Effect of MHD on Casson fluid with Arrhenius activation energy and variable properties

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Exponential variable viscosity; Viscous dissipation; Temperature thermal conductivity; Arrhenius activation energy.

Abstract. The present study investigates MHD Casson nanofluid under the influence of exponential temperature-dependent thermal conductivity and variable viscosity past a stretching surface. After the application of similarity transformations, the governing partial differential equations of the modelled problem are converted into ordinary differential equations and then a solution is achieved with the assistance of the shooting method. The solution obtained with the help of shooting technique is used to analyze the distribution of mass and heat flux over sheet. The effects of various governing parameters on the dimensionless velocity, temperature, and concentration distribution were analyzed and discussed in detail. The simulations of the presented model demonstrated that the surface drag increases as each of the Casson parameter and temperature-dependent thermal conductivity parameter is boosted while the rate of heat transfer is diminished. It is also observed that an increment in the temperature near the surface is noted against the thermal conductivity parameter, whereas the opposite trend is observed away from the surface.

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1. Introduction

When added to the conventional heat transfer fluids, nanoparticles give the underlying idea of heat enhancement agents called nanofluid. The ongoing research on heat transfer improvement in thermal transport processes cannot be conceived without nanofluids, given their numerical applications in science and technology. The development of the field can be found in a number of researches available on the topic of nanofluid. The scope of these studies ranges from experimental to numerical simulations based on mathematical models. Ying et al. [1] numerically handled the Al₂O₃-water nanofluid by considering both single- and two-phased nanofluid models in convectively heated tubes. The numerical results in their study deviated from the experiment at the maximum rate of 20%. Azam et al. [2] introduced the nanofluid into light with temperature-dependent thermal conductivity and heat source/sink. They observed that depression of the unsteadiness parameter led to an increase in Sherwood number. Das et al. [3] numerically evaluated the thermal performance of CuO nanofluids channel flow. A rise of 23% in the heat transfer rate was reported in their study when using nanofluid instead of conventional fluids. Jusoh et al. [4] studied MHD viscoelastic nanofluid and reported dual solution of stagnation point past a porous stretching surface with radiation effect. He found that the accidity in the Deborah number contributed to the upsurge of the drag coefficient. Rasheed et al. [5] analyzed MHD stagnation point in a nanofluid flow with a nonuniform thermal reservoir. Pal and Mandal [6] discussed magnetohydrodynamic stagnation point flow with suction effect and reported that the Sherwood number declined as the Lewis number increased. Abbasi et al. [7] highlighted the bioconvective stagnation point in Maxwell
nanofluid flow past a convectively heated surface. They observed that the energy, concentration, and density profiles of non-convective surfaces were higher than those of convectively heated surfaces. Lund et al. [8] carried out the stability analysis and reported the dual solution of MHD stagnation point Casson fluid. Their main observation was that the sign of the smallest eigenvalue was indicative of the stability of the first solution. Sheikhholeslami et al. [9] analyzed the entropy and thermal behavior of nanomaterial through solar collector involving new tapes and concluded that the energy losses could be reduced using nanofluids and twisted tapes. For more details of nanofluid see [10–17].

Casson fluid is widely applied where the fluid exhibits non-Newtonian behavior with a threshold shear stress. This model is also employed in the field of medical science like blood flow behavior [18]. In addition, it describes the behavior of base fluids that are used as coolants in the food industry such as sodium alginate [19], xanthan gum [20], etc. Owing to its applications, it has been extensively analyzed by many researches on heat transfer. For instance, Hamid [21] examined the Ohmic dissipation effects in a Casson fluid flowing across a vertically eroded needle. He also discussed the dual solutions obtained as a function of needle thickness and remarked that the Nusselt number and frictional effects increased with an increase in the needle thickness. Naqvi et al. [22] investigated the flow around a porous stretching cylinder of a Casson nanofluid. They concluded that an upsurge in the magnetic field intensity and dynamic plastic parameter retarded the flow while the thermal profile of these parameters was enhanced. Kamran et al. [23] offered Keller box solution of a Casson nanofluid over a horizontally elongating sheet slippery in nature and heated convectively. In the present study, the Casson parameter deteriorated the velocity of the fluid and enhanced the temperature.

Magnetohydrodynamics is widely applied to different issues in science and technology on an industrial scale, and the need for it is ever growing, especially in petroleum industry and metallurgical processes [24]. In such cases, the qualitative nature of the products depends on the rate at which the cooling takes place. Since its advent, it has been used in many studies to investigate its impact on the properties of the fluids. Numerous studies on the impact of MHD on the heat transfer rate can be found in the literature. For instance, Hsiao [25] studied the electrical magnetohydrodynamic Maxwell nanofluid with thermal radiation and viscous dissipation effects. Naz et al. [26] investigated the dynamics of hydromagnetic cross nanofluid in the presence of gyrotactic microorganisms. They employed the homotopy analysis method to determine a solution to the boundary layer equations. In another study, Atif et al. [27] looked at the magnetic aspect of tangent hyperbolic nanofluid past a stretching wedge. One of their key results was the rising behavior of the thermal profile with an increase in the Brownian motion, generalized Biot number, and thermophoresis parameter. Recently, Sheikhholeslami et al. [28] conducted a study on flat plate solar collectors and photocatalytic systems in the presence of nanofluid. Using slip conditions, Shah et al. [29] studied the MHD Maxwell nanofluid considering the radiation and chemical reaction effects. According to the findings, the skin friction coefficient increased as the Maxwell parameter increased. Further information can be found in [30–35].

The concept of Arrhenius activation energy is the minimum chemical energy that molecules of a substance require to start a chemical reaction. This quantity of activation energy varies for different chemical substances. An early study of Arrhenius energy was carried out by Bestman [30]. Recently, Nisar et al. [31] examined role of Arrhenius energy in peristaltic flow of Eyring-Powell nanofluid, considering the effect of thermal radiation. The results revealed that the concentration of the nanoparticles in the channel remained intact upon increasing the activation energy parameter. Another study was conducted by Azam et al. [32] who investigated the radiative cross nanofluid axisymmetric flow that developed covalent bonds and evaluated the impact of activation on that bonding. They came up with an interesting result, i.e., the concentration of nanoparticles increased with the elevation of activation energy values. Reddy et al. [33] employed RK-4th order technique to determine the solution of 3D-MHD Eyring–Powell nanofluid flow over a slandering sheet with velocity and thermal slips.

To the best of authors’ knowledge, no study has been conducted on the Casson nanofluid with temperature-dependent thermal conductivity, exponential viscosity, and Arrhenius activation energy yet. This study puts its main focus on four aspects: (a) addressing the heat and mass transfer of Casson nanofluid; (b) examining the impact of Joule heating and viscous dissipation effect; (c) analyzing the effect of exponential temperature-dependent thermal conductivity and variable viscosity on flow and thermal fields; and (d) performing the analysis of the convective heating boundary condition. The system of the resulting ODEs of the modeled problem is solved through shooting technique. Further, the prominent parameters affecting dimensionless quantities such as velocity, concentration, and temperature are graphically represented and investigated in detail.

2. Formulation of problem

A 2D viscous tangent hyperbolic force the use of a sheet elongating at a rate of \( u = ax \) in the \( x- \)
direction. The fluid viscosity and thermal conductivity are assumed temperature dependent. Both temperature and velocity gradients are generated along the \( y \)-direction. Fluid is subjected to a temperature gradient on the surface far away from the sheet which is assumed to be at the constant temperature \( T_{\infty} \). The nanomaterial concentration on the surface far away from the walls and on the surface are regarded as \( C_{\infty} \) and \( C_w \) respectively. Magneto-hydrodynamic impact is evaluated by considering a magnetic field of strength \( B_0 \) in the \( y \)-direction with significant Joule heating. The geometrical setup is demonstrated in Figure 1.

Contained in the above constraints, the governing equations of the problem above modeled are as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \left( 1 + \frac{1}{\beta} \right) \left( \mu_\beta \frac{\partial u}{\partial y} - \frac{\sigma B^2}{\rho} u \right), \tag{2}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho C_p} \frac{\partial}{\partial y} \left( K(T) \frac{\partial T}{\partial y} \right) + \left( 1 + \frac{1}{\beta} \right) \frac{\mu_\beta}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B^2}{\rho C_p} u^2
+ \frac{(\rho C_p)_T}{(\rho C_p)_f} \left[ D_B (\frac{\partial C}{\partial y})^2 + \frac{D_T}{T_{\infty}} \left( \frac{\partial T}{\partial y} \right)^2 \right], \tag{3}
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_{\infty}} \frac{\partial T}{\partial y}^2
- K_r \left( \frac{T}{T_{\infty}} \right)^m (C - C_{\infty}) \left[ \frac{\mu_\beta}{\rho C_p} \right]. \tag{4}
\]

The associated BCs are:

\[
u = u_w, \quad v = 0, \quad -k \frac{\partial T}{\partial y} = h_f (T_f - T),
C = C_w \quad \text{at} \quad y = 0,
\]

\[
u \to 0, \quad T \to T_{\infty}, \quad C \to C_{\infty} \quad \text{as} \quad y \to \infty. \tag{5}
\]

Both thermal conductivity and dynamic viscosity of the fluid are assumed to be the exponential functions of the temperature difference calculated as follows:

\[
\mu_\beta(T) = \mu_\beta^* e^{-\alpha^* (T - T_{\infty})},
K(T) = K^* e^{-b(T - T_{\infty})}.
\]

In the temperature range of 0-400 F, the thermal conductivity changes linearly, thus we have:

\[
K(T) \approx K^*(1 - b(T - T_{\infty})),
\]

where, \( \alpha^* \) is the variable viscosity, \( \epsilon \) thermal conductivity parameters, \( \mu_\beta^* \) and \( K^* \) denote constant coefficients of viscosity and thermal conductivity away from surface; \( b \) and \( \alpha^* \) are the empirical constants that can take positive and negative values [40].

A set of transformations that leads to the non-dimensionalization of the governing equations is given as follows:

\[
\eta = \sqrt{\frac{a}{v}}, \quad \psi = x \sqrt{\alpha v f(\eta)},
\]

\[
\theta(\eta) = \frac{T - T_{\infty}}{T_f - T_{\infty}}, \quad \phi(\eta) = \frac{C - C_{\infty}}{C_w - C_{\infty}}. \tag{6}
\]

While using these transformations, Eq. (1) is satisfied identically and Eqs. (2)-(4) are reduced to:

\[
\left( 1 + \frac{1}{\beta} \right) e^{-\gamma^* (f'' - \gamma f' \theta') - f' - M f'' + f f''} = 0, \tag{7}
\]

\[
\left( 1 + \frac{1}{\beta} \right) e^{-\gamma^* (f'' - \gamma f' \theta') - f' - M f'' + f f''} = 0, \tag{7}
\]

\[
\left( 1 + \frac{1}{\beta} \right) e^{-\gamma^* (f'' - \gamma f' \theta') - f' - M f'' + f f''} = 0, \tag{7}
\]
\[
(1 - \epsilon\theta)\theta'' + Pr \left[ \left( 1 + \frac{1}{\beta} \right) Ec e^{-\gamma\theta} f^{t^1} + f\theta' \right] \\
+ Nt\theta' f' + Nt\theta'' + EcM f^{t^2} \right] - \epsilon\theta'' = 0, \quad (8)
\]

\[
\phi'' + Sc f\phi' - Sc \delta\phi (1 + \theta(\theta_w - 1)) e^{\left( \frac{x}{ax} + \frac{y}{ay} + \frac{z}{az} \right)} \\
+ \frac{Nt}{Nt} \theta''.
\quad (9)
\]

The transformed BCs are:
\[
f(\eta) = 0, \quad f'(\eta) = 1, \quad \theta'(\eta) = \frac{-Bi}{1 - \theta(\eta)}(1 - \theta(\eta)),
\]
\[
\phi(\eta) = 1, \quad \text{at} \quad \eta = 0,
\]
\[
f' \to 0, \quad \theta \to 0, \quad \phi \to 0, \quad \text{as} \quad \eta \to \infty. \quad (10)
\]

In the above equations \( \gamma = -\alpha^*(T_f - T_\infty) \) represents the temperature-dependency viscosity parameter, \( Bi = \frac{\sigma B_i}{\kappa} \) the Biot number, \( Nt = \frac{\Delta D a (\tau) - C}{\rho a} \) the Brownian motion parameter, \( Sc = \frac{\kappa}{\kappa a} \) the Schmidt number, \( Ec = \frac{\alpha^2 x^2}{(a_T T_f - a_T T_\infty)} \) the Eckert number, \( M = \frac{\sigma B_i^2}{\rho a} \) the magnetic number, \( Pr = \frac{\nu}{\alpha} \) the Prandtl number, \( \epsilon = b(T_f - T_\infty) \) the thermal conductivity parameter, \( \theta_w = \frac{T_f - T_\infty}{T_f - T_\infty} \) the temperature ratio parameter, \( E = \frac{E}{aT_\infty} \) the Arrhenius activation energy, and \( \delta = k^2 T \) the rate of chemical reaction.

3. **Physical quantities**

The physical quantities such as surface drag \( C_f \), temperature gradient \( Nu \), and concentration gradient \( Sh \) on the surface are given as follows:
\[
C_f = \frac{\tau_w}{\rho f a_u^2}, \quad Nu = \frac{x \theta_w}{k(T_f - T_\infty)}
\]
\[
Sh = \frac{x j_w}{k(T_f - T_\infty)}.
\]

In the non-dimensional form, we have:
\[
C_f Re_x^{1/2} = (1 - \epsilon\theta) \left( 1 + \frac{1}{\beta} \right) f(0),
\]
\[
Nu_x Re_x^{1/2} = -(1 - \epsilon\theta(0))\theta'(0),
\]
\[
Sh_x Re_x^{1/2} = -\phi'(0),
\]

where \( Re_x = \frac{ax^2}{\nu} \) represents local Reynolds number.

4. **Implementation of the method**

The solution of the below system of Eqs. (12) subject to BCs is achieved using shooting technique [41] along with the fourth-order Runge Kutta method. A suitable domain \([0, \eta_{\text{max}}]\) instead of \([0, \infty]\) was taken into consideration for the simulations such that there will be no fluctuations in the computed results. The criteria for stopping the iterative process are set as follows:
\[
\max \{|\chi_2(\eta_{\text{max}}) - 0|, |\chi_4(\eta_{\text{max}}) - 0|, |\chi_6(\eta_{\text{max}}) - 0|\} < 10^{-6}.
\]

The following new variables are introduced: \( \chi_1 = f, \quad \chi_2 = f', \quad \chi_3 = f'', \quad \chi_4 = \theta, \quad \chi_5 = \theta', \quad \chi_6 = \phi, \quad \text{and} \quad \chi_7 = \phi'. \) Eqs. (7)-(9) were transformed into the following system of the first-order ordinary differential equations:
\[
\chi_1' = \chi_2,
\]
\[
\chi_2' = \chi_3,
\]
\[
\chi_3' = \frac{1}{1 + \frac{1}{\beta}} \left[ \chi_2 - \chi_1 \chi_3 + M \chi_2 \right] e^{\gamma \chi_4} + \gamma \chi_3 \chi_5,
\]
\[
\chi_4' = \chi_5,
\]
\[
\chi_5' = \frac{1}{1 - \epsilon \chi_4} \left[ -Pr(\chi_1 \chi_3 + Nt \chi_5 \chi_7 + Nt \chi_5^2 + \left( 1 + \frac{1}{\beta} \right) \right] Ec e^{\gamma \chi_4} + Ec \chi_3^2 + \epsilon \chi_5^2 \right],
\]
\[
\chi_7' = \chi_7,
\]
\[
\chi_7' = -Sc \chi_1 \chi_7 + Sc \delta \chi_6 (1 + \chi_4(\theta_w + 1))\theta_w e^{\left( \frac{x}{ax} + \frac{y}{ay} + \frac{z}{az} \right)}
\]

with boundary conditions:
\[
\chi_1(\eta) = 0, \quad \chi_2(\eta) = 1,
\]
\[
\chi_3(\eta) = \frac{Bi}{1 - \epsilon\theta(\eta)}(\chi_4(\eta) - 1),
\]
\[
\chi_6(\eta) = 1 \quad \text{at} \quad \eta = 0,
\]
\[
\chi_2 \to 0, \quad \chi_4 \to 0, \quad \chi_6 \to 0, \quad \text{as} \quad \eta \to \infty. \quad (13)
\]

5. **Code validation**

For verification of the correctness of the code, the results from the Skin friction coefficient which were obtained by Nadeem et al. [42], Ahmad and Nazar [43], and Ulah et al. [44] were successfully reproduced. Our simulations are in excellent agreement with the already published results obtained by Nadeem et al. [42].
Table 1. Comparison between the present values of $C_f Re_x^{1/2}$ with the already published values when $m = E = \theta_w = Sc = \delta = \gamma = \epsilon = Nt = Nb = Ec = 0$ and $Pr = 6.8, Bi \to \infty$.

<table>
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<th>$[43]$</th>
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Ahmad and Nazar [43], and Ulah et al. [44] in the literature, as presented in Table 1.

6. Results and discussion

Table 2 presents evaluation of the effect of the pertinent parameters on the skin friction coefficient $C_f Re_x^{1/2}$. According to the growing values for each of the magnetic parameters $M$ and $\gamma$, the temperature-dependent viscosity parameter causes increase in the skin friction coefficient, whereas it is reduced as the Casson parameter $\beta$ is boosted. According to Table 3, an increase in each magnetic number $M$, temperature-dependent viscosity parameter $\gamma$, thermal conductivity parameter $\epsilon$, the Prandtl number $Pr$, Eckert number $Ec$, Brownian motion parameter, and thermophoresis parameter $Nt$ would increase the Nusselt number $Nu_x Re_x^{-1/2}$, however it increases upon increasing Biot number $Bi$.

Figures 2–21 provide a better observation of the effect of the behavior of the physical parameters appearing in Eqs. (1)–(3), caused by a conversion from dimensional PDEs to non-dimensional ODEs, on the velocity $f'$, temperature $\theta(\eta)$, and concentration $\phi(\eta)$ distribution. The velocity profile against the Casson fluid parameter $\beta$ is presented in Figure 2. Accordingly, when $\beta$ gets higher values, the velocity filed is reduced.

The figures show that for $\beta = 0.4$, the velocity is higher than for $\beta = 0.6$, which is true for both $M = 0$ and $M = 3$.

<table>
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Figure 2. Variation in $f'$ due to $\beta$. 

Table 3. Numerical values of $Nu_x Re_x^{-1/2}$ when $m = 1$, $\theta_w = 1.5$, $Sc = 1.2$, $\beta = 0.2$, $\delta = 0.1$ and $E = 0.1$.

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Figure 2. Variation in $f'$ due to $\beta$. 

The figures show that for $\beta = 0.4$, the velocity is higher than for $\beta = 0.6$, which is true for both $M = 0$ and $M = 3$. 

the effect of the behavior of the physical parameters appearing in Eqs. (1)–(3), caused by a conversion from dimensional PDEs to non-dimensional ODEs, on the velocity $f'$, temperature $\theta(\eta)$, and concentration $\phi(\eta)$ distribution. The velocity profile against the Casson fluid parameter $\beta$ is presented in Figure 2. Accordingly, when $\beta$ gets higher values, the velocity filed is reduced. This can be physically expected because the plastic...
dynamic viscosity of the fluid increases upon increasing $\beta$. This increase in return leads to the formation of resistivity in the flow of fluid, which causes a decrease in the fluid velocity. As observed in the same figure, a reduction in the velocity is more prominent under the impact of the magnetic field. Magnetic effects cause resistance in the fluid flow due to which the velocity is significantly subsided in the presence of magnetic effects. Figure 3 evaluates the impact of viscosity parameter $\gamma$ on the non-dimensional $f(\eta)$. According to this figure, $f(\eta)$ is boosted with a reduction in the viscosity parameter. Physically, the higher the viscosity parameter $\gamma$ is, the thicker the particles of the fluid will be, thus leading to a decrease in the velocity.

Figures 4-9 show the variations in temperature distribution against the values of the sundry parameters. According to Figure 4, the temperature increases at higher magnetic parametric values. The impact of the Prandtl number $Pr$ on the temperature is given in Figure 5 where a decreasing trend is observed, as expected. Figure 6 evaluates the impact of parameter $Nb$, the Brownian motion parameter, on the temperature field. The temperature needed for increasing the parametric values of $Nb$ increases mainly due to the abrupt rapid random motion of the species particles present in nanofluid. The Brownian forces resulting from the increasing random motion of particles increase, which enhanced heat transportation. Figure 7 illustrates the effect of thermophoresis on the temperature field. Following the increasing values of $Nt$, the thermophoretic parameter which is the fluid temperature increases. The nanoparticles move from a hotter region to a cooler one because of thermophoretic force, hence more heat transfers in the region of boundary layer. The effect of viscous dissipation is described by Eckert number $Ec$. The Eckert number, $Ec$, represents the relation between the kinetic energy and change in enthalpy. Figure 8 is outlined to visualize the effect of $Ec$ on energy profile. An increase in $Ec$ causes an aecility in the thermal profile. Physically, upon increasing the dissipation, the thermal conductivity is enhanced, thus increasing the temperature profile. The temperature
against the variable thermal conductivity parameter is examined and presented in Figure 9. An increase in the temperature distribution near the boundary is observed and it decreases away from the surface.

Figure 7. Variation in $\theta$ due to $Nt$.

Figure 8. Variation in $\theta$ due to $Ec$.

Figure 9. Variation in $\theta$ due to $\epsilon$.

Figure 10. Variation in $\phi$ due to $M$.

Figure 11. Variation in $\phi$ due to $E$.

Figure 10 evaluates the effect of the magnetic parameter $M$ on the concentration profile $\phi$. As observed, $\phi$ follows a decreasing trend at larger values of $M$. The effect of $E$ on $\phi$ is shown in Figure 11. Following an increase in the value of $E$, $\phi$ is reduced. Figures 12 and 13 show the effect of $Nb$ and $Nt$ on $\phi(\eta)$. As observed, the concentration profile $\phi(\eta)$ increases at higher values of Brownian motion parameter $Nb$. An increase in the Brownian motion leads to an increase in the motion of the small particles inside the flow region. This enhanced chaotic movement intensifies the velocity of the particles that in turn causes an increase in the kinetic energy and the depreciation of the concentration field is observed, which is evident from the figure, whereas an opposite trend in concentration profile $\phi(\eta)$ is depicted for the growing values of $Nt$. Physically, in thermophoresis, the particles apply force on the other particles due to which particles from the hotter region move towards the colder region. Larger values of $Nt$ imply the application of the force on the other particles and, as a result, more fluid mixes.
from the higher-temperature region to a less hotter region. Therefore, this increment in the nanoparticle’s concentration is noticed. The concentration profile $\phi$ against the values of $\delta$ is placed in Figure 14. A decline in $\phi(\eta)$ is noted when $\delta$ increases. An increase is also observed in $\phi$ when the values of $Bi$ and $Ec$ increase, as shown in Figures 15 and 16, whereas it decreases against the increasing values of $Sc$, as shown in Figure 17.
The skin-friction coefficient $C_{f_x}Re_x^{1/2}$ against the values of $M$ and $\beta$ is given in Figure 18. According to this figure, $C_{f_x}Re_x^{1/2}$ is an increasing function of both the parameters $M$ and $\beta$. In the next figure, the effect of $\gamma$ along $\beta$ on $C_{f_x}Re_x^{1/2}$ is shown. The similar behavior of $C_{f_x}Re_x^{1/2}$ is noted against $\gamma$ along with $\beta$, as shown in Figure 19. The Nusselt number $Nu_xRe_x^{-1/2}$ against the values of $Bi$ and $Nb$ is presented in Figure 20, indicating that $Nu_xRe_x^{-1/2}$ is a decreasing function of both the parameters $Bi$ and $Nb$. Figure 21 evaluates the effect of $\gamma$ along $\beta$ on $Nu_xRe_x^{-1/2}$. Similar behavior of $Nu_xRe_x^{-1/2}$ against $\gamma$ along with $\beta$ is also observed.

7. Final remarks

In the present study, prominent parameters affecting velocity, concentration, and temperature distributions are investigated in detail. Some of the key features are as follows:

- An increase in the temperature profile near the surface was observed upon increasing the thermal conductivity parameter; however, an opposite trend was observed far away from the surface;
- An augmentation in the Casson parameter $\beta$ would decrease the velocity profile;
- The temperature distribution was diminished at the growing values of the Prandtl number $Pr$;
- The skin-friction $C_{f_x}Re_x^{1/2}$ was an increasing function of both Casson parameters $\beta$ and $\gamma$ whereas the Nusselt number $Nu_xRe_x^{-1/2}$ decreased for both $\beta$ and $\gamma$;
- There was a decrease in the concentration distribution $\phi(\eta)$ upon increasing the Brownian motion.
parameter $Nb$; however, thermophoresis parameter $Nt$ exhibited opposite behavior.

**Nomenclature**

- $B_0$: Applied magnetic field
- $C_\infty$: Ambient concentration
- $C$: Boundary layer concentration
- $C_p$: Specific heat
- $C_f$: Skin friction coefficient
- $C_0$: Initial reference concentration
- $C_w$: Concentration at wall surface
- $D_T$: Thermophoresis diffusion parameter
- $D_B$: Brownian diffusion coefficient
- $E$: Arrhenius activation energy
- $Ec$: Eckert number
- $h_w$: Local surface heat flux
- $k$: Thermal conductivity
- $Lb$: Bioconvection Lewis number
- $M$: Magnetic number
- $Nu_x$: Nusselt number
- $Nt$: Thermophoresis parameter
- $Nb$: Brownian motion parameter
- $Pr$: Prandtl number
- $Rd$: Thermal radiation parameter
- $q_r$: Radiative heat flux
- $Sc$: Schmidt number
- $T_0$: Initial reference temperature
- $T$: Boundary layer temperature
- $T_w$: Surface temperature
- $T_\infty$: Ambient temperature
- $t$: Time
- $u, v$: Velocity components
- $u_w$: Characteristics velocity
- $We$: Weissenberg number

**Greek letters**

- $\nu$: Kinematic viscosity
- $\rho$: Fluid density
- $\mu$: Dynamic viscosity
- $\sigma$: Electric charge density
- $\phi$: Dimensionless concentration
- $\delta$: Rate of chemical reaction
- $\gamma$: Temperature dependent viscosity parameter
- $\theta$: Dimensionless temperature
- $\epsilon$: Thermal conductivity parameter
- $(\rho C_p)_f$: Heat capacity of the fluid

**Heat capacity of the nanoparticle**

**Dimensionless boundary layer thickness**

**Casson parameter**

**References**


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**Biographies**

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