

A Dempster-Shafer evidence theory for environmental risk assessment in failure modes and effects analysis of Oil and Gas Exploitation Plant

Gholamreza Shams¹, Seyed Morteza Hatefi^{2,*}, Shahla Nemati³

¹ Faculty of Engineering, Shahrekord University, Rahbar Boulevard, PO Box 115, Shahrekord, Iran.
Tel:03832324438, Email: g.shams@sku.ac.ir

^{2,*} Faculty of Engineering, Shahrekord University, Rahbar Boulevard, PO Box 115, Shahrekord, Iran.
Tel:03832324438, Mobile number +989132831193, Email: smhatefi@sku.ac.ir (corresponding author)

³ Faculty of Engineering, Shahrekord University, Rahbar Boulevard, PO Box 115, Shahrekord, Iran.
Tel:03832324438, Email: s.nemati@sku.ac.ir

Abstract: The oil, gas, and petrochemical industries, as one of the largest sources of environmental pollutants, have different types and levels of pollution depending on the type of input materials, process steps, and output products. Various stages of exploration, extraction and processing of oil and gas have many environmental effects, such as those on soil, air, water, creatures, plants, and even humans. In this paper, a failure mode and effects analysis (FMEA) is employed to identify failures and environmental risks in an oil and gas exploitation plant. Dempster–Shafer (DS) theory of evidence is then proposed for environmental risk assessment due to its effectiveness in dealing with uncertain and subjective information. The assessment of experts and the confidence levels of their responses are employed to construct the basic probability assignments (BPA) in DS theory of evidence. Furthermore, a new weighting method is proposed to obtain the discounted BPA which reduces the uncertainty in the information sources and improves the quality of information before combining different sources of information. Finally, the proposed method is applied to an oil and gas exploitation plant to assess environmental risks.

Keywords: Environmental risk assessment, Risk priority number, FMEA, Dempster–Shafer theory of evidence, uncertainty.

1. Introduction

With the advent of technology and increased use of machinery, the risks and incidents have also increased in industrial environments. In gas refineries, operating units are exposed to high temperatures and pressures, so there is a possibility of occurrence of accidents. Given the fact that oil, gas, and petrochemical storage tanks are major infrastructures, as well as due to their enormous environmental hazards, this industry has always been a concern for experts in the field of safety and the environment. Oil storage tanks are the most important industrial facilities that are always at risk of toxic emissions, fires, and explosions. The most common fires and explosions are the most important risks in storage tanks [1]. Also, due to the presence of volatile hydrocarbon contaminants and volatile organic compounds in the refinery, these pollutants are extensively active in the non-saturated soils and can pollute the soil around the reservoirs [2]. These lead to other environmental concerns, such as global warming, ozone depletion, water pollution, and species extinction.

Risk assessment is a systematic approach for identifying hazards and ranking them for decision making, prevention, and mitigation of risks [3-4]. The main objectives of the risk assessment are determining the degree of uncertainty of a studied system and its costs and providing risk and cost reduction solutions [5-6]. Risk assessment can be done in two qualitative and quantitative ways. A quantitative assessment is focused on risk factors and preventive actions, which control and eliminate or prevent failures. The qualitative assessment method requires a scientific approach to decision making, cost justification, risk prevention, and mitigation, and rapid risk control programs [7-8].

Many studies have been conducted on incidents and risks in storage tanks and their products. Chang and Lin [9] studied the incident in industrial storage tanks in the last 40 years. Their

results showed that 74 percent of the incidents occurred in oil refineries, oil terminals, and storage. Wang et al. [10] investigated the effects of earthquakes on liquid gas storage tanks. The results indicated that the use of insulating layers in internal and external walls of reservoir design reduces the potential for vulnerability during an earthquake.

FMEA is one of the useful methods to define, identify, and eliminate potential failures in the oil and petrochemical industries. The risk priority number (RPN) is widely used to determine the priority of failure modes [11-12]. Three risk factors, namely, occurrence (O), severity (S), and detection (D) are used to calculate the RPN of a failure mode. The RPN of a failure mode is calculated by multiplying the values of three factors O, S, and D. Failures with higher RPNs are considered more important and are given higher priority.

RPN-based methods have a major disadvantage in which precise values cannot be assigned to the occurrence, severity, and detection factors. In many cases, these factors are faced with uncertainty, and it is a difficult task to assign the exact numerical value to risk factors. Fuzzy theory has been used in many studies to overcome this problem[13]. In this line of research, Wang et al. [14] obtain a fuzzy rating for the occurrence, severity, and detection of a failure and introduced a fuzzy risk priority number (FRPN), which is calculated by fuzzy weighted geometric mean of three risk factors. Kutlu and Ekmekçioğlu [15] proposed fuzzy FMEA by linguistic variables for risk assessment. Dağsuyu et al. [16] proposed a fuzzy FMEA to identify and prioritize hazards in a sterilization unit of a large hospital. Zhou and Thai [17] applied both grey theory and fuzzy theory on FMEA to obtain risk priority numbers. The authors obtained RPNs for oil tanker equipment failures by both grey theory and fuzzy theory. The authors showed that the results of the two methods are similar. A fuzzy inference system is an efficient

method for risk assessment of failure modes under uncertainty. Application of fuzzy inference system to obtain the risk priority number can be seen in [18-20].

Assessment of experts is one of the most important steps in the risk assessment process. The subjective judgment of experts may lead to unpredictable uncertainty. The existing approaches such as fuzzy set theory and the Bayesian method cannot effectively handle uncertainty [21]. Fuzzy set theory is an effective tool to handle epistemic uncertainty, which comes from a lack of information. As mentioned earlier, applications of fuzzy set theory for risk assessment can be seen in [14-20]. However, fuzzy set theory cannot effectively reflect the conflicting information of multiple sources. The DS method is an efficient tool to support decisions when information is nonspecific, ambiguous, or conflicting.

The Bayesian method is another tool to address uncertainties in the risk assessment process. Applications of this method for risk assessment can be seen [22-25]. The DS method is an extended form of the Bayesian method that has all its advantages. For instance, in the DS method, as in the Bayesian method, existing prior information can be incorporated into the inference of uncertain indices and inferential results. However, the use of prior information in the DS method is not mandatory. This is one of the advantages of the DS method. Second, the DS method, unlike other possible methods such as the Bayesian method, does not require a previous probability calculation. Third, it has a flexible and understandable mass function. Fourth, providing the mass function is easy and convenient. Fifth, the computational complexity of this method is much less than the Bayesian method. All aforementioned discussions show the reasons for choosing the DS theory of evidence for risk assessment under uncertainty.

Several applications of evidence theory can be seen in the concerned literature due to its effectiveness and flexibility in dealing with uncertain and subjective information [26-27]. Recently, valuable studies have been conducted on the use of evidence theory to solve multi-attribute decision-making (MADM) problems. For instance, Liu and Zhang [28] introduced a novel method based on DS evidence theory to eliminate the existing defects of utilizing intuitionistic linguistic numbers in MADM problems. Liu and Gao [29] proposed some applications of intuitionistic fuzzy power Bonferroni mean operators in the context of evidence theory to solve MADM problems. Various models and approaches have been developed based on linguistic intuitionistic fuzzy numbers and DS evidence theory to handle MADM problems under various types of uncertainty. These models and approaches can be seen [30-33].

In the subject literature, evidence theory has become very important to solve risk assessment problems. Yang et al. [34] introduced a modified DS evidence theory to obtain the RPN of failure modes of aircraft engine rotor blades under uncertainty. Su et al. [35] proposed an improved version of the DS evidence theory introduced by Yang et al. [34], in which the basic belief assignments were improved to get more flexible and reasonable results.

Jiang et al. [36] proposed a novel fuzzy evidential method for failure mode and effects analysis. The authors defined fuzzy membership degree for risk assessment of failure modes. Then fuzzy mapping is utilized to obtain the belief structure and generate a basic probability assignment of risk factors. Finally, Dempster-Shafer evidence theory is employed to fuse the evidence of risk factors. Certa et al. [37] employed the DS theory of evidence to deal with the existing uncertainty in the risk assessment process and prioritize the failures of the propulsion system of a fishing vessel.

This paper proposes an evidential model based on the DS theory of evidence for environmental risk assessment in failure modes and effects analysis of the Oil and Gas Exploitation Plant. The proposed evidential model, in addition to the mentioned advantages compared with fuzzy theory and Bayesian network, has several advantages compared to studies that have used the DS evidence theory methods for risk assessment such as Yang et al. [34], Jiang et al. [36], Certa et al. [37], and Hatefi et al. [26]. According to these advantages, the initial motivations of the paper can be stated as follows:

- To effectively deal with various uncertainties involved in the risk assessment process, it introduces a practical way to extract the basic probability assignments from evaluation information of experts by expressing linguistic terms and confidence levels for rating probability, severity, and detection of failures.
- In the process of assessing and ranking failures, the relative weight of risk factors is not taken into consideration. The weight of the three factors is assumed to be equal, but this may not be the case in practical applications. This paper utilizes Deng entropy to determine the relative weight of risk factors and obtain the discounted basic probability assignments for risk factors.

The rest of the paper is organized as follows. In section 2, the FMEA method is introduced briefly. The basic concept of the DS evidence theory is briefly reviewed in section 3. The proposed evidential model for environmental risk assessment, which is constructed based on the Dempster-Shafer evidence theory, is presented in section 4. A new weighting method is proposed in this section for obtaining the discounted basic probability assignment. The environmental risks in an oil and gas exploitation plant are identified in section 5. The proposed

evidential model is applied to the studied case to assess the environmental risks. Section 6 obtains concluding remarks.

2. Failure modes and effects analysis (FMEA)

Nowadays, applications of risk assessment methods in different organizations and industries are increasing, so that various types of qualitative and quantitative methods are developed for risk assessment. These methods are commonly utilized to identify, control, and mitigate the impacts of hazards. Organizations and industries can use these methods to meet their needs. The most important goal of health and safety system management in any organization is to study all methods of risk assessment and select appropriate methods to implement. Therefore, every organization and industry must have an appropriate way of identifying and assessing the risks of occupational safety and health activities as well as products or services [38].

FMEA can be described as a set of organized activities that are used to identify and estimate potential failure modes in a product or process and specify the activities that can eliminate or reduce the chance and probability of potential failures. FMEA is an analytical technique used to identify, mitigate and eliminate potential failures in a system, product design, manufacturing process, or service [38].

In the FMEA method, after identifying the risks, the risk estimation is performed by calculating RPN for each potential failure. RPN is calculated by multiplying three factors of occurrence, severity, and detection by using the traditional score $RPN=S \times O \times D$. These three factors are rated on a scale of 1 to 10. The scales used for these three factors are reported in Table 1. The risk priority number is the basis for prioritizing failure modes. Given that three factors can handle numbers between 1 and 10, the RPN will have a number between 1 and 1000 [34].

Please insert Table 1 here

3. Preliminaries

3.1. DS theory of evidence

The theory of evidence was introduced by Dempster [39] and developed by Shafer [40]. This theory is related to information from multiple sources that can be uncertain, incomplete, and imprecise. The theory of evidence is introduced based on a belief that results from evidence. The belief structure of evidence theory relates to the classical probability model. The following are some of the basic concepts regarding the theory of evidence.

3.1.1 Frame of discernment

Suppose $\theta = \{E_1, E_2, \dots, E_N\}$ be a finite nonempty set of mutually exclusive and exhaustive events.

Notation θ is called the frame of discernment (FOD). The power set Ω is represented by 2^θ which has 2^N elements as follows (Dempster, 1967; Shafer, 1976)[39-40]:

$$2^\theta = \{\emptyset, \{E_1\}, \dots, \{E_N\}, \{E_1, E_2\}, \dots, \{E_1, E_2, \dots, E_i\}, \dots, \theta\} \quad (1)$$

where \emptyset is the empty set. If A is an element belonging to the power set of 2^θ , or $A \in 2^\theta$, then it is called a proposition.

3.1.2 Basic probability assignment (BPA)

A mass function is stated as a mapping from the power set 2^θ to the interval $[0, 1]$. The mass function is mathematically presented as $m(A) : 2^\theta \rightarrow [0, 1]$ and satisfies the following relations.

$$m(\phi) = 0, \sum_{A \in 2^\theta} m(A) = 1 \quad (2)$$

where A is a member of the power set. If the mass function of proposition A is greater than zero, $m(A) > 0$, then A is called a focal element. All focal elements form the body of evidence (BOE). The mass function $m(A)$ expresses that how strongly the evidence supports proposition A . The mass function is also known as a basic probability assignment or a basic belief assignment (BBA).

3.2. Uncertainty measures in Dempster-Shafer framework

Various uncertainty measures are developed in terms of the DS theory of evidence. For instance, Deng [41] developed a new entropy measure called Deng entropy, which is an extended version of Shanon entropy introduced by Shannon [42]. Deng entropy measure is denoted by $E_d(m)$ and formulated as follows:

$$E_d(m) = - \sum_{A \subset X} m(A) \log_2 \frac{m(A)}{2^{|A|} - 1} \quad (3)$$

where $E_d(m)$ denotes Deng entropy, A is a proposition in mass function m , $m(A)$ denotes the mass function of A , $|A|$ denotes the cardinality of proposition A and X is the frame of discernment. In the case where proposition A has a single element, Deng entropy converts to Shanon entropy as follows [42]:

$$E_d(m) = - \sum_{A \subset X} m(A) \log_2 m(A) \quad (4)$$

where A shows a proposition in mass function m , $m(A)$ denotes the mass function of A . Zhou et al. [43] extend Deng entropy and introduced a new belief entropy that considers uncertain

information in a BOE. The belief entropy proposed by Zhou et al. [43] is denoted by $E_{Md}(m)$ and calculated as follows.

$$E_{Md}(m) = - \sum_{A \subset X} m(A) \log_2 \left(\frac{m(A)}{2^{|A|} - 1} \times e^{\frac{|A|-1}{|X|}} \right) \quad (5)$$

where $E_{Md}(m)$ denotes the belief entropy, $|A|$ denotes the cardinality of proposition A and X is the frame of discernment. Term $e^{\frac{|A|-1}{|X|}}$ in the aforementioned belief entropy considers uncertain information in a BOE which has been neglected in Deng entropy [43].

3.3. Discounted BPA

When there are several sources of information for decision-making, information sources must be properly combined and finalized. Since each of these sources has different reliability and the impact of different resource information on the final decision is different, the discounting operator is used to obtain the discounted BPA as follows:

$$\begin{aligned} m^r(A) &= (1-r) \times m(A), \quad A \in \theta \\ m^r(\theta) &= (1-r) + r \times m(\theta) \end{aligned} \quad (6)$$

where A is the focal element of the mass function m . Notations r and $m^r(.)$ denote the coefficient of discounting and the discounted BPA, respectively.

3.4. Dempster combination rule

A combination of two sources of information in the DS theory of evidence is performed by operator \oplus . Let m_1 and m_2 be two basic probability assignments. Dempster combination rule is written as $m = m_1 \oplus m_2$ and formulated as follows [40]:

$$m(A) = \frac{1}{1-k} \sum_{B \cap C = A} m_1(B) m_2(C), \quad k = \sum_{B \cap C = \phi} m_1(B) m_2(C) \quad (7)$$

where k expresses the conflict between two mass functions m_1 and m_2 .

Dempster combination rule can be extended for more than two mass functions. The extended Dempster combination rule can be formulated as follows:

$$m = m_1 \oplus m_2 \oplus \dots \oplus m_L = (((m_1 \oplus m_2) \oplus m_3) \oplus \dots) \oplus m_L \quad (8)$$

3.5 Pignistic probability transformation

The pignistic probability transformation transforms a BPA to a probability distribution to make a decision. Assume that m is a BPA on the frame of discernment θ . The pignistic probability transformation for a singleton $x \in \theta$ can be obtained as follows [28, 30, 44]:

$$BetP\{x\} = \sum_{x \in A \subset \theta} \frac{1}{|A|} \frac{m(A)}{1 - m(\phi)}, \quad m(\phi) \neq 1 \quad (9)$$

where $|A|$ denotes the number of elements of proposition A [29].

4. Environmental risk assessment using D–S evidence theory

In this section, the proposed method for environmental risk assessment based on the DS evidence theory is introduced. The proposed method has the following main steps for environmental risk assessment:

Step1: Determine the linguistic terms of risk factors

In the risk evaluation process, experts score all risk factors for a given environmental failure mode. In the first step, the linguistic terms including severity, occurrence, and detection must be scored based on the experts' opinions. Table 1 is utilized to obtain the ratings of severity, occurrence, and detection of environmental risks.

Step 2: Determine the confidence level for linguistic terms

In the scoring process, experts assign a score for severity, occurrence, and detection based on their experience and subjective judgments. Since judgments may be associated with uncertainty, assigning a linguistic term to score a risk factor is difficult. Therefore, experts prefer to select one or more linguistic terms with appropriate confidence levels to assess risk factors. The numerical scale which states confidence levels is presented in Table 2 [21].

Please insert Table 2 here

Step 3: Convert the experts' opinions to BPAs

According to Table 1, there are ten elements in linguistic terms for rating risk factors. In DS theory of evidence, these elements can be considered as the frame of discernment for severity, occurrence, and detection. Therefore, the frame of discernment is presented as follows:

$$\theta^j = \{1, 2, 3, \dots, 10\}, j = S, O, D \quad (10)$$

The experts' rating for each risk factor including severity, occurrence, and detection, can be considered as focal elements and their respected confidence levels, which are determined based on Table 2, are used as the corresponding mass functions. Suppose that x_{ij} denotes the rating of failure mode i for risk factor j ($j = S, O, D$). For instance, according to Table 1, the respected evaluation levels in x_{ij} are $N, VM, MI, L, MO, S, MA, E, HW$, and HWO for the severity of a failure, and the corresponding confidence levels are $a, b, c, d, e, f, g, h, I$ and J , respectively. Then, the respected BPA for severity can be written as follows [31-32]:

$$\begin{aligned} m(\{N\}) &= a \\ m(\{VM\}) &= b \\ m(\{MI\}) &= c \\ &\vdots \\ m(\{HWO\}) &= J \\ m(\theta) &= 1 - a - b - c - \dots - J \end{aligned}$$

Similarly, the BPAs can be constructed for the occurrence and detection of a failure. As an example, suppose a rating of an expert about severity for the first failure mode (R_1) is low (L) and moderate (MO) with confidence levels 0.2 and 0.3, respectively. Then the respected BPA can be written as: $m(\{L\}) = 0.2, m(\{MO\}) = 0.3, m(\theta) = 0.5$

Step 4: Calculate the discounted BPA based on the new weighted discounting coefficient

In this step, a new weighting method is proposed based on the belief entropy developed by Zhou et al. [43] for weighting BPAs. According to the DS combination rule, two sources of information that may have uncertainty can be combined. It is necessary to consider the quality of information and to reduce uncertainty when combining them. Since the high value of an uncertainty measure reduces the reliability of the results, it is necessary to reduce the uncertainty in the information sources before combining them. Therefore, to reduce uncertainty, the discounted BPA must be obtained based on the weights of that BPA. To introduce the new

weighting method, let BPA_i be the BPA of i th failure mode for a given risk factor such as severity, occurrence, or detection. According to formulation (5), $E_{Md}(BPA_i)$ denotes the uncertainty measure for BPA_i which is calculated by Zhou et al. [43] method. The uncertainty measure needs to be normalized to take a value in the interval $[0, 1]$ according to the following formulation:

$$0 \leq \frac{E_{Md}(BPA_i)}{\max\{E_{Md}(BPA_i)\}} \leq 1 \quad (11)$$

where $\max\{E_{Md}(BPA_i)\}$ is calculated by the following formulation:

$$\max\{E_{Md}(BPA_i)\} = - \sum_i m(F_i) \log_2 \left(\frac{m(F_i)}{2^{|F_i|} - 1} \times e^{\frac{|F_i| - 1}{|X|}} \right) \quad (12)$$

where

$$m(F_i) = \frac{2^{|F_i|} - 1}{\sum_i 2^{|F_i|} - 1} \quad (13)$$

where $E_{Md}(BPA_i)$ denotes the belief entropy introduced by Zhou et al. [43], $\max\{E_{Md}(BPA_i)\}$ denotes the maximum value of the belief entropy, F_i is a proposition in mass function m , $m(F_i)$ is the mass function of F_i , and $|F_i|$ presents the number of elements of F_i . Furthermore, notations X and $|X|$ are the frame of discernment, and its number of elements, respectively.

According to the weight derivation method in Shanon entropy, the normalized form of the degree of diversification can be considered as a weight value [41]. Therefore, according to this weight derivation method, the weight of BPA_i can be calculated by the following formulation:

$$w_i = 1 - \frac{E_{Md}(BPA_i)}{\max\{E_{Md}(BPA_i)\}} \quad (14)$$

where w_i denotes the weight of BPA_i . This weight factor can be used as a coefficient of discounting to obtain the discounted BPA. According to equation (6), the discounted BPA_i can be rewritten as follows:

$$\begin{aligned} m^w(A) &= (1 - w_i) \times m(A), A \in \theta \\ m^w(\theta) &= (1 - w_i) + w_i \times m(\theta) \end{aligned} \quad (15)$$

where w and $m^w(\cdot)$ denote the coefficient of discounting and the discounted BPA, respectively.

Step 5: Combine the discounted BPAs using the Dempster combination rule

In this step, the discounted BPAs are aggregated by the combination rule according to formulations (7) and (8).

Step 6: Apply the pignistic probability transformation

As the combined results for discounted BPAs are in the form of focal elements with mass function values, the pignistic probability transformation is used to convert them into a singleton element. In this step, the pignistic probability transformation is provided for the linguistic terms. To get a numerical value for each risk factor, the probability distribution must be integrated. For doing so, suppose that we have n linguistic terms with ratings L_1, L_2, \dots, L_n for evaluating a risk factor. Furthermore, let P_1, P_2, \dots, P_n be the probability distribution concerning n linguistic terms. Then the aggregated value, which is the mathematical expectation value of the risk factor, is calculated as:

$$\text{Aggregated value} = L_1 P_1 + L_2 P_2 + \dots + L_n P_n \quad (16)$$

Step 7: Calculate the risk priority number

In this step, by utilizing the results obtained in step 7, the risk priority number is calculated for each failure mode by the following equation [45].

$$RPN = S \times O \times D \quad (17)$$

where RPN is the risk priority number of a failure. Notations S , O , and D denote the severity, occurrence, and detection of a failure, respectively.

5. Case study

In this study, environmental risks were first identified by the FMEA in the exploitation plant of one of the oil and gas plants in Ahvaz, Iran. To implement FMEA, a team of experts familiar with the production processes and risk assessment methods completes the worksheet designed for different sections, based on technical experience, interviews with managers, and data collection. Identification of environmental risks and their consequences, survey of design features, the status of the existing environment, identification of pollutants and hazards, and determination of impacts on most affected areas have been done through interviews with experts. Failure modes, causes of failures, failure effects, risk causes, severity, occurrence, and detection, construct the FMEA columns form. As such, the risks are identified and estimated. After identifying the risks using screening, the remaining 11 risks are reported in Table 3. Table 3 shows the results of the implementation of the FMEA method in the studied plant. In this table, failure modes, cases of failure, environmental issues, and environmental impacts are reported.

Please insert Table 3 here

To calculate the environmental RPN, the proposed evidential method introduced in section 4 is applied. According to steps 1 and 2, the rating of risk factors and their respected confidence levels are collected according to the opinions and judgments of experts. For doing so, three

experts who are the top managers in the oil and gas exploitation plant are selected and the respected data are gathered. In the third step of the proposed method, data must be converted into the BPAs for applying Dempster-Shafer evidence theory to obtain RPNs. The BPA values are obtained for three risk factors according to the judgments of the first expert and reported in Table 4.

Please insert Table 4 here

In the fourth step, the weight of BPAs is calculated and then the discounted BPAs are obtained according to formulation (15). For doing so, first, the belief entropy introduced by Zhou et al. [43] is obtained for each BPA according to formulation (5). For example, according to Table 4, consider the BPA of R1 for occurrence. The belief entropy is calculated as follows:

$$BPA(R1) : m(\{M, MH\}) = 0.5, m(\theta) = 0.5$$

$$E_{Md}(m) = 0.5 \times \log_2 \left(\frac{0.5}{2^2 - 1} \times e^{\frac{2-1}{10}} \right) + 0.5 \times \log_2 \left(\frac{0.5}{2^{10} - 1} \times e^{\frac{10-1}{10}} \right) = 6.070$$

The maximum belief entropy introduced by Zhou et al. [43] is calculated based on formulations (12) and (13) for the BPA with the following propositions:

$$\begin{aligned} & (\{NI\}, \frac{2^1 - 1}{58025}), (\{R\}, \frac{2^1 - 1}{58025}), (\{L\}, \frac{2^1 - 1}{58025}), \dots, (\{EH\}, \frac{2^1 - 1}{58025}), \\ & (\{NI, R\}, \frac{2^2 - 1}{58025}), (\{NI, L\}, \frac{2^2 - 1}{58025}), (\{NI, RL\}, \frac{2^2 - 1}{58025}), \dots, (\{VH, EH\}, \frac{2^2 - 1}{58025}) \\ & (\{NI, R, L\}, \frac{2^3 - 1}{58025}), (\{NI, R, RL\}, \frac{2^3 - 1}{58025}), (\{NI, R, M\}, \frac{2^3 - 1}{58025}), \dots, (\{RF, VH, EH\}, \frac{2^3 - 1}{58025}) \\ & \vdots \\ & (\{NI, R, L, RL, M, MH, H, RF, VH, EH\}, \frac{2^{10} - 1}{58025}) \end{aligned}$$

where $\{NI, R, L, RL, M, MH, H, RF, VH, EH\}$ denotes the frame of discernment with respect to the probability of occurrence. The aforementioned propositions show the respected power set. The

elements of the power set are shown as $(F_i, m(F_i))$, in which $m(F_i)$ is calculated by $\frac{2^{|F_i|} - 1}{\sum_i 2^{|F_i|} - 1}$. To

calculate the maximum value of the belief entropy, the expression $-m(F_i) \log_2 \left(\frac{m(F_i)}{2^{|F_i|} - 1} \times e^{\frac{|F_i| - 1}{|X|}} \right)$

must be calculated for all propositions in the power set, and then the sum of all calculated expressions must be obtained according to formula (12). For example, the mentioned expression

for the proposition $(\{NI\}, \frac{2^1 - 1}{58025})$ is calculated as $-\frac{2^1 - 1}{58025} \log_2 \left(\frac{\frac{2^1 - 1}{58025}}{2^1 - 1} \times e^{\frac{1 - 1}{1}} \right) = 0.0003$. After obtaining

this expression for all elements of the power set and summing them, the maximum belief entropy is calculated, whose value becomes 14.406. The weight of $BPA(R1)$ is calculated based on the formulation (14) as follows:

$$w = 1 - \frac{E_{Md}(BPA_i)}{\max\{E_{Md}(BPA_i)\}} = 1 - \frac{6.070}{14.406} = 0.579$$

Similarly, the weights of all BPAs are calculated. The belief entropy introduced by Zhou et al. [43] and the weights of BPAs are obtained and reported in Table 5.

Please insert Table 5 here

After calculating the weights of BPAs, the discounted BPAs can be obtained by formulation (15). The discounted BPAs for the first expert's opinions are reported in Table 6.

Please insert Table 6 here

Similarly, steps 1 to 4 of the proposed method can be applied to the second and third experts' opinions to obtain the discounted BPAs for risk factors. In the fifth step of the proposed method, the discounted BPAs extracted from experts' opinions are combined according to the Dempster

combination rule. For doing so, formulations (7) and (8) are used. The combined discounted BPAs for risk factors are reported in Table 7.

Please insert Table 7 here

In the sixth step, the combined discounted BPAs are converted into the probability distributions by applying the pignistic probability function shown in formulation (9). To get the final score of risk factors, the probability distributions are aggregated based on formulation (16). The respected results are reported in Table 8. Finally, the RPNs are calculated based on formulation (17) and the results are reported in Table 9. According to the results, R2, R7, and R1 gain the first to third ranks among environmental risks. Perforation of the pipeline when surplus water transfers from exploitation gas unit to gravity coal reservoirs (R2), and pump failure (R7) and perforation of the pipeline when transmitting salt oil from exploitation unit to desalination unit are identified as three critical environmental risks.

As mentioned earlier, FMEA, as a risk prevention tool, is a good approach for all industries. This method is a surefire way to anticipate problems and identify the most effective and cost-effective preventive solution. Suggestions for corrective actions to address pipe piercing include: a) provide proper pipeline coverage to prevent corrosion, b) cathode protection to prevent corrosion c) technical inspection of pipelines by the Department of Corrosion and Metals, d) periodic or annual testing of pipelines. The preventive actions are also recommended for pump failure: a) intermediate repair by calibration tool group, b) equipping the unit with spare pumps.

Please insert Table 8 here

Please insert Table 9 here

To highlight the advantages and rationality of the proposed method, it is compared with the FMEA and fuzzy FMEA methods for risk assessment. The traditional ratings for probability,

severity, and detection of a failure are used to obtain the initial data in the FMEA method. Regarding the opinions of experts and Table 1, the linguistic expressions that have gained the highest level of confidence by the experts are considered as primary data for risk factors. In this regard, the numerical ratings corresponding to the mentioned linguistic expressions are obtained. The geometric mean is used to aggregate the assessment rating of experts. Columns first to fourth of Table 10 show the FMEA results including the aggregated value of risk factors and the RPN value of failures.

To obtain fuzzy FMEA results, the linguistic expressions of experts about risk factors are used. Then fuzzy scales introduced in Dağsuyu et al. [16] are utilized to convert the linguistic expressions to the respected triangular fuzzy numbers. The geometric mean is utilized to aggregate the assessment of experts about risk factors. The sixth to eighth columns of Table 10 shows the aggregated fuzzy values of occurrence, severity, and detection, respectively. Multiplication of triangular fuzzy numbers is used to calculate the fuzzy RPN, which is shown in the ninth column of Table 10. Suppose, (α, β, γ) be a triangular fuzzy number, which shows the fuzzy RPN of a failure mode. The following formulation is used to obtain the defuzzified RPN.

$$Defuzzified \ RPN = \frac{[(\gamma - \alpha) + (\beta - \alpha)]}{3} + \alpha \quad (18)$$

According to the following formulation, the defuzzified RPNs are obtained that are reported in the last column of Table 10.

Please insert Table 10 here

Table 11 shows the comparative results of FMEA, fuzzy FMEA, and the proposed evidential model. As mentioned earlier, the conventional RPN and fuzzy RPN are calculated by applying FMEA and fuzzy FMEA methods, respectively. The ranking results of FMEA, fuzzy FMEA, and the proposed evidential methods reveal that R1, R2, R3, and R8 are gained the same ranks by

applying three methods. Furthermore, the remaining failures, except R7, have obtained almost close ranks using three methods. For instance, R4 has the fifth rank by utilizing the FMEA method, and the seventh rank by applying the fuzzy FMEA method, while it has the sixth rank regarding the proposed method.

Please insert Table 11 here

6. Conclusion

In this paper, FMEA is first used to identify environmental risks in the oil and gas exploitation plant. 11 environmental risks with risk causes, environmental risk aspects, and environmental impacts are determined in the studied plant. The evidential model is then proposed based on Dempster-Shafer evidence theory for environmental risk assessment in the oil and gas exploitation plant. The proposed evidential model can effectively handle uncertain and subjective information and enables experts to express their opinions and confidence levels about risk factors. In the evidential model, the basic probability assignments are constructed based on the experts' opinions for risk factors. Furthermore, the new weighting method is developed to obtain discounted BPAs. The main advantages of the proposed weighting method are that it reduces uncertainty in BPAs and improves the quality of information and reliability of results when aggregating multiple BPAs. Finally, the proposed evidential model is employed to assess the environmental risks in the oil and gas exploitation plant.

The effectiveness of the proposed method is illustrated using a real case study in the oil and gas exploitation plant. In future research, the proposed method should be applied to more practice to further verify its feasibility. In this paper, the importance of experts' opinions is considered equally in the risk assessment process. But in real-world applications, decision-makers may have

different knowledge backgrounds, skills, and experiences. For future research, it is suggested to develop an evidential model based on DS evidence theory in which the importance of experts' opinions are considered based on their knowledge backgrounds, skills, and experiences.

Acknowledgments: The authors are grateful for the valuable comments and suggestions of the editor-in-chief and the respected reviewers. These comments enhanced the strength and significance of our paper. This work has been financially supported by the research deputy of Shahrekord University. The grant number was 99GRN31M1759.

References

- [1] Shaluf, I. M., and Abdullah, S. A. "Floating roof storage tank boilover", *Journal of Loss Prevention in the Process Industries*, **24**(1), pp. 1-7 (2011).
- [2] Hocaoglu, S.M., Gursoy Haksevenler, B.H., Pamir Talazana, P., and Aydoner, C. "Assessment of technology modification for olive oil sector through mass balance: A case study for Turkey", *Journal of Cleaner Production*, **188**, pp. 786-795(2018).
- [3] Seyedhoseini, S.M., and Hatefi, M.A. "Two-Pillar Risk Management (TPRM): A Generic Project Risk Management Process", *Scientia Iranica*, **16**(2), pp. 138-148 (2009).
- [4] Rostamzadeh, R., Ghorabae, M.K., Govindan, K., Esmaili, A., and Nobar, H.B.K. "Evaluation of sustainable supply chain risk management using an integrated fuzzy TOPSIS-CRITIC approach", *Journal of Cleaner Production*, **175**, pp. 651-669 (2018).
- [5] Valinejad, F., and Rahmani, D. "Sustainability risk management in the supply chain of telecommunication companies: A case study", *Journal of Cleaner Production*, **203**, pp. 53-67 (2018).
- [6] Naderpour, A., Majrouhi Sardroud, J., Mofid, M., Xenidis, Y., and Pour Rostam, T. "Uncertainty Management in Time Estimation of Construction Projects: A Systematic Literature Review and New Model Development", *Scientia Iranica*, **26**(2), pp. 752-778 (2019).
- [7] Afshari, H., Issa, M.H, and Radvan, A. "Using failure mode and effects analysis to evaluate barriers to the greening of existing buildings using the Leadership in Energy and Environmental Design rating system", *Journal of Cleaner Production*, **127**, pp. 195-203 (2016).

- [8] Hatefi, S. M., and Tamošaitienė, J. “An integrated fuzzy DEMATEL-fuzzy ANP model for evaluating construction projects by considering interrelationships among risk factors”, *Journal of Civil Engineering and Management*, **25**(2), pp. 114-131 (2019).
- [9] Chang, J.I., and Lin, C.C. “A study of storage tank accidents”, *Journal of Loss Prevention in the Process Industries*, **19**(1), pp. 51-59 (2006).
- [10] Wang, D., P. Zhang, P., and Chen, L. “Fuzzy fault tree analysis for fire and explosion of crude oil tanks”, *Journal of Loss Prevention in the Process Industries*, **26**(6), pp. 1390-1398 (2013).
- [11] Ardeshtir, A., Mohajeri, M., and Amiri, M. “Evaluation of Safety Risks in Construction Using Fuzzy Failure Mode and Effect Analysis (FFMEA), ” *Scientia Iranica*, **23**(6), pp. 2546-2556 (2016).
- [12] Arabsheybani, A., Paydar, M.M., and Safaei, A.S. “An integrated fuzzy MOORA method and FMEA technique for sustainable supplier selection considering quantity discounts and supplier's risk”, *Journal of Cleaner Production*, **190**, pp. 577-591 (2018).
- [13] Chen, L. and Deng, Y. “A new failure mode and effects analysis model using Dempster–Shafer evidence theory and grey relational projection method”, *Engineering Applications of Artificial Intelligence*, **76**, pp. 13-20 (2018).
- [14] Wang, Y.M., Chin, K.S., Kwai Poon, G.K., and Yang J.B. “Risk evaluation in failure mode and effects analysis using fuzzy weighted geometric mean”, *Expert Systems with Applications*, **36**(2, Part 1), pp. 1195-1207 (2009).
- [15] Kutlu, A.C., and Ekmekçioğlu, M. “Fuzzy failure modes and effects analysis by using fuzzy TOPSIS-based fuzzy AHP”, *Expert Systems with Applications*, **39**(1), pp. 61-67 (2012).
- [16] Dağsuyu, C., Göçmen, E., Narlı, M., and Kokangül, A. “Classical and fuzzy FMEA risk analysis in a sterilization unit”, *Computers & Industrial Engineering*, **101**, pp. 286-294 (2016).
- [17] Zhou, Q., and Thai, V.V. “Fuzzy and grey theories in failure mode and effect analysis for tanker equipment failure prediction”, *Safety Science*, **83**, pp. 74-79 (2016).
- [18] Rafie, M., and Samimi Namin, F. “Prediction of subsidence risk by FMEA using artificial neural network and fuzzy inference system”, *International Journal of Mining Science and Technology*, **25**(4), pp. 655-663 (2015).
- [19] Ilbahar, E., Karaşan, A., Cebi, S., and Kahraman, C. “A novel approach to risk assessment for occupational health and safety using Pythagorean fuzzy AHP & fuzzy inference system”, *Safety Science*, **103**, pp. 124-136 (2018).

- [20] Jaderi, F., Ibrahim, Z.Z., and Zahiri, M. R. “Criticality analysis of petrochemical assets using risk based maintenance and the fuzzy inference system”, *Process Safety and Environmental Protection*, 121, pp. 312-325 (2019).
- [21] Fei, L., Deng, Y. and Hu, Y. “DS-VIKOR: A New Multi-criteria Decision-Making Method for Supplier Selection”, *International Journal of Fuzzy Systems*, **21**(1) , pp. 157-175 (2019).
- [22] Qiu, G.Q., Huang, S., Zhu, L.L., Su, X.H., and Chen, Y. “Risk Assessment of Multi-State Bayesian Network in an Oil Gathering and Transferring System”, *Procedia Engineering*, **130**, pp. 1514-1523 (2015).
- [23] Barua, S., Gao, X., Pasman, H., and Mannan, M.S. “Bayesian network based dynamic operational risk assessment”, *Journal of Loss Prevention in the Process Industries*, **41**, pp. 399-410 (2016).
- [24] Alauddin, M., Khan, F., Imtiaz, S., and Ahmed, S. “A Probabilistic Risk Assessment of Offshore Flaring Systems Using Bayesian Network”, *Advances in Industrial Safety*, Singapore, Springer Singapore (2020).
- [25] Lan, M., Zhu, J., and Lo, S. “Hybrid Bayesian network-based landslide risk assessment method for modeling risk for industrial facilities subjected to landslides”, *Reliability Engineering & System Safety*, **215**, pp. 107851 (2021).
- [26] Hatefi, S.M., Basiri, M.E., Tamošaitienė, J. “An Evidential Model for Environmental Risk Assessment in Projects Using Dempster–Shafer Theory of Evidence”, *Sustainability*, **11**, pp. 6329 (2019).
- [27] Hatefi, S.M., Asadi, H., Shams, G., Tamošaitienė, J., and Turskis, Z. “Model for the Sustainable Material Selection by Applying Integrated Dempster-Shafer Evidence Theory and Additive Ratio Assessment (ARAS) Method”, *Sustainability*, **13**(18), pp. 10438 (2021).
- [28] Liu, P., and Zhang, X. “Approach to multi-attributes decision making with intuitionistic linguistic information based on dempster-shafer evidence theory”, *IEEE Access*, **6**, pp. 52969-52981 (2018).
- [29] Liu, P., and Gao, H. “Some intuitionistic fuzzy power Bonferroni mean operators in the framework of Dempster–Shafer theory and their application to multicriteria decision making”, *Applied Soft Computing*, **85**, 105790 (2019).
- [30] Liu, P., and Zhang, X. “A new hesitant fuzzy linguistic approach for multiple attribute decision making based on Dempster–Shafer evidence theory”, *Applied Soft Computing*, **86**, pp. 105897 (2020).
- [31] Liu, P., Liu, X., Ma, G., Liang, Z., Wang, C., and Alsaadi, F.E. “A multi-attribute group decision-making method based on linguistic intuitionistic fuzzy numbers and Dempster–

- Shafer evidence theory”, *International Journal of Information Technology & Decision Making*, **19**(02), pp. 499-524 (2020).
- [32] Liu, P., Shen, M., Teng, F., Zhu, B., Rong, L., and Geng, Y. “Double hierarchy hesitant fuzzy linguistic entropy-based TODIM approach using evidential theory”, *Information Sciences*, **547**, pp. 223-243 (2021).
- [33] Liu, P., Zhang, X., and Pedrycz, W. “A consensus model for hesitant fuzzy linguistic group decision-making in the framework of dempster–shafer evidence theory”, *Knowledge-Based Systems*, **212**, 106559 (2021).
- [34] Yang, J., Huang, H.Z., He, L.P., Zhu, S.P., and Wen, D. “Risk evaluation in failure mode and effects analysis of aircraft turbine rotor blades using Dempster–Shafer evidence theory under uncertainty”, *Engineering Failure Analysis*, **18**(8), pp. 2084-2092 (2011).
- [35] Su, X., Deng, Y., Mahadevan, S., and Bao, Q. “An improved method for risk evaluation in failure modes and effects analysis of aircraft engine rotor blades”, *Engineering Failure Analysis*, **26**, pp. 164-174 (2012).
- [36] Jiang, W., Xie, C., Zhuang, M., and Tang, Y. "Failure mode and effects analysis based on a novel fuzzy evidential method", *Applied Soft Computing*, **57**, pp. 672-683 (2017).
- [37] Certa, A., Hopps, F., Inghilleri, R., and La Fata, C.M. “A Dempster-Shafer Theory-based approach to the Failure Mode, Effects and Criticality Analysis (FMECA) under epistemic uncertainty: application to the propulsion system of a fishing vessel”, *Reliability Engineering & System Safety*, **159**, pp. 69-79 (2017).
- [38] Yousefi, S., Alizadeh, A., Hayati, J., and Bagheri, M. “HSE risk prioritization using robust DEA-FMEA approach with undesirable outputs: A study of automotive parts industry in Iran”, *Safety Science*, **102**, pp. 144-158 (2018).
- [39] Dempster, A.P. “Upper and lower probabilities induced by a multivalued mapping”, *Annals of Mathematical Statistics*, **38**(4) , pp. 325–339 (1967).
- [40] Shafer, G. “A Mathematical Theory of Evidence”, Princeton University Press, Princeton (1976).
- [41] Deng, Y. “Deng entropy”, *Chaos, Solitons & Fractals*, **91**, pp. 549-553 (2016).
- [42] Shannon, E.C. “A mathematical theory of communication”, *Bell Systems Technical Journal*, **27** (3), pp. 379–423 (1948).
- [43] Zhou, D., Tang, Y., Jiang, W. “A modified belief entropy in Dempster-Shafer framework”, *PLoS ONE*, **12**(5), e0176832 (2017).

- [44] Liu, W. “Analyzing the degree of conflict among belief functions”, *Artificial Intelligence*, **170**(11), pp. 909-924 (2006).
- [45] Xiao, N., Huang, H.Z., Li, Y., He, L., and Jin, T. “Multiple failure modes analysis and weighted risk priority number evaluation in FMEA”, *Engineering Failure Analysis*, **18**, pp. 1162–1170 (2011).

Biographical notes:

Gholamreza Shams is an Assistant Professor at faculty of engineering, Shahrekord University, Shahrekord, Iran. He received his PhD degree in Civil Engineering at the Delft University, Netherlands, Netherlands. 2014. His current research interests include: Risk management, Sediment Engineering: Flood Engineering, Earth Dams, and Hydraulic Structures.

Seyed Morteza Hatefi is currently an Associate Professor at faculty of engineering, Shahrekord University, Shahrekord, Iran. He received his PhD degree from the School of Industrial Engineering at the University of Tehran, in 2014. He received his BS in Statistics from Shahid Beheshti University, Tehran in 2006, and MS in Industrial Engineering from University of Tehran at Tehran in 2009. His current research interests include: risk management, supply chain network design, logistics systems, multi-criteria decision making, data envelopment analysis, performance measurement and management, and operations research applications.

Shahla Nemati was born in Shiraz, Iran, in 1982. She received her B.S. degree in hardware engineering from Shiraz University, Shiraz, in 2005, her M.S. degree from the Isfahan University of Technology, Isfahan, Iran, in 2008, and her Ph.D. degree in computer engineering from Isfahan University, Isfahan, in 2016. Since 2017, she has been an Assistant Professor with the Computer Engineering Department, Shahrekord University, Shahrekord, Iran. She has written several articles in the fields of data fusion, emotion recognition, affective computing, and audio processing. Her current research interests include data fusion, affective computing, and data mining.

Captions of Tables:

Table 1. Traditional ratings for probability, severity, and detection of a failure (Yang et al. 2011)

Table 2: Confidence levels and their scales (Fei et al., 2019)

Table 3: Failure modes and environmental effects in the oil and gas plant

Table 4: The constructed BPAs based on the first experts' evaluations for risk factors

Table 5: The belief entropy and the weight of BPAs obtained by the first experts' opinions

Table 6: The discounted BPAs based on the first expert's evaluations for risk factors

Table 7: The combined discounted BPAs based on the experts' evaluations

Table 8: BetP and aggregate values for each failure mode under different risk factors

Table 9: Risk priority number and rank of failure modes

Table 10: FMEA and fuzzy FMEA results

Table 11: Comparing results of the FMEA, fuzzy FMEA, and proposed evidential method

Tables:

Table 1. Traditional ratings for probability, severity, and detection of a failure [34]

Rating	Probability of occurrence	Possible failure rate
10	Extremely high (EH): failure almost inevitable	$\geq 1/2$
9	Very high (VH)	$1/3$
8	Repeated failures (RF)	$1/8$

7	High (H)	1/20
6	Moderately high (MH)	1/80
5	Moderate (M)	1/400
4	Relatively low (RL)	1/2000
3	Low (L)	1/15000
2	Remote (R)	1/150000
1	Nearly impossible (NI)	$\leq 1/1500000$

Rating	Severity of effect	Definition
10	Hazardous without warning (HWO)	The highest severity ranking of a failure mode, occurring without warning and consequence is hazardous
9	Hazardous with a warning (HW)	Higher severity ranking of a failure mode, occurring with the warning, the consequence is hazardous
8	Extreme (E)	Operation of system or product is broken down without compromising safe
7	Major (MA)	Operation of system or product may be continued but the performance of system or product is affected
6	Significant (S)	Operation of system or product is continued and performance of system or product is degraded
5	Moderate (MO)	Performance of system or product is affected seriously and the maintenance is needed
4	Low (L)	Performance of system or product is small affected and the maintenance may not be needed
3	Minor (MI)	System performance and satisfaction with minor effect
2	Very minor (VM)	System performance and satisfaction with slight effect
1	No effect (N)	No effect

Rating	Detection	Definition
10	Absolutely uncertainty (AU)	The potential occurring of failure mode cannot be detected in concept, design, and process FMEA/mechanism and subsequent failure mode
9	Very remote (VR)	The possibility of detecting the potential occurring of failure mode is very remote/mechanism and subsequent failure mode
8	Remote (R)	The possibility of detecting the potential occurring of failure mode is remote/mechanism and subsequent failure mode
7	Very low (VL)	The possibility of detecting the potential occurring of failure mode is very low/mechanism and subsequent failure mode
6	Low (L)	The possibility of detecting the potential occurring of failure mode is low/mechanism and subsequent failure mode
5	Moderate (M)	The possibility of detecting the potential occurring of failure mode is moderate/mechanism and subsequent failure mode
4	Moderately high (MH)	The possibility of detecting the potential occurring of failure mode is moderately high/mechanism and subsequent failure mode

3	High (H)	The possibility of detecting the potential occurring of failure mode is high/mechanism and subsequent failure mode
2	Very high (VH)	The possibility of detecting the potential occurring of failure mode is very high/mechanism and subsequent failure mode
1	Almost certain (AL)	The potential occurring of failure mode will be detected

Table 2: Confidence levels and their scales [21]

Confidence level	Scale
Fully convinced	1.0
Almost convinced	0.8
Properly convinced	0.6
Some convinced	0.4
Almost not convinced	0.2
Completely not convinced	0.0
Intermediate values between two adjacent levels	0.9, 0.7, 0.5, 0.3, 0.1

Table 3: Failure modes and environmental effects in the oil and gas plant

Operation		Failure modes	Case of failure	Environmental issues	Environmental effects
Transmission of salt oil from exploitation unit to desalination unit	R1	Perforation of pipeline	Corrosion and decay of pipes	Oil leakage	Soil and groundwater pollution
Transfer of surplus water from exploitation gas unit to gravity coal reservoirs	R2	Perforation of pipeline	Corrosion and decay of pipes	Saltwater leakage	Soil and groundwater pollution
	R3	Perforating tanks	Corrosion and decay of tanks	Oil and saltwater leakage	Soil and groundwater pollution
Transfer of surplus water from the salt reservoir to gravity separator	R4	Perforation of pipeline	Corrosion and decay of pipes	Oil and saltwater leakage	Soil and groundwater pollution
	R5	Perforating tanks	Corrosion and decay of tanks	Oil and saltwater leakage	Soil and groundwater pollution
Transfer of surplus water from sulfur dehydrogenated towers to surplus water tanks	R6	Perforation of pipeline	Corrosion and decay of pipes	Gas and surplus water leakage	Soil, groundwater, and air pollution
	R7	Pump failure	Functional defects or worn out pump	Gas and surplus water leakage	Soil, groundwater, and air pollution
Storage, transportation, and injection of chemicals into the liquid desalination tank	R8	Perforation of pipeline	Corrosion and decay of pipes	Chemical leakage	Soil and groundwater pollution
	R9	Pump failure	Functional defects or worn out	Chemical leakage	Soil and groundwater pollution

pump					
Conduct gas to torches	R10	Incomplete burning of gas in the torches	Torch shutdown	Gas leakage	air pollution
Transfer of effluent to an evaporation pond	R11	Perforation of pipeline	Corrosion and decay of pipes	Wastewater leakage (water and oil)	Soil and groundwater pollution

Table 4: The constructed BPAs based on the first experts' evaluations for risk factors

Failure mode	Occurrence	Severity	Detection
R1	$m(\{M, MH\}) = 0.5, m(\theta) = 0.5$	$m(\{S, MA\}) = 0.6, m(\theta) = 0.4$	$m(\{MH, M\}) = 0.5, m(\theta) = 0.5$
R2	$m(\{H\}) = 0.4, m(\{RF\}) = 0.4, m(\theta) = 0.2$	$m(\{MO\}) = 0.2, m(\{S\}) = 0.4, m(\{MA\}) = 0.2, m(\theta) = 0.2$	$m(\{H, MH\}) = 0.4, m(\theta) = 0.6$
R3	$m(\{R, L\}) = 0.7, m(\theta) = 0.3$	$m(\{MA\}) = 0.8, m(\theta) = 0.2$	$m(\{AL\}) = 0.8, m(\{VH\}) = 0.1, m(\theta) = 0.1$
R4	$m(\{H, RF\}) = 0.6, m(\theta) = 0.4$	$m(\{S, MA\}) = 0.5, m(\theta) = 0.5$	$m(\{VH, H\}) = 0.6, m(\theta) = 0.4$
R5	$m(\{R\}) = 0.2, m(\{RL\}) = 0.4, m(\theta) = 0.4$	$m(\{S\}) = 0.5, m(\{MA\}) = 0.4, m(\theta) = 0.1$	$m(\{AL\}) = 0.8, m(\theta) = 0.2$
R6	$m(\{MH\}) = 0.1, m(\{H\}) = 0.8, m(\{RF\}) = 0.1, m(\theta) = 0$	$m(\{MO\}) = 0.4, m(\{S\}) = 0.6, m(\theta) = 0$	$m(\{AL\}) = 0.2, m(\{VH\}) = 0.7, m(\{H\}) = 0.1, m(\theta) = 0$
R7	$m(\{VH, EH\}) = 0.9, m(\theta) = 0.1$	$m(\{MI, L\}) = 0.4, m(\theta) = 0.6$	$m(\{AL, VH\}) = 0.4, m(\theta) = 0.6$
R8	$m(\{NI\}) = 0.3, m(\{R\}) = 0.3, m(\{L\}) = 0.3, m(\theta) = 0.1$	$m(\{MO\}) = 0.2, m(\{S\}) = 0.8, m(\theta) = 0$	$m(\{H\}) = 0.7, m(\theta) = 0.3$
R9	$m(\{H, RF\}) = 0.4, m(\theta) = 0.6$	$m(\{MI\}) = 0.2, m(\{L\}) = 0.5, m(\{MO\}) = 0.2, m(\theta) = 0.1$	$m(\{VH\}) = 0.3, m(\{H\}) = 0.4, m(\theta) = 0.3$
R10	$m(\{MH\}) = 0.6, m(\theta) = 0.4$	$m(\{MO, S\}) = 0.7, m(\theta) = 0.3$	$m(\{MH\}) = 0.9, m(\theta) = 0.1$
R11	$m(\{M, MH\}) = 1, m(\theta) = 0$	$m(\{MI\}) = 0.1, m(\{L\}) = 0.8, m(\{MO\}) = 0.1, m(\theta) = 0$	$m(\{VH, H\}) = 0.7, m(\theta) = 0.3$

Table 5: The belief entropy and the weight of BPAs obtained by the first experts' opinions

Failure mode	Occurrence		Severity		Detection	
	Belief entropy	weight	Belief entropy	weight	Belief entropy	weight
R1	6.070	0.579	5.315	0.631	6.070	0.579
R2	3.262	0.774	3.662	0.746	6.767	0.530
R3	4.500	0.688	2.462	0.829	1.792	0.876
R4	5.315	0.631	6.070	0.579	5.315	0.631
R5	5.002	0.653	2.231	0.845	2.462	0.829
R6	0.922	0.936	0.971	0.933	1.157	0.920
R7	2.636	0.817	6.767	0.530	6.767	0.530
R8	2.765	0.808	0.722	0.950	3.491	0.758
R9	6.767	0.530	2.631	0.817	4.181	0.710
R10	4.451	0.691	4.500	0.688	1.339	0.907
R11	0.864	0.940	0.922	0.936	4.500	0.688

Table 6: The discounted BPAs based on the first expert's evaluations for risk factors

Failure mode	Occurrence	Severity	Detection
R1	$m(\{M, MH\}) = 0.289, m(\theta) = 0.711$	$m(\{S, MA\}) = 0.379, m(\theta) = 0.621$	$m(\{MH, M\}) = 0.289, m(\theta) = 0.711$
R2	$m(\{H\}) = 0.309, m(\{RF\}) = 0.309, m(\theta) = 0.381$	$m(\{MO\}) = 0.149, m(\{S\}) = 0.298, m(\{MA\}) = 0.149, m(\theta) = 0.402$	$m(\{H, MH\}) = 0.212, m(\theta) = 0.788$
R3	$m(\{R, L\}) = 0.481, m(\theta) = 0.519$	$m(\{MA\}) = 0.663, m(\theta) = 0.337$	$m(\{AL\}) = 0.7, m(\{VH\}) = 0.088, m(\theta) = 0.212$
R4	$m(\{H, RF\}) = 0.379, m(\theta) = 0.621$	$m(\{S, MA\}) = 0.289, m(\theta) = 0.711$	$m(\{VH, H\}) = 0.379, m(\theta) = 0.621$
R5	$m(\{R\}) = 0.131, m(\{RL\}) = 0.261, m(\theta) = 0.608$	$m(\{S\}) = 0.423, m(\{MA\}) = 0.338, m(\theta) = 0.239$	$m(\{AL\}) = 0.663, m(\theta) = 0.337$
R6	$m(\{MH\}) = 0.094, m(\{H\}) = 0.728, m(\{RF\}) = 0.094, m(\theta) = 0.064$	$m(\{MO\}) = 0.373, m(\{S\}) = 0.560, m(\theta) = 0.067$	$m(\{AL\}) = 0.184, m(\{VH\}) = 0.644, m(\{H\}) = 0.092, m(\theta) = 0.08$
R7	$m(\{VH, EH\}) = 0.735, m(\theta) = 0.265$	$m(\{MI, L\}) = 0.212, m(\theta) = 0.788$	$m(\{AL, VH\}) = 0.212, m(\theta) = 0.788$
R8	$m(\{NI\}) = 0.242, m(\{R\}) = 0.242, m(\{L\}) = 0.242, m(\theta) = 0.274$	$m(\{MO\}) = 0.190, m(\{S\}) = 0.760, m(\theta) = 0.05$	$m(\{H\}) = 0.530, m(\theta) = 0.470$
R9	$m(\{H, RF\}) = 0.212, m(\theta) = 0.788$	$m(\{MI\}) = 0.163, m(\{L\}) = 0.409, m(\{MO\}) = 0.163, m(\theta) = 0.264$	$m(\{VH\}) = 0.213, m(\{H\}) = 0.284, m(\theta) = 0.503$
R10	$m(\{MH\}) = 0.415, m(\theta) = 0.585$	$m(\{MO, S\}) = 0.481, m(\theta) = 0.519$	$m(\{MH\}) = 0.816, m(\theta) = 0.184$
R11	$m(\{M, MH\}) = 0.94, m(\theta) = 0.06$	$m(\{MI\}) = 0.094, m(\{L\}) = 0.749, m(\{MO\}) = 0.094, m(\theta) = 0.064$	$m(\{VH, H\}) = 0.481, m(\theta) = 0.519$

Table 7: The combined discounted BPAs based on the experts' evaluations

Failure mode	Occurrence
R1	$m(\{RL\}) = 0.220, m(\{M\}) = 0.218, m(\{MH\}) = 0.048, m(\{RL, M\}) = 0.155, m(\{M, MH\}) = 0.104, m(\theta) = 0.255$
R2	$m(\{H\}) = 0.365, m(\{RF\}) = 0.185, m(\{M, MH\}) = 0.223, m(\theta) = 0.227$
R3	$m(\{NI\}) = 0.036, m(\{R\}) = 0.492, m(\{L\}) = 0.070, m(\{M\}) = 0.203, m(\{R, L\}) = 0.096, m(\theta) = 0.103$
R4	$m(\{M\}) = 0.088, m(\{MH\}) = 0.176, m(\{H\}) = 0.199, m(\{H, RF\}) = 0.3, m(\theta) = 0.237$

R5	$m(\{R\})=0.039, m(\{L\})=0.1, m(\{M\})=0.488, m(\{MH\})=0.139, m(\{L, RL\})=0.049, m(\theta)=0.184$
R6	$m(\{M\}) = 0.110, m(\{MH\}) = 0.082, m(\{H\}) = 0.655, m(\{RF\}) = 0.082, m(\{VH, EH\}) = 0.015, m(\theta) = 0.056$
R7	$m(\{MH\}) = 0.042, m(\{H\}) = 0.337, m(\{VH\}) = 0.173, m(\{RF, VH\}) = 0.062, m(\{VH, EH\}) = 0.284, m(\theta) = 0.102$
R8	$m(\{NI\}) = 0.182, m(\{R\}) = 0.247, m(\{L\}) = 0.280, m(\{RL\}) = 0.052, m(\{M\}) = 0.035, m(\theta) = 0.205$
R9	$m(\{MH\}) = 0.209, m(\{H\}) = 0.265, m(\{RF\}) = 0.042, m(\{H, RF\}) = 0.069, m(\{RF, VH\}) = 0.157, m(\theta) = 0.257$
R10	$m(\{RL\}) = 0.066, m(\{M\}) = 0.394, m(\{MH\}) = 0.214, m(\{RL, M\}) = 0.124, m(\theta) = 0.203$
R11	$m(\{R\}) = 0.032, m(\{L\}) = 0.053, m(\{M, MH\}) = 0.815, m(\{H, RF\}) = 0.048, m(\theta) = 0.052$
Severity	
R1	$m(\{L\}) = 0.165, m(\{MO\}) = 0.083, m(\{S\}) = 0.066, m(\{S, MA\}) = 0.188, m(\{E, HW\}) = 0.188, m(\theta) = 0.309$
R2	$m(\{MO\}) = 0.091, m(\{S\}) = 0.257, m(\{MA\}) = 0.178, m(\{E\}) = 0.102, m(\{HW\}) = 0.026, m(\{S, MA\}) = 0.1, m(\theta) = 0.247$
R3	$m(\{N\}) = 0.053, m(\{VM\}) = 0.374, m(\{MI\}) = 0.053, m(\{S\}) = 0.069, m(\{MA\}) = 0.298, m(\theta) = 0.151$
R4	$m(\{S\}) = 0.046, m(\{MA\}) = 0.1070, m(\{MO, S\}) = 0.114, m(\{S, MA\}) = 0.173, m(\{MA, E\}) = 0.173, m(\theta) = 0.424$
R5	$m(\{MO\})=0.104, m(\{S\})=0.151, m(\{MA\})=0.193, m(\{E\})=0.457, m(\{HW\})=0.057, m(\theta)=0.039$
R6	$m(\{MO\}) = 0.332, m(\{S\}) = 0.497, m(\{MI, L\}) = 0.090, m(\{HW, HWO\}) = 0.022, m(\theta) = 0.060$
R7	$m(\{L\}) = 0.023, m(\{MO\}) = 0.073, m(\{S\}) = 0.099, m(\{MA\}) = 0.582, m(\{MI, L\}) = 0.047, m(\theta) = 0.176$
R8	$m(\{L\}) = 0.006, m(\{MO\}) = 0.642, m(\{LS\}) = 0.330, m(\theta) = 0.022$
R9	$m(\{MI\}) = 0.092, m(\{L\}) = 0.325, m(\{MO\}) = 0.130, m(\{S\}) = 0.121, m(\{MA\}) = 0.121, m(\{L, MO\}) = 0.061, m(\theta) = 0.149$
R10	$m(\{MO\}) = 0.086, m(\{S\}) = 0.103, m(\{MA\}) = 0.374, m(\{E\}) = 0.053, m(\{L, MO\}) = 0.092, m(\{MO, S\}) = 0.140, m(\theta) = 0.151$
R11	$m(\{VM\}) = 0.024, m(\{MI\}) = 0.144, m(\{L\}) = 0.688, m(\{MO\}) = 0.086, m(\{L, MO\}) = 0.028, m(\theta) = 0.030$
Detection	
R1	$m(\{H\}) = 0.067, m(\{MH\}) = 0.141, m(\{MH, M\}) = 0.160, m(\{L, VL\}) = 0.240, m(\theta) = 0.393$
R2	$m(\{VH\}) = 0.023, m(\{H\}) = 0.029, m(\{M\}) = 0.737, m(\{H, MH\}) = 0.045, m(\theta) = 0.166$
R3	$m(\{AL\}) = 0.355, m(\{VH\}) = 0.107, m(\{H\}) = 0.355, m(\{MH\}) = 0.076, m(\theta) = 0.107$
R4	$m(\{VH\}) = 0.129, m(\{H\}) = 0.102, m(\{AL, VH\}) = 0.211, m(\{VH, H\}) = 0.211, m(\theta) = 0.347$
R5	$m(\{AL\})=0.290, m(\{MH\})=0.2051, m(\{H, MH\})=0.221, m(\{MH, M\})=0.137, m(\theta)=0.147$
R6	$m(\{AL\}) = 0.185, m(\{VH\}) = 0.694, m(\{H\}) = 0.051, m(\{AL, VH\}) = 0.042, m(\theta) = 0.028$
R7	$m(\{VH\}) = 0.234, m(\{H\}) = 0.092, m(\{M\}) = 0.127, m(\{AL, VH\}) = 0.116, m(\theta) = 0.430$
R8	$m(\{AL\}) = 0.165, m(\{VH\}) = 0.066, m(\{H\}) = 0.388, m(\{MH\}) = 0.090, m(\{M\}) = 0.060, m(\theta) = 0.230$
R9	$m(\{VH\}) = 0.131, m(\{H\}) = 0.281, m(\{VL\}) = 0.091, m(\{H, MH\}) = 0.188, m(\theta) = 0.309$
R10	$m(\{VH\}) = 0.08628, m(\{H\}) = 0.028, m(\{MH\}) = 0.768, m(\{L, VL\}) = 0.037, m(\theta) = 0.139$
R11	$m(\{MH\}) = 0.115, m(\{M\}) = 0.173, m(\{L\}) = 0.060, m(\{VH, H\}) = 0.212, m(\{MH, H\}) = 0.212, m(\theta) = 0.229$

Table 8: BetP and aggregate values for each failure mode under different risk factors

	Occurrence										Aggregated value
	Bet(NI)	Bet(R)	Bet(L)	Bet(RL)	Bet(M)	Bet(MH)	Bet(H)	Bet(RF)	Bet(VH)	Bet(EH)	
R1	0.026	0.026	0.026	0.323	0.373	0.125	0.026	0.026	0.026	0.026	4.930
R2	0.023	0.023	0.023	0.023	0.023	0.134	0.499	0.207	0.023	0.023	6.732

R3	0.047	0.550	0.128	0.010	0.213	0.010	0.010	0.010	0.010	0.010	3.051
R4	0.024	0.024	0.024	0.024	0.112	0.199	0.373	0.174	0.024	0.024	6.443
R5	0.018	0.058	0.143	0.043	0.506	0.158	0.018	0.018	0.018	0.018	4.838
R6	0.006	0.006	0.006	0.006	0.116	0.087	0.661	0.087	0.013	0.013	6.733
R7	0.010	0.010	0.010	0.010	0.010	0.052	0.347	0.041	0.356	0.152	7.952
R8	0.202	0.268	0.300	0.072	0.055	0.020	0.020	0.020	0.020	0.020	3.022
R9	0.026	0.026	0.026	0.026	0.026	0.235	0.326	0.181	0.104	0.026	6.717
R10	0.020	0.020	0.020	0.148	0.476	0.234	0.020	0.020	0.020	0.020	5.188
R11	0.005	0.037	0.058	0.005	0.413	0.413	0.029	0.029	0.005	0.005	5.352
Severity											
	Bet(N)	Bet(VM)	Bet(MI)	Bet(L)	Bet(MO)	Bet(S)	Bet(MA)	Bet(E)	Bet(HW)	Bet(HWO)	Aggregated value
R1	0.031	0.031	0.031	0.196	0.114	0.192	0.125	0.125	0.125	0.031	5.997
R2	0.025	0.025	0.025	0.025	0.116	0.331	0.252	0.127	0.050	0.025	6.294
R3	0.069	0.390	0.069	0.015	0.015	0.084	0.313	0.015	0.015	0.015	4.297
R4	0.042	0.042	0.042	0.042	0.099	0.232	0.285	0.129	0.042	0.042	6.146
R5	0.004	0.004	0.004	0.004	0.108	0.155	0.197	0.461	0.061	0.004	7.154
R6	0.006	0.006	0.051	0.051	0.338	0.503	0.006	0.006	0.017	0.017	5.491
R7	0.018	0.018	0.041	0.064	0.091	0.116	0.600	0.018	0.018	0.018	6.256
R8	0.002	0.002	0.002	0.008	0.645	0.332	0.002	0.002	0.002	0.002	5.336
R9	0.015	0.015	0.107	0.370	0.175	0.136	0.136	0.015	0.015	0.015	4.898
R10	0.015	0.015	0.015	0.061	0.217	0.188	0.389	0.069	0.015	0.015	6.112
R11	0.003	0.027	0.147	0.705	0.103	0.003	0.003	0.003	0.003	0.003	3.954
Detection											
	Bet(AL)	Bet(VH)	Bet(H)	Bet(MH)	Bet(M)	Bet(L)	Bet(VL)	Bet(R)	Bet(VR)	Bet(AU)	Aggregated value
R1	0.039	0.039	0.106	0.260	0.119	0.159	0.159	0.039	0.039	0.039	5.202
R2	0.017	0.040	0.068	0.039	0.754	0.017	0.017	0.017	0.017	0.017	4.889
R3	0.366	0.118	0.366	0.087	0.011	0.011	0.011	0.011	0.011	0.011	2.528
R4	0.140	0.375	0.242	0.035	0.035	0.035	0.035	0.035	0.035	0.035	3.316
R5	0.305	0.015	0.125	0.398	0.083	0.015	0.015	0.015	0.015	0.015	3.308
R6	0.209	0.718	0.054	0.003	0.003	0.003	0.003	0.003	0.003	0.003	1.944
R7	0.101	0.335	0.135	0.043	0.170	0.043	0.043	0.043	0.043	0.043	3.922
R8	0.188	0.089	0.411	0.113	0.083	0.023	0.023	0.023	0.023	0.023	3.390
R9	0.031	0.162	0.406	0.125	0.031	0.031	0.122	0.031	0.031	0.031	4.101
R10	0.014	0.042	0.042	0.782	0.014	0.033	0.033	0.014	0.014	0.014	4.218
R11	0.023	0.129	0.129	0.244	0.301	0.083	0.023	0.023	0.023	0.023	4.423

Table 9: Risk priority number and rank of failure modes

Failure mode	Aggregated values			RPN	Rank
	Occurrence	Severity	Detection		
R1	4.930	5.997	5.202	153.814	3
R2	6.732	6.294	4.889	207.153	1
R3	3.051	4.297	2.528	33.141	11
R4	6.443	6.146	3.316	131.292	6

R5	4.838	7.154	3.308	114.508	7
R6	6.733	5.491	1.944	71.853	9
R7	7.952	6.256	3.922	195.117	2
R8	3.022	5.336	3.390	54.655	10
R9	6.717	4.898	4.101	134.941	4
R10	5.188	6.112	4.218	133.745	5
R11	5.352	3.954	4.423	93.585	8

Table 10: FMEA and fuzzy FMEA results

Failure mode	FMEA				Fuzzy FMEA				Defuzzified RPN
	O	S	D	RPN	O	S	D	Fuzzy RPN	
R1	4.63	5.78	4.89	130.84	(2.08,4.22,6.26)	(3.42,5.74,7.66)	(2.47,4.72,6.80)	(17.54,114.17,326.26)	152.66
R2	6.82	6.21	3.52	149.29	(5.00,7.00,9.00)	(4.16,6.08,7.88)	(1.44,3.56,5.59)	(30.00,151.44,396.88)	192.77
R3	2.92	6.14	2.29	41.08	(1.44,2.47,4.72)	(3.56,5.59,7.61)	(1.00,2.08,4.22)	(5.13,28.69,151.44)	61.75
R4	6.96	5.23	2.24	81.58	(4.22,6.26,8.28)	(2.47,4.72,6.80)	(1.00,2.08,4.22)	(10.40,61.40,237.49)	103.10
R5	3.74	7.27	2.51	68.22	(1.44,3.56,5.59)	(6.84,8.28,9.65)	(1.44,2.47,4.72)	(14.23,72.60,254.77)	113.87
R6	6.93	5.40	2.08	77.82	(4.93,6.80,8.57)	(2.47,4.72,6.80)	(1.00,2.08,4.22)	(12.16,66.77,245.98)	108.31
R7	8.27	3.27	2.47	66.70	(6.84,8.28,9.65)	(1.44,2.47,4.72)	(1.44,2.47,4.72)	(14.23,50.34,214.89)	93.15
R8	2.88	6.58	2.29	43.46	(1.00,3.00,5.00)	(4.16,6.08,7.88)	(1.00,2.08,4.22)	(4.16,37.95,166.23)	69.45
R9	7.46	4.63	4.19	144.47	(5.85,7.61,9.32)	(2.08,4.22,6.26)	(1.71,3.98,6.08)	(20.80,127.73,354.77)	167.77
R10	5.13	5.36	4.27	117.49	(3.00,5.00,7.00)	(2.47,4.72,6.80)	(1.71,3.98,6.08)	(12.65,93.86,289.69)	132.07
R11	4.98	3.30	3.83	63.05	(2.47,4.72,6.80)	(1.44,2.47,4.72)	(2.08,4.22,6.26)	(7.40,49.07,200.86)	85.77

Table 11: Comparing results of the FMEA, fuzzy FMEA, and proposed evidential method

Failure mode	FMEA		Fuzzy FMEA		Proposed evidential method	
	RPN	Rank	Defuzzified RPN	Rank	RPN	Rank
R1	130.840	3	152.66	3	153.814	3
R2	149.288	1	192.77	1	207.153	1
R3	41.075	11	61.75	11	33.141	11
R4	81.581	5	103.10	7	131.292	6
R5	68.222	7	113.87	5	114.508	7
R6	77.822	6	108.31	6	71.853	9
R7	66.701	8	93.15	8	195.117	2
R8	43.459	10	69.45	10	54.655	10
R9	144.468	2	167.77	2	134.941	4
R10	117.485	4	132.07	4	133.745	5
R11	63.046	9	85.77	9	93.585	8