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Research Note

Convective heat transfer and flow phenomena from a rotating sphere in porous media

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Abstract. The study of flow and convective heat transfer from a rotary sphere in fluid mechanics, astrophysics, and astronaut subjects is important. Today, use of porous media has become widespread because of heat transfer characteristics as well as their lightweight and low volume. Many numerical studies on heat transfer and fluid mechanics in the rotary sphere have been done. The present project studies the phenomena of flow and heat transfer due to the rotation of the sphere at a constant temperature around itself in a porous medium, assuming a laminar, steady and incompressible flow. Analytical solution of the equations used is based on power series and the porosity coefficient is assumed to be between 0 and 1 in this problem. In the spherical coordinate system used here, changes in the azimuthal angle direction are ignored and the body force and pressure gradient for the problem are considered zero. The presence of the porous medium is expected to increase the value of thermal parameters.

1. Introduction

Porous media are used in a variety of industries such as nuclear fuel flasks, optimal insulation of buildings, crude oil, and heat recovery exchangers. Use of porous media is one way among many to improve their heat transfer performance [1]. It is also one of the examples of the application of a cylinder or a rotary sphere in mixers using this rotation in some cases to increase the rate of heat transfer.

Many studies have attempted to measure the natural and forced heat transfer rates around the sphere and other shapes [2–5]. Kishore and Ramteke [6] numerically investigated the phenomenon of heat transfer to spherical particles in Newtonian fluids with velocity slip and uniform thermal boundary conditions at the fluid-solid interface using a Computational Fluid Dynamics (CFD) based in-house solver. New results were obtained in accordance to the following conditions: $Re = 0.1 - 200$ (Reynolds number); $Pr = 1 - 100$ (the Prandtl number); and $\lambda = 0.01 - 100$ (dimensionless slip parameter). The problem of flow and heat transfer between rotating spheres remains of interest to many researchers [7–10]. In a major study, Moghadam and Rahimi [7] numerically studied the heat transfer and flow between two concentric rotating spheres with time-dependent angular velocities. They reported that long delays in the heat transfer of a large portion of the fluid in the annulus were produced by rotation of spheres. Also, they [8] investigated the same problem with constant angular velocities using similarity method and determined the temperature distribution, flow pattern, and heat transfer characteristics. The unsteady free convection flow at large Grashof numbers from a differentially heated rotating sphere was investigated

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by D'Alessio [11]. The analytical study of the author in the form of asymptotic expansion facilitated determining the heat transfer coefficient.

Flow and heat transfer in porous media have been numerically investigated by many researchers [12–14]. Kurdyumov and Linan [15] investigated a sphere in the porous medium in which the surface of the sphere was exposed to a constant heat flux. Merkin [16] studied natural heat transfer for two-dimensional and axial symmetry of objects with any desired shape in a porous medium of saturated fluid. Cheng [17] conducted the natural and forced heat transfer studies on a horizontal cylinder and a sphere saturated with a fluid in a porous medium using similarity solution. Available research works concerning such problems were considered by Sano and Okihara [18], Juncu [19], Gaffar et al. [20], Taherzadeh and Saidi [21], Pepona and Favier [22], Rao et al. [23], and Sano [24]. Chen et al. [25] numerically studied the mixed convection heat transfer from a rotating sphere within an enclosure. According to their results, the heat transfer coefficient increases by rotation. The laminar forced convection of a heated rotating sphere in air was studied by Feng [26] using a three-dimensional immersed boundary-based direct numerical simulation method. The flow structures and the mean Nusselt numbers for flow Reynolds number ranging from 0 to 1000 were obtained. They developed a new equation correlated with the mean Nusselt number of a heated rotating sphere for flows of $0 < Re < 500$. Nigam [27] demonstrated that upon applying the analytical method of power series to the laminar flow problem due to the uniform rotation of a sphere, a solution that corresponds almost to the physical condition of the problem in reality could be found. After integrating and solving several systems of equations, the aforementioned author managed to calculate the velocities in spherical coordinates and the thickness of the hydrodynamic boundary layer. Singh [28] continued the work of Nigam by studying the heat transfer given the laminar flow generated by the uniform rotation of the sphere and then, introduced different forms of power expansions for temperature distribution and thickness of the thermal boundary layer. Next, Kreith et al. [29] demonstrated that experimental and analytical studies of the flow generated by uniform rotation of a sphere in the certain range of Reynolds, Grashof, and Prandtl numbers were consistent with each other.

Based on a review of the previous works, the analytical study of forced convection and flow phenomena caused by a rotating sphere in a porous medium has not been investigated so far. In this paper, flow and heat transfer from a rotating sphere in porous media is analytically investigated. First, the governing equations in the porous medium are simplified for the given problem with respect to the hypotheses of the problem and then, solved by the analytical method of power series in a spherical geometry. It is proved that use of a porous medium increases the values of thermal parameters, such as Nusselt number, relative to the rotating sphere in quiescent water.

2. Mathematical formulation

The geometry discussed for the present problem is a rotating sphere with a constant radius that rotates at the angular velocity of Ω and uniformly around the axis, as shown in Figure 1. The porous medium around this sphere is considered to be a sphere with a radius much larger than the sphere with radius $a_0 (r >> a_0)$.

Due to the geometry of the problem, spherical coordinates are used. Due to the insignificance of velocity in the direction r due to the slow speed, in two directions, the momentum equations are solved only in θ and ϕ directions. Also, due to the symmetry of the considered geometry in ϕ direction, we consider the derivatives of this variable in all zero-governing equations. In this project, the flow is assumed laminar, steady and incompressible and the motion of the sphere is considered to be uniform in its round. The surface temperature of the sphere and the ambient temperature are also assumed constant. The body force is assumed to be zero and the pressure gradient is considered zero due to minor changes to the boundary layer as well as symmetry for the considered problem. The Forchheimer equation is used for mathematical modeling of flow in porous media. A mathematical model can be developed based on the following assumptions: (a) Porous media are isotropic and homogeneous with no contraction or distension; (b) The local thermal equilibrium is considered between solid and liquid phases; (c) The generation of heat due to viscous effects is negligible. The order of terms used in the governing equations can be written as: $u_r \sim O(\partial)$ (due to the insignificance of the velocity in this direction), $u_\theta \sim O(1)$, $u_\phi \sim O(1)$, and $\frac{\partial}{\partial r} \sim O(\delta^{-1})$ (because of the sharp changes in this direction). Hence, the governing equations, regarding all the mentioned

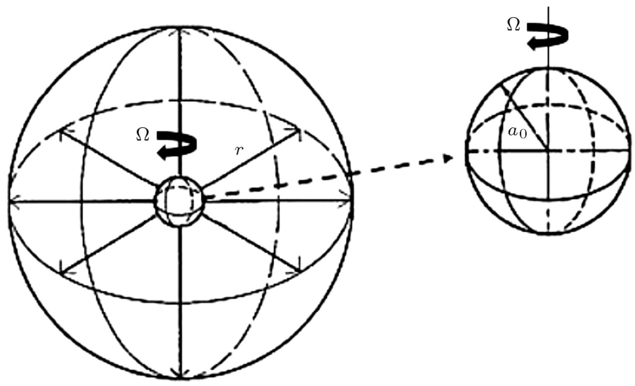

Figure 1. The physical model of the problem.

assumptions, can be written as follows:

$$\frac{\partial u_r}{\partial r}+\frac{1}{a_0}\frac{\partial u_\theta}{\partial \theta}+\frac{\cot\theta}{a_0}u_\theta=0, \tag{1}$$

$$\frac{1}{\varepsilon^2}\left(u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{a_0}\frac{\partial u_\theta}{\partial \theta}-\frac{u_\phi^2}{a_0}\cot\theta\right)=\frac{\nu}{\varepsilon}\left(\frac{\partial^2 u_\theta}{\partial r^2}\right)-\frac{\nu}{K}u_\theta, \tag{2}$$

$$\frac{1}{\varepsilon^2}\left(u_r\frac{\partial u_\phi}{\partial r}+\frac{u_\theta}{a_0}\frac{\partial u_\phi}{\partial \theta}-\frac{u_\theta u_\phi}{a_0}\cot\theta\right)=\frac{\nu}{\varepsilon}\left(\frac{\partial^2 u_\phi}{\partial r^2}\right)-\frac{\nu}{K}u_\phi, \tag{3}$$

$$(\rho C_P)_{eff}\left(u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{a_0}\frac{\partial T}{\partial \theta}\right)=k_{eff}\left(\frac{\partial^2 T}{\partial r^2}\right)+\mu\left[\left(\frac{\partial u_\theta}{\partial r}\right)^2+\left(\frac{\partial u_\phi}{\partial r}\right)^2\right]+\frac{\mu}{K}(u_r^2+u_\theta^2+u_\phi^2). \tag{4}$$

The boundary conditions are as follows:

$$r=0,\qquad T=T_s,\qquad u_r=0,$$
$$u_\theta=0,\qquad u_\phi=a_0\Omega\sin\theta, \tag{5}$$
$$r\to\infty,\qquad T=T_\infty,\qquad u_r=0,$$
$$u_\theta=0,\qquad u_\phi=0, \tag{6}$$

where u_r, u_θ, and u_ϕ are velocity components in directions r, θ, and ϕ, respectively, a_0 is radius of the sphere, porosity coefficient ε, kinematic viscosity ν, permeability coefficient K, fluid density ρ, specific heat capacity C_P at constant pressure, temperature T, thermal conductivity coefficient k, dynamic viscosity μ, and the subscript eff related to effective properties.

The non-dimensional parameters for the present problem are defined as follows:

$$\eta=\frac{T-T_\infty}{T_s-T_\infty},\qquad r^*=\frac{r}{a_0},\qquad u_r^*=\frac{u_r}{4a_0\Omega},$$
$$u_\theta^*=\frac{u_\theta}{4a_0\Omega},\qquad u_\phi^*=\frac{u_\phi}{4a_0\Omega},\qquad Pr=\frac{\nu}{\alpha},$$
$$Re=\frac{4a_0^2\Omega}{\nu},\qquad Da=\frac{K}{a_0^2},$$
$$Eck=\frac{(4a_0\Omega)^2}{C_{P_{eff}}(T_s-T_\infty)}. \tag{7}$$

The non-dimensional boundary conditions are as follows:

$$r^*=0,\qquad \eta=1,\qquad u_r^*=0,$$
$$u_\theta^*=0,\qquad u_\phi^*=\frac{\sin\theta}{4}, \tag{8}$$
$$r^*\to\infty,\qquad \eta=0,\qquad u_r^*=0,$$
$$u_\theta^*=0,\qquad u_\phi^*=0. \tag{9}$$

The governing equations are given below:

$$\frac{\partial u_r^*}{\partial r^*}+\frac{\partial u_\theta^*}{\partial \theta}+u_\theta^*\cot\theta=0, \tag{10}$$

$$\frac{1}{\varepsilon^2}\left(u_r^*\frac{\partial u_\theta^*}{\partial r^*}+u_\theta^*\frac{\partial u_\theta^*}{\partial \theta}-u_\phi^{*2}\cot\theta\right)=\frac{Re}{\varepsilon}\left(\frac{\partial^2 u_\theta^*}{\partial r^{*2}}\right)-\frac{1}{DaRe}u_\theta^*, \tag{11}$$

$$\frac{1}{\varepsilon^2}\left(u_r^*\frac{\partial u_\phi^*}{\partial r^*}+u_\theta^*\frac{\partial u_\phi^*}{\partial \theta}-u_\theta u_\phi^*\cot\theta\right)=\frac{Re}{\varepsilon}\left(\frac{\partial^2 u_\phi^*}{\partial r^{*2}}\right)-\frac{1}{DaRe}u_\phi^*, \tag{12}$$

$$u_r^*\frac{\partial \eta}{\partial r^*}+u_\theta^*\frac{\partial \eta}{\partial \theta}=\frac{1}{RePr}\left(\frac{\partial^2 \eta}{\partial r^{*2}}\right)+\frac{Eck}{Re}\left[\left(\frac{\partial u_\theta^*}{\partial r^*}\right)+\left(\frac{\partial u_\phi^*}{\partial r^*}\right)\right]+\frac{Eck}{ReDa}(u_r^{*2}+u_\theta^{*2}+u_\phi^{*2}). \tag{13}$$

3. Problem solving using power series method

The power series expansions for velocities in spherical coordinates according to the Nigam method [27] are as follows:

$$u_r=\frac{1}{2}(\nu\Omega)^{0.5}(2-3\sin^2\theta)(H_1+H_3\sin^2\theta+H_5\sin^4\theta+...), \tag{14}$$

$$u_\theta=a_0\Omega\cos\theta(F_1\sin\theta+F_3\sin^3\theta+F_5\sin^5\theta+...), \tag{15}$$

$$u_\phi=a_0\Omega\sin\theta(G_1+G_3\sin^2\theta+G_5\sin^4\theta+...). \tag{16}$$

All constants in Relations (14) to (16) are expressed in terms of a dimensionless variable called z. This means we have:

$$H=H(z),\qquad F=F(z),$$
$$G=G(z),\qquad z=\left(\frac{\Omega}{\nu}\right)^{0.5}(r-a_0). \tag{17}$$

By substituting the velocities above into governing equations and collecting powers of sine terms and after simplification, we have:

$$F_1^2-G_1^2+H_1F_1'=\varepsilon F_1''-\left(\frac{\nu\varepsilon^2}{K\Omega}\right)F_1, \tag{18}$$

$$4F_1F_3-2F_1^2-2G_1G_3-1.5H_1F_1'+H_3F_1' +H_1F_3'=\varepsilon F_3''-\left(\frac{\nu\varepsilon^2}{K\Omega}\right)F_3, \quad (19)$$

$$-6F_1F_3+6F_1F_5+3F_3^2-2G_1G_5-G_3^2 -1.5H_3F_1'+H_5F_1'-1.5H_1F_3'+H_3F_3' +H_1F_5'=\varepsilon F_5''-\left(\frac{\nu\varepsilon^2}{K\Omega}\right)F_5, \quad (20)$$

$$2F_1G_1+H_1G_1'=\varepsilon G_1''-\left(\frac{\nu\varepsilon^2}{K\Omega}\right)G_1, \quad (21)$$

$$-2F_1G_1+2F_3G_1+4F_1G_3-1.5H_1G_1'+H_3G_1' +H_1G_3'=\varepsilon G_3''-\left(\frac{\nu\varepsilon^2}{K\Omega}\right)G_3, \quad (22)$$

$$-2F_3G_1+2F_5G_1-4F_1G_3+4F_3G_3+6F_1G_5 -1.5H_3G_1'+H_5G_1'-1.5H_1G_3'+H_3G_3' +H_1G_5'=\varepsilon G_5''-\left(\frac{\nu\varepsilon^2}{K\Omega}\right)G_5. \quad (23)$$

For the energy equation, in addition to the velocity expansions, we consider the following expansion for temperature in accordance with Singh's method [28]:

$$C_{P_{eff}}T=C_{P_{eff}}T_\infty+a_0^2\Omega^2(M_1+M_3\sin^2\theta +M_5\sin^4\theta+...), \qquad M=M(z). \quad (24)$$

By inserting the velocities given in Eqs. (14) to (16) and temperature expansion in Eq. (24) in Eq. (4) and taking into account Relations (17) and (24) as well as simplifying and performing algebraic operations and finally equating the sentences with equal power of $\sin\theta$, on the other side of the equation, we get the following relations to the energy equation (for the aggravation of relations, the constants are not written by the variable z):

$$H_1M_1'=\frac{M_1''}{\mathrm{Pr}}, \quad (25)$$

$$H_1M_3'+H_3M_1'-1.5H_1M_1'+2F_1M_3 =(F_1')^2+(G_1')^2+\left(\frac{\nu}{K\Omega}\right)\left(F_1^2+G_1^2\right) +\frac{M_3''}{\mathrm{Pr}}, \quad (26)$$

$$H_1M_5'+H_3M_3'+H_5M_1'-1.5H_1M_3'-1.5H_3M_1' +4F_1M_5-2F_1M_3+2F_3M_3=2G_1'G_3' +2F_1'F_3'-(F_1')^2+\left(\frac{\nu}{K\Omega}\right)\left(2F_1F_3-F_1^2 +2G_1G_3\right)+\frac{M_5''}{\mathrm{Pr}}. \quad (27)$$

Eqs. (25) to (27) of the three differential equations are obtained by applying the power series expansion to the energy equation.

The boundary conditions governing the problem are expressed in terms of the constants in the expansions of the velocity as follows:

$$\text{at } z=0: \qquad F_1=F_3=F_5=0, \qquad G_1=1, \qquad G_3=G_5=0, \qquad H_1,H_3,H_5=0, \quad (28)$$

$$\text{at } z\to\infty: \qquad F_\to 0, F_3\to 0, F_5\to 0, \qquad G_1\to 0, G_3\to 0, G_5\to 0. \quad (29)$$

In order to keep the hydrodynamic boundary layer flow constant on the surface of the sphere, the following boundary conditions must exist:

$$\text{at } z\to\infty: \qquad F_1'\to 0, F_3'\to 0, F_5'\to 0, \qquad G_1'\to 0, G_3'\to 0, G_5'\to 0. \quad (30)$$

Also, the boundary conditions governing the problem are expressed in terms of the constants in the expansions of temperature as follows:

$$\text{at } z=0: \qquad M_1=\frac{C_{P_{eff}}(T_s-T_\infty)}{{a_0}^2\Omega^2}, \qquad M_3=M_5=0, \quad (31)$$

$$\text{at } z\to\infty: \qquad M_1\to 0, M_3\to 0, M_5\to 0. \quad (32)$$

In order to keep the transfer phenomenon in the thermal boundary layer on the surface of the sphere, the following boundary conditions must exist:

$$M_1'\to 0, M_3'\to 0, M_5'\to 0. \quad (33)$$

The differential equations obtained in Eqs. (18) to (23) and (25) to (27), along with the boundary conditions introduced in Eqs. (28) and (31), are the equations and boundary conditions that we use for the constants in differential equations to solve these equations from the following relations proposed by Nigam [27]:

$$F_1=as(1-s)^2(1+2s)-\frac{1}{2}\partial^2s^2(1-s)^2,$$

$$F_3=bs(1-s)^2(1+2s), \quad F_5=ds(1-s)^2(1+2s), \quad (34)$$

$$G_1=0.5(2+s)(1-s)^2, \quad G_3=cs(1+2s)(1-s)^2,$$

Table 1. Physical properties of the base model.

Ω	ν	K	ε	Pr
0.097 ($\frac{\text{rad}}{\text{s}}$)	0.000001 ($\frac{\text{m}^2}{\text{s}}$)	0.025 (m^2)	0.75	6.98

$$G_5 = es(1+2s)(1-s)^2, \tag{35}$$

$$M_1 = \frac{\left(C_{P_{eff}}(T_s - T_\infty)\right)(2+s_1)(1-s_1)^2}{2a_0{}^2\Omega^2},$$

$$M_3 = A_1 s_1(1+2s_1)(1-s_1)^2 - \frac{1}{2}(0.5835 \cdot \text{Pr}) \Delta_1^2 s_1^2 (1-s_1)^2,$$

$$M_5 = B_1 s_1(1+2s_1)(1-s_1)^2 + \frac{1}{2}(0.3136 \cdot \text{Pr}) \Delta_1^2 s_1^2 (1-s_1)^2. \tag{36}$$

In these relations, we have the dimensionless parameters s and s_1:

$$z = s\partial = s_1 \Delta_1, \tag{37}$$

where ∂ and Δ_1 represent the dimensionless thickness of the hydrodynamic and thermal boundary layers, respectively.

In order to solve the twelve differential equations, we first need to insert $(\Omega, \nu, k, \varepsilon, Pr)$ values per solving time into these equations in accordance with the problem conditions. These values are shown in Table 1.

All of these values, except Pr, calculated for the porous medium, were at $Re = 1000$ for pure liquid water, and the porosity and permeability values were also selected for admission of spherical balls of wood. To facilitate the continuation of operations in Eqs. (34) to (36), we put:

$$s = \frac{z}{\partial}, \qquad s_1 = \frac{z}{\Delta_1}. \tag{38}$$

Given that the constants in the differential equations derived from the continuity can be obtained in terms of other constants, we reduce the number of equations to 9. The resulting relations for H_1, H_3, and H_5 are as follows:

$$H_1 = \frac{z^3}{3} - \frac{4az^5}{5\partial^4} + \frac{3az^4}{2\partial^3} + \frac{z^5}{5\partial^2} - \frac{az^2}{\partial} - \frac{z^4}{2\partial},$$

$$H_3 = -\frac{8bz^5}{5\partial^4} + \frac{3bz^4}{\partial^3} - \frac{2bz^2}{\partial},$$

$$H_5 = -\frac{2bz^5}{5\partial^4} - \frac{12dz^5}{5\partial^4} + \frac{3bz^4}{4\partial^3} + \frac{9dz^4}{2\partial^3} - \frac{bz^2}{2\partial} - \frac{3dz^2}{\partial}. \tag{39}$$

Now, by substituting Eqs. (34) to (36) into the remaining 9 equations, we transform the differential equations into algebraic equations. Of course, at this stage, we must put the $(a_0, C_{P_{eff}}, T_s, T_\infty)$ values into Eq. (36), values of which are based on the boundary conditions and the geometry of the problem, as well as the fluid used, namely, water. These values are shown in Table 2. Upon integration, the calculation results are shown in Table 3.

Table 2. Geometric, physical, and boundary parameters of the base model.

a_0	$C_{P_{eff}}$	T_s	T_∞
0.0508 (m)	4398($\frac{\text{J.kg}}{\text{k}}$)	35(°C)	30(°C)

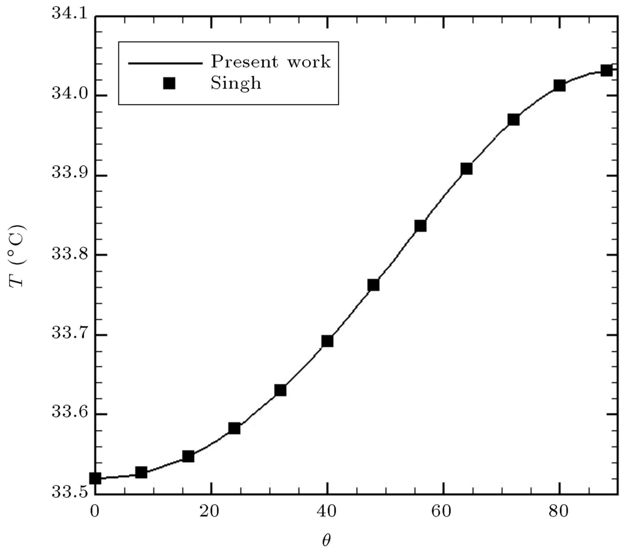

Figure 2. Validation of Singh's results and the present project for distribution of temperature in θ in $s_1 = 0.2$.

4. Results and discussions

4.1. Validation of the results

According to a review of the previous works, the analytical study of the forced convection and flow phenomena caused by a rotating sphere in a porous medium has not been conducted so far. Therefore, in order to validate the results, a comparison between the analytical power series solution carried out by Singh [28] and the results of the present project was made. From Figures 2 and 3, we can compare the validity of the temperature distribution around a quadrant of a sphere at different intervals from the surface of the sphere. These graphs indicate the accuracy and precision of the power series method in the case of this problem.

4.2. Velocity distribution

In Figure 4, the velocity changes in r directions are plotted for different values of s, which represents the radial distance from the surface of the sphere. According to Figure 4 and expansion of the power

Table 3. The obtained values for unknown constants in terms of calculation results.

a	∂	b	c	d	e	Δ_1	A_1	B_1
1.354	2.346	0.383	0.43	0.263	0.197	1.5	333030.54	156206.397

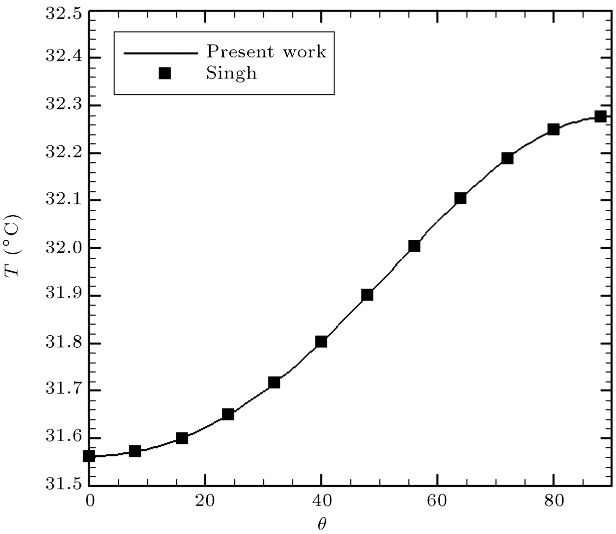

Figure 3. Validation of Singh's results and the present project for distribution of temperature in θ in $s_1 = 0.5$.

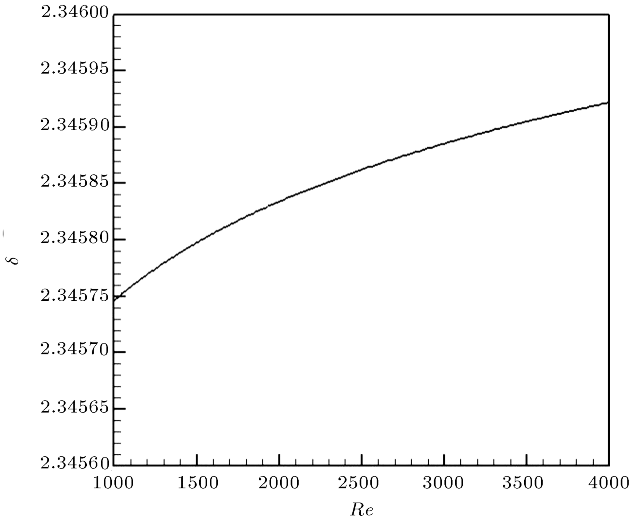

Figure 5. Changes in the dimensionless hydrodynamic boundary layer thickness in Re.

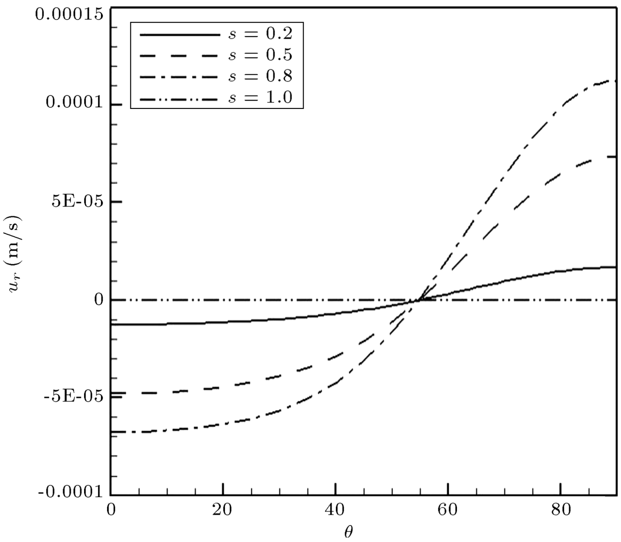

Figure 4. Radial velocity changes in θ direction.

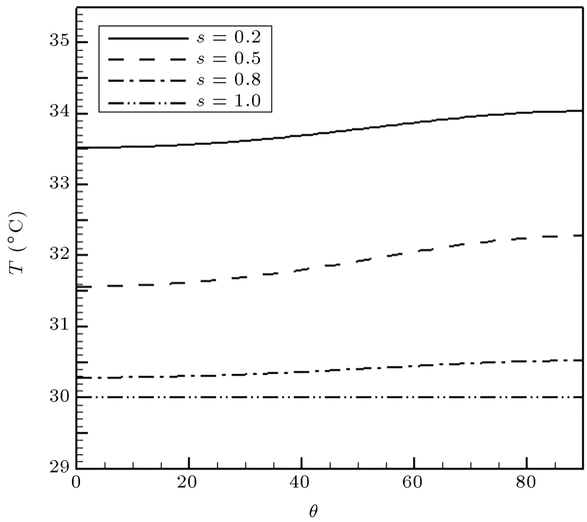

Figure 6. Temperature changes in θ direction.

series considered in Section 3, u_r is negative when $(2 - 3\sin^2\theta) > 0$, positive when $(2 - 3\sin^2\theta) < 0$, and is zero when $(2 - 3\sin^2\theta) = 0$, meaning that $\theta = 54.45°$ in the upper hemisphere and $\theta = 125.15°$ in the lower hemisphere. In Figure 5, both of variables are non-dimensional. Changes in the thickness of the dimensionless hydrodynamic boundary layer in terms of Reynolds number are shown in Figure 5. As can be seen, with increase in the Reynolds number due to the vortex penetration or, in other words, the penetration of the velocity, the thickness of the boundary layer increases.

4.3. Temperature distribution

In Figure 6, temperature variations are plotted in θ at different s_1's, which represent a radial distance from the surface of the sphere. In this figure, temperature changes are shown in relation to θ and radius, and s_1 represents the radius changes within the thermal boundary layer. As can be seen, the temperature approaches the ambient temperature at the boundary of the thermal boundary layer and outside of it. In Figure 7, the temperature distribution in the boundary layer for the quadrant is given, which in fact is equivalent to placing a series of power expansions

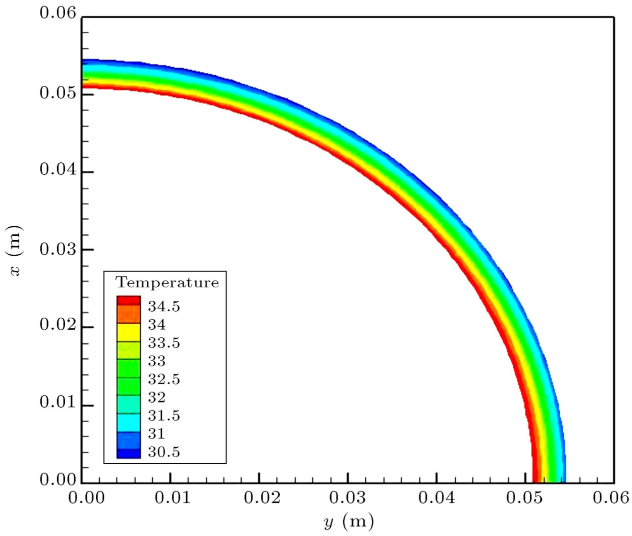

Figure 7. Distribution of temperature within the boundary layer for a quadrant of sphere.

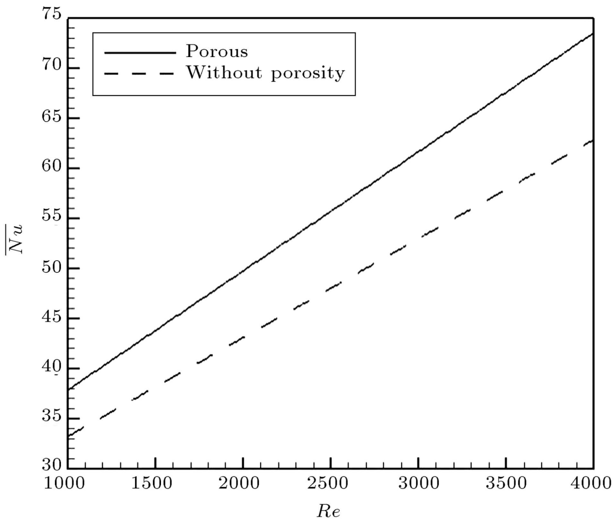

Figure 8. Comparison of Nu average changes in Re in porous media and quiescent water.

for temperature with a constant number and showing homogeneous regions.

4.4. Mean Nusselt number

According to Kreith et al. [29], the final relation for calculating the average Nusselt number on the surface of the sphere is as follows:

$$\overline{Nu}=-\frac{2a_0}{(T_s-T_\infty)}\frac{a_0^2\Omega^{2.5}}{\nu^{0.5}C_{P_{eff}}}\Big[M_1'(0)+\frac{2}{3}M_3'(0)+\frac{8}{15}M_5'(0)+\cdots\Big]. \quad (40)$$

In Figure 8, variation in the mean Nusselt number is compared to that in the Reynolds number for the rotating sphere in the porous media and quiescent water. As it turns out, the presence of a porous medium causes the mean Nusselt number to be higher than ones in the quiescent water at a constant Reynolds number. Also, with increase in the Reynolds number, the mean Nusselt number increases that results from the increase in the gradient of the average temperature inside the boundary layer due to the reduction of the thickness of the thermal boundary layer. This phenomenon results from the approach of the velocity profile behavior to the plug flow velocity profile. In addition, the porous medium itself is a thermal bridge. Also, in a porous medium, the Nusselt number increases due to the increase in the effective heat transfer surface areas.

5. Conclusions

In this study, heat transfer and flow phenomena caused by a rotary sphere in porous media were investigated. Equations governing the problem including consistency equations, momentum, and energy were obtained by using available models for a laminar flow. Finally, after considering the expansions of the power series for velocities and temperature, the equations were solved by using this analytical method. Some of the results are as follows:

- As the Reynolds number increases, the velocity and vortex diffusion increase in the boundary layer and as a result, the thickness of the boundary layer increases;
- With increase in the Reynolds number as the velocity gradient increases in the boundary layer, the temperature gradients increase; given that the overall shape of the temperature distribution is constant, the thickness of the thermal boundary layer decreases, which results in the mean Nusselt number to increase on the surface of the sphere;
- The presence of a porous medium causes the mean Nusselt number to be higher than the ones in the quiescent water at the constant Reynolds number;
- The effect of the rotation of the sphere on the velocity and temperature distribution is only up to the boundary of the boundary layer and is governed by the ambience outside the boundary layer.

Nomenclature

$A_1, B_1, a, b,$	Constant
$c, d, e, F, G,$	Constant
H, M, s, s_1	Constant
a_0	Radius of the sphere (m)
C_P	Specific heat capacity coefficient at constant pressure (kJ/kg.K)

$C_{P_{eff}}$	Effective specific heat capacity coefficient at constant pressure (kJ/kg.K)
$\bar{h}$	Average convection heat transfer coefficient (W/m^2.K)
K	Permeability coefficient (m^2)
k	Conductivity coefficient (W/m.K)
k_{eff}	Effective conductivity coefficient (W/m.K)
Nu	Nusselt number
$\bar{Nu}$	Average Nusselt number $2\bar{h}r/k$
Pr	Prandtl number $\rho C_p \nu/k$
q'''	Heat generation per unit volume W/m^3
r	Radial coordinate (m)
Re	Reynolds number, $2ur/\nu$
T	Temperature (°C)
t	Time (s)
T_∞	Temperature at infinity (°C)
T_s	Temperature on the sphere surface (°C)
u	Velocity (m/s)
z	Dimensionless variable

Greek symbols

Ω	Angular velocity of sphere (rad/s)
∂	Dimensionless hydrodynamic boundary layer thickness
Δ_1	Dimensionless thermal boundary layer thickness
ε	Porosity coefficient
μ	Dynamic viscosity (N.s/m^2)
ν	Kinematic viscosity (m^2/s)
θ	Acute angle subtended at the center of the sphere by any point and the nearest polar Latitude coordinate (°)
ϕ	Meridian angle coordinate (°)
ρ	Density (kg/m^3)
ρ_{eff}	Effective density of the fluid (kg/m^3)

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