Joint determination of purchasing and production lot sizes in an unreliable production system

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Abstract

This paper discusses a production-inventory system under random machine breakdown. Holding safety stock is the common way to mitigate the effect of random machine breakdown on shortages that may occur during machine repair time. Since holding safety stock can be costly, especially for expensive products, this paper investigates an alternative strategy in which it is assumed that the production manager can purchase the same products from a supplier in order to meet the demands that may be lost due to depletion of the inventory after the machine breakdown. The supplier has known lead-time and reliability with the quality assured products. Despite holding safety stock, purchasing occurs only when the machine breakdown happens. The question is about the optimal amount of production and purchasing lot sizes to minimize the total expected costs. The optimality of the model is investigated when failure and repair time follow an exponential distribution, and a computational algorithm for finding the optimal lot sizes is presented. A comparison between the purchasing strategy and holding safety stock is performed through a sensitivity analysis regarding some effective parameters. This study shows that using the purchasing strategy when holding or production cost rises is more beneficial than holding safety stock.

Keywords: Machine breakdown and repair, Economic production quantity, safety stock, Purchasing, lot sizing.

1. Introduction

One of the significant issues for the production managers is the disruptions of the production facility during the production run time. The classical economic production (EPQ) models assume that the production facilities always are failure-free. However, in practical situations, the production system may stop due to age or usage, such as fatigue and corrosion [1]. This disruption may beget an out of stock situation swiftly result in losing the margins that a business could have gained had it met the demand and extend to negatively affecting the future demand of the firm [2]. Since failures are unavoidable, the production manager should have practical solutions to deal with such disruptions. In recent decades, researchers suggested various solutions to mitigate machine breakdowns consequences; among them preventive maintenance and keeping safety stock have been the most common in the literature.

One of the first studies related to implementing of the safety stock policy on stochastic machine breakdown cases was developed by Groenevelt et al. [3]. They addressed the safety stock needed to maintain the designated management service level under exponential failure and general repair time distributions. They showed the expected inventory increase with the failure rate, service level, demand rate, and repair times.

Cheung and Hausman [4] modeled the joint implementation of preventive maintenance and safety stocks in an EPQ system under the general time-to-failure distribution function. They assumed the production and demand rates are equal in an ordinary production period.

Dohi et al. [5] revisited the model of Cheung and Hausman [4] from the theoretical point of view by assuming that the lifetime of the production machine obeys an exponential distribution. They showed that the hazard rate of the repair time distribution could play an important role in to design of optimal control of the preventive maintenance schedule and safety stock. Abboud [6] developed an imperfect EPQ model by assuming that the shortage in the system is partially backlogged. Considering the discrete-time through which the failures and repair times are distributed geometrically, he modeled the inventory-production system as a Markov chain and developed an efficient algorithm to compute the cost function. Giri et al. [7] assumed that the failure rate of the machine is dependent on the production rate, and hence they investigated the optimal values of production rate and production quantity in an imperfect EPQ system. They developed the model, with and without considering safety stock policy. Chelbi and Rezg [8] considered the joint effects of the safety stock and age-based preventive maintenance by developing an analytical model and a numerical procedure to determine stock lot size and the age at which preventive operation must be performed. They showed that the safety stock lot size is sensitive to inventory and maintenance costs variation, whereas the optimal value of the age for preventive maintenance is limited by the constraint of the minimum required availability level. Gharbi et al. [9] also presented a similar study. They demonstrate that how the cost-based measure can be used a basis for determining the optimal buffer stock and the scheduled preventive maintenance period. El-Ferik [10] presented similar research by assuming that the maintenance operation is not performed entirely and cannot restore the system to its primary state. Chakraborty et al. [11] introduced an EPQ model by considering the simultaneous effects of process deterioration, machine breakdown, and preventive maintenance on lot-sizing decisions. Assuming that defective items are not identifiable during the production period, they fixed the warranty cost for the sold items. Sana and Chaudhuri [12] considered the impact of machine breakdown on non-confirming quality items in an imperfect EPQ model, analyzing the joint effect of the preventive maintenance and the variable safety stock on optimal production rate and lot-sizing decisions. Sana [13] develops a model to determine the optimal product reliability and production rate that achieves the most significant total integrated profit for an imperfect manufacturing process. Chiu et al. [14] studied a production system that may produce defective items randomly, and it is also subject to a random machine failure. To prevent the shortage situation from happening, they assumed there is a safety stock level to deal with the possibility of stoppage occurrence in the very earlier stage of the production. Sana [15] presented a three-layer supply chain with perfect and imperfect quality items and considered the impact of business strategies such as order size, production rate, unit production cost, and idle times in different sectors on collaborating marketing systems. Chakraborty and Giri [1] presented the model similar to Sana and Chaudhuri's [12] model with the difference that, they discussed the optimality of the model and suggested a computational method to solve the problem. Sana [16] considered the simultaneous effect of preventive maintenance, keeping buffer stock, and minimal repair warranty in an imperfect production system. Prakash et al. [17] presented a production-inventory model with discrete random machine breakdown and discrete stochastic corrective and preventive repair times. They assumed that the demand rate follows a discrete stochastic distribution. Zhang et al. [18] used a dynamic method for the production of lot-sizing with machine failures in which the average cost is minimized instead of the expected one. Paul et al. [19]presented a disruption recovery model for a single-stage EPQ system under the random stoppage. Their model maximizes the total profit during the recovery time window by generating a revised preventive action plan after the occurrence of disruption. Taleizadeh et al. [20] developed a single-vendor/single-buyer model with random machine breakdown, multiple shipments, and keeping safety stock capability. They assumed both batch lot size and distance between two shipments are identical, and the buyer pays transportation cost. Öztürk [21] investigated optimal production run time on an EPQ system under machine breakdown situations with inspection and rework capability. Nobil et al. [22] considered rework and inspection in an imperfect multi-item single machine production system. Poursolta et al. [23] extended an EPQ model with deteriorating products considering random machine breakdown and stochastic repair time. Pal and Adhikari [24] developed an imperfect EPQ model in which production is executed mainly by the original machine. But when the system faces disruption, the buffer of it continues the production.

In the earlier work stated above, preventive maintenance and holding safety stocks are discussed as the proactive and reactive approaches respectively to deal with the effect of random machine breakdown on the production system's costs and service level. However, sometimes the expensive costs of these approaches are not beneficial for the system due to high product holding or maintenance costs. In such situations, it may be advantageous to use an external supplier to meet the demand while the machine is being repaired. The issue is not necessarily about the final product. It could be related to a standard piece of a final product manufactured by the main supplier in which he/she is responsible for supplying the piece according to a long-term contract. In this case, the supplier can purchase from smaller suppliers with lower reliability to prevent shortages in urgent situations. For example, items such as printed boxes, plastic bottles, or computer chips, could be produced by other manufacturers after minor changes to their production facilities. Although buying items from an external supplier would result in some margin loss, it could compensate for part of the overhead cost, protect the supplier's reputation, and ensure future demand [2]. Purchasing from a supplier is a reactive approach and executed only if machine failure occurs.

Peymankar et al. [2] are the first who considered the purchasing approach in case of machine breakdown. In the proposed model, when machine failure happens, the machine undergoes corrective repair immediately, and during the repair time, the manufacturer has an option to purchase from a supplier with specific reliability. The production time before breakdown and repair time both follow an exponential distribution. The optimal lot sizes for production and purchasing were found through an exhaustive enumeration in a numerical example. They also investigated the effects of revenue sharing and price discount contracts on the optimal lot sizes. Deiranlou et al. [25] also considered the joint effect of holding safety stock, and the purchasing policy, assuming the supplier zero lead-time in machine breakdown case. It is worth noting that the supplier lead-time in [2], and [25] is assumed to be zero that is a somewhat simplified assumption. This implies that the mathematical formulation and conclusion cannot be justified for general situations. Although the supplier lead-time in EPQ systems has been considered in several articles such as [26], in this particular case, none of the articles have examined the lead-time. The significant contribution of this study is to develop a stochastic model in which the supplier has a fixed non-zero lead-time. This assumption brings the model closer to reality; however, it will make the model more complicated. Furthermore, Peymankar et al. [2] just found the optimal lot sizes in a numerical example with an exhaustive numerical search. In this paper, the optimality of the new stochastic model is discussed, and a computational method is presented to find the optimal values.

The rest of this paper is structured as follows. In section 2, the problem state notation and the essential assumptions of the model are defined. This is followed by the development of the mathematical model under general failure and repair time distributions in section 3. In the case of exponential distributions, the optimality of the model is studied, and the solution approach for this case is proposed. A numerical example is presented in section 4 to determine the optimum values of production and purchasing lot sizes and the sensitivity of the essential parameters are examined. A comparison between the purchasing strategy and the safety stock policy is made in this section. Finally, section 5 concludes the paper and proposes directions for future researches.

2. Problem assumptions and notations

We make the following assumptions to develop the proposed model:

(1) The problem concerns a single-machine single-product environment. Time to machine failure and repair time are stochastic variables.

(2) Once the machine is broken down, the corrective repair starts immediately to restore it to its initial working condition.

(3) If the accumulated inventories are enough to meet demand during machine repair, then the next production cycle starts only when the inventory level reaches zero.

(4) If the inventories on hand are depleted during repair time, shortages will occur, and all of them will be lost.

(5) If a machine failure occurs during the production phase, there would be an option to order a lot size from a supplier with a fixed lead-time and reliability. The reliability means that the supplier may not be available with a known probability. This lot size is denoted as purchasing lot size.

The following notations are used throughout the paper:

t_f	random variable denoting machine time to failure
$G(t_f)$	cumulative distribution function corresponding to t_f
D	demand rate (units/time)
$P > D$	production rate (units/time)

t_r	random variable denoting corrective repair time
$H(t_r)$	cumulative distribution function corresponding to t_r
A	fixed setup cost for each production run ($/set up)
$A^{'}$	fixed ordering cost of purchasing ($/order)
θ	reliability of the supplier (%)
L	lead time for purchasing from the supplier
c_h	inventory holding cost ($/unit/time)
c_P	production cost ($/unit)
$c^{'}$	purchasing price from the supplier ($/unit)
c_r	corrective repair cost ($/time)
c_s	lost sale cost ($/unit)
Q	production lot size per cycle (decision variable)
$Q^{'}$	purchasing lot size from the supplier (decision variable)

3. Model formulation

Consider a production-inventory system in which, the production rate (P) and the demand rate (D) are assumed to be constant ($P > D$). Given that the time to failure is stochastic, the best possible situation for the production system is the one where no failure occurs during the production phase ($t_f > Q/P$). In this situation, as shown in Figure 1, the model would be the same as classical EPQ, and we have cycle time and total cost based on equations (1) and (2), respectively.

$$T_1 = \frac{Q}{D} \quad (1)$$

$$TC_1 = A + \frac{c_h (P - D) Q^2}{2P.D} + c_P.Q \quad (2)$$

If machine breakdown occurs during the production phase ($t_f < Q/P$), the corrective repair starts immediately. During the repair time, the accumulated on-hand inventory decreases at a constant D to meet the demand. If the repair is completed before that the on-hand inventory is depleted, the new production cycle is started when the inventory level is reduced to zero. If the on-hand inventory is totally exhausted before completion of the repair, all incurred shortages will be lost.

In order to avoid shortages that may be incurred after machine failure, the production system has an option to order from an external supplier with the lead-time of L. The purchasing is only feasible when the production system makes an order at least L time units before the on-hand inventory is exhausted. We notate the time to exhausting inventory on-hand by t_{idle} . obviously, $t_{idle} = pt_f / D - t_f$. t_{idle} is dependent on t_f and hence it is also stochastic variable. So based on the comparison between t_{idle} and L two conditions may occur. If $t_{idle} < L$ there will be no purchasing from the supplier, otherwise, the production system may or may not purchase from the supplier. The details of these two conditions are provided in sub-section 3.1 and 3.2.

3.1 Condition 1 ($t_{idle} < L$)

In this condition, the supplier lead-time is greater than the idle time; therefore, the production system does not make any purchasing order. If $t_r \le t_{idle}$, the on-hand inventory is enough to cover the demand during the repair and the new cycle starts once the on-hand inventory has been exhausted (Figure 2). For this situation, the cycle time and the total cost of the system are as follows, respectively:

$$T_2 = \frac{P.t_f}{D} \tag{3}$$

$$TC_2 = A + \frac{c_h (P-D) P.t_f^2}{2D} + c_P.P.t_f + c_r.t_r \tag{4}$$

If $t_r > t_{idle}$, the system faces a shortage and loses the demand up to the end of repair time (Figure 3). The corresponding cycle time and total cost for this situation are as equations (5) and (6), respectively.

$$T_3 = t_f + t_r \tag{5}$$

$$TC_3 = A + \frac{c_h (P-D) P.t_f^2}{2D} + c_P.P.t_f + c_r.t_r + c_s (D.t_r - (P-D) t_f) \tag{6}$$

Thus, according to the probability of each situation occurring, the expected length of each cycle and the expected total cost in a cycle in condition 1 are obtained through equations (7) and (8), respectively.

$$E(T_Q)=\int_0^{\frac{Q}{P}}\left\{\int_0^{\frac{(P-D)t_f}{D}}T_2 dH(t_r)+\int_{\frac{(P-D)t_f}{D}}^{\infty}T_3 dH(t_r)\right\}dG(t_f)+\int_{\frac{Q}{P}}^{\infty}T_1 dG(t_f) \tag{7}$$

$$E(TC_Q)=\int_0^{\frac{Q}{P}}\left\{\int_0^{\frac{(P-D)t_f}{D}}TC_2 dH(t_r)+\int_{\frac{(P-D)t_f}{D}}^{\infty}TC_3 dH(t_r)\right\}dG(t_f)+\int_{\frac{Q}{P}}^{\infty}TC_1 dG(t_f) \tag{8}$$

Now, based on the renewal reward theorem [27], the expected total cost per time unit is given by

$$ETC_Q=\frac{E(TC_Q)}{E(T_Q)} \tag{9}$$

3.2 Condition 2 ($t_{idle}>L$)

In this condition, it seems viable to make an order from the supplier. Regarding the repair time and the supplier's reliability, the manufacturing system may face five different situations during the repair. If the repair is completed before the time to order ($pt_f/D-L$, see Figure 4), there is no need to make any order. The formulation of the model for the cycle time and the total cost in this situation are similar to equations (3) and (4), respectively.

If the repair time passes the time to order, purchasing will make sense. However, due to unreliability, the supplier may not be available with the probability of $1-\theta$. In this situation, if

$t_r \le t_{idle}$ (

Figure 5), the system would not face any shortages and cycle time and total cost are obtained similar to equations (3) and (4).

In the previous situation, if $t_r > t_{idle}$, and the system will face shortages (Figure 6) and the corresponding cycle time and total cost for this case are similar to equations (5) and (6).

With the probability of θ the supplier will be available. In this situation, a fixed ordering cost of A' must be paid when making an order amount of Q'; while, the purchasing price for each item is $c^{'}$. As is shown in Figure 7, the ordered lot size can cover the demand during repair time only if $t_r < t_{idle} + Q^{'} / D$.

The corresponding inventory cycle and the total cost of the system for this case are obtained based on equations (10) and (11), respectively.

$$T_4 = \frac{P.t_f}{D} + \frac{Q^{'}}{D} \quad (10)$$

$$TC_4 = A + \frac{c_h (P-D) P.t_f^2}{2D} + \frac{c_h Q^{'2}}{2D} + A^{'} + c_p.P.t_f + c_r.t_r + c^{'}.Q^{'} \quad (11)$$

If $t_r > t_{idle} + Q^{'} / D$, the manufacturer runs out the purchasing quantities before the machine repair is completed and after that, the demands are lost up to the end of the repair. Figure 8 illustrates the inventory path diagram in this situation. So we have cycle time and total cost in a cycle as equations (12) and (13), respectively.

$$T_5 = t_f + t_r \quad (12)$$

$$TC_5 = A + \frac{c_h (P-D) P.t_f^2}{2D} + A^{'} + c_p.P.t_f + \frac{c_h Q^{'2}}{2D} + c_r.t_r + c^{'}.Q^{'} + c_s (D.t_r - (P-D)t_f - Q^{'}) \quad (13)$$

Now, we can obtain the expected cycle time and the expected total cost per cycle in condition 2.

$$E(T_{Q,Q^{'}}) = \int_0^{\frac{Q}{P}} \left\{ \int_0^{\frac{(P-D)t_f}{D}-L} T_2 dH(t_r) + (1-\theta)\left(\int_{\frac{(P-D)t_f}{D}-L}^{\frac{(P-D)t_f}{D}} T_2 dH(t_r) + \int_{\frac{(P-D)t_f}{D}}^{\infty} T_3 dH(t_r) \right) + \theta \left(\int_{\frac{(P-D)t_f}{D}-L}^{\frac{(P-D)t_f}{D}+\frac{Q^{'}}{D}} T_4 dH(t_r) + \int_{\frac{(P-D)t_f}{D}+\frac{Q^{'}}{D}}^{\infty} T_5 dH(t_r) \right) \right\} dG(t_f) + \int_{\frac{Q}{P}}^{\infty} T_1 dG(t_f) \quad (14)$$

$$E(TC_{Q,Q^{'}}) = \int_0^{\frac{Q}{P}} \left\{ \int_0^{\frac{(P-D)t_f}{D}-L} TC_2 dH(t_r) + (1-\theta)\left(\int_{\frac{(P-D)t_f}{D}-L}^{\frac{(P-D)t_f}{D}} TC_2 dH(t_r) + \int_{\frac{(P-D)t_f}{D}}^{\infty} TC_3 dH(t_r) \right) + \theta \left(\int_{\frac{(P-D)t_f}{D}-L}^{\frac{(P-D)t_f}{D}+\frac{Q^{'}}{D}} TC_4 dH(t_r) + \int_{\frac{(P-D)t_f}{D}+\frac{Q^{'}}{D}}^{\infty} TC_5 dH(t_r) \right) \right\} dG(t_f) + \int_{\frac{Q}{P}}^{\infty} TC_1 dG(t_f) \quad (15)$$

We employ the renewal reward theorem [27] to calculate the expected total cost per time unit as follows:

$$ETC_{Q,Q'} = \frac{E(TC_{Q,Q'})}{E(T_{Q,Q'})} \tag{16}$$

In *condition 2* as $t_{idle} > L$ and $t_f < Q/P$, according to the equation $t_{idle} = pt_f / D - t_f$ we have $Q > L.P.D/(P-D)$.

To find the optimal solution for production and purchasing lot sizes, we should minimize the expected total cost per time unit in equations (9) and (16), respectively. Accordingly, ETC_Q is minimized subject to $Q \geq 0$. Furthermore, $ETC_{Q,Q'}$ in minimized subject to $Q' \geq 0$ and $Q > L.P.D/(P-D)$. Finally, the optimal values for Q and Q' are selected based on the lower expected total cost per time unit.

4. The model with exponential failure and exponential repair time

The complexity of the objective function causes the analysis of the model with general failure and general repair time distributions to be difficult. In this section, we consider the optimality of the model in both conditions under exponential failure and repair time distributions as follows.

$$G(t_f) = 1 - e^{-\lambda t_f}$$

$$H(t_r) = 1 - e^{-\mu t_r}$$

For *condition 1*, we proved the convexity of ETC_Q in section *4.1*, but in *condition 2,* due to the complexity of the model, it is difficult to prove the convexity of $ETC_{Q,Q'}$ for any given parameters when Q and Q' are decision variables. For this condition, we first discussed the convexity of the model when each decision variables is known respectively, and then used the algorithm presented in [1] with a few changes for obtaining the optimal solution.

4.1 Condition 1

In this condition, the expected cycle time and the expected total cost per cycle are obtained from Equations (7) and (8) as follows:

$$E(T_Q)=\frac{\lambda.D}{\mu\left(\lambda.D+\mu(P-D)\right)}\left(1-e^{-(\lambda+\frac{\mu(P-D)}{D})\frac{Q}{P}}\right)+\frac{P}{\lambda.D}\left(1-e^{-\frac{\lambda Q}{P}}\right) \tag{17}$$

$$E(TC_Q)=A+\left(\frac{c_P.P}{\lambda}+\frac{c_r}{\mu}\right)\left(1-e^{-\frac{\lambda Q}{P}}\right)+\frac{c_s.\lambda.D^2}{\mu(\lambda.D+\mu(P-D)}\left(1-e^{-(\lambda+\frac{\mu(P-D)}{D})\frac{Q}{P}}\right)+$$
$$\frac{c_h(P-D)P}{\lambda.D}\left(\frac{1}{\lambda}-\left(\frac{1}{\lambda}+\frac{Q}{P}\right)e^{-\frac{\lambda Q}{P}}\right) \tag{18}$$

Therefore, the expected cost per unit time is

$$ETC_Q=A+\left(\frac{c_P.P}{\lambda}+\frac{c_r}{\mu}\right)\left(1-e^{-\frac{\lambda Q}{P}}\right)+\frac{c_s.\lambda.D^2}{\mu(\lambda.D+\mu(P-D)}\left(1-e^{-(\lambda+\frac{\mu(P-D)}{D})\frac{Q}{P}}\right)+$$
$$.\ \frac{c_h(P-D)P}{\lambda.D}\left(\frac{1}{\lambda}-\left(\frac{1}{\lambda}+\frac{Q}{P}\right)e^{-\frac{\lambda Q}{P}}\right)\Bigg/\frac{\lambda.D}{\mu\left(\lambda.D+\mu(P-D)\right)}\left(1-e^{-(\lambda+\frac{\mu(P-D)}{D})\frac{Q}{P}}\right)+\frac{P}{\lambda.D}\left(1-e^{-\frac{\lambda Q}{P}}\right) \tag{19}$$

Property 1. $E(T_Q)$ *is a concave function of Q for all* $Q>0$.

Proof. If $\frac{d^2E(T_Q)}{dQ^2}<0$, the concavity of function $E(T_Q)$ is proved. Now, with second-order partial derivative of Eq. (17) with respect to *Q*, we get

$$\frac{d^2E(T_Q)}{dQ^2}=-\frac{\lambda}{P.D}e^{-\frac{\lambda Q}{P}}-\left(\frac{\lambda(\lambda.D+\mu(P-D))}{\mu.D.P^2}\right)e^{-(\lambda+\frac{\mu(P-D)}{D})\frac{Q}{P}}<0$$

and the proof is completed.

Proposition 1. *Suppose that* $ETC_{Q_2}\le ETC_{Q_1}$ *for two values* Q_1 *and* Q_2. *Then* ETC_Q *is a pseudo-convex function provided* $E(TC_{Q_2})\ge E(TC_{Q_1})+(Q_2-Q_1)\frac{dE(TC_Q)}{dQ}\big|_{Q=Q_1}$

Proof. Since $E(T_Q)>0$ and $E(TC_Q)>0$ for all $Q\ge 0$, therefore, $ETC_{Q_2}\le ETC_{Q_1}$ implies that $\frac{E(T_{Q_2})}{E(T_{Q_1})}\ge\frac{E(TC_{Q_2})}{E(TC_{Q_1})}$. Again, since $E(T_Q)$ is concave from *property 1*, we have

$$E(T_{Q_1})+(Q_2-Q_1)\left[\frac{dE(T_Q)}{dQ}\right]_{Q=Q_1}\ge E(T_{Q_2})$$

Given that $\frac{E(T_{Q_2})}{E(T_{Q_1})}-1\ge\frac{E(TC_{Q_2})}{E(TC_{Q_1})}-1$ we have

$$\frac{(Q_2 - Q_1)\left[\dfrac{dE(T_Q)}{dQ}\right]_{Q=Q_1}}{E(T_{Q_1})} \geq \frac{E(TC_{Q_2}) - E(TC_{Q_1})}{E(TC_{Q_1})} \tag{20}$$

By first-order derivative of ETC_Q with respect to Q at the point of Q_1

$$\left[\frac{dETC_Q}{dQ}\right]_{Q=Q_1} = \frac{E(T_{Q_1})\left[\dfrac{dE(TC_Q)}{dQ}\right]_{Q=Q_1} - E(TC_{Q_1})\left[\dfrac{E(T_Q)}{dQ}\right]_{Q=Q_1}}{\left[E(T_{Q_1})\right]^2}$$

By substituting equivalent $\dfrac{\left[\dfrac{dE(T_Q)}{dQ}\right]_{Q=Q_1}}{E(T_{Q_1})}$ in (20), we have

$$(Q_2 - Q_1).E(T_{Q_1})\left[\frac{dETC_Q}{dQ}\right]_{Q=Q_1} \leq E(TC_{Q_1}) + (Q_2 - Q_1)\left[\frac{dE(TC_Q)}{dQ}\right]_{Q=Q_1} - E(TC_{Q_2})$$

If $E(TC_{Q_2}) \geq E(TC_{Q_1}) + (Q_2 - Q_1)\left[\dfrac{dE(TC_Q)}{dQ}\right]_{Q=Q_1}$

Also $ETC_{Q_2} \leq ETC_{Q_1}$ implies $(Q_2 - Q_1)\left[\dfrac{dETC_Q}{dQ}\right]_{Q=Q_1} \leq 0$. This proves that ETC_Q is a pseudo-convex function provided $E(TC_{Q_2}) \geq E(TC_{Q_1}) + (Q_2 - Q_1)\left[\dfrac{dE(TC_Q)}{dQ}\right]_{Q=Q_1}$ satisfying $ETC_{Q_2} \leq ETC_{Q_1}$.

Proposition 2. *Under Proposition 1, only exists a unique* Q^* *which minimizes* ETC_Q.

Proof. The first derivative of Eq. (19) with respect to Q is $\psi(Q) = \dfrac{dETC_Q}{dQ} = \dfrac{D(Q)}{N(Q)}$ where

$$D(Q) = \left\{\left(c_P + \frac{\lambda.c_r}{\mu.P} + \frac{c_h(P-D)Q}{P.D}\right)e^{-\frac{\lambda Q}{P}} + \frac{c_s.D.\lambda}{\mu.P}e^{-(\lambda+\frac{\mu(P-D)}{D})\frac{Q}{P}}\right\}E(T_Q)$$
$$-\left\{\frac{1}{D}e^{-\frac{\lambda Q}{P}} + \frac{\lambda}{\mu.P}e^{-(\lambda+\frac{\mu(P-D)}{D})\frac{Q}{P}}\right\}E(TC_Q)$$

$$N(Q) = \left\{\frac{\lambda.D}{\mu\left(\lambda.D + \mu(P-D)\right)}\left(1 - e^{-(\lambda+\frac{\mu(P-D)}{D})\frac{Q}{P}}\right) + \frac{P}{\lambda.D}\left(1 - e^{-\frac{\lambda Q}{P}}\right)\right\}^2$$

We have

$$\lim_{Q\to 0} D(Q) = -A\left[\frac{1}{D} + \frac{\lambda}{\mu.P}\right] \quad \text{and} \quad \lim_{Q\to 0} N(Q) = 0$$

It is observed that $\psi(Q) \to -\infty$ as $Q \to 0$. Moreover, $\psi(Q) \to 0$ as $Q \to \infty$. So, given the pseudo-convexity of ETC_Q under *proposition 1*, there exists a unique root Q^* of $\psi(Q) = 0$.

Now, we can obtain the optimal value of Q by solving the Equation $\psi(Q)=0$.

4.2 Condition 2

Assuming the exponential distribution for the random variables in *condition 2*, from Equations (14) and (15), we have

$$E(T_{Q,Q^{'}})=\frac{P}{\lambda.D}\left(1-e^{-\frac{\lambda Q}{P}}\right)+\left(1-e^{-(\lambda+\frac{\mu(P-D)}{D})\frac{Q}{P}}\right)\left(\frac{(1-\theta)\lambda.D+\theta.\lambda.\mu.Q^{'}e^{\mu.L}+\theta.\lambda.De^{-\frac{\mu.Q^{'}}{D}}}{\mu\left(\lambda.D+\mu(P-D)\right)}\right) \tag{21}$$

$$\begin{aligned}E(TC_{Q,Q^{'}})=A+\left(\frac{c_P.P}{\lambda}+\frac{c_r}{\mu}\right)\left(1-e^{-\frac{\lambda Q}{P}}\right)+\frac{c_h(P-D)P}{\lambda.D}\left(\frac{1}{\lambda}-\left(\frac{1}{\lambda}+\frac{Q}{P}\right)e^{-\frac{\lambda Q}{P}}\right)+\\ \left(\frac{(1-\theta)\lambda.c_s.D^2+\theta.\lambda.c_s.D^2.e^{-\frac{\mu.Q^{'}}{D}}+\theta.\mu.\lambda e^{\mu.L}(2A^{'}.D+c_h.Q^{'2}+2c^{'}.Q^{'})}{\mu(\lambda.D+\mu(P-D)}\right)\left(1-e^{-(\lambda+\frac{\mu(P-D)}{D})\frac{Q}{P}}\right)\end{aligned} \tag{22}$$

and the expected cost per unit of time is

$$\begin{aligned}ETC_{Q,Q^{'}}=\Bigg\{A+\left(\frac{c_P.P}{\lambda}+\frac{c_r}{\mu}\right)\left(1-e^{-\frac{\lambda Q}{P}}\right)+\frac{c_h(P-D)P}{\lambda.D}\left(\frac{1}{\lambda}-\left(\frac{1}{\lambda}+\frac{Q}{P}\right)e^{-\frac{\lambda Q}{P}}\right)+\\ \left(\frac{(1-\theta)\lambda.c_s.D^2+\theta.\lambda.c_s.D^2.e^{-\frac{\mu.Q^{'}}{D}}+\theta.\mu.\lambda e^{\mu.L}(2A^{'}.D+c_h.Q^{'2}+2c^{'}.Q^{'})}{\mu(\lambda.D+\mu(P-D)}\right)\left(1-e^{-(\lambda+\frac{\mu(P-D)}{D})\frac{Q}{P}}\right)\Bigg\}\Bigg/\Bigg\{\frac{P}{\lambda.D}\left(1-e^{-\frac{\lambda Q}{P}}\right)\\ +\left(1-e^{-(\lambda+\frac{\mu(P-D)}{D})\frac{Q}{P}}\right)\left(\frac{(1-\theta)\lambda.D+\theta.\lambda.\mu.Q^{'}e^{\mu.L}+\theta.\lambda.De^{-\frac{\mu.Q^{'}}{D}}}{\mu\left(\lambda.D+\mu(P-D)\right)}\right)\Bigg\}\end{aligned}$$

(23)

In this condition, the proof of the objective function convexity is difficult when Q and $Q^{'}$ are decision variables simultaneously. Therefore, we investigate the problem's solution by examining Kuhn-Tucker necessary condition. Assume ρ be the Lagrange coefficient related to the constraint $Q>L.P.D/(P-D)$. We have

$$\frac{\partial ETC_{Q,Q^{'}}}{\partial Q}-\rho E^2(T_{Q,Q^{'}})=0 \tag{24}$$

$$\frac{\partial ETC_{Q,Q^{'}}}{\partial Q^{'}}=0 \tag{25}$$

$$\rho.[Q-L.P.D/(P-D)]=0 \tag{26}$$

Clearly $\rho = 0$ since $Q > L.P.D/(P-D)$. So we can obtain the optimal values of Q and Q' by simultaneous solving of equations (24) ($\rho = 0$) and (25) using the following algorithm based on [1].

Algorithm

Step 1.	Input the parameters value and the accuracy parameter ε_1 and ε_2. Set $i = 0$ and $Q' = Q'^{(0)}$
Step 2.	Set $i = i+1$, $Q' = Q'^{(i-1)}$ and solve Equation (24) for Q. Let the answer be $Q^{(i)}$
Step 3.	Substitute $Q = Q^{(i)}$ and solve Equation (25). Let the answer be $Q'^{(i)}$
Step 4.	Compute $ETC_{Q^{(i)},Q'^{(i)}}$ from Equation (23)
Step 5.	Repeat steps 2 to 4
Step 6.	If $\left|Q'^{(i)} - Q'^{(i-1)}\right| < \varepsilon_1$ and $\left|Q^{(i)} - Q^{(i-1)}\right| < \varepsilon_2$ then $Q'^{*} = Q'^{(i)}$ and $Q^{*} = Q^{(i)}$, and also $ETC_{Q^{*},Q'^{*}} = ETC_{Q^{(i)},Q'^{(i)}}$ .stop, otherwise go to step 5

In the following, we discuss the optimality of the $ETC_{Q,Q'}$ in two situations in which Q and Q' are set to be fixed, respectively. Based on the following discussions, we can easily find the optimal values for Q and Q' in equations (24) and (25), respectively.

4.2.1 When Q' is predetermined

Assuming the purchasing lot size is constant, let $ETC_{Q,Fix} = E(TC_{Q,Fix}) \big/ E(T_{Q,Fix})$ denote the corresponding expected cost per unit time where $E(T_{Q,Q'})$ and $E(TC_{Q,Q'})$ in Equations (20) and (21) turn to $E(T_{Q,Fix})$ and $E(TC_{Q,Fix})$ when Q' is known.

Property 2. *The function $E(T_{Q,Fix})$ is concave.*

Proof. The second order differentiation of $E(T_{Q,Fix})$ with respect to Q is

$$\frac{\partial^2 E(T_{Q,Fix})}{\partial Q^2} = -\frac{\lambda}{P.D} e^{-\frac{\lambda Q}{P}} - \left(\frac{\left(\lambda.D + \mu(P-D)\right)\left(\theta.\lambda.D.e^{-\frac{\mu.Q'}{D}} + (1-\theta)\lambda.D + \theta.\mu.\lambda.Q' e^{\mu.L}\right)}{\mu.D^2.P^2} \right) e^{-(\lambda + \frac{\mu(P-D)}{D})\frac{Q}{P}} < 0$$

So the proof is completed.

Proposition 3. *Let* $ETC_{Q_2,Fix} \leq ETC_{Q_1,Fix}$ *for two distinct values* Q_1 *and* Q_2 *of* Q. *Then* $ETC_{Q,Fix}$ *is a pseudo-convex function provided* $E(TC_{Q_2,Fix}) \geq E(TC_{Q_1,Fix}) + (Q_2 - Q_1)\frac{\partial E(TC_{Q,Fix})}{\partial Q}\Big|_{Q=Q_1}$

Proof. Under *property 2*, the proof of pseudo-convexity of $ETC_{Q,Fix}$ is similar to the method presented in *proposition 1*.

Proposition 4. *Under proposition 3, only exists a unique* Q^*, *which minimizes* $ETC_{Q,Fix}$.

Proof. The first derivative of $ETC_{Q,Fix}$ with respect to Q is $\varphi(Q) = \frac{\partial ETC_{Q,Fix}}{\partial Q} = \frac{U(Q)}{O(Q)}$ where

$$U(Q) = \left\{ \left(c_P + \frac{\lambda.c_r}{\mu.P} + \frac{c_h(P-D)Q}{P.D} \right) e^{-\frac{\lambda Q}{P}} + \left(\frac{(1-\theta)\lambda.c_s.D^2 + \theta.\lambda.c_s.D^2.e^{-\frac{\mu.Q'}{D}} + \theta.\mu.\lambda e^{\mu.L}}{\mu.P.D} \right.\right.$$

$$\left.\left. \frac{(2A'.D + c_h.Q'^2 + 2c'.Q')}{\mu.P.D} \right) e^{-(\lambda + \frac{\mu(P-D)}{D})\frac{Q}{P}} \right\} E(T_{Q,Fix}) - \left\{ \frac{1}{D} e^{-\frac{\lambda Q}{P}} + \left(\frac{(1-\theta)\lambda.D + \theta.\lambda.D.e^{-\frac{\mu.Q'}{D}}}{\mu.P.D} \right.\right.$$

$$\left.\left. \frac{+\theta.\mu.\lambda.Q'.e^{\mu.L}}{\mu.P.D} \right) e^{-(\lambda + \frac{\mu(P-D)}{D})\frac{Q}{P}} \right\} E(TC_{Q,Fix})$$

$$O(Q) = \left\{ \frac{P}{\lambda.D}\left(1 - e^{-\frac{\lambda Q}{P}}\right) + \left(1 - e^{-(\lambda + \frac{\mu(P-D)}{D})\frac{Q}{P}}\right)\left(\frac{(1-\theta)\lambda.D + \theta.\lambda.\mu.Q'e^{\mu.L} + \theta.\lambda.De^{-\frac{\mu.Q'}{D}}}{\mu\left(\lambda.D + \mu(P-D)\right)} \right) \right\}^2$$

We have

$$\lim_{Q\to 0} U(Q) = -A\left[\frac{1}{D} + \frac{\theta.\mu.\lambda.Q'.e^{\mu.L} + (1-\theta)\lambda.D + \theta.\lambda.D.e^{-\frac{\mu.Q'}{D}}}{\mu.P.D} \right] \quad \text{and} \quad \lim_{Q\to 0} O(Q) = 0$$

If $Q \to 0$ then $\varphi(Q) \to -\infty$. Moreover, $\varphi(Q) \to 0$ as $Q \to \infty$. So, given the pseudo-convexity of $ETC_{Q,Fix}$ under *proposition 3*, there exists a unique non-negative root Q^* of $\varphi(Q) = 0$ and the proof is completed.

4.2.2 When Q is predetermined

Suppose that Q is a constant value greater than $L.P.D/(P-D)$. Let $ETC_{Fix,Q'} = E(TC_{Fix,Q'})\big/E(T_{Fix,Q'})$ denote the corresponding expected cost per unit time where $E(T_{Q,Q'})$ and $E(TC_{Q,Q'})$ in Equations (20) and (21) turn to $E(T_{Fix,Q'})$ and $E(TC_{Fix,Q'})$ when Q is known.

Proposition 5. *Under the condition of the equation (23) if* $ETC_{Fix,Q'} < D\left(c_s + \frac{2c_h}{\mu} e^{\mu\left(L+\frac{Q'}{D}\right)}\right)$*, there exists a local minimum* Q'^{*}*, which minimizes* $ETC_{Fix,Q'}$.

Proof. The second-order differentiation of $E(TC_{Fix,Q'})$ and $E(T_{Fix,Q'})$ with respect to Q' respectively are

$$\frac{\partial^2 E(TC_{Fix,Q'})}{\partial Q'^2} = \left(\theta.\lambda.c_s.\mu.e^{-\frac{\mu.Q'}{D}} + 2c_h.\theta.\lambda.Q'e^{\mu.L}\right)K \tag{27}$$

$$\frac{\partial^2 E(T_{Fix,Q'})}{\partial Q'^2} = \left(\frac{\theta.\lambda.\mu}{D} e^{-\frac{\mu.Q'}{D}}\right)K \tag{28}$$

Where $K = \frac{1}{\lambda.D + \mu(P-D)}\left(1 - e^{-(\lambda+\frac{\mu(P-D)}{D})\frac{Q}{P}}\right)$

Under the condition of (24) if

$$E(T_{Fix,Q'})\frac{\partial^2 E(TC_{Fix,Q'})}{\partial Q'^2} - E(TC_{Fix,Q'})\frac{\partial^2 E(T_{Fix,Q'})}{\partial Q'^2} > 0 \tag{29}$$

then, there exists a local minimum Q'^{*}. By substituting Equations (27) and (28) in (29), we have

$$E(T_{Fix,Q'})\left(\theta.\lambda.c_s.\mu.e^{-\frac{\mu.Q'}{D}}+2c_h.\theta.\lambda.Q'e^{\mu.L}\right)K-E(TC_{Fix,Q'})\left(\frac{\theta.\lambda.\mu}{D}e^{-\frac{\mu.Q'}{D}}\right)K>0$$

and after the factorization, we have $ETC_{Fix,Q'}<D\left(c_s+\frac{2c_h}{\mu}e^{\mu\left(L+\frac{Q'}{D}\right)}\right)$. So the proof is completed.

5. Numerical results

In this section, a sample problem with the data set presented in Table (1) is investigated. After solving the model using the algorithm presented in the previous section, we do sensitivity analysis on the main parameters of the given numerical example.

Using computational software MATHEMATICA, We optimized ETC_Q subject to $Q\geq 0$, and the optimal solution is obtained as $Q^*=4683.3$ and $ETC_{Q^*}=15331.8$ as depicted in Figure 9. The second function $ETC_{Q,Q'}$ under constraints $Q>2520$ and $Q'\geq 0$ results in the best solution $Q^*=2963.6$, $Q'^*=357.6$ and $ETC_{Q^*,Q'^*}=14601.4$. The surface generated by $ETC_{Q,Q'}$ over the wide range of values of Q and Q' is shown in Figure 10. By comparing the optimal values of two conditions, the best one is selected

5.1. Sensitivity analysis based on supplier parameters

In this section, we examine the effect of the L and θ on the optimal values of the decision variables and the objective function. As displayed in Figure 11, supplier lead-time plays an essential role in the manufacturer's decision.

In *condition 2*, the increase of lead-time raises the risk of encountering shortages and therefore persuades the production manager to increase the production quantity and consequently reduce the order quantity. In this situation, the expected total cost increases so that for $L>1.8$, ETC_Q

would be less than $ETC_{Q,Q'}$. This means that for high values of *L*, the model employs production as the basis for business acting.

As illustrated in Figure 12, sensitivity analysis on supplier reliability shows that when the supplier is more reliable, the manufacturer decreases purchasing order quantity. Likewise, by decreasing θ, the manufacturer increases the order quantity to confront the risk of inventory stock out and keep an acceptable service level. This is because, in essence, the model is conservative. When the supplier is less reliable, the model suggests the manufacturer purchase larger quantities to hedge against uncertainties and maintain an acceptable fill rate. This balance is obtained through trade-offs between shortage cost, holding cost, and lost sale cost; for more enormous lost sale costs, the model strives to avoid shortages by accumulating more inventory. It is also observed that the more reliable the supplier, the less the expected total cost.

5.2. Sensitivity analysis based on failure and repair rates

It is evident that the manufacturer's decision, to make or do not make purchasing order quantities, is highly dependent on the machine failure rate as well as corrective repair time. This issue is illustrated in Figure 13 and Figure 14.

As the failure rate decreases, the expected cost in both conditions decreases. This reduction is faster in *condition 1*, so that for small values of λ, the expected cost in *condition 1* is lower than *condition 2* (

). In other words, regarding the constant demand rate, by reducing of failure rate, the probability of facing shortages is reduced, and the purchasing strategy is not beneficial for the system.

A similar situation would happen when the corrective repair rate changes. With the increase of the meantime for repair (decrease of μ), the expected cost increases in both conditions, and with the increase of μ, it decreases. The rate of changes is more in condition 1 so that for large values of μ, the manufacturer has no desire to use purchasing strategy (Figure 14).

5.3. Comparison with the safety stock policy

In this section, we compare the purchasing strategy and the safety stock policy. Numerous researchers have explored safety stock policy, and we use the model developed by [7] for evaluation. To make the model comparable with ours, we relax some minor assumptions of the model proposed by [7]. The formulation of the expected cycle time and expected total cost in safety stock policy is presented in Appendix A.

Sensitivity analysis of two policies shows that the production cost and the holding cost have the most impact on the production system to choose the optimal policy. **Error! Reference source not found.** illustrates the fluctuations of the expected cost in two policies based on production cost variations.

As the figure shows, for the chosen data set in the numerical example, the safety stock policy is still preferable to the purchasing policy. With the increase of production cost, as expected, the optimal cost of the system increases in both policies. The increasing rate in the safety stock policy is more than the purchasing policy, so that an increase of more than 12.5% in the unit production cost makes the purchasing policy more beneficial than the safety stock policy. It is worth noting that the supplier lead-time can affect this superiority. According to **Error! Reference source not found.**, for $L = 0.3$, 10% increase in unit production cost and for $L = 0.1$, 7% increase in unit production cost, ensure the production system prefer the purchasing policy.

A similar scenario will happen with the variation of unit holding cost for low values of *L*. as illustrated in Figure 16, the superiority of the safety stock policy remains unchanged if there is an increase of up to 60% in the unit holding cost. Further increase in the unit holding cost makes the purchasing policy more economical for the production system. This is entirely justifiable from the managerial insight because when the unit holding cost exceeds, keeping safety stock imposes staggering costs on the manufacture; therefore, alternative options such as purchasing are more beneficial.

6. Concluding remarks

Considering the importance of responsiveness to the customer in today's world of trade, and organizations' efforts to keep up their credit in the supply chain, process disruption due to machine failure is one of the critical challenges of the production managers. Various contingency plans to deal with such situations have been suggested by researchers, such as preventive maintenance, keeping safety stock, inspection and rework operation. In terms of expensive unit holding costs or low warehouse capacity, emergency replenishment could be a better option than keeping safety stock. In this paper, we investigated the benefit of the purchasing policy as an alternative option to keeping safety stock during a stoppage in the production process caused by corrective maintenance. We also assumed that the external supplier is unreliable and has a predetermined lead-time. Our numerical study has found that the supplier lead-time is a crucial parameter in determining the purchase situation from the market. Moreover, simultaneous analysis of L and other parameters shows that the unit production cost and unit holding cost play an essential role in determining when the purchasing policy is better than relying on keeping safety stock.

Our proposed model could be improved by investigating the joint implementation of purchasing policy and safety stock policy. This might require less safety inventory to protect the company against stock-outs. Moreover, investigating the influence of contractual agreement between manufacturer and supplier on variable lead-time in purchasing strategy could be a fruitful area for a future study. Another avenue for further research is considering the manufacturer and supplier's cash constraints and how the weaker player might be strengthened in this supply chain.

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Biography

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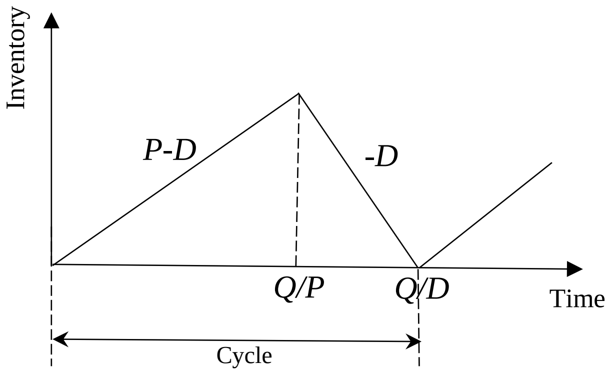

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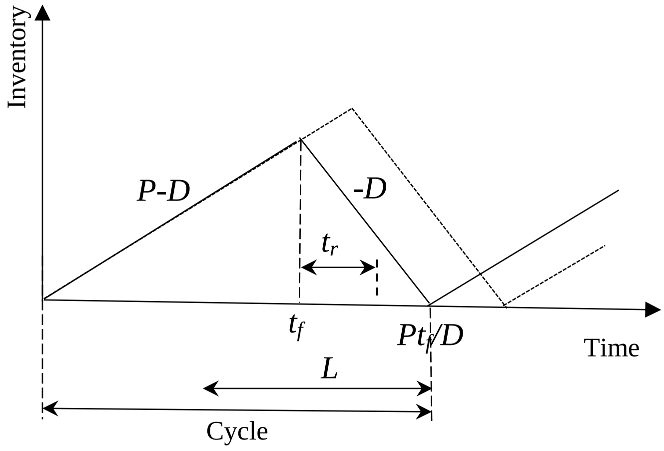

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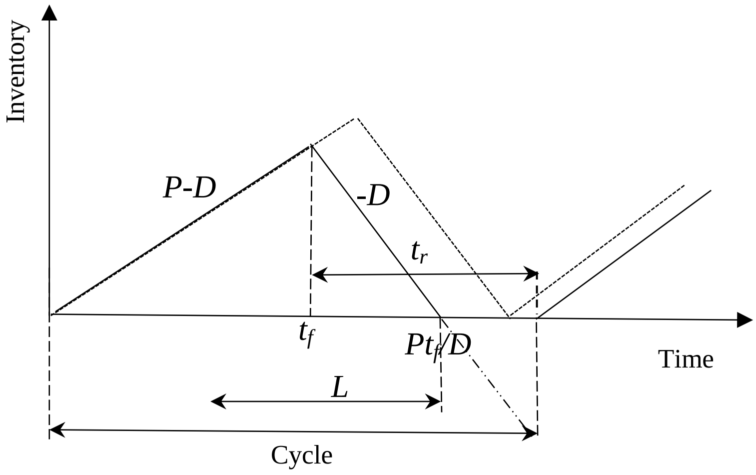

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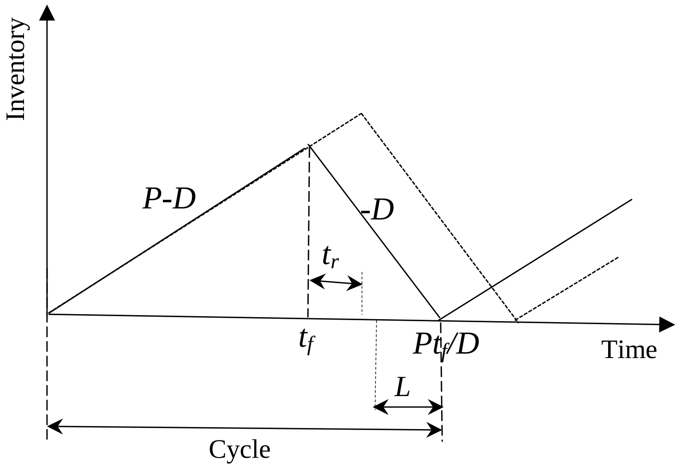

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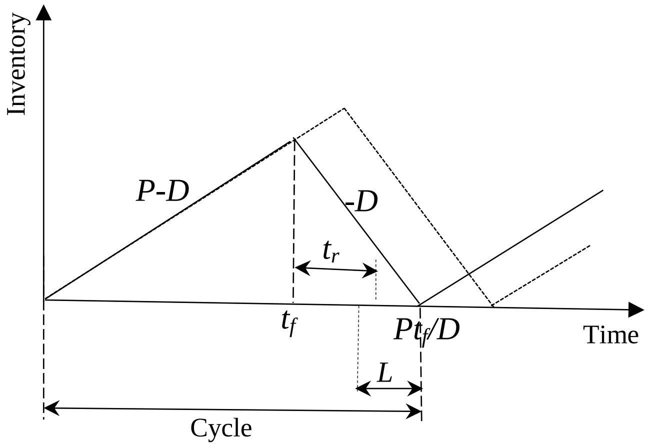

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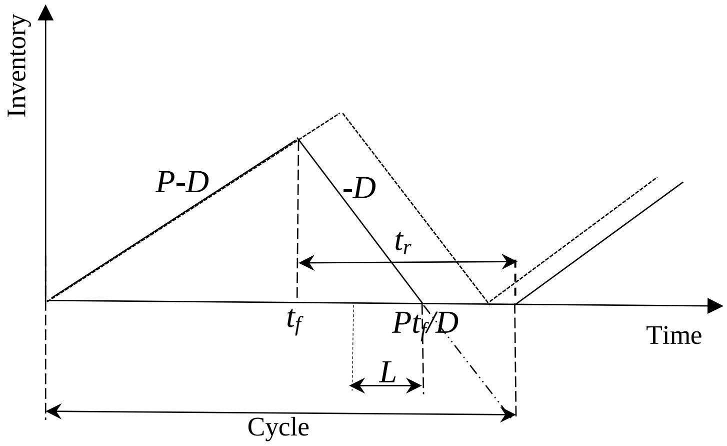

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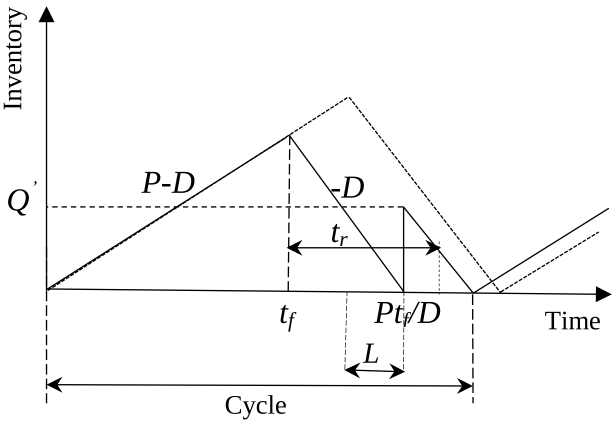

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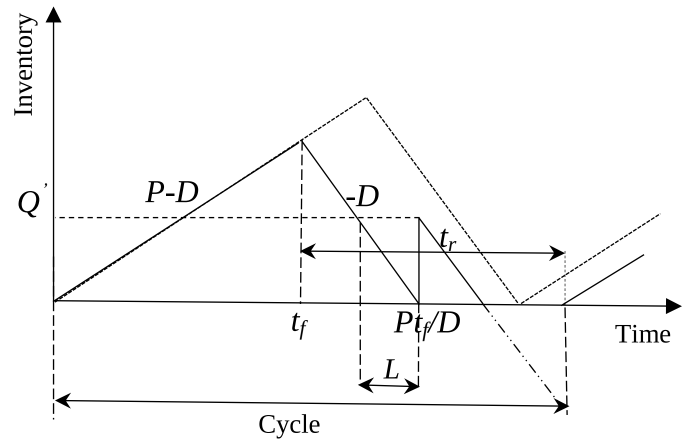

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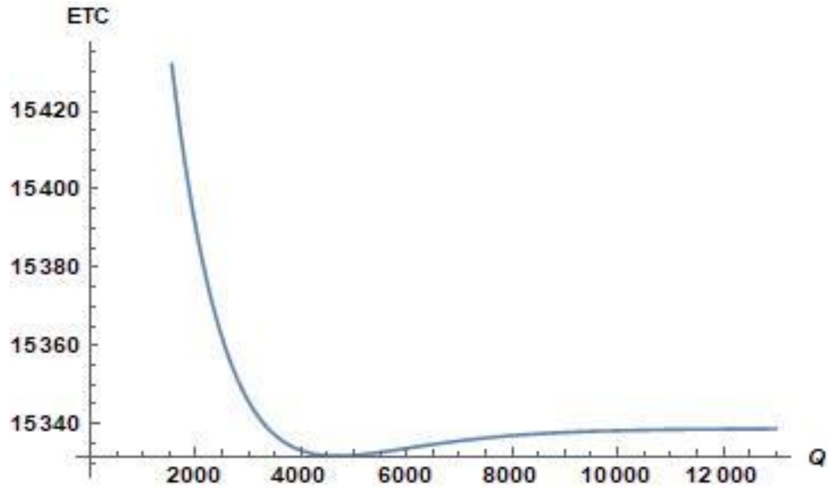


Figure 9

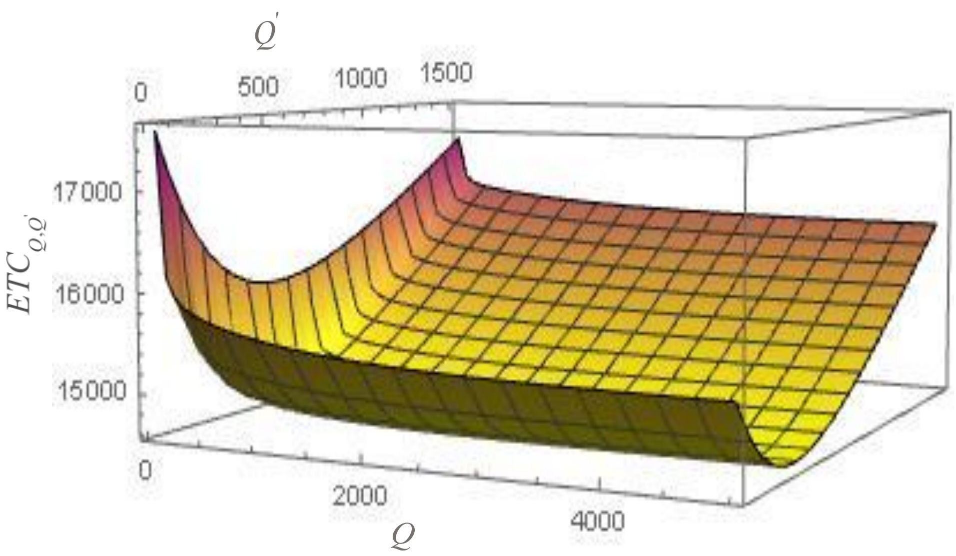
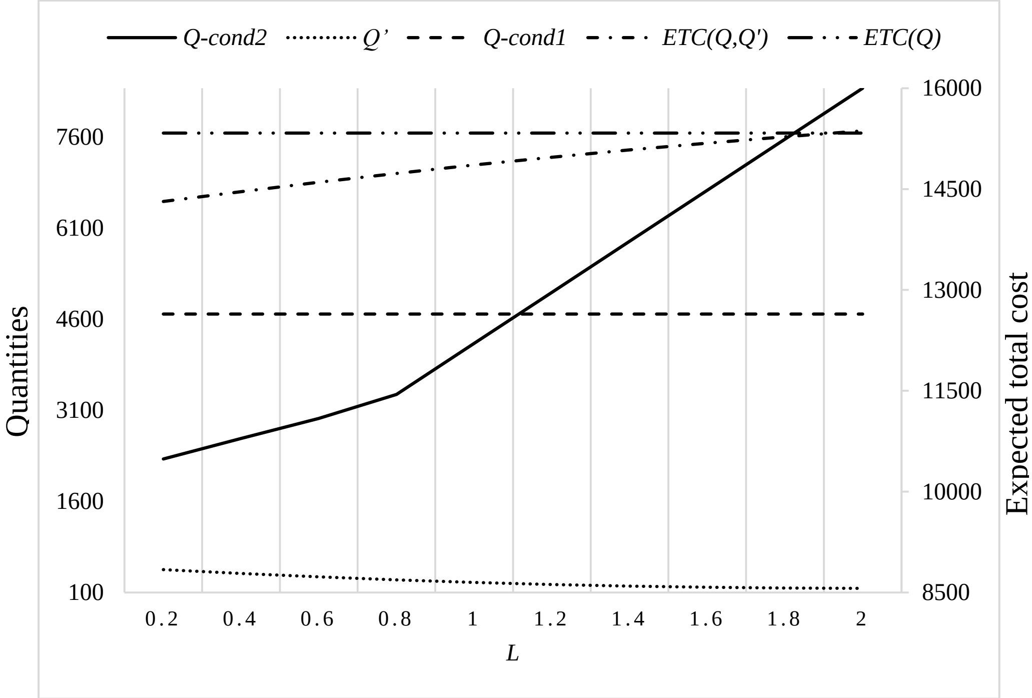


Figure 10



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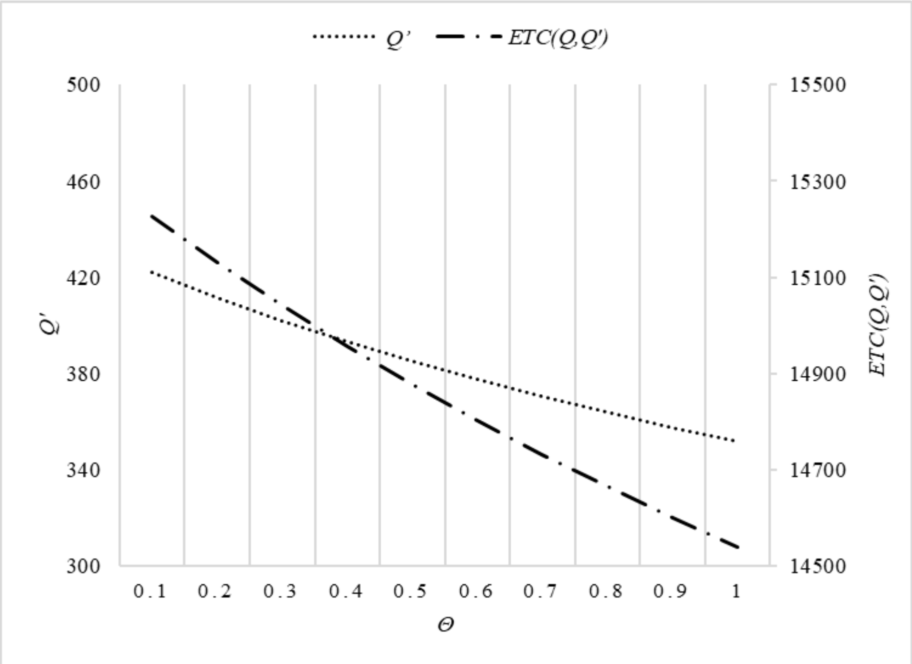

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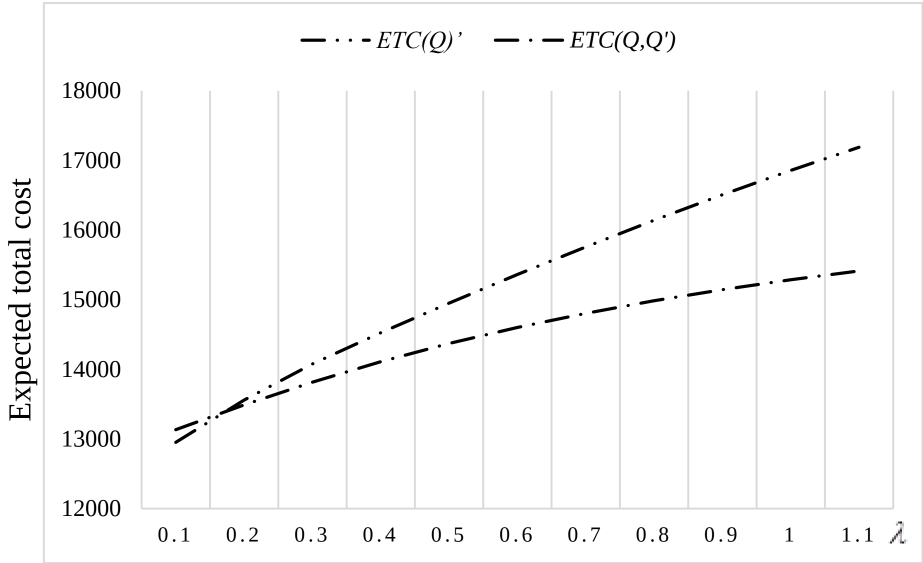

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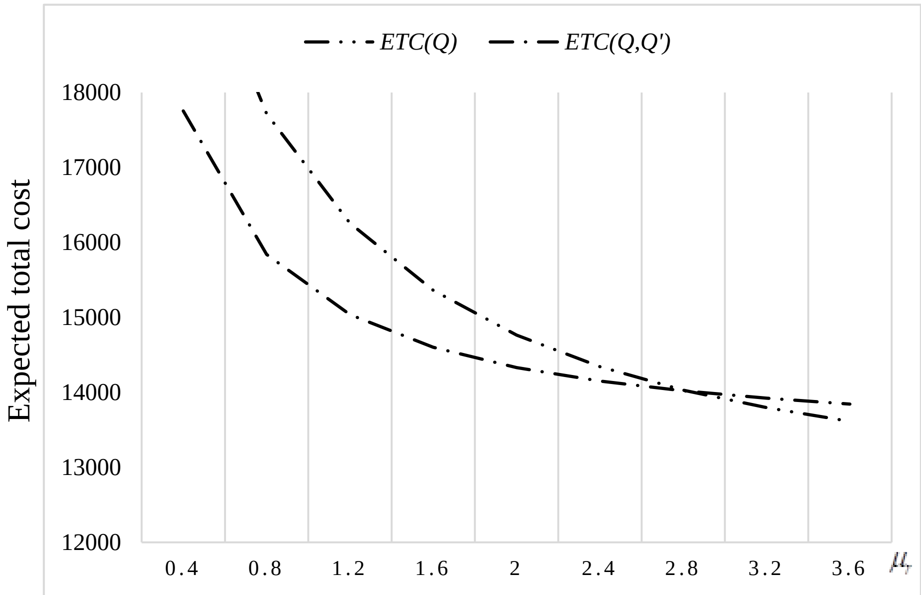

Figure 14

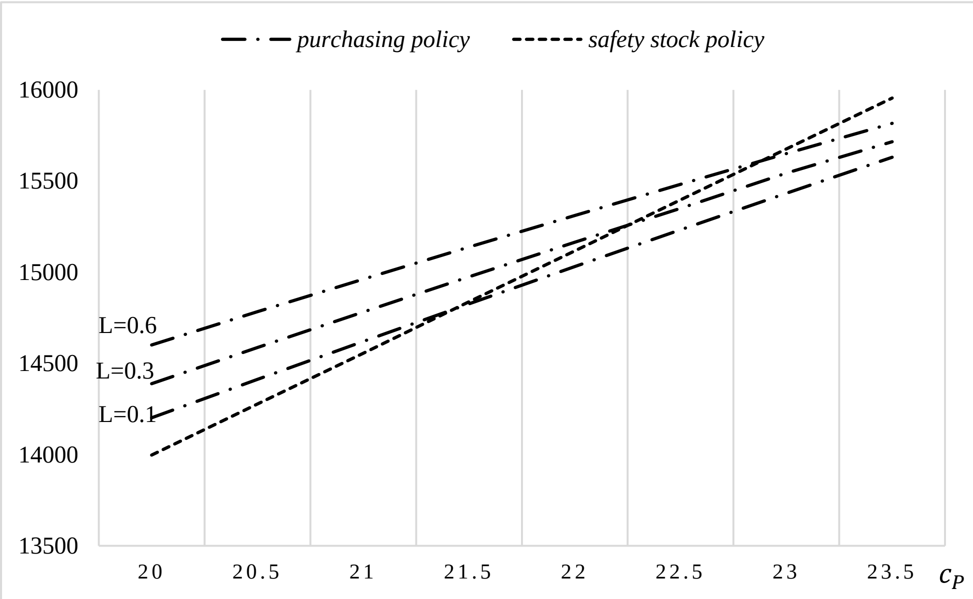

Figure 15

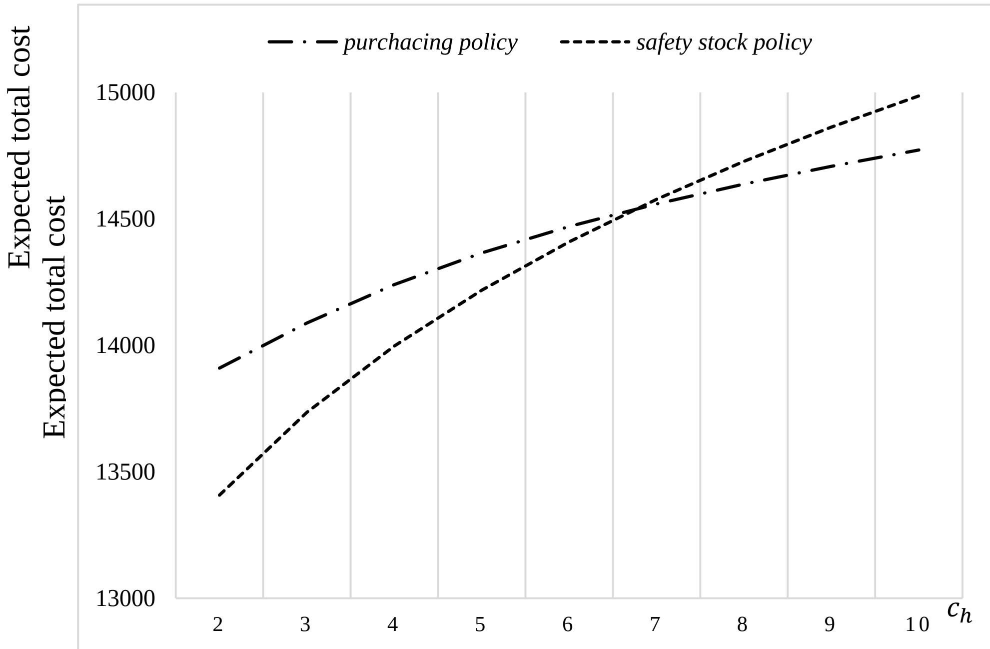

Figure 16

Table 1

parameter	P	D	L	θ	A	$A^{'}$	c_P	$c^{'}$	c_h	c_s	c_r	λ	μ
value	700	600	0.6	0.9	100	120	20	25	4	45	80	0.6	1.6

Appendix A: detailed formulation of *E(T)* and *E(TC)* in safety stock policy

In this appendix we provided details of model in safety stock policy (S_f is the decision variable corresponding to stock level).

$$E(T)=\int_0^{\frac{Q}{P}}\left\{\int_0^{\frac{(P-D)t_f}{D}}\frac{P.t_f}{D}dH(t_r)+\left(\frac{P}{P-D}\right)\int_{\frac{(P-D)t_f}{D}}^{\frac{(P-D)t_f+S_f}{D}}t_r dH(t_r)\right.$$
$$\left.+\int_{\frac{(P-D)t_f+S_f}{D}}^{\infty}\left(t_f+t_r+\frac{S_f}{P-D}\right)dH(t_r)\right\}dG(t_f)+\int_{\frac{Q}{P}}^{\infty}\frac{Q}{D}dG(t_f) \quad \text{Eq. (A.1)}$$

$$E(TC)=A+\int_0^{\frac{Q}{P}}\left\{\int_0^{\frac{(P-D)t_f}{D}}\left(c_h\left(\frac{(P-D)P.t_f^2}{2D}+\frac{S_f.P.t_f}{D}\right)+c_P.P.t_f+c_r.t_r\right)dH(t_r)\right.$$
$$+\int_{\frac{(P-D)t_{fw}}{D}}^{\frac{(P-D)t_f+S_f}{D}}\left(c_h\left(\frac{(P-D)P.t_f^2}{2D}+\frac{S_f.P.t_r}{D}+\frac{P(D.t_r-(P-D)t_f)^2}{2D(P-D)}\right)\right.$$
$$\left.+\frac{c_P.P.D.t_r}{P-D}+c_r.t_r\right)dH(t_r)+\int_{\frac{(P-D)t_f+S_f}{D}}^{\infty}\left(c_h\left(\frac{(P-D)P.t_f^2}{2D}+\frac{S_f.P.t_f}{D}+\frac{S_f^2.P}{2D(P-D)}\right)\right.$$
$$\left.\left.+c_P\left(P.t_f+\frac{S_f.P}{P-D}\right)+c_r.t_r+c_s(D.t_r-(P-D)t_f-S_f)\right)dH(t_r)\right\}dG(t_f)+$$
$$\int_{\frac{Q}{P}}^{\infty}\left(c_h\left(\frac{(P-D)Q^2}{2P.D}+\frac{S_f.Q}{D}\right)+c_P.Q\right)dG(t_f)$$

Eq. (A.2)