Optimizing a fuzzy multi-objective closed-loop supply chain model considering financial resources using meta-heuristic

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Abstract. This paper presents a multi-objective mathematical model to optimize and harmonize a supply chain in order to reduce costs, improve quality, and gain a competitive advantage and position using meta-heuristic algorithms. The purpose of optimization in this field is to enhance both quality and customer satisfaction and reduce the production time and related prices. The present research simultaneously optimized the supply chain in the multi-product and multi-period modes. The presented mathematical model was first validated. The parameters of the proposed algorithm were then adjusted to solve the model using Multi-Objective Simulated Annealing (MOSA) algorithm. To validate the performance of the designed algorithm, some examples were solved based on General Algebraic Modeling System (GAMS). The MOSA algorithm achieved average errors of %0.3, %1.7, and %0.7 for the first, second, and third objective functions, respectively, in the average less than one minute. The average time to solve was 1847 seconds for the GAMS software; however, the GAMS failed to reach an optimal solution for large problems in a reasonable computational time. The average error of the designed algorithm was less than 2% for each of the three objectives under study. These show the effectiveness of the MOSA algorithm in solving the problem introduced in this paper.

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1. Introduction

The business that competes in today’s world is based on the production of goods and services considering both customer needs and cost effectiveness at the same time. In many companies, customer orientation has been adopted to reduce the amount of time spent to meet customer needs and improve the product(s) quality. These companies seek to gain a competitive advantage by effectively managing their purchasing processes and making more beneficial interaction with their suppliers. Coordinating the flow of materials across multiple organizations within each organization is one of the

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major management challenges in the supply chain, and it requires implementation of a variety of technologies and tools to track materials along the route from the source to destination and record information in each step. Given its ability to recover value from the returned and used products, reverse logistics has received considerable attention, making it a key element in the supply chain. The supply chain is a chain that includes all activities related to the flow of goods and conversion of materials from the stage of preparation of raw materials to the stage of delivery of the final goods to the consumer. There are two other streams about the flow of goods: the flow of information and the flow of financial resources and credit. The design of a reverse logistics network is of critical importance because of the need for materials and products to flow in the opposite direction of the supply chain for a variety of reasons. Legal requirements, social responsibilities, environmental concerns, economic interests, and customer awareness have forced manufacturers to produce environmentally-friendly products, reclaim, and collect the returned and used products. Marketing, competitive and strategic issues, and customer's loyalty improvement and the subsequent sales are also the significant motivations behind the reverse logistics. Therefore, different industrial sectors need to improve their structures and activities to meet these challenges. Hence, a decision-making tool for supply chain coordination is presented in this study based on the existing contracts using heuristic algorithms. Adopting the right strategy to improve the supply chain performance brings many benefits and improves productivity in different companies and organizations.

Consideration of the supply chain optimization under different circumstances will decrease costs and improve quality, thus achieving a competitive advantage. Optimization problems in this area seek to enhance the product quality and customer satisfaction and reduce the production time and related costs. Several variables are considered inputs of these kinds of problems.

The objective here is to find the optimal design points fitted with the mentioned objective functions. Given the pricing role in reducing the uncertainty of the returned products and impact of product returns on the number, location, and capacity of facilities needed for product revival in this paper, designing a closed-loop Supply Chain Network (SCN) will be a model considering discounts and financial resource flows. The network of the mentioned model is derived from a study conducted by Ramezani et al. [1]. In a direct direction, the model includes the levels of suppliers, distributors, warehouses, retailers, and customers in which the warehouses are considered separately (allocation of the warehouse to a group of retailers) to make the proposed model more realistic.

In the opposite direction, the network includes the collection, recycling, and disposal centers, all produced in the direct flow of products using materials provided by the suppliers, through distribution centers to the warehouses, and from there to retailers and finally, to customers. The main objective of the current research is to develop a multi-objective contingency optimization model for the closed-loop supply chain design, which involves modeling the closed-loop supply chain problem considering discounts and flow of funds under uncertainty and two secondary objectives of solving the proposed model using the fuzzy perspective and obtaining optimal design points values. The rest of this paper is structured as follows: the theoretical foundations, literature review, and research gap are discussed in Section 2. Then, the solution method is elaborated in Section 3, and the research data analysis as well as the numerical results are presented in Section 4. The results are given in Sections 5 and 6. Finally, the conclusion and future suggestions are presented in Section 7.

2. Literature review

Logistic network design is a part of supply chain planning that primarily focuses on long-term strategic planning [2]. The logistics network design itself is divided into three parts: The forward logistic network design, reverse logistic network design, and integrate forward reverse logistic network (closed-loop).

Forward logistics network: A network of suppliers, manufacturers, distribution centers, and channels among them and customers to obtain raw materials, convert them into finished products, and efficiently distribute them to the customers (Amiri [3]).

Reverse logistics network: The process of efficiently planning, implementing, and controlling the flow of incoming and storing the second-hand goods and the related information in the opposite direction to the traditional supply chain to recover value or disposal [4]. The previous related literature is reviewed in the following.

Peng et al. [5] designed a multi-period forward SCN. They suggested a complex linear programming to solve the problem of explaining the SCN. The proposed multi-period model was designed with two objective functions of optimal distribution and cost reduction. Ramezani et al. [1] presented a multi-objective and multi-product stochastic model for forward/reverse network design under uncertainty. The model objectives include maximizing profits, maximizing customer service levels, and minimizing the total number of defective raw materials purchased from suppliers, thereby determining the locations of the facilities and flows among them in line with capacity constraints. This model is based on this scenario. In this paper, the 𝜖-
constraint method is used to obtain a set of optimal Pareto supply chain configurations.

Hassanzadeh and Zhang [6] presented a multi-objective, multi-product problem in which the communication flow is such that the products first are sent to the demand markets. Then, the products are sent from the demand markets to the collection centers. The product can be improved and then transferred to the production workshops; otherwise, it is transferred to the recycling centers. This problem was solved using two summing weights and ε constraints to convert the two-objective problem into a single-objective one. Vahdani and Sharifi [7] proposed a new mathematical model to design a closed-loop SCN that integrated the network design decisions in both forward and reversed SCNs. They highlighted the uncertainty of the parameters of the proposed model and modeled this uncertainty based on fuzzy parameters. They presented an inexact-fuzzy-stochastic solution methodology to deal with different types of uncertainty in their proposed model.

In this context, Pishvaee et al. [8] developed a feasible multi-objective programming model to design a network of sustainable medical supply chains under uncertainty, considering the conflicting economic, environmental, and social objectives. The present study provides a robust mathematical model for designing a medical needle and syringe supply chain as an essential strategic medical requirement in health systems. To this end, a product and a period were evaluated in this research. A rapid Benders analysis algorithm was also proposed using three efficient acceleration mechanisms that considered the computational complexity of the proposed model solution to solve this model. Moreover, Brailo et al. [9] aimed to optimize the SCN through the Tabu search method. Considering the importance of reducing logistics costs through the supply chain optimization and complexity of realistic problems, the present study aims to implement and evaluate the Tabu search exploratory method to optimize a SCN. According to their research results, the proposed exploratory optimization can be used for networks with complex supply chains that ensure acceptable results on a computer that has been sufficiently optimized.

Qin and Ji [10] designed a reverse logistics network to deal with uncertainty during the recovery process in a fuzzy environment. They formulated a single-objective single-period single-product model to minimize the costs, applied three types of fuzzy programming optimization models based on different decision criteria, and used a hybrid smart algorithm to integrate Genetic Algorithm (GA) with fuzzy simulation in order to solve the proposed models. Yang et al. [11] developed a two-stage optimization method for designing a Multi-Purpose SCN (MP-SCN) with uncertain transportation costs and customer requirements. They developed two objectives for the SCN problem according to the neutral and risky criteria. They also designed an improved Multi-Objective Biogeochemistry-Based Optimization algorithm (MO-BBO) to solve the approximate complicated optimization problem and compare it with the Multi-Objective GA (MO-GA). According to their results, the improved MO-BBO algorithm outperforms MO-GA in terms of solution quality.

By clicking on recent research, Aavik Darestani and Pourasadollah [12] used a multi-objective fuzzy approach to design a closed-loop SCN concerning dynamic pricing. The model objectives include maximizing profits, minimizing delays in delivering goods to customers, and minimizing the return on suppliers’ raw materials. Since the model is multi-objective, the fuzzy mathematical programming approach is used to convert the multi-objective model into a single-objective one in order to solve a large-sized version of the mentioned problem. The results confirm the efficiency and effectiveness of the model. Sarkar et al. [13] provided optimal production delivery policies for suppliers and manufacturers in a constrained closed-loop supply chain for returnable transport packaging through a metaheuristic approach. The model objectives include profit maximization and carbon emissions minimization of the system. A weighted goal programming technique and three distinct meta-heuristic approaches are applied to obtain efficient trade-offs among model objectives. In addition, three heuristic methods including particle swarm optimization, interior point optimization algorithm, and GA were used to determine the best method for the given data. The results provided by the interior-point optimization algorithm and GA were the best ones. The weighted goal programming results while using Single Setup Multi-Delivery (SSMD) policy were compared with the SSMD policy. The findings provided an SSMD policy for supplier and manufacturer-focused decision-making in a proposed Supply Chain Management (SCM) to improve the proper economic sustainability.

Rahimi Sheikh et al. [14] designed a resilience supply chain model by identifying the factors creating instability in the supply chain. Govindan et al. [15] reviewed big data analytics and application for logistics and SCM. Their research also presented a summary of the big data attributes, effective methods for implementation, effective practices for implementation, and evaluation and implementation methods. Their review papers offer various opportunities to improve big data analytics and applications for logistics and SCM. Vanc et al. [16] proposed a new multi-product multi-period mathematical model for integrated production-distribution three-level supply chain. They considered the uncertainty of the model parameters using the Markowitz model and solved the presented model by GA.
Mahmoudi et al. [17] presented a new multi-product, multi-level, and multi-period mathematical model for a reverse logistic network which aimed to minimize the transportation and facilities establishing cost, lessen purchasing from suppliers, and solve the proposed model using a GA. Khorram-Nasab et al. [18] presented an integrated management model for the electronic supply chain of products in gas and oil companies by investigating the effective parameters in the company’s performance. Zahedi et al. [19] designed a closed-loop SCN considering multi-task sales agencies and multi-mode transportation. Their proposed model is comprised of four echelons in the forward direction and five echelons in the backward direction. The model considers several constraints from previous studies and addresses new constraints to explore better real-life problems that employ different transportation modes and rely on sale agency centers. The objective function is to maximize the total profit. This study first considers a distinct cluster of customers based on the product life cycle. The structure of the model is based on linear mixed-integer programming. The proposed model was investigated through a case study regarding the manufacturing industry. The findings of the proposed network revealed that using the attributes of sale agency centers and clusters of customers increased total revenue and the number of returned products.

Srivastava and Rogers [20] researched how to manage various industries of global supply chain risks in India. They believed that in each industry sector, the global supply chain risks and their mitigation strategies differed. They used profile deviation and ideal profile methodology to identify top performers in three industry sectors (Audit, Finance and Consulting, Automotive, and IT and Software) and evaluated their best practices towards managing global supply chain risks. They then found the ‘ideal’ risk mitigation profiles for all the three industries. These findings provide new insights for practitioners as they will serve as a helpful reference tool for Indian executives planning to internationalize.

Jaggi et al. [21] presented a multi-objective production model in the lock industry case study. In the proposed model, an attempt has been made for the production planning problem with multi-products, multi-periods, and multi-machines under a specific environment that attempts to minimize the production cost and maximize the net profit subject to some realistic set of constraints. In a multi-objective optimization problem, objective functions usually conflict with each other, and any improvement in one of the objective functions can be achieved only in concurrence with another objective function. To deal with such situations, the Goal Programming approach was used to obtain an optimal solution to the formulated problem. This optimal solution can only be obtained by achieving the highest degree of each membership goal.

Talwar et al. [22] reviewed big data in supply chain operations and management. Their research is a Systematic Review of the Literature (SRL) to uncover the existing research trends, distill key themes, and identify future research areas. For this purpose, 116 studies were identified and critically analyzed through a proper search protocol. The key outcome of this SRL is the development of a conceptual framework named the Dimensions-Avenues-Benefits (DAB) model for adoption and potential research questions to support novel investigations in the area offering actionable implications for managers working in different verticals and sectors. Maleshwari et al. [23] reviewed the role of big data analytics in SCM. A review from the year 2015–2019 is presented in this study. Further, the significance of DAB in SCM has been highlighted by studying 58 papers, which have been sorted after a detailed study of 200 papers collected through the Web of Science database. Their findings and observations give state-of-the-art insights to scientists and business professionals by presenting an exhaustive list of the progress made and challenges left untackled in the field of DAB in SCM.

Recently, Atabaki et al. [24] used a priority-based Firefly Algorithm (FA) for the network design of a closed-loop supply chain with price-sensitive demand. A mixed-integer linear programming model was developed to make location, allocation, and price decisions maximize total profit regarding capacity and number of opened facilities constraints. The proposed FA uses an efficient solution representation based on the priority-based encoding. Moreover, the algorithm utilizes a backward heuristic procedure for decoding. For large-sized problems, the performance is compared with a differential evolution algorithm, a GA, and an FA relying on the conventional priority-based encoding through statistical tests and a chess rating system. The results indicate the superiority of the proposed approach in both FA structure and encoding-decoding procedure. In the same year, Avakhi Darestani and Hemmati [25] optimized a dual-function closed-loop SCN for corrupt commodities according to the queuing system using three multi-criteria decision-making methods, namely the weighted sum method, ε-constraint method, and the LP-Metrics. The objectives of this study are to minimize total network costs and greenhouse gas emissions. The results indicate a significant difference between the mean of the first and second objective functions and the computational time. According to Zaleta and Socorrás [26], no algorithm can solve the supply chain design problem for large cases in a reasonable time period. Lee and Kwon [27] suggested that although computing power increased and several efficient and powerful software programs were intro-
duced in the market, computing time was still quite long for hundreds of products, customers, and dozens of plants. A research model was developed here based on previous research studies and literature review and gaps identified in modeling and solution methodology.

2.1. Contribution of this work
Overall, this research offers a comprehensive yet multi-objective model for a closed-loop supply chain design, and to make the model more adaptable to the real world, uncertainty in demand and return rates when delivering products to customers is considered. Fuzzy numbers are used to describe these factors and fuzzy mathematical programming for modeling, given the fuzzy capability to interact with uncertainty patterns. The contribution of this study is to present an optimized fuzzy model based on several objective functions and consider discounts and financial flows that show the model is complicated due to the objectives mentioned above and variables mentioned in this environment that have not been explored so far. Since the closed-loop supply chain problem is one of the NP-hard problems, some extraordinary approaches to solving this problem, being part of the paper, contribute to the research literature.

3. Problem modelling
The structure of the studied chain is presented in Figure 1. A transportation system must be considered in this chain for each of the existing connections between the chain members. For this purpose, several predefined transportation systems are investigated, and each of them establishes material connections between different chain members. Moreover, this chain’s key parameters including demand, return rate, and delivery time to customers are assumed uncertain, aiming to get closer to the real situation. The research assumptions can be stated as follows:

- The supply chain understudy is a multi-level, multi-product, and multi-period;
- Discounts are considered in the supply of raw materials;
- The current chain value is considered in the feasibility studies of the chain;
- The problem is based on the demand uncertainty and the delivery amount and time;
- Except for disposal centers, other chain components have limited capacity;
- Hybrid centers can distribute and collect returned goods simultaneously;
- The suppliers’ locations in the chain are fixed;
- The non-deterministic parameters are provided as the triangular fuzzy numbers;
- The problem objectives include maximizing the profit’s present value, minimizing the total weight of the delivery time, and minimizing the defective items received from the suppliers.

A multi-echelon multi-product closed-loop supply chain is designed for this problem. The chain consists of suppliers, manufacturers, distributors, and collection and disposal centers. The ‘suppliers’ location is fixed, but the manufacturing ‘plants’ location must be determined. There is also a set of potential points that

![Figure 1. The SCN of this work.](image-url)
can be distribution, collection, or combination centers. Combination centers can distribute as well as collect simultaneously. The disposal center location should also be determined out of its potential points. Then, a mathematical model was presented in this research.

Moreover, the model of the research was derived from Ramezani et al. [1]. Three objectives were optimized simultaneously in this model. The first objective is to maximize the value of the chain profit; the second objective is to minimize the transition times. The third objective is to minimize defective parts purchased. In this regard, due to the uncertainty of some parameters, the fuzzy theory approach was applied to the mathematical model. Professor Lotfi Asgar Zadeh first introduced fuzzy logic in new computation after setting the fuzzy theory. The fuzzy method is a very efficient method that helps managers control these uncertainties and is, therefore, used in our model to achieve the desired objective. Moreover, the Multi-Objective Simulated Annealing (MOSA) algorithm is used to solve the model due to the complexity of the mathematical model.

3.1. Mathematical model
The proposed mathematical model is presented in the following:

Indices

- $S$: Supplier fixed location ($s = 1, 2, \ldots, S$)
- $i$: Potential locations of plants ($i = 1, 2, \ldots, I$)
- $j$: Potential locations for distribution centers/collection facilities/hybrid centers ($j = 1, 2, \ldots, J$)
- $c$: Customers’ fixed locations ($c = 1, 2, \ldots, C$)
- $k$: Potential centers of goods disposal ($k = 1, 2, \ldots, K$)
- $p$: Products ($p = 1, 2, \ldots, P$)
- $r$: Raw materials ($r = 1, 2, \ldots, R$)
- $l$: Transportation systems ($l = 1, 2, \ldots, L$)
- $t$: Time periods ($t = 1, 2, \ldots, T$).

Parameters

- $a_{cp}^t$: Customer $c$ demand for product $p$ in period $t$
- $PR_{cp}^t$: The selling price of each unit of product $p$ to customer $c$ in period $t$
- $SC_{sr}^t$: Cost of purchasing 1 unit of raw material $r$ from supplier $s$ in period $t$
- $DS_{s}^t$: Discount on purchase of raw materials from supplier $s$ in period $t$
- $MC_{ip}^t$: Production cost per unit of product $p$ in plant $i$ in period $t$
- $OC_{jp}^t$: Operating cost on product $p$ at the collection center $j$ in period $t$
- $IC_{jp}^t$: Inspection and recycling cost per unit of product $p$ at the facility location $j$ in period $t$
- $RC_{ip}^t$: Cost of recovering product $p$ in plant $i$ in period $t$
- $DC_{ip}^t$: Disposal cost per unit of product $p$ at the disposal center $k$ in period $t$
- $HC_{jp}^t$: Maintenance cost per unit of product $p$ in the facilitation center $j$ in period $t$
- $RD_{sr}^t$: The failure rate of raw material $r$ in supplier $s$ in period $t$
- $w_r$: Significance coefficient of raw material $r$
- $FX_{si}^t$: Fixed cost of supplier $s$ selection in period $t$
- $FX_{ij}^t$: Fixed cost of setting up plant $i$ in period $t$
- $FY_{ij}^t$: Fixed cost of setting up facility $j$ in period $t$
- $FZ_{ij}^t$: Fixed cost of setting up a collection center $j$ in period $t$
- $FU_{ij}^t$: Cost of setting up a hybrid center at point $j$ in period $t$
- $FV_{ij}^t$: Fixed cost of setting up a disposal center $k$ in period $t$
- $CS_{sr}^t$: The capacity of supplier $s$ for supplier $r$ in period $t$
- $CX_{i}^t$: Production capacity in plant $i$ in period $t$
- $CY_{j}^t$: The capacity of distribution center $j$ in period $t$
- $CZ_{ij}^t$: The capacity of the collection center $j$ in period $t$
- $CU_{ij}^t$: The capacity of the hybrid center $j$ in period $t$
- $CR_{i}^t$: Plant capacity $i$ to recover products returned in period $t$
- $CV_{k}^t$: The capacity of the disposal center $k$ in period $t$
- $CSI_{sr}^t$: The unit cost of transporting raw material $r$ from supplier $s$ to plant $i$ in period $t$
- $CI_{ij}^t$: The unit cost of transporting product $p$ from plant $i$ to distribution center $j$ in period $t$ with transportation system $l$. 

\[ \text{\ldots} \]
$CJC_{jcl}^t$ The unit cost of transporting product $p$ from the distribution center $j$ to the customer $c$ with the transportation system $l$ in period $t$

$CC_{cjl}^p$ The unit cost of transporting product $p$ from the customer $c$ to the collection center $j$ with the transportation system $l$ in period $t$

$CJ_{jpi}^t$ Cost of transporting product $p$ from the collection center $j$ to the plant $i$ for recovery in period $t$ with the transportation system $l$

$CJK_{jkl}^t$ The unit cost of transporting product $p$ from the collection center $j$ to the disposal center $k$ in period $t$

$TtI_{jlp}^t$ Product transporting time $p$ from plant $i$ to distribution center $j$ in period $t$ with transportation system $l$

$\bar{TJC}_{jlp}^t$ Product transporting time $p$ from distribution center $j$ to customer $c$ with transportation system $l$ in period $t$

$TC_{jcl}^t$ Product transporting time $p$ from customer $c$ to collection center $j$ with transportation system $l$ in period $t$

$TC_{jpi}^t$ Product time $p$ inspected from collection center $j$ to plant $i$ for recovery in period $t$ with transportation system $l$

$n_{rp}$ Raw material consumption coefficient $r$ in product $p$

$m_p$ Rate of capacity utilization in producing product $p$

$R_{rp}$ The return rate of product $p$ from customers

$RX_p$ The reproduction rate of product $p$

$RV_p$ Disposal rate of product $p$

$ir$ Interest rate

$\gamma$ Discount rate

$\beta$ The importance weight of the direct chain and $1 - \beta$ is the important factor of the reverse chain

$BM$ A very large number.

**Variables**

$QSI_{sir}^t$ Amount of raw materials $r$ sent from supplier $s$ to plant $i$ in period $t$

$QI_{jcl}^t$ Quantity of products $p$ sent from plant $i$ to distribution center $j$ with transportation system $l$ in period $t$

$INV_{jlp}^t$ Inventory of products $p$ in the distribution center $j$ at the end of period $t$

$QJC_{jcl}^t$ Amount of products $p$ transferred from the distribution center $j$ to the customer $c$ with the transportation system $l$ in period $t$

$QC_{jcl}^t$ Quantity of products $p$ returned from the customer $c$ to the collection center $j$ with the transportation system $l$ in period $t$

$QJR_{jcl}^t$ Amount of recyclable products $p$ sent from the collection center $j$ to plant $i$ with the transportation system $l$ in period $t$

$QJC_{jkl}^t$ Amount of defective products $p$ sent from the collection center $j$ to the disposal center $k$ in period $t$

$W_s^t$ A binary variable equal to 1 if the supplier $s$ is selected in period $t$

$X_i^t$ A binary variable equal to 1 if plant $i$ is started in period $t$

$Y_j^t$ A binary variable equal to 1 if the distribution center is set up at point $j$ in period $t$

$Z_j^t$ A binary variable equal to 1 if the collection center is set up at point $j$ in period $t$

$U_j^t$ A binary variable equal to 1 if a hybrid center is set up at point $j$ in period $t$

$V_k^t$ A binary variable equal to 1 if the disposal center is set up at point $k$ in period $t$

$A_{ijl}^t$ A binary variable equal to 1 if the transportation system $l$ connects plant $i$ and distribution center $j$ in period $t$

$B_{jcl}^t$ A binary variable equal to 1 if the transportation system $l$ connects the distribution center $j$ to customer $c$ in period $t$

$C_{cjl}^t$ A binary variable equal to 1 if the transportation system $l$ connects customer $c$ to the collection center $j$ in period $t$

$D_{jcl}^t$ A binary variable equal to 1 if the transportation system $l$ connects the collection center $j$ to plant $i$ in period $t$

### 3.2. Mathematical model relationships

The problem consists of three objectives that are presented in detail as follows:

- **Maximize the value of chain profit**: The first objective function maximizes the chain's net present value, derived from the difference between incomes and costs. Eq. (2) is the specified income from the
sale of products in each period. Eq. (3) indicates the total chain costs in each period. These costs include fixed costs of setting up plants and facilities, costs of supply and purchase from suppliers, discounts from suppliers, costs of production and recovery of defective products, operating costs in distribution centers and disposal centers, inventory costs in distribution centers, and transportation costs by different transportation systems in the supply chain.

\[
\text{max } \text{NPV} = \sum_{i} \frac{Income_{it} - Cost_{it}}{(1 + ir)^{t-1}}, \tag{1}
\]

\[
Income_{it} = \sum_{j} \sum_{c} \sum_{p} \sum_{l} OJC_{j, cpl} \cdot PR_{cp}, \tag{2}
\]

\[
Cost_{it} = \sum_{i} FX_{i}^t \cdot (X_{i}^t - X_{i}^{t-1}) + \sum_{j} FY_{j}^t \cdot (Y_{j}^t - Y_{j}^{t-1}) + \sum_{j} FZ_{j}^t \cdot (Z_{j}^t - Z_{j}^{t-1}) + \sum_{j} FU_{j}^t \cdot (U_{j}^t - U_{j}^{t-1}) + \sum_{k} FV_{k}^t \cdot (V_{k}^t - V_{k}^{t-1}) + \sum_{s} FW_{s}^t \cdot W_{s}^t + \sum_{s} \sum_{i} \sum_{r} QSI_{sir}^t \cdot SC_{sr}^t - \sum_{s} \sum_{i} \sum_{r} q_{sir}^t \cdot DS_{s}^t + \sum_{i} \sum_{j} \sum_{p} \sum_{l} QI_{ijpl}^t \cdot DC_{ip}^t + \sum_{j} \sum_{i} \sum_{p} \sum_{l} QI_{ijpl}^t \cdot DC_{ip}^t + \sum_{j} \sum_{c} \sum_{p} \sum_{l} QJC_{j,cpl}^t \cdot OC_{jp}^t + \sum_{c} \sum_{j} \sum_{p} \sum_{l} QJC_{j,cpl}^t \cdot IC_{jp}^t + \sum_{j} \sum_{k} \sum_{p} \sum_{l} QJK_{j,kp}^t \cdot DC_{kp}^t + \sum_{j} \sum_{p} QIV_{jp}^t \cdot HC_{p}^t + \sum_{s} \sum_{i} \sum_{r} QSI_{sir}^t \cdot CSI_{sir}^t + \sum_{i} \sum_{j} \sum_{p} \sum_{l} QI_{ijpl}^t \cdot CI_{ip}^t + \sum_{j} \sum_{c} \sum_{p} \sum_{l} QJC_{j,cpl}^t \cdot CC_{jp}^t + \sum_{j} \sum_{i} \sum_{p} \sum_{l} QJK_{j,kp}^t \cdot CC_{jp}^t + \sum_{j} \sum_{p} QIV_{jp}^t \cdot HC_{p}^t + \sum_{s} \sum_{i} \sum_{r} QSI_{sir}^t \cdot CSI_{sir}^t.
\]

- **Minimize the transition times:** The second objective function minimizes the weighted total of the transmission times in the direct and reverse chains as follows:

\[
\min f_2 = \beta \left( \sum_{i} \sum_{j} \sum_{p} \sum_{l} A_{ij}^t \cdot TIC_{j,ijpl}^t \right) + \sum_{c} \sum_{j} \sum_{p} \sum_{l} B_{c,j}^t \cdot TCC_{j,ijpl}^t + (1 - \beta) \left( \sum_{i} \sum_{j} \sum_{p} \sum_{l} C_{c,j}^t \cdot TCC_{j,ijpl}^t \right) + \sum_{i} \sum_{j} \sum_{p} \sum_{l} D_{t,ijpl}^t \cdot TI_{j,ijpl}^t \right). \tag{4}
\]

- **Minimize defective parts purchased:** The last objective function minimizes the total amount of defective raw materials in suppliers. This goal is to select suppliers that minimize the return of final goods as follows:

\[
\min f_3 = \sum_{s} \sum_{i} \sum_{r} \sum_{t} QSI_{sir}^t \cdot RD_{sir}^t \cdot w_r. \tag{5}
\]

The model’s constraints are presented in Eqs. (6) to (33) as follows. Eq. (6) indicates that the amount of raw materials imported to each plant in each period is equal to the amount of output from that plant in the same period. Eq. (7) ensures that the amount imported for each product in each period to each distribution center and the remaining inventory from the previous period is equal to the amount sent to customers and the remaining inventory at the end of the period.

\[
\sum_{j} \sum_{p} \sum_{l} n_{p,ijpl} QI_{ijpl}^t = \sum_{s} QSI_{sir}^t
\]
\[
+ \sum_{j} \sum_{p} \sum_{l} n_{rp} QJ_{ijpl}^{r}; \quad \forall i, \ j, \ t.
\]

Equation (8) shows that for each product and each period, the amount available in each of the distribution centers or hybrid centers must meet the demand for that product. Equation (9) describes the relationship between customer demand and the amount returned to collection centers and hybrid centers. Equation (10) ensures that the total amount received from customers in collection centers and recyclable centers that can be recycled is equal to the total amount sent from these centers to plants. Equation (11) ensures that the total amount of recyclable goods received from customers at collection centers and recycling centers is equal to the total amount sent to disposal centers:

\[
\sum_{j} \sum_{l} QJC_{ijpl}^{r} = \tilde{d}_{cp}^{r}; \quad \forall c, \ p, \ t.
\]

\[
\sum_{j} \sum_{l} QC_{ijpl}^{r} = D_{cp}^{l}.RR_{p}; \quad \forall c, \ p, \ t.
\]

\[
\sum_{i} \sum_{l} QJ_{ijpl}^{r} = \sum_{c} \sum_{l} QC_{ijpl}^{r}.RX_{p}; \quad \forall j, \ p, \ t.
\]

\[
\sum_{k} QJK_{jlp}^{t} + \sum_{i} \sum_{l} QJI_{ijpl}^{r} = \sum_{c} \sum_{l} QC_{ijpl}^{r}; \quad \forall j, \ p, \ t.
\]

Inequality (12) ensures that suppliers’ raw material does not exceed the suppliers’ capacity. Inequality (13) indicates material capacity constraints in plants similar to suppliers. Inequality (14) indicates that each distribution center’s remaining inventory and the hybrid center should not exceed its capacity. Inequality (15) ensures that the flow of goods from collection centers to plants and disposal centers does not exceed these centers’ capacity. Inequality (16) states that the total amount of goods returned to each plant should not exceed the plant’s recovery capacity. Inequality (17) states that the total amount sent to the disposal centers should not exceed these centers’ capacity. Inequality (18) is the maximum number of facilities that can be established:

\[
\sum_{i} QST_{isr}^{r} \leq CS_{isr}^{r}.O_{isr}^{r}; \quad \forall s, \ r, \ t.
\]

\[
\sum_{j} \sum_{p} \sum_{l} m_{p}.QI_{ijpl}^{r} \leq CX_{i}^{t}.X_{i}^{r}; \quad \forall i, \ t.
\]

\[
\sum_{p} \sum_{l} m_{p}.INV_{j}^{r} + \sum_{c} \sum_{l} \sum_{p} m_{p}.QJC_{ijpl}^{r} \leq CY_{j}^{t}.Y_{j}^{r}
\]

\[
+ CU_{j}^{r}.U_{j}^{r}; \quad \forall j, \ t.
\]

\[
\sum_{j} \sum_{p} \sum_{l} m_{p}.QJ_{ijpl}^{r} \leq C\alpha_{j}^{r}.Z_{j}^{r}
\]

\[
+ CU_{j}^{r}.U_{j}^{r}; \quad \forall j, \ t.
\]

\[
\sum_{j} \sum_{p} \sum_{l} m_{p}.QJI_{ijpl}^{r} \leq CR_{i}^{r}.X_{i}^{r}; \quad \forall i, \ t.
\]

\[
\sum_{j} \sum_{p} m_{p}.QJK_{jlp}^{t} \leq CV_{k}^{r}.V_{k}^{r}; \quad \forall k, \ t.
\]

\[
Y_{j}^{r} + Z_{j}^{r} + U_{j}^{r} \leq 1; \quad \forall j, \ t.
\]

Inequality (19) ensures that raw materials are received from selected suppliers. Inequalities (20) and (21) determine the minimum amount received from each of the selected suppliers so that very small orders are not sent to a particular supplier:

\[
Q_{isr}^{r} \leq W_{isr}^{r}; \quad \forall s, \ r, \ t.
\]

\[
q_{isr}^{r} \leq \frac{1}{2} \left( \sum_{r} O_{is}^{r} \right); \quad \forall s, \ t.
\]

\[
\sum_{i} QST_{isr}^{r} \geq \gamma.CS_{isr}^{r}.Q_{isr}^{r}; \quad \forall s, \ r, \ t.
\]

Inequalities (22) to (25) requires that only one transportation system be used in each chain member:

\[
\sum_{l} A_{ij}^{r} \leq 1; \quad \forall i, \ j, \ t.
\]

\[
\sum_{l} B_{jd}^{r} \leq 1; \quad \forall j, \ c, \ t.
\]

\[
\sum_{l} C_{cjl}^{r} \leq 1 ; \quad \forall c, \ j, \ t.
\]

\[
\sum_{l} D_{jil}^{r} \leq 1; \quad \forall i, \ j, \ t.
\]

Inequalities (26) to (29) indicate that the transportation system is used between the chain members who send goods:
\[ A^i_{jkl} \leq \sum_p Q I^i_{jklp} \quad \forall i, j, l, t, \]  
(26)

\[ B^c_{jcl} \leq \sum_p Q J^c_{jclp} \quad \forall j, c, l, t, \]  
(27)

\[ C^c_{jkl} \leq \sum_p Q C^c_{jklp} \quad \forall c, j, l, t, \]  
(28)

\[ D^j_{jkl} \leq \sum_p Q J^j_{jklp} \quad \forall j, i, l, t. \]  
(29)

Inequalities (30) to (33) indicate that the chain members with no transaction do not send goods to each other:

\[ \sum_p Q I^i_{jklp} \leq BM \cdot A^i_{jkl} \quad \forall i, j, l, t, \]  
(30)

\[ \sum_p Q J^c_{jclp} \leq BM \cdot B^c_{jcl} \quad \forall j, c, l, t, \]  
(31)

\[ \sum_p Q C^c_{jklp} \leq BM \cdot C^c_{jkl} \quad \forall c, j, l, t, \]  
(32)

\[ \sum_p Q J^j_{jklp} \leq BM \cdot D^j_{jkl} \quad \forall j, i, l, t. \]  
(33)

### 3.3. Fuzzification approach and model solution in fuzzy conditions

Each of the non-deterministic parameters is considered as a triangular fuzzy number displayed as \( \tilde{D} = (d_1, d_2, d_3) \). The alpha cut is used to determine the values of \( x \) with an alpha confidence level in its uncertainty. The following equation obtains these values of \( x \):

\[ x_{\alpha} = \{ x : x \in X, \mu_{\alpha}(x) \geq \alpha, \alpha \in [0, 1] \}. \]  
(34)

The lower the alpha, the higher the confidence level and the smaller the confidence interval; in addition, the higher the alpha, the lower the confidence level and the more the confidence interval. Considering the specified alpha level, the range of changes \( x \) can be reduced and the investor can be assured that the investment risk is somewhat reduced. Determining the alpha level or the same level of confidence is the decision-maker’s responsibility and is added as a predefined parameter in the model:

Thus, generally, the fuzzy demand \( \tilde{D} = (d_1, d_2, d_3) \) becomes an interval of \( D = [d^m, d^m] \) considering value for alpha. The following process is then performed to optimize the mathematical model considering the demand interval:

**Step 1.** Set the demand value at the lower limit of \( d^m \) and determine the optimal value of each of the objective functions and name them as \( f_1^m \), \( f_2^m \), and \( f_3^m \).

**Step 2.** Set the demand value at the lower limit of \( d^m \) and determine the optimal value of each of the objective functions and name them as \( f_1^m \), \( f_2^m \), and \( f_3^m \).

**Step 3.** State the optimal amount of each goal using the following equations:

\[ f_1^* = \alpha f_1^m + (1 - \alpha) f_1^i, \]  
(35)

\[ f_2^* = \alpha f_2^m + (1 - \alpha) f_2^i, \]  
(36)

\[ f_3^* = \alpha f_3^m + (1 - \alpha) f_3^i. \]  
(37)

### 3.4. MOSA algorithm

The MOSA is a meta-heuristic algorithm based on the overall structure of Simulation Annulling (SA). Due to the existence of more than one goal for optimization in this algorithm, the superiority of answers in each step is based on the concept of non-dominance. Answer \( x \) is dominant over answer \( y \) if the value of each objective function for answer \( x \) is better than its equivalent for answer \( y \). At each iteration in the MOSA algorithm, the relative dominance of answers over each other is checked after generating a neighborhood answer. If one answer is dominated by the other, we save it in the list of non-dominant answers. Otherwise, the answers are checked based on the probability of Relation (38) and one of them is deleted while the other is used in the next step. Therefore, generally, the main difference between MOSA and SA is how to delete the answers and apply new solutions.

\[ P\{\text{accept}\} = \begin{cases} 1, & \Delta f \leq 0 \\ e^{\lambda f/C}, & \Delta f > 0 \end{cases} \]  
(38)

In the Relation (38), \( P \) is the probability of accepting the next point. \( \Delta f \) represents the changes in the objective function for the established neighborhood, and \( C \) is the control parameter, which is considered equal to the current temperature. A stop criterion is required to complete this algorithm. One criterion for this purpose can be reaching the final temperature. Another criterion is the degree to which the answer does not improve in a certain number of iterations.

In this research, the initial temperature value is 1000 and the temperature reduction rate is equal to 0.01 at the previous stage temperature for the solved examples [28]. In other words, the stopping criterion \( T_{i+1} = 0.99 \times T_i \) is not considered an improvement within the last 100 repetitions or reaching a temperature less than 1.

### 4. Computations and results

First, the proposed mathematical model was validated. In order to determine the validity of the model and
Table 1. Model validation example data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of products</td>
<td>3</td>
</tr>
<tr>
<td>Number of suppliers</td>
<td>3</td>
</tr>
<tr>
<td>Number of factories</td>
<td>4</td>
</tr>
<tr>
<td>Number of distribution, collection, and combination centers</td>
<td>5</td>
</tr>
<tr>
<td>Number of customers</td>
<td>7</td>
</tr>
<tr>
<td>Number of disposal centers</td>
<td>3</td>
</tr>
<tr>
<td>Number of raw materials</td>
<td>2</td>
</tr>
<tr>
<td>Number of transportation systems</td>
<td>2</td>
</tr>
<tr>
<td>Number of periods</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2. Value of objective functions obtained from GAMS software.

<table>
<thead>
<tr>
<th>Objective function</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>First goal (maximizing current value)</td>
<td>165785</td>
</tr>
<tr>
<td>The second objective function (minimizing sending times)</td>
<td>3497</td>
</tr>
<tr>
<td>Third objective function (minimizing defective items)</td>
<td>2794</td>
</tr>
</tbody>
</table>

Since the most important elements of this chain are plants, distribution centers, and recycling and disposal centers, the following outputs regarding location are presented after solving the mathematical model. Then, the supplier selection is determined. The number 0 means no selection, while the number 1 means the supplier selection, which is shown in Table 3. The plant’s location is also indicated in Table 4.

The results related to distribution centers, collection, and hybrid location are shown in Table 5.

Considering that the answers obtained for decision variables are feasible and consistent with the manual analysis, the proposed mathematical model is efficient and valid. The efficiency of the proposed metaheuristic algorithms for solving the desired model is analyzed in the following. First, it is necessary to optimize the value of the algorithm parameters. To do so, the technique of designing experiments will be used based on the Taguchi method.

4.1. Designing experiments for MOSA algorithm parameters

Based on the Taguchi method structure, three values are first proposed for each of the MOSA algorithm parameters. The suggested values are shown in Table 6.

The following modes of the MOSA algorithm are implemented based on the Taguchi L9 scheme, and its outputs are presented in Table 7.

After entering this information into MINITAB software and implementing the Taguchi method, the S/N diagram is presented in Figure 2.

According to the diagram above, the lowest S/N value is appropriate for each parameter. Therefore, the values shown in Table 8 are optimal values relating to the MOSA algorithm, and other examples will be executed with these values.

4.2. Numerical results

It is required to measure the MOSA algorithm’s performance in several examples in different dimensions to evaluate the performance of the introduced algorithm. For this purpose, 11 examples in different dimensions

Table 3. Selected suppliers in an optimal mode.

<table>
<thead>
<tr>
<th>Supplier</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selected/ not selected</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
Table 5. Selected distributors in an optimal mode.

<table>
<thead>
<tr>
<th>Retailer</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selected/not selected</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Distribution center</td>
<td>Hybrid center</td>
<td>0</td>
<td>0</td>
<td>Disposal center</td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Parameters and their values levels for the MOSA algorithm.

<table>
<thead>
<tr>
<th>Solving algorithm</th>
<th>Parameter</th>
<th>Values of each level</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOSA</td>
<td>Number of neighborhood production per iteration (NM)</td>
<td>Level 1</td>
</tr>
<tr>
<td></td>
<td>Initial temperature (T)</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>Temperature reduction coefficient (alpha)</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>Max-iteration</td>
<td>100</td>
</tr>
</tbody>
</table>

Figure 2. Output for the Taguchi method in the MOSA algorithm.

were generated. Information about these examples is provided in Table 9.

In Table 9, $S$ is the number of suppliers, $I$ is the potential plants, $J$ is distribution, collection, and hybrid centers, $C$ is the number of customers, $K$ is the number of potential disposal centers, $P$ is the number of products, $R$ is the number of raw materials, $L$ is the number of transportation systems, and $T$ is the number of studied periods. The examples generated in GAMS software are solved in a time limit of 3600 seconds using the MOSA algorithm. It should be noted that the MOSA algorithm provides several answers in the form of the Pareto boundary. However, GAMS software only presents one answer as the optimal answer. Now, in order to better compare these two solution methods, the answer with the highest value of swarm index as a candidate answer from MOSA is compared with the answer provided by GAMS. The swarm index is calculated as follows:

$$d(k) = \sum_{i=1}^{n} \frac{f_i(k) - f_i(k + 1)}{f_i^{\text{max}} - f_i^{\text{min}}}.$$

In Eq. (39), $d$ is the swarm index value and $k$ is the counter of Pareto boundary responses; $n$ is the number of goals, and $f$ represents the value of the goal function for each goal for the $k$th answer the Pareto boundary. The answer that has the highest value of the swarm index is very close to the other answers. In other words, the answer in the middle of the Pareto border is known as the answer with the highest swarm index. After identifying this answer, the value of each of its objective functions is reported in Table 10 and compared with its equivalent value in GAMS. It should also be noted that the alpha cut method has been used due to the fuzzy amount of demand. In all of the solved examples, the alpha value is assumed to be 0.75. Table 10 summarizes the results of these examples.

According to Table 10, $z_1$ to $z_4$ are the three objective functions obtained from both methods. ‘Time’ is the execution time by both methods. ‘GAP’ provides the error rate of the MOSA algorithm. As can be seen, GAMS software has not been able to solve the last two examples. On the other hand, it has consumed the entire defined time in examples 7, 8, and 9. In other words, the optimization of these examples in GAMS software has been performed for a longer time, but it has stopped after 1 hour due to the time limit of 3600 seconds. The MOSA algorithm solves all the examples presented in less than 1 minute, while the average solution time of GAMS software was 1847 seconds. The following figure compares the solution times of the two methods.

As shown in Figure 3, the solution time increase in GAMS software is much higher than the slope of the
Table 7. Value of answer variable in the Taguchi technique for MOSA.

<table>
<thead>
<tr>
<th>Run order</th>
<th>NM</th>
<th>T</th>
<th>Alpha</th>
<th>Max-iteration</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>21.98</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>33.79</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>28.91</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>27.83</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>26.47</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>15.55</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>48.05</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>19.34</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>20.02</td>
</tr>
</tbody>
</table>

Table 8. The optimal value of MOSA parameters.

<table>
<thead>
<tr>
<th>Solving algorithm</th>
<th>Parameter</th>
<th>Optimal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOSA</td>
<td>Number of neighborhood generation per iteration (NM)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Initial temperature (T)</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>Temperature reduction coefficient (alpha)</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>Max-iteration</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 9. Information of generated problems.

<table>
<thead>
<tr>
<th>Problem</th>
<th>S</th>
<th>I</th>
<th>J</th>
<th>C</th>
<th>K</th>
<th>P</th>
<th>R</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>P2</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>P3</td>
<td>5</td>
<td>6</td>
<td>6</td>
<td>10</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>P4</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>12</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>P5</td>
<td>7</td>
<td>8</td>
<td>10</td>
<td>15</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>P6</td>
<td>8</td>
<td>9</td>
<td>12</td>
<td>20</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>P7</td>
<td>9</td>
<td>10</td>
<td>13</td>
<td>25</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>P8</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>30</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>P9</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>35</td>
<td>10</td>
<td>8</td>
<td>5</td>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>P10</td>
<td>10</td>
<td>15</td>
<td>22</td>
<td>37</td>
<td>10</td>
<td>8</td>
<td>5</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>P11</td>
<td>10</td>
<td>15</td>
<td>25</td>
<td>40</td>
<td>10</td>
<td>8</td>
<td>5</td>
<td>8</td>
<td>15</td>
</tr>
</tbody>
</table>

solution time increase in MOSA. This algorithm has reached the optimal answer for the first and second objective functions regarding the MOSA algorithm error, as seen in example 1. In the third objective function, the general optimal answer is reached in the first four examples. The average MOSA error is 0.3% for the first objective function, 1.7% for the second objective function, and 0.7% for the third objective function, indicating the efficiency of this algorithm in different examples.

4.3. Checking the efficient border of the MOSA algorithm

Since this algorithm optimizes the problem in a multi-objective way and its output includes several answers (the efficient boundary of a multi-objective problem), it is necessary to examine this algorithm’s features in terms of different solutions of the optimal center. Several indicators are provided to evaluate the performance of multi-objective meta-heuristic algorithms. These criteria include Mean Ideal Distance (MID), and

Table 10. The output of solved problems.

<table>
<thead>
<tr>
<th>No. of problem</th>
<th>GAMs</th>
<th>MOSA</th>
<th>GAP (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$z_1$</td>
<td>$z_2$</td>
<td>$z_3$</td>
</tr>
<tr>
<td>P1</td>
<td>96211</td>
<td>1294</td>
<td>671</td>
</tr>
<tr>
<td>P2</td>
<td>114254</td>
<td>2197</td>
<td>948</td>
</tr>
<tr>
<td>P3</td>
<td>135425</td>
<td>3478</td>
<td>1375</td>
</tr>
<tr>
<td>P4</td>
<td>130115</td>
<td>3999</td>
<td>1927</td>
</tr>
<tr>
<td>P5</td>
<td>144287</td>
<td>4875</td>
<td>2348</td>
</tr>
<tr>
<td>P6</td>
<td>149672</td>
<td>5367</td>
<td>2974</td>
</tr>
<tr>
<td>P7</td>
<td>151036</td>
<td>6748</td>
<td>3157</td>
</tr>
<tr>
<td>P8</td>
<td>155324</td>
<td>7015</td>
<td>3644</td>
</tr>
<tr>
<td>P9</td>
<td>160021</td>
<td>7548</td>
<td>4016</td>
</tr>
<tr>
<td>P10</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P11</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Mean</td>
<td>138370.6</td>
<td>4724.56</td>
<td>2340</td>
</tr>
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</table>
Maximum spread or diversity (MD), relative distance from straight answers (SM), and outstanding achievement (RAS). The following is the method of calculating the above indicators:

The MID criterion is used to calculate Pareto’s average distance from the ideal answer or, in some cases, from the origin of the coordinates. In the following relation, it is clear that the lower the value of this criterion, the higher the efficiency of the algorithm. In this relation, Number of Solution (NOS) is the number of answers, g the objectives, and sol the answers.

\[
MID = \frac{1}{NOS} \sum_{sol=1}^{n} \sqrt{\sum_{g=1}^{2} f_{sol,g}^2} \tag{40}
\]

The MD, measures the length of the space cube diameter used by the end values of the objectives for the set of non-dominated solutions. The relation shows the computational procedure of this index. The larger values for the criterion are, the more desired they will be.

\[
MD = \sqrt{\sum_{g=1}^{2} (\max_{sol} f_{sol}^g - \min_{sol} f_{sol}^g)^2} \tag{41}
\]

The SM index calculates how Pareto answers are distributed using the relative distance of consecutive answers.

\[
SM = \frac{\sum_{m=1}^{M} d_m^e + \sum_{k=1}^{4} |d_k - \bar{d}|}{\sum_{m=1}^{M} d_m^e + |4| \bar{d}|} \tag{42}
\]

In this equation, M is the number of objectives and \(d_i\) shows distance. \(d_m^e\) is the distance between the optimal Pareto boundary’s side solutions and the Pareto boundary obtained in the mth objective function. The lower the value of this measure, the better the boundary obtained.

The RAS index, calculated based on the following equation, shows the simultaneous achievement of all objective functions’ ideal values. The lower the value of this index, the higher the efficiency of the algorithm.

\[
RAS = \frac{\sum_{i=1}^{n} |f_{i1}(x) - f_{i1}^{est}(x) + f_{i2}(x) - f_{i2}^{est}(x)|}{n} \tag{43}
\]

Then, for 11 solved examples, MID, MD, SM, and RAS indices are calculated and presented in Table 11 and Figure 4.

The average MID index value for the MOSA algorithm is 150878. Figure 4 shows the trend of this indicator in different examples. The value of this index will increase upon increasing the problem dimensions due to this index’s nature. Accordingly, the MOSA algorithm should increase the value of this index according to the problem dimensions. As can be seen in Figure 4, the MOSA algorithm has done it well.

The average MD index for the MOSA algorithm is 5162. Figure 4 shows the value of this index for different examples. The MD index is not related to the problem dimensions. Therefore, it is expected that this index’s value has a relatively similar trend in different examples. As can be seen, there is a relatively similar
trend in this index in all examples except in examples 7 and 9 (due to algorithm error).

The average of the SM index is 6164. Figure 4 shows the value of this index for different examples. As mentioned before, the lower the value of this index, the better the status. This is well seen in the first six examples, and the small values of this index are given. The sudden increase in this index’s value from examples 9 onwards is due to the enlargement of the problem dimensions and the complexity of finding its optimal boundary.

After running the sample examples, the average value of the RAS index is about 0.204. Figure 4 also shows the value of this index in various examples. Examining the above chart, it is clear that this index’s value, in most examples, was between 0.25 and 0.45. The index’s value does not change much due to averaging this index while increasing the problem dimensions. It should be noted that if the index value is lower, the proximity of the found Pareto boundary to the optimal boundary can be further approved.

4.4. Discussing the results

The numerical results obtained in this study are discussed in this section. After designing the meta-heuristic algorithm, 11 examples were run in different dimensions with the help of this algorithm, and the results are reported separately. The trend of increasing the problem dimensions has affected the objective function’s values and the studied indices, which are briefly expressed below:

1. Increasing the problem dimensions means increasing the limits of the problem indices and increasing the value of each objective function;
2. Based on the comparisons, increasing the problem dimensions leads to a sharp increase in the value of the MID index;
3. If the problem dimensions increase, the SM and MD indices increase relatively. However, it is possible to create fluctuations in these indicators in some problems;
4. Increasing the problem dimensions does not affect the limits of the RAS index values due to the nature of averaging in this index.

Also, it is necessary to compare these results with the findings of similar research in order to prove the superiority of the obtained numerical results. Accordingly, Pishvese et al. [8] (2014) had only evaluated one product and one period, while the present research simultaneously optimizes the supply chain in multi-product and multi-period modes. Therefore, its results will be closer to the real conditions of supply chains. The study of Ramezani et al. [1] is another important research in this field. In this study, the two objectives of increasing profits and increasing service levels were evaluated. In this research, the Epsilon Constraint method was used to solve the problem. Although
the method proposed in this research is inefficient in solving large-scale problems, the method proposed in this research can solve problems on all possible scales [29].

5. Conclusion and further studies

The presented mathematical model was firstly validated. This algorithm’s parameters were initially adjusted to solve the model with the MOSA (Multi-Objective Simulated Annealing) algorithm, and then 11 different examples were designed using this algorithm. The reason for using the MOSA algorithm compared to the SA algorithm to solve the problem was the ability to optimize multiple goals simultaneously. The best way to evaluate this algorithm’s performance was to compare the results’ objective function values obtained from this algorithm with the exact solution value in General Algebraic Modeling System (GAMS) software. For this purpose, 11 examples were produced in different dimensions to evaluate this algorithm ability to solve different examples. Of the 11 examples solved, GAMS only managed to solve 9 of them. However, the proposed algorithm solved all 11 examples with an average error of 0.3% for the first objective function, 1.7% for the second objective function, and 0.7% for the third objective function.

On the other hand, the GAMS software time to solve examples 7, 8, and 9 was precisely 3600 seconds, equivalent to one hour. However, the MOSA algorithm’s average solving time for all the solved examples is 25 seconds, and all the examples are solved in less than 60 seconds. Therefore, it can be concluded that a trade-off is created between the quality of the solutions and time to apply the solution to choose between the MOSA algorithm and GAMS, as shown in Table 10 and (Figure 3). The average time to solution by GAMS software is 1847 seconds and the average time to solution by MOSA is 25 seconds. That is, an average decrease of 730% is created and at the same time, an average error of 0.3% for the first objective function, 1.7%, and 0.7%, for the second and third objective functions should be considered in the MOSA method. The trade-off between the time and the solutions’ quality shows the MOSA algorithm’s outstanding performance in reducing the time to solve the problem ahead and providing near-optimal solutions.

On the other hand, since the MOSA algorithm introduces a set of solutions as the Pareto problem, it is necessary to examine the characteristics of the set of solutions from the Pareto boundary evaluation indices. Accordingly, four different indices have been introduced in this field, and the value of these indices has been calculated for all solved examples. By analyzing the trend of these indices’ values in different examples, it can be well pointed out that the Pareto boundary created by the MOSA algorithm covers well an integrated boundary and the entire Pareto frontal space.

5.1. Implications for researchers

As a planning process, executing and controlling operations, and raw materials storage, supply chain management is critical in various industries during operations and finished products from the starting point to the endpoint of consumption. Hence, optimizing and synchronizing the supply chain was conducted in this research using heuristic algorithms to reduce costs, improve quality, and achieve a competitive advantage and position. The goal of optimization in this area is to improve the quality and ‘customers’ satisfaction and reduce the time of production and its related price. This research aims to design a multi-objective optimization algorithm for multi-period and multi-product reverse logistics problems. First, due to the uncertainty of some parameters and considering the discounts and financial flows, the fuzzy mathematical model was presented; then, the optimal MOSA algorithm was designed to solve it. Three objectives were optimized simultaneously in this model. The first objective was to maximize the value of the chain profit; the second objective was to minimize the transition times. The third objective was to minimize defective parts purchased. The average error of this algorithm for each of the three objectives under study was less than 2%. These illustrate the efficiency of the MOSA algorithm in solving the problem presented in this study. Finally, the performance of the MOSA algorithm compared to the GAMS method shows that GAMS software cannot provide a solution for some large-scale problems, while the MOSA algorithm is well able to provide an optimal solution with minimum error for different conditions. The MOSA algorithm solves all the examples presented in less than 1 minute. However, the average time to solve was 1847 seconds for the GAMS software. The study results are consistent with findings of Lee and Kwon [27] and Braido et al. [9]. The objectives and parameters considered in this study increased in value in terms of complexity and number, but with optimized design, the algorithm achieved an average error of 0.3% for the first objective function, 1.7% for the second objective function, and 0.7% for the third objective function. Also, despite being multi-objective, the convergence time in this study is less than 1 minute, which has also reduced the time compared to previous works [9,11,27], which shows the efficiency of this algorithm compared to previous research. Accordingly, if we look at previous research [1,8], they considered only one product and in one period or used inefficient methods to solve the problem on large scales. While the present research simultaneously optimized the supply chain in the multi-product and multi-period mode,
the results will be closer to the supply chains' actual conditions. Also, the method proposed in this research can solve problems in larger dimensions. Adopting the right strategy to improve supply chain performance brings about many benefits, such as saving energy resources, reducing pollutants, eliminating or reducing waste, creating value for customers, and ultimately improving companies and organizations’ productivity. Since the closed-loop Supply Chain Network (SCN) consists of facilities to achieve this goal and customers’ demand is uncertain, this factor is necessary to find the required number of facilities and the amount of flow transmitted between them.

5.2. Suggestions for future research
The supply chain design problem has become more complex, and more elements are needed today according to the new global regulations and considering the environmental protection rules. It is suggested to use dynamic systems and simulation models to consider different parameters. Supply chain design can also take into account the impact of uncertainties and various parameters on it. Besides, more and more parameters such as financial considerations, risks, and uncertainties can be considered in other models. Other optimization methods and fuzzy programs with different indices can also be considered. Finally, an effective and accurate heuristic solution for larger-size problems can be developed and compared with the method presented here in terms of time and accuracy.

As one of the limitations of this method, the MOSA algorithm requires many initial selections to become an optimal solution method. There should also be a trade-off between the optimization time and the convergence of the final answer so that too much time can reduce the answer’s accuracy. The sensitivity to optimization parameters, which affects algorithm performance quality, is another limitation of this method. Therefore, to resolve the weaknesses of each algorithm, it is suggested to use a combination of different algorithms such as genetics and annealing simulation to optimally solve the multi-objective multi-period and multi-product reverse logistics problem in future research.

References


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