Nurse scheduling problem by considering total number of required nurses as well as nurses' preferences for working shifts: An algorithmic game-theoretic approach

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Abstract

In this paper, nurse scheduling problem (NSP) is studied by minimizing the total number of the required nurses as well as by maximizing the nurses' preferences for working shifts. In this setting, hospital's managers set the total number of the required nurses while nurse-chiefs select the required part-timers and then assign shifts to all nurses including the full-timers and the selected part-timers. Obviously, competition between the managers and nurse-chiefs to make decisions leads to a conflict between their objectives. In this point of view, a two-player game-theoretic framework can be established between them to set decisions. To our knowledge, this study is the first one that develops the game-theoretic approach to solve the NSP. In this setting, four game-theoretic models, including Managers-Stackelberg, Nurses-Stackelberg, Nash, and Centralized, are proposed based on the various competitive and cooperative interactions between the players. Moreover, a mathematical programming model is developed to obtain the equilibrium strategies. It is found that the managers and nurse-chiefs gain their best responses under the Managers-Stackelberg and Nurses-Stackelberg games, respectively. In the Nash game, they make decisions in order to meet their objectives, mostly. Moreover, the equilibrium strategies given by the Managers-Stackelberg and Centralized games are the same.

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1. Introduction

Nowadays, by increasing the need for health services, the governments have to assign a considerable share of their budget to health systems [1]. In this setting, hospitals are considered as the largest organizations in each country to provide health services. As a matter of fact, nurses are taken into account as one of the main cost components in hospitals' annual budget [2]. Therefore, developing an efficient operational plan to make the best use of the available nurses is a crucial decision for policy makers in hospitals [3]. This situation is exacerbated by an acute shortage of the available nurses in many countries [4]. It has been also reported that the major reasons of this nursing shortage are low salary paid to nurses, decline in enrollment at nursing schools, and decreasing the nurses' job satisfaction [5-7]. The problem can be more complicated by considering different factors like the government regulations, labor laws, and desire to accommodate the individual preferences [8, 9].

The consequences of a poor planning for the nurses are often evidenced by decreasing their job satisfaction and by increasing the number of the required nurses to meet the considered requirements [10, 11]. Lu et al. [12] have investigated relationship between the job satisfaction and the professional commitment. They have distributed a structured self-administered questionnaire to 2,197 registered female nurses with an average age of 28.56 years. Regarding the results, there is a positive correlation between the nurses' job satisfaction and the professional commitment to leave their profession. In addition, a discriminant analysis indicated that low job satisfaction is the main reason of 30.5% of the nurses who leave their profession [12]. Thus, considering factors that increase the nurses' job satisfaction is a vital decision for policy makers. In this point of view, assigning the desirable working shifts to the nurses is an effective way to increase their job satisfaction rate [13, 14].

Nurse scheduling problem (NSP) deals with assigning working days, vacations (days-off), and shifts to nursing staffs in hospitals' daily operations [15]. In the NSP, in general, the number of the required nurses is given for each day during planning horizon, and the aim is to assign the shifts to the nurses in order to meet the considered requirements. Several factors such as the workforce laws, the government regulations, and the status of the nurses at the end of the previous planning horizon need to be considered in arranging the nurses scheduling activities [16, 17].

The NSP has been widely studied in the literature. The various mathematical programming models, heuristic and meta-heuristic algorithms, and hybrid methods have been proposed to solve the problem. Earlier researches focused on the shift or days-off scheduling with the aim of minimizing the total nursing costs. In the past two decades, studies have mostly concentrated on the nurse scheduling with the objective of accommodating the individual preferences for the working shifts [18]. Below, several studies are addressed considering the nurses' individual preferences in the NSP.

Several researchers have applied the *mathematical programming models* to solve the NSP. Miller et al. [19] proposed a two-stage mathematical model to balance trade-off between the staffing coverages and the individual preferences. Azaiez and Sharif [20] developed a binary goal programming model to solve a multi-objective nurse scheduling problem. Their model could be used for instances with at most 22 nurses. Al-Yakoob and Sherali [21] presented a mixed integer programming model to minimize the total deviations from a central preference value. Furthermore, Guo et al. [15] formulated the problem as a mixed integer programming model with explicit probability modeling of uncertainty.

Bard and Purnomo [22] tackled to the problem using a Lagrangian-based algorithm to strike a balance between satisfying the individual preferences and minimizing the personnel costs. Maenhout and Vanhoucke [23] developed a branch-and-price algorithm by incorporating multiple objectives of the unit efficiency (i.e., cost) and the personnel job satisfaction (i.e., nurses' preferences). Burke and Curtois [24] proposed a branch and price algorithm in addition to an ejection chain method to solve the NSP. Their approach is sufficiently general to be applied for solving a wide range of the benchmark nurse scheduling instances. Moreover, Jafari et al. [6] considered the NSP by maximizing the nurses' preferences to work in their favorable shifts as well as by minimizing the total surplus nurses to meet the considered demands. They formulated different fuzzy mathematical models to treat uncertainties in the nurses' preferences and the number of the surplus nurses.

There are several studies that have proposed the *heuristic and meta-heuristic algorithms* for the NSP. Gutjahr and Rauner [25] applied an ant colony optimization (ACO) algorithm to maximize the nurses' preferences as well as to minimize the salary paid to the nurses. Majumdar and Bhunia [26] proposed a genetic algorithm by introducing two new crossover and mutation schemes. Furthermore, Hadwan et al. [27] applied a harmony search algorithm to solve the research problem. They found that the proposed algorithm provides very efficient solutions for the benchmark datasets.

Wong et al. [28] provided a two-stage heuristic algorithm in a local emergency department. At the first stage, an initial solution is generated using a simple shift assignment heuristic approach. Then, a sequential local search algorithm is applied at the second stage to improve the quality of the initial schedule by taking into account the nurses' preferences for the working shifts. Legrain et al. [29] presented a simple heuristic algorithm to investigate the scheduling process for two types of the nursing teams, i.e., the regular teams from care units and the float teams that meet the shortages. Moreover, Issaoui et al. [30] provided a three-phase meta-heuristic algorithm based on the variable neighborhood search. First, the shifts are assigned to the nurses. Then, the initial schedule is improved at the second phase. Finally, the patient's satisfaction is maximized at the third phase. Similar studies in this research area are: [31-33].

The *Hybrid methods* are developed by combining the favorable characteristics of the various approaches. Dowsland and Thompson [34] proposed a hybrid method based on the tabu-search algorithm and the network programming model to solve a non-cyclical scheduling problem. Bard and Purnomo [35] balanced the contractual agreements and the management prerogatives via a column generation approach by combining an integer programming model with a heuristic algorithm. He and Qu [36] developed a hybrid constraint programming model based on the column generation approach. Li et al. [37] presented an approach by combining the goal programming model with a meta-heuristic algorithm. Furthermore, Jafari and Salmasi [16] proposed a mathematical programming model as well as a meta-heuristic algorithm based on the simulated annealing (SA) to maximize the nurses' preferences in one of the largest hospitals in Iran, i.e., Milad.

The policy makers in hospitals aim to provide health services via the minimum number of the required nurses, whereas the nurses prefer to work on the desirable working shifts, mostly [38]. This leads to establishing a conflict between the utilities related to the managers and the nursing personnel in hospitals. The *game theory* is known as one of the best applicable mathematical tools to set decisions in conflictive situations [39-42]. In recent years, applications of the game theory have greatly increased in decision-making problems. In the current study, attempts will be made to apply the game-theoretic framework for solving the NSP as a real-case in practice.

In this study, the NSP is considered in one of the largest hospitals in Iran (i.e., Milad) by minimizing the total number of the nurses to meet the daily requirements as well as by maximizing the nurses' preferences for the working shifts. In this setting, in each ward, there are both the full-time and part-time nurses who can work on the shifts and satisfy the number of the required nurses during the planning horizon. The full-timers are arranged in all planning horizons, whereas the part-timers can be scheduled when the requirements cannot be met via the full-timers. The managers set the total number of the scheduled nurses in each ward (including all the full-timers and some of the part-timers), while the nurse-chief related to this ward chooses the required part-timers and assigns the shifts to all nurses.

Clearly, competitive situation established between the managers and nurse-chief leads to a conflict between their objectives. In this point of view, a two-player game-theoretic framework can be developed between them. To our knowledge, this study is the first one that has proposed the game-theoretic approach to solve the NSP. Based on the various competitive and cooperative interactions between the managers and nurse-chief, the equilibrium strategies are obtained under four games, including Managers-Stackelberg, Nurses-Stackelberg, Nash, and Centralized models. Finally, under each game model, a roster is generated by trading off the daily requirements with the individual preferences without violating the constraints considered in the system.

The rest of the paper is organized as follows: In Section 2, the details of the research problem are descripted. A mathematical programming model is developed in Section 3, and the game-theoretic approaches are formulated in Section 4. Section 5 deals with the computational results. Moreover, the conclusions and directions for future studies are provided in Section 6.

2. Problem description

Milad hospital has been constituted from several wards including CCU, ICU, PICU, etc. In each ward, the related nurse-chief arranges the nurse scheduling activities. There exist both the full-time and part-time nurses who can work on the shifts and cover the number of the required nurses during the planning horizons. The full-time nurses are arranged in all planning horizons, while the part-timers are scheduled in some planning horizons to meet the shifts requirements. In this hospital, the managers set the total number of the nurses who should be scheduled in each ward (including all the full-timers and some of the part-timers), whereas the related nurse-chief selects the required part-timers as well as assigns the shifts to all the scheduled nurses. In this setting, information about the nurse availability, demands (requirements), individual requests, and hospital's regulations should be gathered to schedule the nurses, justly.

The considered assumptions to solve the NSP in Milad hospital are as follows:

1. The planning horizon is considered for two weeks.
2. Monday is considered as the first day of each week.
3. Each day consists of three shifts that the nurses can work on them, i.e., morning shift (M) from 8:00 AM to 4:00 PM, evening shift (E) from 4:00 PM to 12:00 PM, and night shift (N) from 12:00 PM to 8:00 AM.
4. The numbers of the required nurses for the shifts on all days of the planning horizon are given and it is assumed these are fixed during the planning horizon.
5. The nurses' preferences for the working shifts during the planning horizon are also considered. At the beginning of the planning horizon, each nurse specifies her preferences to work on a specific shift in each week. In fact, the nurses assign a number to each shift in each week based on their interest to work on that shift during that week. Numbers 3, 1, and 0 correspond to high, medium, and no preferences, respectively. If a nurse prefers to work on a shift, then she assigns number 3 to that shift. When the nurse's interests to work or to do not work on the shift are similar, she assigns number 1 to it. And finally, if the nurse does not like to work on the shift, then she assigns number 0 to that shift.

The constraints incorporated into the model are as follows which must be satisfied:

1. The nurses can work at most on one shift during each day.

2. Considering the schedule of the nurses on the last day of the previous planning horizon, the nurses who work on a night shift on a specific day should be off on the next day.
3. Considering the schedule of the nurses on the last four days of the previous planning horizon, the nurses can work at most on four consecutive days.
4. The full-timers and the selected part-timers respectively work between 64 to 80 hours (i.e., 8 to 10 shifts) and between 32 to 48 hours (i.e., 4 to 6 shifts) during the planning horizon.
5. The shifts requirements on each day during the planning horizon should be met.
6. The annual leave days requested by the nurses should be assigned to them.

Due to the shortage of the available nurses and also to decrease the salary paid to the nurses, the hospital's managers prefer to meet the shifts requirements by minimizing the total number of the scheduled nurses (hereafter referred to as "managers-oriented objective") whereas, from the nurse-chiefs' point of view, the aim is assigning the shifts to the nurses by maximizing the average of the nurses' preferences for the working shifts (hereafter referred to as "nurses-oriented objective"). In this setting, we encounter a bi-objective decision-making problem which is modeled in the next section.

3. Mathematical programming model

In this section, a bi-objective mathematical programming model is developed for the research problem.

Indices and parameters:

A	The number of the available nurses including all full-timers and part-timers
F	The number of the full-time nurses
i	Index of the nurses, where $i = 1, 2, \ldots, A$. Note that sets $\{1, 2, \ldots, F\}$ and $\{F+1, F+2, \ldots, A\}$ refer to the full-timers and part-timers, respectively.
j	Index of the days, where $j = 1, 2, \ldots, 14$
t	Index of the weeks, where $t = 1, 2$

d_m, d_e, d_n The number of the required nurses for the morning, evening, and night shifts on each day, respectively.

r_i = 1 if a night shift has been assigned to nurse i on the last day of the previous planning horizon, and = 0 otherwise.

c_i The number of the consecutive working days assigned to nurse i at the end of the previous planning horizon

$H_{i,j}$ = 1 if nurse i requests for an annual leave on day j, and = 0 otherwise.

$pm_{i,t}, pe_{i,t}, pn_{i,t}$ The preference of nurse i to work respectively on the morning, evening, and night shifts in week t. Numbers 3, 1, and 0 correspond to high, medium, and no preferences, respectively.

Decision variables:

S The total number of the scheduled nurses including all the full-timers and some the part-timers (the managers' decision variable)

n_i = 1 if part-timer i is selected as a scheduled nurse, and = 0 otherwise (the nurse-chief's decision variable).

$M_{i,j}, E_{i,j}, N_{i,j}$ = 1 if a morning, evening, or night shift is respectively assigned to nurse i on day j, and = 0 otherwise (the nurse-chief's decision variable).

$Off_{i,j}$ = 1 if nurse i is considered to be off on day j, and = 0 otherwise (the nurse-chief's decision variable).

Objectives and utility functions:

Z_M The managers-oriented objective, i.e., minimizing the total number of the scheduled nurses

Z_N The nurses-oriented objective, i.e., maximizing the average of the nurses' preferences for the working shifts

U_M The managers' utility function

U_N The nurse-chief's utility function

U_S The utility function of the whole system

Bi-objective mathematical programming model:

$$\min Z_M = S \tag{1}$$

$$\max Z_N = \frac{1}{S} \sum_{i=1}^{A} \sum_{t=1}^{2} \sum_{j=7t-6}^{7t} \left(pm_{i,t}\, M_{i,j} + pe_{i,t}\, E_{i,j} + pn_{i,t}\, N_{i,j} \right) \tag{2}$$

subject to:

$$M_{i,j} + E_{i,j} + N_{i,j} + Off_{i,j} = 1 \qquad i = 1,2,\ldots,A, \quad j = 1,2,\ldots,14 \tag{3}$$

$$Off_{i,1} \geq r_i \qquad i = 1,2,\ldots,A \tag{4}$$

$$N_{i,j} \leq Off_{i,j+1} \qquad i = 1,2,\ldots,A, \quad j = 1,2,\ldots,13 \tag{5}$$

$$\sum_{j=1}^{5-c_i} Off_{i,j} \geq 1 \qquad i = 1,2,\ldots,A \tag{6}$$

$$\sum_{l=0}^{4} Off_{i,j+l} \geq 1 \qquad i = 1,2,\ldots,A, \quad j = 1,2,\ldots,10 \tag{7}$$

$$8 \leq \sum_{j=1}^{14} \left(M_{i,j} + E_{i,j} + N_{i,j} \right) \leq 10 \qquad i = 1,2,\ldots,F \tag{8}$$

$$4n_i \leq \sum_{j=1}^{14} \left(M_{i,j} + E_{i,j} + N_{i,j} \right) \leq 6n_i \qquad i = F+1, F+2,\ldots,A \tag{9}$$

$$\sum_{i=1}^{A} M_{i,j} \geq d_m \qquad j = 1,2,\ldots,14 \tag{10}$$

$$\sum_{i=1}^{A} E_{i,j} \geq d_e \qquad j = 1,2,\ldots,14 \tag{11}$$

$$\sum_{i=1}^{A} N_{i,j} \geq d_n \qquad j = 1,2,\ldots,14 \tag{12}$$

$$Off_{i,j} \geq H_{i,j} \qquad i = 1,2,\ldots,A, \quad j = 1,2,\ldots,14 \tag{13}$$

$$Off_{i,j} \geq 1 - n_i \qquad i = F+1, F+2,\ldots,A, \quad j = 1,2,\ldots,14 \tag{14}$$

$$\sum_{i=F+1}^{A} n_i = S - F \tag{15}$$

$$S \geq F \tag{16}$$

$$S \in int^{+}, \quad n_i \in \{0, 1\} \quad i = F+1, F+2, \ldots, A$$

$$M_{i,j}, E_{i,j}, N_{i,j}, Off_{i,j} \in \{0, 1\} \quad i = 1, 2, \ldots, A, \quad j = 1, 2, \ldots, 14 \tag{17}$$

Objective functions (1) and (2) are respectively related to the managers' and nurse-chief's objectives. Constraints set (3) ensure that the nurses can work at most on one shift on each day. Regarding the constraint 2 presented in Section 2, considering the last night of the previous planning horizon, if the nurses work on a night shift, then they should be off on the next day. Constraints sets (4) and (5) are incorporated into the model to hold this constraint. Constraints sets (6) and (7) meet the constraint 3 concerning the consecutive working days. Furthermore, constraints sets (8) and (9) satisfy the constraint 4 related to the working hours/shifts of the full-timers and the selected part-timers, respectively. Constraints sets (10), (11), and (12) respectively cover the number of the required nurses for the morning, evening, and night shifts during the planning horizon. Incorporating constraints set (13) into the model, the annual leave days requested by the nurses are assigned to them. Moreover, regarding constraints set (14), the unselected part-time nurses are off during the planning horizon. The number of the selected part-timers is specified using constraint (15). Finally, constraint (16) ensures that all of the full-timers are selected to be scheduled during the planning horizon.

The utility functions are applied to normalize the objective functions. The defined utility functions are formulated as follows:

$$U_M = \frac{A - S}{A - S^0} \tag{18}$$

$$U_N = \frac{1}{90S}\Big[3\sum_{i=1}^{F}\sum_{t=1}^{2}\sum_{j=7t-6}^{7t}\left(pm_{i,t}M_{i,j} + pe_{i,t}E_{i,j} + pn_{i,t}N_{i,j}\right) \tag{19}$$

$$+5\sum_{i=F+1}^{A}\sum_{t=1}^{2}\sum_{j=7t-6}^{7t}\left(pm_{i,t}M_{i,j}+pe_{i,t}E_{i,j}+pn_{i,t}N_{i,j}\right)\Big]$$

$$U_S=\alpha U_M+\left(1-\alpha\right)U_N \tag{20}$$

where, S^0 denotes the minimum total number of the required nurses which will be obtained in the next section. Moreover, α and $1-\alpha$ respectively are the weights of the managers' and nurse-chief's utility functions appearing in the utility function of the whole system.

Regarding the above utility functions, one can derive that higher U_M and U_N respectively lead to lower the managers-oriented objective and higher the nurse-oriented objective.

Considering the constraint 4, the full-timers and the part-timers respectively work between 8 to 10 shifts and between 4 to 6 shifts during the planning horizon. Moreover, regarding the assumption 5, the nurses assign numbers 3, 1, and 0 respectively to high, medium, and no preferences for each shift. Thus, upper and lower bounds of the total achievable preferences are respectively equal to 10×3 = 30 and 8×0 = 0 for each full-timer, while corresponding values for each part-timer are equal to 6×3 = 18 and 4×0 = 0, respectively. As a result, U_N is obtained by normalizing objective function Z_N as follows:

$$U_N=\frac{\dfrac{F\sum_{i=1}^{F}\sum_{t=1}^{2}\sum_{j=7t-6}^{7t}\left(pm_{i,t}M_{i,j}+pe_{i,t}E_{i,j}+pn_{i,t}N_{i,j}\right)}{30F}+\dfrac{\left(S-F\right)\sum_{i=F+1}^{A}\sum_{t=1}^{2}\sum_{j=7t-6}^{7t}\left(pm_{i,t}M_{i,j}+pe_{i,t}E_{i,j}+pn_{i,t}N_{i,j}\right)}{18\left(S-F\right)}}{F+\left(S-F\right)}$$

4. Game-theoretic framework

As stated in Section 1, the game theory is a mathematical tool that can be applied in conflictive situations where the decisions made by the players affect their payoffs. In Section 2, the managers' and nurse-chief's decision variables and their objectives were defined. It was also stated that the managers prefer to set the number of the scheduled nurses as small as possible, whereas the nurse-chief chooses the part-timers and then assigns the shifts to the scheduled nurses by maximizing the average of the nurses' preferences for the working shifts.

Obviously, their competitive decisions lead to a conflict between their objectives. In this point of view, a two-player game-theoretic framework can be established between the managers and nurse-chief.

In what follows, four game-theoretic models are proposed based on different competitive and cooperative interactions between the managers and nurse-chief.

4.1. Managers-Stackelberg game model

Under a two-player Stackelberg game, in general, one of the players makes his decisions as the leader and another one acts as the follower. In the Managers-Stackelberg game, at the first-level, the managers as the leader announce the number of the scheduled nurses including all the full-timers and some of the part-timers. Then, at the second-level, the nurse-chief as the follower selects the part-timers and assigns the shifts to the scheduled nurses. The Managers-Stackelberg game model is formulated as follows:

$$\begin{cases} \text{First level: } \underset{S}{\text{minimize}} \text{ objective function (1)} \\ \text{Second level: } \begin{cases} \underset{n,M,E,N,Off}{\text{maximize}} \text{ objective function (2)} \\ \text{subject to:} \\ \text{Constraints set (3)} - \text{(17)} \\ S = S^{*} \text{ made by the managers at the first level} \end{cases} \end{cases}$$

(Problem A)

At the first-level, a similar approach as proposed by Pinedo [43] is applied to minimize the total number of the scheduled nurses. It can be specified based on the constraints discussed in Section 2.

From the constraint 1, each nurse can work at most on one shift on each day. Thus, to meet the constraint 5, the minimum number of the required nurses on each day is equal to $d_m + d_e + d_n$, and clearly, relation $S \geq d_m + d_e + d_n$ should be met. Regarding the constraint 2, if the nurses work on a night shift on a specific day, then they should be considered to be off on the next day. Hence, to cover the demands of the next day, the number of the available nurses on this day should be greater than or equal to the number of the required nurses, i.e., $S - d_n \geq d_m + d_e + d_n$. From the constraint 4, the full-time and part-time nurses can work at most on 10 and 6 shifts during the planning horizon, respectively. So, relation $10F + 6(S - F) \geq 14(d_m + d_e + d_n)$ should hold in order to cover the sum of the required nurses during the planning horizon. Obviously, the number of the scheduled nurses cannot be smaller than the number of the full-timers, i.e., $S \geq F$. Regarding the above explanations, the minimum total number of the scheduled nurses is equal to:

$$S^0 = \max\left\{d_m + d_e + 2d_n, \frac{14(d_m + d_e + d_n) - 4F}{6}, F\right\} \tag{21}$$

Now, by substituting $S^* = S^0$ set by the managers as the number of the scheduled nurses into the mathematical programming model presented in Section 3, the nurse-chief selects the part-timers as well as arranges the nurses' activities at the second-level. This gives the equilibrium decisions under the Managers-Stackelberg game.

4.2. Nurses-Stackelberg game model

In the Nurses-Stackelberg game, at the first-level, the nurse-chief selects the part-time nurses and assigns the shifts to the scheduled nurses by maximizing the average of the nurses' preferences for the working shifts. Then, at the second-level, by considering the schedule of the nurses provided at the first-level, the managers attempt to minimize the total number of the scheduled nurses, if it is possible. The Nurses-Stackelberg game is modelled as follows:

$$\begin{cases} \text{First level:} \begin{cases} \underset{n,M,E,N,Off}{\text{maximize}} \ \text{objective function (2)} \\ \text{subject to:} \\ \text{Constraints set (3)} - (17) \end{cases} \\ \text{Second level:} \begin{cases} \underset{S}{\text{minimize}} \ \text{objective function (1)} \\ \text{subject to:} \\ \text{objective function (2)} = {Z_N}^* \text{ given by the nurse chief at the first level} \end{cases} \end{cases}$$

(Problem B)

In this setting, an algorithm is proposed to set the equilibrium decisions under the Nurses-Stackelberg game. An outline of the algorithm is presented as follows:

Algorithm 1. *The equilibrium generator under the Nurses-Stackelberg game*

Step 0 (Initialization):

Set: $Z_N{}^* = 0$ and $S^0 = \max\left\{ d_m + d_e + 2d_n, \frac{14(d_m + d_e + d_n) - 4F}{6}, F \right\}$.

Step 1 (Iterative):

For $=$ S^0 to A

- Obtain the optimal objective function $Z_N{}'$ and the optimal decision variables n', M', E', N', Off' by solving the following problem:

$$\begin{cases} \mathbf{maximize}_{\boldsymbol{n,M,E,N,Off}} \ \ \mathbf{objective\ function\ (2)} \\ \quad \mathbf{subject\ to}: \\ \quad \mathbf{Constraints\ set\ (3)-(17)} \\ \quad \boldsymbol{S = K} \end{cases}$$

- If $Z_N{}' > Z_N{}^*$, then:

  Set: $S^* = K, Z_N{}^* = Z'_N, n^* = n', M^* = M', E^* = E', N^* = N', Off^* = Off'$

  End

End

Set the obtained decisions as the equilibrium solution of the Nurses-Stackelberg game.

Under the Nurses-Stackelberg game, the managers act as the follower player. Thus, they have to adopt the number of the scheduled nurses which maximizes the average of the nurses' preferences for the working shifts.

4.3. Nash game model

Assume that the players have similar decision powers. In this setting, they make the decisions competitively and simultaneously. This situation establishes a Nash game and the solution obtained from this game is also called the Nash equilibrium. Under the Nash game, the managers and nurse-chief set the number of the scheduled nurses, select the part-timers, and arrange the nurses' activities, simultaneously. The Nash game model is formulated as follows:

$$\begin{cases} \underset{S}{\text{minimize}}\ \text{objective function (1)} \\ \underset{n,M,E,N,Off}{\text{maximize}}\ \text{objective function (2)} \\ \text{subject to:} \\ \text{Constraints set (3)} - (17) \end{cases}$$

(Problem C)

Below, an algorithm is proposed to obtain the equilibrium decisions in the Nash game.

Algorithm 2. *The equilibrium generator under the Nash game*

Step 0 (Initialization):

Set: $S^0 = \max\left\{ d_m + d_e + 2d_n, \frac{14(d_m + d_e + d_n) - 4F}{6}, F \right\}$ and γ as an adequately great number.

Step 1 (Iterative):

For $K = S^0$ to A

- Obtain the optimal objective function $Z_N{}'$ and the optimal decision variables n', M', E', N', Off' by solving the following problem:

$$\begin{cases} \textbf{maximize}_{n,M,E,N,Off} \ \textbf{objective function (2)} \\ \quad \textbf{subject to}: \\ \quad \textbf{Constraints set (3) − (17)} \\ \quad S = K \end{cases}$$

- Calculate U_M and U_N by substituting the obtained optimal decision variables into utility functions (18) and (19).
- If $|U_M - U_N| < \gamma$, then:

  Set: $S^* = K, Z_N^* = Z_N', n^* = n', M^* = M', E^* = E', N^* = N', Off^* = Off'$, and $\gamma = |U_M - U_N|$.

  End

End

Consider the obtained decisions as the equilibrium solution of the Nash game.

To obtain the Nash equilibrium, first, the number of the scheduled nurses is set equal to $S^0 = \max\left\{d_m + d_e + 2d_n, \frac{14(d_m + d_e + d_n) - 4F}{6}, F\right\}$ as the best response given by the managers. Then, the managers' utility function U_M decreases by increasing the total number of the scheduled nurses to increase the nurse-chief's utility function U_N. As a matter of fact, the Nash equilibrium is calculated by minimizing the difference between the managers' and nurse-chief's utility functions.

4.4. Centralized game model

Under the Centralized game, the managers and nurse-chief adopt to act in union and maximize the utility function of the whole system. The Centralized model is formulated as follows:

$$\begin{cases} \underset{S,n,M,E,N,Off}{\text{maximize}} \ U_S = \alpha U_M + (1-\alpha) U_N \\ \text{subject to:} \\ \text{Constraints set } (3) - (17) \end{cases}$$

(Problem D)

Note that the Analytic Hierarchy Process (AHP) approach, proposed by Saati [44], has been applied to estimate weights α and $1-\alpha$ appearing in the utility function of the whole system. For this reason, the importance proportion between the managers' and nurse-chief's utility functions was questioned from 10 managers and 10 nurses randomly selected in Milad hospital. Then, the weights were estimated for each of them applying the AHP approach. Finally, the average of the weights given by the managers and nurses was calculated to estimate α. Based on this approach, the values of α and $1-\alpha$ are equal to 0.55 and 0.45, respectively.

The following algorithm is proposed to obtain the equilibrium solution of the Centralized game.

Algorithm 3. *The equilibrium generator under the Centralized game*

Step 0 (Initialization):

Set: $U_S^{*} = 0$ and $S^0 = \max\left\{ d_m + d_e + 2d_n, \frac{14(d_m + d_e + d_n) - 4F}{6}, F \right\}$.

Step 1 (Iterative):

For $K = S^0$ to A

- Obtain the optimal utility function $U_S{}'$ and the optimal decision variables n', M', E', N', Off' by solving the following problem:

$$\begin{cases} \textbf{maximize}_{\boldsymbol{n,M,E,N,Off}} \ \boldsymbol{U_S = \alpha U_M + (1-\alpha) U_N} \\ \textbf{subject to}: \\ \textbf{Constraint sets } \boldsymbol{(3) - (17)} \\ \boldsymbol{S = K} \end{cases}$$

- If $U_S{}' > U_S{}^*$, then:

  Set: $U_S{}^* = U_S^{'}$, $S^* = K, n^* = n', M^* = M', E^* = E', N^* = N', Off^* = Off'$.

  End

End

Consider the given decisions as the equilibrium solution under the Centralized game.

5. Computational results

This section deals with the computational results obtained from the developed game-theoretic models.

5.1. Experimental evaluation

To evaluate the performance of the developed game models, 10 test problems were generated. The Discrete Uniform distribution was applied to generate the parameters, randomly. The rang of the parameters have been presented in Table 1.

Please insert Table 1 here

Under the test problems, the values of the number of the available nurses, the number of the full-timers, and the number of the required nurses for the working shifts on each day have been also provided in Table 2.

Please insert Table 2 here

Then, all test problems were solved under four game-theoretic frameworks applying the mathematical programming model as well as the algorithms 1, 2, and 3. A Pentium IV 2.1GHz PC with 512MB RAM was used for this reason. The proposed algorithms were coded in MATLAB programming language version 7.6.0.324. Furthermore, IBM ILOG CPLEX optimization Studio version 12.2 was applied to solve the problems using the mathematical model. The results given by solving the test problems have been summarized in Table 3. The averages of the obtained results have been also provided in the last row of Table 3.

The managers and nurse-chief act as the leader under the Managers-Stackelberg and Nurses-Stackelberg games, respectively. As a result, they will receive their best responses (i.e., the optimal objective and utility values) under these two games, respectively. Under the Nash game, players have similar decision powers, and consequently, they make the decisions in order to meet their utilities, mostly. Moreover, due to higher weight for the managers in the utility function of the whole system, the equilibrium decisions are obtained in a similar manner under the Managers-Stackelberg and Centralized games.

Please insert Table 3 here

5.2. Illustrative instance

Now, an example is presented to illustrate the research problem. The values of the parameters have been indicated in Table 4. Note these values have been gathered from one of the wards in Milad hospital with 10 full-timers and 3 part-timers.

Please insert Table 4 here

Without loss of generality, it is assumed that nurses 1-10 and nurses 11-13 are considered as the full-timers and the part-timers, respectively. The number of the required nurses for the morning, evening, and night shifts are 3, 2, and 2, respectively. The schedule of the nurses at the end of the previous planning horizon has been also considered including the night shift on the last day as well as the number of the consecutive working days. For example, nurse 1 works on the night shift on the last day of the previous planning horizon, and the number of the consecutive working days assigned to nurse 5 at the end of the previous planning horizon is equal to 4. Hence, regarding constraints 2 and 3, they should be off on the first day of the current planning horizon.

Also, the nurses' preferences for the working shifts have been considered. For example, nurse-3's preferences to work on the morning, evening, and night shifts during week 1 are high, medium, and low, respectively. Therefore, she prefers to work on the morning shift in this week, mostly. Moreover, nurse 7 requests for an annual leave on days 6 and 7 (i.e., Saturday and Sunday in the first week). Thus, she should be off on these days.

The results obtained from the developed game-theoretic models for the illustrative instance have been summarized in Table 5.

Please insert Table 5 here

As stated in Section 2, the managers prefer to meet the demands by minimizing the total number of the scheduled nurses, while the nurse-chief's objective is assigning the shifts to the nurses by maximizing the average of the nurses' preferences for the working shifts. Under the Managers-Stackelberg and Nurses-Stackelberg games, the managers and nurse-chief respectively act as the leader. So, these two games lead to the best responses for the managers and nurse-chief, respectively. In the Nash game, players have similar decision powers and thus they will receive the objective and utility values which satisfy both of them, mostly. Moreover, under the Centralized game, players adopt the equilibrium strategy which maximizes the utility function of the whole system.

Comparisons of the objective and utility values given by the investigated game models for the illustrative instance have been indicated in Fig. 1 and Fig. 2, respectively.

Please insert Figure 1 here

Please insert Figure 2 here

The schedules given by the various game models for the illustrative instance have been also provided in Table 6.

Please insert Table 6 here

Symbols M, E, N, H, and – denote the morning shift, evening shift, night shift, annual leave day, and off day, respectively. In all schedules, all the constraints considered in Section 2 have been met. The nurses who work on a night shift on a specific day are off on the next day. Each nurse can work at most on four consecutive days. The full-timers work between 8 to 10 shifts while the part-timers work between 4 to 6 shifts during the planning horizon. All requirements have been satisfied and the annual leave days requested by the nurses have been also assigned to them. Furthermore, the nurses' preferences for the working shifts specified in Table 4 have been satisfied, mostly.

Note the numbers of the assigned nurses to the shifts have been given in Morning shift, Evening shift, and Night shift rows. The total working shifts and the total working hours of the nurses have been also indicated in Working shifts and Working hours columns, respectively.

In what follows, a sensitivity analysis is implemented on weight α (appearing in the utility function of the whole system) for investigating its effects on the objective and utility values given by the Centralized game for the illustrative instance. Weight α is changed from 0 to 1 in step sizes of 0.1. The changes in the objective and utility values with α have been exhibited in Fig. 3 and Fig. 4, respectively.

Please insert Figure 3 here

Please insert Figure 4 here

From the above figures, one can observe that in $[0, 0.2)$, higher α leads to lower all the objective and utility values except the managers' utility value. In addition, as α increases in $[0.2,1]$, the utility value of the whole system increases, whereas it has no effect on the other objective and utility values.

6. Summary and conclusion

In this research, the nurse scheduling problem was studied by minimizing the total number of the required nurses as well as by maximizing the nurses’ preferences for the working shifts. In other words, rather than attempting to minimize alone the number of the required nurses, our objective was designed to strike a balance between these two objectives. Several real-world constraints like the labor laws, the governmental regulations, and the status of the nurses at the end of the previous planning horizon were also considered based on our observations in a hospital in Iran, i.e., Milad.

In Milad hospital, in each ward, the related nurse-chief arranges the nurse scheduling activities. In each ward, there exist both the full-time and part-time nurses in order to meet the shifts requirements. In this setting, the managers specify the total number of the scheduled nurses in each ward including all the full-timers and some of the part-timers, while the related nurse-chief selects the required part-timers and then assigns the shifts to all the scheduled nurses.

The competitive decisions, made by the managers and nurse-chief in each ward, lead to a conflict between their objectives. Therefore, a two-player game-theoretic framework can be established between them to make the decisions. For this reason, different game-theoretic models, including Managers-Stackelberg, Nurses-Stackelberg, Nash, and Centralized, were developed based on the various competitive and cooperative interactions between the managers and nurse-chief. Moreover, a mathematical programming model was developed to obtain the equilibrium strategies under the proposed game models. To our knowledge, the current study is the first one which proposes the game theoretical framework to solve the nurse scheduling problem.

Under the Managers-Stackelberg game, the managers as the leader, first, declare the number of the scheduled nurses. Then, the nurse-chief as the follower chooses the part-timers and assigns the shifts to the nurses.

In the Nurses-Stackelberg game, the nurse-chief, first, selects the part-timers and assigns the shifts to the nurses by maximizing the average of the nurses' preferences for the working shifts. Then, the managers minimize the total number of the scheduled nurses by considering the schedule of the nurses at the first level. An algorithm was proposed to obtain the equilibrium decisions under this game.

Under the Nash game, players have similar decision powers. In this game, they specify the number of the scheduled nurses, select the part-timers, and arrange the nurses' activities, simultaneously. An algorithm was also applied in this setting to solve the problem.

Finally, players adopt to act in union under the Centralized game and maximize the utility function of the whole system. An algorithm was proposed in this game to give the equilibrium decisions.

To evaluate the performance of the proposed game-theoretic models, several random test problems were solved under the investigated games. Regarding the obtained results, since the managers and nurse-chief respectively act as the leader in the Managers-Stackelberg and Nurses-Stackelberg games, they will receive their best responses under these two games, respectively. In the Nash game, they make the decisions in order to satisfy their utilities, mostly. Furthermore, the equilibrium strategies given by the Managers-Stackelberg and Centralized games are the same.

Finally, an example was presented to illustrate the research problem. Moreover, a sensitivity analysis was performed on the weight of the managers in the utility function of whole system.

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Figure captions

Fig. 1 Comparisons of the objective values under the developed game models in the illustrative instance

Fig. 2 Comparisons of the utility values under the game models in the illustrative instance

Fig. 3 Changes of the objective values with α in the Centralized game

Fig. 4 Changes of the utility values with α in the Centralized game

Table captions

Table 1 The ranges of the parameters in the test problems generated

Table 2 The values of the parameters in the test problems

Table 3 The results given by solving the test problems

Table 4 The values of the parameters in the illustrative instance

Table 5 The results obtained from the game-theoretic models for the illustrative instance

Table 6 The schedules obtained from the game-theoretic models for the illustrative instance

Figures

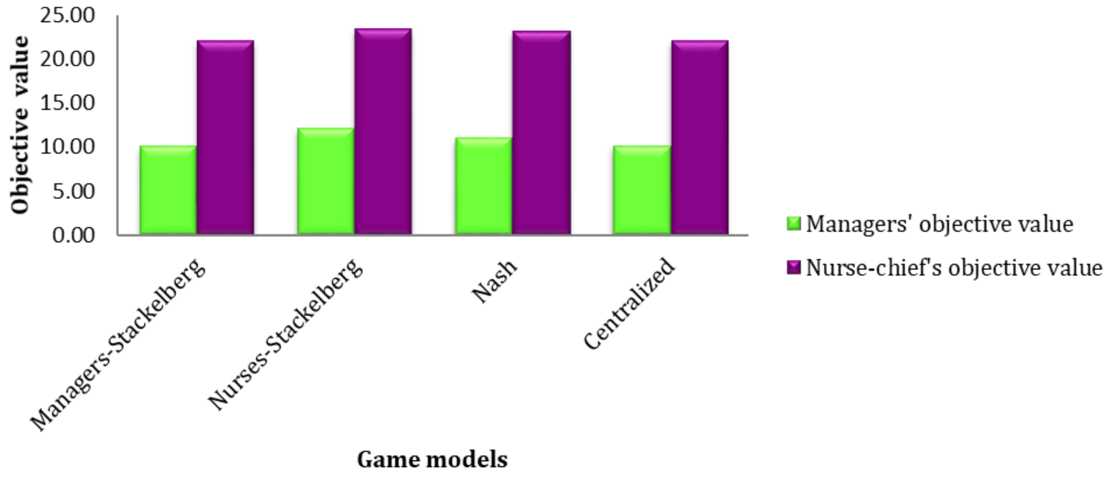

Fig. 1 Comparisons of the objective values under the developed game models in the illustrative instance

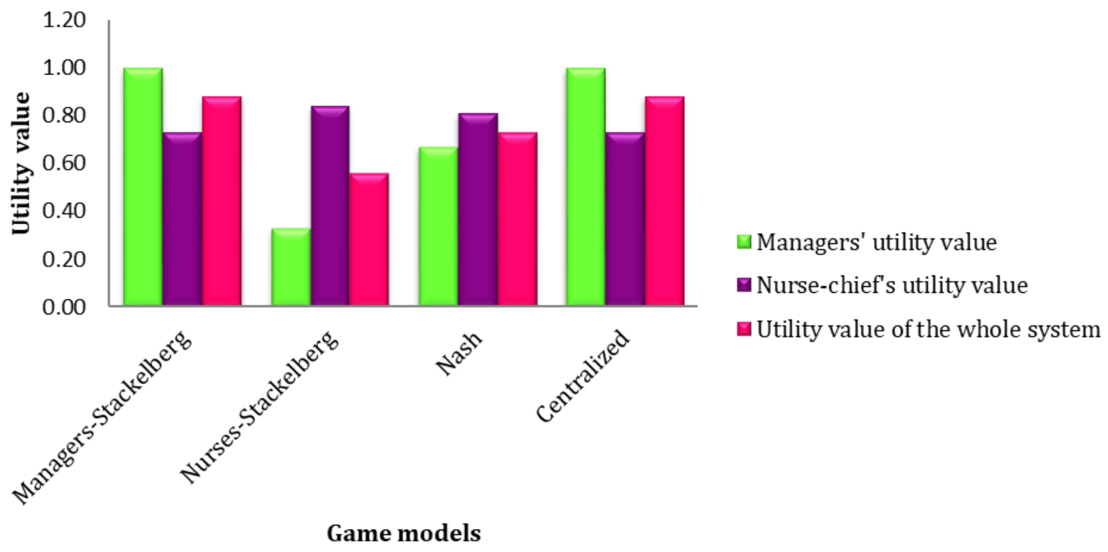

Fig. 2 Comparisons of the utility values under the game models in the illustrative instance

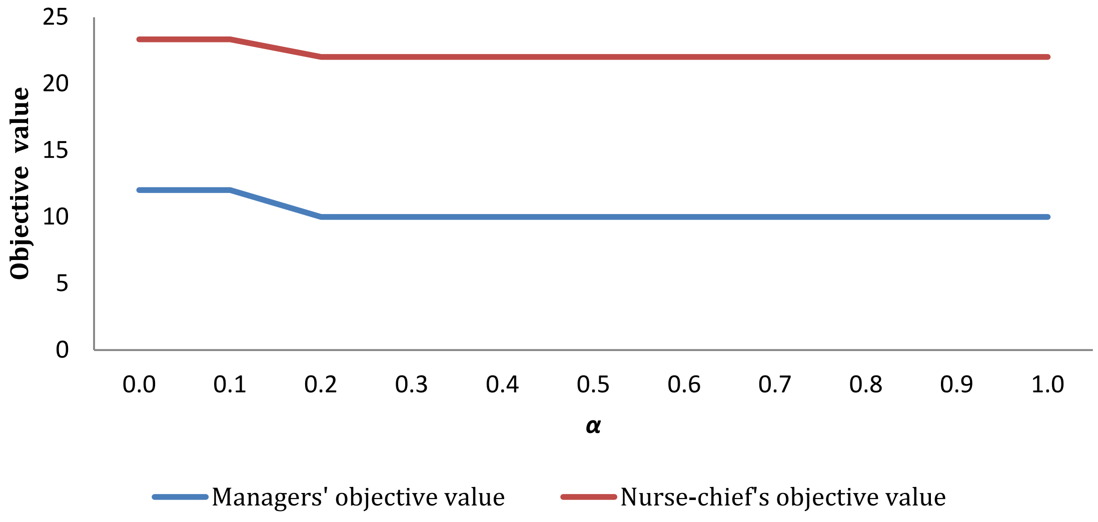

Fig. 3 Changes of the objective values with α in the Centralized game

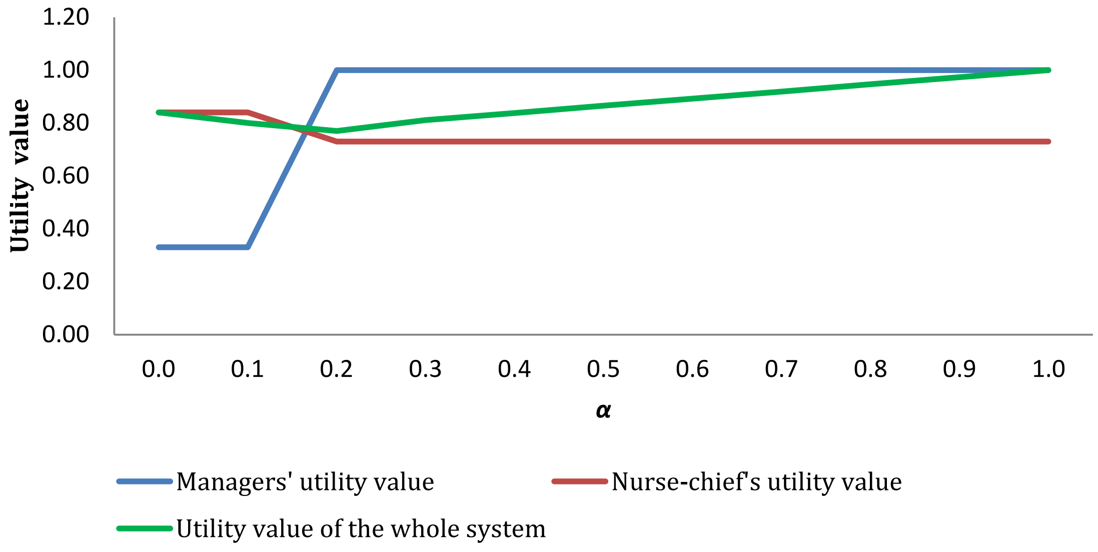

Fig. 4 Changes of the utility values with α in the Centralized game

Tables

Table 1 The ranges of the parameters in the test problems generated

Parameter	A	F	d_m, d_e, d_n	r	c	H	pm, pe, pn
Range	[4, 50]	[1, 30]	[1, 10]	[0, 1]	[0, 4]	[0, 1]	$2^{[0,2]} - 1$

Table 2 The values of the parameters in the test problems

Problem	A	F	d_m	d_e	d_n	S^0
1	4	3	1	1	0	3
2	8	4	2	1	1	7
3	13	11	3	2	1	11
4	16	12	4	3	2	13
5	17	6	4	2	2	15
6	18	11	5	3	2	16
7	20	14	5	4	3	19
8	25	21	6	4	3	21
9	28	20	7	4	4	22
10	30	20	8	5	3	24

Table 3 The results given by solving the test problems

Problem	Managers-Stackelberg game					Nurses-Stackelberg game				
	Z_M	Z_N	U_M	U_N	U_S	Z_M	Z_N	U_M	U_N	U_S
1	3.00	20.67	1.00	0.69	0.86	4.00	20.79	0.00	0.70	0.32
2	7.00	20.86	1.00	0.82	0.92	8.00	20.93	0.00	0.84	0.38
3	11.00	25.54	1.00	0.85	0.93	12.00	25.78	0.50	0.86	0.66
4	13.00	23.69	1.00	0.82	0.92	15.00	23.89	0.33	0.85	0.57
5	15.00	19.27	1.00	0.85	0.93	15.00	19.27	1.00	0.85	0.93
6	16.00	22.33	1.00	0.87	0.94	18.00	22.88	0.00	0.88	0.40
7	19.00	21.68	1.00	0.80	0.91	20.00	21.68	0.00	0.80	0.91
8	21.00	25.43	1.00	0.84	0.93	23.00	25.78	0.50	0.85	0.66
9	22.00	23.14	1.00	0.80	0.91	25.00	23.99	0.50	0.82	0.64
10	24.00	23.13	1.00	0.84	0.93	28.00	23.66	0.33	0.85	0.57
Average of the results	15.10	22.57	1.00	0.82	0.92	16.80	22.87	0.32	0.83	0.60

Problem	Nash game					Centralized game				
	Z_M	Z_N	U_M	U_N	U_S	Z_M	Z_N	U_M	U_N	U_S
1	3.00	20.67	1.00	0.69	0.86	3.00	20.67	1.00	0.69	0.86
2	7.00	20.86	1.00	0.82	0.92	7.00	20.86	1.00	0.82	0.92
3	11.00	25.54	1.00	0.85	0.93	11.00	25.54	1.00	0.85	0.93
4	14.00	23.57	0.67	0.82	0.74	13.00	23.69	1.00	0.82	0.92
5	15.00	19.27	1.00	0.85	0.93	15.00	19.27	1.00	0.85	0.93
6	16.00	22.33	1.00	0.87	0.94	16.00	22.33	1.00	0.87	0.94
7	19.00	21.68	1.00	0.80	0.91	19.00	21.68	1.00	0.80	0.91
8	22.00	25.59	0.75	0.85	0.80	21.00	25.43	1.00	0.84	0.93
9	23.00	23.41	0.83	0.80	0.82	22.00	23.14	1.00	0.80	0.91
10	25.00	23.26	0.83	0.85	0.84	24.00	23.13	1.00	0.84	0.93
Average of the results	15.50	22.62	0.91	0.82	0.87	15.10	22.57	1.00	0.82	0.92

Table 4 The values of the parameters in the illustrative instance

Number of the full-time nurses: $F = 10$					Number of the available nurses: $A = 13$			
Demand			d_m		d_e		d_n	
			3		2		2	
Nurse	r_i	c_i	$pm_{i,1}$	$pm_{i,2}$	$pe_{i,1}$	$pe_{i,2}$	$pn_{i,1}$	$pn_{i,2}$
1	1	1	3	3	0	0	0	0
2	0	0	3	3	1	1	0	1
3	0	1	3	3	1	1	0	1
4	0	1	0	0	1	1	0	3
5	0	4	0	0	3	3	1	0
6	0	0	1	3	3	3	0	1
7	0	1	3	3	1	0	1	1
8	0	3	3	3	0	1	0	0
9	0	1	0	1	1	0	1	0
10	0	0	3	3	3	3	0	0
11	1	1	3	3	3	1	3	3
12	0	1	3	1	3	3	3	3
13	0	0	3	3	0	0	0	0

Nurse	$H_{i,j}$													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	1	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	1	1	0	0	0	0	0	1	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	1
11	0	0	0	0	0	0	0	0	0	0	1	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	1	0	0	0	0	0	0	0	0	0

Table 5 The results obtained from the game-theoretic models for the illustrative instance

Game	Number of the nurses			Objective values		Utility values		
	A	F	S^0	Z_M	Z_N	U_M	U_N	U_S
Managers-Stackelberg	13	10	10	10.00	22.00	1.00	0.73	0.88
Nurses-Stackelberg	13	10	10	12.00	23.33	0.33	0.84	0.56
Nash	13	10	10	11.00	23.09	0.67	0.81	0.73
Centralized	13	10	10	10.00	22.00	1.00	0.73	0.88

Table 6 The schedules obtained from the game-theoretic models for the illustrative instance

Schedule generated by the Managers-Stackelberg game																
Nurse	Day														Working shifts	Working hours
	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14		
1	H	H	M	M	M	N	-	M	M	M	M	-	M	M	10	80
2	M	M	M	M	-	M	M	N	-	M	N	-	N	-	10	80
3	M	M	M	-	M	M	M	N	-	N	-	M	N	-	10	80
4	E	E	N	-	E	N	-	E	N	H	E	N	-	N	10	80
5	-	E	E	N	-	E	N	-	E	E	-	E	E	E	10	80
6	E	N	-	E	E	E	E	-	E	N	-	N	-	E	10	80
7	M	N	-	M	N	H	H	M	M	M	N	-	H	M	9	72
8	N	-	-	N	-	M	M	M	M	-	M	M	M	M	10	80
9	N	-	N	-	N	-	N	-	N	-	M	M	M	N	9	72
10	-	M	E	E	M	-	E	E	-	E	E	E	E	H	10	80
11	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0
12	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0
13	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0
Morning shift	3	3	3	3	3	3	3	3	3	3	3	3	3	3		
Evening shift	2	2	2	2	2	2	2	2	2	2	2	2	2	2		
Night shift	2	2	2	2	2	2	2	2	2	2	2	2	2	2		

Schedule generated by the Nurses-Stackelberg game																
Nurse	Day														Working shifts	Working hours
	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14		
1	H	H	M	M	M	M	-	M	M	M	M	-	M	M	10	80
2	M	M	M	-	M	M	M	N	-	M	M	-	N	-	10	80
3	M	M	-	M	M	M	M	-	M	N	-	M	N	-	10	80
4	E	N	-	N	-	N	-	E	N	H	E	N	-	N	9	72
5	-	E	E	E	-	-	E	E	E	-	E	E	E	E	10	80
6	E	-	E	E	E	E	-	M	E	E	-	E	M	-	10	80
7	-	M	M	M	M	H	H	M	M	M	N	-	H	M	9	72
8	-	M	M	M	-	M	M	M	M	-	M	M	-	M	10	80
9	N	-	N	-	N	-	N	-	N	-	N	-	M	N	8	64
10	M	E	M	-	E	E	E	M	-	E	M	M	-	H	10	80
11	-	N	-	N	-	N	-	N	-	N	H	N	-	-	6	48
12	N	-	N	-	N	-	N	-	-	-	-	-	E	E	6	48
13	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0
Morning shift	3	4	5	4	4	4	3	5	4	3	4	3	3	3		
Evening shift	2	2	2	2	2	2	2	2	2	2	2	2	2	2		
Night shift	2	2	2	2	2	2	2	2	2	2	2	2	2	2		

Table 6 The schedules obtained from the game-theoretic models for the illustrative instance (continued)

Schedule generated by the Nash game

Nurse	Day: Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Working shifts	Working hours
	1	2	3	4	5	6	7	8	9	10	11	12	13	14		
1	H	H	M	M	M	-	M	M	M	-	M	M	M	M	10	80
2	M	M	-	M	M	-	M	N	-	M	M	M	N	-	10	80
3	M	M	M	-	M	M	M	-	-	M	M	M	N	-	10	80
4	N	-	E	N	-	N	-	E	N	H	N	-	E	N	9	72
5	-	N	-	E	E	E	N	-	E	-	E	E	E	E	10	80
6	E	E	E	E	-	E	E	-	E	E	-	E	-	E	10	80
7	M	M	N	-	N	H	H	M	M	N	-	N	H	M	9	72
8	-	M	M	M	-	M	-	M	M	M	-	M	M	M	10	80
9	N	-	N	-	N	-	N	-	N	-	N	-	M	N	8	64
10	E	E	-	-	E	M	E	E	-	E	E	E	M	H	10	80
11	-	N	-	N	-	N	-	N	-	N	H	N	-	-	6	48
12	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0
13	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0
Morning shift	3	4	3	3	3	3	3	3	3	3	3	4	4	3		
Evening shift	2	2	2	2	2	2	2	2	2	2	2	3	2	2		
Night shift	2	2	2	2	2	2	2	2	2	2	2	2	2	2		

Schedule generated by the Centralized game

Nurse	Day: Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Working shifts	Working hours
	1	2	3	4	5	6	7	8	9	10	11	12	13	14		
1	H	H	M	M	M	N	-	M	M	M	M	-	M	M	10	80
2	M	M	M	M	-	M	M	N	-	M	N	-	N	-	10	80
3	M	M	M	-	M	M	M	N	-	N	-	M	N	-	10	80
4	E	E	N	-	E	N	-	E	N	H	E	N	-	N	10	80
5	-	E	E	N	-	E	N	-	E	E	-	E	E	E	10	80
6	E	N	-	E	E	E	E	-	E	N	-	N	-	E	10	80
7	M	N	-	M	N	H	H	M	M	M	N	-	H	M	9	72
8	N	-	-	N	-	M	M	M	M	-	M	M	M	M	10	80
9	N	-	N	-	N	-	N	-	N	-	M	M	M	N	9	72
10	-	M	E	E	M	-	E	E	-	E	E	E	E	H	10	80
11	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0
12	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0
13	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0
Morning shift	3	3	3	3	3	3	3	3	3	3	3	3	3	3		
Evening shift	2	2	2	2	2	2	2	2	2	2	2	2	2	2		
Night shift	2	2	2	2	2	2	2	2	2	2	2	2	2	2		

Biography

Hamed Jafari was born in Iran in 1987. He received B.E., M.E., and Ph.D degrees in Industrial Engineering from Guilan University, Sharif University of Technology, and Isfahan University of Technology in Iran, in 2009, 2011, and 2016, respectively. His main areas of research interest are Operational Research, Game Theory, and Health Systems. Now, he is Assistant Professor in Isfahan University of Technology in Iran and works on the applications of the game theoretical approaches on the various supply chain structures.