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Monitoring lifetime data for a failure censoring reliability test with replacement using Shewhart type and exponentially weighted moving average control charts

H. Rasay*

Department of Industrial Engineering, Kermanshah University of Technology, Kermanshah, Postal Code: 6715685420, Iran.

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Abstract. Lifetime data has two distinguishing features: They are usually censored and have non-normal distributions. Generally speaking, censoring schemes can be classified into (i) censoring schemes with replacement and (ii) censoring schemes without replacement. Replacement during a life testing means that once observing a failure item, it is replaced by a new one. While there is a relatively large body of literature regarding control charts under censoring schemes without replacement, i.e., the second type schemes, designing control charts for the first type schemes has not yet received considerable attention. In this paper, two types of control charts are developed to monitor the lifetime data of an exponentially distributed item. To obtain the information of the lifetime data, failure censoring is conducted: n items are randomly selected and put on the test simultaneously. The test continues until observing r ($r \leq n$) failures. During the test, once observing a failure time, it is replaced by a new one. Development of Shewhart type and Exponentially Weighted Moving Average (EWMA) control charts under failure censoring with replacement is the main novelty of the current study. Comparative and simulation studies are conducted. Average Run Length (ARL) curves are also derived with respect to different parameters of the process.

1. Introduction

Lifetime as a quality characteristic is one of the main concerns in reliability engineering. To acquire the information regarding the lifetime of a specific item, conducting life testing is almost inevitable. On the other hand, life testing is usually expensive and time consuming. Thus, different approaches have been developed to alleviate the problems of life testing including, but not limited to, censoring schemes, truncated and accelerated life testing. Left, right, progressing, interval and hybrid censoring are among the censoring schemes. An observation is said to be censored if the exact failure time is unknown [1]. Right censoring is one of the most popular life testing approaches which

*. *Tel.: 098 8338305005*
E-mail address: Hasan.Rasay@gmail.com (H. Rasay)

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is classified into type I or time censoring and type II or failure censoring. In a failure censoring scheme, n items are simultaneously placed on the test and the test continues until observing s ($s \leq n$) failures. Generally speaking, and in another aspect, the censoring schemes can be classified into two major groups: (i) censoring schemes with replacement and (ii) censoring schemes without replacement. Replacement during a censoring scheme means that once observing a failure item during the test that item is replaced by a new one. In other words, the total number of inspected items during the test remains constant. Accordingly, replacement can be applied in both the failure censoring and time censoring schemes. It is commonly assumed the time needed to replace a failure item by a new one is negligible. While there is a relatively large body of literature regarding control charts under censoring schemes without replacement, i.e., the second type schemes, designing control charts for the first type schemes has not yet received considerable attention.

Some researchers have developed control charts for monitoring Time Between Events (TBE) [2–4]. Generally speaking, monitoring the lifetime data is different from the monitoring TBE. In development of control charts for monitoring TBE, the property of gamma distribution is usually applied [5]. The lifetime data has two distinguishing characteristics: They are usually censored and have non-normal distributions. Accordingly, development of statistical tools, i.e., control charts, acceptance sampling plans, applicable for lifetime data is more demanding.

Various control charts have been developed to monitor the lifetimes under different censoring schemes. In [6], a variable control chart is proposed under failure censoring for Weibull distribution. Xu and Jeske [7] developed an Exponentially Weighted Moving Average (EWMA) control chart under time censoring for Weibull distributed items. Faraz et al. [8] proposed Shewhart type control charts for monitoring shape and scale parameters of the Weibull distribution. Also, some control charts are developed under truncated life testing. In [9], an np control chart is designed under truncated life testing of Weibull distributed lifetimes. Aslam et al. [10] proposed an attribute control chart under truncated life testing for Pareto distribution.

It is widely discussed in the literature of quality engineering that Shewhart control charts are not effective in detecting small shifts of the process [11]. To overcome this problem, other control charts have been proposed including EWMA and Cumulative Sum (CUSUM) control charts among others. While the statistic of a Shewhart control chart is computed based on the information of the current sample, in an EWMA control chart, to compute the statistic of the chart, not only the information of the current sample, but also the information of the previous samples is taken into consideration. Accordingly, EWMA charts are sometimes called memory-based control charts.

Some t-charts are developed by Rasay and Arshad [12] to monitor the lifetime data under failure censoring. Under the assumption of Weibull distribution for the lifetime data, in [13], a Conditional Expected Value (CEV) control chart is proposed. Comparative studies are conducted for real cases such as lifetime of aeronautical bearings. To evaluate the lifetime data of Weibull distributed items, in [14] a quick switching sampling plan is proposed which minimizes the average failure items during the test. T-control charts based on modified multiple dependent state sampling are designed under the assumption of exponential distribution for data [15]. X-bar control chart is proposed in [16] to monitor the inverse Rayleigh distribution while repetitive group sampling is employed. Yousaf et al. [17] provides Bayesian estimations for a transformed Weibull distribution under type I right censoring. In the following, Table 1 summarizes some of the research in the scope of the current study.

As Table 1 shows, in developing the control charts under censoring schemes, it is commonly assumed that censoring is conducted without replacement. In this paper, two types of control charts are proposed to monitor the lifetime under failure censoring with replacement while the items are assumed exponentially distributed. The first control chart is a Shewhart type while the other is an EWMA control chart. Replacement during the life tests leads to more speed in acquiring the necessary data. In the proposed approach of this study, by replacing a failure item by a new one during the failure censoring test, the data is obtained more rapidly and consequently the statistics of the proposed control charts can be computed faster. This in turn increases the speed of the control chart in detecting the possible shifts of the process.

The rest of the paper is organized as follows: Section 2 describes a preliminary regarding the proposed failure censoring with replacement. In Section 3, a Shewhart type control chart is developed to monitor the lifetime. Section 4 proposes an EWMA chart, and Section 5 conducts some simulation and sensitivity studies regarding the Average Run Length (ARL) of the control charts. To clarify the performance of the control charts, Section 6 presents two examples. Managerial insights and real world applications of the proposed control charts are discussed in Section 7. Finally, Section 8 concludes the paper.

2. Conducting failure censoring with replacement

Let denote the random variable of lifetime of an item by T. It is assumed that T follows an exponential distribution with the following Probability Density

Table 1. Some research in the scope of designing control charts to monitor the lifetime data.

Reference	**Type of censoring**	**Type of the control chart**	**Lifetime distribution**
[5]	Type I right censoring (without replacement)	CUSUM	Weibull
[6]	Failure censoring (without replacement)	Shewhart type control chart	Weibull
[21]	Truncated life test (without replacement)	Attribute control chart	Weibull
[22]	Accelerated life test (without replacement)	Case-selectin control chart	Log-normal
[10]	Truncated life test (without replacement)	Attribute control chart	Pareto distribution
[23]	Type II censoring (without replacement)	Shewhart type control chart	Exponential distribution
[24]	Truncated life test (without replacement)	Attribute control chart	Weibull
[7]	Type I censoring (without replacement)	EWMA	Weibull
Current study	Failure censoring (with replacement)	Shewhart and EWMA control charts	Exponential

Function (PDF):

$$f(t;\theta)=\frac{e^{-t/\theta}}{\theta}. \tag{1}$$

Accordingly, the mean and variance of T are θ and θ^2, respectively. In some literature, PDF of exponential distribution is stated as follows:

$$f(t;\lambda)=\lambda e^{-\lambda t}, \tag{2}$$

while λ is commonly called failure rate. Comparing Eqs. (1) and (2) yields that $\lambda=\frac{1}{\theta}$. Designing control charts for monitoring variable T is desirable. It should be noted that monitoring the lifetime is equivalent to monitoring the failure rate λ or θ. In this regard, conducting life testing is necessary. The life testing is conducted as follows: select n items randomly and put them on the test simultaneously. The test continues until observing r ($r\leq n$) failure items. During the test, once observing a failure item, it is replaced by a new one. In other words, the total number of failure items during the test remains n. Also, it is assumed that, the time needed to replace a failure item by a new one is insignificant. This scheme of life testing is called failure censoring with replacement.

Let $N(t_0)$ denote the total number of failure items observed after passing t_0 time units from the start of the test. As each failure item is immediately replaced by a new item, $N(t_0)$ follows a Poisson distribution with the following distribution:

$$P(N(t_0)=r)=\frac{e^{-\lambda n t_0}(n\lambda t_0)^r}{r!}; \quad r=0,1,2,\cdots. \tag{3}$$

In other words, $N(t_0)$ is a Poisson process with rate $n\lambda$ per time unit. In the proposed control charts, in each sampling point, the following hypothesis test is conducted:

$$H:\begin{cases}H_0:\lambda=\lambda_0\\ H_1:\lambda\neq\lambda_0\end{cases} \tag{4}$$

This hypothesis test leads to a two-sided control chart. Under the assumptions provided in this section, Section 3 is about the designing a Shewhart type control chart, while Section 4 is about the designing of an EWMA control chart.

3. Shewhart type control chart

It is widely discussed by the researchers of the quality engineering that if T follows an exponential distribution of Eq. (1), transformed variable $T^*=T^{1/\beta}$ follows a Weibull distribution with shape parameter β and scale parameter $\theta^{1/\beta}$. Also, for $\beta=3.6$, T^* approximately has normal distribution [18,19]. Using the property of Weibull distribution, the mean and variance of T^* can be obtained as follows:

$$\mu=\theta^{1/\beta}\Gamma\left(1+\frac{1}{\beta}\right), \tag{5}$$

$$\sigma^2=\theta^{2/\beta}\left[\Gamma\left(1+\frac{2}{\beta}\right)-\left(\Gamma\left(1+\frac{1}{\beta}\right)\right)^2\right]. \tag{6}$$

As discussed in the previous section, at the ith inspection time, n items are randomly selected and simultaneously put on the life testing. The test continues until observing r failure items. During the test, once observing a failure item, it is replaced by a new one. Suppose that the following data is obtained for the failure times of r items: $t_{i,1}, t_{i,2}, \cdots, t_{i,r}$. After transforming the failure times by $\beta=3.6$, the mean of the transformed variables is computed as follows:

$$\bar{t}_i^*=\frac{\sum_{j=1}^{r} t_{i,j}^*}{r}; \qquad i=1,2,\cdots, \tag{7}$$

while $\bar{t}_i^*=t_{i,j}^{1/3.6}$. According to the property of normal distribution, $\bar{t}_i^*$ follows a normal distribution with the following mean and variance:

$$\mu=\left(\frac{\theta}{n}\right)^{\frac{1}{3.6}}\Gamma\left(1+\frac{1}{3.6}\right), \tag{8}$$

$$\sigma^2=\frac{(\theta/n)^{2/3.6}}{r}\left[\Gamma\left(1+\frac{2}{3.6}\right)-\left(\Gamma\left(1+\frac{1}{3.6}\right)\right)^2\right]. \tag{9}$$

At the ith inspection time, $\bar{t}_i^*$ is plotted on the control chart as the chart statistic. For type I error α, and target value θ_0, Upper Control Limit (UCL) and Lower Control Limit (LCL) of the chart are computed as follows:

$$LCL=(\theta_0/n)^{1/3.6}$$
$$\left(\Gamma\left(1+\frac{1}{3.6}\right)-k\sqrt{\frac{\Gamma(1+\frac{2}{3.6})-(\Gamma(1+\frac{1}{3.6}))^2}{r}}\right), \tag{10}$$

$$UCL=(\theta_0/n)^{1/3.6}$$
$$\left(\Gamma\left(1+\frac{1}{3.6}\right)+k\sqrt{\frac{\Gamma(1+\frac{2}{3.6})-(\Gamma(1+\frac{1}{3.6}))^2}{r}}\right). \tag{11}$$

In Eqs. (10) and (11), k is the coefficient of the control chart. ARL of the chart in the in-control state, ARL_0, is the inverse of type I error $ARL_0=\frac{1}{\alpha}$, while type I error is computed as follows:

$$\alpha=P(\bar{t}_i^*>UCL|\theta=\theta_0)+P(\bar{t}_i^*<LCL|\theta=\theta_0)=2[1-\Phi(k)], \tag{12}$$

while $\Phi(.)$ is Cumulative Distribution Function (CDF) of standard normal distribution. In the following of this section, the ARL of the control chart in the out-of-control state, ARL_1, is computed. Suppose that the value of θ shifts from θ_0 to $\theta_1=\delta\theta_0$, and δ determines the magnitude of the shift. The value of δ less than 1 denotes the deterioration of the process, while the value of δ larger than 1 indicates an improvement of the process. As the detection of the deterioration of the process may be more important from the viewpoint of some managers, in some research, designing one-sided control charts for lifetime is taken into consideration [20]. Anyway, the probability of type II error of the control chart can be computed as follows:

$$\beta=P(LCL<\bar{t}_i^*<UCL|\theta_1=\delta\theta_0)=\Phi(A_1)-\Phi(A_2), \tag{13}$$

while A_1 and A_2 can be computed as follows:

$$A_1 = \frac{UCL - (\delta\theta_0/n)^{1/3.6}\Gamma(1+\frac{1}{3.6})}{(\delta\theta_0/n)^{1/3.6}\sqrt{\frac{\Gamma(1+\frac{2}{3.6})-(\Gamma(1+\frac{1}{3.6}))^2}{r}}}, \quad (14)$$

$$A_2 = \frac{LCL - (\delta\theta_0/n)^{1/3.6}\Gamma(1+\frac{1}{3.6})}{(\delta\theta_0/n)^{1/3.6}\sqrt{\frac{\Gamma(1+\frac{2}{3.6})-(\Gamma(1+\frac{1}{3.6}))^2}{r}}}. \quad (15)$$

Thus, ARL of the out-of-control state is computed as follows:

$$ARL_1 = \frac{1}{1-\beta}. \quad (16)$$

4. EWMA control chart

In this section, designing an EWMA control chart is discussed. For the smoothing parameter ω, at time point ith, the EWMA statistic is computed as follows:

$$z_i = \omega\bar{t}_i^* + (1-\omega)z_{i-1}. \quad (17)$$

For a large value of i, the EWMA statistic follows a normal distribution with the following mean and variance:

$$\mu = \left(\frac{\theta}{n}\right)^{\frac{1}{3.6}}\Gamma(1+\frac{1}{3.6}), \quad (18)$$

$$\sigma^2 = \frac{\omega}{2-\omega}\frac{(\theta/n)^{2/3.6}}{r}\left[\Gamma\left(1+\frac{2}{3.6}\right)-\left(\Gamma\left(1+\frac{1}{3.6}\right)\right)^2\right]. \quad (19)$$

The UCL and LCL of this chart are computed as follows:

$$LCL = (\theta_0/n)^{1/3.6}\left(\Gamma\left(1+\frac{1}{3.6}\right) - k\sqrt{\frac{\omega}{r(2-\omega)}\left[\Gamma\left(1+\frac{2}{3.6}\right)-\left(\Gamma\left(1+\frac{1}{3.6}\right)\right)^2\right]}\right), \quad (20)$$

$$UCL = (\theta_0/n)^{1/3.6}\left(\Gamma(1+\frac{1}{3.6}) + k\sqrt{\frac{\omega}{r(2-\omega)}\left[\Gamma\left(1+\frac{2}{3.6}\right)-\left(\Gamma\left(1+\frac{1}{3.6}\right)\right)^2\right]}\right). \quad (21)$$

The probability of type II can be computed as follows:

$$\beta = P(LCL < z_i < UCL|\theta = \delta\theta_0)$$
$$= \Phi(A_1) - \Phi(A_2), \quad (22)$$

while A_1 and A_2 can be computed as follows:

$$A_1 = \frac{UCL - (\delta\theta_0/n)^{1/3.6}\Gamma(1+\frac{1}{3.6})}{(\delta\theta_0/n)^{1/3.6}\sqrt{\frac{\omega}{r(2-\omega)}\left[\Gamma(1+\frac{2}{3.6})-(\Gamma(1+\frac{1}{3.6}))^2\right]}}, \quad (23)$$

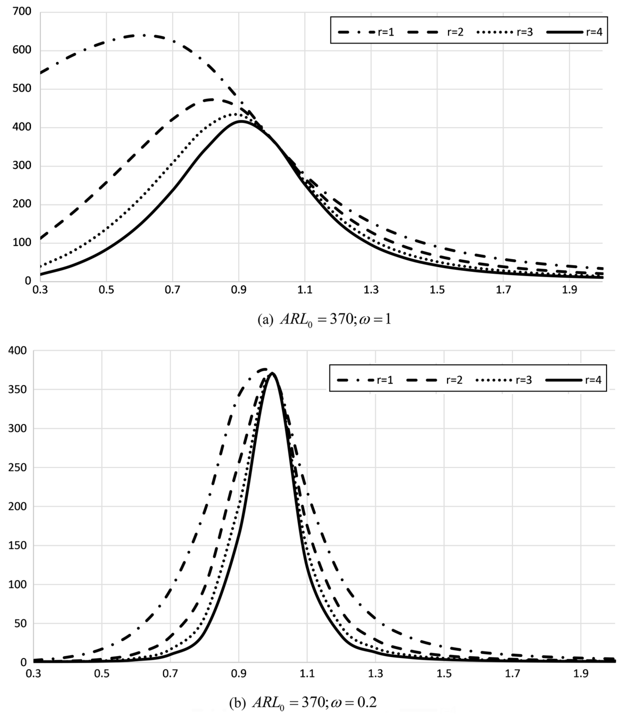
$$A_2 = \frac{LCL - (\delta\theta_0/n)^{1/3.6}\Gamma(1+\frac{1}{3.6})}{(\delta\theta_0/n)^{1/3.6}\sqrt{\frac{\omega}{r(2-\omega)}\left[\Gamma(1+\frac{2}{3.6})-(\Gamma(1+\frac{1}{3.6}))^2\right]}}. \quad (24)$$

It should be noted for the value of $\omega = 1$, the EWMA chart reduces to the Shewhart type control chart proposed in the previous section. Using Eq. (16), ARL_1 can be computed.

5. Analyzing ARL

In this section, some analyses are conducted regarding the ARL of the proposed control charts. The results of these analyses are presented in Figures 1 and 2. Figure 1 displays the ARL curves of the control charts for different values of r. The following observations can be inferred from this figure:

- Increasing r improves the power of the control charts in detection of the process shift. For example, the



(a) $ARL_0 = 370; \omega = 1$

(b) $ARL_0 = 370; \omega = 0.2$

Figure 1. Average run length for different values of r.

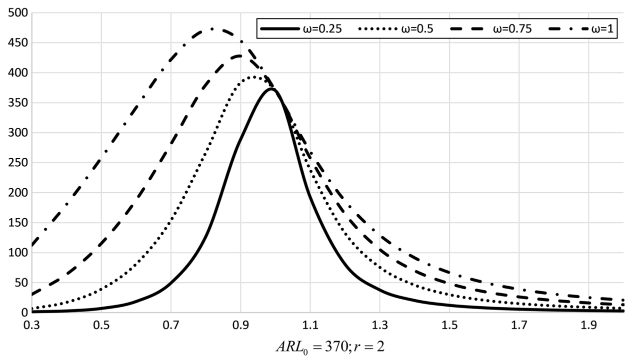


Figure 2. Average run length for different values of smoothing parameter.

control chart with $r = 3$ has a better performance than its counterpart with $r = 2$, i.e, the ARL curve of the control chart with $r = 3$ is below the ARL of the control chart with $r = 2$;

- For $\delta = 1$, the ARL in the in-control state is considered $ARL_0 = 370$. In this point, the ARL curves of the charts cross each other. In a control chart, it is desirable that the curve of ARL becomes maximum while there is no shift in the process, i.e., for the value of $\delta = 1$. Obviously, this is not the case here. For example, for the control chart corresponds to $r = 4$, the ARL is maximum around $\delta = 0.9$. This phenomenon is not desirable in a control chart and it is stated as the bias of the ARL curve [4]. While the ARL curve is bias, for the downward shifts, as the value of δ decreases from $\delta = 1$, the ARL curve first increases then decreases;
- With increasing r, the bias of the ARL curves decreases;
- The control charts have more ability in detection of the upward shift than detection of the downward shifts. In other words, the ARL curve is not symmetric.

Figure 1(b) displays the ARL curve while the smoothing parameter $\omega = 0.2$. Figure 2 shows the ARL curves for different values of smoothing parameter. Comparing Figure 1(a) with (b), and from Figure 2, the following observations can be interpreted:

- Decreasing the smoothing parameter significantly decreases the ARL, and consequently improves the ability of the control chart in detection of the process shift;
- Decreasing the smoothing parameter leads to the reduction of the bias of the ARL curves. So that, for example, for $\omega = 0.2$, the maximum of the ARL curve is around $\delta = 1$;
- With a decrease of the smoothing parameter, the asymmetric of the ARL curve decreases. It means that for a smaller value of ω, the difference between the ability of the control chart in detection of the upward and downward shifts decreases.

6. Simulation based examples

In this section, two examples are discussed to illustrate the applications of the proposed control charts. The first example is about the upward shift while the second is about the downward shift. The data of the examples are randomly generated using MINITAB 16 software. The control charts are also drawn in this software. In this software, the random data from different distributions including exponential distribution can be generated according to desired parameters. Thus to conduct the simulation, under the parameters assumed for the in-control state, a random data set is generated and for the out-of-control state, another one is generated. Using Eqs. (7) and (17) the statistics of the Shewhart and EWMA charts are computed and the charts are drawn.

6.1. Example 1

In this example, the performance of the control charts for detection of an upward shift is analyzed. It is used simulated data for the analyses of this section. First, 30 samples are selected while the mean of the exponential distribution is 2000 hours. Then, another 30 samples are selected while the mean of the process changes from 2000 to 3000 hours. To conduct the life testing, 5 items are simultaneously placed on the test. The charts corresponding to the values of $r = 3$ are displayed in Figures 3 and 4. Figure 4 shows a Shewhart type control chart, while Figure 3 shows an EWMA chart with $\omega = 0.4$. The other parameters of the control charts are illustrated in Figures 3 and 4. As can be seen, in this example, only the EWMA chart can identify the shift of the process.

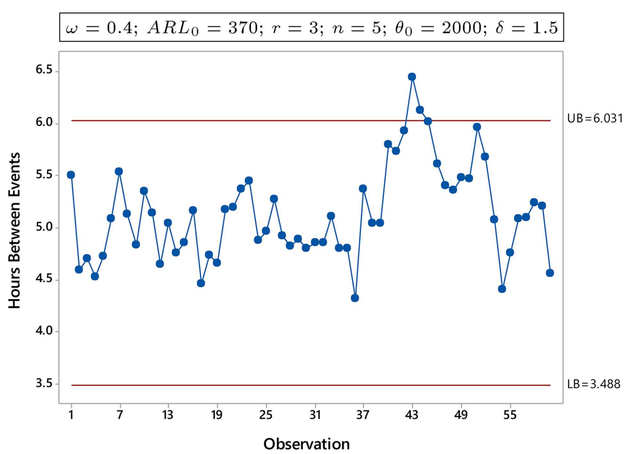


Figure 3. Detection of an upwards shift using EWMA chart.

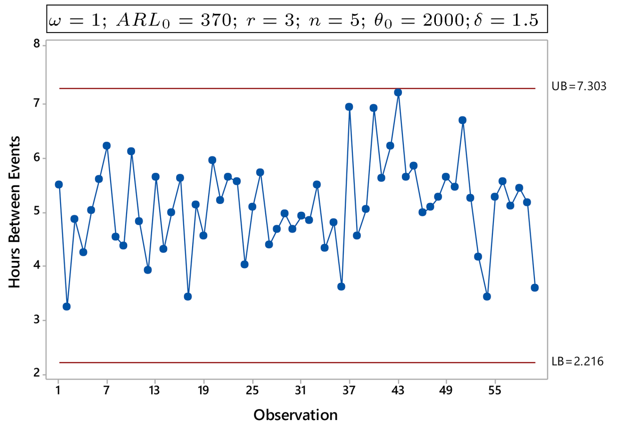


Figure 4. Shewhart type control chart for detection of an upwards shift.

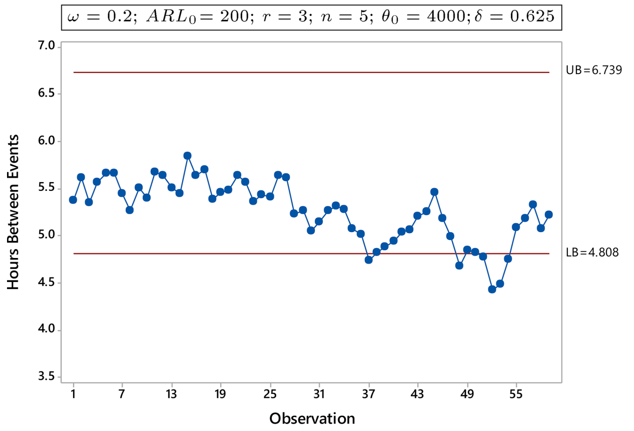


Figure 5. Detection of a downward shift using EWMA chart.

6.2. Example 2

In this example, the performance of the proposed control charts in detection of the process deterioration (downward shifts) is analyzed. Thus, 30 samples are selected under $\theta = 4000$. The next 30 observations are generated under $\theta = 2500$. The parameters of the chart are as follows:

$$ARL_0 = 200;\ \ r = 3;\ \ n = 5;\ \ \theta_0 = 4000;\ \ \delta = 0.625.$$

As Figures 5 and 6 show, only EWMA chart can detect the downward shift of the process.

7. Managerial implications and real applications

In the previous section, the applications of the proposed charts were shown using two simulated studies. In many real-world applications, TBE is properly assumed to follow exponential distribution. Hence, development of control charts to monitor TBE has a paramount importance. In this direction, Ref. [4] employed the data of the registered earthquakes in the Mount St. Helens region of Washington State and the data of Urinary Tract Infections.

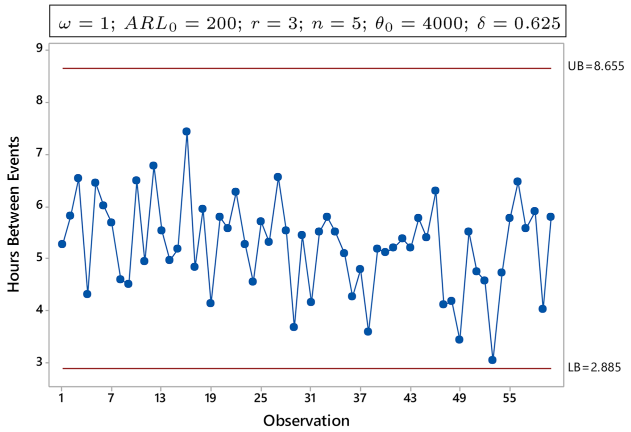


Figure 6. Shewhart type control chart for detection of an upwards shift.

Another application of the failure censoring is k-out-of-n system which means that the system has n components and it fails if at least r components fail. TBE can be assumed as a special case for k-out-of-n systems and the proposed control charts have applications in this scope as well. In [6], data from manufacturing car company in Korea are employed to show the applications of a variable control chart under failure censoring. The data are about the time (in months) until the service is required due to a certain subsystem failure of passenger car.

Evaluation and monitoring the lifetime data as a quality characteristic involve many costs and also it is time consuming. Many of the tests in this scope are destructive which make the subject more challenging. Hence, designing and employing suitable statistical methods have attracted the attentions of many researchers. Employing the proposed control charts in this paper effectively prevents from producing low quality items, i.e., items with lower lifetime mean. They can detect the out-of-control state and warn the quality engineers/mangers to adjust their manufacturing process. The proposed charts provide effective statistical tools simply employed by the mangers and engineers to monitor the lifetime data and continuously improve the quality. Furthermore, using the proposed failure censoring, the costs of life tests noticeably decrease. Replacement during conducting the failure censoring significantly decreases the time of the test so that the data necessary to compute the statistic of the test acquire more efficiently.

8. Conclusion

In this paper, two types of control charts are proposed for monitoring lifetime data under failure censoring with replacement, while the lifetime is assumed to follow an exponential distribution. The first chart is a Shewhart type and the second is an Exponentially

Weighted Moving Average (EWMA) control chart. According to the relationship between exponential and Weibull distribution, the observations of the failure times during life testing are transformed to an approximately normal distribution, and accordingly, the control charts limits are derived. The equations to compute the Average Run Length (ARL) in the out-of-control state are also derived. According to our analyses, the ARL curves of the control charts are bias and asymmetric. The bias of the ARL curves means that the ARL is not maximum while there is no shift in the process. This phenomenon negatively affects the performance of the control charts in detection of the out-of-control state. Also, the ARL curves are asymmetric meaning that the control charts have more ability in detection of the upward shift. According to the analyses, as the value attributed to the weight of the past observations in the EWMA control charts increases, both phenomena, i.e., asymmetric and bias of ARL curve, are alleviated to a large extent. Designing unbiased control charts so that the ARL curve becomes maximum while there is no shift in the process is a fruitful direction to extend the current study. Extending the current study while time censoring is employed for the life test can be considered as another direction for future works.

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Biography

Hasan Rasay received his PhD in Industrial Engineering from Yazd University, Yazd, Iran. Currently, he is an Assistant Professor of Industrial Engineering at Kermanshah University of Technology, Kermanshah, Iran. His areas of interest include: reliability, quality control, quality engineering and operations research. He has publications in famous journals such as European journal of Industrial Engineering, Computer and industrial Engineering, Communication in Statistic and Quality Engineering among others.