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Product acceptance determination based on EWMA yield index using repetitive and MDS sampling schemes

M. Aslam[a,*], M. Azam[b], R.A.K. Sherwani[c], C.-H. Yen[d], and C.-H. Jun[e]

a. *Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah 21551, Saudi Arabia.*
b. *Department of Statistics and Computer Sciences, University of Veterinary and Animals Sciences, Lahore 54000, Pakistan.*
c. *College of Statistical and Actuarial Sciences, University of the Punjab, Lahore, Pakistan.*
d. *Department of Industrial Engineering and Management Information, Huafan University, Taipei, Taiwan.*
e. *Department of Industrial and Management Engineering, POSTECH, Pohang 37673, Republic of Korea.*

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Abstract. This study presents a repetitive group sampling plan and a multiple dependent state sampling plan based on the EWMA (exponentially weighted moving average) yield index for product acceptance. The proposed plans utilize the current and previous information through EWMA statistic to reach a decision of lot sentencing. A non-linear optimization model is developed to determine the plan parameters of the proposed plans for various specified conditions. The performance of the proposed plans over several existing sampling plans is analyzed, showing that the proposed plans are efficient in reducing the sample size for lot sentencing. For industrial application, a real example is given to demonstrate the implementation of the proposed plans.

1. Introduction

Acceptance sampling plans have been widely used in the manufacturing industry for lot sentencing, including inspection of raw material, semi-products, and final products. Customers would perform the inspection of goods when there is a need to verify the quality of goods submitted by suppliers. Mostly, the 100% inspection of goods is not feasible because of costs, time, destructive test, and so on. Therefore, the application of acceptance sampling plans is almost inevitable. A well-designed sampling plan should have high probability of acceptance for lots with good quality and low probability of acceptance for lots with bad quality. Generally, the parameters of a well-design sampling plan are determined by minimizing the sample size while satisfying the principle of two points on operating characteristic curve.

Attribute sampling plans and variable sampling plans have been developed for various situations in the literature. The former is used when data of interest is obtained for the counting process and the latter is used when data of interest is obtained for the measurement process. Attribute sampling plans are easier to apply than variable sampling plans, while variable sampling plans are more informative than attribute sampling plans. Therefore, both plans have been considered to be important for lot sentencing. Some researchers favor the attribute sampling plan, while others favor the variable sampling plan. More detailed information about the applications of two types of sampling plans can be seen in [1–27].

Most of variable acceptance sampling plans in the

*. *Corresponding author.*
E-mail addresses: aslam_ravian@hotmail.com (M. Aslam); mazam72@yahoo.com (M. Azam); rehan.stat@pu.edu.pk (R.A.K. Sherwani); jimyen@cc.hfu.edu.tw (C.-H. Yen); chjun@postech.ac.kr (C.-H. Jun)

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literature use only the current information to make a decision about the submitted lot. This type of plans is called "memoryless" plan. The efficiency of variable sampling plans can be increased by utilizing the current and previous information about the lot. The EWMA statistic is very popular in designing the control chart as it gives more weight to the current data and decreasing weight to the past data. According to [28], the control charts based on EWMA can detect smaller shifts in the process. More details about the EWMA statistic can be seen in [29]. Due to the advantages of the EWMA statistic, researchers found it interesting to design a sampling inspection plan using the mentioned statistic. Recently, Aslam et al. [30] proposed a variable sampling plan using the EWMA statistic when the quality of interest follows normal distribution with known or unknown standard deviation. They compared the efficiency of the proposed plan with the existing plans. Yen et al. [31] designed the sampling plan for a yield index using the EWMA statistic.

It is important to note that the inspection cost is directly proportional to the sample size. Therefore, some other flexible sampling schemes such as a Repetitive Group Sampling Plan (RGSP) [32], Multiple Dependent State Sampling (MDSS) plan [33], and Multiple Dependent State Repetitive Plan (MDSRP) [34] are proposed. These sampling schemes provide a smaller sample size than the plan based on a single sampling scheme. By exploring the literature and to the best of authors' knowledge, works on RGSP and MDSS with EWMA yield index have not been proposed yet. As mentioned, RGSP and MDSS schemes have been proved to be more economical than the single sampling plan. In addition, the EWMA yield index not only considers the quality of the current lot and the preceding lots, but also provides an exact measure on the process yield. For these motivations, the designing of MDSS and RGSP based on the EWMA yield index is proposed.

The rest of this paper is organized as follows. The yield index S_{pk} is introduced in Section 2. In Sections 3 and 4, the designing of RGSP and MDSS is presented, respectively. In Section 5, advantages of the proposed plans are described. Section 6 provides an example to illustrate the proposed methodology. Finally, conclusions are made in Section 7.

2. The yield index S_{pk}

According to [31], "process yield has been a standard criterion used in the manufacturing industry as a common measure on process performance". The process yield index is defined as the proportion of products manufactured within the given specification limits. The product manufactured beyond the specification limits incurs the extra cost of rework. As mentioned by [35], "traditionally, products manufactured within the specification limits are considered to be equally conforming and those outside the specification limits are considered to be equally nonconforming". The process yield is defined mathematically as:

$$Y = \int_{LSL}^{USL} f(x)dx, \tag{1}$$

where LSL is the Lower Specification Limit, USL the Upper Specification Limit, and $f(x)$ the probability density function (pdf) of the quality variable of interest X. The disadvantage of Eq. (1) is that it is unable to differentiate the products within the specification limit and beyond the specification limits [35]. Boyles [36] developed the capability index, called yield index S_{pk} which is helpful to provide the exact measure of process yield. This index is defined as follows:

$$S_{pk} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{USL-\mu}{\sigma}\right)+\frac{1}{2}\Phi\left(\frac{\mu-LSL}{\sigma}\right)\right\} \tag{2}$$

where $\Phi(.)$ is the cumulative distribution function (cdf) of the standard normal distribution, Φ^{-1} the inverse function of the standard normal distribution, μ the mean, and σ the standard deviation of the process. This index is more efficient than other capability indices since it provides a one-to-one relationship to the process yield [31]. Yen et al. [31] reported the process yield and non-conformities in Parts Per Million (PPM), when $S_{pk} = 1.0(0.1)2.0$ and popular performance requirements are 1.00, 1.33, 1.50, 1.67, and 2.00.

In practice, μ and σ are unknown and they are estimated from the data. The best linear unbiased estimator of μ is $\overline{X} = \sum_{i=1}^{n} X_i/n$ and that of σ^2 is $S^2 = \sum_{i=1}^{n}(X_i - \overline{X})/(n-1)$. The estimator $\widehat{S}_{pk}$ is expressed as follows:

$$\widehat{S}_{pk} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{USL-\overline{X}}{S}\right)+\frac{1}{2}\Phi\left(\frac{\overline{X}-LSL}{S}\right)\right\}. \tag{3}$$

As the exact distribution of this estimator is analytically intractable, Lee et al. [37] derived the approximately normal distribution of $\widehat{S}_{pk}$ as given below:

$$\widehat{S}_{pk} \sim N\left(S_{pk}, \frac{a^2+b^2}{36n\phi^2(3S_{pk})}\right), \tag{4}$$

where $\phi(.)$ is the pdf of the standard normal distribution, and

$$a = \frac{1}{\sqrt{2}}\left\{3C_p(2-C_a)\phi\left(3C_p(2-C_a)\right),\right.$$
$$\left. + 3C_pC_a\phi(3C_pC_a)\right\},$$

$$b = \phi\left(3C_p(2-C_a)\right) - \phi(3C_pC_a),$$

$$d = (USL - LSL)/2,$$

$$C_p = (USL - LSL)/6\sigma,$$

$$C_a = 1 - |\mu - M|/d.$$

3. Designing of the proposed plan using repetitive group sampling

The Repetitive Group Sampling Plan (RGSP) attracted the researchers due to its simplicity as compared to the double sampling plan or the sequential sampling plan. The RGSP is proven to be more efficient than the single and double sampling schemes [8,19,21,38,39,40]. The proposed RGSP based on the EWMA version of yield index is stated as follows:

Step 1. Take a random sample of size from a submitted lot at time t. Compute sample mean $\overline{X} = \sum_{i=1}^{n} X_i/n$ and sample standard deviation $S = \sqrt{\sum_{i=1}^{n}(X_i - \overline{X})/(n-1)}$. Calculate the yield index $\widehat{S}_{pkt}$ as in Eq. (3) for the current lot. Choose the smoothing constant $\lambda(0 < \lambda < 1)$ and compute the following EWMA yield index:

$$\widehat{S}_{pk}^{EWMA_t} = \lambda\widehat{S}_{pkt} + (1-\lambda)\widehat{S}_{pk}^{EWMA_{t-1}}.$$

Step 2. The decision about the submitted lot is stated as follows:

(a) Accept the lot if $\widehat{S}_{pk}^{EWMA_t} \geq k_a$;

(b) Reject the lot if $\widehat{S}_{pk}^{EWMA_t} < k_r$;

(c) If $k_r \leq \widehat{S}_{pk}^{EWMA_t} < k_a$, repeat Step 1.

The proposed RGSP is based on three parameters: sample size n, acceptance value k_a, and rejection value k_r. The proposed RGSP is the extension of the plan designed by [31] because it reduces to the latter when $k_a = k_r$. The Operating Characteristic (OC) function of the proposed RGSP is derived as follows. The lot acceptance probability based on a single sample is given as:

$$P\{\widehat{S}_{pk}{}^{EWMA_t} \geq \mathrm{k}_a\} = 1 - \Phi \left(\frac{k_a - S_{pk}}{\sqrt{[\lambda/(2-\lambda)]\,(a^2+b^2)\{36n\phi^2(3S_{pk})\}^{-1}}}\right). \tag{5}$$

The probability of repetitive sampling for a submitted lot is given as follows:

$$P\left\{k_r \leq \widehat{S}_{pk}^{EWMA_t} < k_a\right\} = \Phi \left(\frac{k_a - S_{pk}}{\sqrt{[\lambda/(2-\lambda)]\,(a^2+b^2)\{36n\phi^2(3S_{pk})\}^{-1}}}\right) - \Phi\left(\frac{k_r - S_{pk}}{\sqrt{[\lambda/(2-\lambda)]\,(a^2+b^2)\{36n\phi^2(3S_{pk})\}^{-1}}}\right). \tag{6}$$

Therefore, the OC function is given by Eqs. (7) and (8) as shown in Box I. The Average Sample Number (ASN) of the proposed RGSP is given by Eq. (9) as shown in Box II.

The sampling plan, which provides protection to producer and consumer, is considered as efficient. Let α be the producer's risk and β be the consumer's risk. The producer desires that the lot acceptance probability be larger than $1-\alpha$ when the quality level of lot is at the acceptance quality level ($AQL = S_{AQL}$), while the consumer desires the lot acceptance probability to be smaller than β when quality level of lot is at the limiting quality level ($LQL = S_{LQL}$). The three plan parameters of the proposed plan are obtained using the following non-linear optimization model:

$$\mathrm{MIN}_{n,k_a,k_r}\mathrm{ASN}(S_{LQL}. \tag{10a}$$

Subject to:

$$\pi_A(S_{AQL}) \geq 1-\alpha, \tag{10b}$$

$$\pi_A(S_{pk}) = \frac{P\{\widehat{S}_{pk}^{EWMA_t} \geq k_a\}}{1 - P\{k_r \leq \widehat{S}_{pk}^{EWMA_t} < k_a\}}, \tag{7}$$

$$\pi_A(S_{pk}) = \frac{1 - \Phi\left(\frac{k_a - S_{pk}}{\sqrt{[\lambda/(2-\lambda)](a^2+b^2)\{36n\phi^2(3S_{pk})\}^{-1}}}\right)}{1 - \left\{\Phi\left(\frac{k_a - S_{pk}}{\sqrt{[\lambda/(2-\lambda)](a^2+b^2)\{36n\phi^2(3S_{pk})\}^{-1}}}\right) - \Phi\left(\frac{k_r - S_{pk}}{\sqrt{[\lambda/(2-\lambda)](a^2+b^2)\{36n\phi^2(3S_{pk})\}^{-1}}}\right)\right\}}. \tag{8}$$

Box I

$$\mathrm{ASN}(\mathrm{S}_{pk}) = \frac{n}{1-\left\{\Phi\left(\frac{k_a-S_{pk}}{\sqrt{[\lambda/(2-\lambda)](a^2+b^2)\{36n\phi^2(3S_{pk})\}^{-1}}}\right)-\Phi\left(\frac{k_r-S_{pk}}{\sqrt{[\lambda/(2-\lambda)](a^2+b^2)\{36n\phi^2(3S_{pk})\}^{-1}}}\right)\right\}}. \tag{9}$$

Box II

$$\pi_A(S_{LQL}) \leq \beta, \quad 0 < k_r < k_a. \tag{10c}$$

The plan parameters of the proposed RGSP at different values of λ, α, and β are presented for $(S_{AQL}, S_{LQL}) = (1.33, 1.0)$, as shown in Table 1. In Table l, the plan parameters are given for $(S_{AQL}, S_{LQL}) = (1.50, 1.33)$. The smoothing constant lambda is a weight on the current information as compared to past information. Thus, a small value of lambda (around $0.1 - 0.3$) is recommended as long as the process is stable. A larger value of lambda may be used if the user seeks to reflect more weight on the current lot. In fact, in our tables, we compare the parameter values for various values of lambda, which is the same as the sensitivity analysis.

From Tables 1 and 2, we find that the plan parameters have the following trends:

1. At fixed values of α and λ, ASN decreases as β increases;
2. At fixed values of α and β, ASN increases as λ increases;
3. At fixed values of β and λ, ASN decreases as α increases;
4. The acceptance constant and rejection constant are both larger than 1 given the quality levels of $(S_{AQL}, S_{LQL}) = (1.33, 1.0)$ and $(S_{AQL}, S_{LQL}) = (1.50, 1.33)$.

It is noted that the EWMA statistic considers the effect of accumulating the past sample sizes and the current one simultaneously. Therefore, the normal approximation even for a very small sample size can be justified.

Table 1. Parametric values of the proposed RGSP and various λ under $(S_{AQL},\ S_{LTPD}) = (1.33,\ 1.0)$.

		$\lambda = 0.1$				$\lambda = 0.3$				$\lambda = 0.5$				$\lambda = 1.0$			
α	β	n	k_a	k_r	ASN	n	k_a	k_r	ASN	n	k_a	k_r	ASN	n	k_a	k_r	ASN
0.010	0.010	3	1.2310	1.0316	4.69	12	1.2064	1.0609	15.55	21	1.2155	1.0483	29.33	49	1.2604	1.0028	95.03
	0.025	3	1.2044	1.0348	4.54	11	1.1846	1.0502	14.99	20	1.1908	1.0446	28.29	50	1.2167	1.0132	88.18
	0.050	3	1.1721	1.0359	4.40	10	1.1684	1.0376	14.45	19	1.1688	1.0381	27.39	50	1.1878	1.0155	84.60
	0.075	3	1.1575	1.0388	4.24	11	1.1393	1.0532	14.03	19	1.1487	1.0397	26.39	50	1.1669	1.0166	81.70
	0.100	3	1.1356	1.0400	4.06	10	1.1369	1.0396	13.60	20	1.1295	1.0480	25.70	50	1.1513	1.0174	79.07
0.025	0.010	3	1.2244	1.0771	3.72	9	1.2420	1.0597	12.22	16	1.2513	1.0488	23.10	49	1.2448	1.0535	68.88
	0.025	2	1.2507	1.0118	3.60	8	1.2203	1.0448	11.69	15	1.2235	1.0439	22.08	47	1.2137	1.0518	65.96
	0.050	2	1.2167	1.0161	3.42	7	1.2091	1.0269	11.19	14	1.1979	1.0360	21.09	44	1.1921	1.0452	62.84
	0.075	2	1.1917	1.0216	3.22	7	1.1835	1.0289	10.74	13	1.1856	1.0273	20.16	42	1.1750	1.0404	60.14
	0.100	2	1.1726	1.0245	3.08	7	1.1659	1.0321	10.28	14	1.1578	1.0419	19.37	40	1.1655	1.0336	58.27
0.050	0.010	2	1.2879	1.0486	2.98	7	1.2712	1.0607	9.78	13	1.2753	1.0538	18.78	39	1.2745	1.0573	55.47
	0.025	2	1.2432	1.0605	2.78	6	1.2557	1.0408	9.24	12	1.2524	1.0507	17.55	35	1.2514	1.0471	52.09
	0.050	2	1.2005	1.0680	2.62	6	1.2182	1.0498	8.63	11	1.2243	1.0435	16.36	34	1.2181	1.0489	49.08
	0.075	2	1.1783	1.0722	2.51	6	1.1928	1.0525	8.30	10	1.2125	1.0314	15.50	32	1.2009	1.0438	46.35
	0.100	2	1.1574	1.0743	2.42	5	1.2014	1.0237	7.89	10	1.1926	1.0346	14.87	31	1.1849	1.0416	44.28
0.075	0.010	2	1.2763	1.0837	2.57	6	1.2975	1.0624	8.50	11	1.3026	1.0549	16.16	34	1.2940	1.0625	48.13
	0.025	2	1.2383	1.0934	2.46	5	1.2820	1.0398	7.87	9	1.2916	1.0290	14.99	31	1.2652	1.0589	44.41
	0.050	2	1.1951	1.1002	2.34	5	1.2395	1.0523	7.26	9	1.2477	1.0423	13.72	25	1.2623	1.0268	41.25
	0.075	2	1.1704	1.1054	2.25	4	1.2513	1.0130	6.91	8	1.2391	1.0262	12.91	27	1.2177	1.0501	38.68
	0.100	2	1.1524	1.1056	2.19	4	1.2273	1.0192	6.53	8	1.2160	1.0310	12.29	24	1.2164	1.0317	36.79
0.100	0.010	2	1.2733	1.1085	2.38	5	1.3264	1.0520	7.57	10	1.3146	1.0642	14.29	28	1.3277	1.0483	43.11
	0.025	2	1.2295	1.1202	2.28	5	1.2767	1.0688	6.97	9	1.2892	1.0591	13.12	27	1.2851	1.0606	39.07
	0.050	2	1.1917	1.1277	2.18	4	1.2745	1.0377	6.33	8	1.2615	1.0501	11.93	23	1.2705	1.0408	35.83
	0.075	2	1.1666	1.1293	2.12	4	1.2430	1.0473	5.93	7	1.2553	1.0304	11.18	23	1.2375	1.0513	33.38
	0.100	2	1.1475	1.1348	2.04	4	1.2173	1.0539	5.61	7	1.2304	1.0383	10.54	21	1.2306	1.0374	31.72

Table 2. Parametric values of the proposed RGSP and various λ under $(S_{AQL},\ S_{LTPD}) = (1.50,\ 1.33)$.

		$\lambda = 0.1$				$\lambda = 0.3$				$\lambda = 0.5$				$\lambda = 1.0$			
α	β	n	k_a	k_r	ASN	n	k_a	k_r	ASN	n	k_a	k_r	ASN	n	k_a	k_r	ASN
0.010	0.010	18	1.4536	1.3634	23.90	49	1.4718	1.3446	80.55	49	1.5574	1.2568	285.07	–	–	–	–
	0.025	16	1.4466	1.3550	23.03	48	1.4534	1.3459	76.73	50	1.5245	1.2710	222.80	–	–	–	–
	0.050	14	1.4440	1.3456	22.21	46	1.4384	1.3441	73.37	50	1.5017	1.2791	189.58	–	–	–	–
	0.075	13	1.4314	1.3408	21.11	46	1.4262	1.3452	70.56	49	1.4880	1.2801	176.18	50	1.7658	0.9986	7770.81
	0.100	15	1.4144	1.3532	20.67	48	1.4150	1.3500	68.14	50	1.4733	1.2863	159.90	50	1.7456	1.0105	5901.13
0.025	0.010	12	1.4855	1.3489	19.16	45	1.4772	1.3595	64.36	49	1.5420	1.2917	157.11	–	–	–	–
	0.025	12	1.4659	1.3537	18.10	40	1.4637	1.3537	60.19	49	1.5119	1.3027	132.36	50	1.7917	1.0198	5249.46
	0.050	9	1.4711	1.3288	17.41	35	1.4546	1.3439	57.12	50	1.4869	1.312 6	116.47	50	1.7431	1.0503	2816.93
	0.075	11	1.4366	1.3505	16.32	33	1.4478	1.3410	54.46	50	1.4762	1.3154	110.75	50	1.7093	1.0711	1870.25
	0.100	10	1.4339	1.3436	15.58	32	1.4391	1.3401	51.98	49	1.4600	1.3170	102.44	50	1.6836	1.0878	1363.50
0.050	0.010	10	1.5053	1.3499	16.12	37	1.4882	1.3633	52.50	50	1.5296	1.3221	108.03	49	1.8229	1.0277	4195.87
	0.025	9	1.4854	1.3485	14.58	30	1.4860	1.3478	48.96	48	1.5037	1.329 3	94.26	50	1.7498	1.0797	1694.63
	0.050	9	1.4638	1.3546	13.52	31	1.4584	1.3583	44.91	50	1.4739	1.340 9	85.31	50	1.6982	1.1153	921.48
	0.075	8	1.4574	1.3465	12.65	30	1.4466	1.3581	42.60	45	1.4685	1.334 2	79.90	50	1.6658	1.1343	673.50
	0.100	7	1.4591	1.3357	12.00	23	1.4589	1.3330	40.22	50	1.4447	1.348 9	75.29	49	1.6471	1.1422	558.11
0.075	0.010	9	1.5143	1.3541	14.12	30	1.5112	1.3549	46.74	49	1.5280	1.3358	91.55	50	1.7875	1.0758	1841.45
	0.025	9	1.4807	1.3666	12.61	26	1.4970	1.3488	42.53	50	1.4936	1.3525	79.10	49	1.7343	1.1099	973.62
	0.050	7	1.4825	1.3460	11.52	22	1.4885	1.3401	38.12	48	1.4735	1.3561	72.32	49	1.6787	1.1467	552.28
	0.075	6	1.4825	1.3348	10.67	24	1.4629	1.3548	35.81	39	1.4774	1.3380	67.18	50	1.6451	1.1712	408.30
	0.100	7	1.4497	1.3556	10.09	20	1.4682	1.3382	33.65	41	1.4601	1.3478	63.60	49	1.6230	1.1791	344.18
0.100	0.010	9	1.5067	1.3687	12.63	29	1.5115	1.3646	42.08	49	1.5231	1.3508	80.01	50	1.7693	1.1039	1148.90
	0.025	7	1.5086	1.3500	11.45	25	1.4973	1.3602	37.50	49	1.4952	1.3635	71.61	50	1.7061	1.1490	575.27
	0.050	6	1.4947	1.3438	10.13	19	1.5026	1.3370	34.03	40	1.4883	1.3519	63.20	50	1.6570	1.1821	363.88
	0.075	6	1.4773	1.3506	9.39	18	1.4913	1.3378	31.36	36	1.4812	1.3460	58.39	50	1.6239	1.2011	282.56
	0.100	6	1.4651	1.3553	8.89	17	1.4793	1.3375	28.85	31	1.4833	1.3328	54.61	49	1.6023	1.2118	236.02

Note: (-) shows that plan parameters do not exist.

4. Designing of the proposed plan using multiple dependent state sampling

Most available sampling plans only consider the present state of a lot, that is, accepting or rejecting a lot is based on the present lot quality. To overcome this issue, Wortham [33] first introduced the concept of multiple dependent state sampling plan, which accepts or rejects a lot based on not only the quality of current lot but also the quality of preceding lots. This sampling plan can be used in case when lots are submitted serially. In this section, a Multiple Dependent State Sampling (MDSS) plan is proposed based on yield index by referring to [10]. The proposed procedure is given as below:

Step 1. Take a random size from the submitted lot at time t. Compute the sample mean $\overline{X}$ and the sample standard deviation S. Calculate the yield index $\widehat{S}_{pkt}$. Choose the smoothing constant $\lambda (0 < \lambda < 1)$ and compute the following EWMA yield index:

$$\widehat{S}_{pk}^{EWMA_t} = \lambda \widehat{S}_{pkt} + (1-\lambda)\widehat{S}_{pk}^{EWMA_{t-1}}.$$

Step 2. The decision about the submitted lot is made as follows:

(a) Accept the lot if $\widehat{S}_{pk}^{EWMA_i} \geq k_a$;

(b) Reject the lot if $\widehat{S}_{pk}^{EWMA_i} < k_r$;

(c) If $k_r \leq \widehat{S}_{pk}^{EWMA_t} < k_a$, then accept the lot when "i" preceding lots have been accepted under the condition of $\widehat{S}_{pk}^{EWMA_i} \geq k_a$. Otherwise, reject the lot.

This proposed plan is based on four plan parameters: sample size n, acceptance value k_a, rejection value k_r, and the number of preceding lots accepted i. This plan is a generalization of [31]. It reduces to [31] when $i = 1$.

The OC function $\pi_A^{MDS}(S_{pk})$ of the proposed plan is derived as follows:

$$\pi_A^{MDS}(S_{pk}) = P\{\widehat{S}_{pk}^{EWMA_t} \geq k_a\} + P\{k_r \leq \widehat{S}_{pk}^{EWMA_t} < k_a\} [P\{\widehat{S}_{pk}^{EWMA_t} \geq k_a\}]^i. \tag{11}$$

Based on Eqs. (5) and (6), Eq. (11) can be rewritten as follows:

Table 3. Parametric values of the proposed MDSS with $i = 2$ and various λ under $(S_{AQL},\ S_{LTPD}) = (1.33,\ 1.0)$.

		$\lambda = 0.1$			$\lambda = 0.3$			$\lambda = 0.5$			$\lambda = 1.0$		
α	β	n	k_a	k_r	n	k_a	k_r	n	k_a	k_r	n	k_a	k_r
0.010	0.010	7	1.1408	1.0470	24	1.1425	1.0308	44	1.1413	0.8150	118	1.0896	0.2163
	0.025	7	1.1232	0.4290	20	1.0921	0.3687	40	1.1303	1.1074	106	1.0718	0.3919
	0.050	6	1.1078	0.9677	18	1.1143	0.3131	34	1.1140	0.3586	97	1.0740	0.6892
	0.075	5	1.1057	0.8330	17	1.1069	0.3868	31	1.1046	0.9396	83	1.0476	0.4856
	0.100	5	1.0831	0.3859	15	1.0686	0.4479	24	1.0433	0.5578	83	1.0652	0.6948
0.025	0.010	6	1.1538	0.2089	19	1.1116	0.3038	37	1.1566	0.6270	94	1.0977	0.7688
	0.025	5	1.1412	0.5697	17	1.1399	0.2905	28	1.0870	0.6040	93	1.0756	0.2602
	0.050	5	1.1203	0.3403	15	1.1247	0.2978	28	1.1253	0.8013	76	1.0720	0.5543
	0.075	4	1.1164	0.2359	13	1.1171	0.5982	25	1.1187	1.0925	72	1.0756	0.3059
	0.100	4	1.1040	1.0221	13	1.1105	0.5218	23	1.1079	0.7568	66	1.0500	0.7414
0.050	0.010	5	1.1713	0.4557	17	1.1176	0.6720	31	1.1056	0.5222	81	1.1155	0.8023
	0.025	4	1.0868	0.7080	14	1.1579	1.1218	26	1.1559	0.4821	72	1.1062	0.2422
	0.050	4	1.1346	0.5305	12	1.1396	1.0217	23	1.1421	0.4871	66	1.1412	0.8882
	0.075	4	1.1155	0.3401	9	1.0663	0.3381	20	1.1319	1.0018	60	1.1305	1.1083
	0.100	3	1.1192	0.4539	9	1.0679	0.4897	17	1.0634	0.3570	55	1.1207	0.7415
0.075	0.010	5	1.1668	0.7052	15	1.1784	0.9264	24	1.1166	0.8453	74	1.1145	0.4362
	0.025	4	1.0970	0.2206	12	1.1665	1.1513	23	1.1648	0.5036	65	1.1200	0.8358
	0.050	3	1.1521	1.0901	11	1.1555	1.1057	20	1.1485	0.3481	57	1.1522	0.2263
	0.075	3	1.1478	0.6586	9	1.1408	1.0988	16	1.0692	0.6708	51	1.1406	0.7873
	0.100	3	1.1397	0.9972	9	1.1346	0.2394	13	1.0586	0.6134	46	1.1319	1.1207
0.100	0.010	4	1.1893	0.2224	13	1.1890	0.5144	25	1.1221	0.4619	67	1.1176	0.5009
	0.025	4	1.0984	0.9058	11	1.1766	0.3300	21	1.1726	0.2567	61	1.1760	0.4242
	0.050	3	1.1530	0.9804	8	1.0821	0.7265	17	1.1607	1.0693	51	1.1611	0.7972
	0.075	3	1.1337	0.6054	8	1.1512	0.9869	15	1.1520	0.4270	45	1.1518	1.0959
	0.100	3	1.1113	0.3142	8	1.1462	0.7625	14	1.1409	1.1138	41	1.1408	0.8490

$$\pi_A^{MDS}(S_{pk}) = \left(1-\Phi\left(\frac{k_a - S_{pk}}{\sqrt{[\lambda/(2-\lambda)]\,(a^2+b^2)\{36n\phi^2(3S_{pk})\}^{-1}}}\right)\right) + \left\{\Phi\left(\frac{k_a - S_{pk}}{\sqrt{[\lambda/(2-\lambda)]\,(a^2+b^2)\{36n\phi^2(3S_{pk})\}^{-1}}}\right) - \Phi\left(\frac{k_r - S_{pk}}{\sqrt{[\lambda/(2-\lambda)]\,(a^2+b^2)\{36n\phi^2(3S_{pk})\}^{-1}}}\right)\right\} \left[1-\Phi\left(\frac{k_a - S_{pk}}{\sqrt{[\lambda/(2-\lambda)](a^2+b^2)\{36n\phi^2(3S_{pk})\}^{-1}}}\right)\right]^i. \quad (12)$$

For specified values of i, the three plan parameters of the proposed plan can be obtained using the following non-linear optimization model:

$$MIN_{n,k_a,k_r} = n, \quad (13a)$$

Subject to:

$$\pi_A^{MDS}(S_{AQL}) \geq 1-\alpha, \quad (13b)$$

$$\pi_A^{MDS}(S_{LQL}),\ 0 < k_r < k_a. \quad (13c)$$

Based on Eqs. (13a) to (13c), the plan parameters of the proposed plan are given in Tables 3 and 4. It is noted the behavior of plan parameters in Tables 3 and 4 is the same as those in Tables 1 and 2. The codes are shown in a supplementary file.

5. Advantages of proposed plans

In this section, we will compare the proposed plans with the existing plans in terms of ASN. To justify the performance of the proposed plans, we will set the same values of specified parameters for all sampling plans. It is noted that a sampling plan which provides the smaller values of ASN is considered to be more efficient.

5.1. The proposed RGSP vs [31] plan

First, we compare the efficiency of the proposed RGSP with the plan developed by Yen et al. [31]. For

Table 4. Parametric values of the proposed MDSS with $i = 2$ and various λ under $(S_{AQL},\ S_{LTPD}) = (1.50, 1.33)$.

		$\lambda = 0.1$			$\lambda = 0.3$			$\lambda = 0.5$			$\lambda = 1.0$		
α	β	n	k_a	k_r	n	k_a	k_r	n	k_a	k_r	n	k_a	k_r
0.010	0.010	35	1.3910	0.6271	130	1.4099	0.5094	240	1.3750	0.4431	–	–	–
	0.025	29	1.3711	0.4445	101	1.3833	0.3868	178	1.3778	1.1928	564	1.3817	0.4521
	0.050	27	1.3717	0.2591	97	1.3959	0.3900	156	1.3636	0.7519	419	1.3660	0.6833
	0.075	27	1.3714	0.4755	88	1.3902	0.7449	131	1.3608	1.1404	414	1.3635	0.6965
	0.100	25	1.3856	0.3119	82	1.3854	1.1077	131	1.3622	0.7275	384	1.3549	0.3745
0.025	0.010	33	1.4174	0.7593	107	1.3947	0.6782	190	1.3815	0.6351	505	1.3941	0.5732
	0.025	28	1.4105	1.0627	93	1.4103	0.7763	160	1.3772	1.1674	452	1.3742	0.6754
	0.050	21	1.3770	1.1337	79	1.4025	0.8973	140	1.3688	0.3110	365	1.3745	0.5507
	0.075	19	1.3612	0.6385	71	1.3972	0.3064	135	1.3974	0.5162	365	1.3699	1.0159
	0.100	18	1.3571	0.4443	65	1.3925	0.5371	122	1.3925	0.6613	288	1.3572	0.7418
0.050	0.010	28	1.4248	1.1738	76	1.3913	0.7866	176	1.4244	1.3386	436	1.3945	0.6401
	0.025	24	1.4167	0.6921	75	1.3924	0.4058	135	1.3852	0.5877	337	1.3870	0.6285
	0.050	20	1.4101	0.9152	65	1.4099	0.5746	121	1.3876	1.0446	299	1.3782	0.4950
	0.075	18	1.4027	0.3582	58	1.4043	1.0083	102	1.3793	0.5640	283	1.3661	0.9708
	0.100	16	1.3991	0.7180	53	1.3997	0.7612	100	1.3995	1.1916	254	1.3603	0.3850
0.075	0.010	25	1.4042	1.0690	75	1.4015	0.8104	142	1.3983	0.8159	364	1.3947	1.1042
	0.025	21	1.4223	0.2300	67	1.3972	0.9966	111	1.3978	0.2840	356	1.3942	0.2860
	0.050	17	1.4157	0.4025	57	1.4153	0.5212	97	1.3869	0.5452	285	1.3834	0.2813
	0.075	15	1.4099	1.1529	50	1.4103	0.7559	92	1.3821	0.5867	232	1.3675	0.7466
	0.100	14	1.3657	0.6306	45	1.4049	0.7363	86	1.4058	0.9944	193	1.3628	1.1439
0.100	0.010	23	1.4349	0.5255	76	1.4350	0.7251	114	1.4027	0.5687	325	1.4031	0.2773
	0.025	18	1.4008	0.8426	51	1.3940	0.5098	112	1.3842	0.8445	273	1.3942	1.1308
	0.050	16	1.4190	0.3956	51	1.4203	0.9728	96	1.3968	1.1037	256	1.3772	1.0291
	0.075	14	1.4152	0.9481	45	1.4153	1.2424	84	1.4152	0.7762	222	1.3845	1.0823
	0.100	12	1.4102	0.3612	40	1.4094	0.2601	75	1.4099	1.3538	203	1.3672	0.5838

Note: (-) shows that plan parameters do not exist.

comparison, we selected $\lambda = 0.1,\ 0.3$ and $(S_{AQL}, S_{LQL}) = (1.33, 1.0)$. The values of ASN for both sampling plans are shown in Table 5.

From Table 5, we see that the proposed RGSP provides smaller ASN than the plan in [31]. For example, given $\alpha = 0.010$, $\beta = 0.010$, and $\lambda = 0.1$, ASN of the proposed RGSP is 4.25, while that of the existing plan is 7. Thus, the proposed plan is more efficient than the sampling plan in [31].

5.2. The proposed RGSP vs traditional RGSP

Now, we will compare the efficiency of the proposed RGSP with memoryless (traditional) sampling plan. The proposed plan reduces to traditional RGSP when $\lambda = 1$. We selected the same values of all the parameters for this comparison. For comparison, we selected $\lambda = 0.1, 0.3$ and $(S_{AQL}, S_{LQL}) = (1.33, 1.0)$. The values of the ASN for both plans are shown in Table 6.

From Table 6, we see that the proposed RGSP provides smaller ASN than the traditional RGSP plan. For example, given $\alpha = 0.010$, $\beta = 0.010$, and $\lambda = 0.1$, ASN of the proposed RGSP is 4.25, while that of the existing plan is 81.74. Thus, the proposed plan is more efficient than the traditional RGSP plan.

5.3. The proposed MDSS plan vs [31]

To compare the efficiency of the proposed MDSS plan

Table 5. Comparison of the proposed RGSP with existing plan [31].

α	β	$\lambda = 0.1$		$\lambda = 0.3$	
		Existing	**Proposed**	**Existing**	**Proposed**
0.010	0.010	7	4.25	24	14.22
	0.025	7	3.98	21	13.24
	0.050	6	3.82	18	12.51
	0.075	5	3.69	17	12.00
	0.100	5	3.59	16	11.62

Table 6. Comparison of the proposed RGSP with traditional RGSP.

α	β	$\lambda = 0.1$		$\lambda = 0.3$	
		Existing	**Proposed**	**Existing**	**Proposed**
0.010	0.010	81.74	4.25	81.74	14.22
	0.025	75.63	3.98	75.63	13.24
	0.050	71.02	3.82	71.02	12.51
	0.075	68.02	3.69	68.02	12.00
	0.100	65.69	3.59	65.69	11.62

Table 7. Comparison of proposed MDSS plan with existing plan [31].

α	β	$\lambda = 0.1$		$\lambda = 0.5$	
		Existing	**MDSS plan**	**Existing**	**MDSS plan**
0.010	0.010	39	25	245	240
	0.025	34	29	210	178
	0.050	29	27	182	156
	0.075	27	27	165	131
	0.100	25	25	153	131

with that of the existing plan [31], we again set the same values of all plan parameters for all sampling plans. The plan parameters of both the proposed sampling plans are shown in Table 7. For comparison, we selected $\lambda = 0.1, 0.5$ and $(S_{AQL}, S_{LQL}) = (1.50, 1.33)$.

For example, given $\alpha = 0.010$, $\beta = 0.025$, and $\lambda = 0.5$, ASN of the proposed MDSS plan is 178, while that of the existing plan is 210. Thus, the proposed MDSS plan is more efficient than the existing plan.

6. Applications in industry

We illustrate the application of the proposed plan with the help of color STN displays data where each pixel is divided into R, G, and B sub-pixels [31]. Suppose that the membrane thickness of each pixel is variable of interest. The engineer wants to inspect the quality of color STN displays using the proposed plans. The target value T, specification limits USL, and LSL of membrane thickness are set as $T = 12000$Å, $USL = 12500$Å, and $LSL = 11500$Å, respectively. In the contract, suppose $\alpha = 0.05$, $\beta = 0.10$, $\lambda = 1.0$, and $(S_{AQL},\ S_{LQL}) = (1.33, 1.0)$. Based on the predetermined values, if the proposed MDSS plan with two accepted preceding lots is applied, we find that the corresponding parameters (n, k_a , k_r) of the sampling plan are (55, 1.1207, 0.7415) from Table 3. Hence, the 55 inspected samples will be taken from one lot randomly to make lot sentencing. The sample data used are the same as those in [31].

Yen et al. [31] demonstrated that the data were well fitted to a normal distribution with the help of a normal probability plot. From the sample data in [31], we can obtain $\overline{X} = 11715.2, S = 49.21$ and the calculated value of S_{pk} is 1.5072. Assume that $\widehat{S}_{pk}^{EWMA_{i-1}} = 1.1052$; then, $\widehat{S}_{pk}^{EWMA_i}$ is calculated as 1.5072 with $\lambda = 1.0$.

Therefore, the lot will be accepted by the consumer since the value of $\widehat{S}_{pk}^{EWMA_i}$, 1.5072, is larger than the critical acceptance value 1.1207 significantly.

7. Conclusion remarks

This study designed two sampling plans for the quality characteristic of interest with normal distribution. The necessary measures for both sampling plans were presented. A detailed study was provided to demonstrate the advantages of the proposed plans. From a comparative study, we concluded that the proposed RGSP plan outperformed other plans using single sampling, repetitive sampling, and the proposed MDSS plans. To illustrate the application of the proposed plan, color STN displays data of a real case were presented. The proposed plans could be applied to circumstances with expensive inspections. For scope of application in industries, they can be applied in the mobile industry, automobile industry, electronic industry, and so on. For the direction of future research, the proposed plan using a cost model can be considered. Also, this study can be extended for the quality characteristic of interest with non-normal distribution.

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Supplementary data is available at:

file:///C:/Users/SHAMILA/AppData/Local/Temp/Supplementry%20file.pdf

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Biographies

Muhammad Aslam introduced the area of Neutrosophic Statistical Quality Control (NSQC) for the first time. He is the founder of Neutrosophic Inferential Statistics (NIS) and NSQC. His contribution is the development of neutrosophic statistics theory for the inspection, inference, and process control. He originally developed the theory in these areas under Neutrosophic Statistics.

Muhammad Azam holds his Master's degree in Statistics from Islamia University Bahawalpur in 1996 with distinction (Gold Medalist). He completed his MPhil from QAU, Islamabad in 2006 and PhD from University of Innsbruck Austria in 2010. He has been involved in teaching for various institutes for the last 21 years. He started his career as a lecturer in Statistics from Punjab Education Department and served there for 13 years. In 2010, he joined the Forman Christian College University Lahore as an Assistant Professor and served there for five years. In 2015, he joined as an Associate Professor and the Chairman of the Department of Statistics and Computer Science, UVAS, Lahore. On January 04, 2018, Dr. Azam was selected as a Professor of Statistics and he also continued working as the Chairman of the Department till March 13, 2018. On March 14, 2018, he was assigned the responsibility as the Dean of Faculty of Life Sciences Business Management (FLSBM). He has published more than 80 research articles mostly published in impact factor international journals. He has attended a number of national and international conferences/workshops. His research interests include survey sampling, statistical quality control, decision trees, and ensemble classifiers. He has produced 25

MPhil students. Currently, 4 PhD and 4 MPhil research students are working under his supervision.

Rehan Ahmad Khan Sherwani was born in Lahore, Pakistan in 1981. He received the Master's degree in Statistics and the PhD degree from the University of the Punjab, Pakistan. He is currently working as an Assistant Professor at the College of Statistics, University of the Punjab, Lahore. He has numerous publications in peer-reviewed national and international research journals. His areas of specialization include regression analysis, multilevel models, structural equation models, mixed models, and their applications. He is a member of Boards of Studies of University of the Punjab and GC University Faisalabad, Pakistan. He has also worked as a member of Punjab Technical Committee for Census 2017; a member of Dean of the Faculty of Science, Purchase Committee; a Focal Person of QEC Faculty of Science, University of the Punjab; a Coordinator of MSc Biostatistics Programme, and a Coordinator of MSc Business Statistics and Management Programme. He is also a member of National Curriculum Review Committee by HEC for the subject of Statistics.

Ching-Ho Yen received the MS degree in Statistics from National Tsing Hua University in 2000, and the PhD degree in Industrial Engineering and Management from National Yang Ming Chiao Tung University in 2007. He is currently a Professor at the Department of Industrial Engineering and Management Information, Huafan University, Taipei, Taiwan. He is interested in statistics, quality control, and data mining.

Chi-Hyuck Jun received the BS degree in Mineral and Petroleum Engineering from Seoul National University, the MS degree in Industrial Engineering from the Korea Advanced Institute of Science and Technology, and the PhD degree in Operations Research from the University of California at Berkeley. Since 1987, he has been with the Department of Industrial and Management Engineering, Pohang University of Science and Technology (POSTECH), where he is currently a Professor Emeritus at the Department of Industrial and Management Engineering. He is interested in data mining and reliability/quality.