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Roll restoring coefficients of planing boats' maneuver using $2D + t$ approach

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Abstract. The importance of maneuverability as a key feature of marine craft safety is widely recognized. Mathematical modeling together with Maneuver Hydrodynamic Coefficients (MHCs) is employed for the maneuverability simulation. Generally, experimental, analytical, and numerical methods are used to extract MHCs, with the $2D + t$ approach being used recently. In this study, roll restoring MHCs of planing hulls are evaluated by the $2D+t$ approach. The running attitude of planing boats alters during any kind of maneuver due to forward speed change. This study presents a simple and applicable Planar Motion Mechanism (PMM) procedure considering running attitude for the extraction of MHCs. In this procedure at a given forward speed, the planing hull is restrained to PMM apparatus in a fixed running attitude resulting from a conventional resistance test at the same forward speed. This procedure is employed by the $2D + t$ method for prismatic planing hulls in a set of forward speeds in the roll condition. This resulted in three regression formulae for Y_ϕ, K_ϕ, and N_ϕ as a function of deadrise angle and the Froude number. The results of this study can be used directly in the simulation of maneuvers via a mathematical model. Moreover, this approach can be followed in future work for other MHCs related to sway and yaw motions in future work.

1. Introduction

Planing boats were first used in maritime applications, resulting in the development of fast, maneuverable, and agile marine craft. Despite the fact that planing boats are more maneuverable than ships, their accidents are devastating. Therefore, the maneuverability of planing boats has a significant impact on their safety. Mathematical models, Computational Fluid Dynamic (CFD) methods, and free-running model tests are commonly used to evaluate the maneuverability of planing boats. Maneuver Hydrodynamic Coefficients (MHCs) measured by Planar Motion Mechanism (PMM) tests are often required for the simulation of maneuvers based on mathematical models. Hydrodynamic forces and MHCs can be calculated using CFD software and semi-analytical approaches based on potential theory, such as the $2D + t$ approach.

Numerical simulation, mathematical modeling, experimental, and sea trial methods have all been used to test ship maneuverability and MHCs. Sutulo and Soares [1] recently looked into empirical approaches for predicting ship maneuvers. They came to the conclusion that using universal empirical methods in a broad sense can lead to unacceptably significant disparities, and that this method should be utilized with caution and preferably on prototype ships. Yasukawa [2] conducted a captive model test for a car carrier in the proximity of a sloped bank with variations in water depth, hull-to-bank distance, hull drift angle, and heel angle to study course stability. Taimuri

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et al. [3] presented a modular mathematical model and a technique for estimating maneuver trajectories and motion time histories of single- and twin-screw propulsion ships the MHCs of which were extracted by PMM tests or semi-empirical relations. Ni et al. [4] proposed the mathematical model for the heave and pitch motion in regular waves to improve the maneuverability of the maritime simulator. The multi-parameter conformal mapping method was adopted to solve the hydrodynamic problem of the ship transverse sections, and then the integration of the hydrodynamic coefficients and the wave exciting forces for the ship hull was obtained using the strip method. Yiew et al. [5] developed a real-time method to simulate vessel maneuvering in waves. MHCs of the KRISO Container Ship (KCS) hull were estimated as a benchmark using URANS-CFD generated maneuvers in regular waves over a range of incidence angles, wavelengths, and Fr. The estimated wave loads, together with rudder and propeller forces were prescribed in the mathematical maneuvering model. Wicaksono et al. [6] presented a mix of available empirical relationships that can be used as a tool to calculate ship maneuvering motion using a modular mathematical model based on the Maneuvering Modeling Group (MMG) model. To validate the calculation tool, full-scale sea experiments such as straight running and zig-zag were carried out in Osaka Bay using a 17-m twin-screw passenger ferry.

The above-reviewed literature is only part of the latest research on ship maneuverability. Regarding the maneuverability of planing boats, several investigations on mathematical modeling and extraction of MHCs using experimental and numerical approaches have been carried out. Henry [7] investigated planing hulls maneuverability by static testing of prismatic models to extract hydrodynamic coefficients due to sway velocity. Brown and Klosinski [8,9] considered the stability of planing hulls by conducting static tests at several drift angles in a range of running attitudes and roll angles. Lewandowski [10–12] presented many empirical formulae for MHCs of planing hulls including roll-induced hydrodynamic coefficients and hydrodynamic coefficients for roll coupled with sway and yaw motions. Plante et al. [13] extracted MHCs of planing boats by performing hundreds of PMM tests, however, the researchers could not find the procedures of these tests.

Ikeda et al. [14] conducted a thorough study of hydrodynamic forces acting on a planing hull through static tests considering the change in running attitude. Katayama et al. [15] showed that MHCs of planing hulls are not constant during the maneuver and change as craft speed changes. They concluded that sway and yaw speeds and accelerations affect the running attitudes of planing hulls during the maneuver. Moreover, experimental tests by Katayama et al. [16] on the turning diameter of a planing hull also supported their achievement [15]. Katayama et al. [17] presented a 3 DOF mathematical model for maneuverability of planing boats. They extracted MHCs under fixed and free-roll conditions and employed the relevant MHCs at each instant.

Morabito [18] employed a two-dimensional oblique impact model to the three-dimensional planing body using slender body theory to estimate side force as a function of yaw angle. The results were compared to Lewandowski [19] empirical formula, which showed some discrepancy between experiment and empirical formula. Yasukawa et al. [20] presented a 4-DOF equation of motion used on a high-speed ship at a Fr range of 0.6 to 1.0. They introduced a method for extracting MHCs that involves changing the running attitude. The MHCs were calculated as a function of rise-up and trim angle. Hajizadeh et al. [21] simulated planing boat maneuvers such as the straight-line maneuver, course-changing, and turning maneuvers using MHCs introduced by Lewandowski [19]. Zeraatgar et al. [22] investigated the surge added mass of planing hulls by model experiments as well as approximated by the quasi-analytical method. They concluded that the surge added mass coefficient of a planing boat can reach up to 10% of the craft mass.

Numerical methods can be considered as a good option for this goal. Tascon et al. [23] studied the application of a slender body for the computation of the hydrodynamic forces acting on a planing-hull. They used Star-CCM+® commercial CFD software to obtain the force distribution on a rolled wedge impacting the water surface at vertical and horizontal velocities. Then the forces on the 2-Dimensional (2D) wedge were integrated along the length of the craft, giving the hydrodynamic forces. Ghadimi and Panahi [24] analyzed a stepped and non-stepped planing hull in steady yawed condition by Ansys-CFX to investigate hydrodynamic forces and moments acting on the boat at different yaw angles and C_v. Some researchers widely used CFD methods to model ship motions in the horizontal plane to determine their hydrodynamic coefficients [25,26]. Tavakoli and Dashtimanesh [27] developed a mathematical model based on a $2D + t$ approach to simulate the PMM test. Tavakoli and Dashtimanesh [28] developed a mathematical model for simulating planing boats steady turning, in which maneuvering forces and moments acting on the ship are calculated using $2D + t$-theory. To solve the motion equations, it is assumed that the craft is free in 6 DOF and the motion is strongly coupled. Algarin and Bula [29] developed a mathematical model using $2D+t$ theory to study the maneuverability of planing boats by simulating turning and zig-zag tests with 6 DOF. They investigated the effect of the main design parameters such as deadrise angle, LCG, VCG, and forward speed on the maneuverability of the planing hull. Sadati et al. [30] carried out several accelerating, stopping, and turning maneuver full-scale tests on two planing boats. They investigated the effects of forward speed, steering angle, and operation regimes on the turning ability of planing boats.

In summary, PMM tests are performed by ships in free-running attitude, where attitude changes during PMM tests and ship maneuvers are negligible. Regarding the maneuver of planing boats, changes in attitude due to changes in forward speed result in highly speed-dependent MHCs. The above literature review shows that the calculation of planing hull MHCs and determination of the running attitude condition during PMM testing is an ongoing research topic. Additionally, ITTC has not recommended any particular PMM test procedure for planing model [31].

In this study, two concepts are employed to calculate roll-restoring MHCs of planing hulls. The first concept concerns the simplification of running attitude change during a maneuver operation. The simplification is that the running attitude in any type of maneuver is equal to the running attitude on the straight path provided their forward speeds are equal. This simplification leads to a single attitude for a given forward speed for a planing boat during the maneuver. Therefore, forces acting on a hull during maneuvers and MHCs are calculated at each forward speed and the corresponding attitude. This concept also allows for a forward speed PMM test to be performed in which the model is restrained in the running attitude resulting from the resistance test at the same forward speed. To cover a range of maneuver cases, a PMM test should be conducted for a range of forward speeds. Interpolation of MHCs between calculated forward speeds or regression formula extracted from calculated MHCs is convenient for the rest of forward speeds.

The second concept simplifies a 3D hull for summing a set of rolled 2D wedges to evaluate the hydrodynamic coefficient. A roll angle results in an asymmetric hull, which can be simulated as the sum of a number of 2D asymmetric wedge sections entering the water. After the vertical and transverse forces on asymmetric wedges in the water entry problem are known, the roll restoring MCHs of the planing hull is estimated.

The combination of the above two concepts solves a complicated problem, which is the main result of this study. In addition, this PMM method is used for multiple prismatic planing hulls at multiple forward speeds, yielding regression formulae for Y_ϕ, K_ϕ, and N_ϕ as a function of β and Fr. The inferred regression formulas can be used directly in the maneuver simulation. Finally, the PMM method of this study can be used for other MHCs related to sway and yaw velocities and acceleration.

2. Calculation of the hydrodynamic coefficients using the $2D + t$ approach

2.1. Mathematical model of planing boats maneuver

A set of MHCs should be found in hydrodynamic coordinates by performing or simulating PMM tests to simulate the maneuver through a mathematical model such as 4 DOF in general form based on the available mathematical model [19–21] system [19] as follows:

$$\begin{aligned}
&\Delta \cdot \dot{u} - \Delta \cdot v \cdot r = X_{\dot{u}}\dot{u} - R_t + X_{\text{thrust}} + f_X\left(u, \dot{u}\right),\\
&\Delta \cdot \dot{v} + \Delta \cdot u \cdot r = Y_v v + Y_{\dot{v}}\dot{v} + Y_\phi \phi + Y_r r + Y_{\dot{r}}\dot{r}\\
&\qquad + Y_{\text{rudder}} + f_Y\left(v, \dot{v}, r, \dot{r}, \phi\right),\\
&I_{xx}\dot{p} - I_{xz}\dot{r} = K_\phi \phi + K_p p + K_{\dot{p}}\dot{p} + K_v v + K_r r\\
&\qquad + K_{\text{rudder}} + f_K\left(v, r, \phi, p, \dot{p}\right),\\
&I_{zz}\dot{r} - I_{xz}\dot{p} = N_r r + N_{\dot{r}}\dot{r} + N_\phi \phi + N_v v + N_{\dot{v}}\dot{v}\\
&\qquad + N_{\text{rudder}} + f_N\left(v, \dot{v}, r, \dot{r}, \phi\right).
\end{aligned} \tag{1}$$

Ship PMM tests are performed at free-running attitude, ignoring attitude change during PMM tests and ship maneuvers. Regarding the maneuver of planing boats, changes in running attitude due to changes in forward speed result in highly speed-dependent MHCs. At each forward speed during the maneuver, a planing boat has a specific running attitude, whereby forces in all directions and consequently MHCs are calculated according to the same attitude and forward speed. According to the literature reviewed, the running attitude (rise-up and trim angles) is considered as two additional parameters, and PMM tests were repeated in a set of the pair of rise-up and dynamic trim angles, which extremely increases the test runs. Another idea is that planing boat PMM tests are performed in a free-running attitude, which is the opposite of the logic of captive model testing.

Basically, when a planing boat is moving in a straight path in a steady-state, the running attitude is a function of its forward speed. In this study, it is assumed that in any type of maneuver, at a given forward speed, the running attitude is the same as the running attitude resulting from moving in a straight path. Therefore, at a given forward speed, in each maneuver, the craft attitude is assumed to be the result of a conventional resistance test.

According to the above assumption, the PMM test procedure employed in the present paper reduces the number of tests. In this procedure, the hydrodynamic forces and MHCs of sway, roll, and yaw are measured where the model is restrained vertically at the same rise-up and trim angle resulting from conventional resistance test at a given forward speed. The PMM test must be conducted for a range of forward speeds that the planing boats may encounter during the maneuver. Interpolation of MHCs between calculated forward speeds or regression formula extracted from calculated MHCs is convenient for the rest of forward speeds. Using this PMM procedure, MHCs in sway, roll, and yaw motion can be extracted as a function of running attitude and forward speed, resulting in coupling between surge, heave and pitch motions and sway, roll, and yaw motions. The method is employed in this study for some roll restoring hydrodynamic coefficients.

2.2. Outline of the $2D+t$ approach

Referring to Eq. (1) three hydrodynamic coefficients Y_ϕ, K_ϕ, and N_ϕ appear as a function of roll displacement, which should be calculated under the condition that the hull is restrained at a specific running attitude for a given forward speed based on the PMM procedure.

Figure 1 shows a planing hull at a specific running attitude for a given forward speed and fixed roll angle leading to forces and moments in sway, roll and yaw directions. This is the static roll test employed for roll restoring MHCs. The described problem can be redefined as the integration of a set of asymmetry water entry of 2D wedge sections.

The $2D+t$ is a kind of strip theory in the time domain. This means that a 3D body is defined as a finite number of sections (strips) at each instant of time. The $2D+t$ method is widely used for motions of planing boats in waves. This method extends the applicability of the handy strip theory to unsteady problems. To determine the pressure distribution during the symmetric entry of 2D wedge, Wagner [32] developed an analytical formula based on potential flow theory and energy conservation as follows:

$$\frac{P}{\rho}=-\dot{w}\sqrt{c^2-y^2}+\frac{wc\dot{c}}{\sqrt{c^2-y^2}}-\frac{w^2}{2}\frac{y^2}{c^2-y^2}, \qquad (2)$$

where P is pressure, ρ is the flow density, c is the half wetted beam and w is the vertical impact speed of 2D wedge. Toyama [33] extended Wagner method for asymmetric entry as follows:

$$\frac{P(\zeta)}{\rho}=-\dot{w}\sqrt{1-\zeta^2}+\frac{w\dot{c}(1+\mu\zeta)}{\sqrt{1-\zeta^2}}-\frac{w^2}{2}\frac{\zeta^2}{1-\zeta^2}, \quad (3)$$

where $\zeta=(y-\mu c)/c$ and μ are defined as asymmetric parameters. Algarin and Tascon [34] further developed the method and proposed pressure on each side of the wedge as follows:

$$\frac{P}{\rho}=-\dot{w}\sqrt{c^2-(-\mu+y)^2}+\frac{w\left(c\dot{c}+(-\mu+y)\dot{\mu}\right)}{\sqrt{c^2-(-\mu+y)^2}} -\frac{w^2}{2}\frac{(-\mu+y)^2}{c^2-(-\mu+y)^2}, \qquad (4)$$

where c and $\dot{c}$ are the wetted beam and its rate, respectively; also μ and $\dot{\mu}$ are defined as asymmetry parameter and its rate as follows [34]:

$$c=\frac{1}{2}(c_1+c_2), \qquad \dot{c}=\frac{1}{2}\left(\dot{c}_1+\dot{c}_2\right), \qquad (5)$$

$$\mu=\frac{1}{2}(c_1-c_2), \qquad \dot{\mu}=\frac{1}{2}\left(\dot{c}_1-\dot{c}_2\right), \qquad (6)$$

where c_1 and c_2 are the wetted beams on sides 1 and 2, respectively (see Figure 2).

An asymmetric wedge of a planing hull can experience three states: both sides unwetted, side 1 wetted, and side 2 unwetted and both sides wetted.

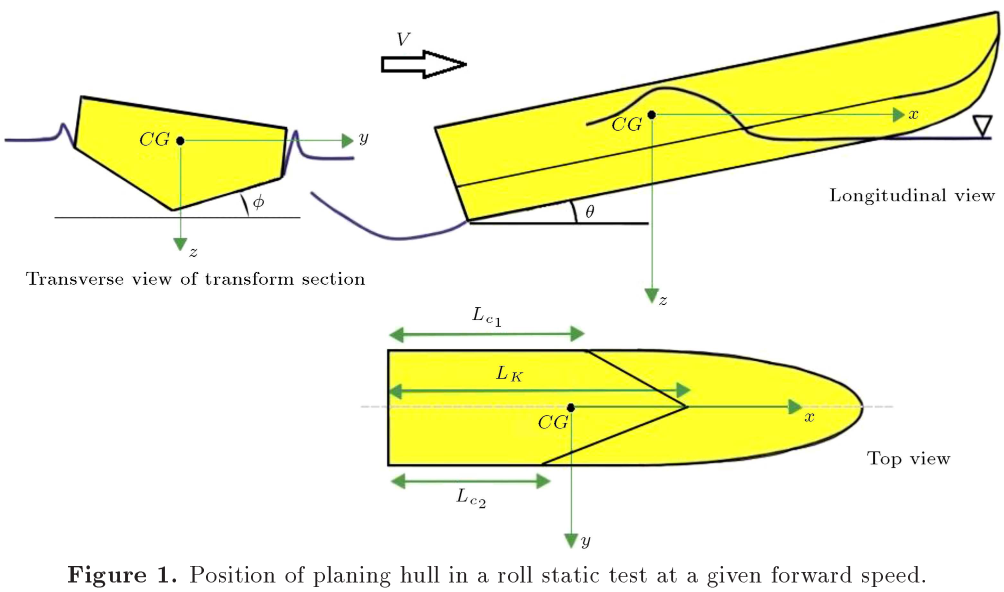

Figure 1. Position of planing hull in a roll static test at a given forward speed.

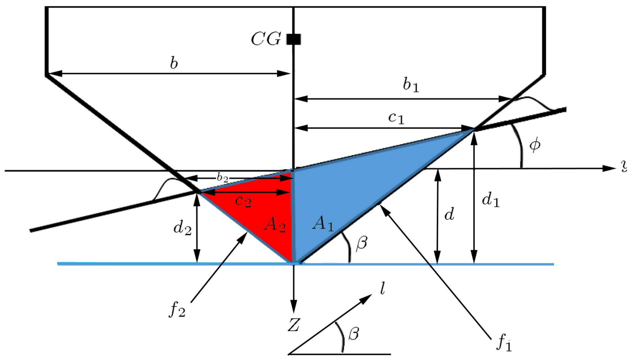

Figure 2. Definition of parameters of wedge for asymmetric water entry problem.

State 1: Both sides unwetted

For a wedge section, it is assumed that the asymmetry does not affect the jet velocity, the wetted beam and its ratio and asymmetry parameter and its ratios are estimated as follows [34]:

$$c=\frac{\pi}{4}wt\left(\frac{1}{\tan(\beta_1)}+\frac{1}{\tan(\beta_2)}\right),$$

$$\dot{c}=\frac{\pi}{4}w\left(\frac{1}{\tan(\beta_1)}+\frac{1}{\tan(\beta_2)}\right), \quad (7)$$

$$\mu=\frac{\pi}{4}wt\left(\frac{1}{\tan(\beta_1)}-\frac{1}{\tan(\beta_2)}\right),$$

$$\dot{\mu}=\frac{\pi}{4}w\left(\frac{1}{\tan(\beta_1)}-\frac{1}{\tan(\beta_2)}\right), \quad (8)$$

where $\beta_1=\beta+\phi$ and $\beta_2=\beta-\phi$ are the deadrise angles of sides 1 and 2, respectively. The pressure distribution is calculated by Eq. (4).

State 2: Side 1 wetted and side 2 unwetted

The boundary condition for the flow separation from chine in side 1 is $P=0$ in $y=b_1$ assuming constant velocity entry ($w=$ constant and $\dot{w}=0$), should be replaced in Eq. (4) for further action [34]:

$$\frac{P}{\rho}=\frac{w(c\dot{c}+(-\mu+b_1)\dot{\mu})}{\sqrt{c^2-(-\mu+b_1)^2}}-\frac{w^2}{2}\frac{(-\mu+b_1)^2}{c^2-(-\mu+b_1)^2}=0. \quad (9)$$

By simplifying Eq. (9), the following is obtained:

$$\frac{2\,(c\dot{c}+(-\mu+b_1)\dot{\mu})\,\sqrt{c^2-(-\mu+b_1)^2}}{(-\mu+b_1)^2}=w. \quad (10)$$

Finally, by integrating Eq. (10), Eq.(11) is obtained:

$$\frac{2}{3}\frac{[c^2-(-\mu+b_1)^2]^{3/2}}{(-\mu+b_1)^2}-2\sqrt{c^2-(-\mu+b_1)^2}$$
$$+2c\ln\left|\frac{c+\sqrt{c^2-(-\mu+b_1)^2}}{(-\mu+b_1)}\right|=w(t-t_1), \quad (11)$$

where t_1 is the instant when flow separation occurs on side 1. On the other hand, Eqs. (5) and (6) yield [34]:

$$c_2+\mu=c, \qquad \dot{c}_2+\mu=\dot{c}. \quad (12)$$

Assuming that the flow separation from the chine on side 1 does not affect the wetted beam of side 2 [34]:

$$c_2=\frac{\pi}{2}\frac{wt}{\tan\beta_2}, \qquad \dot{c}_2=\frac{\pi}{2}\frac{w}{\tan\beta_2}. \quad (13)$$

Eqs. (11), (12), and (13) are solved iteratively to calculate c, $\dot{c}$, μ and $\dot{\mu}$. The pressure distribution is calculated by Eq. (4).

State 3: Both sides wetted

Boundary conditions for the flow separation from chine on side 2 is $P=0$ at $y=b_1$. Assuming constant velocity entry ($w=$ constant and $\dot{w}=0$), the boundary condition should be replaced in Eq. (4) for further action [34]:

$$\frac{P}{\rho}=\frac{w\,(c\dot{c}+(-\mu+b_2)\dot{\mu})}{\sqrt{c^2-(-\mu+b_2)^2}}-\frac{w^2}{2}\frac{(-\mu+b_2)^2}{c^2-(-\mu+b_2)^2}=0. \quad (14)$$

By simplifying Eq. (14) the following is obtained:

$$\frac{2\,(c\dot{c}+(-\mu+b_2)\dot{\mu})\,\sqrt{c^2-(-\mu+b_2)^2}}{(-\mu+b_2)^2}=w. \quad (15)$$

Finally, by integrating Eq. (15), Eq. (16) is obtained:

$$\frac{2}{3}\frac{[c^2-(-\mu+b_2)^2]^{3/2}}{(-\mu+b_2)^2}-2\sqrt{c^2-(-\mu+b_2)^2}$$
$$+2c\ln\left|\frac{c+\sqrt{c^2-(-\mu+b_2)^2}}{(-\mu+b_2)}\right|=w(t-t_2), \quad (16)$$

where t_2 is the moment when the flow separates from side 2. When the flow separation occurs on both sides, c and μ are calculated by solving Eq. (16) [34]:

$$\frac{2}{3}\frac{[c^2-(-\mu+b_1)^2]^{3/2}}{(-\mu+b_1)^2}-2\sqrt{c^2-(-\mu+b_1)^2}$$
$$+2c\ln\left|\frac{c+\sqrt{C^2-(-\mu+b_1)^2}}{(-\mu+b_1)}\right|=w(t-t_1),$$
$$\frac{2}{3}\frac{[c^2-(-\mu+b_2)^2]^{3/2}}{(-\mu+b_2)^2}-2\sqrt{c^2-(-\mu+b_2)^2}$$
$$+2c\ln\left|\frac{c+\sqrt{c^2-(-\mu+b_2)^2}}{(-\mu+b_2)}\right|=w(t-t_2). \quad (17)$$

$\dot{c}$ and $\dot{\mu}$ are calculated by solving Eq. (18) [34]:

$$\frac{2\,(c\dot{c}+(-\mu+b_1)\dot{\mu})\,\sqrt{c^2-(-\mu+b_1)^2}}{(-\mu+b_1)^2}=w,$$
$$\frac{2\,(c\dot{c}+(-\mu+b_2)\dot{\mu})\,\sqrt{c^2-(-\mu+b_2)^2}}{(-\mu+b_2)^2}=w. \quad (18)$$

Finally, pressure distribution is calculated by Eq. (4).

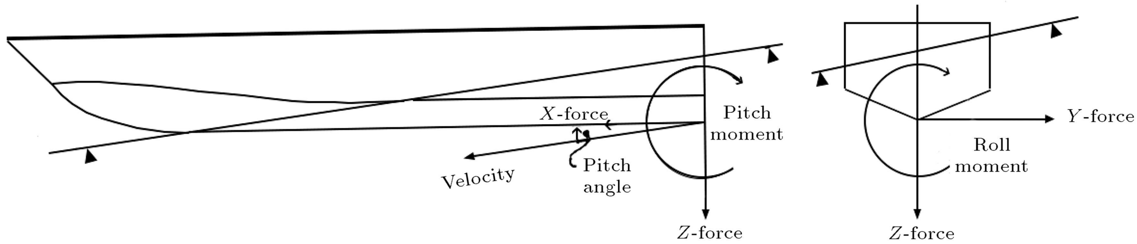

Figure 3. Positive direction of forces and moments according to the measurement of Brown and Klosinski [9].

2.2.1. Integration of 2D hydrodynamic forces

Each section condition (chine-wet or unwet) is determined along the wetted keel length for a given forward speed of the craft at a given roll angle. The pressure evaluated by Eq. (4) is integrated around the section and forces (f_1 and f_2) and center of forces of both sides of each section (y_{c_1}, z_{c_1}, y_{c_2}, and z_{c_2}) are extracted as follows (see Figure 3):

$$f_1 = \int_0^{b_1/\cos\beta} P.dl \qquad \text{and} \qquad f_2 = \int_0^{b_2/\cos\beta} P.dl, \quad (19)$$

$$y_{c_1} = \frac{\int_0^{b_1/\cos\beta} P.l.d}{f_1}\cos\beta \qquad \text{and}$$

$$y_{c_2} = \frac{\int_0^{b_2/\cos\beta} P.l.dl}{f_2}\cos\beta, \quad (20)$$

$$z_{c_1} = \frac{\int_0^{b_1/\cos\beta} P.l.dl}{f_1}\sin\beta, \qquad \text{and}$$

$$z_{c_2} = \frac{\int_0^{b_2/\cos\beta} P.l.dl}{f_2}\sin\beta, \quad (21)$$

$$\vec{f} = \vec{f_1} + \vec{f_2},$$

$$f_{y_1} = -f_1 \cdot \sin\beta \qquad \text{and} \qquad f_{y_2} = f_2 \cdot \sin\beta,$$

$$f_{z_1} = f_1 \cdot \cos\beta \qquad \text{and} \qquad f_{z_2} = f_2 \cdot \cos\beta,$$

$$m_x = -\, f_{y_1} \cdot (VCG - z_{c_1}) - f_{y_2} \cdot (VCG - z_{c_2}) - f_{z_1} \cdot y_{c_1} + f_{z_2} \cdot y_{c_2}, \quad (22)$$

where $\vec{f}$ is resultant force, f_{y_1}, f_{y_2}, f_{z_1}, and f_{z_2} are force components, m_x is roll moment of each section. The total forces in sway, roll, and yaw directions induced by the roll angle are approximated by the slender body theory with the integration of sections forces along the wetted keel length:

$$f_y = f_{y1} + f_{y2} \rightarrow Y = \int_0^{L_k} f_y \cdot dx, \quad (23)$$

$$K = \int_0^{L_k} m_x \cdot dx, \quad (24)$$

$$N = \int_0^{L_k} x \cdot f_y \cdot dx, \quad (25)$$

where x is longitudinal distance between each section.

2.3. Verification of 2D + t approach

PMM tests were conducted by Brown and Klosinski [8,9] for several planing hulls with the constant deadrise angles of 10°, 20°, and 30° at different speed coefficients of 1.5, 3, and 4. The specification of planing hulls is presented in Table 1. All models had identical buoyancy tested in combination of several speed coefficients, trim and roll angles. The coordinate system defined by Brown and Klosinki [8,9], shown in Figure 3, is used in this study to validate the calculation of roll-induced forces and moments. To validate the $2D + t$ approach, calculated forces and moments are compared to PMM test results from Brown and Klosinki [8,9] as shown in Tables 2 to 6.

As shown in Tables 2 to 6, the sway force absolute error of the $2D+t$ method ranges from 7.7% to 12.9%. The roll moment error is between 7% and 16.18% and the yaw moment error is between 8.9% and 16.98%. The source of error is the $2D + t$ approach which simplifies a 3D pressure distribution to integrate the 2D section pressure along the wetted length. Additionally, it may be contributed by inaccuracy associated with

Table 1. Specification of Brown and Klosinski planing model [8,9].

Parameter	Quantity
L (m)	1.27
B (m)	0.229
Δ (kg)	5.212
LCG (m)	0.572
VCG (m)	0.105

Table 2. Planing model of 30 degree deadrise angle at $C_v = 3$, $\theta = 6$, and $\phi = 10$.

Parameter	Asymmetric $2D + t$	Experiment	Error (%)
Y (N)	–8.9735	–10.2309	+12.3
K (N.m)	–1.3027	–1.1253	–15.8
N (N.m)	–3.6733	–4.1624	+11.8

Table 3. Planing model of 30 degree deadrise angle at $C_v = 4$, $\theta = 6$, and $\phi = 10$.

Parameter	Asymmetric $2D + t$	Experiment	Error (%)
Y (N)	–11.7831	–13.5226	+12.9
K (N.m)	–1.6919	–1.5185	–11.4
N (N.m)	–3.3800	–3.8776	+12.8

Table 4. Planing model of 20 degree deadrise angle at $C_v = 3$, $\theta = 6$, and $\phi = 10$.

Parameter	Asymmetric $2D + t$	Experiment	Error (%)
Y (N)	–5.2217	–4.8486	–7.7
K (N.m)	–1.0444	–0.9762	–7.0
N (N.m)	–1.9334	–1.7761	–8.9

Table 5. Planing model of 20 degree deadrise angle at $C_v = 4$, $\theta = 3$, and $\phi = 10$.

Parameter	Asymmetric $2D + t$	Experiment	Error (%)
Y (N)	–5.6787	–6.4942	+12.6
K (N.m)	–1.1136	–1.3287	+16.18
N (N.m)	–3.0614	–3.6878	+16.98

Table 6. Planing model of 20 degree deadrise angle at $C_v = 4$, $\theta = 6$, and $\phi = 10$.

Parameter	Asymmetric $2D + t$	Experiment	Error (%)
Y (N)	–7.9456	–7.0727	–12.3
K (N.m)	–1.6590	–1.5456	–7.3
N (N.m)	–1.6753	–1.4507	–15.48

the calculated 2D section pressure. The error range is due to a phenomena of impact nature which may be regarded fairly acceptable.

3. Calculation of roll restoring MHCs by $2D + t$ approach

3.1. Specifications of planing hulls

To extract the roll restoring MHCs, prismatic hulls are a good choice for parametric studies and generalizations. Four hulls are considered, following the Fridsma [35] body lines with 15°, 20°, 25°, and 30° deadrise angles. The bodylines of the models are shown in Figure 4 and hulls specifications are given in Table 7.

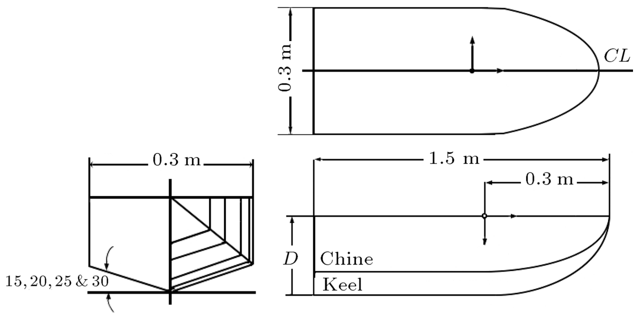

Figure 4. Bodylines of the models.

3.2. Determination of running attitude

According to the PMM method for calculating MHCs (Section 2.1), the running attitude of planing boats must first be extracted by drag testing or some other method at any forward speed. Therefore, the planing models dynamic trim and wetted keel length calculate as a symbol of their running attitude by Savitsky method [36] for a range of forward speeds which are the input parameters for simulation of static roll test by $2D + t$ approach. Table 8 shows calculation results for four forward speeds, say 4, 5, 6, and 7 m/s.

Table 7. Model specification.

Parameter	Quantity
L (m)	1.5
B (m)	0.3
Δ (kg)	16.45
β (degree)	15, 20, 25, and 30
LCG (m)	0.614
VCG (m)	0.0882

Table 8. The results of resistance tests for planing models.

β (degree)	V (m/s)	θ (degree)	L_k (m)
15	4	4.0106	1.3885
	5	4.0333	1.2317
	6	3.6294	1.1704
	7	3.1848	1.1563
20	4	4.2436	1.4403
	5	4.3176	1.2806
	6	3.9657	1.2242
	7	3.4833	1.2119
25	4	4.4922	1.4697
	5	4.6249	1.3258
	6	4.2576	1.2681
	7	3.8133	1.2606
30	4	4.7576	1.4777
	5	4.9571	1.3686
	6	4.6164	1.3106
	7	4.1775	1.3042

3.3. Calculation of MHCs

Based on the $2D + t$ approach, a computer code is developed in MATLAB software, where the inputs are V, L_k, θ, and ϕ and the outputs are roll-induced forces and moments in sway, roll, and yaw directions. At a forward speed of 4, 5, 6, and 7 m/s for prismatic hulls of 15°, 20°, 25°, and 30° deadrise angles at roll angles of 0°, 2°, 4°, 6°, 8°, and 10°, the sway, roll, and yaw forces and moments are calculated. Figures 5 to 10 show sway, roll, and yaw force versus ϕ at different conditions. All these computations are done in the hydrodynamic coordinate system (x, y, z) as shown in Figure 2.

As can be seen in Figures 5, 6, and 7, Y, K, and N increase as ϕ increases for a prismatic hull at given

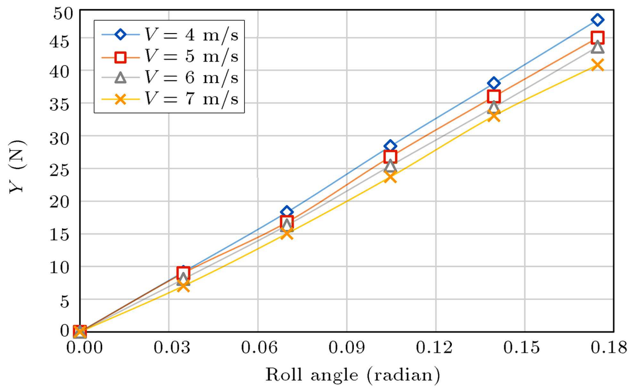

Figure 5. Sway force versus roll angle at deadrise angle of 20°.

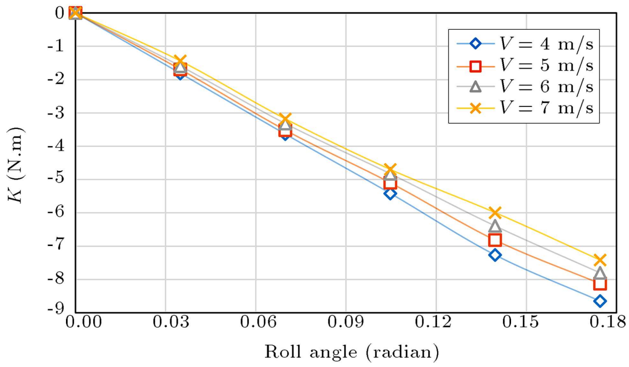

Figure 6. Roll moment versus roll angle at deadrise angle of 20°.

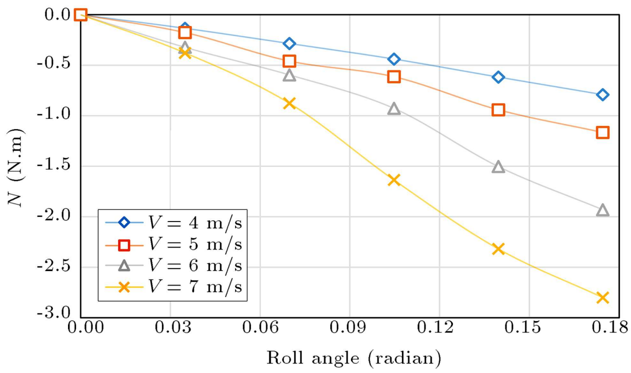

Figure 7. Yaw moment versus roll angle at deadrise angle of 20°.

forward speeds. Also, as forward speed increases, N increases while Y and K decrease at a given roll angle.

As can be seen in Figures 8, 9, and 10, Y, K, and N rapidly increase as ϕ increases for prismatic hulls at a given forward speed. Also, as the deadrise angle increases, N increases while Y and K decrease at a given roll angle.

The expressions for Y, K, and N are odd functions of ϕ; that is, the coefficients of the terms in the expansion with odd powers are only non-zero [37]. The expansion of Y or K or N as a function of ϕ is typically as follows [37]:

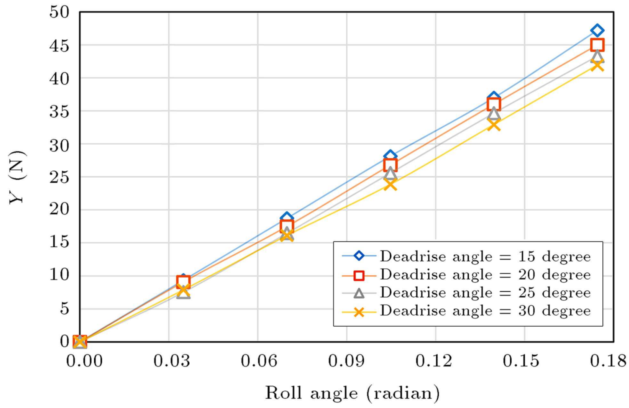

Figure 8. Sway force versus roll angle at forward speed of 5 m/s.

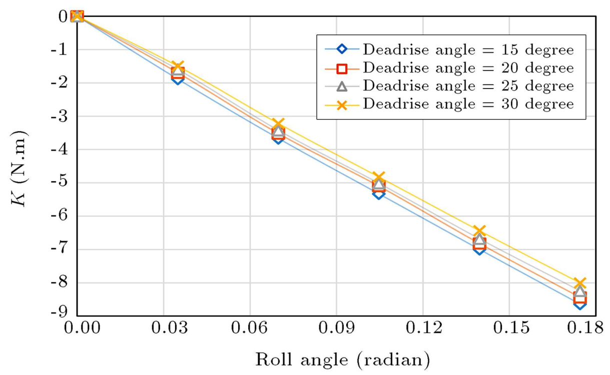

Figure 9. Roll moment versus roll angle at forward speed of 5 m/s.

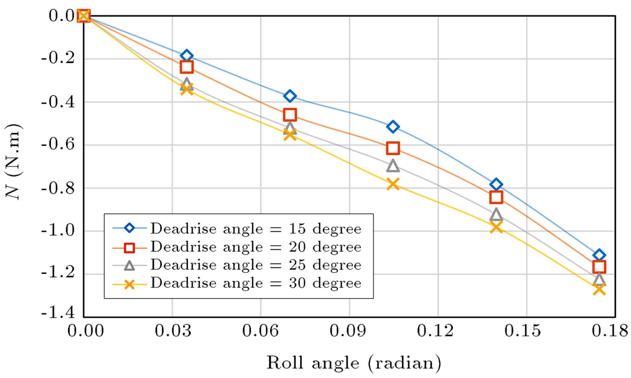

Figure 10. Yaw moment versus roll angle at forward speed of 5 m/s.

$$Y(\phi) = d_1\phi + d_3\phi^3 + d_5\phi^5 + \cdots \tag{26}$$

where d_1, d_3, and d_5 are K_ϕ, $K_{\phi\phi\phi}$, and $K_{\phi\phi\phi\phi\phi}$, respectively.

Figures 5 and 6 illustrate that the sway force and roll moment for a 20° deadrise angle hull are linear functions of roll angle throughout a range of speeds. Figures 8 and 9 also show that a linear relationship is maintained for deadrise angles of 15, 25, and 30 degrees. Figures 7 and 10 show that the yaw moment is not linear in respect to the roll angle. However, the level of nonlinearity is relatively small which may be approximated by a mean linear line. Considering the above details, the MHCs, Y_ϕ, K_ϕ, and N_ϕ are calculated as shown in Table 9.

Table 9. Roll-induced MHCs.

β (degree)	V (m/s)	F_n (–)	Y_ϕ (N)	K_ϕ (N.m)	N_ϕ (N.m)
15	4	1.043	284.11	–52.891	–4.5552
	5	1.303	268.81	–49.281	–6.1339
	6	1.564	255.13	–48.091	–10.627
	7	1.825	240.52	–46.079	–15.022
20	4	1.043	274.48	–51.49	–4.5592
	5	1.303	257.78	–48.406	–6.3872
	6	1.564	250.52	–45.304	–11.07
	7	1.825	238.16	–44.283	–16.847
25	4	1.043	257.78	–49.743	–4.6821
	5	1.303	251.26	–47.524	–6.6359
	6	1.564	239.57	-45.093	–11.561
	7	1.825	227.62	-41.792	–17.804
30	4	1.043	246.8	–49.069	–4.7198
	5	1.303	239.39	–46.192	–7.4524
	6	1.564	226.04	–42.866	–13.681
	7	1.825	219.44	–41.309	–20.448

Table 9 shows that when forward speed increases, the absolute value of Y_ϕ and K_ϕ decreases while the absolute value of N_ϕ increases. Additionally, by increasing the deadrise angle at given forward speed, Y_ϕ and K_ϕ decrease while N_ϕ increases.

3.4. Regression formulae for roll-induced MHCs

For prismatic planing hulls, the roll-induced restoring hydrodynamic coefficients are functions β of and Fr, according to the calculations. Based on the above results, many functions are examined in SPSS, a well-known statistical software, for approximation of Y_ϕ, K_ϕ, and N_ϕ. Finally, three regression formulae are concluded for Y_ϕ, K_ϕ, and N_ϕ as follows:

$$Y_\phi = 376.4371 - 1.7316\beta - 74.7067\text{Fr} - 0.044075\beta^2 + 1.8179\text{Fr}^2 + 1.2195\beta\text{Fr}, \tag{27}$$

$$K_\phi = -72.3876 + 0.3397\beta + 17.0559\text{Fr} - 0.0053575\beta^2 - 3.6228\text{Fr}^2 + 0.1267\beta\text{Fr}, \tag{28}$$

$$N_\phi = -27.8387 + 0.7656\beta + 33.9055\text{Fr}$$

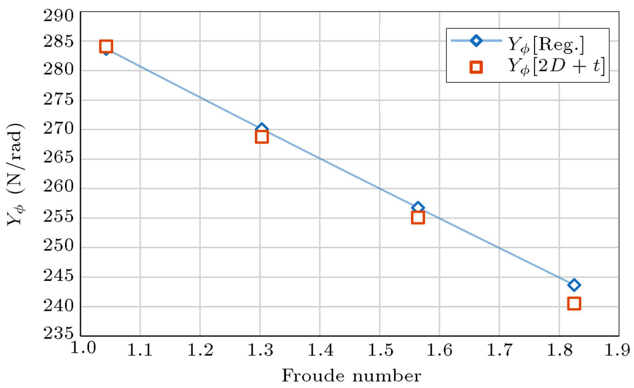

Figure 11. Y_ϕ versus Froude number for deadrise angle of 15°.

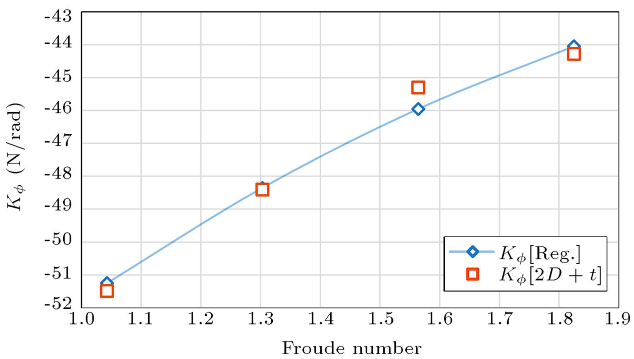

Figure 12. K_ϕ versus Froude number for deadrise angle of 20°.

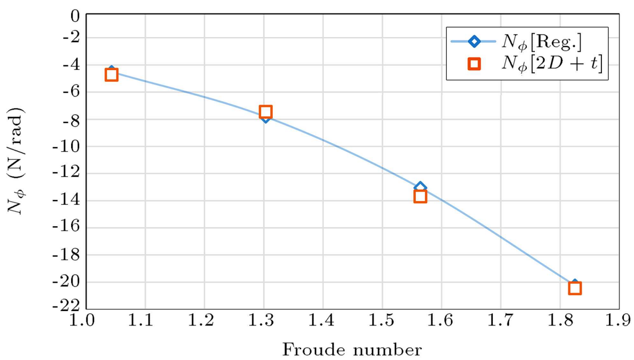

Figure 13. N_ϕ versus Froude number for a 30° deadrise angle.

$$-0.00648225\beta^2 - 14.2761\text{Fr}^2 - 0.4359\beta\text{Fr}. \quad (29)$$

The aforementioned regression formulae are compared to the $2D+t$ results in Table 9 to ensure that they are correct. The regression formulae and hydrodynamic coefficients calculated using the $2D+t$ technique are compared in Figures 11 to 13. The regression equations appear to be appropriate.

4. Conclusions

In this study, roll-restoring Maneuver Hydrodynamic Coefficients (MHCs) of planing hulls are calculated by the $2D+t$ approach based on the presented Planar Motion Mechanism (PMM) test procedure. The craft attitude is determined from the conventional resistance test for consideration of running attitude in the simulation of static roll tests at a given forward speed. Conclusions from this study are as follows:

- The 2D water entry method, combined with the $2D+t$ approach, can be used to simplify a complex problem like calculating the hydrodynamic forces exerted on planing boats and MHCs due to roll angle;
- The absolute values of Y_ϕ and K_ϕ decrease as forward speed increases, while N_ϕ increases. Furthermore, increasing deadrise angle at given forward speed, decreases Y_ϕ and K_ϕ while increases N_ϕ;
- The three regression formulae developed in this study for Y_ϕ, K_ϕ, and N_ϕ are in fair agreement with other methods and can be used directly in maneuver simulation;
- This study may encourage others to use the PMM approach to calculate other MHCs such as sway and yaw-induced hydrodynamic coefficients in the future.

Nomenclature

B	Breadth of planing boats (m)
CG	Center of gravity of planing boats
I_{xx}	Roll inertia moment (kg.m^2)
I_{zz}	Yaw inertia moment (kg.m^2)
I_{xz}	Yaw-roll inertia moment (kg.m^2)
L	Overall length of planing boats (m)
LCG	Longitudinal Center of Gravity from transom (m)
VCG	Vertical Center of Gravity from keel (m)
β	Dead-rise angle (degree)
Δ	Mass of planing boats (kg)
C_v	Speed coefficient ($C_v = \sqrt{V/Bg}$)
Fr	Froude number (Fr $= V/\sqrt{gL}$)
L_k	Wetted keel length (m)
p	Roll velocity (radian/s)
$\dot{p}$	Roll acceleration (radian/s^2)
r	Yaw velocity (radian/s)
$\dot{r}$	Yaw acceleration (radian/s^2)
u	Surge velocity (m/s)
$\dot{u}$	Surge acceleration (m/s^2)
V	Forward speed (m/s)

v	Sway velocity (m/s)
$\dot{v}$	Sway acceleration (m/s^2)
ϕ	Roll angle or displacement (degree or radian)
θ	Dynamic trim angle (degree or radian)
K	Moments imposed on planing hull in roll direction (N.m)
K_{rudder}	Rudder-imposed control force in roll direction (N.m)
N	Moments imposed on planing hull in yaw direction (N.m)
N_{rudder}	Rudder-imposed control force in yaw direction (N.m)
$R_t(u)$	Resistance force (N)
X	Force imposed on planing hull in surge direction (N)
X_{thrust}	Thrust force in surge direction (N)
Y	Force imposed on planing hull in sway direction (N)
Y_{rudder}	Rudder-imposed control force in sway direction (N)
f_K	Nonlinear terms of hydrodynamic force in roll direction
f_N	Nonlinear terms of hydrodynamic force in yaw direction
f_X	Nonlinear terms of hydrodynamic force in surge direction
f_Y	Nonlinear terms of hydrodynamic force in sway direction
K_p	Roll moment change due to roll velocity change (N.m.s)
$K_{\dot{p}}$	Roll moment change due to roll acceleration change (N.m.s^2)
K_r	Roll moment change due to yaw speed change (N.m.s)
K_v	Roll moment change due to sway speed change (N.s)
K_ϕ	Roll moment change due to roll displacement change (N.m)
N_r	Yaw moment change due to yaw speed change (N.m.s)
$N_{\dot{r}}$	Yaw moment change due to yaw acceleration change (N.m.s^2)
N_v	Yaw moment change due to sway speed change (N.s)
$N_{\dot{v}}$	Yaw moment change due to sway acceleration change (N.m.s)
N_ϕ	Yaw moment change due to roll displacement change (N.m)
$X_{\dot{u}}$	Surge force change due to surge acceleration change (kg)
Y_r	Sway force change due to yaw velocity change (N.s)
$Y_{\dot{r}}$	Sway force change due to yaw acceleration change (N.s^2)
Y_v	Sway force change due to sway speed change (N.s/m)
$Y_{\dot{v}}$	Sway force change due to sway acceleration change (kg)
Y_ϕ	Sway force change due to roll displacement change (N)

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