An economic-statistical production quantity model under quality-maintenance policy for imperfect manufacturing systems with interaction effect among assignable causes

Ali Salmasnia*

Department of Industrial Engineering, Faculty of Engineering, University of Qom, Iran

Email: a.salmasnia@qom.ac.ir, Tel: +989122865720

Behnam Abdzadeh

Department of Industrial Engineering, Faculty of Engineering, University of Qom, Iran

Email: behnamabdzadeh@gmail.com, Tel: +98 9376260833

Mohammad Reza Maleki

Industrial Engineering Group, Golpayegan College of Engineering, Isfahan University of Technology, Golpayegan, 87717-67498, Iran

Email: m.maleki@iut.ac.ir, Tel: +989124071032

Abstract

This study integrates production and maintenance planning with statistical process monitoring in the presence of dependent multiple assignable causes. To adapt the model to the reality, two assumptions are considered: (1) the assignable causes (ACs) are dependent, and (2) the occurrence of ACs can affect both process mean and variability. Given the second assumption, a non-central chi-square (NCS) chart is used to monitor the process. Since the occurrence rate of ACs increases over time, a non-uniform sampling scheme is presented to reduce the out-of-control time period. A sensitivity analysis is presented to explore how the number of AC types influences the cost terms. The results indicate that the more AC types, the higher quality loss and maintenance costs are imposed on the manufacturer. Moreover, three comparative studies are conducted for confirming the effectiveness of the model. The first comparative study shows that the total cost will be less than its real value when the interdependency among the ACs is ignored. The second comparison shows that the NCS chart outperforms the $\bar{X}-R$ in detecting the process disturbances and leads to a less quality loss cost. Eventually, the last one represents that employing the non-uniform sampling strategy leads to a significant cost savings.

Keywords: Production planning, Maintenance planning, Non-central chi-square chart, Correlated assignable causes, Non-uniform sampling strategy.

1. Introduction

To date, several models have been presented to analyze the production systems. The economic production quantity (EPQ) as one of the most known production models which is firstly introduced by Taft [1]. Similar to the traditional models, this model considers two unrealistic assumptions: (1) all the produced items conform to the customer requirements; (2) the machine doesn't degrade during the production cycle. However, the process may go to the out-of-control conditions during the production cycle due to different reasons, and consequently, some no-confirming items may be produced. Hence, a proper tool is required to improve the outcome of the production process continuously by detecting assignable causes (ACs). Control charts, as the most important tools of statistical process monitoring (SPM), can effectively detect the ACs during the production process (Salmasnia et al. [2], and Salmasnia et al. [3]). On the other hand, notwithstanding the considered assumptions in the EPQ model, the machines may cease working in a time period due to exhaustion. In this regard, appropriate maintenance policies can play a fundamental role in preventing and postponing the machine breakdowns.

Although, the production planning is in a close relationship with SPM and maintenance planning, these concepts are rarely investigated monolithically. Several studies such as Rahim and Ohta [4], Pan et al. [5], and Gunay and Kula [6] attempted to integrate the production planning and SPM topics. Another category of researches such as Emami-Mehregani et al. [7], Zahedi-Hosseini et al. [8], Si et al. [9], and Liu et al. [10] focused on optimizing production and maintenance planning without considering the SPM concept. A large number of researches proposed integrated models of SPM and maintenance planning. Examples include Mehrafrooz and Noorossana [11], Liu et al. [12], Xiang [13], Atashgar and Abdollahzadeh [14], Salmasnia et al. [15], and Salmasnia et al. [16].

As a pioneer research, Ben-Daya and Makhdoum [17] optimized the SPM, maintenance and production planning simultaneously. They investigated the impact of three types of maintenance policies on optimizing the EPQ and economic design of the control chart. Then, Ben-Daya [18] proposed an integrated model of three mentioned concepts and assessed the impact of maintenance operations on process output. Lam and Rahim[19] presented a similar model by taking into account both uniform and non-uniform sampling schemes. Jafarian-

Namin et al. [20] presented an integrated model of production planning, control chart, and maintenance planning for processes with autocorrelated quality characteristics under a delay monitoring policy. Salmasnia et al. [21] developed an integrated model of the mentioned topics under multiple ACs. Salmasnia et al. [22] proposed an integrated production and maintenance planning model by considering a control chart with variable parameters. Eventually, Salmasnia et al. [23] presented a joint model of production and maintenance planning under an adaptive control chart.

In an array of production processes like the process considered in the study of Salmasnia et al. [21], the hazard rate is an increasing function of time. In such processes, using the traditional sampling approach with fixed sampling intervals (uniform sampling) is not functional, because the integrated hazard rate in different sampling intervals is not equal to one another. Hence, several researchers have presented non-uniform sampling strategy in which the samples are taken in a way that the integrated hazard rate in different intervals is equal. In this field, Banerji and Rahim [24] developed a non-uniform sampling scheme for the processes with increasing hazard rate. Rahim [25] used a non-uniform sampling scheme in an integrated model of inventory planning and control chart design. In addition, among the other researches regarding the non-uniform sampling strategy, we can refer to Ben-Daya and Makhdoum [17], Chen and Yang [26], Moghadam et al. [27], and Salmasnia et al. [28].

Quite the contrary to many studies mentioned in the paragraphs above, in a production process, various ACs such as the low-quality of raw materials, incorrect setting up, sudden shocks, and electricity fluctuations, may shift the process to an out-of-control condition. Considering the multiple ACs, Duncan [29] developed a model for the economic design of $\bar{X}$ control chart. Afterwards, Chen and Yang [26] presented an economic design model considering multiple ACs when the in-control time period follows the Weibull distribution, like the other researches such as Yu and Hou [30] and Asadzadeh and Khoshalhan [31]. In the mentioned researches, it was assumed that the ACs are independent of each other while in real conditions, the occurrence of an AC may reduce or increase the occurrence probability of the others. Nenes et al. [32] presented an economic-statistical model for the adaptive control charts. They used the Markov chain approach to account for the interacting ACs. Moreover, Tasias and Nenes [33] presented a model similar to the previous models for simultaneous monitoring of the mean and variance parameters employing variable parameters (VP) charts.

The $\bar{X}$ control chart is used to detect the ACs affecting the process mean while the R chart is utilized to identify the ACs affecting the process variability. These charts can be simultaneously employed to detect the ACs affecting both mean and variance parameters. Examples include Saniga [34], Costa [35], Rahim and Costa [36], and Ohta et al. [37]. The $\bar{X}-R$ charts are not effective in detecting small shifts and make no advantage of recognizing the shift type. Consequently, Costa and Rahim [38] presented the non-central chi-square chart for monitoring the mean and variance parameters simultaneously. After presenting NCS chart, an array of studies have been conducted on these charts among which Costa and Rahim [39], Costa and De Magalhaes [40], and Tsai et al. [41] are the most notable ones. Despite the effectiveness of the NCS chart, this monitoring scheme has not been combined with maintenance planning and inventory control topics. Concerning the mentioned gaps in the literature, the novelties of this study are:

1- To bring the model closer to the real manufacturing systems, multiple ACs and interdependency among them are taken into account;

2- A non-uniform sampling scheme is developed for faster detection of the occurred ACs;

3- The process mean and variability are monitored simultaneously to improve the customer satisfaction.

The rest of this study organized as follows: In Section 2, the problem is defined. In Section 3, the mathematical programming model is explained. Afterwards, in Section 4, the solution approach is described. The optimization results, sensitivity analysis and comparative studies are presented in Section 5. Eventually, in Section 6, conclusions are made, and some suggestions for future studies are provided.

2. Problem definition

We assume that the manufacturing process starts with an in-control condition, and s types of ACs can shift the process to out-of-control situations. Salmasnia et al. [21], assumed that when i^{th} AC occurs, any other ACs won't happen until the end of the production cycle. To make the proposed model more practical, it is assumed that the occurrence of one AC does not prevent the occurrence of the other ones. As mentioned before, in this research, it's assumed that each AC can affect both the process mean and variance. Thus, an appropriate monitoring scheme is required to monitor the mean and variance parameters together. This study employs the NCS chart which is more efficient than the $\bar{X}-R$ chart (Costa and Rahim [38]), particularly in detecting small and mediocre shifts.

Due to the fact that the time-to-occurrence of each AC follows the Weibull distribution, a non-uniform sampling strategy is used for more efficiency of SPM. It is worth mentioning that similar to Salmasnia et al. [21], the production process is divided into three different scenarios. In the first scenario, the process is completely in-control, and no AC occurs during the production cycle. Thus, the planned maintenance operation is carried out at the end of the production cycle. In the second scenario, due to the occurrence of an AC, the process shifts to an out-of-control condition. In this scenario, the out-of-control condition is detected by the control chart before the end of the production cycle, and consequently corrective maintenance is carried out. It this scenario, other ACs may take place before the detection of the shift which results in larger shifts in process mean and variance. In the third scenario, the process shifts to the out-of-control condition because of one AC. However, the shift is not detected by the control chart until the end of the production cycle. In this scenario, until the end of the production cycle, other ACs may occur, and at the end of the production cycle, corrective maintenance is carried out.

The aim of this study is to determine the optimum values of the number of sampling intervals between two planned maintenance operations (k), and the length of production cycle (W_{k+1}) along with the NCS chart parameters including the control limit coefficient (L_{CS}), the length of the first sampling interval (h_1), non-centrality parameter (d) and the sample size (n) in a way that the expected total cost consisting of SPM costs, inventory costs, and maintenance costs is minimized.

2.1. Notations

Before developing the mathematical programming model, the notations used for problem formulation are presented in Table 1.

Please insert Table 1 here

2.2. Assumptions

Since this study is an extension of Salmasnia et al. [21], this subsection only focuses on the assumptions that are different from the mentioned paper. The rest of the assumptions that are considered in Salmasnia et al. [21], and do not violate the following assumptions, are also applied in this research.

1. Production cycle is monitored by means of a non-uniform sampling scheme. This scheme is conducted in a way that the integrated hazard rates are the same during all sampling periods. According to the non-uniform sampling scheme, random samples each of size *n* are taken from the process at points $h_1, h_1+h_2, h_1+h_2+h_3, \ldots$. Denoting W_j as the time of taking the j^{th} sample, the sampling intervals and sampling points are computed as follows.

$$W_j = j^{\frac{1}{\nu}} h_1 \tag{1}$$

$$h_j = j^{\frac{1}{\nu}} h_1 - (j-1)^{\frac{1}{\nu}} h_1 \tag{2}$$

2. The quality characteristic of interest follows a normally distribution, and the occurrence of the i^{th} AC changes both mean and variance parameters as below:

$$\mu_i = \mu_0 + \delta_i \sigma_0 \tag{3}$$

$$\sigma_i = \psi_i \sigma_0 \tag{4}$$

3. The occurrence probability of one AC is under the impact of another AC occurred before that. The time-to-occurrence of AC type *u* (A_u) when the process is under the impact of AC type *i* (A_i) follows a Weibull distribution with size and shape parameters $\lambda_{i,u}$ and ν, respectively. Accordingly, we have $f_{i,u}(t) = \lambda_{i,u} \nu t^{\nu-1} e^{\lambda_{i,u} t^{\nu}}$.
4. The state $i = 0$ denotes the in-control condition ($\psi_0 = 1, \delta_0 = 0$).
5. The occurrence of one AC does not prevent the occurrence of the others, which leads to larger shifts in process mean and variability. That is to say, the posterior process state is always worse than its prior state $\lambda_{i,u} = 0 \ (\forall u < i)$.
6. The time-to-occurrence of the earliest AC, when the process is under the impact of i^{th} assignable cause type is a Weibull variable with parameters $\lambda_i = \sum_{u=i+1}^{m} \lambda_{i,u}$ and ν ($f_i(t) = \lambda_i \nu t^{\nu-1} e^{\lambda_i t^{\nu}}$) with respect to the assumptions (2) and (4).
7. In the developed Markov chain, at time *t* the process will be in its i^{th} state, if the process is under the impact of i^{th} AC type (if the process is in-control, the process is assumed to be in state $i = 0$).
8. The state *y* is referred as an intermediate state between the prior state *i* and the posterior state *u* when the process initially goes from state *i* to state *y* and then goes

from state y to state u. It is assumed that the probability of existing more than one intermediate state in a sampling interval is negligible.

To calculate the expected total cost in the proposed model, the following definitions are required:

a) q_{ij} $(\forall i \in \{1,2,...,s\}$, $j \in \{1,2,...,k+1\})$: The probability that the process shifts from the in-control to an out-of-control condition due to the occurrence of A_i during h_j .

$$q_{ij} = \int_{W_{j-1}}^{W_j} f_{0,i}(t)\,dt \tag{5}$$

b) $\tau_{ij}(\forall i \in \{1,2,...,s\}$, $j \in \{1,2,...,k+1\})$: The expected in-control time period within h_j , given that A_i has occurred during the j^{th} sampling interval.

$$\tau_{ij} = \frac{\int_{W_{j-1}}^{W_j} (t - W_{j-1})\, f_{0,i}(t)dt}{q_{ij}} \tag{6}$$

c) $\tau_i(\forall i \in \{1,2,...,s\})$: The expected in-control time period within a given sampling interval in which A_i occurs.

$$\tau_i = \sum_{j=1}^{k} \tau_{ij} q_{ij} = \sum_{j=1}^{k} \int_{W_{j-1}}^{W_j} (t - W_{j-1})\, f_{0,i}(t)dt \tag{7}$$

d) $q_0(a,b)$: The probability that the earliest AC occurs during the interval $[a,b]$

$$q_0(a,b) = \int_a^b f_0(t)\,dt \tag{8}$$

3. Model description

In this Section, initially, a Markov chain is developed to model interdependencies among the ACs. Afterwards, different cost elements related to the model are calculated.

3.1. Developing a Markov chain to model interdependence among assignable causes

As it was previously discussed, the mentioned process can be under the impact of different ACs in a production cycle. Hence, depending on the time period that the process spends in each state (the time after the occurrence of one AC to the occurrence of next one), the manufacturer incurs costs. To calculate the expected time period in each state, the model needs to calculate steady-state probabilities, and for this end, a Markov chain is developed.

In the general concept of the Markov chain, the time is classified into three different periods of past, present, and future. The main characteristic of the Markov process is that the future of this process is not dependent on the route which it has traversed in the past, and it only depends on its present position. Generally, a Markov process is categorized based upon two factors:

(1) Time parameter which can be discrete or continuous. When time is discrete, it can be interpreted that the process behavior is studied in specific points.

(2) Set of values which Markov characteristic can have that is named system state.

Markov chain is a particular case of Markov process in which both system state and time parameter can only take discrete values. If the process is investigated only in the sampling points, and the system state is defined based on the assumption (8), we can consider it as a Markov chain. The system state in each sampling point is merely dependent on its state in the previous one, because the time origin of Weibull distribution is considered as the start of the process. In the rest of this section, the elements of the transition probability matrix are calculated, and the steady-state probabilities are determined by using this matrix.

The probability of transition from state i to state u denoted by P_{iu} is equal to the probability that the process is in state i at the beginning of a sampling interval, and is in state u at the end of that interval. This probability is under the impact of a Weibull distribution, which is not a memoryless distribution. Equivalently, the probability of transition from state i to state u differs in different sampling periods. Hence, the following formula is utilized based on the law of total probability to calculate the transition state.

$$P_{iu} = \sum_{j=1}^{k+1} \Pr\left((i \to u) \cap (W_{j-1} < t < W_j)\right) \tag{9}$$

Given Equation (9), the probability of transition from state $i-1$ to state i is obtained using Equations (10) and (11). This probability calculates the intersection between two events: (1) when the process is under the impact of A_{i-1}, A_i happens before the other ACs. (2) after the occurrence of A_i, no other AC occurs. It is mentionable that the probability of transition from state $i-1$ to state i in the first sampling interval is larger than zero, only if the prior state be state 0. This condition always occurs because as it was previously pointed out, the process starts from its in-control condition. As a result, a little difference exists between Equations (10) and (11).

$$P_{i-1,i}=\sum_{j=1}^{k+1}\left(\int_{W_{j-1}}^{W_j}\frac{\lambda_{i-1,i}}{\lambda_{i-1}}\,f_{i-1}(t)\left[e^{-\lambda_i(W_j)^{v}}\right]dt\right)\qquad for\ \ i-1=0 \tag{10}$$

$$P_{i-1,i}=\sum_{j=2}^{k+1}\left(\int_{W_{j-1}}^{W_j}\frac{\lambda_{i-1,i}}{\lambda_{i-1}}\,f_{i-1}(t)\left[e^{-\lambda_i(W_j)^{v}}\right]dt\right)\qquad for\ \ i-1\neq 0 \tag{11}$$

To determine the probability of transition from state *i* to state $u>i+1$ based on the assumption (9), we consider two scenarios. In first scenario, the process directly goes from state *i* to state *u* while in the second one it transfers from state *i* to an intermediate state and transfers from the intermediate state to state *u*. Based on this, the probability of P_{iu} $(u>i+1)$ are given by Equations (12) and (13).

$$P_{i,u}=\sum_{j=1}^{k+1}\left[\left(\int_{W_{j-1}}^{W_j}\frac{\lambda_{i,u}}{\lambda_i}\,f_i(t)\left[e^{-\lambda_u(W_j)^{v}}\right]dt\right)+\right.$$
$$\left.+\left(\sum_{y=i+1}^{u-1}\int_{W_{j-1}}^{W_j}\frac{\lambda_{i,y}}{\lambda_i}\,f_i(t).P'_{y,u}(t,W_j)dt\right)\right]\qquad for\ \ i=0 \tag{12}$$

$$P_{i,u}=\sum_{j=2}^{k+1}\left[\left(\int_{W_{j-1}}^{W_j}\frac{\lambda_{i,u}}{\lambda_i}\,f_i(t)\left[e^{-\lambda_u(W_j)^{v}}\right]dt\right)+\right.$$
$$\left.+\left(\sum_{y=i+1}^{u-1}\int_{W_{j-1}}^{W_j}\frac{\lambda_{i,y}}{\lambda_i}\,f_i(t).P'_{y,u}(t,W_j)dt\right)\right]\qquad for\ \ i\neq 0 \tag{13}$$

It should be pointed out that $P'_{y,u}(t,W_j)$ equals to the probability that the process is in state *y* at time *t*, and it directly transfers from state *y* to state *u* during $\left[t,W_j\right]$, and remains in state *u* until W_j. This is formulated as follow:

$$P'_{y,u}(t,W_j)=\int_{t'=t}^{W_j}\frac{\lambda_{y,u}}{\lambda_y}\,f_y(t'\,|\,t'>t)\left[e^{-\lambda_u(W_j)^{v}}\right]dt \tag{14}$$

Ultimately, $P_{i,i}$ equals to the probability that the process remains in state *i* from the beginning to the end of the sampling interval. In other words, this probability equals to the probability of not transferring from state *i* to any other state.

$$P_{i,i}=1-\sum_{u\neq i}P_{i,u} \tag{15}$$

After calculating transitions probabilities based on Equations (10)-(15), the transition probabilities matrix is attainable in the following form:

$$\mathbf{P} = \begin{bmatrix} P_{0,0} & P_{0,1} & P_{0,2} & \cdots & P_{0,s} \\ 0 & P_{1,1} & P_{1,2} & \cdots & P_{1,s} \\ 0 & 0 & P_{2,2} & \cdots & P_{2,s} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & P_{s,s}=1 \end{bmatrix} \tag{16}$$

Then, based on matrix $\mathbf{P}$, the steady-state probabilities can be obtained as follows:

$$\boldsymbol{\pi} = \boldsymbol{\pi}_{(0)} \times \mathbf{P} \tag{17}$$

where $\boldsymbol{\pi} = [\pi_0, \pi_1, \ldots, \pi_s]$ and $\boldsymbol{\pi}_{(0)}$ are the steady-state probabilities vector and the initial probabilities vector, respectively. As previously mentioned, the production process starts with the in-control condition and therefore we have $\boldsymbol{\pi}_{(0)} = [1, 0, \ldots, 0]_{1\times(s+1)}$. It should be noted that regarding to the previous explanations, the steady-state probability for state i ($i = 1, 2, \ldots, s$) given that the process is out-of-control is obtained as Equation (18).

$$\pi'_i = \frac{\pi_i}{1-\pi_0} \tag{18}$$

where π'_i denotes the steady-state probability for state i given that the process is out-of-control.

3.2. Features of scenarios

Scenario1: According to Figure 1, under this scenario, the process remains in-control during the production cycle and is restored to the as-good-as-new condition by planned maintenance after the end of the cycle. Therefore, the expected in-control and out-of-control time periods under this scenario are attainable through equations below:

$$E(T_{in} \mid S_1) = W_{k+1} = (k+1)^{\frac{1}{\nu}} h_1 \tag{19}$$

$$E(T_{out} \mid S_1) = 0 \tag{20}$$

The occurrence probability of this scenario, similar to Salmasnia et al. [21], equals to the probability that the earliest AC occurs after W_{k+1}.

$$P(S_1) = 1 - F_0(W_{k+1}) = e^{-\lambda_0 (k+1) h_1^V} \quad (21)$$

Please insert Figure 1 here

Scenario 2: Under this scenario, the process mean and variance go to the out-of-control condition during the production cycle due to the occurrence of AC. The NCS chart cannot necessarily detect it in the first sample after the occurrence of the shift. However, before the end of the cycle, the shift is recognized by the control chart. Based on Figure 2, a difference between this model and Salmasnia et al. [21] is that the process can be under the impact of different ACs in an out-of-control time period. In other words, after the occurrence of the earliest AC, other ACs can occur which deteriorate the process. Consequently, the expected in-control and out-of-control time periods are calculated by Equations (22) and (23).

$$E(T_{in} \mid S_2) = \int_0^{W_k} t f_0(t \mid 0 < t < w_{k+1}) dt \quad (22)$$

$$E(T_{out} \mid S_2) = \sum_{i=1}^{m} \frac{\lambda_{0,i}}{\lambda_0} \left(\sum_{j=1}^{k} \sum_{r=1}^{k+1-j} \left[\left(q_0(W_{j-1}, W_j) \right) (w_{r+j-1} - w_{j-1}) \beta^{r-1} (1-\beta) \right] - \tau_i \right) + nE + T_1 \quad (23)$$

Please insert Figure 2 here

It should be noted that β is obtained by Equation (24). Because in this model, the out-of-control time period can be under the impact of different ACs, the steady-state probabilities should be taken into account in calculations.

$$\beta = \sum_{i=1}^{m} \pi_i' \beta_i \quad (24)$$

In the above equation, β_i is the probability of Type II error when the process is under the impact of A_i. According to the features of the NCS chart discussed by Costa and Rahim [38], β_i can be obtained using Equation (25). It is remarkable that the average run length (*ARL*) in out-of-control condition can be calculated using Equation (26).

$$\beta_i = 1 - P(Y_l > L_{CS} \sigma_0^2) = 1 - P(Y_l / \sigma_i^2 > L_{CS} / \gamma_i^2) \quad (25)$$

$$ARL_1 = 1/(1-\beta) \quad (26)$$

where Y_l is the NCS statistic at l^{th} sample. According to Equation (23) and the steady-state probabilities, the expected time length that the process is under the impact of i^{th} assignable cause is obtained as:

$$E(Ta_i \mid S_2) = \frac{\pi_i}{1-\pi_0} E(T_{out} \mid S_2) \tag{27}$$

where Ta_i is the time interval that process remains in state $i = 1, 2, \ldots, s$. The occurrence probability of scenario 2, equals to the occurrence probability of the earliest AC before taking k^{th} sample, given that at least one alarm issues in previous samples. If the earliest AC occurs at time *t*, the maximum number of samples taken from an out-of-control process will be obtained as $Sa(t) = k - \left\lfloor (t/h_1)^{1/v} \right\rfloor$ (see Proof 1).

Proof 1. The maximum number of taken samples in a production cycle is equal to *k*. Furthermore, with considering the following mathematical inference, the number of samples taken before *t* is obtained as $\left\lfloor (t/h_1)^{1/v} \right\rfloor$. Moreover, the maximum number of samples taken during the out-of-control time period is calculated as $Sa(t) = k - \left\lfloor (t/h_1)^{1/v} \right\rfloor$.

$$W_j < t < W_{j+1} \Rightarrow j^{1/v} h_1 < t < (j+1)^{1/v} h_1 \Rightarrow j^{1/v} < \frac{t}{h_1} < (j+1)^{1/v}$$
$$\Rightarrow j < \left(\frac{t}{h_1}\right)^{1/v} < (j+1) \Rightarrow j = \left\lfloor \left(\frac{t}{h_1}\right)^{1/v} \right\rfloor$$

Finally, the occurrence probability of scenario 2 is computed through Equation (28).

$$P(S_2) = \int_0^{W_k} f_0(t)(1-\beta^{Sa(t)})dt \tag{28}$$

Scenario 3: In this scenario, during the production cycle, the process shifts to an out-of-control condition. However, the NCS chart does not detect the occurrence of AC until the end of the production cycle. As it can be seen in Figure 3, similar to scenario 2, in this scenario the process can be under the impact of different ACs. During the implementation of planned maintenance, the out-of-control state is detected and therefore, the planned maintenance is replaced by the corrective maintenance. The expected in-control and out-of-control time periods in this scenario are calculated based on equations below:

$$E(T_{in} \mid S_3) = \int_0^{W_{k+1}} t f_0(t \mid 0 < t < W_{k+1}) dt \tag{29}$$

$$E(T_{out} \mid S_2) = W_{k+1} - E(T_{in} \mid S_3) = (k+1)^{\frac{1}{v}} h_1 - E(T_{in} \mid S_3) \tag{30}$$

Therefore, the expected time that the process spends in each of states $i = 1, 2, \ldots, s$ is:

$$E(Ta_i \mid S_3) = \frac{\pi_i}{1-\pi_0} E(T_{out} \mid S_3) \tag{31}$$

Based on Equation (32), $P(S_3)$ equals to the probability that the earliest AC occurs before W_{k+1} but no alarm issues after its occurrence.

$$P(S_3) = \int_0^{W_{k+1}} f_0(t)\beta^{Sa(t)}dt \tag{32}$$

Please insert Figure 3 here

3.3. Model costs

3.3.1 Quality loss cost

Each AC leads to different quality loss cost per unit. According to the expected time that the process is under the impact of each AC, the expected quality loss cost under the out-of-control condition can calculated. The expected quality loss cost under in-control condition is computed similar to Salmasnia et al. [21]. Hence, the quality loss cost in each scenario is obtained as Equations (33)-(35). Then, the expected total quality loss cost is obtained as Equation (36).

$$E(C_Q \mid S_1) = Q_{in}.p.E(T_{in} \mid S_1) = Q_{in}.p.(k+1)^{\frac{1}{\nu}} h_1 \tag{33}$$

$$E(C_Q \mid S_2) = Q_{in}.p.E(T_{in} \mid S_2) + p.\sum_{i=1}^{s} Qout_i.E(Ta_i \mid S_2) \tag{34}$$

$$E(C_Q \mid S_3) = Q_{in}.p.E(T_{in} \mid S_3) + p.\sum_{i=1}^{s} Qout_i.E(Ta_i \mid S_3) \tag{35}$$

$$E(C_Q) = \sum_{z=1}^{3} E(C_Q \mid S_z) \times P(S_z) \tag{36}$$

3.3.2. Sampling cost

As mentioned, the process remains in-control during the production cycle in scenario 1. On the other hand, the NCS chart does not detect the assignable causes in scenario 3. Therefore, the number of samples taken in both scenarios 1 and 3 is a constant value denoted by k. In contrast, in scenario 2, the process stops after taking $r_{in} + r_{out}$ samples where r_{in} and r_{out} indicate the expected number of samples taken when the process is in-control and out-of-control, respectively.

$$r_{in} = \sum_{j=1}^{k} (j-1) \times q_0(W_{j-1}, W_j) \tag{37}$$

$$r_{out} = (\sum_{j=1}^{k} (\int_{w_{j-1}}^{w_j} f_0(t)dt))(\sum_{r=1}^{k+1-j} r(1-\beta)\beta^{r-1}) \tag{38}$$

The sampling cost is affected by two factors: (1) the fixed and variable costs of each sample; and (2) the expected number of taken samples. According to the given explanations, the sampling cost for each scenario and the total sampling cost are obtained by using equations below:

$$E(C_S \mid S_1) = (C_F + nC_V)k \tag{39}$$

$$E(C_S \mid S_2) = (r_{in} + r_{out})(C_F + nC_V) \tag{40}$$

$$E(C_S \mid S_3) = k(C_F + nC_V) \tag{41}$$

$$E(C_S) = \sum_{z=1}^{3} E(C_S \mid S_z) \times P(S_z) \tag{42}$$

3.3.3. Maintenance cost

In this sub-section, first the expected maintenance cost under $z^{th}; z = 1,2,3$ scenario, $E(C_M \mid S_z)$, including the maintenance activity implementation and the false alarm evaluation is calculated. Note that, the false alarm cost is obtained via the multiplication of evaluation cost of each false alarm by the expected number of issued false alarms in its corresponding scenario. Since the process remains in-control during the production cycle in scenario 1, the maintenance cost which is given in Equation (43) consists of the planned maintenance cost and the false alarm cost.

$$E(C_M \mid S_1) = \frac{k \times C_Y}{ARL_0} + C_{PM} \tag{43}$$

As noted, the process goes to an out-of-control state due to the occurrence of assignable causes in scenarios 2 and 3. As a consequence, we replace k by r_{in} to calculate the expected number of false alarms in these scenarios. Besides, the corrective maintenance cost is imposed in both scenarios 2 and 3 which is dependent on the type of the assignable cause. Accordingly, the expected maintenance cost in scenarios 2 and 3 can be obtained as Equations (44) and (45), respectively:

$$E(C_M \mid S_2) = \frac{r_{in} \times C_Y}{ARL_0} + \sum_{i=1}^{s} \pi_i' \times Ccm_i \tag{44}$$

$$E(C_M \mid S_3) = \frac{r_{in} \times C_Y}{ARL_0} + \sum_{i=1}^{s} \pi_i' \times Ccm_i \tag{45}$$

It is worth mentioning that, the value of r_{in} in scenarios 2 and 3 is obtained by Equations (37) and (47), respectively. Moreover, the value of in-control *ARL* is attained as $ARL_0 = 1/\alpha$ where α is calculated based on Costa and Rahim [38].

$$\alpha = P(Y_l > L_{CS}\sigma_0^2) = P(Y_l / \sigma_0^2 > L_{CS}) \tag{46}$$

$$r_{in} = \sum_{j=1}^{k+1} (j-1) \times q_0(W_{j-1}, W_j) \tag{47}$$

Finally, the expected maintenance cost for a given production cycle is calculated based on the values of $E(C_M \mid S_z); z = 1,2,3$ and their corresponding occurrence probabilities as follows:

$$E(C_M) = \sum_{z=1}^{3} E(C_M \mid S_z) \times P(S_z) \tag{48}$$

3.3.4. Inventory holding cost and setup cost

According to the classic EPQ model, the inventory holding cost and the setup cost can be obtained according to Equations (49) and (50).

$$IHC = \frac{B \times W_{k+1} \times (p - D_d)}{2} \tag{49}$$

$$C_{setup} = \frac{D \times A'}{p \times W_{k+1}} \tag{50}$$

where W_{k+1} denotes the end time of a perfect production cycle.

3.4. Objective function and constraints

According to the previous explanations, the objective function and the model constraints are given as follow:

$$Min\ ETC = C_{setup} + IHC + E(C_Q) + E(C_S) + E(C_M) \tag{51-1}$$

$s.t:$

$$1 \le n \le u_0 \tag{51-2}$$

$$ARL_0 > ARL_L \tag{51-3}$$

$$ARL_1 < ARL_U \tag{51-4}$$

$$W_{k+1} \geq CT \tag{51-5}$$

$$d > 0, h_1 > 0,\ L_{CS} > 0,\ n \in N^+,\ k \in N^+ \tag{51-6}$$

The proposed mathematical model aims to minimize the expected total cost which is defined in Equation (51-1). Note that, the cost objective function contains five terms of the setup, inventory holding, quality loss, sampling and the maintenance costs. The proposed mathematical model includes five constrains. The constraint (51-2) guaranties that the sample size does not exceed a pre-determined value u_0. The statistical constraints (51-3) and (51-4) ascertain the in-control and out-of-control performance of NCS chart, respectively. Constraint (51-5) ensures the process continuity by selecting a lower bound for the production length of a perfect cycle. Finally, constraints (51-6) defines the acceptable domain of the decision variables.

The economic production quantity (EPQ) expressed by Equation (52) depends on the production rate (p) and the cycle length (w_{k+1}). As it can be seen, the value of w_{k+1} is a function of two decision variables h_1 and k obtained by solving the mathematical programming (51).

$$EPQ = W_{k+1} \times p = (k+1)^{1/v} \times h_1 \times p \tag{52}$$

4. Solution approach

Regarding to the high complexity of the proposed mathematical model, this paper employs the particle swarm optimization (PSO) as a famous population-based algorithm for optimizing the mathematical programming (51). The PSO algorithm is one of the most popular methods among the meta-heuristic algorithms which finds near-optimal solutions in a reasonable computation time. This algorithm has been widely used by some researchers in the existing literature such as Jafarian-Namin et al. [42], Salmasnia et al. [43], and Matin et al. [44]. In PSO algorithm, each solution in feasible space is known as a particle which contains two features including the position and velocity (particle direction). It combines the local and global searching mechanisms to improve its search effectiveness. In the first iteration, the position and velocity vectors are selected randomly. Afterwards, the search process continues by updating particles based on three factors of force of inertia (w), global best ($gbest$), and personal best ($pbest$). Note that, the $gbest$ indicates the best solution observed so far while the $pbest$ is the best decision variable vector experienced by the i^{th} particle. That is to say, the particle's velocity vector is updated based on a compromise among three vectors: (1) the current velocity, (2) moving towards its $pbest$, and (3) moving towards the $gbest$. The vector of velocity and position are updated in each iteration, after finding the two best values. The PSO's searching process in a two-dimensional feasible space is illustrated in Figure 4.

Please insert Figure 4 here

In Figure 4, x_i^t, v_i^t and $pbest_i^{t-1}$ represent the position, velocity and personal best vectors of the i^{th} particle in t^{th} iteration while $gbest^t$ vector denotes the best vector of decision variables during the past t iterations. Moreover, r_g and r_p are two random vectors whose elements are selected uniformly from the interval $[0,1]$. Furthermore, c_1 and c_2 are cognition and social learning parameters, respectively and are determined through a trial-and-error process subject to $c_1 + c_2 = 4$.

Particle representation is a substantial factor in the PSO algorithm. A five-dimensional vector is used in this research for representing the particles, which is indicated by Equation (53).

$$x_i^t = [n, h_1, L_{CS}, d, k] \tag{53}$$

The particle representation includes two discrete decision variables n, k and three continuous ones including h_1, L_{CS}, and d. As mentioned before, the initial values of the continuous decision variables are randomly generated from a uniform distribution between their corresponding lower and upper limits. However, to determine the initial value of discrete decision variables, firstly a random value is produced from a uniform distribution within the interval $[0,1]$. Then, the initial values of the discrete variables n and k are obtained using equations (54) and (55).

$$n = Min(n_{\min} + \lfloor (n_{\max} - n_{\min} + 1) \times R_1 \rfloor, n_{\max}) \tag{54}$$

$$k = Min(k_{\min} + \lfloor (k_{\max} - k_{\min} + 1) \times R_2 \rfloor, k_{\max}) \tag{55}$$

Where $n_{\min}$, $n_{\max}$, $k_{\min}$ and $k_{\max}$ denote the lower and upper limits of n and k, respectively while R_1 and R_2 are two random numbers from $U(0,1)$. Eventually, vector of decision variables is updated according to PSO procedure, and the algorithm stops when it meets the stopping criterion. The PSO flowchart summarizing its computational procedure is depicted in Figure 5.

Please insert Figure 5 here

5. Experimental results

In this section, first in sub-section 5.1 a numerical example is presented. Then, in sub-section 5.2 a sensitivity analysis is provided to analyze how the changes in the number of ACs affects the cost terms. In sub-section 5.3, the presented model is compared with a model in which the multiple ACs are independent from each other. In sub-section 5.4, the ability of NCS chart is evaluated against with the $\bar{X}-R$ chart. Eventually, in sub-section 5.4, the performance of uniform and non-uniform sampling strategies are compared.

5.1. Numerical example

In this sub-section, an industrial example borrowed from Chen and Yang [26] is introduced to highlight the application of the proposed mathematical model. In this example, the company under investigations sells a specific food product to a wholesaler in packages marked with the specific weight. According to Bisgaard et al. [45], it is assumed that the quality characteristic of the interest is the weight of packages and ACs can change both mean and variability parameters. The values of parameters which are independent from the type of ACs are given in Table 2.

Please insert Table 2 here

In addition to the parameters above, there are some other parameters affected by the type of ACs. According to the sixth assumption of sub-section 2.1, parameters δ_i and ψ_i for $i=1,2,\ldots,s$ are introduced as follow:

$$\delta_i=\delta_0+i.\delta_v \tag{56}$$

$$\psi_i=\psi_0+i.\psi_v \tag{57}$$

where $\delta_0=0$ and $\psi_0=1$. Knowing that $Qout_i$ and Ccm_i are proportional to the shift magnitude, the following equations are used to calculate them.

$$Ccm_i=Ccm_B+(i-1).Ccm_v \tag{58}$$

$$Qout_i=Qout_B+0.5\times(i-1)\times Qout_B \tag{59}$$

Since the quality loss cost per unit under in-control condition should be proportional to the out-of-control condition, it is assumed that $Q_{in}=0.2Qout_B$ which is independent from the type of AC. To determine λ_{iu}, in case that the process is under the impact of A_i, it is assumed that the more u is closer to i, the larger is the occurrence rate of A_u ,. Therefore:

$$\lambda_{iu} = \begin{cases} \dfrac{\lambda_B}{2^{u-i-1}} & u > i \\ 0 & other \end{cases} \tag{60}$$

To determine the parameters dependent on ACs, the values of δ_v, ψ_v, Ccm_v, Ccm_B, $Qout_B$ and λ_B are defined based upon Table 3. Then, by using these values and according to Equations (56)-(60), the values of parameters dependent on ACs are computed.

Please insert Table 3 here

As mentioned at the beginning of this subsection, the numerical example is related to the food industry. In this industry, the production of a nonconforming items can result in significant problems such as food poisoning. As a result, the quality loss cost per item in both the in-control and out-of-control conditions is remarkable in comparison with the other costs presented in Tables 2 and 3. Accordingly, the findings of this research can also be employed in the pharmaceutical or military industries since the non-conforming products impose excessive costs on the manufacturer. Assuming $s = 6$, we solve the proposed mathematical programming (51) using the PSO algorithm. The obtained results for decision variables are $(n, h_1, L_{CS}, k, d) = (11, 1.4910, 26.40, 44, 0.25179)$ while ETC, ARL_{in}, ARL_{out} and are obtained as 38118.29, 104.71, and 1.38.

5.2. Sensitivity analysis on the number of assignable cause types

Here, we investigate how the number assignable cause types affect different cost terms. To do this, given the values allocated to the parameters, by using PSO algorithm, the model is solved for $s = 1, 2, \ldots, 6$ (where *s* is the number of AC types) and the results are summarized in Table 4. As can be also seen in Figure 6, the expected quality loss cost increases with a rise in the number of ACs. The reason is that increasing the number of ACs, decreases not only the expected in-control time period, but also the out-of-control condition includes more states with higher costs of quality loss. Besides, according to Figure 7, with an increase in the number of ACs, the maintenance cost rises with a slowing-down slope. Its reason is the rise in the steady-state probability for states with higher costs of corrective maintenance. On the other side, it can be seen from Figure 8 that the expected sampling cost has both ascending and descending trends. This cost is proportional to the sample size and the number of sampling points in a process cycle. When *s* increases from 3 to 4, although the number of samplings rises, the expected sampling cost reduces, and its reason is a considerable decrease in the sampling size from 12 to 6.

Please insert Figure 6 here

Please insert Figure 7 here

Please insert Figure 8 here

Recall that the inventory holding cost, the setup cost, and economic production quantity are calculated based on Equations (49), (50) and (52) and depend only on the production run length. For all values of $s = 1, 2, \ldots, 6$, since the production length is obtained equal to 10, the inventory holding cost, the setup cost and EPQ are the same as can be seen in Table 5. Consequently, with an increase in the number of ACs, the quality loss and maintenance costs have ascending trend while the inventory holding and setup costs remain fixed. Only the sampling cost has a descending trend in some cases, which in comparison with the other costs has a smaller share of the expected total cost. Hence, as can be observed from Figure 9, the expected total cost rises with the increase in *s*.

Please insert Figure 9 here

Please insert Table 4 here

Please insert Table 5 here

5.3. Comparing two models with dependent and independent assignable causes

In this sub-section, the model with dependent ACs (called as dependent model) is compared with the model in which that ACs are independent (named as independent model). It is assumed that the independent ACs occur in a process whose parameters are calculated based on sub-section 5.1. However, in the case of the independent model, the occurrence rate of each AC under the in-control condition is calculated based on Equation (61).

$$\lambda_i = \frac{\lambda_B}{2^{i-1}} \tag{61}$$

The results obtained by solving two dependent and independent models using the PSO algorithm for ACs 1 to 6 are given in Table 6. As it can be inferred from Table 6, the cost of the independent model is less than the dependent model. This can leads to misleading interpretations because the independency assumption among the ACs is far from the reality. The most significant factor that causes difference between the costs of these two models is the steady-state probabilities. As mentioned earlier, in the dependent model, π_i' indicates the probability that the process is under the impact of A_i given that the process is out-of-control.

However, in an independent model, similar to Salmasnia et al. [21], π_i' is substituted by λ_i / λ_0 where $\lambda_0 = \sum_{i=1}^{s} \lambda_i$.

Please insert Table 6 here

As can be seen in Table 6, under the single AC scenario, the optimum value in both models are equal, and the expected total costs are approximately equal. The nuance in the expected total costs of the two models is due to the difference in calculations methods, which are explained below the Equation (23). However, for $s = 2,3,\ldots,6$ the expected total cost of the proposed model (dependent model) is higher, which is closer to the reality. The reason is that in the dependent model, the occurrence probability of the worse conditions is higher. In Figure 10, the comparison between the two expected total costs is illustrated graphically.

Please insert Figure 10 here

As can be seen in Figure 11, the coefficient of control limit (L_{CS}) is usually larger in the proposed model than the independent model. That is because of the higher occurrence probability of large shifts in the proposed model which reduces the probability of Type II error and raises the L_{CS} . On the other hand, since the occurrence probability of large shifts in process mean and standard deviation is lower in the independent model, sampling is more easy-going. Consequently, the values of h_1 and n are usually larger and smaller, respectively compared to the dependent model.

Please insert Figure 11 here

5.4. Comparing non-central chi-square control chart with $\bar{X} - R$ chart performances

In this sub-section, the performance of the proposed model is compared with the competing one which uses the combined $\bar{X} - R$ chart based on the previously generated examples. To do so, it is initially considered four levels for each of δ_v , ψ_v , and λ_B , $Qout_B$, C_Y as given in Table 7. Then, the numerical examples are produced by using a Taguchi design consisting of 16 experiments as can be seen in Table 8. It is notable that the rest of the parameters in all examples, based on Table 2, are the same.

Please insert Table 7 here

Please insert Table 8 here

For fair comparison between the NCS and $\bar{X}-R$ charts, we assimilate the in-control costs for both methods and calculate costs of out-of-control state for the process. To accomplish this, first the considered example is solved with the proposed model, which considers NCS chart, and the decision variables and cost values are calculated. In order to compute the optimum values of the model considering $\bar{X}-R$ chart (hereafter referred as $\bar{X}-R$ model), the values of h_1 and k are considered equal to those of the proposed method. Moreover, we try to make α the same in both models so that the in-control costs in mentioned models become the same. Afterwards, the optimum values of n, $L_{\bar{X}}$ and L_R are determined such that the total cost in the $\bar{X}-R$ model is minimized. The results gained from solving two models can be seen in Tables 9 and 10. As can be seen in Tables 9 and 10, in all examples, the NCS chart has better performance than its competing chart. While the expected in-control costs are the same in both models with an acceptable approximation, the expected out-of-control costs in $\bar{X}-R$ model is higher than the proposed model. In Table 11, the amount of improvement in costs by using the NCS control chart is demonstrated.

In order to clarify the higher performance of the proposed model, it is essential to investigate the performance of the two control charts based on the different shifts in process mean and standard deviation. It can be inferred from Table 12 that the NCS chart has a better performance in detecting both mean and variance shifts. The superiority of the non-central chi-square chart over the $\bar{X}-R$ is more significant when smaller shifts are induced. In other words, as δ_v and ψ_v increases, the difference between the NCS and $\bar{X}-R$ charts reduces. This fact is illustrated in Figures 12-14 graphically. It can be seen in Figure 14 that for each δ_v, the improvement percentage decreases with an increase in ψ_v.

Please insert Figure 12 here

Please insert Figure 13 here

Please insert Figure 14 here

Please insert Table 9 here

Please insert Table 10 here

Please insert Table 11 here

Please insert Table 12 here

5.5. Comparing uniform sampling with non-uniform sampling

The examples that are produced in section 5.3 are used to carry out this comparison. The optimal decision variables of the proposed model which uses non-uniform sampling, are given in Table 9. Furthermore, the sampling interval in the uniform sampling scheme (h_F) is fixed on h_1. Then k is determined in the form of $k = \lceil W(k+1)/h_F \rceil$ in order to satisfy constraint 51-5. Considering the value of h_1 and k, and by assuming that the rest of the decision variables are based upon Table 10, the costs of the model are calculated in case of uniform sampling. Since in this comparison, W_{k+1} usually is not the same in the uniform and the non-uniform sampling scheme, the expected cost per time unit $(ECTU = ETC / W(k+1))$ is used to compare two methods. It is s notable that $Eout_p$ means the ratio of the expected out-of-control time to the expected production cycle time and is calculated as follow:

$$Eout_p = \frac{P(S_2).E(T_{out} \mid S_2) + P(S_3).E(T_{out} \mid S_3)}{\left(P(S_1) + P(S_3)\right).W(k+1) + P(S_2).\left(E(T_{in} \mid S_2) + E(T_{out} \mid S_2)\right)} \tag{62}$$

As it can be seen in Table 13, using the non-uniform sampling strategy reduces the expected costs per time unit from 14.1 to 32.5 percent in different examples. In other words the use of the non-uniform sampling strategy instead of the uniform one leads to a considerable savings in the annual manufacturer costs. The reason for this is non-uniform sampling higher ability in detecting shifts. Based on the data given in Table 13, non-uniform sampling reduces the ratio of the out-of-control time to the production cycle time. Because the more the time of process elapses and the occurrence probability of shift increases, the more the possibility of in-time detecting of shifts using non-uniform sampling and smaller sampling periods becomes. Hence, although sampling costs are lower in the case of uniform sampling, this approach will result in a delay in detecting shifts and raises costs of quality loss, which finally increases production costs. Figure 15 compares the costs of these two sampling schemes graphically.

Please insert Figure 15 here

Please insert Table 13 here

6. Conclusion

In this research, an integrated model of production and maintenance planning, as well as statistical process monitoring was established with consideration of dependency among the multiple assignable causes. To improve the applicability of the proposed model, it was assumed that the occurrence of ACs can affect both the process mean and variability.

Therefore, a NCS chart was utilized for monitoring the process parameters simultaneously. Furthermore, a non-uniform sampling scheme was employed such that the integrated failure rate be similar over all of the sampling intervals. Eventually, the performance of the proposed model was investigated by providing a numerical example. Then, a sensitivity analysis on the number of assignable cause types was provided. Furthermore, three comparative studies were presented to evaluate the features of the proposed mathematical programming. The results confirmed that with increasing the number of ACs, the expected total cost increments because of arising quality loss and maintenance costs. Then, the presented model was compared with a model with independent ACs. The results showed that the independency assumption of ACs brings misleading results for producers and underestimates the production costs. Afterwards, the efficiency of the NCS chart was studied by comparing the presented model with a corresponding model in which the $\bar{X}-R$ chart is employed. The obtained results implied that in the case of small shifts in process mean and standard deviation, compared to the $\bar{X}-R$ chart, the use of the NCS control chart reduces the production costs significantly. Furthermore, the costs of the model were compared in the case of using uniform and non-uniform sampling schemes. The resulting values showed that using non-uniform sampling strategy can reduce the ETCU from 14.1 to 32.5 percent. The reason behind this is that as time goes on, the sampling intervals become smaller in the non-uniform scheme, and shift detection is accelerated.

Finally, some practical recommendations for industrial managers are presented as:

- Ignoring the correlation among the multiple assignable causes may results in underestimating the process critical condition by the quality practitioner. Moreover, in this situation, the expected cost imposed on the system is estimated less than the reality.
- The use of NCS chart imposes less quality loss costs on manufacturer because of its superiority over the $\bar{X}-R$ chart in detecting out-of-control conditions.
- In mechanical processes, employing a non-uniform sampling strategy instead of a uniform one reduces the quality loss cost due to a smaller ratio of out-of-control time interval to the cycle time.

References

1. Taft, E. "The most economical production lot", *Iron Age*, **101**(18), pp. 1410-1412 (1918).
2. Salmasnia, A., Rahimi, A., and Abdzadeh, B. "An integration of NSGA-II and DEA for economic–statistical design of T^2-Hotelling control chart with double warning lines", *Neural Computing and Applications*, **31**(2), pp. 1173-1194 (**2019)**.
3. Salmasnia, A., Namdar, M., and Noroozi, M. "Robust design of a VP-NCS chart for joint monitoring mean and variability in series systems under maintenance policy", *Computers & Industrial Engineering*, **124**, pp. 220-236 (2018).
4. Rahim, M., and Ohta, H. "An integrated economic model for inventory and quality control problems", *Engineering Optimization*, **37**(1), pp. 65-81 (2005).
5. Pan, E., Jin, Y., and Wang, Y. "Integration of economic production quantity in optimization design of control chart based on loss function and random process shift", *Journal of Manufacturing Technology Management*, **22**(7), pp. 929-946 (2011).
6. Gunay, E,. and Kula, U. "Integration of production quantity and control chart design in automotive manufacturing", *Computers & Industrial Engineering*, **102**, pp. 374-382 (2016).
7. Emami-Mehrgani, B., Neumann, W. P., Nadeau, S., and Bazrafshan, M. "Considering human error in optimizing production and corrective and preventive maintenance policies for manufacturing systems", *Applied mathematical modelling*, **40**(3), pp. 2056-2074 (2016).
8. Zahedi-Hosseini, F., Scarf, P., and Syntetos, A. "Joint maintenance-inventory optimisation of parallel production systems", *Journal of Manufacturing Systems*, **48**, pp. 73-86 (2018).
9. Si, G., Xia, T., Zhu, Y, Du, S., and Xi, L. "Triple-level opportunistic maintenance policy for leasehold service network of multi-location production lines", *Reliability Engineering & System Safety*, **190**, pp. 106519 (2019).
10. Liu, X., Wang, W., and Peng, R. "An integrated preventive maintenance and production planning model with sequence- dependent setup costs and times", *Quality and Reliability Engineering International*, **33**(8), pp. 2451-2461 (2017).
11. Mehrafrooz, Z., and Noorossana, R. "An integrated model based on statistical process control and maintenance", *Computers & Industrial Engineering*, **61**(4), pp. 1245-1255 (2011).
12. Liu, L., Yu, M., Ma, Y., and Tu, Y. "Economic and economic-statistical designs of an X control chart for two-unit series systems with condition-based maintenance", *European Journal of Operational Research*, **226**(3), pp. 491-499 **(2013).**
13. Xiang, Y. "Joint optimization of X control chart and preventive maintenance policies: a discrete-time Markov chain approach", *European Journal of Operational Research*, **229**(2), pp. 382-390 (2013).
14. Atashgar, K., and Abdollahzadeh, H.A. "Joint reliability and imperfect opportunistic maintenance optimization for a multi- state weighted k- out- of- n system considering economic dependence and periodic inspection", *Quality and Reliability Engineering International*, **33**(8), pp. 1685-1707 (2017).
15. Salmasnia, A., Namdar, M., and Abdzadeh, B. "An integrated quality and maintenance model for two-unit series systems", *Communications in Statistics-Simulation and Computation*, **49**(4), pp. 886-917 (2019).
16. Salmasnia, A., Abdzadeh, B., and Rahimi, A. "Joint optimisation of double warning T 2-Hotelling chart and maintenance policy with multiple assignable causes", *Journal of Statistical Computation and Simulation*, **90**(3), pp. 465-488 (2020).

17. Ben-Daya, M., and Makhdoum, M. "Integrated production and quality model under various preventive maintenance policies", *Journal of the operational Research Society*, **49** (8), pp. 840-853 (1998).
18. Ben-Daya, M. "Integrated production maintenance and quality model for imperfect processes" *IIE Transactions*, **31**(6), 491-501 (1999).
19. Lam, K., and Rahim, M. "Joint Determination of Economic Design of X-Control Charts, Economic Production Quantity, and Production Run Length for a Deteriorating Production System", *Frontiers in Statistical Quality Control*, 7, 7, 58 (**2004)**.
20. Jafarian-Namin, S., Fallahnezhad, M.S., Tavakkoli-Moghaddam, R., Salmasnia, A., and Fatemi Ghomi, S.M.T. "An integrated quality, maintenance and production model based on the delayed monitoring under the ARMA control chart", *Journal of Statistical Computation and Simulation*, 10.1080/00949655.2021.1904241, (2021).
21. Salmasnia, A., Abdzadeh, B., and Namdar, M.A. "joint design of production run length, maintenance policy and control chart with multiple assignable causes", *Journal of Manufacturing Systems*, **42**, pp. 44-56 (2017).
22. Salmasnia, A., Kaveie, M., and Namdar, M. "An integrated production and maintenance planning model under VP-T^2 Hotelling chart", *Computers & Industrial Engineering*, **118**, pp. 89-103 (2018).
23. Salmasnia, A., Soltani, F., Heydari, E., and Googoonani, S. "An integrated model for joint determination of production run length, adaptive control chart parameters and maintenance policy", *Journal of Industrial and Production Engineering*, **36**(6), pp. 401-417 (2019).
24. Banerjee, P.K., and Rahim, M. "Economic design of–control charts under weibull shock models", *Technometrics*, **30**(4), pp. 407-414 (1988).
25. Rahim, M. "Joint determination of production quantity, inspection schedule, and control chart design", *IIE transactions*, **26**(6), pp. 2-11 (1994).
26. Chen, Y.S., and Yang, Y.M. "Economic design of x-control charts with Weibull in-control times when there are multiple assignable causes", *International Journal of Production Economics*, **77** (1), pp. 17-23 (2002).
27. Moghadam, M.B., Khadem, Y., Fani, S., and Pasha, M.A. "Effects of non-normality on economic and economic statistical designs of-control charts with multiple assignable causes and Weibull in-control times", *Communications in Statistics-Simulation and Computation*, **47**(7), pp. 2055-2069 (2018).
28. Salmasnia, A., Hajihosseini, Z., Namdar, M., and Mamashli, M. "A joint determination of production cycle length, maintenance policy, and control chart parameters considering time value of money under stochastic shift size", *Scientia Iranica*, **27**(1), pp. 427-447 (2018).
29. Duncan, A.J. "The economic design of-charts when there is a multiplicity of assignable causes", *Journal of the American Statistical Association*, 66(333), pp. 107-121 (1971).
30. Yu, F.J, and Hou, J. "Optimization of design parameters for control charts with multiple assignable causes", *Journal of Applied Statistics*, **33**(3), pp. 279-290 (2006).
31. Asadzadeh, S., and Khoshalhan, F. "Multiple-objective design of an $\overline{X}$ control chart with multiple assignable causes, *The International Journal of Advanced Manufacturing Technology*, **43** (3-4), pp. 312 (2009).
32. Nenes, G., Tasias, K.A., and Celano, G.A. "General model for the economic-statistical design of adaptive control charts for processes subject to multiple assignable causes", *International Journal of Production Research*, **53**(7), pp. 2146-2164 **(2015).**

33. Tasias, K.A., and Nenes, G. "Economic− statistical design of Vp control schemes for joint monitoring of mean and variance in the presence of multiple assignable causes", *Quality Technology & Quantitative Management,* **15**(4), pp. 484-506 (2018).
34. Saniga, E.M. "Economic statistical control-chart designs with an application to $\bar{X}$ and *R* charts", *Technometrics,* **31**(3), pp. 313-320 (1989).
35. Costa, A.F. "Joint economic design of X and R control charts for processes subject to two independent assignable causes", *IIE transactions,* **25**(6), pp. 27-33 (1993).
36. Rahim, M., and Costa, A.F. "Joint economic design of X and R charts under Weibull shock models", *International Journal of Production Research,* **38**(13), pp. 2871-2889 (2000).
37. Ohta, H., Kimura, A., and Rahim, A. "An economic model for $\bar{X}$ and R charts with time-varying parameters. *Quality and Reliability Engineering International,* 18(2), pp. 131-139 (2002).
38. Costa, A., and Rahim, M. "Monitoring process mean and variability with one non-central chi-square chart", *Journal of Applied Statistics,* **31**(10), pp. 1171-1183 (2004).
39. Costa, A.F.B., and Rahim, M. "The non-central chi-square chart with two-stage samplings", *European journal of operational research,* **171**(1), pp. 64-73 (2006).
40. Costa, A.F., and De Magalhaes, M.S. "An adaptive chart for monitoring the process mean and variance", *Quality and Reliability Engineering International,* **23**(7), pp. 821-831 (2007).
41. Tsai, T.R., Chiang, J.Y., and Chang, S.I. "Economic design of two-stage non-central chi-square charts for dependent variables", *Computers & Industrial Engineering,* **61**(4), pp. 970-980 (2011).
42. Jafarian-Namin, S., Nezhad, F., Tavakkoli-Moghaddam, R., Salmasnia, A., and Abooie, M.H. "An integrated model for optimal selection of quality, maintenance, and production parameters with autocorrelated data", doi: 10.24200/SCI.2021.56484.4745
43. Salmasnia, A., Emamjomeh, E., and Maleki, M.R. "Integration of production planning, maintenance scheduling and non-central chi-square chart parameters with random failures and multiple assignable causes", *Journal of Advanced Manufacturing Systems*. doi: 10.1142/S0219686721500414, (2022).
44. Matin, H.N., Salmasi, N., and Shahvari, O. "Makespan minimization in flowshop batch processing problem with different batch compositions on machines", *International Journal of Production Economics,* **193**, pp. 832-844 (2017).
45. Bisgaard, S., Hunter, W.G., and Pallesen, L. "Economic selection of quality manufactured product", *Technometrics,* **26**(1), pp. 9-18 (1984).

Figures and Tables

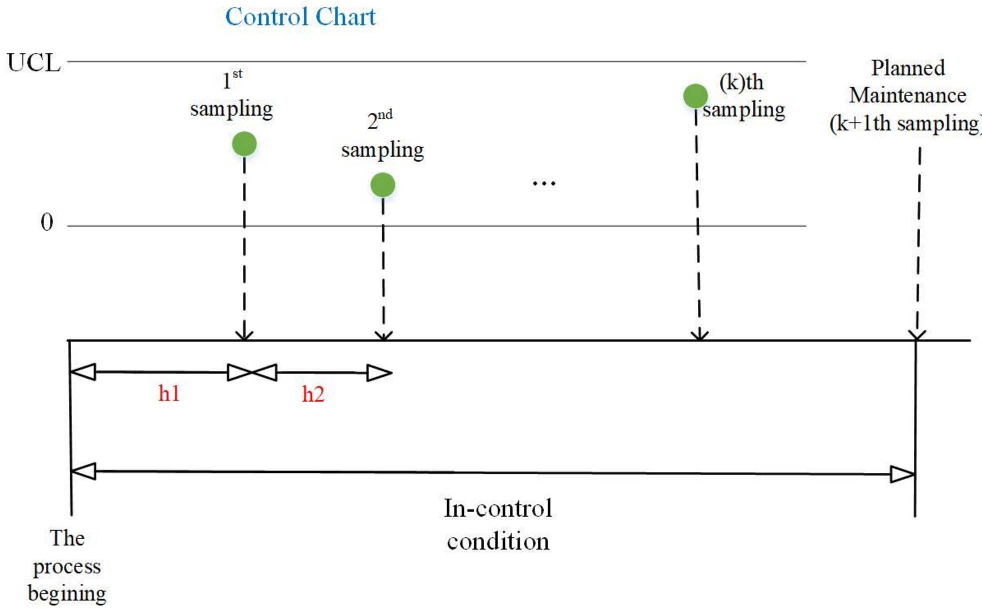


Figure 1. Illustration of scenario 1

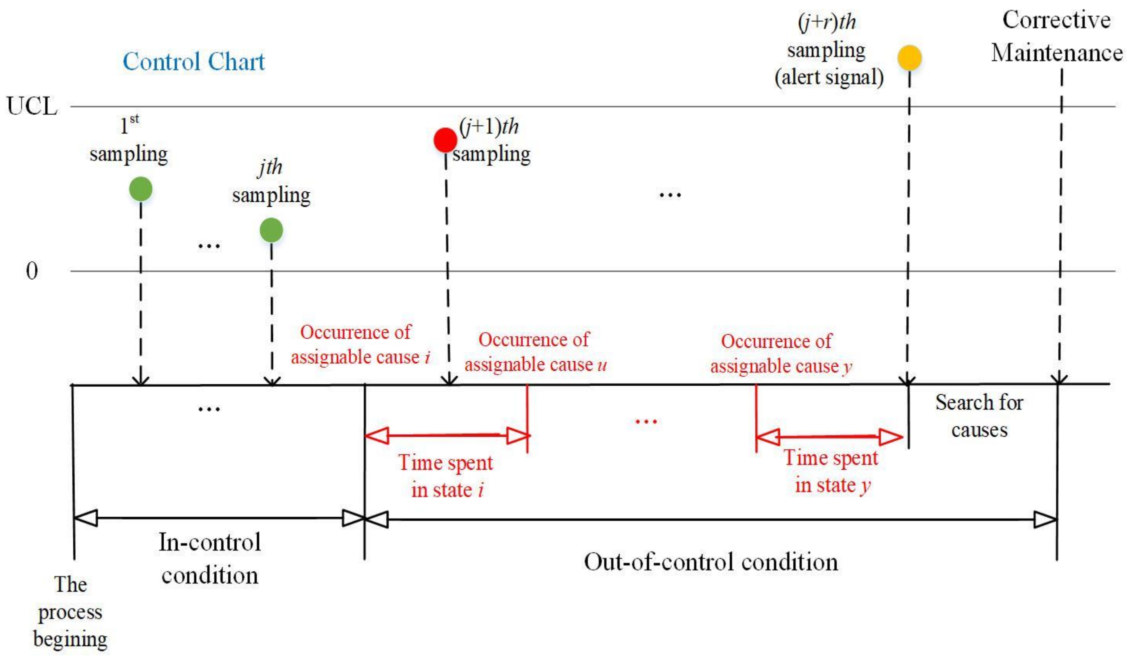


Figure 2. Illustration of scenario 2

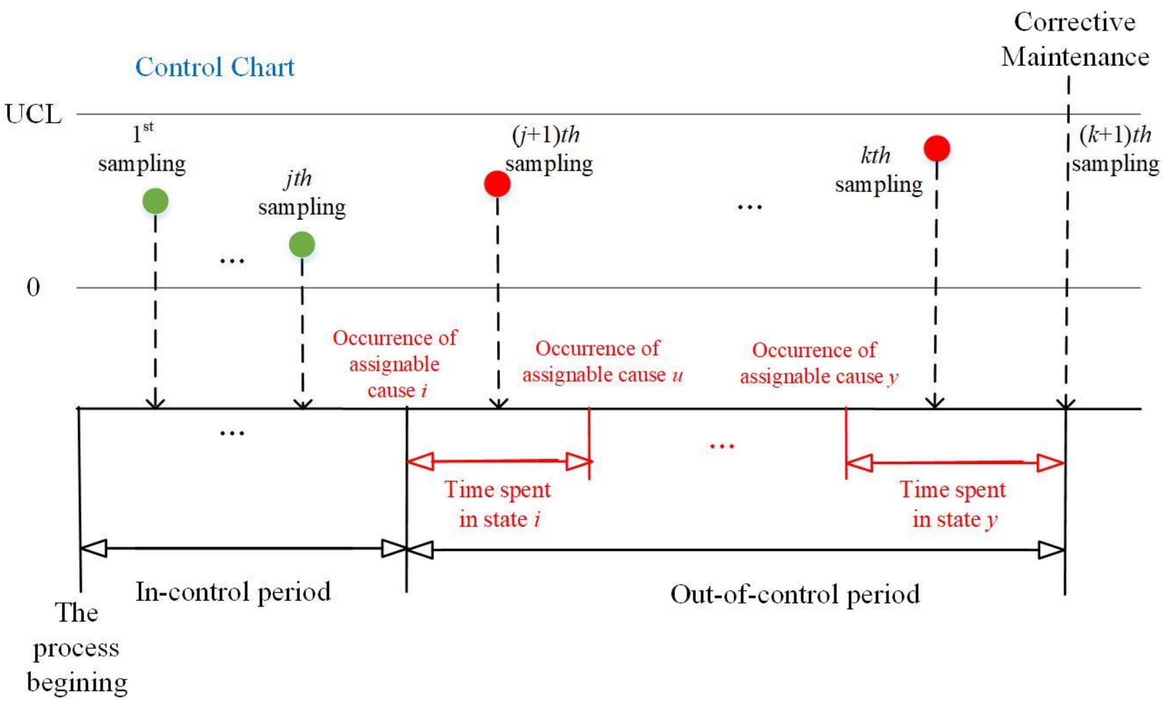


Figure 3. Illustration of scenario 3

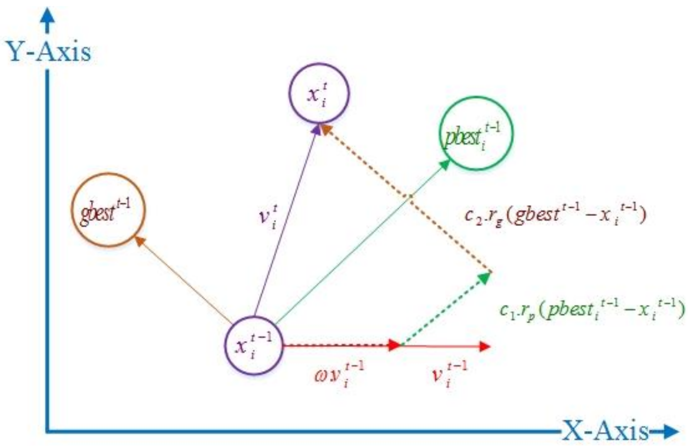


Figure 4. The PSO's searching process

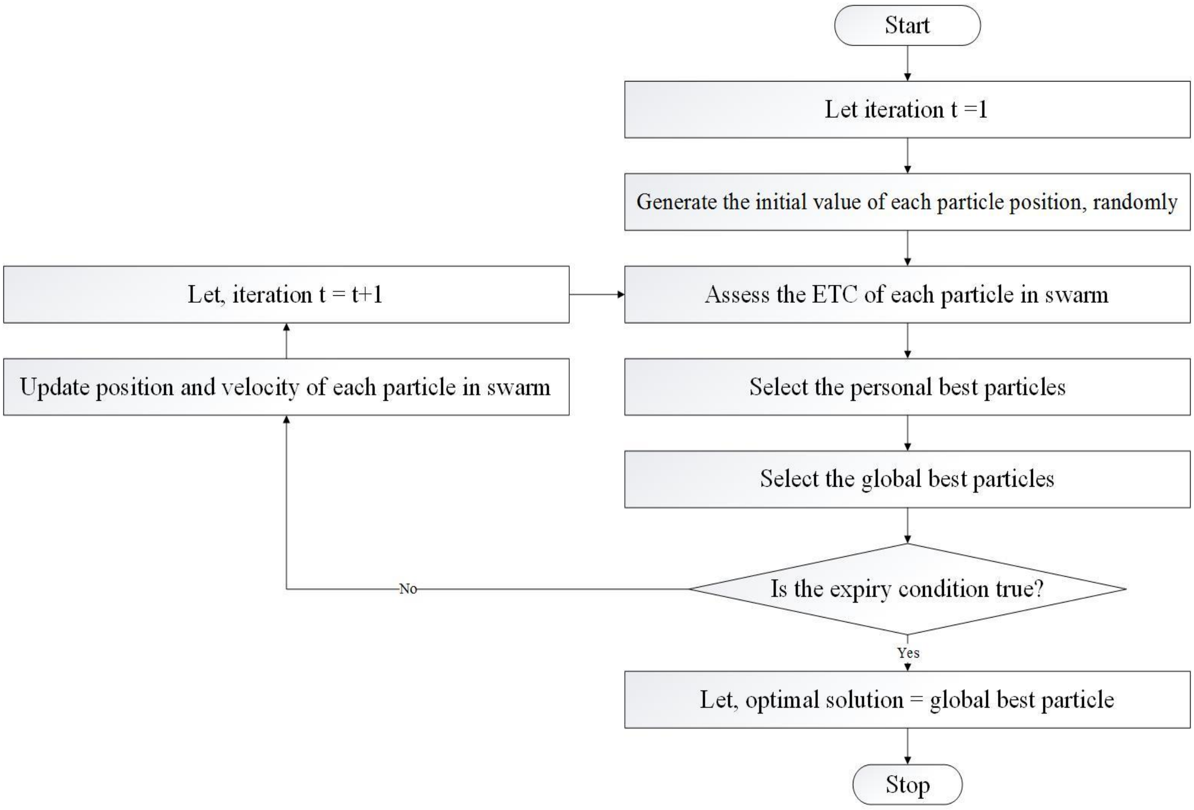


Figure 5. The graphical representation of PSO algorithm

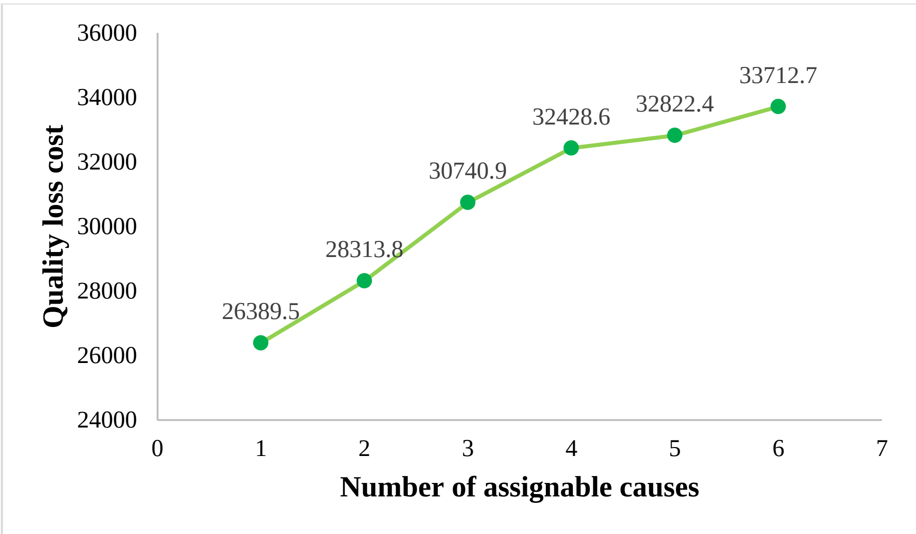


Figure 6. Effect of *s* on the quality loss cost

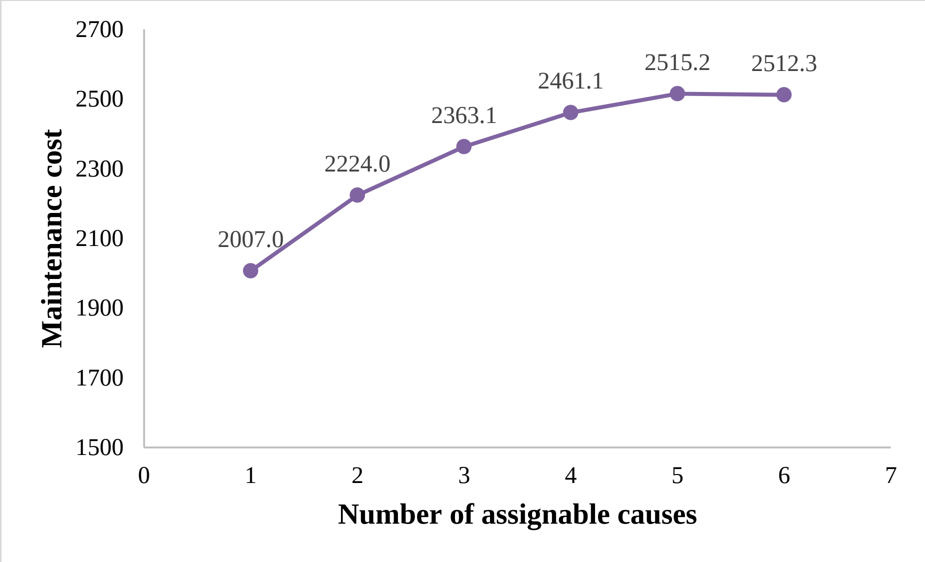


Figure 7. Effect of *s* on the maintenance cost

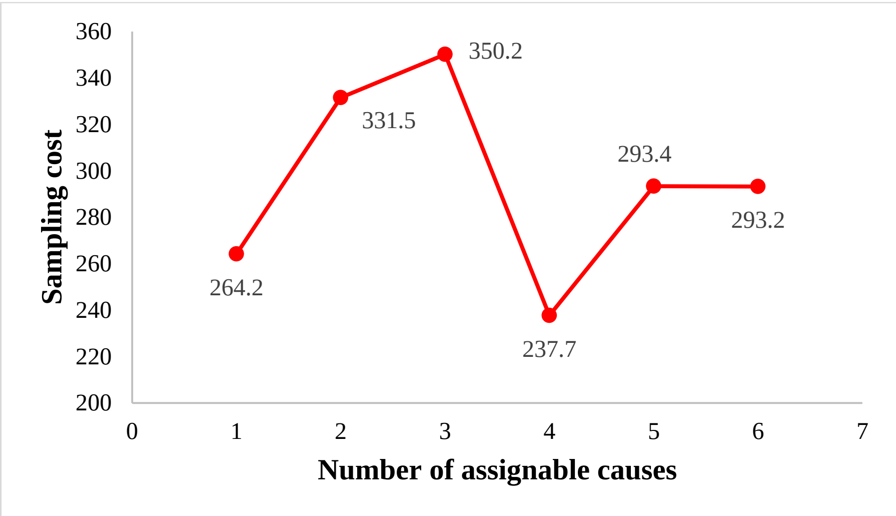


Figure 8. Effect of *s* on the sampling cost

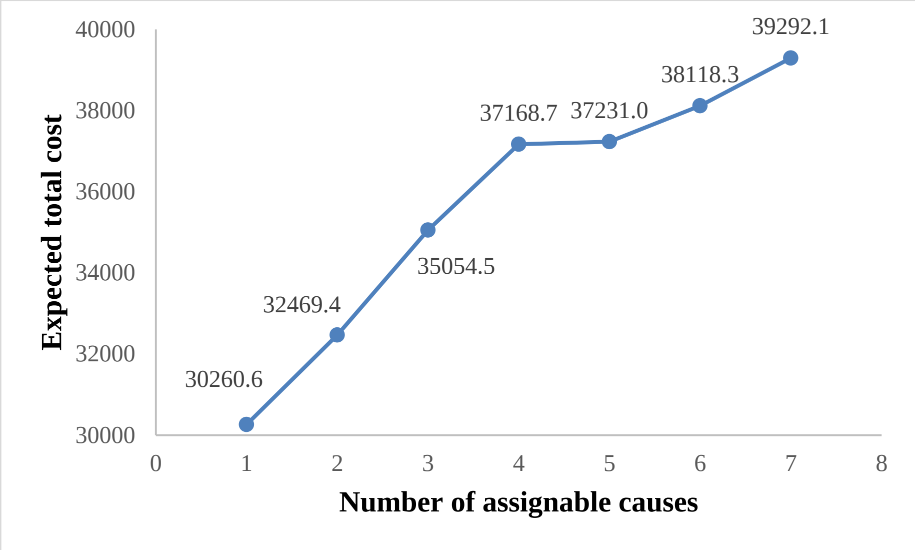


Figure 9. Effect of *s* on the sampling cost

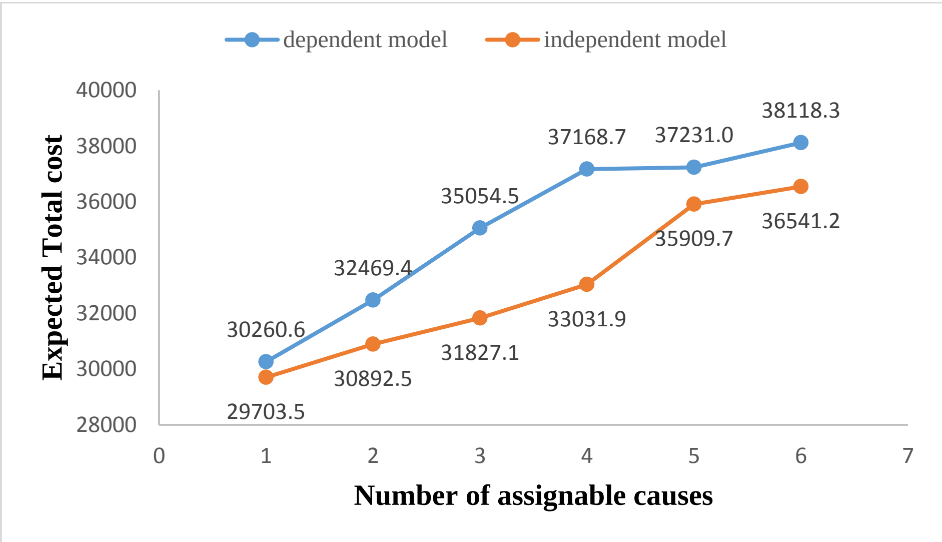
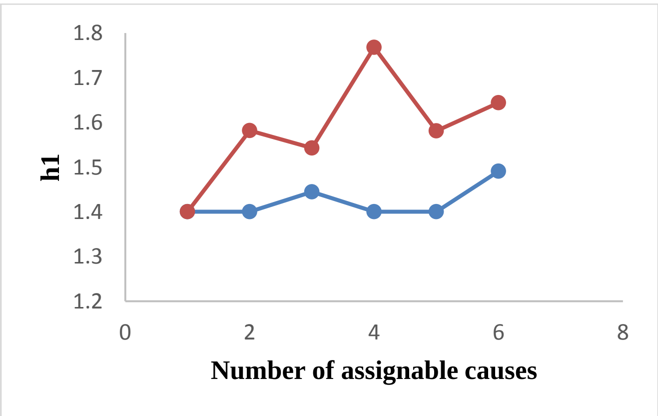
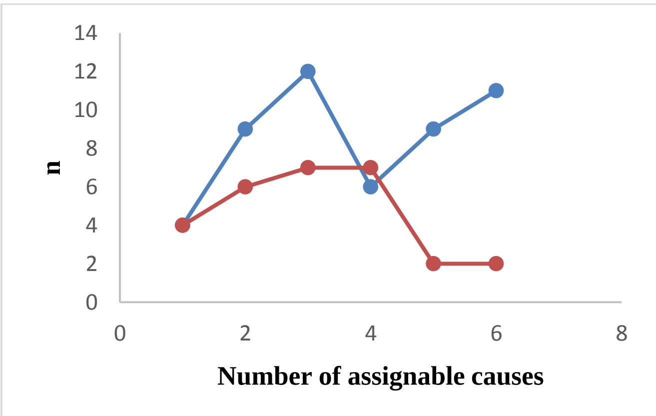


Figure 10. Comparison between the dependent and the independent models





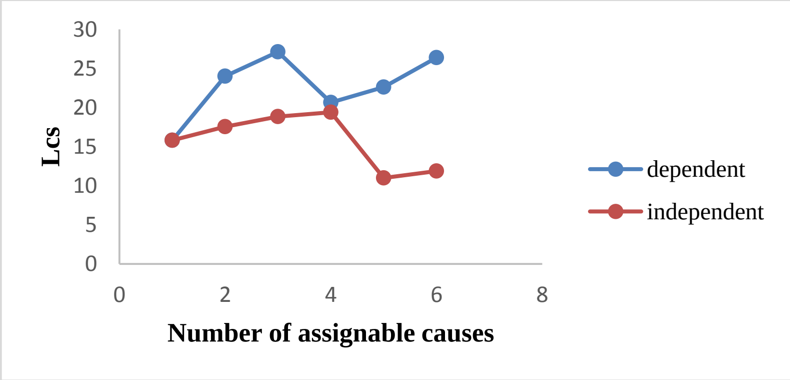


Figure 11. Comparison between decision variables in the dependent and the independent models

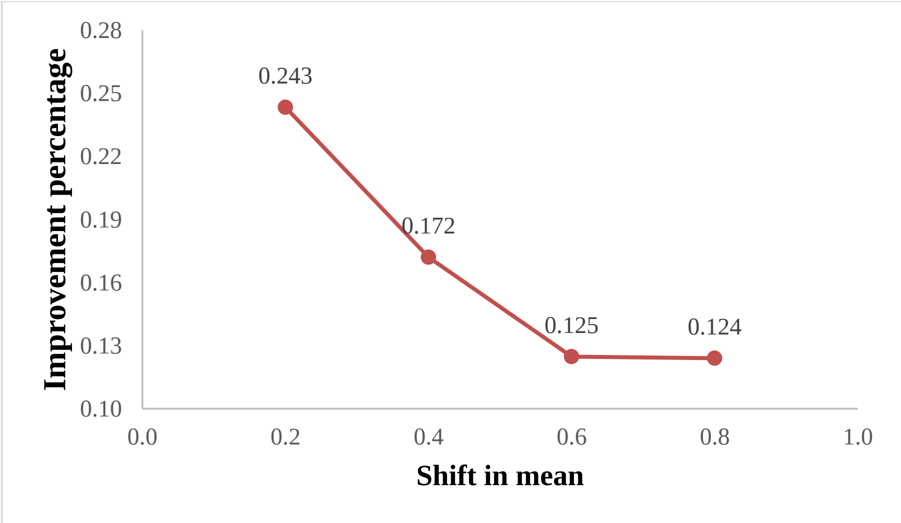


Figure 12. Effect of shift in the mean on the cost improvement

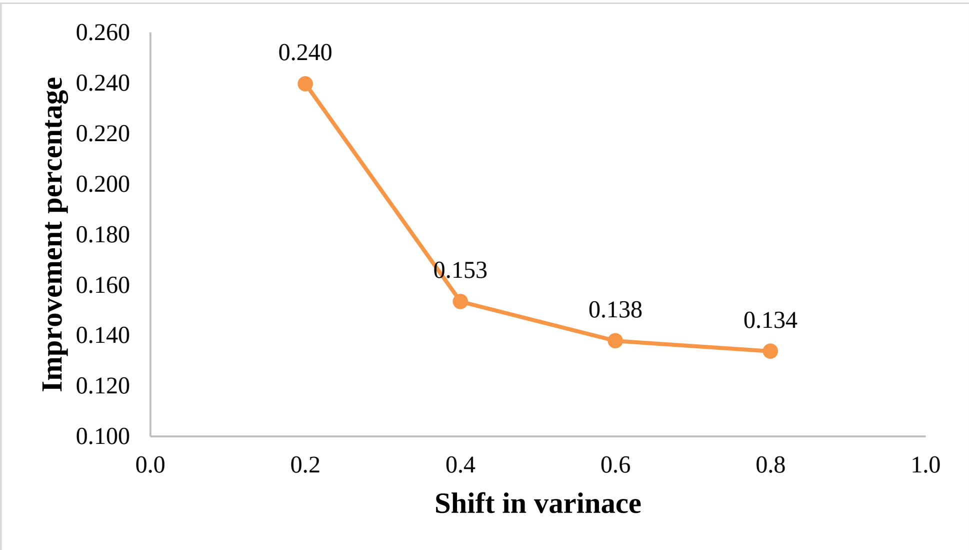


Figure 13. Effect of shift in the variance on the cost improvement

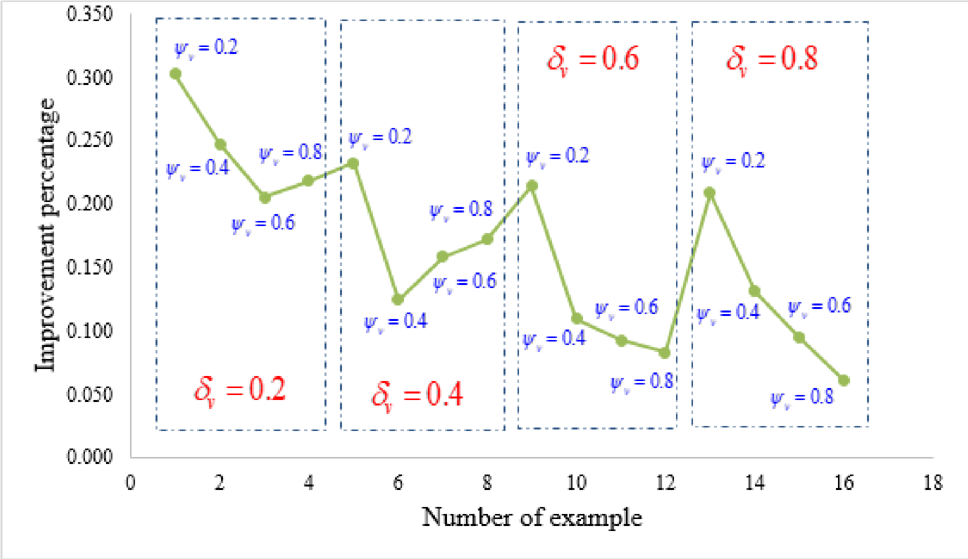


Figure 14. Trend of the cost improvement for different shifts in mean and variance

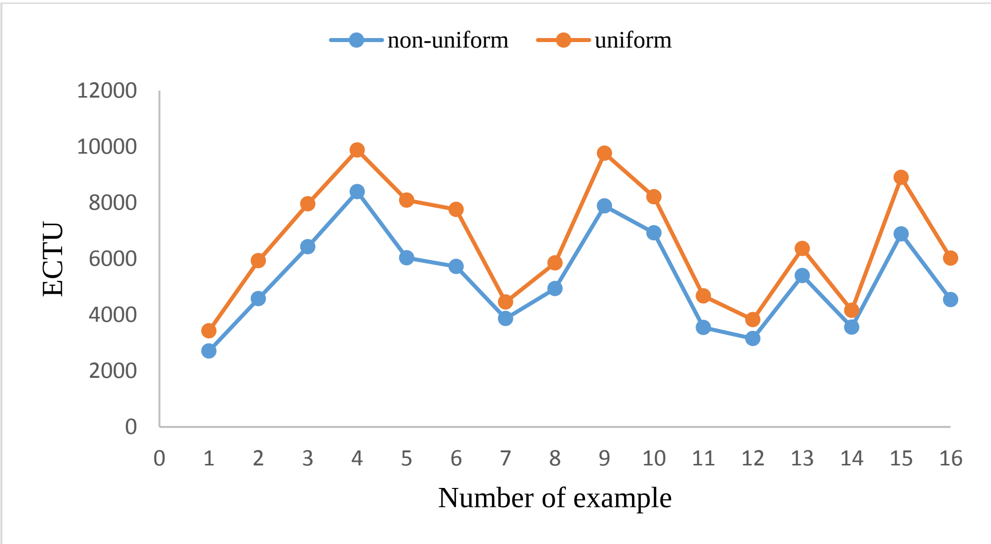


Figure 15. Comparison of ECTU in the uniform and non-uniform sampling schemes

Table 1. Notations

Notation	Description
Indices	
i,u,y	Index of assignable causes
j,r,l	Index of sampling intervals
z	Index of scenarios
Decision variables	
h_1	The length of the first sampling interval
k	The number of sampling intervals in a perfect production cycle
L_{CS}	The control limit coefficient of non-central chi-square chart
n	The sample size
d	The non-centrality parameter of NCS chart
Independent parameters	
A'	The setup cost of production cycle
ARL_L	The lower bound of average run length when the process is in-control
ARL_U	The upper bound of average run length when the process is out-of-control
B	The inventory holding cost per unit per time unit
Ccm_i	The corrective maintenance cost when the process is under the impact of i^{th} assignable cause
C_F	The fixed sampling cost
C_V	The variable sampling cost
C_{PM}	The planned maintenance cost
CT	The lower bound of a perfect cycle time
C_Y	The false alarm cost
D	The annual demand rate
D_d	The daily demand rate
E	The time to take and record a random sample
k_L	The lower bound of k
p	The production rate
A_i	The i^{th} type of assignable cause
Q_{in}	The quality loss cost per unit when the process is in-control
$Qout_i$	The quality loss cost per unit when the process the process is under the impact of A_i
$Qout_B$	The fixed term of $Qout_i$ for each $i = 1,2,\ldots,s$
s	Number of assignable cause types
T_1	The time to detect and validate of the assignable cause
u_0	The upper bound of the sample size
$\lambda_{i,u}$	The occurrence rate of A_u when the process is under the impact of A_i
μ_0	The in-control process mean
δ_i	The shift size in the process mean when the process is under the impact of A_i
δ_v	The variable term of δ_i
μ_i	The process mean when the process is under the impact of A_i
σ_0	The in-control standard deviation
ψ_i	The shift size in the process standard deviation when the process is under the impact of A_i
ψ_v	The variable term of ψ_i

σ_i	The process standard deviation when the process is under the impact of A_i
ν	Shape parameter of the Weibull distribution
$\boldsymbol{\pi}$	The vector of steady-state probabilities
π_i	The steady-state probability for state *i*
π_i'	The steady-state probability for state *i* given that the process is out-of-control
$\boldsymbol{\pi}_{(0)}$	The vector of initial probabilities
Other parameters	
ARL_0	The average run length when the process is in-control
ARL_1	The average run length when the process is out-of-control
Ccm_i	The corrective maintenance cost when the process is under the impact of A_i
Ccm_B	The fixed term of the corrective maintenance cost
Ccm_v	The variable term of the corrective maintenance cost
C_{setup}	The expected setup cost
$E(C_{in})$	The expected total cost when the process is in-control
$E(C_{out})$	The expected total cost when the process is out-of-control
$E(C_M)$	The expected maintenance cost
$E(C_M \mid S_z)$	The expected maintenance cost given that the scenario *z* occurs
$E(C_Q)$	The expected quality loss cost
$E(C_Q \mid S_z)$	The expected quality loss cost given that the scenario *z* occurs
$E(C_S)$	The expected sampling cost
$E(C_S \mid S_z)$	The expected sampling cost given that the scenario *z* occurs
$ECTU$	The expected cost per time unit
ETC	The expected total cost per production cycle
IHC	The expected inventory holding cost
$Eout_p$	The ratio of the expected out-of-control time interval to the expected production cycle length
EPQ	Economic production quantity
$E(Ta_i \mid S_z)$	The expected time length that the process is under the impact of A_i given that the scenario *z* occurs
$E(T_{in} \mid S_z)$	The expected in-control time length given that the scenario *z* occurs
$E(T_{out} \mid S_z)$	The expected out-of-control time length given that the scenario z occurs
$f_{i,u}(t)$	The probability density function of time-to-occurrence of A_u when the process is under the impact of A_i
$f_i(t)$	The probability density function of time-to-occurrence of the earliest assignable cause when the process is under the impact of A_i
h_F	The time between two successive samples in uniform sampling strategy
h_j	The time between the $(j-1)^{th}$ and j^{th} samples in non-uniform sampling strategy
L_R	The control limit coefficient in *R* chart
$\mathbf{P}$	The transition probabilities matrix
$P_{i,u}$	The transition probability from state *i* to state *u*
$P(S_Z)$	The occurrence probability of scenario *z*
$P'_{y,u}(t,t')$	The probability that the process is in state y in time *t*, and it directly transfers to state *u* during $[t,t']$, and remains in state *u* until t'
q_{ij}	The probability that the process shifts from the in-control to an out-of-control condition due to the

	occurrence of A_i during h_j.
$q_0(a,b)$	The probability that the earliest assignable causes occurs during the interval $[a,b]$
r_{in}	The expected number of samples taken when the process is in-control
r_{out}	The expected number of samples taken when the process is out-of-control
$Sa(t)$	The maximum number of samples taken under the out-of-control condition when the earliest assignable cause occurs at time t
t	The random variable of time-to-shift
W_j	The time of taking the j^{th} sample (the end of the j^{th} sampling interval)
Y_l	The non-central chi-square chart statistic for the l^{th} sample
α	The probability of Type I error
β_i	The probability of Type II error given that the process is under the impact of A_i
β	The probability of Type II error
$\phi(.)$	The cumulative distribution function of standard normal distribution
λ_0	The occurrence rate of the earliest assignable cause under in-control condition
λ_i	The occurrence rate of the earliest assignable cause when the process is under the impact of A_i
λ_B	The base value for calculation of $\lambda_{i,u}$
τ_i	The expected in-control time length within a given sampling interval in which A_i occurs
τ_{ij}	The expected in-control time length within h_j given that A_i occurs

Table 2. The values of the independent parameters of the type of assignable causes

Parameter	D_d	p	E	C_V	C_F	C_{PM}	C_Y
Value	80	100	0.01	1	5	1300	1000
Parameter	CT	ARL_U	ARL_L	T_1	A'	B	D
Value	10	10	100	1.25	60	10	10000

Table 3. The values of parameters dependent on assignable causes

Parameter	λ_B	$Qout_B$	Ccm_B	Ccm_v	ψ_v	δ_v
Value	0.01	100	2000	500	0.5	0.25

Table 4. Obtained optimal values from solving the proposed model

Number of assignable causes	Optimal value of decision variables					Optimal Cost				Constraints		
	n	h_1	L_{CS}	k	d	ETC	$E(C_Q)$	$E(C_S)$	$E(C_M)$	ARL_{in}	ARL_{out}	$W(k+1)$
1	4	1.4003	15.81	50	0.4596	30260.63	26389.47	264.16	2006.99	100.17	4.56	10.00014
2	9	1.4004	24.02	50	0.33095	32469.38	28313.79	331.52	2224.03	102.79	1.70	10.00091
3	12	1.4449	27.12	47	0.17919	35054.52	30740.86	350.17	2363.065	102.38	1.37	10.01066
4	6	1.4004	20.65	50	0.49554	37168.71	32428.56	237.67	2461.13	104.35	1.83	10.00051
5	9	1.4003	22.61	50	0.20526	37231.05	32822.44	293.38	2515.22	101.60	1.48	10.00013
6	11	1.4910	26.40	44	0.25179	38118.29	33712.73	293.23	2512.25	104.71	1.38	10.00186

Table 5. Items without change when the number of assignable causes increases

Number of assignable causes	Optimal values		
	EPQ	IHC	C_{setup}
1	1000.01	1000.01	599.99
2	1000.09	1000.09	599.95
3	1001.07	1001.07	599.36
4	1000.05	1000.05	599.97
5	1000.01	1000.01	599.99
6	1000.18	1000.18	599.89

Table 6. Comparison of the dependent model with the independent model

Dependent model

Number of assignable causes	Optimal value of decision variables					Probability of stability in the out-of-control states					
	n	h_1	L_{CS}	k	d	π'_1	π'_2	π'_3	π'_4	π'_5	π'_6
1	4	1.4003	15.81	50	0.4596	1					
2	9	1.4004	24.02	50	0.33095	0.579	0.421				
3	12	1.4449	27.12	47	0.17919	0.479	0.293	0.210			
4	6	1.4004	20.65	50	0.49554	0.478	0.261	0.153	0.108		
5	9	1.4003	22.61	50	0.20526	0.477	0.251	0.136	0.080	0.056	
6	11	1.4910	26.40	44	0.25179	0.478	0.248	0.131	0.071	0.042	0.030

Number of assignable causes	Optimal value of decision variables					Probability of stability in the out-of-control states					
	n	h_1	L_{CS}	k	d	λ_1 / λ_0	λ_2 / λ_0	λ_3 / λ_0	λ_4 / λ_0	λ_5 / λ_0	λ_6 / λ_0
1	4	1.4003	15.81	50	0.45964	1					
2	6	1.5819	17.56	39	0.20974	0.677	0.333				
3	7	1.5430	18.84	41	0.13909	0.571	0.286	0.143			
4	7	1.7679	19.41	31	0.22722	0.533	0.267	0.133	0.067		
5	2	1.5813	10.99	39	0.46414	0.516	0.258	0.129	0.065	0.032	
6	2	1.6413	11.88	36	0.58674	0.508	0.254	0.127	0.063	0.032	0.016

Table 7. Levels of parameters for generating examples

level	C_Y	$Qout_B$	λ_B	ψ_v	δ_v
1	400	50	0.01	0.2	0.2
2	600	75	0.02	0.4	0.4
3	800	100	0.03	0.6	0.6
4	1000	125	0.04	0.8	0.8

Table 8. Examples produced by the Taguchi design

Examples	C_Y	$Qout_B$	λ_B	ψ_v	δ_v
1	400	50	0.01	0.2	0.2
2	600	75	0.02	0.4	0.2
3	800	100	0.03	0.6	0.2
4	1000	125	0.04	0.8	0.2
5	1000	100	0.02	0.2	0.4
6	800	125	0.01	0.4	0.4
7	600	50	0.04	0.6	0.4
8	400	75	0.03	0.8	0.4
9	600	125	0.03	0.2	0.6
10	400	100	0.04	0.4	0.6
11	1000	75	0.01	0.6	0.6
12	800	50	0.02	0.8	0.6
13	800	75	0.04	0.2	0.8
14	1000	50	0.03	0.4	0.8
15	400	125	0.02	0.6	0.8
16	600	100	0.01	0.8	0.8

Table 9. The obtained results for the suggested model

Examples	Optimal decision variables					In-control		Out-of-control		Total cost
	n	h_1	L_{CS}	k	d	α	$E(C_{in})$	β	$E(C_{out})$	ETC
1	10	1.4750	25.57	45	0.3257	0.01	6801.9	0.66602	12753.4	21155.5
2	9	1.4437	29.90	57	0.6614	0.00999	7299.8	0.473	20505.3	29405.2
3	7	1.4003	18.00	50	0.1690	0.01	7903.8	0.33203	26295.6	35799.5
4	10	1.4004	24.56	50	0.2435	0.00996	8539.5	0.13958	32207	42346.6
5	5	1.4007	20.12	50	0.6253	0.00999	9656.8	0.64476	28278.3	39535.2
6	6	1.5076	17.16	43	0.1440	0.01	16110.9	0.48940	25973	43684
7	10	1.4006	28.46	50	0.4962	0.00995	3490.9	0.20582	14541.3	19632.3
8	9	1.4003	22.35	50	0.1792	0.01	5956.8	0.14079	19589	27145.8
9	10	1.4004	28.63	50	0.50504	0.00997	9841.5	0.32020	32674.8	44116.3
10	10	1.4004	27.97	50	0.47129	0.00998	6825.3	0.22406	26784.9	35210.2
11	7	1.4003	21.35	50	0.36795	0.00837	9989.5	0.26282	15102.4	26691.9
12	7	1.4005	19.69	50	0.25748	0.00989	4978.2	0.17191	13052.1	19630.3
13	8	1.4005	24.84	50	0.51120	0.01	5165	0.25631	20656.1	27421.1
14	9	1.4004	25.90	50	0.45891	0.00992	4081.9	0.17723	13959.2	19641.2
15	5	1.4013	25.92	50	0.97225	0.00991	11916.2	0.27854	29536.5	43053.0
16	4	1.4743	18.34	45	0.67062	0.00963	12944.2	0.29681	19590.8	34135.1

Table 10. The obtained results for the $\bar{X}-R$ model

Examples	Optimal decision variables					In-control		Out-of-control		Total cost
	n	h_1	$L_{\bar{X}}$	L_R	k	α	$E(C_{in})$	β	$E(C_{out})$	ETC
1	12	1.4750	3.43	5.31	45	0.01001	6874.8	0.97605	21870.9	30345.8
2	20	1.4437	2.60	6.58	57	0.01000	7436.3	0.83814	30038.1	39074.4
3	34	1.4003	2.60	6.82	50	0.00999	8149.2	0.67298	35298.8	45048.0
4	29	1.4004	2.58	7.33	50	0.00996	8668.3	0.66025	43896.4	54164.7
5	19	1.4007	3.96	5.61	50	0.00999	9842.1	0.85862	40090.6	51532.9
6	30	1.5076	4.10	5.91	43	0.01000	16538.3	0.65618	31781.1	49919.4
7	20	1.4006	3.65	5.66	50	0.00995	3558.7	0.64409	18177.2	23335.9
8	26	1.4003	3.97	5.82	50	0.01001	6111.5	0.60969	25071.3	32782.7
9	22	1.4004	3.38	5.74	50	0.00998	9950.7	0.74523	44588.2	56138.9
10	21	1.4004	3.12	5.75	50	0.00999	6899.8	0.49305	31043.5	39543.4
11	15	1.4003	3.99	5.52	50	0.00836	10155.9	0.62233	17660.3	29416.2
12	23	1.4005	2.79	6.00	50	0.00989	5188.7	0.33411	14619.0	21407.8
13	27	1.4005	3.63	5.85	50	0.01001	5293.8	0.68704	27751.2	34645.1
14	41	1.4004	3.74	6.12	50	0.00993	4372.9	0.30403	16642.0	22614.9
15	11	1.4013	3.91	5.23	50	0.00992	11995.2	0.56341	33958.6	47554.1
16	15	1.4743	2.60	6.59	45	0.00962	13148.9	0.44059	21629.3	36378.2

Table 11. Improvement percentage in ETC when using non-central chi-square chart

Example	1	2	3	4	5	6
Percentage of improvement	0.303	0.274	0.205	0.218	0.233	0.125
Example	7	8	9	10	11	13
Percentage of improvement	0.159	0.172	0.214	0.110	0.093	0.083
Example	13	14	15	16		
Percentage of improvement	0.209	0.131	0.095	0.062		

Table12. Average of improvement percentage in ETC for each δ_v and ψ_v when using non-central chi-square chart

δ_v	0.2	0.4	0.6	0.8
Average improvement percentage	0.243	0.172	0.125	0.124
ψ_v	0.2	0.4	0.6	0.8
Average improvement percentage	0.240	0.153	0.138	0.134

Table 13. Comparison of the uniform and non-uniform sampling schemes

Example	Non-uniform sampling					Uniform sampling					Percentage of improvement
	h_1	k	$Eout_p$	$E(C_S)$	$ECTU$	h_F	k	$Eout_p$	$E(C_S)$	$ECTU$	
1	1.4750	45	0.203	317.41	2114.793	1.4750	6	0.325	67.52	2833.238	0.254
2	1.4437	57	0.272	199.76	2939.945	1.4437	6	0.409	55.66	4317.492	0.319
3	1.4003	50	0.308	127.30	3579.784	1.4003	7	0.414	43.54	4625.642	0.226
4	1.4004	50	0.337	119.14	4234.278	1.4004	7	0.408	45.24	4974.239	0.149
5	1.4007	50	0.286	160.04	3952.341	1.4007	7	0.442	47.27	5859.353	0.325
6	1.5076	43	0.185	213.92	4368.382	1.5076	6	0.315	46.07	6077.732	0.281
7	1.4006	50	0.340	120.57	1962.749	1.4006	7	0.419	46.37	2285.593	0.141
8	1.4003	50	0.299	143.81	2714.578	1.4003	7	0.381	47.14	3219.543	0.157
9	1.4004	50	0.310	158.72	4411.266	1.4004	7	0.412	53.91	5657.386	0.220
10	1.4004	50	0.342	121.05	3520.802	1.4004	7	0.422	46.69	4233.507	0.168
11	1.4003	50	0.172	262.46	2669.182	1.4003	7	0.293	53.32	3404.477	0.216
12	1.4005	50	0.249	172.52	1962.675	1.4005	7	0.354	46.50	2379.525	0.175
13	1.4005	50	0.341	105.62	2741.610	1.4005	7	0.425	40.95	3301.980	0.170
14	1.4004	50	0.301	144.51	1963.899	1.4004	7	0.387	47.68	2308.064	0.149
15	1.4013	50	0.252	145.46	4302.104	1.4013	7	0.369	40.10	5661.891	0.240
16	1.4743	45	0.173	178.42	3413.366	1.4743	6	0.291	35.71	4575.246	0.254

Brief technical biographies

Ali Salmasnia is currently an Associate Professor of Industrial Engineering in University of Qom, Qom, Iran. His research interests include quality engineering, reliability, applied multivariate statistics and multi-criterion decision making. He is the author or co-author of various papers published in *Journal of Manufacturing Systems*, *Computers and Industrial Engineering*, *Applied Soft Computing*, *Neurocomputing*, *Applied Mathematical Modelling*, *Expert Systems with Applications*, *Quality Technology and Quantitative Management*, *Journal of Information Science*, *Neural Computing and Applications*, *Applied Stochastic Models in Business and Industry*, *IEEE Transactions on Engineering Management*, *International Journal of Information Technology and Decision Making*, *Operational Research*, *TOP*, *Quality and Reliability Engineering International*, *Journal of Statistical Computation and Simulation*, *International Journal of Advanced Manufacturing Technology*, *Communications in Statistics-Simulation and Computation*, *Arabian Journal for Science and Engineering*, *Journal of Industrial and Business Economics*, *International Journal of Modeling and Simulation*, and *Scientia Iranica.*

Behnam Abdzadeh is currently a PhD candidate of Industrial Engineering in Iran University of Science and Technology. His research interest include statistical process monitoring, project management, and reliability engineering. He is the author or co-author of papers published in *Journal of Manufacturing Systems, Neural Computing and Applications, Journal of Statistical Computation and Simulation*, *Communications in Statistics-Simulation and Computation, Scientia Iranica, Journal of Industrial and Systems Engineering*, *Journal of Industrial Engineering and Management Studies,* and *Journal of Modelling in Engineering*.

Mohammad Reza Maleki is an Assistant Professor of Industrial Engineering at Golpayegan College of Engineering in Isfahan University of Technology. His research interests include statistical process monitoring, profile monitoring, and reliability engineering. He has been the author or co-author of many papers published in high-ranked journals such as *Computers and Industrial Engineering*, *Quality and Reliability Engineering International*, *Communications in Statistics–Simulation and Computation*, *Communications in Statistics–Theory and Methods*, *Transactions of the Institute of Measurement and Control*, *Journal of Industrial and Business Economics*, *Arabian Journal for Science and Engineering*, *Journal of Advanced Manufacturing Systems*, *International Journal of Modeling and Simulation* and *Scientia Iranica*.