Q} - h_r Q\left(\beta - \frac{1}{2}\right) - \frac{D}{Q}\alpha e^{-\theta L} - \frac{\sigma^2 L h_r}{4(1-\beta)Q}. \tag{11}$$

Taking the first partial derivatives of Eq. (11) with respect to Q and L, we have:

$$\frac{\partial TP_R(Q,L)}{\partial Q} = \frac{AD}{Q^2} - \left(\beta - \frac{1}{2}\right)h_r + \frac{D}{Q^2}\alpha e^{-\theta L} + \frac{\sigma^2 L h_r}{4(1-\beta)Q^2}, \tag{12}$$

$$\frac{\partial TP_R(Q,L)}{\partial L} = \frac{\theta D}{Q}\alpha e^{-\theta L} - \frac{\sigma^2 h_r}{4(1-\beta)Q}. \tag{13}$$

Now, the necessary conditions for the optimality of $TP_R(Q,L)$ include the following:

$$\frac{\partial TP_R(Q,L)}{\partial Q} = 0, \qquad \frac{\partial TP_R(Q,L)}{\partial L} = 0.$$

The following equations are obtained based on Eqs. (12) and (13):

$$\frac{AD}{Q^2} + \frac{D}{Q^2}\alpha e^{-\theta L} - \left(\beta - \frac{1}{2}\right)h_r + \frac{\sigma^2 L h_r}{4(1-\beta)Q^2} = 0,$$

$$\frac{\theta D}{Q}\alpha e^{-\theta L} - \frac{\sigma^2 h_r}{4(1-\beta)Q} = 0.$$

Followed by solving the above two systems of equations, the lot size and the lead time are given by:

$$Q = \sqrt{\frac{AD + D\alpha e^{-\theta L} + \frac{\sigma^2 L h_r}{4(1-\beta)}}{\left(\beta - \frac{1}{2}\right)h_r}}, \tag{14}$$

$$L_\beta = \frac{1}{\theta}\log\frac{4\theta D\alpha(1-\beta)}{\sigma^2 h_r}. \tag{15}$$

Followed by substituting Eq. (15) into Eq. (10), K_β is written as:

$$K_\beta = \frac{\sigma^2}{4(1-\beta)\theta Q}\log\frac{4\theta D\alpha(1-\beta)}{\sigma^2 h_r} - (1-\beta)Q. \tag{16}$$

Taking different second partial derivatives of Eq. (11) with respect to Q and L, we have:

$$\frac{\partial^2 TP_R(Q,L)}{\partial Q^2} = -\frac{2AD}{Q^3} - \frac{2D}{Q^3}\alpha e^{-\theta L} - \frac{2\sigma^2 L h_r}{4(1-\beta)Q^3},$$

$$\frac{\partial^2 TP_R(Q,L)}{\partial L^2} = -\frac{D}{Q}\alpha\theta^2 e^{-\theta L},$$

$$\frac{\partial^2 TP_R(Q,L)}{\partial Q \partial L} = -\frac{D}{Q^2}\alpha\theta e^{-\theta L} + \frac{\sigma^2 h_r}{4(1-\beta)Q^2}.$$

Now, the optimality condition of $TP_R(Q,L)$ can be verified. Obviously, we have:

$$\frac{\partial^2 TP_R(Q,L)}{\partial L^2} = -\frac{D}{Q}\alpha\theta^2 e^{-\theta L} < 0.$$

For $0.5 < \beta < 1$ and based on Eq. (15), we have:

$$\left(\frac{\partial^2 TP_R(Q,L)}{\partial Q^2}\right)_{(Q,L_\beta)} = -\left(\frac{2AD}{Q^3} + \frac{2D}{Q^3}\alpha e^{-\theta L} + \frac{2\sigma^2 L h_r}{4(1-\beta)Q^3}\right) < 0,$$

and:

$$\left(\frac{\partial^2 TP_R(Q,L)}{\partial Q^2}\frac{\partial^2 TP_R(Q,L)}{\partial L^2} - \left(\frac{\partial^2 TP_R(Q,L)}{\partial Q \partial L}\right)^2\right)_{(Q,L_\beta)} = \frac{D^2\alpha^2\theta^2 e^{-2\theta L}}{Q^4} + \frac{2AD^2\alpha\theta^2 e^{-\theta L}}{Q^4} + \frac{D\alpha\theta h_r \sigma^2 e^{-\theta L}}{2(1-\beta)Q^4} + \frac{D\alpha\theta\sigma^2 h_r e^{-\theta L}}{2(1-\beta)Q^4}(\theta L - 1) > 0.$$

It can be concluded that $TP_R(Q,L)$ is a concave function in Q and L.

Eqs. (14), (15), and (16) give the optimal values for the lot size, lead time, and safety stock, respectively. If we substitute these values into Eq. (11), we can calculate the optimal profit for the retailer.

2.2.2. The manufacturer's inventory model

The setup cost for the ith cycle is $\tilde{S}_i$. When the vendor fabricates the first Q units, he or she sends these units to the retailer and then, the manufacturer delivers every Q/D unit of time until the inventory level drops to zero (Figure 2). Therefore, the average inventory per unit time is computed as follows:

$$I_m = \left\{\left[mQ\left(\frac{Q}{P} + (m-1)\frac{Q}{D}\right) - \frac{m^2Q^2}{2P}\right] - \left[\frac{Q^2}{D}(1 + 2 + \cdots + (m-1))\right]\right\}\frac{D}{mQ} = \frac{Q}{2}\left[(m-1) + (2-m)\frac{D}{P}\right].$$

Thus, the manufacturer's holding cost per unit of time is $h_m \frac{Q}{2}\left[(m-1)+(2-m)\frac{D}{P}\right]$; manufacturer's revenue is wD where w is the wholesale price and D is the demand; and manufacturer's manufacturing cost is cD where w is the fabrication cost and D is the demand.

The expected amount of the defective items in a run of size mQ with the given probability of θ_1 under which the process can go out of control is given by $\frac{m^2Q^2\theta_1}{2}$. Consequently, the defective cost per unit of time is $\frac{xmQD\theta_1}{2}$; and opportunity cost of quality improvement investment is $yb\left[\log\left(\frac{\theta_0}{\theta_1}\right)\right]$.

As a result, manufacturer's expected profit for all cycles is equal to the revenue minus the total inventory cost, which is comprised of the setup, manufacturing, holding, and rework costs as well as the quality improvement investment.

$$\begin{aligned}TP_M(m) =&(w-c)D - \frac{D}{mQ}\frac{\sum_{i=1}^m \tilde{S}_i}{m}\\ &- \frac{h_mQ}{2}\left[(m-1)+(2-m)\frac{D}{P}\right]\\ &- \frac{xmQD\theta_1}{2} - yb\left[\log\left(\frac{\theta_0}{\theta_1}\right)\right].\end{aligned} \tag{17}$$

The manufacturer decides on the optimal number of lots m^* reacting to the buyer's optimal lot size Q^* and optimal lead time L^*.

The optimal number of lot m^* has upper $m_{\max}$ and lower $m_{\min}$ bounds. To compute these bounds, the following cases of learning should be taken into account.

(i) Maximum learning in the setup cost, i.e., $\tilde{S}_i \to S_{\min}$. Eq. (17) turns into the following one:

$$\begin{aligned}TP_M(m) =&(w-c)D - \frac{DS_{\min}}{mQ}\\ &- \frac{h_mQ}{2}\left[(m-1)+(2-m)\frac{D}{P}\right]\\ &- \frac{xmQD\theta_1}{2} - y\left[b\log\left(\frac{\theta_0}{\theta_1}\right)\right].\end{aligned} \tag{18}$$

If we regard the above equation as a continuous function of m, then: $\frac{\partial^2 TP_M}{\partial m^2} = -\frac{2D}{m^3Q}S_{\min} < 0$, indicating that $TP_M(m)$ is concave with an unique $m_{\min}$ determined by setting $\frac{\partial TP_M}{\partial m} = 0$ to get:

$$m_{\min} = \frac{1}{Q}\sqrt{\frac{DS_{\min}}{\frac{h_m}{2}\left[1-\frac{D}{P}\right] + xD\theta_1/2}}. \tag{19}$$

(ii) No learning in the setup cost, i.e., $\tilde{S}_i = S_1$. Eq. (17) turns into:

$$\begin{aligned}TP_M(m) =&(w-c)D - \frac{DS_1}{mQ}\\ &- \frac{h_mQ}{2}\left[(m-1)+(2-m)\frac{D}{P}\right]\\ &- \frac{xmQD\theta_1}{2} - yb\left[\log\left(\frac{\theta_0}{\theta_1}\right)\right].\end{aligned} \tag{20}$$

If we regard the above equation as a continuous function of m, then $\frac{\partial^2 TP_M}{\partial m^2} = -\frac{2D}{m^3Q}S_1 < 0$, indicating that $TP_M(m)$ is concave with a sole $m_{\max}$ calculated by solving $\frac{\partial TP_M}{\partial m} = 0$ to obtain:

$$m_{\max} = \frac{1}{Q}\sqrt{\frac{DS_1}{\frac{h_m}{2}\left[1-\frac{D}{P}\right] + xD\theta_1/2}}. \tag{21}$$

The optimal number of lots m^* that maximizes Eq. (17) is enclosed between the bounds given in Eqs. (19) and (21).

The retailer in the non-coordinated supply chain selects its optimal policy (Q^*, L^*) and then, the manufacturer determines the optimal numbers of shipments m^*. Thus, the total profit of the system is calculated as follows:

$$TP_{NSC}(Q^*, m^*, L^*) = TP_R(Q^*, L^*) + TP_M(m^*). \tag{22}$$

2.2.3. The centralized supply chain model

In a centralized model, all supply chain participants are parts of the same corporation called vertical integration. It is assumed that there is a sole decision-maker in a centralized system who has access to the entire information and determines the maximum joint expected total profit of the centralized model. Nevertheless, this situation of control mechanism is not applied unless both manufacturers and retailers pursue the same objective, i.e., maximization of the joint expected profit rather than their own profit separately. It is also assumed that both manufacturers and retailers in the inventory model are in a long-term strategic partnership; hence, they are willing to cooperate and share information with each other in order to obtain benefits for both parties. This is the reason why they jointly determine the best policy for the centralized inventory model. The optimal policy of this coordinated system is found by solving the following problem:

$$\begin{aligned}TP_{SC}(Q,m,L) = (p-c)D - \frac{D}{mQ}\frac{\sum_{i=1}^m \tilde{S}_i}{m}\\ - h_m\frac{Q}{2}\left[(m-1)+(2-m)\frac{D}{P}\right]! - xmQD\theta_1/2\\ - yb\left[\log\left(\frac{\theta_0}{\theta_1}\right)\right] - \frac{AD}{Q} - \frac{D}{Q}\alpha e^{-\theta L}\end{aligned}$$

$$-\left(\beta-\frac{1}{2}\right)h_rQ-\frac{\sigma^2Lh_r}{4(1-\beta)Q}. \tag{23}$$

Now, the necessary conditions for the optimality of $TP_{SC}(Q,m,L)$ for the fixed value of 'm' are given below:

$$\frac{\partial TP_{SC}(Q,m,L)}{\partial Q}=0, \qquad \frac{\partial TP_{SC}(Q,m,L)}{\partial L}=0.$$

Hence, the following equation can be obtained:

$$\frac{D}{mQ^2}\frac{\sum_{i=1}^{m}\tilde{S}_i}{m}-\frac{h_m}{2}\left[(m-1)+(2-m)\frac{D}{P}\right]$$

$$-\frac{xmD\theta_1}{2}+\frac{AD}{Q^2}+\frac{D\alpha e^{-\theta L}}{Q^2}$$

$$-\left(\beta-\frac{1}{2}\right)h_r+\frac{\sigma^2Lh_r}{4(1-\beta)Q^2}=0,$$

$$\frac{D}{Q}\alpha\theta e^{-\theta L}-\frac{\sigma^2h_r}{4(1-\beta)Q}=0.$$

Followed by solving the above two systems of equations, the lot size and lead time can be obtained as follows:

$$Q=\sqrt{\frac{\frac{D}{m}\frac{\sum_{i=1}^{m}\tilde{S}_i}{m}+AD+D\alpha e^{-\theta L}+\frac{\sigma^2Lh_r}{4(1-\beta)}}{\frac{h_m}{2}\left[(m-1)+(2-m)\frac{D}{P}\right]+xmD\theta_1/2+\left(\beta-\frac{1}{2}\right)h_r}}, \tag{24}$$

$$L_\beta=\frac{1}{\theta}\log\frac{4\theta D\alpha(1-\beta)}{\sigma^2h_r}. \tag{25}$$

The main objective here is to maximize $TP_{SC}(Q,m,L)$. The following results were obtained by taking different derivatives of Eq. (23) partially with respect to Q and L:

$$\frac{\partial^2TP_{SC}(Q,m,L)}{\partial Q^2}=-\frac{2D}{mQ^3}\frac{\sum_{i=1}^{m}\tilde{S}_i}{m}-\frac{2AD}{Q^3}$$

$$-\frac{2D}{Q^3}\alpha e^{-\theta L}-\frac{\sigma^2Lh_r}{2(1-\beta)Q^3}, \tag{26}$$

$$\frac{\partial^2TP_R(Q,L)}{\partial L^2}=-\frac{D}{Q}\alpha\theta^2e^{-\theta L}, \tag{27}$$

$$\frac{\partial^2TP_R(Q,L)}{\partial Q\partial L}=-\frac{D}{Q^2}\alpha\theta e^{-\theta L}+\frac{\sigma^2h_r}{4(1-\beta)Q^2}. \tag{28}$$

Now, the optimality condition of $TP_{SC}(Q,m,L)$ can be validated. Clearly, there is:

$$\frac{\partial^2TP_R(Q,L)}{\partial L^2}=-\frac{D}{Q}\alpha\theta^2e^{-\theta L}<0.$$

For $0.5<\beta<1$ we have:

$$\left(\frac{\partial^2TP_{SC}(Q,m,L)}{\partial Q^2}\right)_{(Q,L_\beta)}<0,$$

$$\left(\frac{\partial^2TP_R(Q,L)}{\partial Q^2}\frac{\partial^2TP_R(Q,L)}{\partial L^2}\right.$$

$$\left.-\left(\frac{\partial^2TP_R(Q,L)}{\partial Q\partial L}\right)^2\right)_{(Q,L_\beta)}=\frac{2D^2\alpha\theta^2e^{-\theta L}}{mQ^4}\frac{\sum_{i=1}^{m}\tilde{S}_i}{m}$$

$$+\frac{D^2\alpha^2\theta^2e^{-2\theta L}}{Q^4}+\frac{2AD^2\alpha\theta^2e^{-\theta L}}{Q^4}$$

$$+\frac{D\alpha\theta h_r\sigma^2e^{-\theta L}}{2(1-\beta)Q^4}+\frac{D\alpha\theta\sigma^2h_re^{-\theta L}}{2(1-\beta)Q^4}(\theta L-1)>0. \tag{29}$$

Therefore, it can be concluded that $TP_{SC}(Q,m,L)$ is a concave function in Q and L. Obviously:

$$\frac{\partial^2TP_{SC}(Q,m,L)}{\partial m^2}=-\frac{6D}{m^4Q}\sum_{i=1}^{m}\tilde{S}_i<0.$$

Therefore, $TP_{SC}(Q,m,L)$ is concave in m for the fixed values of Q and L, indicating that there must be an optimal m^{**} to satisfy the following relation:

$$TP_{SC}(Q,m,L)(m^{**}-1)\leq TP_{SC}(Q,m,L)(m^{**})$$

$$\geq TP_{SC}(Q,m,L)(m^{**}+1). \tag{30}$$

3. Numerical illustration

In order to better understand the mechanism of the inventory model, consider the following case whose required data was derived hypothetically from the literature of inventory.

Suppose that ABC is a telecommunication firm, assuming a joint venture of two companies called A and B to manufacture smartphones. This firm has been established due to the strategic tie-up between A and B. While Company A (known as manufacturer) fabricates the smartphones, Company B (known as retailer) deals with the marketing and sales department.

The manufacturing capacity of Company A is $P=3000$ units per year, and its manufacturing cost per unit is $c=\$80$. For Company A, the wholesale price per unit is $w=\$90$, and the holding cost of each unit per year is $h_m=\$30$. Given that no manufacturing system is perfect, Company A invested a capital amount of $(W(\theta_1))$ in improving the process quality (reducing out-of-control probability from θ_0 to θ_1), considering $\theta_0=0.0002$, $\theta_1=0.00016$, and $\delta=0.0025$. The fraction opportunity cost of the capital invested by Company A is $y=0.1$ per \$ per year, and the cost of rework for defective items per unit is $x = \$10$. Customers' demands for smartphone at the end of Company B is $D = 600$ units a year. In this regard, Company B employs 'Job Training Programme' to train its employees. As observed, the learning rate is $\varphi = 0.89$ and the forgetting exponent is $\gamma = 0.11$ of its employees. Company B calculated $S_1 = \$1000$ per setup, $S_{\min} = \$400$ per setup, and holding cost per unit per year as $h_r = \$26$. In addition, the ordering cost is $A = \$200$ per order and as observed, there is a lead time between placing and receiving the order with parameter $\sigma = 7$ units/week. Company B whose priority is their customer's satisfaction used crashing cost to reduce the lead time as much as possible with parameter $\alpha = 105$, $\theta = 1.4$. It also sets the retail price as $p = \$100$ per unit. Now, a problem may arise, i.e., how to determine the optimal number of shipments for Company A and optimal order quantity, lead time, and safety stock for Company B in two different situations: when they work as individual entities and when they develop an integrated system.

3.1. Decentralized system

In case the system is decentralized, the retailer dominates the manufacturer. In this case, the retailer decides about the terms and conditions based on which the manufacturer has to develop the optimal policy to maximize the total profit of the system. As observed in Table 3, in the case of decentralized system, the number of shipments does not have any effect on the retailer's decision (see Eqs. (14) and (15)). In this case, the retailer's optimal policy is as follows:

- Order quantity = 109 units;
- Lead time = 1.22 weeks;
- Safety stock = 2.66 units;
- Profit = \$3255 per year.

As observed in Table 3, as the manufacturer increases the number of shipments to fulfill the retailer's demand, their profit increases first and then decreases. In this case, the optimal strategy for the manufacturer to maximize the total profit of the system is suggested below:

- Number of shipments = 2;
- Profit = \$1507 per year. So, the total profit of the system is \$4763 per year.

3.2. Centralized system

In the case of a centralized system, both retailer and manufacturer work as a single entity. As observed in Table 3, the retailer's profit from his point of view is maximum when the number of shipments is 2; however, the manufacturer assumes his profit maximum with a single shipment. In order to remain in the system, the retailers need to make a compromise concerning their interest. In this situation, the optimal solution is given below and also shown in Table 3:

- **Retailer** Order quantity = 216 units, Lead time = 1.22 weeks, safety stock = 2.66 units, and profit = \$2597 per year.
- **Manufacturer:** Number of shipments = 1, profit = \$2470 per year.
- **Integrated system:** Profit = \$5067 per year.

Majors findings from Table 3 can be summarized as follows:

- According to Table 3, when the retailer and manufacturer work as a single entity, the retailer's overall profit decreased by 20%, while the manufacturer's profit increased by 63%;
- The profit in a centralized system increased by 6%, compared to that in a decentralized one. Apparently, the centralized system outweighs its decentralized counterpart. Although the retailers in the centralized system lose interest to remain in the supply chain, they are aware that they are losing to form the system. Therefore, for a better coordination, the manufacturer must give an incentive to the retailer in the form of either sharing the transportation cost or reducing the cost so that the retailer keeps the interest to coordinate with each other. In doing so, the efficiency of the system increases which is the main objective of all members of the supply chain. This result provides managerial insight for different players for better coordination.

Table 3. Summary of the results under decentralized and centralized decisions.

Players	Parameters	Decentralized			Centralized		
		$m=1$	$m=2$	$m=3$	$m=1$	$m=2$	$m=3$
Retailer	L^*	1.22	1.22	1.22	1.22	1.22	1.22
	Q^*	109	109	109	216	125	92
	K^*	2.66	2.66	2.66	2.66	2.66	2.66
	TP_R^*	3255	3255	3255	2597	3230	3215
Manufacturer	TP_M^*	151	1508	1046	2470	1598	1197
Supply Chain	TP_{SP}^*	3406	4763	4301	5067	4829	4413

3.3. Sensitivity analysis

Different parameters such as the service level constraint, quality of the product, and learning-forgetting play significant roles in collaboration with different members in the supply chain. It is relevant to discuss how the changes in these input data affect the total profit of the supply chain.

3.3.1. The effect of learning-forgetting on the setup cost

Figures 3 and 4 show the effect of learning-forgetting in the setup cost on the order quantity and profit of the supply chain. For the fixed value of m, the ordering quantity decreases and the profit of the supply chain increases due to the combined effect of learning-forgetting, implying that the manufacturer exhibits greater flexibility to change while producing smaller lots through setup cost reduction programs.

According to Figure 5, to achieve a higher level of quality of the product, additional cost is required; hence, the profit of the supply chain would decrease. Given that it is up to the decision-maker to determine how to use this information to make a tradeoff between the quality level and profit of the system according to the market requirement.

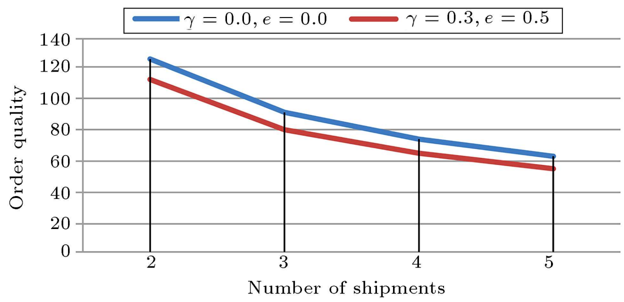

Figure 3. The effect of learning-forgetting on order quantity.

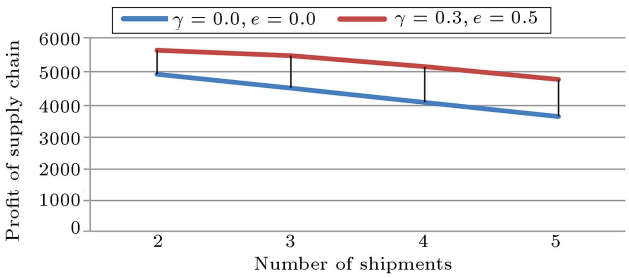

Figure 4. Effect of learning-forgetting on profit of supply chain.

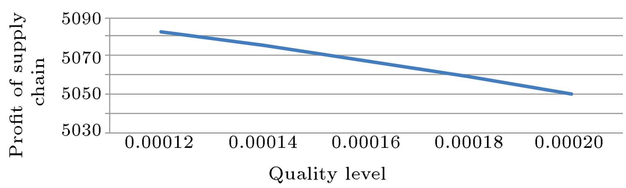

Figure 5. Quality level vs profit of supply chain.

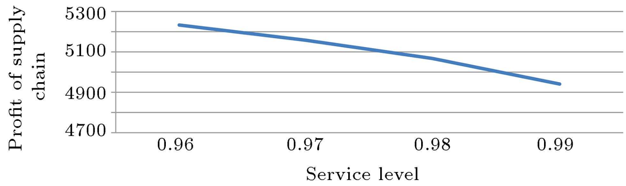

Figure 6. Service level vs profit of supply chain.

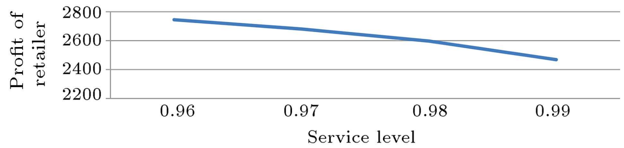

Figure 7. Service level vs profit of retailer.

3.3.2. Effect of service level constraint on profit of supply chain

The effect of the service level constraint on the profit of the supply chain is illustrated in Figure 6. As observed in this figure, under the centralized decision model, the profit of the supply chain decreases with an increase in the service level constraint. In other words, the higher the service level, the lower the supply chain profit. In addition, upon increasing the service level from 0.96 to 0.97 and from 0.98 to 0.99, the profits of the system would decrease by 1.4% and 2.5%, respectively. This observation gives the needed direction to the decision-makers to come to the conclusion that the higher service level is beneficial to any organization or they must work out other policies.

3.3.3. Effect of service level constraint on profit of retailer

The effect of the service level constraint on the retailer's profit in the centralized model is shown in Figure 7 where the retailer's profit in the centralized decision model decreases upon increasing the service level constraint. In other words, the higher the service level, the lower the retailer's profit.

4. Conclusion

At the present time, organizations have recognized the importance of the centralized decision. In this direction, the organizations use the lead time and the service level as competitive advantages to differentiate themselves from others in the market. The lead time and the service level are essential elements in any inventory management system. In practical situations, the lead time is controllable by adding a crashing cost.

This paper developed a supply chain model for the retailer and the manufacturer to the optimal lot size, the lead time, the safety stock, and the number of shipments under the effect of learning-forgetting phenomenon on the setup cost. The fabrication process of manufacturer was not perfect and the certain level of the product quality could be attained with the additional cost. The proposed inventory model demonstrated that the centralized decision was better than decentralized one. To ensure a better coordination for the supply chain, different players must adopt an incentive-based mechanism in order that they maintain the interest in coordinating with each other. It was also observed that if all players decided to increase the service level, then they must make a compromise about their profit. A similar behavior was observed in the case of product quality. Due to the effect of learning-forgetting, the order quantity decreased and the profit of the centralized system increased.

There are many potential avenues for further research. In this study, a single-retailer, single-manufacturer, and single-product supply chain was considered, but in the future version of the present study, this inventory model can be extended to the case of multiple manufacturers, multiple retailers and multiple products. From the literature, it was observed that uncertainties were associated with demand and different cost parameters. Thus, considering these parameters as imprecise parameters might be an interesting research problem.

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