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Stability analysis of sloping grounds in unsaturated soils using the method of stress characteristics

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Abstract. An extension of slip line theory to unsaturated soils is presented in this research and applied to the problem of stability of slopes in unsaturated soils. The matric suction was then introduced and applied to governing equations based on the well-known Bishop effective stress concept. In this regard, the van Genuchten model was utilized to estimate the effective stress parameter required for measuring effective stress. The influence of the soil matric suction on the stability of slopes was investigated for a variety of soils under steady-state evaporation and infiltration, i.e., distribution of the matric suction was assumed to remain constant with time. In addition, a measure of stability in terms of some stability factors was introduced.

1. Introduction

Classical soil mechanics deals with a variety of typical problems including the bearing capacity of shallow and deep foundations [1–11], earth pressure problems [12–17], and stability of slopes [18–23]. In most of these problems, the soil is regarded as either completely saturated or dry. However, in practice, the soil above the ground water table is unsaturated while the non-zero matric suction often exists. This suction gives rise to the greater strength of the soil structure than that in saturated and dry states. One of the interesting subjects in the geotechnical engineering is the stability of slopes in either saturated or unsaturated state. The latter is relatively less addressed than other typical classical problems in geotechnical engineering [24–28]. However, this is a practical problem in many countries around the world, where the soil in those areas rarely experiences full saturation. Some regions like Hong Kong and Malaysia are devoid of flat lands to perform any construction for which cut slopes have proved inevitable [29]. Many of such slopes are of unsaturated soils and they require meticulous attention. Moreover, it is imperative to gain a behavioral knowledge of unsaturated soils to predict the potential danger and take appropriate measures.

Slope stability problems are usually approached using the limit equilibrium method, limit analysis method, and the method of stress characteristics. In the limit equilibrium approach, a slip line is considered and soil above this slip line is often assumed to be a rigid material and divided into several blocks. Depending on the assumption of the distribution and magnitude of the inter-slice forces, the factor of safety is obtained in terms of force and moment equilibrium along the slip surface. The surface with the least safety factor is regarded as a critical slip surface [24,30–32]. Failure load in limit analysis method is reported to be approximately between two bounds. The lower and upper bound theorems determine these two limits [33–37]. The method of stress characteristics, on the

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other hand, considers soil as a continuous matter. Governing equations are obtained by simultaneously combining the equilibrium and yield surface equations. Sokolovskii [38] assumed the body to be in the state of limiting equilibrium and combined the equilibrium and yield equations for constant cohesion and internal friction angle, thus presenting a solution to some typical geotechnical problems including the bearing capacity and stability of slopes. Using this method, Harr [39] solved some problems in plasticity of soil. Later, Bolton and Lau [40] used the method to present the second and third bearing capacity factors for strip and circular foundations. Kumar and Mohan Rao [41] employed this method to determine the bearing capacity under static and seismic conditions. Martin [42,43] applied this method to conduct a rather precise estimation of the third bearing capacity factor. Thanh and Russell [44] applied this method to unsaturated soils to investigate the effect of suction on limiting passive strength of the soil behind the retaining wall. Veiskarami and Zanj [45] investigated the stability of sheet-pile walls subjected to seepage flow by slip lines and finite element methods. Johari et al. [46] evaluated the reliability analysis of seismic ultimate bearing capacity of strip footing using the slip line method combined with random field theory. Each method is clearly characterized by its own restrictions and advantages. The greatest advantage of the slip line method compared to other methods is its independence from the soil constitutive model and its compatibility with real soil behavior, e.g., in detection of failure mechanisms and estimation of limit loads.

The current study considers the soil to be obeying the Mohr-Coulomb yield criterion. The method of stress characteristics is then extended to the problem of stability of slopes of unsaturated soils. The shear strength of soils is related to the effective stress using Bishop's [47] widely employed effective stress expression for unsaturated soils. The effective stress is described as the summation of two independent stress state tensors as follows:

$$\sigma'_{ij} = (\sigma_{ij} - u_a\delta_{ij}) + \chi(u_a - u_w)\delta_{ij}. \quad (1)$$

In this equation, σ_{ij} is a component of the stress tensor, σ'_{ij} the effective stress, u_a the pore air pressure, and u_w the pore water pressure. The multiplier χ is Bishop's well-known parameter called effective stress, which is associated with the degree of saturation, but not necessarily in a linear fashion. Many researchers have presented a number of relevant mathematical relationships so as to estimate the value of effective stress parameter. In this regard, Vanapalli et al. [48], Khalili and Khabbaz [49], Khalili et al. [50], and Vaunat and Casini [51] proposed significant correlations.

The main objective of this research is to estimate a factor of stability against failure for slopes of unsaturated soils, implementing the method of stress characteristics undergoing some modifications to encompass the unsaturated condition. In connection with this point, van Genuchten [52] model is used to estimate parameter χ. The suction is assumed to be in steady-state evaporation and infiltration. The factor of stability is defined in a meaningful manner to be applicable for engineering purposes. Since no flow of water is considered through the media, the matric suction varies linearly with distance from ground water table and also there will be no variation of matric suction on the horizontal plane. This might seem an unrealistic assumption as the entire field could be affected by the rainfall pattern. However, the variations caused by rainfall are mostly seasonal and gradual over time. The present study in this respect could be regarded in a particular fixed instance in time. The boundary conditions consist of a traction-free surface of the slope. Governing equations are formed using the finite difference numerical solution strategy, which is well described in the literature (e.g., Bolton and Lau [40]; Anvar and Ghahramani [53]). The numerical results are then compared with conventional data, wherever possible. The term conventional in this paper represents a state in which stresses are computed irrespective of matric suction (i.e., fully saturated or dry). Five types of soils with different van Genuchten parameters including Air Entry Value (AEV) were the subjects of this study. The AEV is the matric suction separating saturated state from unsaturated state. It is in fact defined as the matric suction value exceeded before air recedes into the soil pores [54]. Analyses have been carried out for different slope angles and slope heights in conventional and unsaturated states. It has been found that increase in the matric suction tends to increase the stability of a slope in an unsaturated soil. This increase is particularly significant for fine-grained soils.

Another issue that should be investigated in separate research is the deformation problem which is often followed by surface settlement. This issue is out of the scope of this work. There are some relevant researches that can be found in the literature in this regard [44,55].

2. Soil model

As already stated, the soil in this study was assumed to be following a rigid, perfectly plastic behavior consisting of the assumptions used for the method of stress characteristics in compliance with the Mohr-Coulomb yield criterion. The suction is introduced as a cohesion intercept or the so-called apparent cohesion. It is important to note that both Bishop's effective stress parameter and the matric suction vary in the media and hence, the cohesion intercept changes with

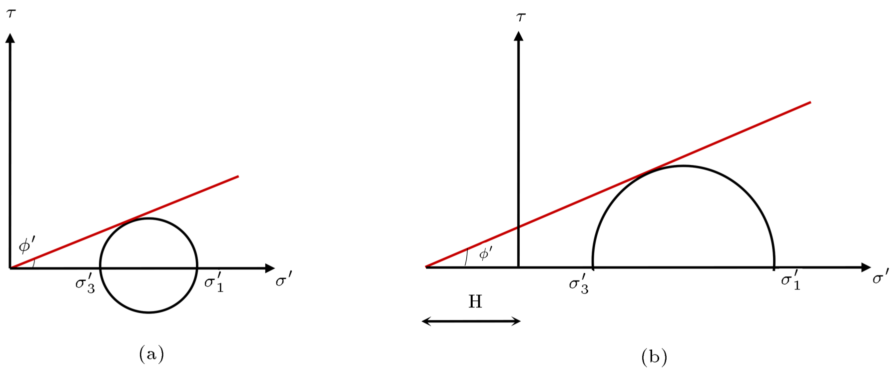


Figure 1. Mohr stress circle for (a) saturated (conventional analysis) and (b) unsaturated soils.

depth. Therefore, this makes the results different from those formerly done by assuming a constant cohesion intercept in the media using the method of stress characteristics, e.g., Jahanandish et al. [8]. Furthermore, the main point of the present study is to use the method of stress characteristics for slope stability problems rather than other subjects such as bearing capacity and retaining walls, e.g., Thanh and Russell [44]. The soil itself shows no cohesion when it is subject to conventional analysis. Figure 1 shows the Mohr stress circle corresponding to these two states. For unsaturated state, we have:

$$\frac{(\sigma_1' - \sigma_3')}{2} = \left(H + \frac{\sigma_1' + \sigma_3'}{2}\right) \sin\phi', \tag{2}$$

where, σ_1' and σ_3' are principal stresses; ϕ' is the effective angle of internal friction; and c' is the apparent cohesion which is defined below:

$$c' = -\chi u_w \tan\phi', \tag{3}$$

$$H = c' \cot\phi'. \tag{4}$$

Note that the pore air pressure is assumed to be the atmospheric pressure.

The van Genuchten model has been used to estimate Bishop's effective stress parameter χ. This model relates χ and the matric suction as follows:

$$\chi = \frac{1}{[1 + (\alpha h)^n]^m}, \tag{5}$$

where h is the matric suction and α, m, and n are the model parameters in the Soil Water Retention Curve (SWRC) of soils. The SWRC diagrams allow showing the parameters such as degree of saturation, coefficient of hydraulic conductivity, normalized gravimetric water content (which in this paper is Bishop's effective stress parameter using van Genuchten formula), etc. in terms of matric suction.

The equilibrium equations in two dimensions for the plane strain problem of any arbitrary material in the absence of horizontal body forces (e.g. without a seismic force) are as follows:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau}{\partial z} = 0, \qquad \frac{\partial \tau}{\partial x} + \frac{\partial \sigma_z}{\partial z} = \gamma, \tag{6a}$$

$$\frac{\partial \sigma'_x}{\partial x} + \frac{\partial \tau}{\partial z} = 0 \qquad \frac{\partial \tau}{\partial x} + \frac{\partial \sigma'_z}{\partial z} = \gamma_e. \tag{6b}$$

The parameters γ and γ_e in these equations are body forces acting in the vertical direction which in this case are soil density in conventional analysis and effective soil density, i.e., unsaturated analysis, respectively. This system of equations can be used by augmenting the yield criterion as the third equation to render it into a solvable system of three equations with three unknowns. The procedure is studied and the corresponding solution can be found through the method of stress characteristics. Soil obeying the Mohr-Coulomb yield criterion possesses two directions along which slip will occur when the limiting state of equilibrium is reached. These directions are the well-known characteristics of the stress in the limiting equilibrium (i.e., at yield). In this state, θ is the angle between the major principal stress direction and the positive x axis. Slip directions form an angle of $\mu = \frac{\pi}{4} - \frac{\phi'}{2}$ with θ. These directions are called *Characteristics Directions* or *Slip Lines* and are depicted in Figure 2.

By introducing the stress invariant $\bar{\sigma}$, the effective mean stress, as $\bar{\sigma} = (\sigma_x' + \sigma_z')/2$, the normal and shear

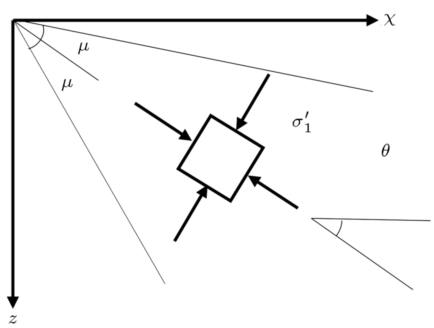


Figure 2. Slip lines in Cartesian coordinates system.

stresses can be found in terms of parameters $\bar{\sigma}$ and θ as follows:

$$\sigma'_x = (\overline{\sigma} + H)(1 + \sin\phi' \cos 2\theta) - H, \tag{7a}$$

$$\sigma'_z = (\overline{\sigma} + H)(1 - \sin\phi' \cos 2\theta) - H, \tag{7b}$$

$$\tau = (\overline{\sigma} + H)\sin\phi' \sin 2\theta, \tag{7c}$$

where σ'_x and σ'_z are effective normal stresses and τ is the shear stress along the soil element. By substituting Eq. (7) into Eq. (6) and considering the variation in c' and ϕ' within the field, the set of stress characteristics equations can be found as follows.

Along the positive *characteristics* direction: $\frac{dz}{dx} = \tan(\theta + \mu)$

$$d\overline{\sigma} + 2(\overline{\sigma}\tan\phi' + c')d\theta = \gamma_e(\tan\phi' dx + dz) + (\overline{\sigma} - H)\left(\frac{\partial\phi'}{\partial z}dx - \frac{\partial\phi'}{\partial x}dz\right) + \left(\frac{\partial c'}{\partial z}dx - \frac{\partial c'}{\partial x}dz\right). \tag{8}$$

Along the negative *characteristics* direction: $\frac{dz}{dx} = \tan(\theta - \mu)$

$$d\overline{\sigma} - 2(\overline{\sigma}\tan\phi' + c')d\theta = -\gamma_e(\tan\phi' dx - dz) - (\overline{\sigma} - H)\left(\frac{\partial\phi'}{\partial z}dx - \frac{\partial\phi'}{\partial x}dz\right) + + + + + - \left(\frac{\partial c'}{\partial z}dx - \frac{\partial c'}{\partial x}dz\right). \tag{9}$$

In this study, the internal friction angle is taken constant as an independent variable and since the ground water level is constant, variations of apparent cohesion occur in the z direction only. In addition, the negative pore water pressure is assumed to increase linearly above the groundwater level. Thus, Eqs. (8) and (9) yield the following equations:

$$d\overline{\sigma} + 2(\overline{\sigma}\tan\phi' + c')d\theta = \gamma_e(\tan\phi' dx + dz) + \left[\chi(-\gamma_w dx) + u_w\frac{\partial\chi}{\partial z}\right]\tan\phi', \tag{10}$$

$$d\overline{\sigma} - 2(\overline{\sigma}\tan\phi' + c')d\theta = -\gamma_e(\tan\phi' dx - dz) - \left[\chi(-\gamma_w dx) + u_w\frac{\partial\chi}{\partial z}\right]\tan\phi'. \tag{11}$$

3. Geometry and boundary conditions

The geometry of the problem is illustrated in Figure 3. Ground water level in this paper has been taken constant and located at the toe of the slope. As mentioned previously, since there is no flow of water considered within the soil media, based on the theories of classical soil mechanics, the soil above the water table is subject to the negative pore pressure which varies in a fashion slightly proportional to the distance above the ground water table. This assumption is quite reasonable for most soils in normal conditions, i.e., in the absence of other types or sources of pressure or highly non-homogeneous nature. Parameter h, shown in Figure 3, is the matric suction. The boundary condition involves the surface of traction free of the sloping ground. However, since the computer program is incapable of starting from zero value of traction, a negligible value of surcharge q, acting on the horizontal projection of that surface, is taken into account simply for the purpose of running the program. Having tangent and normal values of surcharge pressure along the slope and the Mohr-Coulomb yield criterion (Figures 3 and 4), the unknowns $\overline{\sigma'}$, θ, x, and z can be specified along the slope surface as follows:

$$\sigma'_v = q\cos\alpha, \tag{12}$$

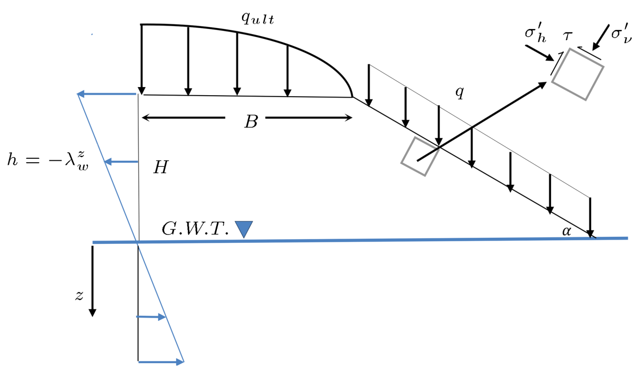


Figure 3. Geometry of the problem and definition of parameters.

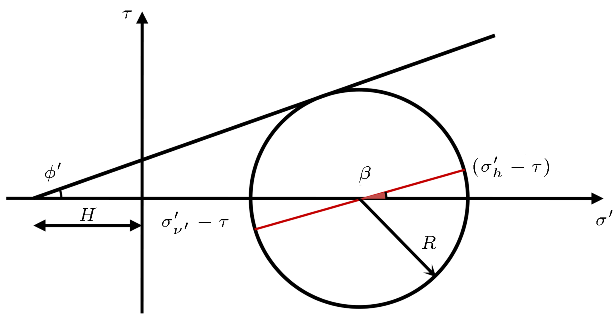


Figure 4. Application of the Mohr-Coulomb yield criterion to finding parameters σ' and θ along the slope surface.

$$\tau = q \sin\alpha. \tag{13}$$

To find σ'_h, we have:

$$R = \tau_{\max} = \frac{1}{2}(\sigma'_h - \sigma'_v) = \left[H + \frac{1}{2}(\sigma'_h + \sigma'_v)\right]\sin\phi', \tag{14}$$

$$\overline{\sigma'} = \frac{1}{2}(\sigma'_h + \sigma'_v), \tag{15}$$

$\tau_{\max}$ represents the maximum shear stress which is equal to the radius of the Mohr stress circle. Then, by utilizing the Mohr stress circle, the magnitude of θ along the slope will be found accordingly:

$$\theta = \alpha + \frac{\beta}{2}, \tag{16}$$

where β is shown in Figure 4:

$$\beta = \tan^{-1}\frac{2\tau}{(\sigma'_h - \sigma'_v)}. \tag{17}$$

Another important issue, which is one of the constrains of this paper, is the spatial variability of soil layers as it could affect the outcome of the work, especially in slopes of short height where the plastic zone may fall in more than one layer. However, as the central assumption in this study, the slopes are assumed to be relatively high and hence, only shallow failure mechanisms are possible. In addition, the soil layer was assumed to be homogeneous and hence, the spatial variability of the soil which could affect the results was not considered. This issue, however, could be investigated in a separate study in the future by other researchers.

4. Methodology and analyses

Conventional and unsaturated analyses were carried out on five types of soil with different van Genuchten parameters, for which a computer code was developed. The algorithm and the code were verified several times for standard bench mark problems and other conditions reported in published papers. For instance, the bearing capacity of a flat ground as a special case was examined and the results were favorably consistent with those obtained by the algorithm (and the code) developed in this paper [8,41,53,56,57]. The process of verifications is laid out and discussed in the next sections. In all cases, the soil cohesion was neglected, although for computational purpose, a small value of soil cohesion was assumed in calculations (e.g., 0.1 kPa). It is known that slopes of cohesionless soils are unstable where the slope inclination angle exceeds the effective angle of internal friction. Also, the van Genuchten model was utilized in order to express Bishop's effective stress parameter in terms of matric suction. In this regard, the van Genuchten parameters of these five types of soil were collected using the *Soil Vision* software (*Soil Vision* is an information bank for soils containing a series of available laboratory test data including the particle size distribution, the Atterberg limit, mechanical and engineering properties of soils, and so on). The data of the soils for this study are tabulated in Table 1. Analyses were applied to three different slope angles, four different slope heights, and also two states of conventional and unsaturated conditions. In addition, the hydraulic gradient, if any, is assumed to be very low and the effect of the water flow is neglected. Thus, the total number of 24 analyses on each soil type was carried out.

Initiating from the known state of stress in the sloping ground, the slip lines were extended to the flat ground at the top and the bearing pressure was found as a measure of stability. Once the stress characteristics network was completed within the soil media, the distribution of the limiting stresses was done for the two conventional and unsaturated states. In the case of a few methods such as the limit equilibrium method, although the pressure distribution beneath the footing is assumed to be uniform, the pressure distribution cannot be uniform in most other methods, e.g., the method of stress characteristics, lower bound limit analysis, finite element methods, etc. This is the character of these methods which employ field equations (not only equilibrium equations for rigid blocks) and hence, the distributions of the stress or displacement fields make the pressure distribution inevitably non-uniform.

Table 1. Air entry value and van Genuchten parameters of the soils investigated in this study.

Type	Soil type 1	Soil type 2	Soil type 3	Soil type 4	Soil type 5
	SP	Loamy sand	Loamy clay	ML	CL
Air entry value (kPa)	0.43	5.1	6.5	25	72
α van Genuchten	0.121	0.024	0.052	0.0027	0.0036
n van Genuchten	0.685	1.094	1.447	0.486	1.643
m van Genuchten	1.066	0.543	0.309	0.372	0.696

Many approaches similar to those of Brinch Hansen [3], Bolton and Lau [40], Kumar [6], etc. are characterized by a non-uniform pressure distribution. A comparison, however, is possible once the "average" of the pressures is calculated and used for this purpose. Therefore, in all these methods, the bearing capacity implicitly means the "average" pressure (although not uniformly distributed) beneath the footing.

The parameter *stability factor* or some measure of the bearing ratio was defined here as the ratio of the bearing pressure obtained along a flat ground in the unsaturated condition to that calculated in the conventional condition. In other words, it indicates how stable or precarious a slope is when the soil is unsaturated. The greater the bearing ratio, the more reliable it is against the failure, and vice versa. Therefore, in order to be able to compare these two states, the widths through which analyses were done along the flat ground must be the same in both states. It was observed that the suction would lead to higher stability, especially when the slope inclination angle approached the angle of the effective internal friction. In this state, the stability factors were greater than those of smaller slope angles.

5. Results

As stated before, five types of soils with different van Genuchten parameters were selected. The diagram demonstrating the variations of apparent cohesion and Bishop's effective stress parameter associated with five soil samples is presented in Figure 5. Since the highest slope dealt with in this study was 30 m, these diagrams were plotted for this range. By making use of the van Genuchten parameters, one can easily get to SWRC (Figure 6). All soils under study had an effective internal friction angle ranging between $\phi' = 28°$ and $\phi' = 32°$, taken from the *Soil Vision* databank. This slight difference was neglected; therefore, an average value was assigned to all soil types equal to $\phi' = 32°$. Analyses were carried out in a condition characterized by slope angles of $\alpha = 20°$, $25°$, and $30°$ and heights of $H = 5$ m, 10 m, 20 m, and 30 m for both conventional and unsaturated conditions. The final results of the study are presented in Figures 7–9 and Tables 2 to 6. The parameter "AEV" noted in the figures is the Air Entry Value.

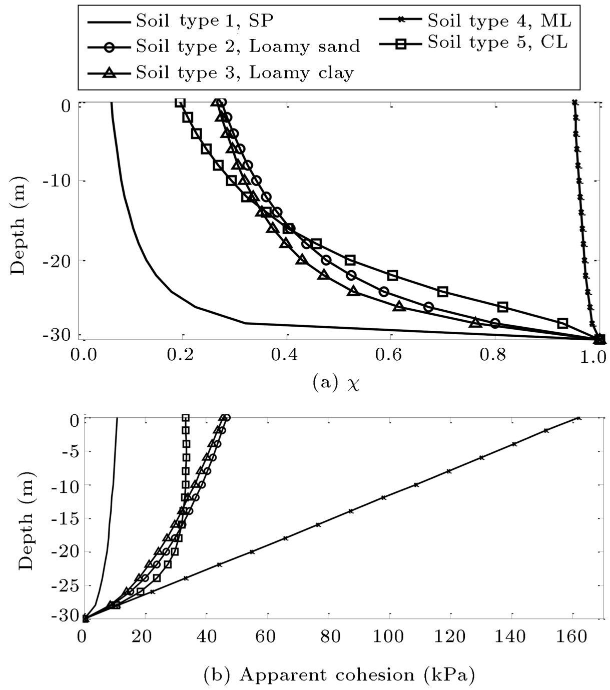


Figure 5. Variation of (a) Bishop's effective stress parameter and (b) apparent cohesion with distance from the groundwater table.

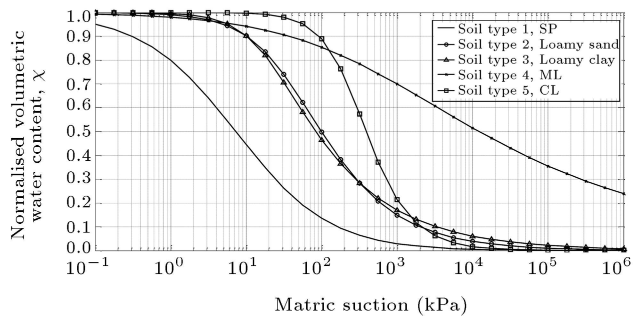
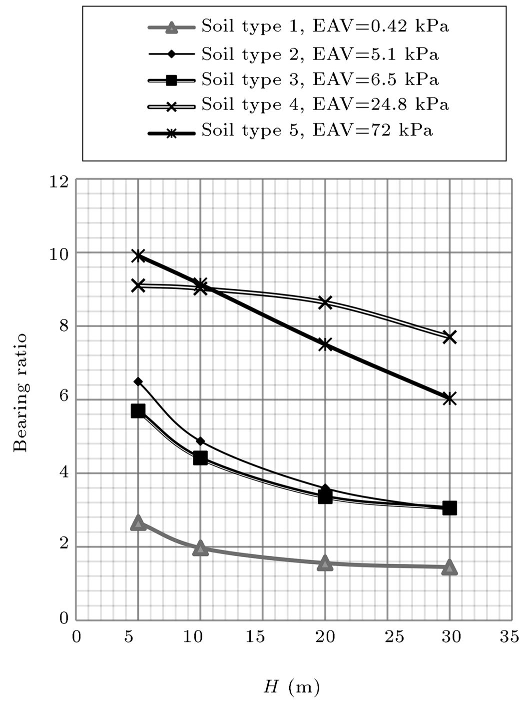


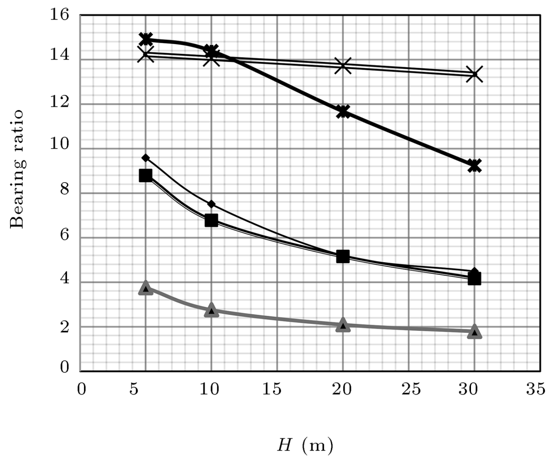
Figure 6. Soil Water Retention Curve (SWRC) curves of soil types 1–5.

Table 2. Results corresponding to soil Type 1.

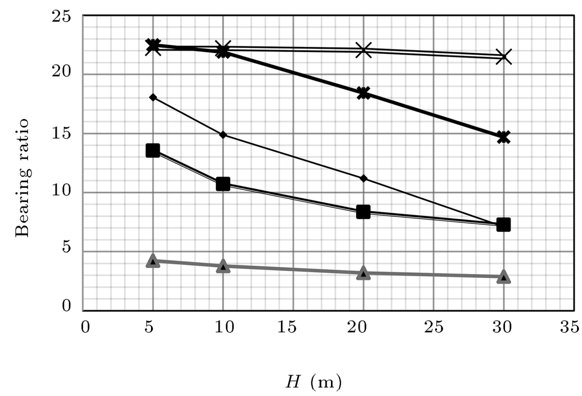
	Soil type 1, AEV=0.43 kPa								
	$\alpha = 20°$			$\alpha = 25°$			$\alpha = 30°$		
	Bearing pressure			Bearing pressure			Bearing pressure		
	$\phi' = 30°$ and $c' = 0$			$\phi' = 30°$ and $c' = 0$			$\phi' = 30°$ and $c' = 0.5$ kPa		
Height (m)	Conventional (kPa)	Unsaturated (kPa)	Bearing ratio	Conventional (kPa)	Unsaturated (kPa)	Bearing ratio	Conventional (kPa)	Unsaturated (kPa)	Bearing ratio
5	86.8	231.2	2.66	44.93	168.88	3.75	34.46	145.79	4.23
10	207.08	408.86	1.97	100.27	275.41	2.75	57.34	216.54	3.78
20	481.18	755.34	1.56	240.37	501.9	2.09	101.06	323.31	3.2
30	757.39	1097.52	1.45	364.19	650.42	1.79	142.4	410.82	2.88



(a) Slope of $\alpha = 20°$



(b) Slope of $\alpha = 25°$



(c) Slope of $\alpha = 30°$

Figure 7. Variation of bearing ratio with slope height for slopes of different angles.

6. Verification and comparisons

Results of the comparison between conventional and unsaturated analyses were confirmed through comparative study. Given the lack of sufficient comparable data to verify the results obtained for unsaturated cases, comparisons were made only for conventional states. Results of the conventional analyses are in good agreement with those of Meyerhof [2], Brinch Hansen [3], Vesić [4], and Kumar [6] when the angle of slope is zero, i.e., the bearing capacity problem. As illustrated in Table 7, the results were also compared with those obtained by Kumar [6] with different slope angles (Tables 8 and 9 and Figure 10). Parameters P_u and χ in the diagram of Figure 10 indicate the bearing pressure beneath the foundation and the distance from the foundation's edge (relative to the singularity point), respectively. As observed, there is good agreement between the results of Kumar [6] and the present study. The bearing pressure begins at values very close to zero at one edge and increases linearly along the footing base.

7. Discussion

Upon delving further into the results of this study, it is discernible that the bearing ratio, as a measure of slope stability, increases with increase in the slope inclination angle in all soil types (Figure 7). This sounds quite reasonable since the steeper the slope, the more precarious and unstable it is, especially when the slope angle approaches the angle of effective internal friction. However, the matric suction tends to boost the soil cohesion and contribute to a greater level of stability. Therefore, a steep slope with an inclination angle near the internal friction angle has a higher ratio of stability when it is in the unsaturated state. This evidence is given in Figure 8 with the slopes inclined at an angle equal to $\alpha = 30°$ exhibiting the largest bearing ratios. Since the type-1 soil is a granular soil, the reported stability factors are significantly smaller than those of the other soil types.

It is also observed that the bearing ratio decreases with increase in the height of the slope (Figures 8 and 9). Although the matric suction increases with the increasing distance from the ground water table (which in this study is constantly at the same level as toe of the slope), this increase cannot make up for the adverse effect of the height. As is shown in Figure 7, the bearing ratio corresponding to type-4 soil hardly varies with increase in the slope height (especially for the slope angle of $\alpha = 30°$). This is mostly due to the higher value of the apparent cohesion with respect to the other soil types. according to Figure 5, the apparent cohesion value corresponding to type-4 soil is approximately between 3 to 17 times of those of the other types.

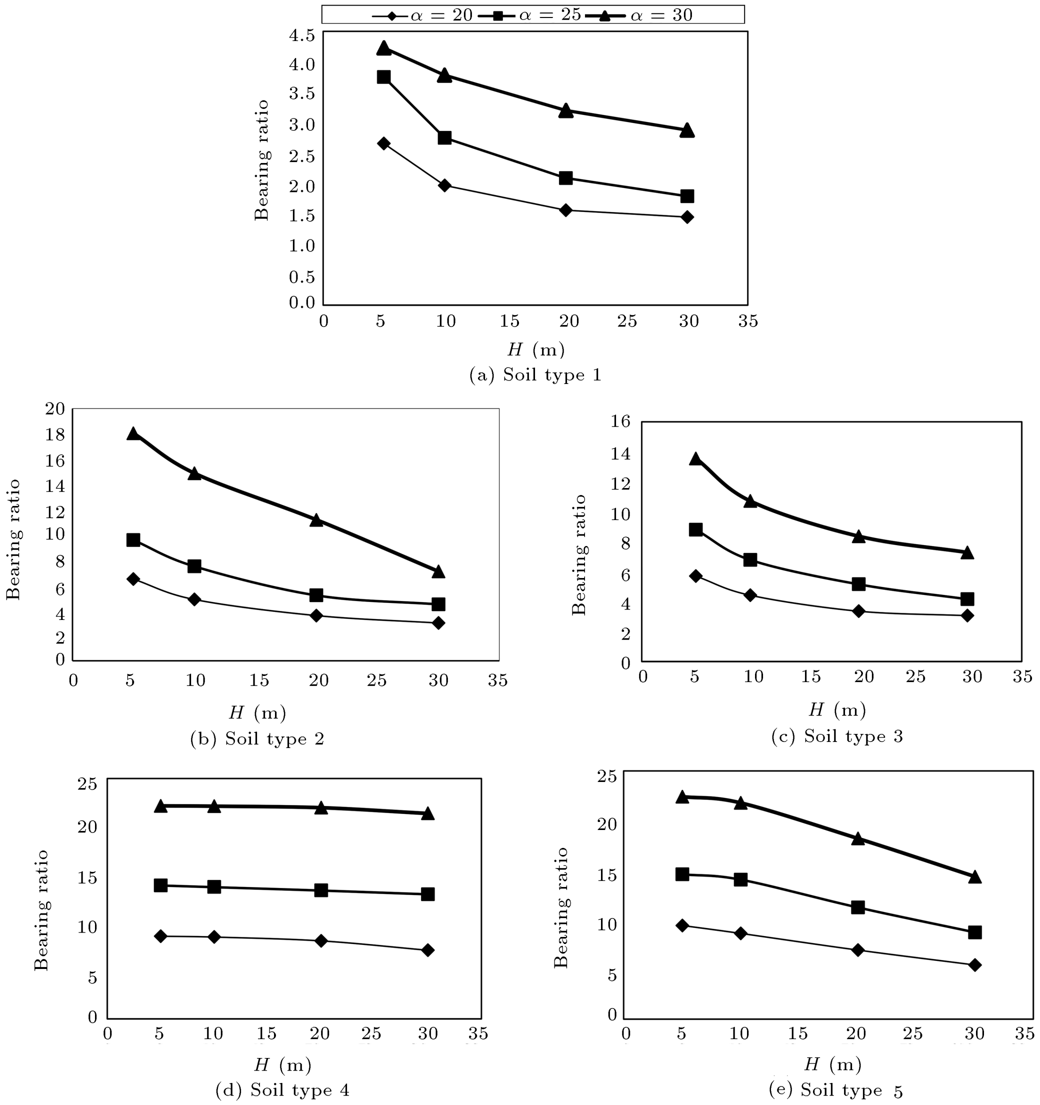


Figure 8. Variation of the bearing ratio with height of the slope.

Table 3. Results corresponding to soil type 2.

	Soil type 2, AEV=5.1 kPa								
	$\alpha = 20°$			$\alpha = 25°$			$\alpha = 30°$		
	Bearing pressure			Bearing pressure			Bearing pressure		
	$\phi' = 30°$ and $c' = 0.1$ kPa			$\phi' = 30°$ and $c' = 0.1$ kPa			$\phi' = 30°$ and $c' = 0.1$ kPa		
Height (m)	**Conventional (kPa)**	**Unsaturated (kPa)**	**Bearing ratio**	**Conventional (kPa)**	**Unsaturated (kPa)**	**Bearing ratio**	**Conventional (kPa)**	**Unsaturated (kPa)**	**Bearing ratio**
5	73.45	476.57	6.49	40.73	390	9.58	18.84	340.01	18.04
10	167	812.99	4.87	87.69	657.53	7.5	37.1	551.86	14.87
20	382	1374.6	3.59	202.21	1055.69	5.2	76.3	852.82	11.18
30	620	1873.23	3	308.08	1385.67	4.49	160	1137.24	7.09

This leads to a significantly stronger structure of the mentioned soil and greater resistance against failure. The effect of the AEV as an independent parameter was also investigated. In contrast to our expectation, it was shown that there was no specific pattern that could be considered common between the AEV and the bearing ratio for the soils investigated in this study. Figure 9 shows the bearing pressure versus the AEV for three different slope angles and four different slope heights. Other van Genuchten parameters appear to be of greater importance, as type-4 soil with a smaller AEV than type-5 soil (almost one third) enjoys

Table 4. Results corresponding to soil type 3.

	Soil type 3, AEV=6.5 kPa								
	$\alpha = 20°$			$\alpha = 25°$			$\alpha = 30°$		
	Bearing pressure			Bearing pressure			Bearing pressure		
	$\phi' = 30°$and $c' = 0.1$ kPa			$\phi' = 30°$and $c' = 0.1$ kPa			$\phi' = 30°$and $c' = 0.1$ kPa		
Height (m)	Conventional (kPa)	Unsaturated (kPa)	Bearing ratio	Conventional (kPa)	Unsaturated (kPa)	Bearing ratio	Conventional (kPa)	Unsaturated (kPa)	Bearing ratio
5	81.5	464.44	5.69	43.64	383.68	8.79	24.78	335.73	13.54
10	179	789.59	4.41	92.7	628.28	6.78	50.32	538.7	10.7
20	401.99	1350.03	3.36	198.43	1023.08	5.15	102.23	854.06	8.35
30	609.53	1861.22	3.05	327.59	1363.26	4.16	153.43	1115.65	7.271

Table 5. Results corresponding to soil type 4.

	Soil type 4, AEV=25 kPa								
	$\alpha = 20°$			$\alpha = 25°$			$\alpha = 30°$		
	Bearing pressure			Bearing pressure			Bearing pressure		
	$\phi' = 30°$and $c' = 0.1$ kPa			$\phi' = 30°$and $c' = 0.1$ kPa			$\phi' = 30°$and $c' = 0.1$ kPa		
Height (m)	Conventional (kPa)	Unsaturated (kPa)	Bearing ratio	Conventional (kPa)	Unsaturated (kPa)	Bearing ratio	Conventional (kPa)	Unsaturated (kPa)	Bearing ratio
5	63.45	578.1	9.1	35.01	498.35	14.23	20.99	445.61	22.23
10	124.14	1119.6	9.02	68.66	965.78	14.06	38.87	863.02	22.2
20	250.1	2160.1	8.64	135.21	1855.42	13.72	74.93	1652.79	22.05
30	410.17	3159.6	7.7	202.83	2707.29	13.34	111.93	2404.69	21.48

Table 6. Results corresponding to soil type 5.

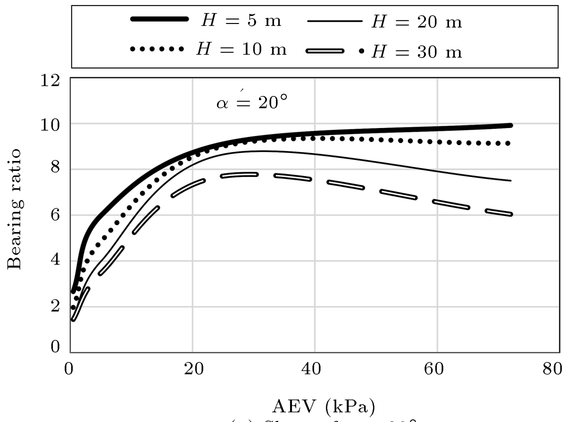
	Soil type 5, AEV=72 kPa								
	$\alpha = 20°$			$\alpha = 25°$			$\alpha = 30°$		
	Bearing pressure			Bearing pressure			Bearing pressure		
	$\phi' = 30°$and $c' = 0.1$ kPa			$\phi' = 30°$and $c' = 0.1$ kPa			$\phi' = 30°$and $c' = 0.1$ kPa		
Height (m)	Conventional (kPa)	Unsaturated (kPa)	Bearing ratio	Conventional (kPa)	Unsaturated (kPa)	Bearing ratio	Conventional (kPa)	Unsaturated (kPa)	Bearing ratio
5	62.8	622.34	9.91	36.14	538.66	14.90	21.44	482.4	22.5
10	130.11	1188.24	9.11	71	1021.16	14.38	41.77	914.33	21.89
20	283.83	2128.21	7.49	153.93	1795.95	11.67	86.2	1587.27	18.41
30	473.9	2859.5	6.03	255.47	2359.4	9.23	140.31	2060.55	14.68

Table 7. Comparison of bearing capacity factors, N_q and N_c.

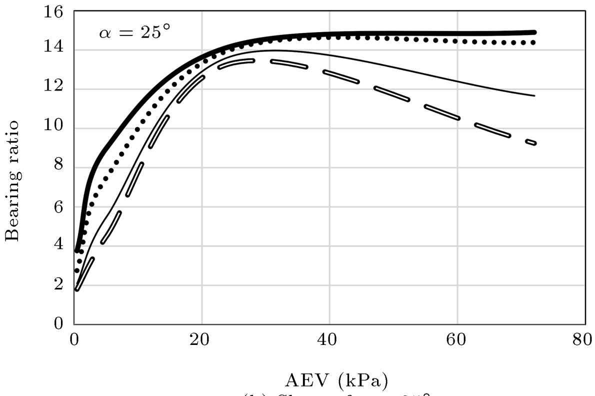
	N_q			N_c		
ϕ'	Meyerhof (1963) [2], Hansen (1970) [3]	Kumar and Mohan Rao (2003) [41] (single side mechanism)	Present study	Meyerhof (1963) [2], Hansen (1970) [3]	Kumar and Mohan Rao (2003) [41] (single side mechanism)	Present study
0	1	1	1	5.14	5.14	5.14
5	1.57	1.57	1.57	6.52	6.52	6.53
10	2.47	2.47	2.47	8.34	8.34	8.35
15	3.94	3.94	3.94	10.97	10.97	10.99
20	6.4	6.4	6.4	14.84	14.84	14.86
25	10.7	10.7	10.75	20.8	20.8	20.9
30	18.4	18.4	18.49	30.14	30.14	30.29
35	33.3	33.3	33.46	46.13	46.13	46.36
40	64	64	64.31	75.08	75.08	75.45

Table 8. Comparison of bearing capacity factors, N_q, on the sloping ground.

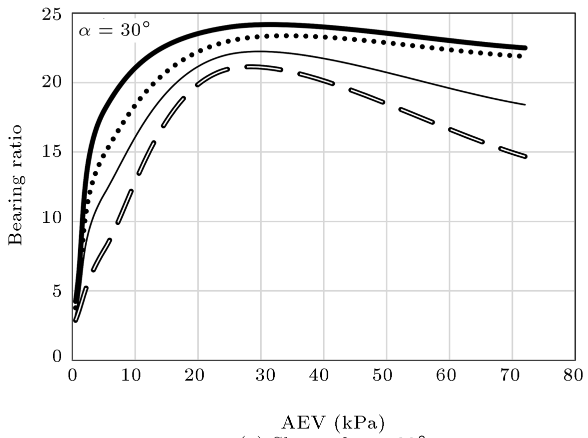
N_q					
	$\phi' = 30°$			$\phi' = 40°$	
α	Kumar and Mohan Rao (2003) [41]	Present study	α	Kumar and Mohan Rao (2003) [41]	Present study
0	18.4	18.49	0	65	65.15
10	12.8	13.16	15	33.5	34.68
20	7.9	8.48	30	13.1	15.18



(a) Slope of $\alpha = 20°$



(b) Slope of $\alpha = 25°$



(c) Slope of $\alpha = 30°$

Figure 9. Variation of bearing ratio with air entry value for slopes.

apparent cohesion of almost three times the type-5 soil within the 30-meter range of the slope height.

Figures 11 and 12 underscore the existence of slight rotation in the mixed zone's characteristics directions in the unsaturated case which does not exist in the conventional state. This rotation stands out more clearly in slopes with the angle of = 30°, since the slope angle is equal to the angle of the effective internal friction in this study. Figure 12 represents the proof

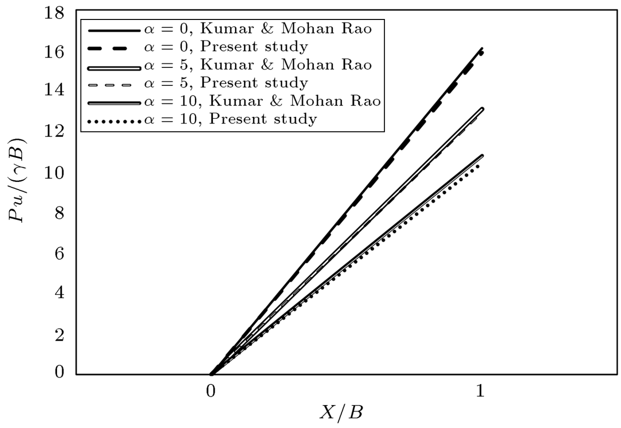


Figure 10. Comparison of values of $p_u/\gamma B$ vs x/B.

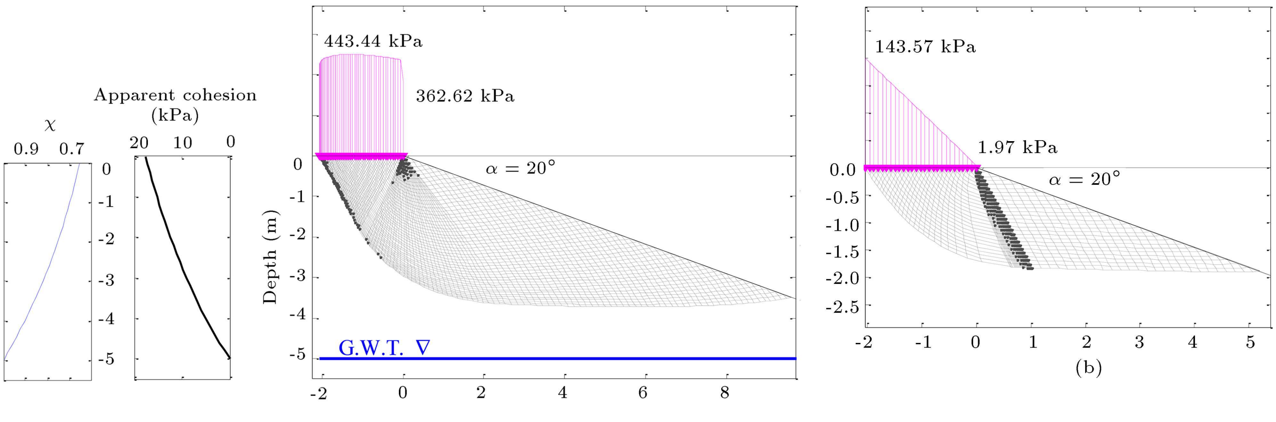
that the failure surface of a conventional analysis is almost approaching a thin narrow strip. However, due to the presence of apparent cohesion in the unsaturated state, the failure surface is developed which makes it smoother.

8. Conclusions

The effect of the matric suction on the stability of slopes was theoretically explored using the method of stress characteristics. This research was performed to introduce a model for predicting the bearing ratio as a measure of slope stability, which is the ratio of bearing pressure along a flat ground on an unsaturated medium to that of conventional state based on the solution of plasticity problems in soil mechanics. The method was capable of modeling the soil matric suction. Variation of the matric suction with depth down to the ground water level was linear. Bishop's effective stress theory in unsaturated soils was also employed to compute the shear strength and the bearing capacity of soil. In this regard, van Genuchten model was utilized to relate the matric suction to Bishop's effective strength parameter. The results of the conventional analyses were validated with respect to the works of Meyerhof, Brinch Hansen and Vesić bearing capacity and also Kumar and Mohan Rao. Since there were no significant comparable results in the unsaturated state, the comparison in this state was precluded. It was observed that in the presence of the matric suction, the slope became more stable and failure surfaces turned smoother. It was also observed that in contrast to general expectations, the air entry value parameter did not affect the general behavior of the unsaturated soil samples in this study and other van Genuchten parameters were of greater significance due to their direct effect on Bishop's effective stress parameter and, consequently, the apparent cohesion. The soil type and the height of the slope were also found to be important factors in the general stability of a slope. In general, soils exhibiting higher apparent

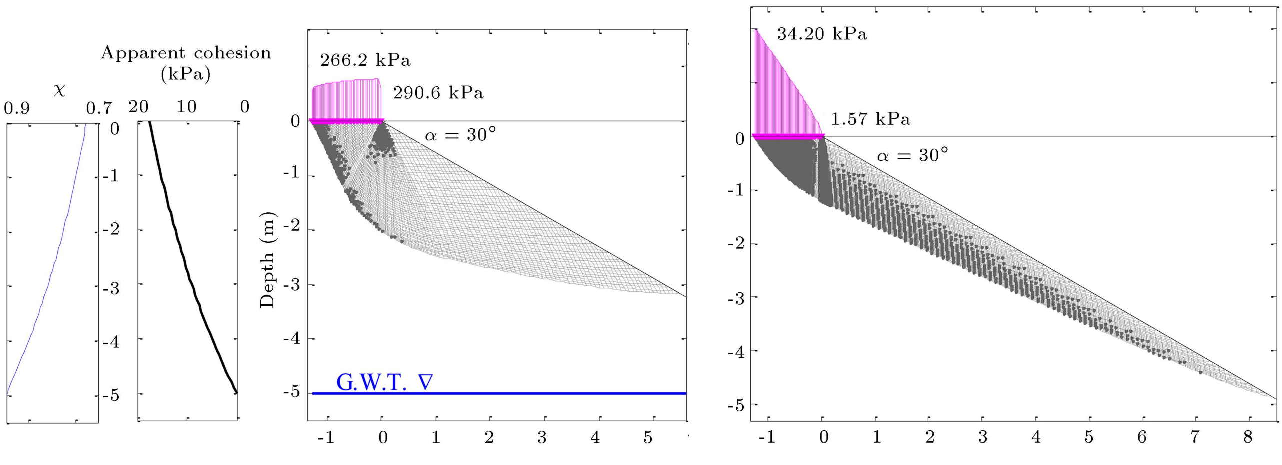
Table 9. Comparison of bearing capacity factors, N_c, on the sloping ground.

	N_q				
	$\phi' = 30°$			$\phi' = 40°$	
α	Kumar and Mohan Rao	Present study	α	Kumar and Mohan Rao	Present study
0	30.14	30.29	0	75	76.45
10	24.4	24.41	15	48	48.53
20	19.5	19.61	30	30.5	30.72



(a)

Figure 11. Slip lines for soil type 2, slope height = 5 m, slope of angle $\alpha = 20$, corresponding to (a) unsaturated analysis and (b) conventional analysis.



(a) (b)

Figure 12. Slip lines for soil type 2, slope height = 5 m, slope of angle $\alpha = 30$, corresponding to (a) unsaturated analysis and (b) conventional analysis.

cohesion have a higher tendency towards stability for a slope with a constant height. Thus, granular soils are less stable than other soils. On the other hand, the slope height has an opposite influence on stability as it tends to highly contribute to destabilizing the slope regardless of the slope angle.

For future studies, some suggestions could be made. For instance, the effect of the spatial variability of the soil as well as the effect of infiltration as a result of rain pattern can be taken into account. Also, the variation of the matric suction in the horizontal direction due to the flow of water may be examined. In addition, a more efficient way of solving the governing equations, especially for spatially variable soil layers, can be developed in relation to the numerical solution techniques.

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