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Multi-period home health care routing and scheduling problem with the medical grouping of patients

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Abstract. Home health care service has significant importance in modern societies. In most of the active institutions in this field, the traditional procedure is used for planning and managing health personnel and determining patient visit sequence. This procedure usually causes an increase in costs and reduces patients' satisfaction. This paper, for the first time, groups the patients in a model according to the level of emergency and discriminating in their examination. Considering dependency and independence of patient visits to each other, assuming multi-depot and multi-period issues are attractive aspects of the proposed model. The model is solved with General Algebraic Modeling System (GAMS) for small scale and two variable neighborhood search algorithm and simulated annealing algorithm are used to solve large scale problems and their performances are compared. The results indicate minimizing total cost and also increasing patients' satisfaction by the proposed model.

1. Introduction

The World Health Organization has stated that the rate of the care-dependent elderly person in western and European countries will strongly increase within the next ten years [1]. The official reports from this organization also exposed that the elderly people prefer to receive care at their homes, not at the health centers [2].

These reasons stimulate the Home Health Care (HHC) center to provide a full range of services at patients' homes by using a group of personals [3]. In this way, while reducing health system costs, The HHC centers also reduce the number of occupied beds in hospitals [4]. Especially in the current situation where the COVID-19 pandemic has infected all medical centers, the HHC centers, by providing medical services at home, prevent the elderly from getting sick [5]. Also, as reported by Fathollahi-Fard et al. [6], the HHC services are one of the important factors in the economic growth of western countries such as the USA. The HHC centers provide more than 2.22 million jobs opportunities in 2019, employing more than 1.51 million workers in the process in the USA [7].

In most of the active centers in this field, the traditional procedure is used for planning and managing health personnel and determining patient's visit sequence. In other words, an experienced nurse who has the role of coordinator performs this assignment [8]. This procedure usually causes an increase in costs and reduces patients' satisfaction. Also, the managers of HHC centers considering that quality of services and patients' satisfaction has a meaningful effect on their

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institution revenue [9], to make an optimal decision and reach the predetermined aims such as economic goals, an encounter with complicated and important limitations. Some of these aims such as minimizing the related cost of personnel movement, minimizing the related cost of assigning personnel to the patients, minimizing traveled distance by the personnel, and minimizing spent time for personnel movement and visiting patients are common aims in routing optimization problems as well as routing-scheduling optimization problems, which are considered in HHC optimization. Some researchers have considered these aims are as single [10–15], and others applied them in combination with other particular HHC aims. In addition, there are several and different limitations. The time window for personnel and patients, rules corresponding with personnel work hour, patients' preferences in assigning personnel to them, assigning personnel to patients as to their qualitative level, ability, and specialty are some of the basic limitations of HHC problems pointed in several papers [11,12,16–18]. Combining these aims and limitations with other conditions conducts a situation near to the real world condition.

Fernandez et al. [19] for the first time, examined assigning issues of nurses to different areas in the frame of a single period probable model. The objective of their model was to minimize the distance that each of the personnel travels to visit patients. Their paper, investigating the behavior of a set of nurses working in the UK, used an estimation method to solve the proposed model. Also Begur et al. [11], for the first time, planned HHC activity in the frame of a multi-period model, minimizing the duration time of personnel movement in a health care institution in Birmingham. They introduced a mixed-integer programming model and solved it with a heuristic method.

The necessity of the simultaneous presence of several persons to provide a service to a patient and time dependency of some services to each other asked by a patient was investigated by Bredström and Rönnqvist [20]. In this paper, they assume some services asked by some patients and old individuals must be done, with a time pause, before or after other services. Also, they assume the simultaneous presence of several nurses at the patient's home to provide a special service to him/her. Their model made conditions to distribute the workload of visiting the patients as balanced between personnel. They used a heuristic method for their model. These conditions have been indicated in Mankowska et al. [21]. Latter paper, dealing with routing and assignment problems, divides patients into two categories: (1) Patients require a single nurse service and (2) Patients require the presence of two nurses. The second category also is divided into two subcategories, those who require the simultaneous presence of two nurses at home and patients who require the presence of two nurses with a time pause between them. Also, they developed an adaptive Variable Neighborhood Search (VNS) algorithm to solve their model. Along with this, Dohn et al. [22] suggested a branch and price algorithm for scheduling simultaneous tasks. Their model aimed to increase the number of visits assigned to each person.

Rasmussen et al. [23] proposed a model to minimize transportation cost, dissatisfaction resulted from the inappropriate assignment of personnel to the patients and several unperformed visits. They introduce five different types of possible dependencies between services asked by patients and, to solve their model, they used branch and price method.

Nowak et al. [24] made conditions by focusing on this issue that establishing a permanent relationship between personnel and patients improves patient's medical process, provide conditions a patient is assigned to certain personnel as much as possible, so the patient feels better and personnel encompasses patient's medical process completely. Also, they assume the duration of visits has an opposite relationship with the number of visits in the week.

Hiermann et al. [25] deal with the optimal assignment of home cares to patients. They proposed a two phases approach. The random solution generated in the first phase improves during the second phase applying each of four metaheuristic; VNS, memetic algorithm, Simulated Annealing (SA), and scatter search. The memetic algorithm had better performance than other algorithms. Minimizing overtime and personnel movement costs were among the aims of their model. Braekers et al. [16] developed a bi-objective mathematical model. The first objective minimizes the total cost and the second objective maximizes the patient's convenience. Dynamic metaheuristic based on local searches is used to solve the model. Shi et al. [8] presented an HHC routing and scheduling problem by considering stochastic travel and service times. Different metaheuristics including genetic algorithm, SA, Bat algorithm, and firefly algorithm are used. Cappanera et al. [26] developed a robust optimization model for HHC center planning. A non-standard cardinality-constraint robust approach was used to address the model. Liu et al. [27] presented a bi-objective mixed integer mathematical model for planning of nursing visits and the daily traveling route scheduling. The first objective of their model is to minimize the total operation cost of the healthcare center and the second one is to maximize the patient's satisfaction. Also, an ϵ-constraint method is adopted to solve the model. Also, Lin et al. [28] presented jointly rostering, routing, and rerostering for HHC medical services. Their model has two objective functions; one objective minimizes the total overtime costs of all nurses and the other minimizes the total transportation costs of personnel. A hybrid metaheuristic based on harmony search algorithm and genetic algorithm is used to solve their model.

In another different study, Demirbilek et al. [29] presented a multi-period HHC scheduling considering working time balancing in a dynamic environment. Also, Tanoumand and Ünlüyurt [30] presented a resource-constrained HHC vehicle routing problem for providing medical services by nurses and aids at patients' homes. By categorizing patients according to their needs, they presented a model for minimizing the transportation cost and proposed a branch-and-price algorithm to solve it.

To read more about the HHC planning problem, refer to recent review literature [31,32].

This paper, for the first time, presents grouping the patients according to their physical and medical condition. Due to the lack of specialists cooperating with HHC institutions, overlapping time window of some patients, and necessity of providing services to more needy patients, it may happen situations that giving service to all patients is not possible. For this reason, in our model, patients are divided into three categories: "A" (most important patients), "B"(important patients), and "C"(ordinary patients). The emphasis of the model is to provide services to groups "A", "B", and "C", respectively. The model is solved with General Algebraic Modeling System (GAMS) for small scale and two algorithms, VNS and SA, are used for large scale problems. The performances of these algorithms are compared. Finally, some managerial insights are deployed to achieve sustainability for HHC institutions.

The paper has the following structure: Section 2 defines the problem. Section 3 introduces the mathematical model of the problem. Section 4 presents solution methods. Section 5 deals with numerical examples and finally, Section 6 is devoted to the conclusion and future suggestions.

2. Definition of the problem

The model proposed in this paper is denoted on the graph $G = (V, A)$ that $V = \{0_p, 1, 2, 3, ...I, n_p\}$ includes personnel homes (0_p and n_p) and patients' homes $(1, 2, 3, ..., I)$ are nodes of this graph and $A = \{(i, j)|i, j \in V, i \neq j\}$ are relative routes between patients and personnel's homes or the same arcs of the graph. Patients in each of days, need one or two visits and finally, in the examined time interval, they require a certain number of visits represented by f_i. The medical plan of each patient is determined at the beginning of the week and, based on the medical process of the patient and the medical group to which he/she belongs (A, B, or C), planning is done. On the other hand, HHC institution has some employees that some of them are permanently in access and others are one or several days in a week in access; parameter of γ_p^d indicates this issue. The institution must determine how many employees and with which quality is needed every day. This is done concerning the number of patients and their medical group. Due to employee shortages, some patients' requests may not be processed. The model presented in this paper, by grouping patients, tries to ensure that patients with medical priorities are served by the institution.

Generally, the personnel, after beginning the work shift, starts movement from his/her home, by their vehicle (private car, public transportation, bike or walking) to visit the patients, and finally, they come back home at the end of their work shift.

2.1. Parameters

p	Set of personnel, $P = \{1.....\|P\|\}$
d	Set of time periods, $T = \{1.....\|T\|\}$
c_{ij}^p	Cost of movement personnel p concerning the mode of transportation from node i to node j
Ca_i^p	Cost of visiting the patient i by the personnel p
Sc_p	Cost of daily employment of personnel p
Cf_p	Cost of overtime unit of personnel p
γ_p^d	Equal to 1 if personnel p is in access in day d; otherwise 0
β_i^p	Equal to 1 if personnel p can visit patient i; otherwise 0
q_p^d	Personnel p is allowed to start his/her activity in day d after this time
${q'}_p^d$	Personnel p must finish his/her activity in day d before this time
lt	Legal time of activity of each person in a day
pt	The maximum time that each person is allowed to do overtime in a day
La_i	Penalty cost of not responding to patient i according to the patient's medical group
s_i^d	Equal to 1 if patient i needs service in day d; otherwise 0
η_i	A value between 0 and 1 that shows the importance of patient i according to his/her medical group
e_i^d	The earliest time asked by patient i in day d
u_i^d	Ideal time desirable for patient i in day d

l_i^d The latest time asked by patient i in day d

ts_{ij}^p Movement time from patient i's home to patient j's home added to patient j's visit time by personnel p

$V_{ij}^{\min}$ The minimum time interval between two visits of patient i (i and j are two different nodes in one geographical area which means they represent the home of a patient)

$V_{ij}^{\max}$ The maximum time interval between two visits of patient i

f_i The number of required visits of patient i in the planned time period

2.2. Variables

x_{ij}^{pd} Equal to 1 if personnel p moves from patient i's home to patient j's home in day d; otherwise 0

w_i Equal to 1 if the institution contracts with patient i to give him/his service; otherwise 0

g_i^d Equal to 1 if one person visits patient i in day d; otherwise 0

t_i^{pd} Starting time of the visit of patient i by personnel p in day d

Ot_p^d Overtime of personnel p in day d

3. Mathematical model

Considering the parameters and variables defined in the previous section, the following gives the mathematical model of the problem:

$$\text{Min } Z=\sum_i\sum_j\sum_p\sum_d c_{ij}^p\,x_{ij}^{pd}+\sum_{i\in\{0_p\}}\sum_j\sum_p\sum_d Sc_p\,x_{ij}^{pd}+\sum_i(1-w_i)\,La_i+\sum_i\sum_{j\in V\backslash\{0_p,n_p\}}\sum_p\sum_d Ca_j^p\,x_{ij}^{pd}+\sum_p\sum_d Cf_p\,Ot_p^d, \tag{1}$$

$$\sum_{i\in\{0_p\}}\sum_j x_{ij}^{pd}\le 1 \qquad \forall p,d, \tag{2}$$

$$\sum_i\sum_{j\in\{n_p\}} x_{ij}^{pd}\le 1 \qquad \forall p,d, \tag{3}$$

$$\sum_i x_{ik}^{pd}=\sum_j x_{kj}^{pd} \qquad \forall k\in V\backslash\{0_p,n_p\},p,d, \tag{4}$$

$$\sum_j x_{ij}^{pd}\le\gamma_p^d\,\beta_i^p\,s_i^d \qquad \forall i\in V\backslash\{0_p,n_p\},p,d, \tag{5}$$

$$\sum_j\sum_p x_{ij}^{pd}+(1-g_i^d)=1 \quad \forall i\in V\backslash\{0_p,n_p\},p,d, \tag{6}$$

$$\left(\eta_i(u_i^d-e_i^d)+e_i^d\right)\sum_j x_{ij}^{pd}\le t_i^{pd}\le\left(\eta_i(u_i^d-l_i^d)+l_i^d\right)\sum_j x_{ij}^{pd} \quad \forall i\in V\backslash\{0_p,n_p\},p,d, \tag{7}$$

$$q_p^d\le t_i^{pd}\le {q'}_p^d \qquad \forall i\in V\backslash\{0_p,n_p\},p,d, \tag{8}$$

$$t_i^{pd}+ts_{ij}^p\,x_{ij}^{pd}\le t_j^{pd}+l_i^d(1-x_{ij}^{pd}) \quad \forall (i,j)\in V\backslash\{0_p,n_p\},p,d, \tag{9}$$

$$e_i^d\,g_i^d+\sum_p t_{ij}^{pd}+V_{ij}^{\min}\le\sum_p t_j^{pd}+l_j^d\,g_i^d\le e_i^d\,g_i^d+\sum_p t_{ij}^{pd}+V_{ij}^{\max} \qquad \forall (i,j)\in\delta,d, \tag{10}$$

$$x_{ij}^{pd}=0 \qquad \forall i,j(i=j),p,d, \tag{11}$$

$$\sum_d s_i^d\,g_i^d=f_i\,w_i \qquad \forall i, \tag{12}$$

$$\sum_i\sum_j ts_{ij}^p\,x_{ij}^{pd}-lt=Ot_p^d \qquad \forall p,d, \tag{13}$$

$$Ot_p^d\le pt \qquad \forall p,d, \tag{14}$$

$$x_{ij}^{pd},w_i,g_i^d\in\{0,1\} \qquad \forall i,j,p,d, \tag{15}$$

$$t_i^{pd},Ot_p^d\ge 0 \qquad \forall i,p,d. \tag{16}$$

The first term of the objective function (1) is the transportation cost between available nodes. The second term is the number of personnel. The third term represents the penalty cost of not meeting a patient's demand. It should be noted that different groups of patients have different penalty costs. The fourth term computes patient visit cost, which varies according to the patients' medical group and the level of specialization of the employee. Finally, the fifth term computes personnel's overtime cost in the planned period. Sets of Constraint (2) and (3) determine origin and destination movement. Each personnel starts his movement from his home and finally comes back home. The set of Constraint (4) shows the balance of the flow. The set of Constraint (5) guarantees personnel is assigned to a patient on a particular day if the following conditions are met: (1) The patient asks to visit on that day and (2) the qualified personnel is in access on that day. The set of Constraint (6) indicates either a patient is visited by available personnel or he would not be visited in the planning period at all. The set of Constraint (7) determines the personnel's arrival time at the patient home. This set of constraints consider the patient's medical group and his treatment importance level. Parameter η_i represents how much arrival time is near or far from the ideal time of the patient. The value of this parameter would be higher for a more important group and this causes a higher satisfaction level of the patient. The set of Constraint (8) shows starting time of service for each patient. This set considers the time window of personnel. The set of Constraint (9) considers two consecutive visits by one person in a day. The set of Constraint (10) corresponds to the consecutive visits in a day for one patient. Some patients need two visits in one day, these two visits must be performed with a minimum and maximum time pause from each other. The set of Constraint (11) guarantees movement is not done from one home to the same home. The set of Constraint (12) represents that the institution determines which of the patients is to be given service. If the institution decides to visit a patient in a day, the service must be given to him during the planning period completely, and otherwise, no longer he would be given service. Sets of Constraints (13) and (14) compute personnel overtime and sets of Constraints (15) and (16) determine the type of decision variables.

4. Solution methods

The proposed model is a multi-depot traveling salesman problem that there is a possibility of several services in several times to one patient in one day where these services may be dependent or independent from each other. Based on Toth and Vigo [33], the studied problem in this paper is NP-hard. Hence to solve the problem in a small scale, GAMS software is applied and, in a large scale, VNS and SA have been applied.

4.1. Proposed VNS algorithm

VNS algorithm systematically searches the solution space by changing the neighbors of a solution. This algorithm starts with a primary solution and then it chooses a neighbor randomly among the nearest neighbors ($N_l(l = 1, ..., l_{\max})$). Therefore, the search is done to obtain the local optimal solution. In the case the obtained solution is better than the best solution found up to now, the best solution is updated; then another random neighbor is generated from the same neighbor; otherwise, another neighborhood structure N_{l+1} is searched. All the above-mentioned remarks are repeated until condition $l = l_{\max}$ is met.

Many researchers have utilized neighborhood search algorithm in the HHC field [21,25,34,35], so this paper applies the VNS algorithm as a solution method.

The steps of the proposed algorithm are the construction of the initial solution, local search, and solution representation. The Following explains in detail.

4.1.1. Constructing the initial solution

In the first step, to begin the algorithm, a series of primary solutions including the primary route and each exit time of each person from his home is determined. The algorithm applies the primary route based on Clarke and Wright [36] , which is a well-known method in the literature of vehicle routing problems.

Step 1. Concerning the parameters of the model, for each period, personnel p starts his route from 0_p (the personnel p's home), who has the smallest index among the current personnel in the period. Then we form sub tours $0_p - i - 0_p$ for each node $i \in V$.

Step 2. For each pair of nodes $i \in V$ and $j \in V$ if $i \neq j$, $i \neq 0_p$, and $j \neq 0_p$, saving value, computed from Eq. (17), is obtained. This is performed to merge the pair of sub tours $0_p - i - 0_p$ and $0_p - j - 0_p$ and converting to tour (route) $0_p - i - j - 0_p$.

$$s_{i_k\,j_k} = c^p_{i0_p} + c^p_{0_p j} - c^p_{ij} \qquad \forall p, i, j \in V, i \neq j, i \neq 0_p, j \neq 0_p\,. \tag{17}$$

Step 3. The saving values of each region are sequenced in descending order.

Step 4. The task of merging sub tours for arcs of $i - 0_p$ and $0_p - j$ is performed if two following conditions are met simultaneously: Firstly, personnel p can visit patients i and j. Secondly, the sum of visiting time of each of the available nodes in route and the personnel's movement time does not exceed the sum of legal time and personnel's overtime. It should be noted that each person must fulfill service at the maximum level, which is the sum of legal time and overtime. Also each personnel exit from his home in q^d_p.

Step 5. If some of the nodes remain, the next person that has the smallest index between the remaining personnel on that day is selected and then we go to Step 4. The above procedure is continued until all the available personnel is assigned in that period or all the patients are given service.

So far the obtained solution (s_0) is considered as the initial solution ($s = s_0$) and objective function $f_d(s)$ is computed. Then the next stage is begun.

$$f_d(s) = L = \sum_{p} \sum_{(i,j)\in V} d_{ij}\, x^{pd}_{ij} \qquad \forall d, \tag{18}$$

where d_{ij} is the distance between node i and j and L is the length of the route.

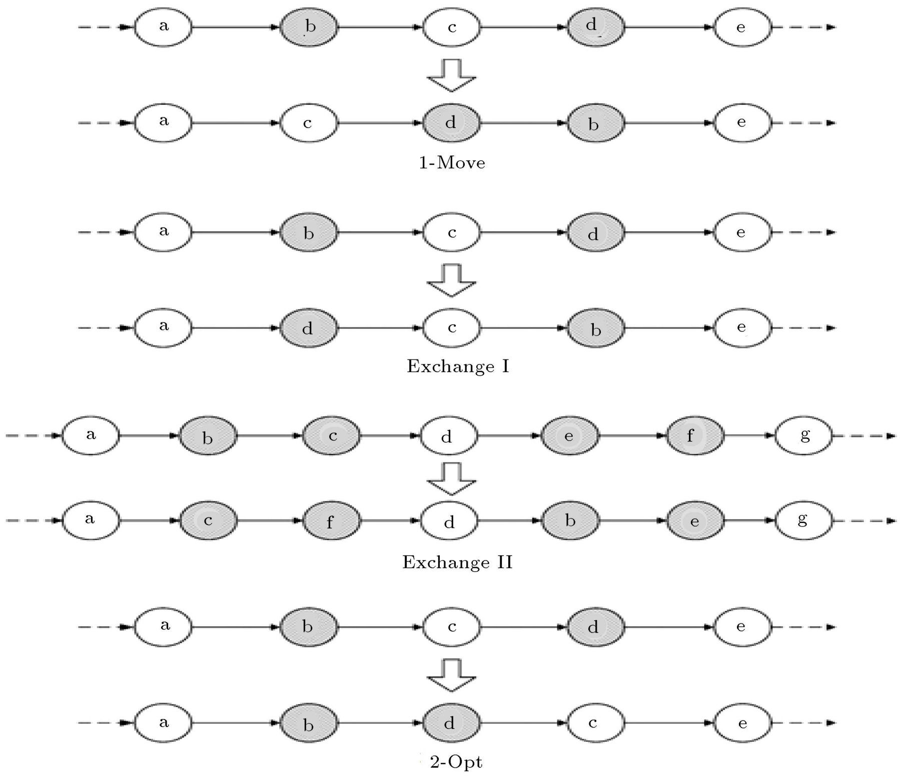

Figure 1. Operators representation.

4.1.2. Local search

In the local search stage, two inter-route operators and four intra-route operators are used to improving the current solution.

The first neighborhood operator of inter-route: One node is chosen randomly from one route for substitution. This route is defined as the origin route. Among the other routes, the route which has the following most term is chosen as the target route:

$$\sum_{i \in R} (l_i^d - (t_i^{pd} + ts_{ij}^p)) \qquad R \text{ is selected route.} \qquad (19)$$

The node selected from the origin route (for example 'a') is placed before the nearest node of the target route. If the sum of the patients visiting time of this route exceeds the sum of the legal hours and overtime of personnel, one of the target route nodes (for example 'b') chosen randomly to be transformed to the origin route in a way that in excess of the target route visit is removed and also the limitation of personnel's activity of origin route is not violated. Node 'b' is entered before the nearest node of the origin route.

The second neighborhood operator of inter-route: One node is chosen from each route randomly (for example 'a'). Among the routes which have the empty capacity as much as visiting time requested by patient 'a', one route is chosen randomly and node 'a' is placed before the nearest node belonging to the target route.

The four intra-route operators used in the proposed VNS algorithm are: *1-Move, 2-Opt, Exchange I,* and *Exchange II* as illustrated in Figure 1.

Two compositions of $C1$ and $C2$ constructed from four inside route operators defined in Table 1 were shown to have better performance among other developed compositions.

At first, one of two inter-route operators is implemented randomly and then one of two compositions defined in Table 1 for each route is randomly implemented.

Now the solution (s') is obtained. If $f_d(s) \geq f_d(s')$ the obtained neighborhood is replaced with the current solution: $s = s'$

If $F^* \succ f_d(s)$ then $F^* = f_d(s), s = s'$.

4.1.3. Solution representation

If the number of iterations exceeds the maximum iterations of the algorithm, the algorithm terminates and the best solution found so far is displayed.

4.2. SA algorithm

This algorithm applied for the first time by Kirkpatrick et al. [37] is the repetitive and random algorithm that has been used in solving combinatorial optimization problems widely. The following describes the SA algorithm developed for the problem under study.

Table 1. Compositions of operators used for local search.

Composition	Details
$C1$	In this composition, at first, the algorithm generates a random number between 1 to 4 and then: 1) If the number is equal to 1, operator *1-Move* is conducted. 2) If the number is equal to 2, operator *2-Opt.* is conducted. 3) If the number is equal to 3, operator *Exchange I* is conducted. 4) If the number is equal to 4, operator *Exchange II* is conducted.
$C2$	In this composition, at first, the algorithm generates a random number between 1 to 4 and then: 1) If the number is equal to 1, only operator *Exchange I* is conducted. 2) If the number is equal to 2, at first operator *Exchange I* is conducted and then *1-Move* is conducted. 3) If the number is equal to 3, operator *Exchange I*, *1-Move*, and *2-Opt.* are conducted respectively. 4) If the number is equal to 4, operator *Exchange II*, *1-Move*, *2-Opt.*, and *Exchange II* are conducted respectively.

4.2.1. Generation of initial solution
The procedure used here is the same defined for the VNS algorithm.

4.2.2. Local search
An inside route operator and two inter-route operators are used. The inter-route operators were the same used in the VNS algorithm. The inside route operator is introduced as follows:

The neighborhood operator of the inside route: One node is chosen from each route randomly. This node is displaced inside the same route considering the patient time window limitations until the route length (L) is reduced. Then this process is implemented for the other routes.

The solution obtained is called s' and then updates the s and F^* as explained in the VNS algorithm. In each iteration of the algorithm, the objective function is compared with the best objective function found so far. If the former is better, the best solution is updated; otherwise, it would not change.

4.2.3. Objective functions
The objective function considered for the infeasible solution is the sum of all the reversed time intervals in personnel's movement routes:

$$TWV(s)=\sum_{p}\left[\sum_{i\in V}\Big(\sum_{j\in V}(t_i^{pd}+ts_{ij}^{p})\,x_{ij}^{pd}-l_i^{d}\Big)^{+}\right]. \quad (20)$$

Also the objective function considered for feasible solution is the total travelled distance by the personnel:

$$f_d(s)=\sum_{p}\sum_{(i,j)\in V} d_{ij}\,x_{ij}^{pd}. \quad (21)$$

4.2.4. Solution representation
The algorithm terminates when the final temperature is reached. When this condition is met, the best solution found is displayed.

5. Numerical experiment

We have used 24 instances with a different number of employees (personnel) and patients which are tested 10 times per instance. Each problem consists of some patients and personnel. In the smallest size of the problem, there are 8 patients and 2 personnel and, in the largest size of the problem, there are 350 patients and 40 personnel. All the patients' and employees homes have been chosen randomly in 150*150 space of distance unit. The distance between two homes is assumed as Euclidean metric that traveled by the personnel. Three different movement methods including walking, public transportation, and private vehicle are considered. Five service types $s=\{1,...,5\}$ have been considered. Two services are implemented just by the expert personnel and the others by all the personnel. As it was explained before, there is the possibility that some patients need two services in one day. These patients constitute 20% of all the patients. The value of $V_{ij}^{\min}$ is random in the range of 60 to 120, and the value of $V_{ij}^{\max}$ has been assumed randomly in $[2V_{ij}^{\min}, 4V_{ij}^{\min}]$. In addition, the patients are classified into 3 categories "A", "B", and "C". In each instance, 30% of patients are from group "A", 50% of patients are from group "B" and the remaining patients are from group "C". The value η_i is 0.5, 0.3, and 0 for groups "A", "B", and "C", respectively. Also, the value of La_i is 500, 350, and 100 for different groups, respectively. The patient time window was taken in the range of 45 to 100, and the length of service time is computed randomly in the range of 15 to 30.

The proposed model was implemented by GAMS software 24.1.2 with a time limitation of 7200 seconds and it has been conducted on the computer Intel(R) Core i3-2328M CPU with 2.2 GHz and 4 GB RAM.

In the conducted experiments, it was observed by ignoring Constraint Eq. (9); the relaxed problem

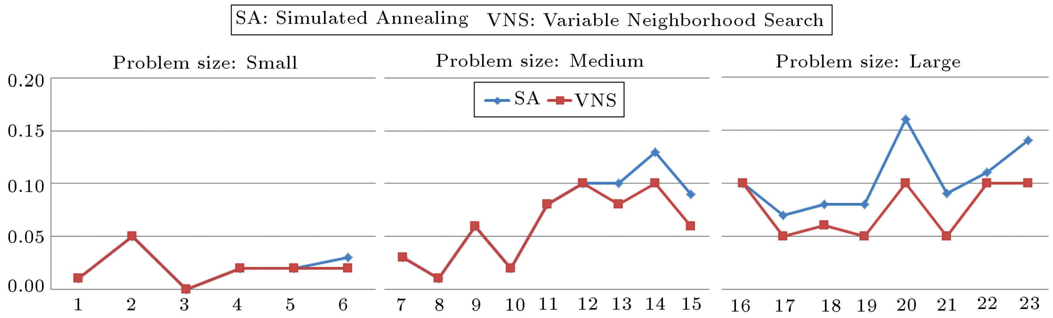

Figure 2. The gap between two metaheuristics objective functions and the lower bound of GAMS in different problem sizes.

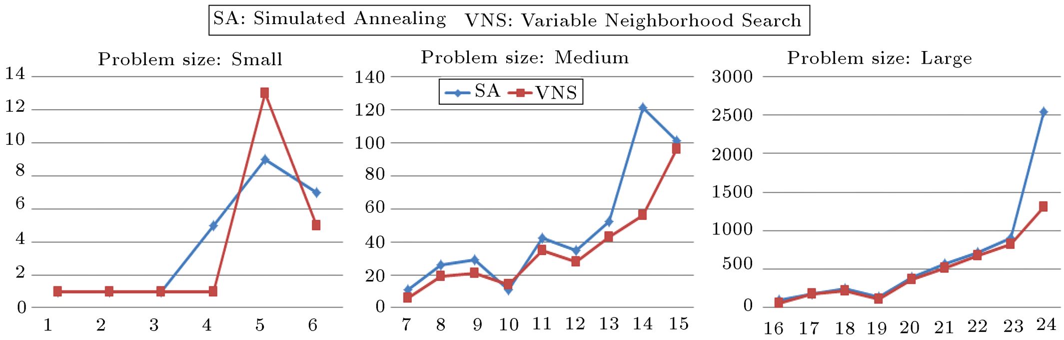

Figure 3. Comparison of solution times in two metaheuristic algorithms in different problem sizes.

can reach an ideal objective function in a short time. Table 2 gives the results of numerical experiments in different size problems. For the instances the model has reached the optimal solution within a time less than 7200 seconds, the results of the relaxed problem have been compared with those obtained by GAMS software and for other instances, results of each metaheuristic algorithm have compared with those obtained by lower bound of relaxed problem. The statistical result, after 10 independent runs, in terms of the best, mean, worst, and the Standard Deviation (STD) obtained from the objective value by two metaheuristic algorithms are given in Table 2. A comparison of the two algorithms is presented below from different perspectives.

Figure 2 illustrates the gap between the value of the objective function gained from two metaheuristic algorithms and the lower bound of GAMS for instance 1 to 23 in different problem sizes. As can be seen, the average gap in the small size of problem is 0.022 and 0.02 for SA and VNS, respectively. This value is equal to 0.069 and 0.06 for medium and 0.104 and 0.077 for large size of problems for SA and VNS, respectively. Also, in the worst case, the gap value in SA is 0.16 and in VNS is 0.1. Thus, the VNS algorithm shows better performance in different size problems.

Figure 3 is about the solution time of SA and VNS algorithms in different problem sizes. As can be seen, in all but one instance, the VNS performed better than the SA in different size problems.

In order to compare the performance of SA and VNS algorithms in different problem sizes, Table 3 is presented. In this table, the number of instances that SA and VNS algorithms could reach the best solution in different sizes of the problem is reported in the first and second columns, respectively. The average of STD obtained in different instances in each size of the problem is also compared.

Also, the worst solution found in VNS is better than SA In all but one instance (instance 3 in small size of the problem). Therefore, as can be seen, due to less STD and better solutions, VNS algorithm, especially in large size of the problem, has better performance than SA algorithm and it seems reliable to solve the HHC routing optimization problems in different sizes.

5.1. Case study

According to official statistics, in 1994, 10% of the country's population was made up of the elderly and over 60 years of age. This proportion is expected to reach 33% in the next 35 years. Caring for this group, which will include about a third of the community's population, besides the consideration to care about patients, children, pregnant women, and disabled people is important. This has been the focus of attention in recent years in Iran, and several HHC centers have been operating in this area.

The Sepidgoster HHC center is one of these institutions founded in 2008, providing 24-Hour services to

Table 2. Results of numerical experiments.

Problem size	Relaxed problem		GAMS		SA						VNS					
	LB	GAP	Z^*	CPU (S)	Best	Mean	Worst	STD	CPU (S)	GAP	Best	Mean	Worst	STD	CPU (S)	GAP
Small	1055.342	0.00	1056.492	2	1065.938	1065.940	1065.943	0.00105	1	0.01	1065.938	1065.940	1065.941	0.00082	1	0.01
	1641.683	0.04	1710.970	19	1723.919	1723.920	1723.922	0.00085	1	0.05	1723.920	1723.921	1723.922	0.00092	1	0.05
	1433.801	0.00	1433.801	707	1433.801	1433.801	1433.802	0.00088	1	0.00	1433.801	1433.801	1433.802	0.00088	1	0.00
	1407.630	0.01	1424.960	92	1435.828	1435.830	1435.832	0.00117	5	0.02	1435.828	1435.830	1435.831	0.00091	1	0.02
	2040.751	0.02	2074.103	75	2081.688	2081.690	2081.692	0.00089	9	0.02	2081.689	2081.691	2081.692	0.00090	13	0.02
	1644.662	0.01	1661.626	996	1693.999	1694.000	1694.001	0.00106	7	0.03	1677.549	1677.550	1677.551	0.00106	5	0.02
Medium	1673.453	0.02	1714.922	1438	1723.688	1723.690	1723.692	0.00122	11	0.03	1723.687	1723.690	1723.692	0.00118	6	0.03
	2735.124	0.00	2735.124	452	2762.468	2762.470	2762.472	0.00106	26	0.01	2762.468	2762.470	2762.472	0.00104	19	0.01
	2120.750	0.05	2236.213	5203	2248.098	2248.100	2248.103	0.00105	29	0.06	2248.098	2248.100	2248.101	0.00085	21	0.06
	2168.193	0.01	2194.881	5571	2211.548	2211.550	2211.553	0.00126	11	0.02	2211.548	2211.550	2211.551	0.00105	14	0.02
	3106.042	0.03	3211.267	6483	3354.517	3354.519	3354.522	0.00129	42	0.08	3354.518	3354.520	3354.521	0.00093	35	0.08
	3048.231	–	–	7200	3353.098	3353.100	3353.101	0.00096	35	0.10	3353.098	3353.100	3353.101	0.00100	28	0.10
	4046.402	–	–	7200	4451.008	4451.011	4451.013	0.00105	52	0.10	4370.119	4370.120	4370.121	0.00103	43	0.08
	3951.063	–	–	7200	4464.718	4464.720	4464.721	0.00111	121	0.13	4346.188	4346.190	4346.191	0.00110	56	0.10
	6195.405	–	–	7200	6752.979	6752.980	6752.983	0.00111	101	0.09	6567.118	6567.120	6567.122	0.00100	96	0.06
Large	5426.474	–	–	7200	5969.118	5969.120	5969.122	0.00113	95	0.10	5969.117	5969.120	5969.121	0.00103	57	0.10
	7150.032	–	–	7200	7650.539	7650.540	7650.541	0.00107	179	0.07	7507.537	7507.540	7507.543	0.00105	182	0.05
	7085.640	–	–	7200	7652.497	7652.499	7652.502	0.00135	253	0.08	7510.788	7510.790	7510.792	0.00108	225	0.06
	9022.291	–	–	7200	9744.068	9744.070	9744.072	0.00119	142	0.08	9473.408	9473.410	9473.411	0.00100	115	0.05
	11196.112	–	–	7200	12987.517	12987.519	12987.521	0.00155	401	0.16	12315.710	12315.710	12315.712	0.00107	375	0.10
	16308.463	–	–	7200	17776.198	17776.200	17776.201	0.00106	569	0.09	17123.900	17123.900	17123.901	0.00101	518	0.05
	24438.261	–	–	7200	27126.527	27126.530	27126.533	0.00144	715	0.11	26882.120	26882.120	26882.121	0.00110	678	0.10
	26677.411	–	–	7200	30412.227	30412.230	30412.232	0.00116	906	0.14	29345.200	29345.200	29345.201	0.00098	820	0.10
	–	–	–	7200	35523.329	35523.331	35523.332	0.00157	2543	–	35001.520	35001.520	35001.522	0.00125	1301	–

SA: Simulated Annealing; VNS: Variable Neighborhood Search; LB: Lower Bound; STD: Standard Deviation.

Table 3. Statistical comparison between SA and VNS.

The problem size	SA	VNS
• Small size of the problem		
Best solution	3 (from 6 instances)	2 (from 6 instances)
STD	0.00098	0.00092
• Medium size of the problem		
Best solution	1 (from 9 instances)	4 (from 9 instances)
STD	0.00112	0.00102
• Large size of the problem		
Best solution	0 (from 9 instances)	9 (from 9 instances)
STD	0.00128	0.00106

SA: Simulated Annealing; VNS: Variable Neighborhood Search; STD: Standard Deviation.

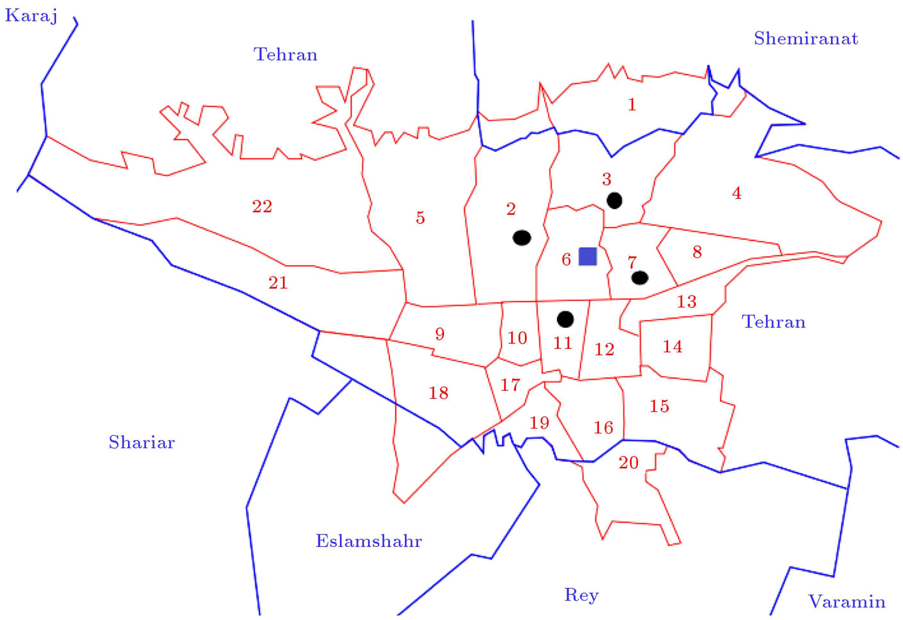

Figure 4. Institution's activities.

to patients, elderly people, and other people who require medical and nursing care at home.

The institute has 220 active nursing and psychiatric personnel, and it allocates them according to the patients' needs, the quality, and the ability of their personnel. The institute, which operates in Tehran's 6th region, has had the highest volume of service in the 2nd, 3rd, 7th, and 11th regions in the last month. This has shown in Figure 4.

The activities of this institution often include the care of the elderly, children, and disabled patients. since care for some elderly or children often takes more than a day, only those patients with a service time of less than an hour have been investigated. Also, patients who require more than one nurse simultaneously have not been studied. Table 4 which includes one-day service demand for regions 2, 3, 7, and 11 is as follows:

Table 5 provides the information about the medical personnel of the institute.

5.2. Sensitivity analysis

Giving service to the patients is different from giving service to some customers in other industries and patients' medical state should be taken into account. In this situation, there is not possible to give service to all the patients, at first those are selected who are in the emergency state (group A). Hence other patients would be visited as the second priority. This issue not only influences patients satisfaction significantly but also decreases system costs. At present, this issue has not been studied in the literature at the knowledge of authors. Two scenarios for study the effect of grouping the patients in Sepidgoster HHC center are compared in Table 6. The first scenario is without grouping and the last considering 3 medical groups for the patients; A, B, and C. Data in Table 6 corresponds to one working shift.

What will be seen in the following corresponds to the first example. The second example behaves the same as example 1. As it is observed in scenario 1,

Table 4. Information of patients' demand.

Region	No. of visits less than one hour	No. of visits with interdependent services	No. of patients from group A	No. of patients from group B	No. of patients from group C
2	5	3	1	2	2
3	7	1	2	2	3
7	5	2	2	3	0
11	8	2	2	4	2
Total	25	8	7	12	7

Table 5. Information of HHC personnel.

Personnel	Quality level of personnel	No. of days which personnel is present in the institute	Average cost for each visit	Permissible overtime (min)
Nurse	Level 1	5 days	20000	60
	Level 2	7 days	15000	0
	Level 3	7 days	10000	0
General physician	Level 1	3 days	40000	60
	Level 2	4 days	30000	0
	Level 3	4 days	25000	0
Specialist	Level 1	2 days	70000	120
	Level 2	3 days	50000	60

Table 6. The grouping patients effect on health care cost and patients' satisfaction.

	Example 1		Example 2	
	Scenario 1	Scenario 2	Scenario 1	Scenario 2
La_i	450	350,450,550	150	150,300,400
Total number of patients	25	25	75	75
Number of patients serviced by personnel	12	14	53	68
Percent of visited patients from group "A" by the total number of patients in this group	55%	100%	21.5%	100%
Percent of visited patients from group "B" by the total number of patients in this group	25%	62.5%	30%	86.5%
Percent of visited patients from group "C" by the total number of patients in this group	62.5%	0%	75%	12%
Total cost (*1000)	14504	10225	128536	79550
Cost of daily employment of personnel	270	315	4151	9873
Penalty cost of not responding to patients	5850	4150	39981	7463
Total cost of personnel overtime	120	60	359	117

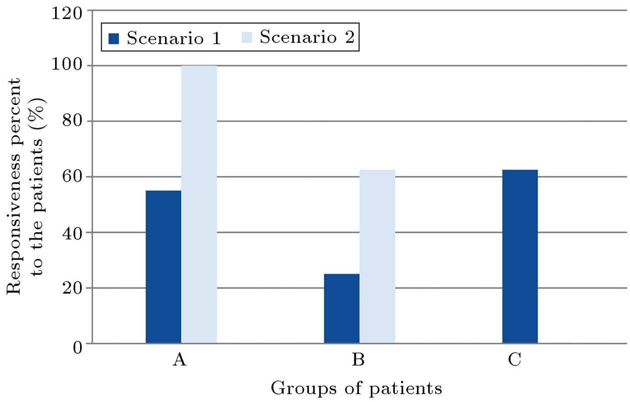

Figure 5. The effect of grouping on patients' satisfaction.

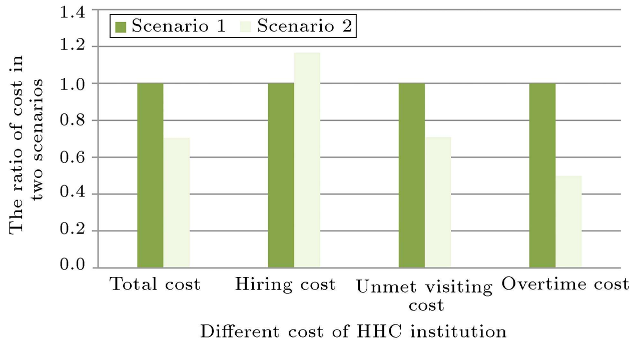

Figure 6. The effect of grouping on different HHC cost.

the cost of 450 units of money for not visiting each patient is paid. In scenario 2, 400, 450, and 500 units of money are paid for not giving service to groups "A", "B", and "C", respectively. Figure 5 denotes the effect of medical grouping of the patients on responsiveness to them.

Figure 6 illustrates the cost ratio of scenario 1 to scenario 2. It is observed that the hiring cost in scenario 2 is higher than scenario 1 but another cost is higher for scenario 1. The excess hiring cost of scenario 2 is due to the employment of expert personnel for responsiveness to more emergency patients. The total cost of scenario 1 is considerably higher than scenario 2, the ratio is 1.4.

Now the effect of increasing La_i in different variables of the proposed model is discussed. Here 5 experiments with different La_i have been conducted and variations related to the number of personnel and number of visited patients, cost of visiting patients by the personnel and cost of unmet visits, and costs related to the personnel employment and overtime have been examined. Figures 7–9 represent these studies respectively. The analysis has been conducted on one daily shift data of the institution.

As it is observed, since penalty cost has increased from 475 to 485, to prevent the increase in the number of unmet visits, the model decides to employ two new personnel (Figure 7). So this decision decreases the cost of unmet visits and increases the number of not

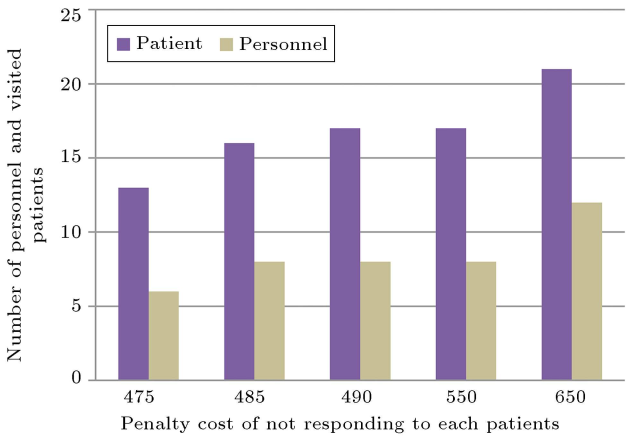

Figure 7. The effect of penalty cost on the number of personnel and visited patients.

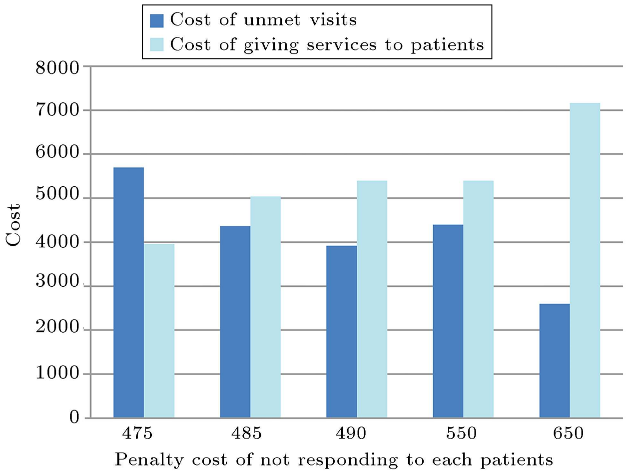

Figure 8. The effect of penalty cost on the service cost and unmet visits.

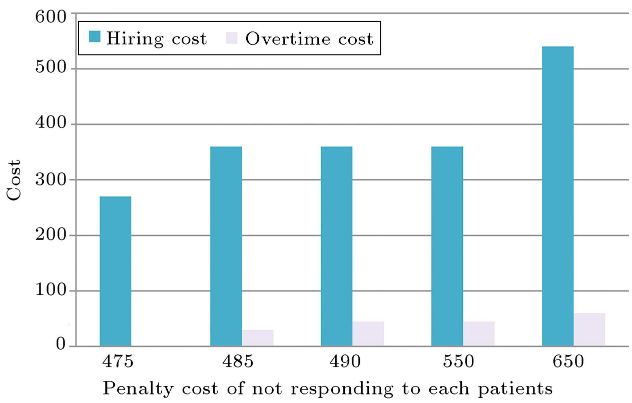

Figure 9. The effect of penalty cost on overtime and hiring cost.

visited patients, the costs related to giving service are increasing service cost (Figure 8). This decision increases hiring and overtime costs (Figure 9).

Increasing penalty cost from 485 to 490 makes the system not employ new personnel. In this case, the model by increasing personnel's overtime has responded to another patient's demand. The cost of giving services has increased and the cost of unmet services has decreased.

An increase in penalty cost from 490 to 550 has not made any change in the model decision-making. In this case, only unmet visits have increased. In the other words, the cost of unmet visits prefers employing new personnel.

Finally, by increasing penalty cost from 550 to 650, the model decides to employ 4 new personnel and responds to the visit of 21 patients. This decision decreases unmet visit cost, increases employment, overtime, and servicing costs. The above remarks denote that different decision-making is required under different groups made for patients.

6. Conclusion and future suggestions

This paper proposed a new mathematical model for routing and scheduling problem to plan and manage the personnel of Home Health Care (HHC). The necessity of proper service with the best quality at the most appropriate time to the patients is a permanent concern of HHC managers. This paper paid attention to this concern and classified the patients into different groups to provide service accordingly with priority to improve patients' satisfaction. A comprehensive analysis of the effect of grouping the patients on the corresponding costs and patients satisfaction was made. To solve the problem, two metaheuristic algorithms, Variable Neighborhood Search (VNS) and Simulated Annealing (SA) were developed. Statistical results are also reviewed for each algorithm in terms of mean, worst, best and Standard Deviation (STD) in all problem sizes, the VNS showed better performance. Considering the problem under uncertain conditions and developing other algorithms to solve the model can be considered as future research.

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