A possibilistic programming approach for biomass supply chain network design under hesitant fuzzy membership function estimation

Hossein Gitinavard [a], Mohsen Akbarpour Shirazi [b, *], Mohammad Hossein Fazel Zarandi [c]

[a] *Department of Industrial Engineering and Management Systems, Amirkabir University of Technology, 424 Hafez Ave., Tehran, Iran*

(gitinavard@aut.ac.ir, (+98)21-64545370)

[b] *Department of Industrial Engineering and Management Systems, Amirkabir University of Technology, 424 Hafez Ave., Tehran, Iran*

(akbarpour@aut.ac.ir, (+98)21-64545370)

[c] *Department of Industrial Engineering and Management Systems, Amirkabir University of Technology, 424 Hafez Ave., Tehran, Iran*

(zarandi@aut.ac.ir, (+98)21-64545390)

* Corresponding author: Mohsen Akbarpour Shirazi (mobile number: (+98)912-2198433)

Abstract

The recognition of membership function by knowledge acquisition from experts is an important factor for many fuzzy mathematical programming models. Meanwhile, hesitant fuzzy set theory as a known and popular modern fuzzy set by assigning some discrete membership degrees under a set could appropriately deal with imprecise information in decision-making problems. Thus, the hesitant fuzzy membership function (HFMF) estimation could help users of the mathematical programming approaches to provide a powerful solution in continuous space problems. Therefore, this study proposes a possibilistic programming approach based on Bezier curve mechanism for estimating the HFMF. In the process of possibilistic programming approach, an optimization model is presented to tune the primary parameters of Bezier curve by the goal of minimizing the sum of the squared errors (SSE) between the empirical data and fitted HFMF. After that, the efficiency and applicability of the proposed approach is checked by proposing a novel mathematical model for biomass supply chain network design problem. Finally, a computational experiment and validation procedure about the biomass supply chain network design is provided to peruse the verification and validation of the proposed approaches.

Keywords: Hesitant fuzzy set theory, Preference-based characteristic function, Bezier curve-based mechanism, Membership function estimation, Biomass supply chain network design.

1. Introduction

Membership function estimation from the training data and knowledge acquisition is one of the crucial issues concerning with the fuzzy set theory application [1]. Hence, there are no rules or guidelines that can be considered to select a suitable membership function generation method [2]. Consequently, several methods such as heuristic methods [3, 4], probabilistic technique [5], statistical data based technique [6], classification approaches [7], artificial neural network-based methods [8], and computer-aided design [9] have been proposed to construct the membership function, suitably. Thereby, there is a lack of consensus on the single best method to apply for generating the membership function. In this sake, the choice of the technique relates to the particular problem and every technique has the capability and potential underlying the theory or assertion [10, 11].

Meanwhile, the heuristic approaches are inflexible; they cannot utilize training data set that leads the applicability of these approaches to limited, particularly in high dimensions. Besides, the probabilistic techniques appropriately work in some conditions in which the possibility distribution for each object is known. Hence, the statistical data-oriented approaches as well as classification methods required a lot of known and precise information/data to establish smooth membership function. Moreover, artificial neural network-based methods can be used in case of supervised and unsupervised learning algorithms in which the supervised learning algorithms need the labeled training data set, and the unsupervised learning algorithms have complicated procedure to implement [10, 12].

However, the review of the tailored membership function generation techniques indicates that it is worthwhile to exploit the membership function based on an appropriate technique which has the following desirable characteristics:

1. *Accurate*: The membership function should be found to deal with both a vague situation and ambiguity condition. Meanwhile, the lack of appropriate information/data and qualitative evaluation lead to the vague situation in which the ambiguity occurs when the experts are doubtful about their judgments.
2. *Flexible*: In the procedure of extending the membership function, a wide range of available membership functions such as triangular, sigmoidal, trapezoidal, and Gaussian can be considered.
3. *Data preservation*: The membership function should be established based on both data information and knowledge acquisition which can be achieved by an expert's preference based on Delphi or brainstorming frameworks.
4. *Computationally affordable*: The provided approach should be easily extended, adjusted, and tuned regarding the computer applications. Furthermore, computer graphics can be utilized to facilitate the procedure of membership functions estimation by allowing users to direct and easy manipulation of various shapes [13].
5. *Easy to use*: Once a new membership function has been established, it should be easy for a given x to find $\mu_{\tilde{E}}(x)$.

Consequently, this study considers Bezier curve and surface theory from the computer-aided design to handle the aforementioned characteristics in an efficient and practical way of membership function estimation under hesitant fuzzy set environment. In this case, generating a hesitant fuzzy membership function (HFMF) could help the users for solving the mathematical programming model to deal with what imprecise information that experts are suspicious about their preferences. Furthermore, a possibilistic mathematical programming model by the aims of minimizing the sum of the squared errors (SSE) between the empirical data, and fitted HFMF is proposed based on a possibilistic programming approach to tune and adjust the control points of Bezier curve mechanism.

Hence, the estimated HFMF is utilized to address with uncertain demand parameter of the proposed mathematical programming model for biomass supply chain network design problem. In sums, the merits and novelties of this paper are explained as follows: (1)

develops the Bezier curve mechanism based on hesitant fuzzy information to formulate the computational procedure of HFMF estimation; (2) proposes a possibilistic mixed integer programming model to adjust the control points of Bezier curve mechanism; (3) proposes a novel mathematical programming model for multi-feedstock multi-bioproduct supply chain network design under disruption risks to check the verification of the presented process of HFMF estimation; (4) provides computational experiments and comparative analysis about the two-echelon multi-product supply chain network design by focuses on multi-feedstock to peruse the validation of the elaborated approach.

The rest of this study is organized as follows. In section 2, the review of the literature about the elaborated approaches for membership function generation and the survey on biomass supply chain network design problem are provided. In section 3, the proposed mathematical programming model regarding the problem description of two-echelon biomass supply chain network design is presented. Moreover, the process of the proposed Bezier curve-based possibilistic programming approach is explained in section 4, in details. In this section, the basic definition and properties for Bezier curve are defined, and then the mathematical and possibilistic programming frameworks are proposed. In section 5, the proposed mathematical programming model for multi-feedstock multi-bioproduct supply chain network design problem is implemented to a computational experiment to represent the verification of this model. Moreover, a comparative analysis is considered to ensure the validity of the proposed approaches. Finally, some concluding remarks and future works/suggestion are described in section 6.

2. Literature review

In this section, the review of the literature about the manipulated methods for membership function generation is provided, firstly. In addition, the proposed approach has been developed based on hesitant fuzzy setting information that its literature is reviewed. Furthermore, the proposed possibilistic programming mechanism that is presented in this study is applied to a real case of biomass supply chain network design to show its validity. Consequently, a survey of the biomass supply chain network design literature is done to represent the novelty of the proposed mathematical programming model.

2.1. Background of membership function generation

Membership functions estimation/generation can be done objectively or subjectively. In case of an objectively approach, some incomplete data points exist such that the membership function can be estimated by fitting a fine-tune curve to the incomplete data set. In the case of subjectively approach, the incomplete data points in the form of membership values pairs have not existed, or some qualitative variables must be evaluated. Since this case of membership function estimation is based on experts' judgments, it can be named preference-based characteristic function. Hence, the presence of experts or value-membership pairs can be elicited in an interacting way with experts [14, 15]. Thereby, a membership function that established underlying this case, and then its boundary parameters are tuned based on an optimization approach is defined as possibilistic programming membership function estimation [16].

However, some efforts have been made to generate the membership function to address with incomplete information. Meanwhile, Zimmermann and Zysno [17] presented empirical research by focusing on vagueness modeling problem, in which the vague concepts were quantitatively represented by determining their membership functions. Chen and Otto [18] extended an efficient and simple constrained interpolation scheme based on measurement theory to appropriately fit a membership function to a set of known value-membership pairs. In this sake, Marchant [19] checked the trapezoidal membership function that it prepared the knowledge of experts based on some measurement-theoretic axiomatization conditions. Huynh et al. [20] elaborated an integrated approach based on context models and modal logic to estimate the membership functions of fuzzy expressions. Chen and Tsai [21] presented a methodology to generate the membership function to address with Iris data classification problem. In their study, the fuzzy rules were elicited from training instance sets based on the boundary shift value, correlation coefficient threshold, and center shift value.

Furthermore, Sanchez et al. [22] presented an integrated framework based on c-means algorithms and artificial neural networks to generate membership functions for fuzzy systems. Sami and Badie [23] gave a flexible framework based on meta-function and case-based reasoning process to create the fuzzy membership functions. Jain and Khare [24] defined a Bezier curve mechanism to establish the membership functions for vehicular and meteorological parameters of urban vehicular exhaust emissions modeling problem. Moreover, Bouhentala et al. [25] proposed an integrated approach based on clustering Gustafson–Kessel algorithm and envelope detection algorithm to provide the local linearization and interval-valued membership function estimation, respectively.

The literature review shows that an uncertainty modeling based on mathematical optimization model for dealing with vagueness and hesitant conditions is not presented by researchers regarding the best of our knowledge. To address the issue, this study extended the Bezier curve mechanism based on hesitant fuzzy set theory to establish a HFMF. Furthermore, possibilistic programming mixed integer programming approach is presented to optimally adjust the control points of Bezier curve mechanism.

2.2. Overview of hesitant fuzzy set theory

The HFSs theory among the other fuzzy sets theories has been presented as a helpful tool by considering some membership degrees that are defined by experts under a set to cover all aspects of hesitant or ambiguous situations [26]. Thereby, Wu et al. [27] extended an integer mathematical programming approach based on additive consistency to predict missing values of incomplete hesitant fuzzy linguistic preference relations. Wan et al. [28] manipulated a hesitant fuzzy mathematical programming model for integrated group decision analysis with incomplete criteria weight information and hesitant fuzzy truth degrees. In addition, Song and Li [29] presented a group decision analysis-based hesitant fuzzy linguistic preference relations and mathematical programming method.

Furthermore, Li and Wang [30] elaborated a mathematical programming model based probabilistic hesitant fuzzy preferences distance measure to compute the consistency index and recognizing the missing values during the decision making process. Zhang et al. [31] presented a multiplicative consistency-based interval-valued hesitant fuzzy preference relation and mathematical programming model to optimize the consensus and consistency of group decision procedure. Hence, Rashid and Sindhu [32] proposed a linear programming model based on interval-valued hesitant fuzzy information to compute the unknown criteria weights. Wei et al. [33] developed a linear assignment model to compute the optimal preference candidates ranking regarding a set of criteria importance and criteria-wise rankings based on the Hesitant Euclidean distance and HFSs information.

The literature review of hesitant fuzzy set theory in field of mathematical programming model represents that the proposed approaches are focused on discrete solution space that are utilized the distance measures to reach a precise value among the several HFSs. Moreover, all studies have been developed their mathematical programming model for multi-criteria decision-making problems. Therefore, assigning a set of hesitant fuzzy membership degrees for a wide range of homogenous values such as demand parameter, required a HFMF. This membership function could help the mathematical programming developers to cover all aspects of their imprecise problem with hesitancy degrees recognition. However, this study presents a possibilistic mixed integer programming model based on Bezier curve mechanism and the hesitant fuzzy subjective/objective information to predict a suitable HFMF.

2.3. Review of biomass supply chain network design

The concerns about the climate changes, high energy demand, volatile fuel price, resource depletion, food crisis, global warming, air pollution, and global economic turbulence have forced the countries focused on clean sources and renewable energies [34, 35]. Meanwhile, bioethanol and biodiesel are the most commonly utilized liquid biofuels that can be regarded as a suitable substitutions for gasoline and fossil fuels, respectively [36]. Therefore, efficient design of biomass supply chain network can play an important role to enhance the competencies of biofuels via fossil fuels such as its performance, customer satisfaction, productivity and responsiveness, coordination of consecutive echelons, and environmental impact reduction [37]. Hence, many authors have implemented various methodologies including geographic information system, simulation, and mathematical programming model [38].

Hence, Bai et al. [39] proposed both cooperative and non-cooperative Stackelberg leader-follower game frameworks in bi-level approach to cope with possible business partnership scenarios among biofuel manufacturers and feedstock suppliers. Tong et al. [40] presented a robust mixed-integer linear fractional mathematical programming model to provide production planning, integration strategy selection, and biofuel supply chain network design, simultaneously. Moreover, Li and Hu [41] elaborated a two-stage stochastic mathematical programming model to design decentralized bio-oil gasification supply chain by the goal of maximizing the biofuel producers' annual profit. Mohseni and Pishvaee [42] presented a robust programming model by the aim of cost reduction to specify the optimal tactical and strategic decisions of microalgae-based supply chain network design.

Furthermore, Ghaderi et al. [35] elaborated a multi-objective robust possibilistic nonlinear programming approach with social and environmental life cycle perspectives to design switchgrass-based bioethanol supply chain network. Kesharwani et al. [43] manipulated a multi-objective mixed integer nonlinear programming model to develop a four-layer biofuel supply chain towards sustainability aspects. Besides, Fattahi and Govindan [44] proposed a multi-stage mathematical programming model to sustainable biofuel supply chain design under stochastic environment and disruption risk. However, interested readers can refer to an in-depth review of the biofuel supply chain network design that is represented by Ghaderi et al. [36].

The survey of the literature indicates the gaps in biomass supply chain network design that the aim of this study fills the gaps. Meanwhile, this paper presents a novel possibilistic mathematical programming model for the multi-feedstock multi-bioproduct supply chain network design problem under imprecise information and disruption risks. This proposed approach considers the wide range of feedstocks as first, second, and third generations of biomass to produce the different types of bio-products demands. Thereby, the limited cultivation area and the harvesting site locations are depended to disruption risks such as floods, wildfires, etc.

3. Problem definition and formulation

In this section, the problem definition for two-echelon biofuel supply chain network design is explained. Then, the proposed mathematical model is presented regarding the assumptions.

3.1.Problem description

In this section, the scope of the problem is explained to represent the specific and unique features of the multi-feedstock multi-bioproduct supply chain network design problem under disruption risks. As indicated in Figure 1, a two-echelon biomass supply chain network including harvesting sites (*h*), biorefineries (*i*), and customers (*m*) is provided in which the amount of the harvested area (CJ_{hbp}) for feedstock cultivation (*b*) is limited. In addition, the amount of stored feedstock at biorefinery (Sb_{ibpt}) is concerned with harvested feedstock (H_{hbpt}) in terms of disruption risks (RS_{rht}). The biomass supply chain may be disrupted regarding the flood, wildfires, hurricanes, etc. which could destroy some parts of harvesting sites. To address the issue, each season is provided as a time period, and the problem is planned for a one-year time horizon. Then, these risks could affect processed feedstock (w_{ibpkt}), bio-product production (*p*), and consequently the demand satisfaction (D_{mpt}). Meanwhile, the harvesting site location should be installed (x_{hbp}) where the risk of natural disasters is low that could affect the installation location of biorefineries ($y_{ikll'}$) with an optimal capacity of storage (CB_{ilt}) and production ($CP_{ikl't}$). These capacities of the biorefinery can be expanded for feedstock storage (CEB_{ilt}) and bio-products production ($CEP_{ikl't}$) by changing the currently established devices such as pipes, furnace, pumps, and new production line installation. Therefore, a large amount of bio-products (ep_{ipt}) is produced concerning with conversion rate of feedstocks to bio-products by a specific technology (β_{bpk}). However, some main assumptions regarding the real situations and reasonable assumptions that are defined in literature are inspired to simplify the proposed mathematical programming model. Hence, these assumptions are defined as follows:

- The studied biomass supply chain is a two-echelon, multi-period, multi-feedstock, and multi-product.
- The shortage is not allowable, and all bio-product demands must be supplied in each period.
- The bio-product demands are considered as uncertain information under hesitant fuzzy environment.
- The customer zones for bio-products are fixed and known.

– The candidate locations for harvesting sites and biorefineries are known, but the most suitable location between them should be optimally specified regarding the proposed mathematical programming model.
– All feedstocks regarding their conversion rate can be used for producing the bio-products.
– Required capacities of harvesting feedstock, stored feedstock, and produced bio-product are not fixed and computed in each period regarding the considered continuous decision variables.
– Transportation of bio-products is performed via a single transportation mode.
– The yields of feedstock are not related to the time period to prevent the continuous yielding of feedstocks.

{Please insert here Figure 1}

3.2. Model formulation

In this section, the proposed mixed integer mathematical programming model is formulated for multi-feedstock multi-bioproduct supply chain network design problem. Therefore, the considered sets, parameters, binary and positive variables are explained as follows:

Sets

i	Index of biorefinery sites location; $\left(i=1,2,...,I\right)$
h	Index of harvesting sites location; $\left(h=1,2,...,H\right)$
m	Index of bio-product customers; $\left(m=1,2,..,M\right)$
k	Index of conservation technologies; $\left(k=1,2,..,K\right)$
r	Index of disruption risks types; $\left(r=1,2,..,R\right)$
l	Index of biorefinery capacities for bio-products production; $\left(l=1,2,..,L\right)$
l'	Index of biorefinery capacities for feedstock storage; $\left(l'=1,2,..,L'\right)$
b	Index of feedstock types; $\left(b=1,2,..,B\right)$
p	Index of bio-products; $\left(p=1,2,..,P\right)$
t	Index of time period; $\left(t=1,2,..,T\right)$

Parameters

λ_{hbpt}	Rate of harvested feedstock *b* for producing the product *p* at location *h* in period *t*
β_{bpk}	Conversion rate of feedstock *b* for producing bio-product *p* with conversion technology *k*
α_b	Determination rate of feedstock *b*
RS_{rht}	The risk priority number of disruption risk type *r* for location *h* in period *t* where $RS_{rht}=Pos_{rht}El_{rht}$ in which Pos_{rht} is the possibility of occurred accident and El_{rht} is the expected loss of the accident
D_{mpt}	Demand of costume *m* for bio-product *p* in period *t*

LA_{hbp}	Minimum farm area allocated for harvesting feedstock *b* to produce bio-product *p* at location *h*
UA_{hbp}	Maximum farm area available for harvesting feedstock *b* to produce bio-product *p* at location *h*
LB_{il}	Lower bound of storage capacity for biorefinery with size *l* at location *i*
UB_{il}	Upper bound of storage capacity for biorefinery with size *l* at location *i*
$LP_{ikl'}$	Lower bound of production capacity for biorefinery with size l' and conversion technology *k* at location *i*
$UP_{ikl'}$	Upper bound of production capacity for biorefinery with size l' and conversion technology *k* at location *i*
$FCI_{ill'}$	Fixed cost of opening biorefinery at location *i* with storage capacity *l* and production capacity l'
FCH_{hbp}	Fixed cost of harvesting for feedstock *b* to produce bio-product *p* at location *h*
VCB_{il}	Variable cost per unit storage capacity for biorefinery *i* with size *l*
$VCP_{ipkl'}$	Variable cost per unit production capacity for biorefinery *i* with size l' for bio-product *p* by conversion technology *k*
VCH_{hbp}	Variable cost of harvesting for feedstock *b* to produce bio-product *p* at location *h*
TCH_{hib}	Transportation cost of feedstock *b* from harvesting site *h* to biorefinery location *i*
TCE_{imp}	Transportation cost of bio-product *p* from biorefinery *i* to customer *m*
CH_{hbp}	Harvesting cost of feedstock *b* to produce bio-product *p* at location *h*
CC_{ibpk}	Production cost of bio-product *p* from feedstock *b* with conversion technology *k* at biorefinery location *i*
ICb_{ib}	Inventory holding cost of feedstock *b* at biorefinery location *i*
ICE_{ip}	Inventory holding cost of bio-product *p* at biorefinery location *i*
ESC_{ilt}	Expansion cost of storage capacity for biorefinery *i* with size *l* in period *t*
$EPC_{ipkl't}$	Expansion cost of production capacity for biorefinery *i* with size l' for bio-product *p* by conversion technology *k* in period *t*

Binary variables

x_{hbp}	1 if harvesting site selected for feedstock *b* to produce bio-product *p* at location *h*; 0 otherwise
$y_{ikll'}$	1 if biorefinery is installed with storage capacity *l* and production capacity l' by conversion technology *k* at location *i*; 0 otherwise

Positive variables

H_{hbpt}	Amount of harvested feedstock *b* for producing bio-product *p* at location *h* in period *t*
CJ_{hbp}	Amount of harvested area for feedstock *b* for producing bio-product *p* at location *h*
w_{ibpkt}	Amount of processed feedstock *b* for producing bio-product *p* with conversion technology *k* at harvesting site location *h* to biorefinery *i* in period *t*
Tb_{hibpt}	Amount of shipped feedstock *b* for producing bio-product *p* from harvesting site with location *h* to biorefinery *i* in period *t*
TP_{impt}	Amount of shipped bio-product *p* from biorefinery *i* to customer *m* in period *t*
Sb_{ibpt}	Amount of stored feedstock *b* for producing bio-product *p* at biorefinery *i* in period *t*
SP_{ipt}	Amount of stored bio-product *p* at biorefinery *i* in period *t*
ep_{ipt}	Amount of produced bio-product *p* at biorefinery *i* in period *t*
CB_{ilt}	Storage capacity of biorefinery *i* with size *l* in period *t*

$CP_{ipkl't}$	Production capacity of biorefinery i with size l' for bio-product p by conversion technology k in period t
CEB_{ilt}	Storage capacity expansion with size l at biorefinery i in period t
$CEP_{ipkl't}$	Production capacity expansion with size l' at biorefinery i for bio-product p by conversion technology k in period t

Therefore, the proposed mathematical model by the goal of minimizing the total cost of bio-products supply chain network design is presented as follows:

$$\text{Min} \quad Z = Z_{FC} + Z_{VC} + Z_{TC} + Z_{PC} + Z_{IC} + Z_{EC} \tag{1}$$

$$Z_{FC} = \sum_i \sum_k \sum_l \sum_{l'} FCI_{ill'} y_{ikll'} + \sum_h \sum_b \sum_p FCH_{hbp} x_{hbp} \tag{2}$$

$$Z_{VC} = \sum_i \sum_l \sum_t VCB_{il} CB_{ilt} + \sum_i \sum_p \sum_k \sum_{l'} \sum_t VCP_{ikl'} CP_{ipkl't} + \sum_h \sum_b \sum_p VCH_{hbp} CJ_{hbp} \tag{3}$$

$$Z_{TC} = \sum_i \sum_h \sum_b \sum_p \sum_t TCH_{hib} Tb_{hibpt} + \sum_i \sum_m \sum_p \sum_t TCE_{imp} TP_{impt} \tag{4}$$

$$Z_{PC} = \sum_h \sum_b \sum_p \sum_t CH_{hbp} H_{hbpt} + \sum_i \sum_b \sum_p \sum_k \sum_t CC_{ibpk} w_{ibpkt} \tag{5}$$

$$Z_{IC} = \sum_i \sum_b \sum_p \sum_t ICb_{ib} Sb_{ibpt} + \sum_i \sum_p \sum_t ICE_{ip} SP_{ipt} \tag{6}$$

$$Z_{EC} = \sum_i \sum_l \sum_t ESC_{ilt} CEB_{ilt} + \sum_i \sum_p \sum_k \sum_{l'} \sum_t EPC_{ipkl't} CEP_{ipkl't} \tag{7}$$

Subject to:

$$H_{hbpt} \geq \sum_i Tb_{hibpt} \qquad \forall h, b, p, t \tag{8}$$

$$H_{hbpt} \leq (1 - RS_{rht}) \lambda_{hbpt} CJ_{hbp} \qquad \forall h, b, p, r, t \tag{9}$$

$$x_{hbp} LA_{hbp} \leq CJ_{hbp} \leq x_{hbp} UA_{hbp} \qquad \forall h, b, p \tag{10}$$

$$\sum_h Tb_{hibpt} + (1 - \alpha_b) Sb_{ibp(t-1)} = \sum_k w_{ibpkt} + Sb_{ibpt} \qquad \forall i, b, p, t \tag{11}$$

$$ep_{ipt} + SP_{ip(t-1)} = \sum_m TP_{impt} + SP_{ipt} \qquad \forall i, p, t \tag{12}$$

$$ep_{ipt} \leq \sum_k \sum_b \beta_{bpk} w_{ibpkt} \qquad \forall i, p, t \tag{13}$$

$$\sum_i TP_{impt} = D_{mpt} \qquad \forall m, p, t \tag{14}$$

$$\sum_b \sum_p Sb_{ibpt} \leq \sum_l CB_{ilt} \qquad \forall i, t \tag{15}$$

$$CB_{ilt} = CB_{il(t-1)} + CEB_{ilt} \qquad \forall i, l, t \tag{16}$$

$$y_{ikll'} LB_{il} \leq CB_{ilt} \leq y_{ikll'} UB_{il} \qquad \forall i, k, l, l' \tag{17}$$

$$\sum_k \sum_b w_{ibpkt} \leq \sum_k \sum_{l'} CP_{ipkl't} \qquad \forall i, p, t \tag{18}$$

$$CP_{ipkl't} = CP_{ipkl'(t-1)} + CEP_{ipkl't} \qquad \forall i, p, k, l', t \tag{19}$$

$$y_{ikll'} LP_{ipkl'} \le CP_{ipkl't} \le y_{ikll'} UP_{ipkl'} \qquad \forall i, p, k, l, l' \tag{20}$$

$$SP_{ip0} = 0 \quad \forall i, p \tag{21}$$

$$Sb_{ibp0} = 0 \quad \forall i, b, p \tag{22}$$

$$x_{hbp}, y_{ikll'} \in \{0,1\} \quad \forall h, b, p, i, k, l, l' \tag{23}$$

$$H_{hbpt}, Tb_{hibpt}, CJ_{hbp}, w_{ibpkt}, Sb_{ibpt}, TP_{impt}, SP_{ipt}, ep_{ipt}, CB_{iklt}, CEB_{iklt}, CP_{ikl't}, CEP_{ikl't} \ge 0 \qquad \forall h, i, m, b, p, k, l, l', t \tag{24}$$

The objective function which is represented by Eq. (1) minimizes the total cost of multi-feedstock multi-bioproduct supply chain network design. This equation is established based on six components as Eqs. (2)-(7) that are fixed opening costs (Z_{FC}), variable opening costs (Z_{VC}), transportation costs (Z_{TC}), feedstock harvesting and bio-product production costs (Z_{PC}), inventory holding costs (Z_{IC}), and capacity expansion costs of storage and production (Z_{EC}), respectively.

Furthermore, constraint (8) ensures that the amount of shipped harvested feedstock from the harvesting site to the biorefinery is limited to the amount of harvested feedstock. Constraint (9) determines the amount of harvested feedstock regarding the limited harvested area. In this constraint, the amount of harvested feedstock is concerned with risks of disruption as floods, wildfires, hurricanes, etc. Also, constraint (10) binds the harvested area to the minimum allocated and maximum available area for cultivation. Constraints (11) and (12) demonstrate the mass balance on harvesting feedstock and bio-products at biorefinery, respectively. Hence, constraint (13) computes the amount of bio-products produced regarding the feedstock and technology types. Constraint (14) guarantees that the bio-products demand is supplied in each period. The storage capacity constraint of biorefinery is defined by Eq. (15). Moreover, constraints (16) and (17) handle the storage capacity expansion and its lower and upper bounds for installed biorefineries. Constraint (18) limits the produced bio-products regarding the production capacity of established biorefineries. Thereby, constraints (19) and (20) take into account the production capacity expansion and its lower and upper bounds for established biorefineries. Constraints (21) and (22) determine the initial inventory levels of bio-products and feedstocks at biorefinery, respectively. Finally, the binary and positive variables are defined by constraints (23) and (24).

4. Proposed Bezier curve-based possibilistic programming approach

In this section, Bezier curve mechanism is tailored regarding the hesitant fuzzy set theory to finding the structure of HFMF. Then, a possibilistic programming optimization model is presented to adjust the primary parameters of Bezier curve by minimizing the sum of squared errors between the empirical data and fitted membership function. However, the procedures of the aforementioned proposed approaches are explained based on the following sections.

4.1. Bezier curve mechanism

Bezier curve and surfaces theory is one of the main progresses in computer-aided design that could establish a smooth curve based on a mathematical foundation to along the neighborhood of set of control points [45]. In addition, to represent the formal expressions and its characteristics, the following definition and properties are presented:

Definition. A Bezier curve considering the n+1 control points ($C_k \triangleq (C_0, C_1, ..., C_n)$) is represented as follows:

$$f(t,n,C) \triangleq \sum_{k=0}^{n} C_k B_{n,k}(t) \tag{25}$$

where $C_k \triangleq (x_k, y_k)^T$, $t \in [0,1]$, $B_{n,k}(t) = \begin{pmatrix} n \\ k \end{pmatrix} (1-t)^{n-k} t^k$ is the Bernstein polynomial. Furthermore, $f(t,n,C) \in R^2$ in which $f(t,n,C) = \left[f_x(t,n,C_x), f_y(t,n,C_y) \right]^T$ that $(C_x, C_y) \triangleq \left[(x_0, y_0), (x_1, y_1), ..., (x_n, y_n) \right]^T$.

Property 1. The Bezier curve $f(t,n,C)$ represented based on $t \in [0,1]$, lies in the polygon convex hull founded by the control points $C_k \triangleq (C_0, C_1, ..., C_n)$. Meanwhile, this property satisfies that the Bezier curve will not fall out the control polygon.

Property 2. The Bernstein polynomial ($B_{n,k}(t)$) reaches its maximum at $t = \frac{k}{n}$. If the control point C_k is shifted, then the Bezier curve is often affected in the region about the parameter $t = \frac{k}{n}$. This property guarantee that the Bezier curve established by exaggerating the target shape utilizing the control polygon.

Property 3. The first and last control points that are interpolated by Bezier curve are defined as $f(0,n,C) = C_0$ and $f(1,n,C) = C_n$, respectively.

4.2. Mathematical framework of HFMF estimation

The proposed mathematical framework of the HFMF estimation regarding Bezier curve properties is founded to establish its membership function in terms of generality. In this case, the following conditions are usually required to establish the HFMF.

Condition 1 [46, 47]**.** Let $\tilde{E}$ be a hesitant fuzzy set on the universe of discourse X which $h_E(x)$ is a set of membership degrees that mapped from universe of discourse X to [0, 1]. Thereby, the mathematical representation of hesitant fuzzy set is presented as follows [48]:

$$\tilde{E} = \{< x, h_E(x) > | \, x \in X\} \tag{26}$$

On the other hand, the $h_{\tilde{E}}(x_t) = \left(x_t, \left\{ \gamma_1, \gamma_2, ..., \gamma_{l_{x_t}} \right\} \right)$ can be defined as a set of membership degrees for a hesitant fuzzy number (x_t) that some hesitant fuzzy membership degrees (γ_λ) with set length of $\lambda = 1, 2, ..., l_{x_t}$ is devoted.

Condition 2 [49]**.** Consider $h_m(x_t)$ and $h_n(x_t)$ as two hesitant fuzzy set with different set length that are shown by l_m and l_n, respectively. Thereby, the length of the sets should be the same by adding some membership degrees based on expert's risk preferences to utilize the hesitant fuzzy set theory, appropriately. To address the issue, the risk preferences of experts are considered in three categories as risk-neutral ($\varphi_n(x_t)$), risk-seeking ($\varphi_s(x_t)$), and risk-averse ($\varphi_a(x_t)$) that are obtained as follows:

$$\varphi_n(x_t) = \frac{\sum_{\lambda=1}^{l_{x_t}} \gamma_\lambda}{l_{x_t}} \tag{27}$$

$$\varphi_s(x_t) = \text{Max}_\lambda \{\gamma_\lambda\} \tag{28}$$

$$\varphi_a(x_t) = \text{Min}_\lambda \{\gamma_\lambda\} \tag{29}$$

Condition 3. The normality of the hesitant fuzzy numbers ($h_{\tilde{E}}^N(x_t)$) should be satisfied based on following relation.

$$h_{\tilde{E}}^N(x_t) = \frac{\gamma_\lambda}{\sup_{x_t}(\gamma_\lambda)} \quad \forall \lambda = 1, 2, ..., l_{x_t} \tag{30}$$

Condition 4. A hesitant fuzzy set $\tilde{E}$ is convex if the following relation for $x_1, x_2 \in X_t$ and $\delta \in [0,1]$ is established.

$$\mu_{\tilde{E}}(\delta x_1 + (1-\delta)x_2) \geq \min\{\mu_{\tilde{E}}(x_1), \mu_{\tilde{E}}(x_2)\} \tag{31}$$

Therefore, the relation of $\frac{1}{l_{x_t}}\sum_{\lambda=1}^{l_{x_t}} \gamma_\lambda(x_t) \geq \min\left\{\frac{1}{l_{x_{t-1}}}\sum_{\lambda=1}^{l_{x_{t-1}}} \gamma_\lambda(x_{t-1}), \frac{1}{l_{x_{t+1}}}\sum_{\lambda=1}^{l_{x_{t+1}}} \gamma_\lambda(x_{t+1})\right\}$ should be checked to ensure that the consistency of experts' judgments.

Meanwhile, the parametric HFMF ($\mu_{\tilde{E}}^\lambda(x_t)$) is defined as follows in which its monotonic structure is schematically represented in Figure 2.

$$\mu_{\tilde{E}}^\lambda(x_t) = \begin{cases} 0 & \forall\ x_t < \theta_L^\lambda - \alpha^\lambda \\ \mu_{\tilde{E}}^{\lambda L}(x_t) & \forall\ \theta_L^\lambda - \alpha^\lambda < x_t < \theta_L^\lambda \\ 1 & \forall\ \theta_L^\lambda \leq x_t \leq \theta_R \\ \mu_{\tilde{E}}^{\lambda R}(x_t) & \forall\ \theta_R^\lambda < x_t < \theta_R^\lambda + \beta^\lambda \\ 0 & \forall\ x_t \geq \theta_R^\lambda + \beta^\lambda \end{cases} \tag{32}$$

where the left and right spreads are represented as θ_L^λ and θ_R^λ, respectively. In addition, $\theta^\lambda = x_t\left(\mu_{\tilde{E}}^\lambda(x_t) = 1 | 1 \leq \lambda \leq l_{x_t}\right)$ is a value with hesitant fuzzy full membership degree, and the left and right values with full hesitant fuzzy membership degrees are defined by $\theta_L^\lambda = \underset{t}{\mathrm{Max}}\left\{x_t \mid \mu_{\tilde{E}}^{\lambda L}(x_t)=1, 1\le\lambda\le l_{x_t}, x_t < x_t(\theta^\lambda)\right\}$ and $\theta_R^\lambda = \underset{t}{\mathrm{Min}}\left\{x_t \mid \mu_{\tilde{E}}^{\lambda R}(x_t)=1, 1\le\lambda\le l_{x_t}, x_t > x_t(\theta^\lambda)\right\}$ in which $\mu_{\tilde{E}}^{\lambda L}(x_t)$ and $\mu_{\tilde{E}}^{\lambda R}(x_t)$ are the non-decreasing and non-increasing hesitant fuzzy membership values, respectively. In addition, four cases may occur that are anticipated as follows:

Case 1. If $\mu_{\tilde{E}}'^{\lambda}(x_t') < \mu_{\tilde{E}}^{\lambda}(x_t)$ for $x_t' < x_t$, then the non-decreasing hesitant fuzzy membership values ($\mu_{\tilde{E}}^{\lambda L}(x_t)$) are established.

Case 2. If $\mu_{\tilde{E}}'^{\lambda}(x_t') > \mu_{\tilde{E}}^{\lambda}(x_t)$ for $x_t' < x_t$, then the non-increasing hesitant fuzzy membership values ($\mu_{\tilde{E}}^{\lambda R}(x_t)$) are constructed.

Case 3. If $\left\{\theta_L^\lambda \le \left\|\mu_{\tilde{E}}^{\lambda}(x_t)=1\right\| \le \theta_R^\lambda \mid \exists\left(x_t^L \le \theta_L^\lambda\right) \in \mu_{\tilde{E}}^{\lambda L}(x_t), \left(x_t^R \ge \theta_R^\lambda\right) \in \mu_{\tilde{E}}^{\lambda R}(x_t)\right\}$, then the monotonic structure of hesitant fuzzy membership values can be represented as Figure 2 (a).

Case 4. If $\left\{\left\|\mu_{\tilde{E}}^{\lambda}(x_t)=1\right\|=1 \mid \exists x_{t-1}^L \in \mu_{\tilde{E}}^{\lambda L}(x_t), x_{t+1}^R \in \mu_{\tilde{E}}^{\lambda R}(x_t)\right\}$, then the monotonic structure of hesitant fuzzy membership values can be indicated as Figure 2 (b).

{Please insert here Figure 2}

The left and right HFMFs regarding their n+1 control points can be defined as following parametric expressions, respectively.

$$\left[x_t, \mu_{\tilde{E}}^{\lambda L}(x_t)\right]^T = \vec{\mu}_{\tilde{E}}^{\lambda L}\left(t, n_L, C_L^\lambda\right) \triangleq \sum_{k=0}^{n_L}\left(\sum_{\lambda}^{l_{x_t}} C_{L,k}^\lambda B_{n_L,k}(t)\right) \tag{33}$$

$$\left[x_t, \mu_{\tilde{E}}^{\lambda R}(x_t)\right]^T = \vec{\mu}_{\tilde{E}}^{\lambda R}\left(t, n_R, C_R^\lambda\right) \triangleq \sum_{k=0}^{n_R}\left(\sum_{\lambda}^{l_{x_t}} C_{R,k}^\lambda B_{n_R,k}(t)\right) \tag{34}$$

where $t \in [0,1], C_{L,k}^\lambda \triangleq \left(x_{L,k}, y_{L,k}^\lambda\right)^T$ and $C_{R,k}^\lambda \triangleq \left(x_{R,k}, y_{R,k}^\lambda\right)^T$ are the kth Bezier control points in λth length of the hesitant fuzzy set for the left and right HFMFs, respectively. Consequently, for two-dimensional space, the aforementioned procedure is represented as $\vec{\mu}_{\tilde{E}}^{\lambda L}\left(t, n_L, C_L^\lambda\right) = \left[f_x\left(t, n_L, x_{L,k}\right), f_y\left(t, n_L, y_{L,k}^\lambda\right)\right]^T$ and $\vec{\mu}_{\tilde{E}}^{\lambda R}\left(t, n_R, C_R^\lambda\right) = \left[f_x\left(t, n_R, x_{R,k}\right), f_y\left(t, n_R, y_{R,k}^\lambda\right)\right]^T$, where:

$$C_{L,k}^\lambda = \left[x_{L,k}, y_{L,k}^\lambda\right]^T \triangleq \left[\left(x_{L,0}, \left\{y_{L,0}^1, \ldots, y_{L,0}^{l_{x_t}}\right\}\right)^T, \ldots, \left(x_{L,n_L}, \left\{y_{L,n_L}^1, \ldots, y_{L,n_L}^{l_{x_t}}\right\}\right)^T\right] \tag{35}$$

$$C_{R,k}^{\lambda}=\left[x_{R,k},y_{R,k}^{\lambda}\right]^T \triangleq \left[\left(x_{R,0},\left\{y_{R,0}^{1},...,y_{R,0}^{l_{x_t}}\right\}\right)^T,...,\left(x_{R,n_R},\left\{y_{R,n_R}^{1},...,y_{R,n_R}^{l_{x_t}}\right\}\right)^T\right] \quad (36)$$

4.3. Proposed possibilistic programming approach

In this approach, a direct interactive mechanism for knowledge acquisition is provided to define the hesitant fuzzy membership degrees for each point in the reference set from experts. Meanwhile, structured group decision-making methods such as Delphi or brainstorming can be used for data gathering to establish the HFMF. However, the proposed possibilistic programming approach prepares a procedure for constructing HFMF from knowledge acquisition by specifying the control point numbers and their locations $\left(x_t,\mu_{\tilde{E}}^{\lambda}\left(x_t\right)\right)$ in the solution space.

In this case, the left and right side of HFMF can be appraised independently from each other. Thereby, a mathematical programming model is manipulated for estimating the left side (monotonically non-decreasing portion) of a HFMF. Moreover, a similar procedure can be considered for estimating the right side (monotonically non-increasing portion) of a HFMF.

Consider $\xi_{L,i}=\left(\hat{x}_{L,i},\hat{y}_{L,i}^{\lambda}\right)^T$ for $i=1,...,m_L$ as the given data points, where m_L are the data point numbers and $\hat{y}_{L,i}^{\lambda}$ is a set of hesitant fuzzy membership degrees with a set length of l_{x_i} that is provided by a group of experts through the knowledge acquisition process for the *i*th value of $\hat{x}_{L,i}\in X$. Suppose there are at least 3 data points ($m_L\geq 3$) which are sorted in increasing order and consider $n_L+1=m_L$ as the control points that are defined in Eq. (25). In addition, let $x_{L,k}$ and $y_{L,k}^{\lambda}$ ($\lambda=1,...,l_{x_i},k=0,...,n_L$) as decision variables that obtained from the proposed possibilistic programming approach for tuning the locations of $\xi_{L,i}$ in the solution space. Also, t_i ($i=1,...,m_L$) is the parameter value for *i*th data point of Bezier curve.

Consequently, some decision variables are clearly known before performing the proposed optimization model. In this case, the first control point is $C_{L,0}^{\lambda}=\left(\hat{x}_{L,1},0\right)^T\ \forall\lambda$, and the last one is $C_{L,n_L}^{\lambda}=\left(\hat{x}_{L,m_L},1\right)^T\ \forall\lambda$. Furthermore, $x_{L,0}=\hat{x}_{L,1}$, $y_{L,0}^{\lambda}=0$, $x_{L,n_L}=\hat{x}_{L,m_L}$, $y_{L,n_L}^{\lambda}=1$, $t_1=0$, and $t_{m_L}=1$. Finally, the proposed possibilistic programming approach is handled regarding the following mathematical programming model by aims of minimizing the SSE between the empirical data and fitted HFMF.

$$\text{Min}\quad \sum_{i=2}^{m_L-1}\sum_{\lambda=1}^{l_{x_i}}\left(\hat{y}_{L,i}^{\lambda}-\sum_{k=0}^{n_L}y_{L,k}^{\lambda}\binom{n_L}{k}t_i^k\left(1-t_i\right)^{n_L-k}\right)^2 \quad (37)$$

$$\sum_{k=0}^{n_L}x_{L,k}\binom{n_L}{k}t_i^k\left(1-t_i\right)^{n_L-k}=\hat{x}_{L,i}\quad \forall i=2,...,m_L-1 \quad (38)$$

$$t_i\leq t_{i+1}\quad \forall i=1,...,m_L-1 \quad (39)$$

$$x_{L,k}\leq x_{L,k+1}\quad \forall k=0,...,n_L-1 \quad (40)$$

$$\sum_{\lambda=1}^{l_{x_t}} y_{L,k}^{\lambda} \le \sum_{\lambda=1}^{l_{x_t}} y_{L,k+1}^{\lambda} \quad \forall k=0,...,n_L-1 \tag{41}$$

$$\hat{x}_{L,1} \le x_{L,k} \le \hat{x}_{L,m_L} \quad \forall k=1,...,n_L-1 \tag{42}$$

$$0 \le y_{L,k}^{\lambda} \le 1 \quad \forall \lambda=1,...,l_{x_i}, k=1,...,n_L-1 \tag{43}$$

$$0 \le t_i \le 1 \quad \forall i=2,...,m_L-1 \tag{44}$$

The objective function by goals of minimizing the SSE among the empirical data and fitted HFMF is defined by Eq. (37). Moreover, for $t_i \in [0,1]$ constraint (38) is constructed such that Bernstein polynomial equation is equal to the first coordinates of a control point. Equations (39)-(41) are the basic constraints for the left side of HFMF. Finally, constraints (42)-(44) ensure the acceptable range of both elements of control points and parameter value of Bezier curve.

4.4. Procedures of the proposed mathematical framework

In sums, proposed mathematical framework for estimating the HFMF is concluded based on the following steps:

Step 1. Construct the control points matrix ($\wp$) based on hesitant fuzzy subjective/objective information regarding Eqs. (35) and (36) as follows:

$$\wp = \left[x_k, y_k^{\lambda}\right]^T \triangleq \begin{array}{c} \\ x_0 \\ x_1 \\ \vdots \\ x_k \\ \vdots \\ x_n \end{array} \begin{array}{c} \begin{array}{cccc} \lambda=1 & \lambda=2 & \cdots & \lambda=l_{x_t} \end{array} \\ \begin{bmatrix} (x_0, y_0^1) & (x_0, y_0^2) & \cdots & (x_0, y_0^{l_{x_t}}) \\ (x_1, y_1^1) & (x_1, y_1^2) & \cdots & (x_1, y_1^{l_{x_t}}) \\ \vdots & \vdots & \ddots & \vdots \\ (x_k, y_k^1) & (x_k, y_k^2) & \cdots & (x_k, y_k^{l_{x_t}}) \\ \vdots & \vdots & \ddots & \vdots \\ (x_n, y_n^1) & (x_n, y_n^2) & \cdots & (x_n, y_n^{l_{x_t}}) \end{bmatrix}^T \end{array} \tag{45}$$

Step 2. The hesitant fuzzy set values of $\wp$ with different set length should be equal to the maximum set length based on condition 2.

Step 3. Check the consistency of the experts' judgments regarding the following necessary and sufficient conditions:

– ***Necessary condition:*** If $\frac{1}{l_{x_t}}\sum_{\lambda=1}^{l_{x_t}}\gamma_\lambda(x_t) \not\ge \min\left\{\frac{1}{l_{x_{t-1}}}\sum_{\lambda=1}^{l_{x_{t-1}}}\gamma_\lambda(x_{t-1}), \frac{1}{l_{x_{t+1}}}\sum_{\lambda=1}^{l_{x_{t+1}}}\gamma_\lambda(x_{t+1})\right\}$, then the experts should modify their judgments for x_t or its hesitant fuzzy membership degrees should be changed by adding the value of $\left|\left(\min\left\{\frac{1}{l_{x_{t-1}}}\sum_{\lambda=1}^{l_{x_{t-1}}}\gamma_\lambda(x_{t-1}), \frac{1}{l_{x_{t+1}}}\sum_{\lambda=1}^{l_{x_{t+1}}}\gamma_\lambda(x_{t+1})\right\}\right) - \frac{1}{l_{x_t}}\sum_{\lambda=1}^{l_{x_t}}\gamma_\lambda(x_t)\right|$ to $\gamma_{l_{x_t}}(x_t)$.

– ***Sufficient condition:*** $\left[x_t, \mu_{\tilde{E}}^{\lambda}(x_t)\right]^T = \begin{cases} \left[x_t, \mu_{\tilde{E}}^{\lambda L}(x_t)\right]^T = \left(\theta_L^{\lambda} - \alpha^{\lambda} \le x_t^L \le x_{t+1}^L \le \theta_L^{\lambda}, \left\{\gamma_1 \le \gamma_2, \ldots, \le \gamma_{l_{x_t}}\right\}\right) \\ \left[x_t, \mu_{\tilde{E}}^{\lambda R}(x_t)\right]^T = \left(\theta_R^{\lambda} \le x_t^L \le x_{t+1}^L \le \theta_R^{\lambda} + \beta^{\lambda}, \left\{\gamma_1 \ge \gamma_2, \ldots, \ge \gamma_{l_{x_t}}\right\}\right) \end{cases}$.

Step 4. Normalize the consistent hesitant fuzzy group control points matrix based on condition 3.

Step 5. Define the left and right hesitant fuzzy control points to establish the Bezier curve mechanism as $\left\{\theta_L^{\lambda} \le \left\|\mu_{\tilde{E}}^{\lambda}(x_t) = 1\right\| \le \theta_U^{\lambda} \mid \exists\left(x_t^L \le \theta_L^{\lambda}\right) \in \mu_{\tilde{E}}^{\lambda L}(x_t), \left(x_t^R \ge \theta_U^{\lambda}\right) \in \mu_{\tilde{E}}^{\lambda R}(x_t)\right\}$.

Step 6. Compute the left and right HFMFs based on Bezier-Bernstein equations by relations of (35) and (36).

Step 7. Adjust the hesitant fuzzy control points regarding the proposed possibilistic programming approach based on equations (37)-(44).

Step 8. In this step, the equivalent auxiliary crisp (EAC) model should be founded by inspiration from Pishvaee and Torabi [50] study. Meanwhile, the methods of Parra et al. [51] and Jiménez et al. [52] are hybridized to convert the proposed hesitant fuzzy mathematical model into an EAC model.

Thereby, an α*-cut* of a hesitant fuzzy number $\tilde{E}$ is defined by $E_{\alpha} = \left\{x \in X \middle| \mu_{\tilde{E}}^{\lambda}(x_t) \ge \alpha\right\}$. Since $\mu_{\tilde{E}}^{\lambda}$ is upper semi-continuous, the α*-cuts* are bounded and closed intervals that are denoted by $E_{\alpha} = \left[\mu_{\tilde{E}}^{-1\lambda L}(\alpha), \mu_{\tilde{E}}^{-1\lambda R}(\alpha)\right]$. However, the hesitant fuzzy expected interval (HFEI) and the hesitant fuzzy expected value (HFEV) of $\tilde{E}$ can be defined as follows:

$$HFEI\left(\tilde{E}\right) = \left[E_1, E_2\right] = \left[\int_0^1 \mu_{\tilde{E}}^{-1\lambda L}(\alpha)\, d\alpha, \int_0^1 \mu_{\tilde{E}}^{-1\lambda R}(\alpha)\, d\alpha\right] \tag{46}$$

$$HFEV\left(\tilde{E}\right) = \frac{E_1 + E_2}{2} \tag{47}$$

Then, inspired by the proposed ranking method of Jiménez [53], the degree in which $\tilde{R}$ is bigger than $\tilde{F}$ that is founded as follows:

$$\mu_M\left(\tilde{R}, \tilde{F}\right) = \begin{cases} 1 & \text{if} \quad E_1^R - E_2^F > 0 \\ \dfrac{E_2^R - E_1^F}{E_2^R - E_1^F - \left(E_1^R - E_2^F\right)} & \text{if} \quad 0 \in \left[E_1^R - E_2^F, E_2^R - E_1^F\right] \\ 0 & \text{if} \quad E_2^R - E_1^F < 0 \end{cases} \tag{48}$$

Therefore, EAC of two types of constraint as $\tilde{R}_i x \ge \tilde{F}_i \quad \forall i$ and $\tilde{R}_i x = \tilde{F}_i \quad \forall i$ can be obtained based on following relations, respectively.

$$\left[(1-\alpha) E_2^{R_i} + \alpha E_1^{R_i}\right] x \ge \alpha E_2^{F_i} + (1-\alpha) E_1^{F_i} \qquad \forall i \tag{49}$$

$$\left[\left(1 - \frac{\alpha}{2}\right) E_2^{R_i} + \frac{\alpha}{2} E_1^{R_i}\right] x \ge \frac{\alpha}{2} E_2^{F_i} + \left(1 - \frac{\alpha}{2}\right) E_1^{F_i} \qquad \forall i \tag{50}$$

$$\left[\frac{\alpha}{2}E_2^{R_i}+\left(1-\frac{\alpha}{2}\right)E_1^{R_i}\right]x\le\left(1-\frac{\alpha}{2}\right)E_2^{F_i}+\frac{\alpha}{2}E_1^{F_i}\qquad\forall i \tag{51}$$

Furthermore, the processes of the proposed mathematical framework for estimating the HFMF are depicted in Figure 3.

{Please insert here Figure 3}

5. Computational experiments and validation

In this section, two practical examples are provided to indicate the performance of the presented model and the efficiency of the presented solution framework. Besides, a comparative analysis is considered to show the validity of the proposed approach by comparing the obtained results from the triangular fuzzy approach and certain condition with the proposed hesitant fuzzy mechanism. Meanwhile, as indicated in Table 1, two test problems are established, and their sizes are represented.

As expressed in the problem description, the bio-products demand is uncertain as it is obtained from the hesitant fuzzy distribution. The other required parameters are defined as crisp values. In addition, corresponding hesitant fuzzy distribution of bio-products demand is defined as $D_{mpt}=\{<x,h_{D_{mpt}}(x)>|\,x\in R\}=\{<(10,50),[0,1]>|\,x\in R\}$ in which the demand values regarding their membership degrees are reported in Table 2.

{Please insert here Table 1}

{Please insert here Table 2}

The presented mixed integer mathematical programming model and proposed possibilistic programming model are coded in GAMS 24.1.3 optimization software by CPLEX solver. In addition, the extended Bezier curve mechanism is coded in MATLAB R2013a to compute the hesitant fuzzy control points. Hence, all results are performed on Intel Core i7-3610 M 2.30 GHz computer with 6 GB RAM. Thereby, the HFMF is obtained regarding the developed Bezier curve mechanism-based possibilistic programming model that is depicted in Figure 4. Hence, the risk preference of each expert is considered as risk-seeking ($\varphi_s(x_t)$) that the length of hesitant fuzzy membership degrees sets is converted to three elements.

{Please insert here Figure 4}

Thus, the proposed multi-feedstock multi-bioproduct supply chain network design model regarding the proposed solution method is converted to EAC model as follows:

$$\text{Min} \quad Z = Z_{FC} + Z_{VC} + Z_{TC} + Z_{PC} + Z_{IC} + Z_{EC} \tag{52}$$

s.t.

$$\sum_i TP_{impt} \geq \frac{\alpha}{2} E_2^{\tilde{D}_{mpt}} + \left(1 - \frac{\alpha}{2}\right) E_1^{\tilde{D}_{mpt}} \quad \forall m, p, t \tag{53}$$

$$\sum_i TP_{impt} \leq \left(1 - \frac{\alpha}{2}\right) E_2^{\tilde{D}_{mpt}} + \frac{\alpha}{2} E_1^{\tilde{D}_{mpt}} \quad \forall m, p, t \tag{54}$$

Constraints (2)-(13) and (15)-(24) (55)

where Eqs. (53) and (54) are the EAC models of constraint (14) in which the bio-products demand is handled under hesitant fuzzy set environment ($\sum_i TP_{impt} = \tilde{D}_{mpt} \quad \forall m, p, t$). For instance, the aforementioned EAC model regarding the HFEI ($HFEI\left(\tilde{D}_{mpt}\right) = \left[20.09, 38.26\right]$) and HFEV ($HFEV\left(\tilde{D}_{mpt}\right) = 29.17$) measures of the considered hesitant fuzzy bio-products demand is converted to:

$$\text{Min} \quad Z = Z_{FC} + Z_{VC} + Z_{TC} + Z_{PC} + Z_{IC} + Z_{EC} \tag{56}$$

s.t.

$$\sum_i TP_{impt} \geq \frac{\alpha}{2} \times \left(38.26\right) + \left(1 - \frac{\alpha}{2}\right) \times \left(20.09\right) \quad \forall m, p, t \tag{57}$$

$$\sum_i TP_{impt} \leq \left(1 - \frac{\alpha}{2}\right) \times \left(38.26\right) + \frac{\alpha}{2} \times \left(20.09\right) \quad \forall m, p, t \tag{58}$$

Constraints (2)-(13) and (15)-(24) (59)

Thus, the obtained results from both the test problems are presented in Table 3 regarding the different α-levels. As indicated in this table, increasing the α-levels lead to the worst objective function. Furthermore, the test problems are solved based on triangular fuzzy modeling and crisp model to ensure the validity of the proposed hesitant fuzzy mechanism. Although the performance of two approaches is balanced, the objective function values of the proposed approach are lower than the considered triangular fuzzy approach as popular fuzzy set theory. Also, solving the proposed mixed integer mathematical programming model under certain condition represents that both fuzzy set theories can lead to better solutions by lower objective functions. Moreover, with regard to the computational time, the triangular fuzzy approach could find the optimum solution a little quicker than the proposed approach in most of the cases. Consequently, the proposed hesitant fuzzy mechanism could be appropriately dealt with the uncertain situation by considering the vagueness and hesitancy conditions, simultaneously.

{Please insert here Table 3}

Also, the trends of three approaches are compared to represent the verification of the proposed approach. Thus, the optimum values of some important variables which are obtained from the second test problem by minimum satisfaction level (α=0.6) are compared. Therefore, the total amount of harvested feedstock for producing bio-products at all locations in each period ($\sum_h \sum_b \sum_p H_{hbpt} \quad \forall t$) and the total amount of produced bio-products at all bio-refineries in each period ($\sum_i \sum_p ep_{ipt} \quad \forall t$) are depicted in Figures 5 and 6, respectively. As represented in these figures, the trends of three approaches are similar and approved the obtained results from the proposed hesitant fuzzy mechanism. Moreover, the obtained results from the proposed approach indicated that the proposed hesitant fuzzy mechanism can be more robust than the other two approaches. This improvement may be obtained from the unique feature of hesitant fuzzy set theory that allows experts to express their judgments by assigning some membership degrees for an uncertain parameter under a set. This feature can be dealt with imprecise information and the experts doubt about their judgments, simultaneously.

{Please insert here Figure 5}

{Please insert here Figure 6}

Furthermore, to indicate the validity of the extended Bezier curve mechanism and the proposed possibilistic programming model, the trends of both approaches are depicted in Figure 7. As it can be seen in this figure, the proposed possibilistic programming model adjusts the locations of the hesitant fuzzy control points and implements the normalization procedure in the process of HFMF estimation. Consequently, the proposed possibilistic programming model can be more reliable than the other approaches regarding minimizing the SSE among the empirical data and fitted HFMF.

{Please insert here Figure 7}

6. Conclusions, limitations, and future suggestions

In recent years, hesitant fuzzy set theory is mostly used to solve the group decision-making problems by assigning some membership degrees for an object under a set to decrease both hesitancy and uncertainty. Meanwhile, utilizing this theory for coping with imprecise and unreliable information in case of mathematical programming approaches requires a membership function to reach an acceptable solution in continuous space. This study is discussed about this open problem to estimate the HFMF by knowledge acquisition from experts. Thus, Bezier curve mechanism as computer-aided design is considered to propose a mathematical programming model based on the possibilistic programming approach by aims of minimizing the SSE between the empirical data and fitted HFMF. In this case, the proposed procedure of the HFMF estimation is checked in the field of biomass supply chain network design problem. To address the issue, a mathematical programming approach is proposed in which the bio-products demand is uncertain and followed from HFMF. To represent the verification and validation of this study, a computational experiment about the biomass supply chain network design and a comparative analysis are provided, respectively. In a comparative analysis, the obtained results from the proposed approach are compared with the triangular fuzzy approach and certain situation to guarantee the validation of the proposed approach. Consequently, the obtained results from the comparative analysis show that the proposed approach can lead to a precise solution and robust results in comparison with the two other approaches. Moreover, the proposed possibilistic programming model is compared with an extended Bezier curve mechanism to approve its validity. As observed, the obtained results from the proposed possibilistic programming model are reliable than the extended Bezier curve mechanism to minimize the SSE among the empirical data and fitted HFMF.

Although the proposed approach can suitably deal with the incomplete and imprecise information, the number of control points is integer and unknown that could increase the complexity of the problem, dramatically. Fortunately, the number of control points that is required for real-world cases is small. Therefore, to address this limitation, considering the number of control points as a parameter can help users to solve broad range of nonlinear problems instead of facing with most complex mixed integer nonlinear programming models, directly.

For future suggestions, the proposed Bezier curve-based possibilistic programming approach can be considered to develop the membership function-based techniques such as possibilistic chance constraint programming method, robust possibilistic optimization modeling, etc. Furthermore, the proposed HFMF can be utilized to design hesitant fuzzy inferences systems and appropriately establish the hesitant fuzzy rules. Moreover, in case of biomass supply chain network design, each layer of the two-echelon network can be considered as an agent of machine learning approach, in which the biorefinery layer can use an enhanced case of the proposed mathematical programming model as multi-objective optimization model to increase the flexibility and efficiency of the supply chain network design process.

References

1. Medasani, S., J. Kim, and R. Krishnapuram, ‘‘An overview of membership function generation techniques for pattern recognition’’. *International Journal of approximate reasoning*. **19**(3-4), pp. 391-417 (1998).
2. Bilgiç, T. and I.B. Türkşen, *Measurement of membership functions: theoretical and empirical work*, in *Fundamentals of fuzzy sets*. 2000, Springer. p. 195-227.
3. Uddin, M.S., M. Miah, M.A.-A. Khan, et al., ‘‘Goal programming tactic for uncertain multi-objective transportation problem using fuzzy linear membership function’’. *Alexandria Engineering Journal*. **60**(2), pp. 2525-2533 (2021).

4. Ashtari, P., R. Karami, and S. Farahmand-Tabar, ‘‘Optimum geometrical pattern and design of real-size diagrid structures using accelerated fuzzy-genetic algorithm with bilinear membership function’’. *Applied Soft Computing*. **110**, pp. 107646 (2021).
5. Xu, P., B. Liu, X. Hu, et al., ‘‘State-of-Charge Estimation for Lithium-ion Batteries Based on Fuzzy Information Granulation and Asymmetric Gaussian Membership Function’’. *IEEE Transactions on Industrial Electronics*, (2021).
6. Gao, J., J. Yao, and L. Chen, ‘‘The Statistical Methods of Membership Function in Structural Serviceability Failure Criterion’’. *KSCE Journal of Civil Engineering*, pp. 1-8 (2021).
7. Ubukata, S., A. Notsu, and K. Honda, ‘‘Objective function-based rough membership C-means clustering’’. *Information Sciences*. **548**, pp. 479-496 (2021).
8. Lin, F.-J., C.-I. Chen, G.-D. Xiao, et al., ‘‘Voltage Stabilization Control for Microgrid with Asymmetric Membership Function Based Wavelet Petri Fuzzy Neural Network’’. *IEEE Transactions on Smart Grid*, (2021).
9. Pelalak, R., A.T. Nakhjiri, A. Marjani, et al., ‘‘Influence of machine learning membership functions and degree of membership function on each input parameter for simulation of reactors’’. *Scientific Reports*. **11**(1), pp. 1-11 (2021).
10. Meng, F., C. Tan, and X. Chen, ‘‘Multiplicative consistency analysis for interval fuzzy preference relations: A comparative study’’. *Omega*. **68**, pp. 17-38 (2017).
11. Yang, L., X. Zhou, and Z. Gao, ‘‘Credibility-based rescheduling model in a double-track railway network: a fuzzy reliable optimization approach’’. *Omega*. **48**, pp. 75-93 (2014).
12. Anighoro, A. and J.r. Bajorath, ‘‘Compound ranking based on fuzzy three-dimensional similarity improves the performance of docking into homology models of G-protein-coupled receptors’’. *ACS omega*. **2**(6), pp. 2583-2592 (2017).
13. Krampe, V., P. Edme, and H. Maurer. *A Suitable Objective Function for Optimizing the Experimental Design for Seismic Full Waveform Inversion*. in *82nd EAGE Annual Conference & Exhibition*. 2020. European Association of Geoscientists & Engineers.
14. Turksen, I., ‘‘Measurement of membership functions and their acquisition’’. *Fuzzy Sets and systems*. **40**(1), pp. 5-38 (1991).
15. Klir, G. and B. Yuan, *Fuzzy sets and fuzzy logic*. Vol. 4. 1995: Prentice hall New Jersey.
16. Medaglia, A.L., S.-C. Fang, H.L. Nuttle, et al., ‘‘An efficient and flexible mechanism for constructing membership functions’’. *European Journal of Operational Research*. **139**(1), pp. 84-95 (2002).
17. Zimmermann, H.-J. and P. Zysno, ‘‘Quantifying vagueness in decision models’’. *European Journal of Operational Research*. **22**(2), pp. 148-158 (1985).
18. Chen, J.E. and K.N. Otto, ‘‘Constructing membership functions using interpolation and measurement theory’’. *Fuzzy Sets and systems*. **73**(3), pp. 313-327 (1995).
19. Marchant, T., ‘‘A measurement-theoretic axiomatization of trapezoidal membership functions’’. *IEEE Transactions on Fuzzy Systems*. **15**(2), pp. 238-242 (2007).
20. Huynh, V.-N., Y. Nakamori, T.B. Ho, et al. *A context model for constructing membership functions of fuzzy concepts based on modal logic*. in *International Symposium on Foundations of Information and Knowledge Systems*. 2002. Springer.
21. Chen, S. and F. Tsai, ‘‘A new method to construct membership functions and generate fuzzy rules from training instances’’. *International journal of information and management sciences*. **16**(2), pp. 47 (2005).
22. Sanchez, A., R. Alvarez, J. Moctezuma, et al. *Clustering and artificial neural networks as a tool to generate membership functions*. in *Electronics, Communications and Computers, 2006. CONIELECOMP 2006. 16th International Conference on*. 2006. IEEE.
23. Sami, M. and K. Badie. *Generating Fuzzy Membership Functions through a Meta-Function: An Experience Mining Approach*. in *Automation Congress, 2006. WAC'06. World*. 2006. IEEE.
24. Jain, S. and M. Khare, ‘‘Construction of fuzzy membership functions for urban vehicular exhaust emissions modeling’’. *Environmental monitoring and assessment*. **167**(1-4), pp. 691-699 (2010).
25. Bouhentala, M., M. Ghanai, and K. Chafaa, ‘‘Interval-valued membership function estimation for fuzzy modeling’’. *Fuzzy Sets and Systems*. **361**, pp. 101-113 (2019).

26. Mousavi, M., H. Gitinavard, and S. Mousavi, ‘‘A soft computing based-modified ELECTRE model for renewable energy policy selection with unknown information’’. *Renewable and Sustainable Energy Reviews*. **68**, pp. 774-787 (2017).
27. Wu, P., H. Li, J.M. Merigo, et al., ‘‘Integer programming modeling on group decision making with incomplete hesitant fuzzy linguistic preference relations’’. *IEEE Access*. **7**, pp. 136867-136881 (2019).
28. Wan, S.-P., Y.-L. Qin, and J.-Y. Dong, ‘‘A hesitant fuzzy mathematical programming method for hybrid multi-criteria group decision making with hesitant fuzzy truth degrees’’. *Knowledge-Based Systems*. **138**, pp. 232-248 (2017).
29. Song, Y. and G. Li, ‘‘A mathematical programming approach to manage group decision making with incomplete hesitant fuzzy linguistic preference relations’’. *Computers & Industrial Engineering*. **135**, pp. 467-475 (2019).
30. Li, J. and Z.-x. Wang, ‘‘A programming model for consistency and consensus in group decision making with probabilistic hesitant fuzzy preference relations’’. *International Journal of Fuzzy Systems*. **20**(8), pp. 2399-2414 (2018).
31. Zhang, Y., J. Tang, and F. Meng, ‘‘Programming model-based method for ranking objects from group decision making with interval-valued hesitant fuzzy preference relations’’. *Applied Intelligence*. **49**(3), pp. 837-857 (2019).
32. Rashid, T. and M.S. Sindhu. *Application of linear programming model in multiple criteria decision making under the framework of interval-valued hesitant fuzzy sets*. in *International Conference on Intelligent and Fuzzy Systems*. 2020. Springer.
33. Wei, G., F.E. Alsaadi, T. Hayat, et al., ‘‘A linear assignment method for multiple criteria decision analysis with hesitant fuzzy sets based on fuzzy measure’’. *International Journal of Fuzzy Systems*. **19**(3), pp. 607-614 (2017).
34. Azadeh, A., R. Babazadeh, and S. Asadzadeh, ‘‘Optimum estimation and forecasting of renewable energy consumption by artificial neural networks’’. *Renewable and Sustainable Energy Reviews*. **27**, pp. 605-612 (2013).
35. Ghaderi, H., A. Moini, and M.S. Pishvaee, ‘‘A multi-objective robust possibilistic programming approach to sustainable switchgrass-based bioethanol supply chain network design’’. *Journal of Cleaner Production*. **179**, pp. 368-406 (2018).
36. Ghaderi, H., M.S. Pishvaee, and A. Moini, ‘‘Biomass supply chain network design: an optimization-oriented review and analysis’’. *Industrial crops and products*. **94**, pp. 972-1000 (2016).
37. Babazadeh, R., J. Razmi, M. Rabbani, et al., ‘‘An integrated data envelopment analysis–mathematical programming approach to strategic biodiesel supply chain network design problem’’. *Journal of Cleaner Production*. **147**, pp. 694-707 (2017).
38. Bai, Y., T. Hwang, S. Kang, et al., ‘‘Biofuel refinery location and supply chain planning under traffic congestion’’. *Transportation Research Part B: Methodological*. **45**(1), pp. 162-175 (2011).
39. Bai, Y., Y. Ouyang, and J.-S. Pang, ‘‘Biofuel supply chain design under competitive agricultural land use and feedstock market equilibrium’’. *Energy Economics*. **34**(5), pp. 1623-1633 (2012).
40. Tong, K., F. You, and G. Rong, ‘‘Robust design and operations of hydrocarbon biofuel supply chain integrating with existing petroleum refineries considering unit cost objective’’. *Computers & Chemical Engineering*. **68**, pp. 128-139 (2014).
41. Li, Q. and G. Hu, ‘‘Supply chain design under uncertainty for advanced biofuel production based on bio-oil gasification’’. *Energy*. **74**, pp. 576-584 (2014).
42. Mohseni, S. and M.S. Pishvaee, ‘‘A robust programming approach towards design and optimization of microalgae-based biofuel supply chain’’. *Computers & Industrial Engineering*. **100**, pp. 58-71 (2016).
43. Kesharwani, R., Z. Sun, and C. Dagli, ‘‘Biofuel supply chain optimal design considering economic, environmental, and societal aspects towards sustainability’’. *International Journal of Energy Research*. **42**(6), pp. 2169-2198 (2018).
44. Fattahi, M. and K. Govindan, ‘‘A multi-stage stochastic program for the sustainable design of biofuel supply chain networks under biomass supply uncertainty and disruption risk: A real-life

case study''. *Transportation Research Part E: Logistics and Transportation Review*. **118**, pp. 534-567 (2018).

45. Farin, G., *Curves and surfaces for computer-aided geometric design: a practical guide*. 2014: Elsevier.
46. Torra, V. and Y. Narukawa. *On hesitant fuzzy sets and decision*. in *Fuzzy Systems, 2009. FUZZ-IEEE 2009. IEEE International Conference on*. 2009. IEEE.
47. Torra, V., ''Hesitant fuzzy sets''. *International Journal of Intelligent Systems*. **25**(6), pp. 529-539 (2010).
48. Xia, M. and Z. Xu, ''Hesitant fuzzy information aggregation in decision making''. *International Journal of Approximate Reasoning*. **52**(3), pp. 395-407 (2011).
49. Xu, Z. and X. Zhang, ''Hesitant fuzzy multi-attribute decision making based on TOPSIS with incomplete weight information''. *Knowledge-Based Systems*. **52**, pp. 53-64 (2013).
50. Pishvaee, M.S. and S.A. Torabi, ''A possibilistic programming approach for closed-loop supply chain network design under uncertainty''. *Fuzzy Sets and systems*. **161**(20), pp. 2668-2683 (2010).
51. Parra, M.A., A.B. Terol, B.P. Gladish, et al., ''Solving a multiobjective possibilistic problem through compromise programming''. *European Journal of Operational Research*. **164**(3), pp. 748-759 (2005).
52. Jiménez, M., M. Arenas, A. Bilbao, et al., ''Linear programming with fuzzy parameters: an interactive method resolution''. *European Journal of Operational Research*. **177**(3), pp. 1599-1609 (2007).
53. Jiménez, M., ''Ranking fuzzy numbers through the comparison of its expected intervals''. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*. **4**(04), pp. 379-388 (1996).

Figures captions:

Figure 1. Multi-feedstock multi-bioproduct supply chain network under disruption risks
Figure 2. The schematically representation of parametric HFMF
 (a) If exist more than one x_t that their membership degrees are equal to 1.
 (b) If exists only one x_t with membership degree 1.
Figure 3. Processes of the HFMF estimation
Figure 4. The HFMF of bio-products demand
Figure 5. Comparative analysis regarding the total harvested feedstocks in each period
Figure 6. Comparative analysis regarding the total produced bio-products in each period
Figure 7. Extended Bezier curve mechanism versus the proposed possibilistic programming model

Tables captions:

Table 1. The test problems sizes by defining the values of sets
Table 2. The bio-products demand value and their hesitant fuzzy membership degrees
Table 3. The summary of the obtained results for both test problems regarding to different α-levels

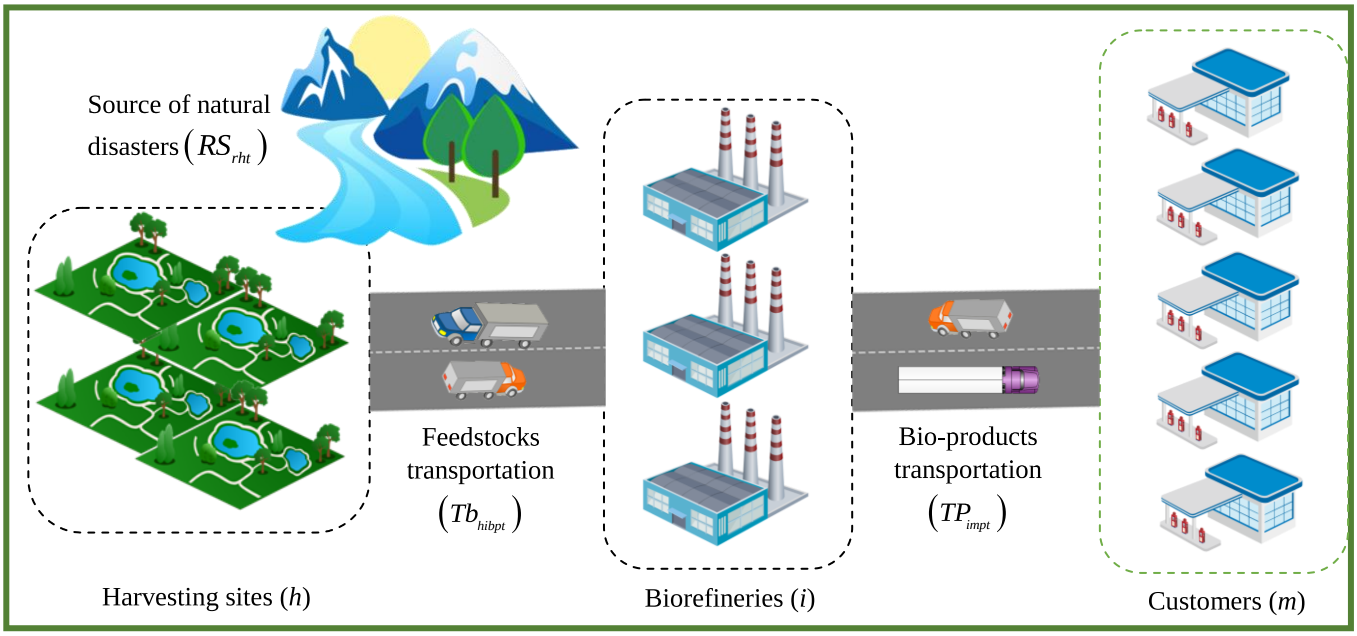

Figure 1. Multi-feedstock multi-bioproduct supply chain network under disruption risks

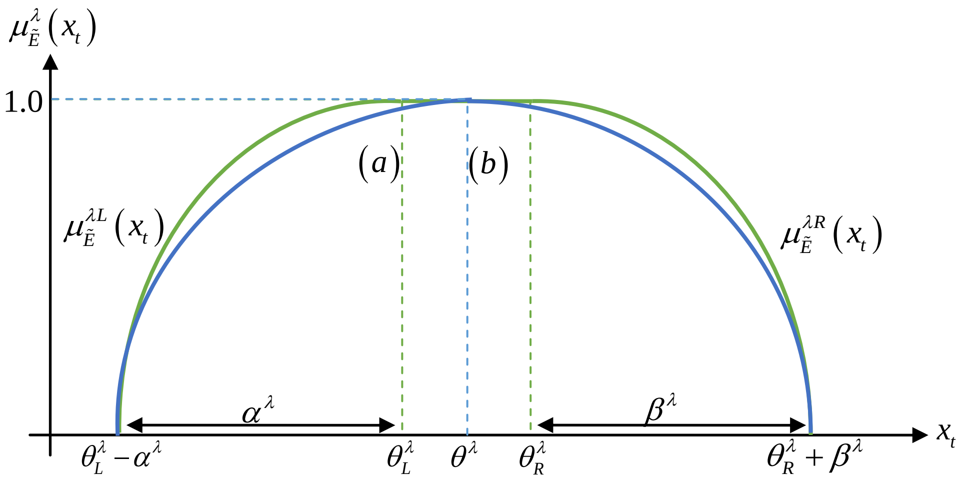

Figure 2. The schematically representation of parametric HFMF
(a) If exist more than one x_t that their membership degrees are equal to 1.
(b) If exists only one x_t with membership degree 1.

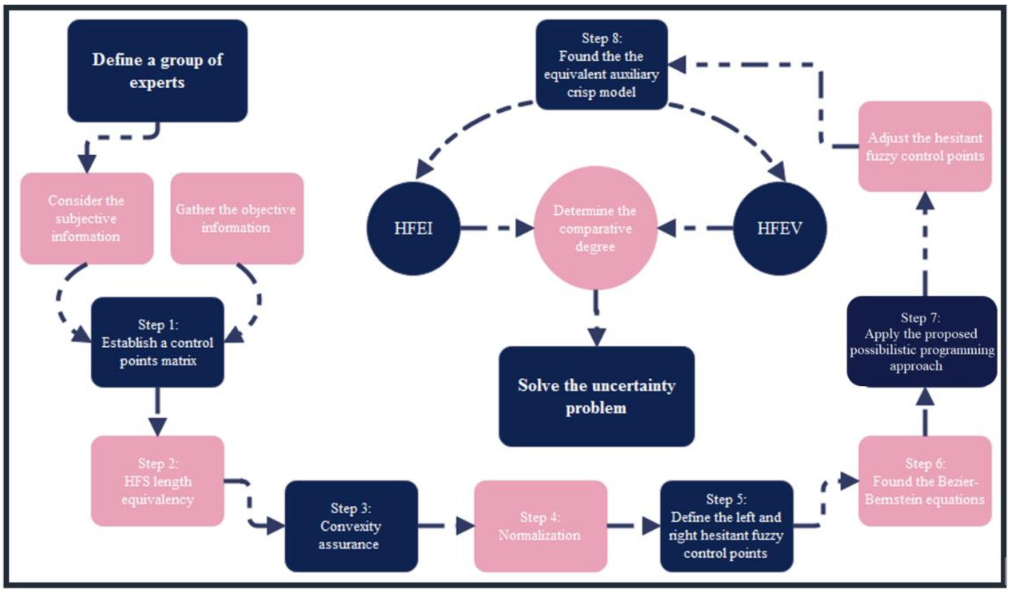

Figure 3. Processes of the HFMF estimation

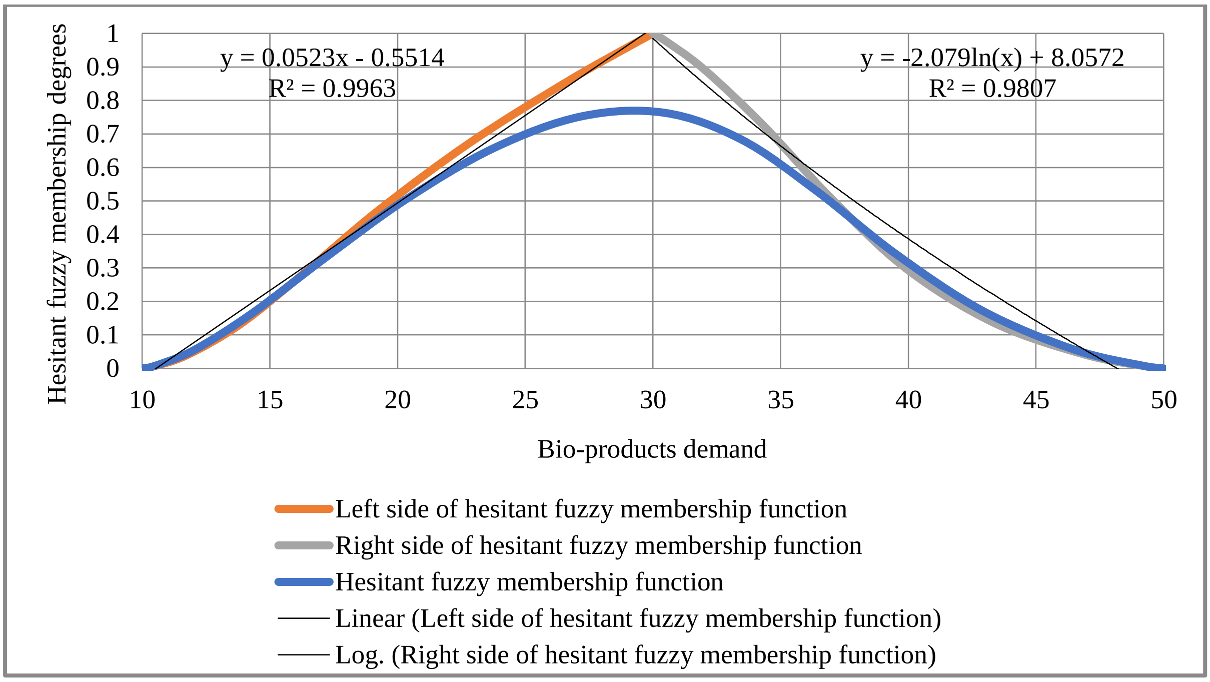

Figure 4. The HFMF of bio-products demand

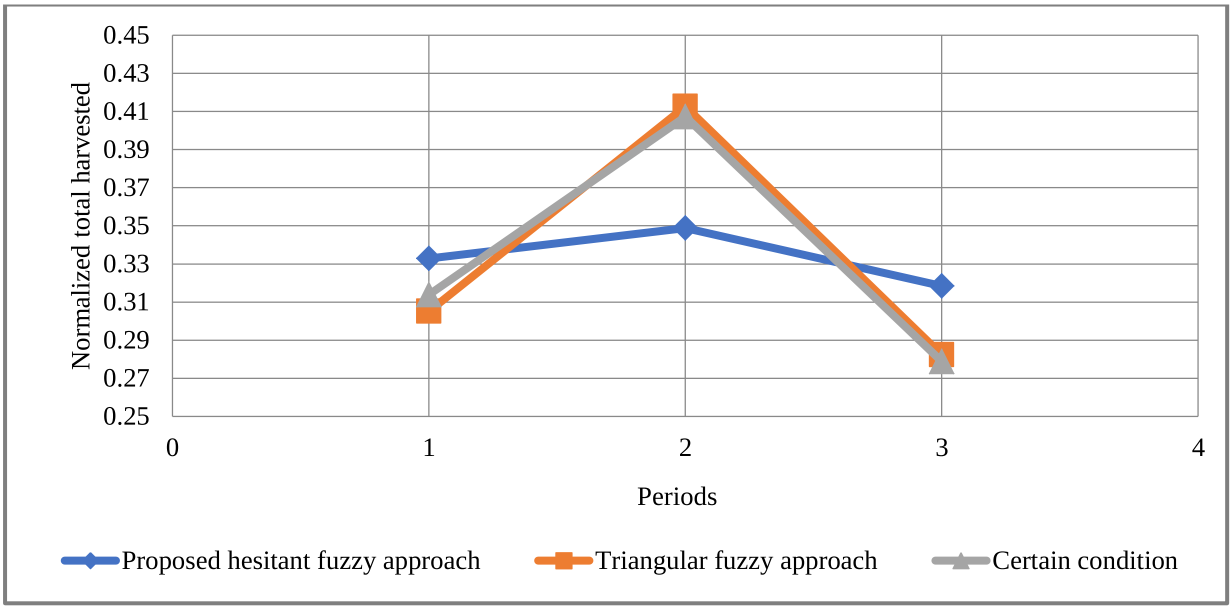

Figure 5. Comparative analysis regarding the total harvested feedstocks in each period

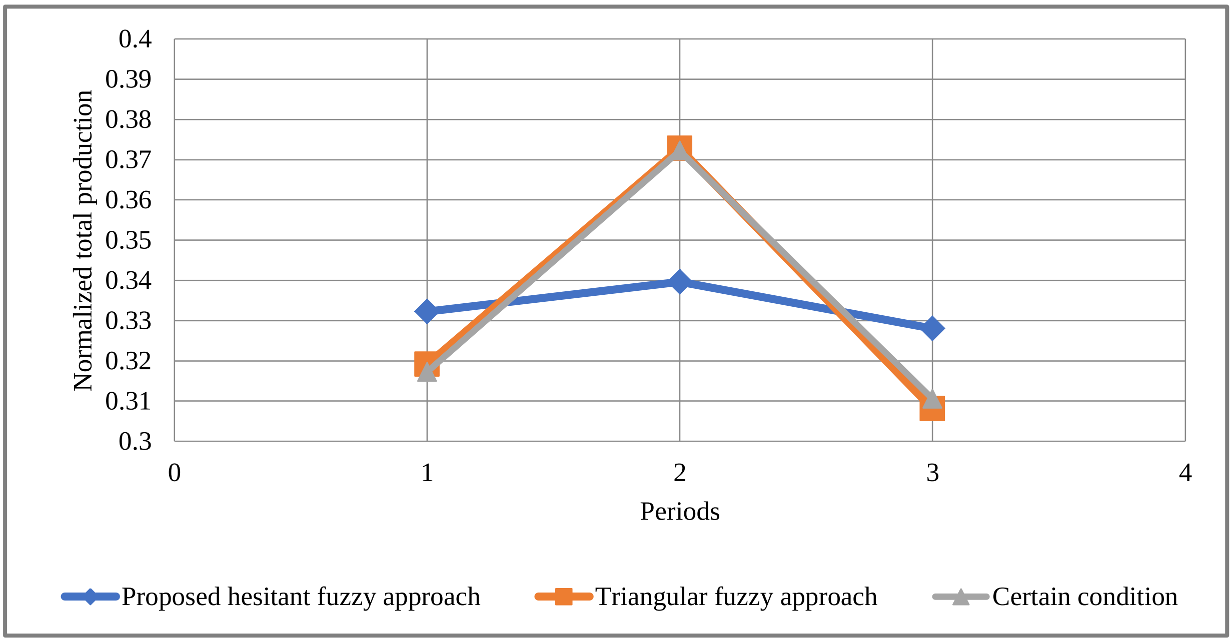

Figure 6. Comparative analysis regarding the total produced bio-products in each period

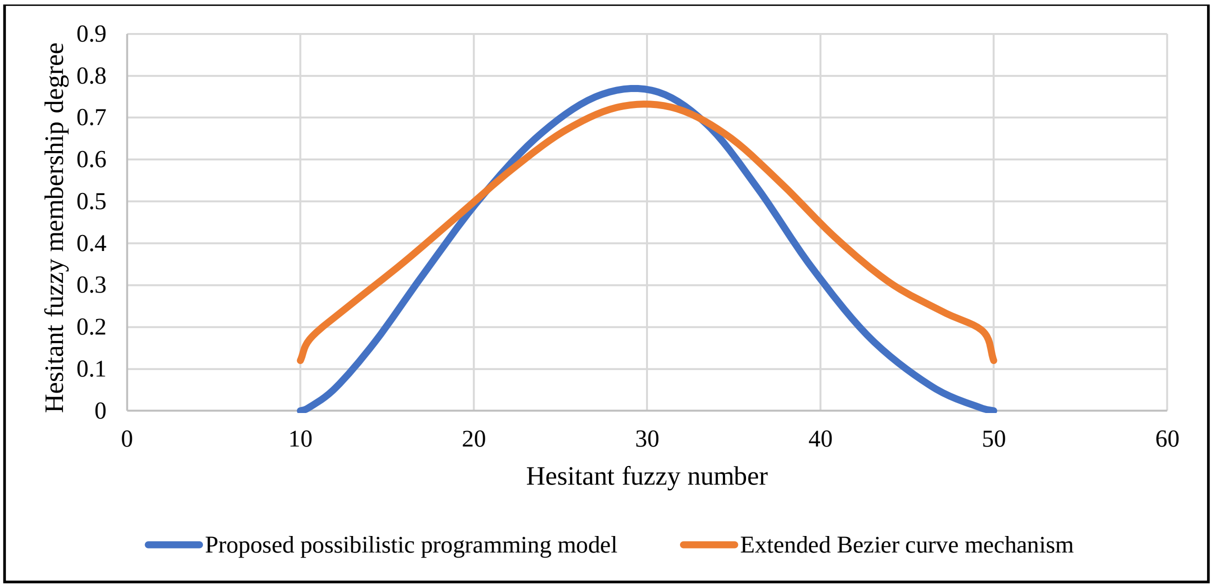

Figure 7. Extended Bezier curve mechanism versus the proposed possibilistic programming model

Table 1. The test problems sizes by defining the values of sets

Test problem no.	No. of biorefinery site location (i)	No. of harvesting site location (h)	No. of bio-product customers (m)	No. of conservation technologies (k)	No. of disruption risks types (r)
1	2	3	3	2	2
2	3	4	4	2	2
Test problem no.	**No. of biorefinery capacities for bio-products production (l)**	**No. of biorefinery capacities for feedstock storage (l')**	**No. of feedstock types (b)**	**No. of bio-products (p)**	**No. of time period (t)**
1	2	2	2	2	2
2	2	2	3	3	3

Table 2. The bio-products demand value and their hesitant fuzzy membership degrees

	Demand value (x)	Hesitant fuzzy membership degrees ($h_{D_{mpt}}(x)$)
D_{mpt}	10	{0.10, 0.15, 0.20}
	20	{0.30, 0.50}
	30	{0.7, 0.85, 0.90}
	40	{0.50, 0.30, 0.20}
	50	{0.10, 0.20}

Table 3. The summary of the obtained results for both test problems regarding to different α-levels

Test problem no.	α-level	Proposed approach		Triangular fuzzy approach		Certain condition	
		Z-values	CPU time (s)	*Z*-values	CPU time (s)	*Z*-values	CPU time (s)
1	0.6	72817.07	0.10	75050.22	0.09	76315.01	0.31
	0.7	73604.99	0.11	75177.62	0.13		
	0.8	74392.91	0.12	75305.01	0.11		
	0.9	75180.83	0.09	75432.40	0.12		
	1	75968.75	0.11	75559.79	0.10		
2	0.6	102873.95	0.16	103868.75	0.15	104052	1.09
	0.7	103102.62	0.15	103893.12	0.14		
	0.8	103334.29	0.15	103917.50	0.16		
	0.9	103565.95	0.32	103941.87	0.29		
	1	103797.62	0.16	103966.25	0.18		

Hossein Gitinavard is currently Ph.D candidate at Department of Industrial Engineering and Management Systems, Amirkabir University of Technology, Tehran, Iran. He received B.Sc. and M.Sc. degrees from the School of Industrial Engineering, University of Tehran and School of Industrial Engineering, Iran University of Science and Technology, respectively. His main research interests include fuzzy sets theory, multi-criteria decision-making under uncertainty, artificial neural networks, and applied operations research. He has published several papers in reputable journals and international conference proceedings.

Mohsen Akbarpour Shirazi is an associate professor at the Department of Industrial Engineering and Management Systems, Amirkabir University of Technology. He received a PhD in Systems Engineering from Amirkabir University of Technology. His focus is on systems engineering and optimization, and his research in this area focuses on complex systems modeling and the analysis and design of structured systems. He has published more than 150 papers in scientific journals and conferences in the area of modeling and developing system, engineering problem solving, and mathematical modeling of large-scale and complex problems in the field of logistics and supply chain.

Mohammad Hossein Fazel Zarandi is professor at the Department of Industrial Engineering at Amirkabir University of Technology, Tehran, Iran and a member of the Knowledge-Information Systems Laboratory at University of Toronto, Canada. His main research interests focus on big data analytics, artificial intelligence, data modeling, soft intelligent computing, deep learning, fuzzy sets and systems, meta-heuristics, and optimization. Professor Fazel Zarandi has published over 25 books and Book Chapters, more than 300 scientific journal papers, more than 200 refereed conference papers, and several technical reports in the above areas, most of which are also accessible on the web. He has taught several courses in big data analytics, data modeling, fuzzy systems engineering, decision support systems, information systems, artificial intelligence and expert systems, systems analysis and design, scheduling, deep learning, simulations, and multi-agent systems at several universities in Iran and North America.