Coordinating a closed-loop green supply chain for remanufactured product under competition

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Abstract The success of any remanufacturing system depends on proper supply of used product. However, in the real-world, many remanufacturers are unable to collect used products directly from customers. This article considers a closed-loop green supply chain consisting of a single remanufacturer, two competing retailers and two competing collectors for dealing with used and remanufactured products. Considering the effect of acquisition prices and greening level on collection quantity, and that of selling prices and greening level on the market demand, closed-form solutions are derived for a centralized model and two decentralized models depending on collectors' cooperation and competition. Optimal results of the decentralized models are compared to find the best decentralized policy. Moreover, a novel multi-link two-part tariff contract among the channel members is proposed to resolve the channel coordination issue. The results indicate that (*i*) cooperation between the collectors is only beneficial for themselves while competition between them is favorable from the viewpoint of the whole supply chain; (*ii*) utilization ratio has a noteworthy effect on the performance of the supply chain; (*iii*) the proposed contract assists the collectors with getting more than double transfer price and the remanufacturer to improve its profit over 21% compared to the decentralized setting.

Keywords: Closed-loop supply chain; remanufacturing; greening level; collectors' competition; two-part tariff contract; game theory.

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1 Introduction

A few decades ago, there was no enthusiasm among human beings about product remanufacturing. Most of the end-of-life products were stored in the stack, and sometimes there was no way without burning them. No doubt, these actions caused environmental damage. Now, the governments, NGOs, business personal, academicians have begun to look into this matter and they are trying to find out the way to reduce these garbages. Companies are working on the prevention of more wastes by creating constructive and worthwhile systems. As a result, the study of the closed-loop supply chain (CLSC) which integrates both forward and reverse logistics, plays a key role in sustainable development and protecting the environment (Giri and Masanta [1]). Forward logistics comprises product design, new product development, procurement and production, marketing, sales, etc. whereas reverse logistics embraces all those activities which are needed for closing the loop viz. used products recovery, inspection, sorting, remanufacturing, remarketing, and reselling (Guide and Van Wassenhove [2], Eydi et al. [3]). According to Ginsburg [4], remanufacturing of end-of-life products can reduce the manufacturing costs by up to 40-65%. So, there are many companies such as IBM, Ford, Caterpillar, Zara, H&M, HP, Xerox, Canon, etc. which entertain product remanufacturing besides producing new products (Wei et al. [5]). Although these manufacturers have started implementing remanufacturing in their production planning, it is not always possible to recycle all the collected used products. For instance, in Europe, the plastic wastes collection was increased by 19% in 2018 from the 2006 baseline but only 32% of those wastes were used for recycling (Bac [6]). Accordingly, a detailed investigation is required on how to procure an adequate quantity of used products and recycle those products.

In addition to product remanufacturing, green innovation is another important area of research in supply chain management. As social and environmental problems like global warming, earthquake, sea-level rise, ozone layer hole, desert storms, unseasonal rainfall, etc. are increasing day by day, the governments of developed countries are executing numerous principles and guidelines to implement zero-waste manufacturing shortly. Consumers are also thinking about using environment-friendly (green) products. Motivated by more government pressure and growing consumers' environmental awareness, and to survive in today's competitive business environment, each manufacturer wants to attract the attention of those customers who care about green products. For instance, Dell and HP have promoted the safe disposal of their products to decrease the environmental impact. In addition, they are committed to use renewable energy sources and adopt zero-waste manufacturing technologies (Chen and Akmalul'Ulya [7]). As the greening level of the product plays an important role in customers' satisfaction, companies tend to invest some extra amount for product greening. According to the survey by the European Commission, more than 80% European prefers to purchase green product (Nouira et al. [8]). In this regard, the appropriate pricing strategy of the green product plays an important role in the performance of the CLSC. Nowadays, multiple retailers sell a product in the potential market. In this situation, factors like self-price elasticity, cross-price elasticity affect the purchasing behavior of many potential customers. As the retailers' pricing decisions and the manufacturer's decision on product greening level have a direct impact on the market, the simultaneous impact of these factors has become an important concern to supply chain managers.

A reverse supply chain (RSC) is a part of CLSC whose main purpose is to collect end-of-use products from customers for proper disposal or capturing their residual value. So, the RSC requires a provocative return policy during product remanufacturing to make an optimum profit. Besides improving environmental sustainability, a higher rate of collection of used products enhances the profitability of the supply chain. Accordingly, it is important to encourage the customers to return the used products following incentive schemes such as paying off acquisition prices. Collecting efforts through financial incentives gains the commendation of the customers and in turn, increases the market demand (Ke and Cai [9]). Similar to the acquisition price, the environmental effort (green innovation) of the remanufacturer can influence those customers in returning used products who don't have sufficient knowledge about the importance of used product return. There are some manufacturers/remanufacturers such as HP, Dell, Apple, Xerox, Canon, Kodak, etc. who prefer to contract with the third-party collectors for collecting end-of-life products (Liu et al. [10]). Plastic, glass, metal, and paper are well-known products that are collected by third-party collectors after the end-of-use and these can be remanufactured more than once. In the real world, there exist multiple collectors for collecting used products, and so there is competition among them. In an RSC, the strategies of multiple collectors are very important from both the economic and environmental viewpoints. Therefore, a detailed analysis is needed to find out the influence of competing collectors on supply chain decisions.

In the traditional business, each member (manufacturer/remanufacturer, retailer, collector) of the supply chain makes its own decisions independently. The horizontal/vertical cooperation and non-cooperation among the channel members can affect supply chain performance. Cooperation may improve the decisions and profitability of some specific members who work cooperatively, but it leads to poor performance for other members and the whole supply chain. If the members optimize their decisions jointly, where a centralized decision-maker optimizes all the decisions, then the supply chain as a whole is benefitted. On the other hand, for a decentralized system, an effective coordination contract is indispensable for promoting the performance of all the supply chain members and the whole supply chain. Although through channel coordination, the surplus profit, which is the difference between the profits of the centralized policy and the decentralized policy, is divided among the channel members to enhance their profitability, there is no proper rule for distributing the surplus profit. So, it is assumed that the channel members divide the surplus profit according to their bargaining powers for enhancing profitability and achieving a win-win outcome for all the channel members. One of the keys to the success of any business is to integrate operations across all aspects of business streams within or outside the companies' boundaries. The Supply Chain Operations Reference (SCOR) Model, developed and endorsed by the Supply Chain Council, integrates all the operations from "the supplier's supplier to the customer's customer" (Supply Chain Operations Reference Model. Supply Chain Council. October 7, 2004.) and it can be utilized as a typical language of initiatives for enterprises to portray the supply chains and the correspondence between them (Wang et al. [11]).

Motivated by the above issues, this study considers a CLSC which integrates collection of used products through competing collectors, green innovation while remanufacturing, selling of remanufactured products through competing retailers, and channel coordination. Assuming that the return quantity is affected by the acquisition prices offered by the collectors and the greening level of the product, and market demand is influenced by the selling prices and greening level of the product, we investigate the impact of competitive gaming behaviors (competition and cooperation) of the third-party collectors on the performance of CLSC. We model the CLSC which consists of a single remanufacturer, two competing retailers and two competing collectors under a remanufacturer-led Stackelberg game with the remanufacturer as the Stackelberg leader and the other members as followers. The centralized policy is determined as the benchmark case to compare with the decentralized policies. Finally, a novel multi-link two-part tariff contract is developed to coordinate the supply chain. Precisely this study is carried out to find answers to the following research questions:

- What would be the optimal pricing strategies and greening level under consideration of different policies?
- Which behavior of the collectors is the best from the viewpoint of all members of the supply chain including the customers?
- How does the competition between the retailers and the collectors affect used product collection, product greening, and profitability of the channel members?
- How does the proposed contract work? Can it provide better profit to all the channel members?

The main contribution of the present study is two-fold. First, we consider a multi-level supply chain in which the remanufacturer acts as the Stackelberg leader, and two competing collectors and two competing retailers act as followers. The remanufacturer puts effort into green innovation which influences both the supply of used products and the market demand of the remanufactured products. The competing collectors compete with each other on the acquisition price of the used products and the competing retailers compete with each other on the selling price of the remanufactured products. Under this setting, we investigate the effect of competing collectors' competition and cooperation behavior to find the most effective behavior. Second, a novel multi-link (the links between the remanufacturer and each of two competing collectors in the reverse channel, and the links between the remanufacturer and each of two competing retailers in the forward channel) TPT contract is proposed to improve the performance of the supply chain from the viewpoint of the channel individuals, consumers and the environment.

The rest of the study is countenanced as follows: The following section provides the review of related literature. Section 3 represents the notations and assumptions used in this paper. In Section 4, the proposed models are formulated under the centralized structure and two decentralized structures, and a comparison of their analytical results are provided. A multi-link two-part tariff contract is developed in Section 5. Section 6 deals with the numerical illustration followed by the sensitivity analysis. Section 7 concludes the article with some important managerial insights and future research directions. All proofs and additional symbols are given in Appendices. The framework of the study is portrayed in Figure 1.

<< **Insert Figure 1 about here** >>

2 Literature review

This section attempts to summarize the most pertinent and recent studies of CLSCs and to distinguish between the current study and previous studies. To this end, the literature review is provided in three parts viz. competition in forward and/or reverse channel, green innovation in the supply chain, and supply chain coordination.

2.1 *Competition in forward and/or reverse channel*

In today's competitive business, there are lots of competition in all parts (such as at the time of producing, selling, collecting used products) of the business. In this subsection, we'll review those literature which focus on the competition in forward and/or reverse channel. Savaskan and Van Wassenhove [12] investigated the interaction between the products' pricing decision in forward channel and manufacturer's reverse channel selection where the manufacturer may directly collect the used products or the retailer can collect the used products. Yang and Zhou [13] analyzed the pricing and quantity decisions in a supply chain which consists of a manufacturer and two competitive retailers under duopolistic retailers' different competitive behavior viz. Collusion, Cournot and Stackelberg game. David and Adida [14] studied competition and coordination issues in a two-channel supply chain where the supplier sells the product through the direct channel as well as through multiple retailers and claimed that the existence of a large number of retailers is preferable for the supplier. For channel coordination, they proposed a linear quantity discount contract and showed that it can soothe the inefficiency of the supply chain. Li et al. [15] established Stackelberg game models for analyzing the characteristics of two collection channels viz. formal and informal collection under three government mechanisms and demonstrated that appropriate governance mechanisms should be implemented for controlling informal collection. For the seek of profit maximization and social benefit, Li et al. [16] discussed the effect of coordination strategies in a three-echelon RSC consisting of a single collector, a single remanufacturer and two competing retailers. Giri et al. [17] proposed five models such as centralized, manufacturer-led decentralized, retailer-led decentralized, third party-led decentralized policies and Nash game, and analyzed the pricing and return product collection decisions in a dual-channel CLSC where in the forward channel the manufacturer sells the product through the traditional retail channel and online channel, and in the reverse channel, the manufacturer collects the used product through the e-tail channel and the third party collector.

Batarfi et al. [18] examined the impact of different return policies on supply chain system behavior before and after dual-channel adoption, and demonstrated that a more generous return policy provides higher supply chain profitability. Lan et al. [19] analyzed the competition and coordination in a three-tier supply chain under demand uncertainty in which the manufacturer sells the product to the retailer through two competing distributors. Through game-theoretic modeling approach, Nazari et al. [20] analyzed the optimal pricing and inventory policy in a CLSC with multiple manufacturers and multiple sales channels. In order to focus on collaborative product quality improvement, Chakraborty et al. [21] addressed cost-sharing contract in a supply chain comprising two competing manufacturers and a common retailer. By introducing the retail channel as an option for collecting used products together with the e-tail channel in a green CLSC, Mondal et al. [22] extended the work of Giri et al. [17]. Mondal and Giri [23] investigated the retailers' competition and cooperation in the forward channel of a green CLSC under government intervention and cap-and-trade policy. They found that cooperation is only profitable for the retailers but competition is beneficial for the manufacturer, consumers and the entire supply chain. Recently, Ranjbar et al. [24] assessed the optimal pricing and collection strategies with competitive dual recycling channels (through retailer and TPC) under various channel-leaderships, viz. manufacturer-led, retailer-led and TPC-led Stackelberg games. Unlike the existing literature, in this paper, we discuss the competition between two retailers in the forward channel and competition between two collectors in the reverse channel.

2.2 *Green innovation in the supply chain*

In recent years, due to environmental degradation and climate change, a large number of consumers have moved towards eco-friendly products and they are willing to pay more price for those products. At the same time, governments of developed countries and NGOs compel the manufacturer to produce green products. Since the market demand is proportional to the greening level of the product, to gain a competitive advantage, the manufacturer has no choice but to produce green products. A comprehensive review of the green supply chain is presented by Srivastava [25]. Swami and Shah [26] proposed a sustainable supply chain consisting of a single manufacturer and a single retailer in which both the manufacturer and the retailer exert effort in green production. Inspired by the companies that are doing significant work in reducing carbon footprint through product redesign, Ghosh and Shah [27] explored the effect of cost sharing contract through both bargaining and without bargaining in a supply chain where the manufacturer undertakes green initiatives. Zhang et al. [28] focused on the performance of a green supply chain in a dynamic environment where the unit production cost is collectively influenced by the cost of learning and the inefficiency of operations. Through the game-theoretical approach, Jamali and Rasti-Barzoki [29] explored the pricing strategies for a non-green product and its substitute green product under two dual-channel supply chains each of which includes a traditional retail channel and a direct online channel. They showed that increasing public awareness about green products is beneficial for the supply chain. Giri et al. [30] analyzed pricing, warranty period and greening level of a product in a two-echelon CLSC. The customers can return the defective products within the warranty period, a portion of which is refurbished and sent back to customers while the remaining portion is sold in the secondary market after remanufacturing and the same portion of the new product is given to the respective customers. Wei et al. [5] formulated three two-period supply chain models depending on the manufacturer's collection options of used products, and focused on the interaction between greening and remanufacturing strategies. Zhang and He [31] investigated the optimal policies for new and green remanufactured products with short-life-cycle under consideration of consumer behavior. Ma [32] considered a two-period green CLSC and compared various cooperation modes among manufacturer, retailer and collector for improving channel performance. Considering price, advertisement and uncertain demand, Khorshidvand et al. [33] proposed a hybrid modeling approach for multi-objective optimization in a green and sustainable CLSC. In another study, Khorshidvand et al. [34] focused on revenue management in a multi-level multi-channel supply chain. In this study, we investigate the effects of greening level on both the market supply and the market demand.

2.3 *Supply chain coordination*

Supply chain coordination together with appropriate decision strategies is one of the most well-organized approaches for achieving optimal supply chain profitability. Several contracts are used for coordinating supply chains and settling channel conflicts. For instance, revenue sharing contract (Giri et al. [30]), cost sharing contract (Ghosh and Shah [27], Jiaping et al. [35]), two-part tariff (TPT) contract (Swami and Shah [26], Hong et al. [36], Lipan et al. [37]), quantity discount contract (David and Adida [14], Heydari et al. [38]), etc. have been used as stimulant mechanisms for coordinating supply chain. According to Cachon [39], a specific set of operations needs to be executed for better performance of the supply chain, and it can be achieved if the firms are coordinated by contracts in such a way that the purposes of each firm are combined with the purpose of the whole supply chain. Hong et al. [36] investigated optimal pricing, advertising and collection decisions under the centralized and three decentralized models depending on the collection options of used products, and showed that a simple TPT contract can coordinate the supply chain. Zhang and Ren [40] investigated the effect of revenue-and-expense sharing, and TPT contract on the performance of a CLSC consisting of an original manufacturer, a third-party remanufacturer and a retailer. Zheng et al. [41] established a RSC model consisting of a collector and a remanufacturer for making pricing, collection effort and contract design decisions under complete and incomplete information scenarios, and showed that the incomplete information scenario may lead to efficiency loss. They also showed that the TPT contract can efficiently promote the channel performance. Lipan et al. [37] proposed a dual-recycling mode in a two-echelon RSC and investigated the competition between the recyclable dealer and the recycler. They considered a TPT contract and a profit sharing contract to coordinate the RSC and create a win-win situation. Jiaping et al. [35] considered a dual-channel CLSC in which the retailer provides service for returning used products in the offline channel. They combined revenue sharing and cost sharing contracts for improving the profitability of the supply chain, and encouraging the retailer to exert effort in recycling used products. Raj et al. [42] analyzed five different contracts viz. wholesale price, linear TPT, revenue sharing, greening-cost sharing, and revenue- and greening-cost sharing contracts and demonstrated the effects of these contracts on optimal decisions and profitability of the supply chain. Modak et a. [43] developed a socially responsible CLSC considering social donation as a CSR activity and collection of used products as environmental sustainability. They used the TPT contract for coordinating the supply chain and improving channel performance. Rahmani et al. [44] investigated the effect of channel coordination and horizontal cooperation in two different RSCs each of which consists of a remanufacturer, a retailer and a collector, and the remanufacturer puts effort into quality improvement of used products. Wang et al. [45] developed two CLSC models under revenue sharing and TPT contracts to explore the effect of consumer behavior on the performance of the supply chain. They showed that the TPT contract can coordinate the supply chain, while the revenue sharing contract cannot. Recently, Taleizadeh et al. [46] explored the effect of cost sharing contract on pricing strategy, quality improvement effort in a CLSC under green technologies investment and cap and trade policy. Contrary to the existing literature, in this research, a multi-link TPT contract is developed to coordinate the remanufacturer, two competing collectors and two competing retailers simultaneously.

2.4 *Research gaps and our contributions*

From the above literature review, we have the following observations: Firstly, several studies have been focused on competition in supply chain, but most of them are confined in either forward channel or reverse channel. Although, Giri et al. [17] and Mondal et al. [22] dealt with dual forward and dual reverse situations, but they didn't consider competition between the retailers in the forward logistics and competition between the collectors in the reverse logistics. This is the first attempt to investigate competition between the retailers while selling products and that between the collectors while collecting used products. Secondly, most of the previous literatures considered green innovation while producing products from fresh raw materials, and their effect on market demand. Although Rahmani et al. [44] assumed quality improvement of used products by the remanufacturer, they overlooked the effect of greening level in product return. To the best of our knowledge, the effect of greening on both the used products collection and the market demand has not yet been explored by the researchers. Finally, channel coordination being essential for both economic and environmental considerations, numerous efforts have been made on channel coordination but those are limited to two-level supply chains. Although Rahmani et al. [44] considered channel coordination in two different three-level supply chains, they ignored competitions between retailers (while selling) and between collectors (while collecting) in a supply chain. This research for the first time proposes channel coordination in a multi-level situation (remanufacturer, two competing retailers and two competing collectors).

3 Notations and assumptions

Notations:

w_i	unit wholesale price of the remanufacturer to the retailer i ($i = 1, 2$).
p_i	unit selling price of the retailer i ($i = 1, 2$).
a_i	unit acquisition price of the collector i ($i = 1, 2$).
m_i	unit transfer price paid by the remanufacturer to the collector i ($i = 1, 2$).
θ	level of green innovation.
D_{0i}	basic demand of the remanufactured product to the retailer i ($i = 1, 2$).

R_{0i}	basic supply of the used product to the collector i $(i = 1, 2)$.
D_i	demand of the remanufactured product to the retailer i $(i = 1, 2)$.
R_i	supply of the used product to the collector i $(i = 1, 2)$.
D	total market demand.
R	total supply quantity.
α	self-price sensitivity to market demand.
β	cross-price sensitivity to market demand.
γ	greening level sensitivity to market demand.
δ	self-acquisition price sensitivity to supply quantity.
η	cross-acquisition price sensitivity to supply quantity.
μ	greening level sensitivity to supply quantity.
c	unit remanufacturing cost of used products.
c_I	unit inspection cost of the used product.
τ	utilization ratio of used products, $0 \leq \tau \leq 1$.
λ	green innovation investment efficiency coefficient.
Π_{RM}	profit of the remanufacturer.
Π_{Ri}	profit of the retailer i $(i = 1, 2)$.
Π_{Ci}	profit of the collector i $(i = 1, 2)$.
Π	profit of the whole supply chain.

Assumptions:

1. The return quantity of used products is deterministic and linearly dependent on the acquisition prices and greening level of the product. It can be expressed as $R_i(a_i, a_j, \theta) = R_{0i} + \delta a_i - \eta a_j - \mu\theta; i = 1, 2$ and $j = 3 - i$, where δ, η and μ are self-acquisition price sensitivity, cross-acquisition price sensitivity and green level sensitivity to the return quantity, respectively. We assume that the effect of the self-acquisition price is greater than that of the cross-acquisition price *i.e.* $\delta > \eta$.

2. The remanufacturer only sells the remanufactured products. So, the total supply of used products must be greater than or equal to the total market demand *i.e.* $R \geq D$. It is also assumed that all the collected used products may not be suitable for remanufacturing. The remanufacturer inspects the collected used products and finds that a fraction (τ) of the collected used products qualifies for remanufacturing, where $0 \leq \tau \leq 1$. Therefore, $D = \tau R$.

3. The market demand is deterministic and linearly dependent on the selling prices and

greening level of the product. We take the market demand as $D_i(p_i, p_j, \theta) = D_{0i} - \alpha p_i + \beta p_j + \gamma\theta; i = 1, 2$ and $j = 3 - i$, where α, β and γ are self-price sensitivity, cross-price sensitivity and green sensitivity coefficient, respectively. Similar to the acquisition price, here also we assume that the effect of self-price is greater than that of the cross price *i.e.* $\alpha > \beta$.

4. The greening level investment cost does not affect the unit manufacturing cost. It is a one-time investment. As the greening level expands, it turns out to be progressively expensive. The greening level investment cost is considered as a quadratic function given by $\lambda\theta^2$ (Ghosh and Shah [27], Chen et al. [47]). This quadratic function not just captures the phenomenon of expanding marginal expense, yet additionally mirrors the fact that the investment cost of the underlying greening level per unit is diminishing.

4 Model formulation

We consider a closed-loop green supply chain consisting of a single remanufacturer, two competing collectors and two competing retailers. The two collectors collect the used products from consumers by paying acquisition prices a_1 per unit and a_2 per unit, respectively and transfer those to the remanufacturer in return for transfer prices m_1 per unit and m_2 per unit, respectively. After collection, the collected used products are inspected by the remanufacturer at c_I per unit. A fraction (τ) of the inspected products qualifies for remanufacturing, and the remanufacturing cost is c per unit. The remanufacturer sells the remanufactured products with a greening level θ to the two retailers at w_1 per unit and w_2 per unit, respectively. Then the retailers sell those products to the potential customers at p_1 per unit and p_2 per unit, respectively (see Figure 2).

<< **Insert Figure 2 about here** >>

The profit function for the remanufacturer is given by

$$\Pi_{RM}(w_1, w_2, m_1, m_2, \theta) = (w_1 - c)D_1 + (w_2 - c)D_2 - (m_1 + c_I)R_1 - (m_2 + c_I)R_2 - \lambda\theta^2 \quad (1)$$

Here, the first two terms indicate the profit obtained from selling the product to the retailers. Third and fourth terms denote the costs due to used products collection and inspection. The last term denotes the extra cost for producing green product.

The profit function for the retailer i and the collector i are given by

$$\begin{aligned} \Pi_{Ri}(p_i) &= (p_i - w_i)D_i,\ i = 1, 2. && (2) \\ \Pi_{Ci}(a_i) &= (m_i - a_i)R_i,\ i = 1, 2. && (3) \end{aligned}$$

We develop different models under centralized and decentralized policies. As we are interested in competition and cooperation of the collectors, in the decentralized policy, we study collectors' competitive and cooperative behaviors in a Stackelberg game model where the remanufacturer act as the Stackelberg leader and the retailers and the collectors act as the followers. While obtaining the optimal results under decentralized policies, we use backward induction method in which the downstream members (retailers and collectors) first decide their decisions simultaneously for given decisions of the upstream member (remanufacturer), and then the upstream member optimizes its own decisions.

4.1 Centralized policy (C-Model)

In this policy, the remanufacturer, the retailers and the collectors work jointly to optimize their decisions such as acquisition prices a_1 and a_2 , selling prices p_1 and p_2, and the greening level θ, through maximizing the total profit of the supply chain system. The internal transfer prices w_i and m_i (For the rest of the paper, we use $i = 1, 2$ and $j = 3 - i$.) do not play any role (see Figure 2(a)). The profit function for the centralized model is given by

$$\Pi^C(p_1, p_2, a_1, a_2, \theta) = (p_1 - c)D_1 + (p_2 - c)D_2 - (a_1 + c_I)R_1 - (a_2 + c_I)R_2 - \lambda\theta^2 \tag{4}$$

Hence, the C-Model can be presented as follows:

$$\begin{cases} \max\limits_{(p_1, p_2, a_1, a_2, \theta)} \Pi^C(p_1, p_2, a_1, a_2, \theta) \\ \text{subject to } D^C = \tau R^C \end{cases}$$

Using the first order conditions for optimality of $\Pi^C(p_1, p_2, a_1, a_2, \theta)$, the equilibrium solution for the centralized policy can be obtained as given in the following proposition:

Proposition 1. *If* $\lambda > \frac{(\alpha-\beta)\mu^2+(\delta-\eta)\gamma^2}{2(\alpha-\beta)(\delta-\eta)}$*, the centralized policy has the following unique solution*

$$p_i^{*C} = \frac{D_{0i}\Phi_4 + D_{0j}\Phi_5 - 2(\alpha+\beta)\big(\Phi_2 - 2c\tau(\delta-\eta)\big)\big(\lambda\tau(\alpha-\beta)(\delta-\eta) - \gamma\Phi_1\big)}{2(\alpha+\beta)\Psi_1}, \tag{5}$$

$$a_i^{*C} = \frac{-R_{0i}\Phi_6 - R_{0j}\Phi_7 - 2(\delta+\eta)\big(\lambda(\alpha-\beta)(\delta-\eta) + \mu\Phi_1\big)\big(2c(\alpha-\beta) - \tau\Phi_3\big)}{2(\delta+\eta)\Psi_1}, \tag{6}$$

$$\theta^{*C} = \frac{\Phi_1\big((\alpha-\beta)\Phi_2 + (\delta-\eta)\tau\Phi_3\big)}{\Psi_1}. \tag{7}$$

Proof. See Appendix A.

4.2 Remanufacturer-led decentralized policy

In this subsection, we consider different decentralized policies with the remanufacturer as the Stackelberg leader, and the retailers and the collectors as the followers. With a common remanufacturer, the collectors can work independently or jointly. Depending on this, in the next subsection, we consider two models – competition between the collectors and cooperation between the collectors.

4.2.1 Competition between the collectors (N-Model)

In this model, the collectors work independently and so there is competition between them. The remanufacturer as the Stackelberg leader first optimizes the wholesale prices, transfer prices and greening level of the product simultaneously knowing the decisions of the retailers and the collectors. Then the retailers and the collectors optimize their decisions such as selling prices and acquisition prices, respectively. Hence, the N-Model (see Figure 2(b)) can be presented as follows:

$$\begin{cases} \max\limits_{(w_1,w_2,m_1,m_2,\theta)} \Pi_{RM}^N(w_1,w_2,m_1,m_2,\theta,\tilde{a_i}(m_1,m_2,\theta),\tilde{p_i}(w_1,w_2,\theta)) \\ \text{subject to } D^N = \tau R^N \\ \text{where } \tilde{a_i}(m_1,m_2,\theta),\tilde{p_i}(w_1,w_2,\theta) \text{ are obtained from solving} \\ \begin{cases} \max\limits_{(a_i)} \Pi_{Ci}^N(a_i) \\ \max\limits_{(p_i)} \Pi_{Ri}^N(p_i) \end{cases} \end{cases}$$

At first, we determine the best responses of the collectors and the retailers by solving the first order necessary conditions for optimality of the objective functions of the collectors and retailers simultaneously. As $\frac{\partial^2\Pi_{Ci}^N}{\partial a_i^2} = -2\delta < 0$ and $\frac{\partial^2\Pi_{Ri}^N}{\partial p_i^2} = -2\alpha < 0$, there exists unique response for the collectors and the retailers which are given by

$$\tilde{a_i}^N(m_i,m_j,\theta) = \frac{2\delta(-R_{0i}+\delta m_i+\mu\theta) + \eta(-R_{0j}+\delta m_j+\mu\theta)}{4\delta^2-\eta^2}, \tag{8}$$

$$\tilde{p_i}^N(w_i,w_j,\theta) = \frac{2\alpha(D_{0i}+\alpha w_i+\gamma\theta) + \beta(D_{0j}+\alpha w_j+\gamma\theta)}{4\alpha^2-\beta^2}. \tag{9}$$

With these reactions of the collectors and retailers, the remanufacturer will optimize its decisions. The equilibrium solutions can be obtained as given in the following proposition:

Proposition 2. *If* $\lambda > \frac{(\alpha-\beta)(2\alpha-\beta)\delta\mu^2+(\delta-\eta)(2\delta-\eta)\alpha\gamma^2}{2(\alpha-\beta)(2\alpha-\beta)(\delta-\eta)(2\delta-\eta)}$ *then, at the equilibrium, the decisions of the remanufacturer, the retailers and the collectors in the N-Model are given respectively by*

$$\begin{aligned}
\theta^{*N} &= \frac{\alpha\delta\Phi_1\big((\alpha-\beta)\Phi_2+(\delta-\eta)\tau\Phi_3\big)}{\Psi_2}, && (10)\\
w_i^{*N} &= \frac{D_{0i}\alpha\Xi_1+D_{0j}\Xi_2-2\delta(\alpha+\beta)\big(\Phi_2-2c\tau(\delta-\eta)\big)\big(\lambda\tau(\alpha-\beta)(2\alpha-\beta)(\delta-\eta)-\alpha\gamma\Phi_1\big)}{2(\alpha+\beta)\Psi_2}, && (11)\\
m_i^{*N} &= \frac{-R_{0i}\delta\Xi_3-R_{0j}\Xi_4-2\alpha(\delta+\eta)\big(\lambda(\alpha-\beta)(\delta-\eta)(2\delta-\eta)+\delta\mu\Phi_1\big)\big(2c(\alpha-\beta)-\tau\Phi_3\big)}{2(\delta+\eta)\Psi_2}, && (12)\\
p_i^{*N} &= \frac{D_{0i}\alpha\Xi_5+D_{0j}\Xi_6-2\alpha\delta(\alpha+\beta)(2\alpha+\beta)\big(\Phi_2-2c\tau(\delta-\eta)\big)\big(\lambda\tau(\alpha-\beta)(\delta-\eta)-\gamma\Phi_1\big)}{2(\alpha+\beta)(2\alpha+\beta)\Psi_2}, && (13)\\
a_i^{*N} &= \frac{-R_{0i}\delta\Xi_7-R_{0j}\Xi_8-2\alpha\delta(\delta+\eta)(2\delta+\eta)\big(\lambda(\alpha-\beta)(\delta-\eta)+\mu\Phi_1\big)\big(2c(\alpha-\beta)-\tau\Phi_3\big)}{2(\delta+\eta)(2\delta+\eta)\Psi_2}. && (14)
\end{aligned}$$

Proof. See Appendix B.

4.2.2 Cooperation between the collectors (J-Model)

In this model, the two collectors work jointly to optimize their decisions and maximize their total profit. Similar to the N-Model, here also the remanufacturer as the Stackelberg leader first optimizes the wholesale prices, the transfer prices and the greening level of the product simultaneously knowing the decisions of the retailers and the collectors. Then the retailers and the collectors optimize their decisions such as selling prices and acquisition prices, respectively. The profit functions of the retailers, the collectors and the remanufacturer are given by

$$\begin{aligned}
\Pi_{Ri}^{J}(p_i) &= (p_i-w_i)D_i; && (15)\\
\Pi_{C}^{J}(a_1,a_2) &= \sum_{i=1}^{2}(m_i-a_i)R_i; && (16)\\
\Pi_{RM}^{J}(w_1,w_2,m_1,m_2,\theta) &= (w_1-c)D_1+(w_2-c)D_2-(m_1+c_I)R_1-(m_2+c_I)R_2-\lambda\theta^2. && (17)
\end{aligned}$$

Hence, the J-Model (see Figure 2(c)) can be presented as follows:

$$\begin{cases}
\max\limits_{(w_1,w_2,m_1,m_2,\theta)} \Pi_{RM}^{J}(w_1,w_2,m_1,m_2,\theta,\tilde{a_i}(m_1,m_2,\theta),\tilde{p_i}(w_1,w_2,\theta))\\
\text{subject to } D^J=\tau R^J\\
\text{where } \tilde{a_i}(m_1,m_2,\theta),\tilde{p_i}(w_1,w_2,\theta) \text{ are obtained from solving}\\
\begin{cases}
\max\limits_{(a_1,a_2)} \Pi_{C}^{J}(a_1,a_2)\\
\max\limits_{(p_i)} \Pi_{Ri}^{J}(p_i)
\end{cases}
\end{cases}$$

At first, we determine the best responses of the collectors and the retailers by solving the first order necessary conditions for optimality of the objective functions of the collectors and retailers simultaneously and those responses are given by

$$\begin{aligned} \tilde{a_i}^J(m_i, m_j, \theta) &= \frac{(\delta+\eta)\big(m_i(\delta-\eta)+\mu\theta\big)-(R_{0i}\delta+R_{0j}\eta)}{2(\delta^2-\eta^2)}, && (18)\\ \tilde{p_i}^J(w_i, w_j, \theta) &= \frac{2\alpha(D_{0i}+\alpha w_i+\gamma\theta)+\beta(D_{0j}+\alpha w_j+\gamma\theta))}{4\alpha^2-\beta^2}. && (19) \end{aligned}$$

With these reactions of the collectors and the retailers, the remanufacturer will optimize its decisions. The equilibrium solution can be obtained as given in the following proposition:

Proposition 3. *If* $\lambda > \frac{(\alpha-\beta)(2\alpha-\beta)\mu^2+2(\delta-\eta)\alpha\gamma^2}{4(\alpha-\beta)(2\alpha-\beta)(\delta-\eta)}$ *then, at the equilibrium, the decisions of the remanufacturer, the retailers and the collectors in the J-Model are given respectively by*

$$\begin{aligned} \theta^{*J} &= \frac{\alpha\Phi_1\big((\alpha-\beta)\Phi_2+(\delta-\eta)\tau\Phi_3\big)}{\Psi_3}, && (20)\\ w_i^{*J} &= \frac{D_{0i}\alpha\Delta_1+D_{0j}\Delta_2-2(\alpha+\beta)\big(\Phi_2-2c\tau(\delta-\eta)\big)\big(\lambda\tau(\alpha-\beta)(2\alpha-\beta)(\delta-\eta)-\alpha\gamma\Phi_1\big)}{2(\alpha+\beta)\Psi_3}, && (21)\\ m_i^{*J} &= \frac{-R_{0i}\Delta_3-R_{0j}\Delta_4-2\alpha(\delta+\eta)\big(2\lambda(\alpha-\beta)(\delta-\eta)+\mu\Phi_1\big)\big(2c(\alpha-\beta)-\tau\Phi_3\big)}{2(\delta+\eta)\Psi_3}, && (22)\\ p_i^{*J} &= \frac{D_{0i}\alpha\Delta_5+D_{0j}\Delta_6-2\alpha(\alpha+\beta)(2\alpha+\beta)\big(\Phi_2-2c\tau(\delta-\eta)\big)\big(\lambda\tau(\alpha-\beta)(\delta-\eta)-\gamma\Phi_1\big)}{2(\alpha+\beta)(2\alpha+\beta)\Psi_3}, && (23)\\ a_i^{*J} &= \frac{-R_{0i}\Delta_7-R_{0j}\Delta_8-4\alpha(\delta+\eta)\big(\lambda(\alpha-\beta)(\delta-\eta)+\mu\Phi_1\big)\big(2c(\alpha-\beta)-\tau\Phi_3\big)}{4(\delta+\eta)\Psi_3}. && (24) \end{aligned}$$

Proof. The proof is omitted as it is similar to that of Proposition 2.

Proposition 4. *At the equilibrium, the greening levels of the N-Model and the J-Model follow the relationship* $\theta^N > \theta^J$, *and if* $\lambda > max\{\frac{\gamma\Phi_1}{\tau(\alpha-\beta)(\delta-\eta)}, \frac{\alpha[\gamma^2\tau^2(\delta-\eta)^2-\mu^2(\alpha-\beta)^2]}{2\tau^2(\alpha-\beta)(2\alpha-\beta)(\delta-\eta)^2}\}$ *then the wholesale prices, selling prices, transfer prices and acquisition prices follow the relationships:* $w_i^J > w_i^N$, $p_i^J > p_i^N$, $m_i^J > m_i^N$ *and* $a_i^N > a_i^J$.

Proof. See Appendix C.

Proposition 4 shows that when the collectors collect the used products independently and simultaneously, the remanufacturer produces products with higher greening level and sells those products to the retailer with lower wholesale price whenever λ exceeds a threshold value. As a result, the retailer also sells those products to potential customers at lower selling price. While collecting used products from customers, the collectors pay a higher acquisition price to collect more used products when they work independently but in this case, the remanufacturer pays a lower transfer price. Higher acquisition prices and lower transfer prices produce a loss to the collectors. So, the collectors prefer to work cooperatively to mitigate profit loss. But their joint decisions again produce a loss to the remanufacturer and the retailers. So, the remanufacturer and the retailers want the collectors to work independently. In the next section, we present a multi-link TPT contract to solve this problem.

5 Multi-link two-part tariff contract (T-Model)

The multi-link two-part tariff contract is a new approach to coordinate a supply chain consisting of more than two members. Similar to the traditional TPT contract, this contract also helps to increase the profit of all the channel members compared to other decentralized models. Under the proposed contract, the remanufacturer guarantees to sell the products with lower wholesale price w_i^T to the retailers and at the same time increases the transfer price m_i^T for collecting used products. In return, he charges fixed payments (*i.e.* franchise fees) F_{Ri} and F_{Ci} from the retailer i and collector i, respectively. As the N-Model gives the best performance, we consider the proposed contract for that model. Under the proposed contract, the profit functions of the remanufacturer, the retailers and the collectors are given by

$$\Pi^T_{RM}(w_1, w_2, m_1, m_2, \theta) = (w_1^T - c)D_1^T + (w_2^T - c)D_2^T - (m_1^T + c_I)R_1^T - (m_2^T + c_I)R_2^T - \lambda\theta^2 + F_{RM}; \tag{25}$$

$$\Pi^T_{Ri}(p_i) = (p_i^T - w_i^T)D_i^T - F_{Ri}, \tag{26}$$

$$\Pi^T_{Ci}(a_i) = (m_i^T - a_i^T)R_i^T - F_{Ci}, \tag{27}$$

$$\text{where } F_{RM} = F_{R1} + F_{R2} + F_{C1} + F_{C2}$$

Similar to the N-Model, here also we first determine the best responses of the collectors and the retailers by solving the first order necessary conditions for optimality of the objective functions of the collectors and retailers simultaneously and the responses are obtained as

$$a_i^T(m_i, m_j, \theta) = \frac{2\delta(-R_{0i} + \delta m_i + \mu\theta) + \eta(-R_{0j} + \delta m_j + \mu\theta)}{4\delta^2 - \eta^2}, \tag{28}$$

$$p_i^T(w_i, w_j, \theta) = \frac{2\alpha(D_{0i} + \alpha w_i + \gamma\theta) + \beta(D_{0j} + \alpha w_j + \gamma\theta))}{4\alpha^2 - \beta^2}. \tag{29}$$

For channel coordination, the above acquisition prices and transfer prices will be equal to those of the centralized policy. So, equating $a_i^T = a_i^C$ and $p_i^T = p_i^C$, we get

$$w_i^T = \frac{D_{0i}\Gamma_1 + D_{0j}\Gamma_2 - 2(\alpha+\beta)\big(\Phi_2 - 2c\tau(\delta-\eta)\big)\big(\lambda\tau(\alpha-\beta)(2\alpha-\beta)(\delta-\eta) - \gamma\Phi_1\big)}{2\alpha(\alpha+\beta)\Psi_1}, \tag{30}$$

$$m_i^T = \frac{-R_{0i}\Gamma_3 - R_{0j}\Gamma_4 - 2(\delta+\eta)\big(\lambda(\alpha-\beta)(\delta-\eta)(2\delta-\eta) + \delta\mu\Phi_1\big)\big(2c(\alpha-\beta) - \tau\Phi_3\big)}{2\delta(\delta+\eta)\Psi_1}. \tag{31}$$

$$\theta^T = \theta^C. \tag{32}$$

Putting these values in Equations (25), (26) and (27), we can get the respective profits of the remanufacturer, retailers and collectors which are given by

$$\begin{aligned}
\Pi^T_{RM} &= (w^T_1 - c)D^C_1 + (w^T_2 - c)D^C_2 - (m^T_1 + c_I)R^C_1 - (m^T_2 + c_I)R^C_2 - \lambda(\theta^C)^2 + F_{RM}; && (33)\\
\Pi^T_{Ri} &= (p^C_i - w^T_i)D^C_i - F_{Ri}, && (34)\\
\Pi^T_{Ci} &= (m^T_i - a^C_i)R^C_i - F_{Ci}, && (35)\\
\text{where } F_{RM} &= F_{R1} + F_{R2} + F_{C1} + F_{C2}.
\end{aligned}$$

From the above one can note that, $\Pi^T = \Pi^T_{RM} + \sum_{i=1}^2(\Pi^T_{Ri} + \Pi^T_{Ci}) = \Pi^C$ *i.e.* sum of the profits of the remanufacturer, the retailers and the collectors is equal to that of centralized policy. Therefore, the proposed multi-link TPT contract coordinates the supply chain.

The supply chain members accept a contract only when their individual profits after the contract are higher than or equal to those before the contract. As the proposed contract coordinates the supply chain, the remanufacturer, the retailers and the collectors will accept the contract if $\Pi^T_{RM} \geq \Pi^N_{RM}$, $\Pi^T_{Ri} \geq \Pi^N_{Ri}$ and $\Pi^T_{Ci} \geq \Pi^N_{Ci}$.

Proposition 5. *A win-win situation under the proposed contract is permissible if the franchise fees follow the relations:*

$$\begin{aligned}
F_{RM} &\geq F^{min}_{RM};\ F_{R1} \leq F^{max}_{R1};\ F_{R2} \leq F^{max}_{R2};\ F_{C1} \leq F^{max}_{C1};\ F_{C2} \leq F^{max}_{C2}.\\
\text{where } F^{min}_{RM} &= (w^N_1 - c)D^N_1 + (w^N_2 - c)D^N_2 - (m^N_1 + c_I)R^N_1 - (m^N_2 + c_I)R^N_2 - \lambda(\theta^N)^2\\
&\quad -(w^T_1 - c)D^C_1 - (w^T_2 - c)D^C_2 + (m^T_1 + c_I)R^C_1 + (m^T_2 + c_I)R^C_2 + \lambda(\theta^C)^2\\
F^{max}_{R1} &= (p^C_1 - w^T_1)D^C_1 - (p^N_1 - w^N_1)D^N_1;\\
F^{max}_{R2} &= (p^C_2 - w^T_2)D^C_2 - (p^N_2 - w^N_2)D^N_2;\\
F^{max}_{C1} &= (m^T_1 - a^C_1)R^C_1 - (m^N_1 - a^N_1)R^N_1;\\
F^{max}_{C2} &= (m^T_2 - a^C_2)R^C_2 - (m^N_2 - a^N_2)R^N_2.
\end{aligned}$$

Proof. See Appendix D.

Proposition 5 gives the maximum limits of franchise fee which the retailers and collectors are willing to pay for achieving win-win outcome. Now, from $F_{RM} = F_{R1} + F_{R2} + F_{C1} + F_{C2}$, we get $F^{min}_{RM} \leq F_{RM} = F_{R1} + F_{R2} + F_{C1} + F_{C2} \leq F^{max}_{R1} + F^{max}_{R2} + F^{max}_{C1} + F^{max}_{C2} (= F^{max}_{RM}, \text{say})$. Therefore, to achieve win-win outcome for all the channel members, the franchise fee (F_{RM}) charged by the remanufacturer from the retailers and collectors should be such that $F_{RM} \in [F^{min}_{RM}, F^{max}_{RM}]$. After coordination, the supply chain generates some amount of surplus profit $\triangle_{sp} = \Pi^C - \Pi^N$, which the channel members can share according to their bargaining power. By doing so, the collectors and the retailers manage to reduce the maximum limit of franchise fee.

6 Numerical illustration and sensitivity analysis

To validate the proposed models numerically, in this section, we employ three numerical examples to calculate the optimal decisions and profitability of the channel members. In Example 1, we consider hypothetical parameter-values that are consistent with parameter-values used in the literature like Rahmani et al. [44], with some legitimate changes following the assumptions and optimality conditions of our study. We check all the conditions which are necessary for the existence and uniqueness of the optimal solution. For instance, for the N-Model with parameter-values given in Example 1, the principle minors associated with the remanufacturer's objective function are $|M_1| = -1.50 < 0$, $|M_2| = 2.23 > 0$, $|M_3| = -7.96 < 0$, $|M_4| = 24.13 > 0$, and $|H^N| = -0.13 < 0$. It clearly shows that the corresponding Hessian matrix is negative definite for the chosen data set. So, there exists a unique optimal solution for the remanufacturer's problem. We use MATHEMATICA 9.0 software for obtaining the optimal results numerically following the analytical results derived for the proposed models.

Example 1. In this example, we consider a supply chain which deals with disposable cameras. The remanufacturer in this case only remanufactures the used cameras. He contracts with two collectors for collecting used cameras. While remanufacturing, he focuses on the greening of the products such as doing away with PVC, doing away with toxic flame retardants, using biodegradable plastic, lowering energy consumption, etc. The initial supplies for the used cameras to the two collectors are $R_{01} = 15$ cameras/day and $R_{02} = 12$ cameras/day, respectively. The collectors collect those used cameras from the customers by paying acquisition prices. The self-acquisition price sensitivity to the market supply is $\delta = 4$ and cross-acquisition price sensitivity is $\eta = 2.5$. The supply of used products is also affected by the greening level of the product with a sensitivity factor $\mu = 0.65$. After collection, the remanufacturer inspects the used cameras with a cost of $c_I = \$0.7$/camera and $\tau = 70\%$ of the collected cameras can be available for remanufacturing. The remanufacturer then remanufactures the remanufacturable cameras with a remanufacturing cost of $c = \$2$/camera, and sells these products to the potential customers through two competing retailers. The basic market demand for the retailers are $D_{01} = 100$ cameras/day and $D_{02} = 80$ cameras/day, respectively. The market demand is affected by the selling prices and the greening level of the products where the self-price sensitivity, the cross-price sensitivity and the green level sensitivity parameters are $\alpha = 1.5$, $\beta = 0.15$, $\gamma = 1.0$, respectively. The green investment efficiency cost coefficient for the remanufacturer is $\lambda = 0.3$. The parameter-values for the above supply chain problem are summarised in Table 1.

<< **Insert Tables 1-2 about here** >>

The optimal results of the proposed models are displayed in Table 2 which shows that collectors' competitiveness behavior has a significant impact on the optimal decisions and profitability of the channel members. When the collectors collect the used products independently, they offer higher acquisition prices to collect more used products. The remanufacturer enjoys the competition between the collectors. He gives a comparatively lower transfer price than the situation when the collectors work jointly. As the customers are showing interest in returning used products, the remanufacturer also produces higher green products and sells those to retailers with lower wholesale prices. As a result, the retailers also sell those products to customers with lower selling prices. Due to lower selling price and higher green product, the market demand increases and the profits of the remanufacturer and the retailers become higher. But the profit of the collectors becomes lower as they pay higher acquisition price and in return get a lower transfer price. So, the collectors always want to work jointly to maximize their profits. But cooperation between the collectors produces a loss to both the remanufacturer and the retailers. So, they force the collectors to work independently. While comparing these decentralized models with the centralized model, we note that the centralized model is more environment-friendly (as it produces higher green products) and preferable for the customers (as they can get higher acquisition price while returning used products and they have to pay lower selling price while purchasing these products). Due to higher green products and lower selling prices, the total profit of the supply chain is higher in the centralized model.

Under the proposed multi-link TPT contract, the remanufacturer offers the marginal prices \$18.44/camera and \$18.29/camera to the collectors, which are more than double compared to the N-Model, and sells the product to the retailers at wholesale prices \$45.97/camera and \$46.57/camera which are respectively 19% and 8% lower than the wholesale price in the N-Model. The maximum limits of franchise fee of the collectors and the retailers are \$110.24, \$95.29, \$232.51 and \$61.51, respectively, and the minimum limit of franchise fee of the remanufacturer is \$306.68. Therefore, the bargaining range of franchise fees for the remanufacturer to achieve a win-win outcome for all the channel members is [306.68, 499.55]. The surplus profit obtained after coordination is \$192.88 which is divided among the channel members according to their bargaining powers. By doing so, the collectors and the retailers manage to reduce their maximum limits of franchise fees. Assuming the bargaining powers of the remanufacturer, each retailer and each collector as 30%, 20% and 15%, respectively, profits of the remanufacturer, the retailers and the collectors after bargaining are obtained as \$972.31, \$171.89, \$97.22, \$101.92 and \$97.12, respectively. Therefore, the remanufacturer can enhance its profit by more than 21% compared to the N-Model by applying the proposed multi-link TPT contract.

Example 2. In this example, we consider a local remanufacturer remanufacturing printer toner cartridges and exerting efforts in greening (lowering energy consumption, reducing the dependency on petroleum that would have been used in the manufacturing of new cartridges, etc.) these products. The data collected from a reliable source by the survey of the Kolkata Municipal daily market, Kolkata, West Bengal, India show that the initial supplies for the used cartridges to the collectors are $R_{01} = 12$ cartridges/day and $R_{02} = 10$ cartridges/day, respectively, and the basic market demand for the retailers are $D_{01} = 80$ cartridges/day and $D_{02} = 70$ cartridges/day, respectively. The inspection cost and remanufacturing cost are $c_I =$ Rs. 15/cartridge and $c =$ Rs. 100/cartridge. It is noted that the remanufacturer can remanufacture $\tau = 90\%$ of the collected cartridges. Here also we consider hypothetical parameter-values following the assumptions and optimality conditions of our study and those parameters are presented in Table 1.

The optimal results for different models are presented in Table 2. When the collectors go for joint decisions, their acquisition prices are lower and they get higher transfer prices from the remanufacturer. Higher transfer prices force the remanufacturer to set higher wholesale prices which again force the retailers to sell the products at higher selling prices. As a result, the joint decision is only profitable for the collectors. Since the remanufacturer is the leader of the supply chain, he signs a contract with the retailers and the collectors for improving his profit. Under the proposed contract, the remanufacturer sells the product at comparatively lower wholesale prices to the retailers and offers higher transfer prices to the collectors. The maximum limits of franchise fees for the retailers are Rs. 1762.57 and Rs. 969.93, and for the collectors are Rs. 872.28 and Rs. 788.60 for achieving a win-win situation. The bargaining range of franchise fees for the remanufacturer to achieve a win-win outcome for all the channel members is [2533.60, 4393.38]. If the surplus profit (Rs. 1859.90) obtained through coordination is divided among the channel members according to their bargaining power (50% for remanufacturer, 15% for each retailer and 10% for each collector), profits of the remanufacturer, the retailers and the collectors after bargaining will be Rs. 20949.10, Rs. 2046.0, Rs. 1750.44, Rs. 1202.12 and Rs. 1164.89, respectively.

Example 3. This is an independent example considered for checking the validity of the results of the above two examples. The parameter-values for this example are also given in Table 1. It is observed that the optimal results of different models for this example validate the results of the above two examples and follow the same trend. So, we present the optimal results in Table 2 without explaining more details.

<< **Insert Table 3 about here** >>

Table 3 presents the optimal results for the proposed models when the remanufacturer remanufactures all the collected used cameras. As the remanufacturer can remanufacture all the collected used cameras, without any hesitation he can improve the greening level of the product and offer higher transfer prices while collecting used cameras from the collectors. The collectors also pay higher acquisition prices to the customers. Due to the higher greening level of the product, they may demand higher prices. Higher acquisition prices influence the customers to return more used cameras whereas a higher greening level of the remanufactured cameras attracts the attention of the customers. As a result, the profits of all the channel members as well as the whole supply chain increase. So, at the time of collection of used cameras, the collectors expect that the remanufacturer would remanufacture all the used cameras at a comparatively lower cost.

<< **Insert Figures 3-6 about here** >>

Now, we perform the sensitivity analysis of some parameters by considering the parameter-values of Example 1.

Effect of λ – It is obvious that if the green improvement related cost increases, the producers have to lower the greening level of the product in order the maintain their profitability, which is reflected in Figure 3(a). It shows that the greening level decreases in all three models. As the coordinated model gives a higher green product, the effect of the green improvement cost coefficient (λ) on that model is slightly higher than the other two models. Due to the lower greening level of the product, the remanufacturer has to sell the product at a lower wholesale price. As a result, the retailers also sell it to potential customers with lower selling prices. Besides selling the remanufactured product at lower wholesale prices, the remanufacturer pays lower transfer prices to the collectors while collecting used products. So, the collectors have to pay lower acquisition prices to the customers to collect used products from them. Due to lower acquisition prices, the return quantity decreases, and due to lower green product, the market demand also decreases. As a result, the profits of all the channel members and the whole supply chain decrease (see Figure 3(b)). So, in order to encourage the remanufacturer, the government should provide some amount of subsidies.

Effect of η – Figure 4 represents the effects of competition parameter η on the greening level, the return quantity, the market demand and the total profit of the supply chain while collecting used products from the potential customers. Figure 4(a) shows that the greening level of the product decreases in all three models when η increases. It also shows that the coordinated model always provides higher green products while the joint decision of the two collectors forces the remanufacturer to produce lower green products. It further indicates that, for lower values of η, the N-Model and the J-Model provide almost the same level of green product. Similar to the greening level of the product, the return quantity and the market demand of the product also follow the same pattern with η, which are exhibited in figures 4(b) and 4(c). As both the return quantity and the market demand decrease, profits of all the channel members, as well as the whole supply chain also decrease with η (see Figure 4(d)). From Figure 4, we can note that, to achieve more improvement in the greening level of the remanufactured product, and the profitability of the channel members, it is better to reduce the competition between the collectors. Further, the proposed multi-link TPT contract has a great advantage in increasing the greening level of the remanufactured product as well as the profitability of the channel members. Therefore, the supply chain members should make decisions under the proposed contract to increase the greening level of the remanufactured product and their profitability.

Effect of β – Figure 5 represents the effects of competition parameter β on the greening level, the return quantity, the market demand and the total profit of the supply chain while selling remanufactured products in the potential market. Figure 5(a) shows that the greening level of the product increases in all the three models when β increases. It also shows that coordinated model always provides higher green product while joint decision of the two collectors forces the remanufacturer to produce lower green products. Similar to the greening level of the product, the return quantity and the market demand of the product also follow the same pattern with β, which are exhibited in figures 5(b) and 5(c). As both the return quantity and the market demand increase, profits of all the channel members as well as the whole supply chain also increase with β (see Figure 5(d)). Therefore, from Figure 5, it is clear that, in order to achieve more improvement in greening level of the remanufactured product, and the profitability of the whole supply chain, the retailers should be highly competitive.

Effect of lump-sum fees – Figure 6(a) shows the effect of lump-sum fees on the profit of the retailers and the collectors under N-Model and the proposed contract, whereas Figure 6(b) shows the effect of lump-sum fees on the remanufacturer's profit. As the lump-sum fees of the retailers and the collectors increase, profits of the retailers and the collectors decrease but the profit of the remanufacturer increases. We take different sets of values of F_{Ri} and F_{Ci} and investigate those values of lump-sum fee for which a win-win situation for all the channel members exists. From Figure 6(a), we note that, for the set of values ($F_{R1} = 140, F_{R2} = 30, F_{C1} = 60, F_{C2} = 70$), (160, 40, 70, 80), (180, 50, 80, 90) and (215, 55, 85, 95), the retailers and the collectors achieve win-win outcome; for the set of values (235, 60, 105, 100), the retailer 2 and the collector 1 can achieve win-win outcome whereas the profits of the retailer 1 and the collector 2 under the proposed contract become lower than those of N-Model *i.e.* they fail to achieve a win-win outcome. For the set of values (255, 70, 115, 110), profits of all the channel members under the proposed contract become lower than those of N-Model. So, none of them achieve a win-win outcome. From Figure 6(b), we note that, for the first set of values, the remanufacturer fails to achieve the win-win outcome. For the set of values (160, 40, 70, 80), (180, 50, 80, 90) and (215, 55, 85, 95) all the channel members can achieve a win-win outcome. So, for these set of values, the proposed contract is helpful for all the channel members.

7 Conclusions

The growing interest in product remanufacturing and today's competitive business scenario lead to developing the present closed-loop green supply chain model with a single remanufacturer, two competing retailers and two competing collectors for dealing with remanufactured products. The collectors collect the used products from customers and send those to the remanufacturer for remanufacturing where the market supply is assumed to be changed with the acquisition prices and the greening level of the product. The remanufacturer first inspects the collected used products and differentiates the remanufacturable and the non-remanufacturable items. The remanufacturable items are remanufactured by the remanufacturer while the non-remanufacturable items are disposed of. The remanufacturer then sells the remanufactured products to the potential customers through the competing retailers where the market demand is assumed to be dependent on the selling prices and the greening level of the product. We have studied the centralized policy and two decentralized policies depending on the competition and the cooperation of the collectors and considering the remanufacturer as the Stackelberg leader. It is observed that the profit of the centralized policy is much higher than that of a decentralized policy. The cooperation between the collectors provides more profit to themselves whereas the competition between them provides higher profit to the other channel members as well as improves the greening level of the product. To overcome this problem, a novel coordination contract (multi-link TPT contract) is proposed. Besides coordinating the supply chain, the proposed contract also provides a win-win outcome to all the channel members and improves the greening level of the product.

The present study reveals some important insights. Firstly, the utilization ratio of the used products has a noteworthy impact on the optimal decisions as well as the profitability of the channel members. That is, the quantity of ingredients expelled from the used products is one of the key factors in managing the reverse supply chain. The collectors should maintain some criteria while collecting used products and allow those products which satisfy a certain quality level so that the remanufacturer can remanufacture most of the collected products. Secondly, the consumers' environmental awareness can influence the performance of the supply chain as the socially responsible consumers force the remanufacturer to produce higher green products and at the same time compel them to collect more used products. These findings clearly indicate that it may be helpful for the supply chain to have more socially responsible consumers. Thirdly, in any remanufacturing system, the supply market plays an important role. Our results indicate that the proposed multi-link TPT contract can increase the collection quantity of used products significantly compared to the decentralized scenario. This outcome is consistent with Rahmani et al. [44]. Fourthly, the competition between the collectors for collecting used products has a significant effect on the performance of the supply chain. One may expect that the cooperative behavior of the collectors may increase the collection quantity. But our results demonstrate that cooperative behavior is profitable only for the competing members while their competitiveness is beneficial for other supply chain members and the customers. Therefore, to enjoy the competitive advantage of the collectors, the environment-friendly customers and the remanufacturer should force the collectors to work independently not cooperatively. Rahmani et al. [44] didn't consider any competition in a single supply chain. So, we think our study can provide better outcomes than the above-mentioned literature. Fifthly, similar to the collectors, the competitive behavior of the retailers in the forward channel while selling the remanufactured product can also influence the performance of the supply chain significantly. Our results indicate that the performance of the supply chain tends to improve with the increase of retailers' competitiveness. This outcome is consistent with Yang and Zhou [13]. Sixthly, if the green investment-related cost becomes higher, then the remanufacturer may shift himself from green improvement to the collection of more used products for improving environmental sustainability. But our results show that if the supply chain manager follows the proposed contract, then the remanufacturer can enhance the greening level of the product as well as improve the collection of used products without worrying about the increase of the green investment-related cost. Finally, although the performance of the supply chain is better when both the retailers and the collectors work independently, it fails to compete with the centralized policy, which strengthens the importance of channel coordination. The proposed multi-link TPT contract can solve this problem by coordinating multiple links viz. the links between the remanufacturer and each of two competing collectors in the reverse channel, and the links between the remanufacturer and each of two competing retailers in the forward channel.

Although the current study provides significant insights into the field of supply chain management, limitations due to prevailing assumptions of the study can be taken for future research. This study is based on both deterministic demand and return. It can be extended by considering probabilistic demand which will be more realistic. We have assumed only two retailers and two collectors. Consideration of multiple retailers and multiple collectors will be an interesting extension. We have considered the supply chain for the remanufactured products only. Consideration of new and remanufactured products will be another extension. One can examine the effect of other power structures (retailer-led Stackelberg, collector-led Stackelberg, vertical Nash) and other coordination contracts (revenue sharing, cost sharing, profit sharing, etc.) to extend the present study. Government activities such as subsidy, cap-and-trade policy, etc. can also be taken into consideration in our proposed model for future study.

Conflict of interest

The authors declare that they have no conflict of interest.

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Appendices

According to our assumptions, the profit functions of the remanufacturer, retailers and collectors are given as follows:

$$\Pi_{RM}(w_1, w_2, m_1, m_2, \theta) = (w_1 - c)D_1 + (w_2 - c)D_2 - (m_1 + c_I)R_1 - (m_2 + c_I)R_2 - \lambda\theta^2; \tag{36}$$

$$\Pi_{Ri}(p_i) = (p_i - w_i)D_i; \tag{37}$$

$$\Pi_{Ci}(a_i) = (m_i - a_i)R_i. \tag{38}$$

Appendix A

Proof of Proposition 1

The centralized model is

$$\max_{(p_1,p_2,a_1,a_2,\theta)} \Pi^C(p_1, p_2, a_1, a_2, \theta) = (p_1 - c)D_1 + (p_2 - c)D_2 - (a_1 + c_I)R_1 - (a_2 + c_I)R_2 - \lambda\theta^2$$

subject to $D^C = \tau R^C$

So, we consider the Lagrangian function as

$$\Pi^C_L = \Pi^C + L(\tau R^C - D^C), \tag{39}$$

where $L(\geq 0)$ is the Lagrangian multiplier.

The corresponding Hessian matrix is given by

$$H^C = \begin{pmatrix} \frac{\partial^2\Pi^C_L}{\partial p_1^2} & \frac{\partial^2\Pi^C_L}{\partial p_1\partial p_2} & \frac{\partial^2\Pi^C_L}{\partial p_1\partial a_1} & \frac{\partial^2\Pi^C_L}{\partial p_1\partial a_2} & \frac{\partial^2\Pi^C_L}{\partial p_1\partial\theta} \\ \frac{\partial^2\Pi^C_L}{\partial p_2\partial p_1} & \frac{\partial^2\Pi^C_L}{\partial p_2^2} & \frac{\partial^2\Pi^C_L}{\partial p_2\partial a_1} & \frac{\partial^2\Pi^C_L}{\partial p_2\partial a_2} & \frac{\partial^2\Pi^C_L}{\partial p_2\partial\theta} \\ \frac{\partial^2\Pi^C_L}{\partial a_1\partial p_1} & \frac{\partial^2\Pi^C_L}{\partial a_1\partial p_2} & \frac{\partial^2\Pi^C_L}{\partial a_1^2} & \frac{\partial^2\Pi^C_L}{\partial a_1\partial a_2} & \frac{\partial^2\Pi^C_L}{\partial a_1\partial\theta} \\ \frac{\partial^2\Pi^C_L}{\partial a_2\partial p_1} & \frac{\partial^2\Pi^C_L}{\partial a_2\partial p_2} & \frac{\partial^2\Pi^C_L}{\partial a_2\partial a_1} & \frac{\partial^2\Pi^C_L}{\partial a_2^2} & \frac{\partial^2\Pi^C_L}{\partial a_2\partial\theta} \\ \frac{\partial^2\Pi^C_L}{\partial\theta\partial p_1} & \frac{\partial^2\Pi^C_L}{\partial\theta\partial p_2} & \frac{\partial^2\Pi^C_L}{\partial\theta\partial a_1} & \frac{\partial^2\Pi^C_L}{\partial\theta\partial a_2} & \frac{\partial^2\Pi^C_L}{\partial\theta^2} \end{pmatrix} = \begin{pmatrix} -2\alpha & 2\beta & 0 & 0 & \gamma \\ 2\beta & -2\alpha & 0 & 0 & \gamma \\ 0 & 0 & -2\delta & 2\eta & 0 \\ 0 & 0 & 2\eta & -2\delta & \mu \\ \gamma & \gamma & 0 & \mu & -2\lambda \end{pmatrix}$$

The leading principle minors are $|M_1| = -2\alpha < 0$, $|M_2| = 4(\alpha^2 - \beta^2) > 0$, $|M_3| = -8\delta(\alpha^2 - \beta^2) < 0$, $|M_4| = 16(\alpha^2 - \beta^2)(\delta^2 - \eta^2) > 0$, and $|H^C| = -16(\alpha + \beta)(\delta + \eta)[(\delta - \eta)\big(2\lambda(\alpha - \beta) - \gamma^2\big) - (\alpha - \beta)\mu^2] < 0$ if $\lambda > \frac{(\alpha-\beta)\mu^2+(\delta-\eta)\gamma^2}{2(\alpha-\beta)(\delta-\eta)}$. Therefore, the Hessian matrix is negative definite if $\lambda > \frac{(\alpha-\beta)\mu^2+(\delta-\eta)\gamma^2}{2(\alpha-\beta)(\delta-\eta)}$.

Thus, we find that Π^C_L is jointly concave in $p_i,\ a_i,\ \theta$. So, the optimal solution can be determined by using KKT condition, *i.e.* $\frac{\partial\Pi^C_L}{\partial p_i} = 0$, $\frac{\partial\Pi^C_L}{\partial a_i} = 0$, $\frac{\partial\Pi^C_L}{\partial\theta} = 0$, $L(\tau R^C - D^C) = 0$, $(\tau R^C - D^C) = 0$, and $L \geq 0$. Now, from $\frac{\partial\Pi^C_L}{\partial p_i} = 0$, $\frac{\partial\Pi^C_L}{\partial a_i} = 0$, $\frac{\partial\Pi^C_L}{\partial\theta} = 0$, we can get the values of $p_i,\ a_i,\ \theta$ in terms of L which are given by

$$\begin{aligned}
p_i^C &= \frac{D_{0i}\Omega_1 + D_{0j}\Omega_2 + (\alpha+\beta)\big(2(c+L)[2(\delta-\eta)\big(\lambda(\alpha-\beta)-\gamma^2\big) - 2\mu^2(\alpha-\beta)] - \gamma\mu[\Phi_2 + 2L\tau(\delta-\eta)]\big)}{4(\alpha+\beta)[(\delta-\eta)\big(2(\alpha-\beta)\lambda-\gamma^2\big) - (\alpha-\beta)\mu^2]}, \\
a_i^C &= \frac{-R_{0i}\Omega_3 - R_{0j}\Omega_4 + (\delta+\eta)\gamma\mu[\Phi_3 - 2L(\alpha-\beta)] - 2(c_I - L\tau)[(\delta-\eta)\big(2\lambda(\alpha-\beta)-\gamma^2\big) - 2(\alpha-\beta)\mu^2]}{4(\delta+\eta)[(\alpha-\beta)\big(2(\delta-\eta)\lambda-\mu^2\big) - (\delta-\eta)\gamma^2]}, \\
\theta^C &= \frac{\gamma(\delta-\eta)\Phi_3 - \mu(\alpha-\beta)\Phi_2 - 2L(\alpha-\beta)(\delta-\eta)(\gamma+\mu\tau)}{2[2(\alpha-\beta)(\delta-\eta)\lambda - \big((\alpha-\beta)\mu^2 + (\delta-\eta)\gamma^2\big)]}.
\end{aligned}$$

where $\Omega_1 = [(\delta-\eta)(4\alpha\lambda-\gamma^2) - 2\alpha\mu^2], \Omega_2 = [(\delta-\eta)(4\beta\lambda+\gamma^2) - 2\beta\mu^2], \Omega_3 = [(\alpha-\beta)(4\delta\lambda-\mu^2) - 2\delta\gamma^2]$,

$\Omega_4 = [(\alpha-\beta)(4\eta\lambda-\mu^2) - 2\eta\gamma^2]$.

Now, from $\tau R^C - D^C = 0$, we get $L = \frac{\Phi_3[(\alpha-\beta)\big(2\lambda(\delta-\eta)-\mu^2\big)+\tau\gamma\mu(\delta-\eta)] - \Phi_2[\tau(\delta-\eta)\big(2\lambda(\alpha-\beta)-\gamma^2\big)+\gamma\mu(\alpha-\beta)]}{\Psi_1}$

Putting this value of L in the above equations, the optimal values of the decision variables can be obtained as given in Proposition 1.

Appendix B

Proof of Proposition 2

We use backward induction method during the calculation of decentralized models. So, the followers *i.e.* the collectors and the retailers first determine their decisions. As $\frac{\partial^2 \Pi_{Ci}^N}{\partial a_i^2} = -2\delta < 0$ and $\frac{\partial^2 \Pi_{Ri}^N}{\partial p_i^2} = -2\alpha < 0$, there exists unique response for the collectors and the retailers which are given below:

$$\begin{aligned}
\tilde{a_i}^N(m_i, m_j, \theta) &= \frac{2\delta(-R_{0i} + \delta m_i + \mu\theta) + \eta(-R_{0j} + \delta m_j + \mu\theta)}{4\delta^2 - \eta^2}, && (40) \\
\tilde{p_i}^N(w_i, w_j, \theta) &= \frac{2\alpha(D_{0i} + \alpha w_i + \gamma\theta) + \beta(D_{0j} + \alpha w_j + \gamma\theta)}{4\alpha^2 - \beta^2}. && (41)
\end{aligned}$$

Putting these decisions in the remanufacturer's profit function, we get

$$\begin{aligned}
\Pi_{RM}^N &= \sum_{i=1}^{2} \frac{(w_i - c)\alpha[2\alpha D_{0i} + \beta D_{0j} - (2\alpha^2-\beta^2)w_i + \alpha\beta w_j + (2\alpha+\beta)\gamma\theta]}{4\alpha^2 - \beta^2} \\
&\quad - \sum_{i=1}^{2} \frac{(m_i + c_I)\delta[2\delta R_{0i} + \eta R_{0j} + (2\delta^2-\eta^2)m_i - \delta\eta m_j - (2\delta+\eta)\mu\theta]}{4\delta^2 - \eta^2} - \lambda\theta^2 \qquad (42)
\end{aligned}$$

So, the remanufacturer's problem becomes

$$\begin{aligned}
&\max_{(w_1, w_2, m_1, m_2, \theta)} \; \Pi_{RM}^N(w_1, w_2, m_1, m_2, \theta) \\
&\text{subject to } D^N = \tau R^N
\end{aligned}$$

Now, we consider the Lagrangian function as

$$\Pi_L^N = \Pi_{RM}^N + L(\tau R^N - D^N), \qquad (43)$$

where $L(\geq 0)$ is the Lagrangian multiplier.

The corresponding Hessian matrix is given by

$$H^N = \begin{pmatrix} \frac{\partial^2 \Pi_L^N}{\partial w_1^2} & \frac{\partial^2 \Pi_L^N}{\partial w_1 \partial w_2} & \frac{\partial^2 \Pi_L^N}{\partial w_1 \partial m_1} & \frac{\partial^2 \Pi_L^N}{\partial w_1 \partial m_2} & \frac{\partial^2 \Pi_L^N}{\partial w_1 \partial \theta} \\ \frac{\partial^2 \Pi_L^N}{\partial w_2 \partial w_1} & \frac{\partial^2 \Pi_L^N}{\partial w_2^2} & \frac{\partial^2 \Pi_L^N}{\partial w_2 \partial m_1} & \frac{\partial^2 \Pi_L^N}{\partial w_2 \partial m_2} & \frac{\partial^2 \Pi_L^N}{\partial w_2 \partial \theta} \\ \frac{\partial^2 \Pi_L^N}{\partial m_1 \partial w_1} & \frac{\partial^2 \Pi_L^N}{\partial m_1 \partial w_2} & \frac{\partial^2 \Pi_L^N}{\partial m_1^2} & \frac{\partial^2 \Pi_L^N}{\partial m_1 \partial m_2} & \frac{\partial^2 \Pi_L^N}{\partial m_1 \partial \theta} \\ \frac{\partial^2 \Pi_L^N}{\partial m_2 \partial w_1} & \frac{\partial^2 \Pi_L^N}{\partial m_2 \partial w_2} & \frac{\partial^2 \Pi_L^N}{\partial m_2 \partial m_1} & \frac{\partial^2 \Pi_L^N}{\partial m_2^2} & \frac{\partial^2 \Pi_L^N}{\partial m_2 \partial \theta} \\ \frac{\partial^2 \Pi_L^N}{\partial \theta \partial w_1} & \frac{\partial^2 \Pi_L^N}{\partial \theta \partial w_2} & \frac{\partial^2 \Pi_L^N}{\partial \theta \partial m_1} & \frac{\partial^2 \Pi_L^N}{\partial \theta \partial m_2} & \frac{\partial^2 \Pi_L^N}{\partial \theta^2} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{2\alpha(2\alpha^2-\beta^2)}{4\alpha^2-\beta^2} & \frac{2\alpha^2\beta}{4\alpha^2-\beta^2} & 0 & 0 & \frac{\alpha\gamma}{2\alpha-\beta} \\ \frac{2\alpha^2\beta}{4\alpha^2-\beta^2} & -\frac{2\alpha(2\alpha^2-\beta^2)}{4\alpha^2-\beta^2} & 0 & 0 & \frac{\alpha\gamma}{2\alpha-\beta} \\ 0 & 0 & -\frac{2\delta(2\delta^2-\eta^2)}{4\delta^2-\eta^2} & \frac{2\delta^2\eta}{4\delta^2-\eta^2} & \frac{\delta\mu}{2\delta-\eta} \\ 0 & 0 & \frac{2\delta^2\eta}{4\delta^2-\eta^2} & -\frac{2\delta(2\delta^2-\eta^2)}{4\delta^2-\eta^2} & \frac{\delta\mu}{2\delta-\eta} \\ \frac{\alpha\gamma}{2\alpha-\beta} & \frac{\alpha\gamma}{2\alpha-\beta} & \frac{\delta\mu}{2\delta-\eta} & \frac{\delta\mu}{2\delta-\eta} & -2\lambda \end{pmatrix}$$

The leading principle minors are $|M_1| = -\frac{2\alpha(2\alpha^2-\beta^2)}{4\alpha^2-\beta^2} < 0$, $|M_2| = \frac{4\alpha^2(\alpha^2-\beta^2)}{4\alpha^2-\beta^2} > 0$, $|M_3| = -\frac{8\alpha^2\delta(\alpha^2-\beta^2)(2\delta^2-\eta^2)}{(4\alpha^2-\beta^2)(4\delta^2-\eta^2)} < 0$, $|M_4| = \frac{16\alpha^2\delta^2(\alpha^2-\beta^2)(\delta^2-\eta^2)}{(4\alpha^2-\beta^2)(4\delta^2-\eta^2)} > 0$, and
$|H^N| = -\frac{16\alpha^2\delta^2(\alpha+\beta)(\delta+\eta)\left((\delta-\eta)(2\delta-\eta)[2\lambda(2\alpha-\beta)(\alpha-\beta)-\alpha\gamma^2]-\delta\mu^2(\alpha-\beta)(2\alpha-\beta)\right)}{(2\alpha-\beta)(2\delta-\eta)(4\alpha^2-\beta^2)(4\delta^2-\eta^2)} < 0$, if
$\lambda > \frac{(\alpha-\beta)(2\alpha-\beta)\delta\mu^2+(\delta-\eta)(2\delta-\eta)\alpha\gamma^2}{2(\alpha-\beta)(\delta-\eta)(2\alpha-\beta)(2\delta-\eta)}$. Thus the Hessian matrix is negative definite if
$\lambda > \frac{(\alpha-\beta)(2\alpha-\beta)\delta\mu^2+(\delta-\eta)(2\delta-\eta)\alpha\gamma^2}{2(\alpha-\beta)(\delta-\eta)(2\alpha-\beta)(2\delta-\eta)}$.
Thus, we find that Π_L^N is jointly concave in $w_i,\ m_i,\ \theta$. So, the optimal solution can be determined by using KKT condition, *i.e.* $\frac{\partial \Pi_L^N}{\partial w_i} = 0$, $\frac{\partial \Pi_L^N}{\partial m_i} = 0$, $\frac{\partial \Pi_L^N}{\partial \theta} = 0$, $L(\tau R^N - D^N) = 0$, $(\tau R^N - D^N) = 0$, and $L \geq 0$. Now, from $\frac{\partial \Pi_L^N}{\partial w_i} = 0$, $\frac{\partial \Pi_L^N}{\partial m_i} = 0$, $\frac{\partial \Pi_L^N}{\partial \theta} = 0$, we can get the values of $w_i,\ m_i,\ \theta$ in terms of L which are given by

$$w_i^N = \frac{\begin{pmatrix} D_{0i}\Theta_1 + D_{0j}\Theta_2 - \gamma\delta\mu(\alpha+\beta)(2\alpha-\beta)[\Phi_2 + 2L\tau(\delta-\eta)] \\ +2(c+L)(\alpha+\beta)[2(\delta-\eta)(2\delta-\eta)\big(\lambda(\alpha-\beta)(2\alpha-\beta)-\alpha\gamma^2\big) - 2\delta\mu^2(\alpha-\beta)(2\alpha-\beta)] \end{pmatrix}}{4(\alpha+\beta)[2(\alpha-\beta)(\delta-\eta)(2\alpha-\beta)(2\delta-\eta)\lambda - \big((\alpha-\beta)(2\alpha-\beta)\delta\mu^2 + (\delta-\eta)(2\delta-\eta)\alpha\gamma^2\big)]}, \tag{44}$$

$$m_i^N = \frac{\begin{pmatrix} -R_{0i}\Theta_3 - R_{0j}\Theta_4 + (\delta+\eta)(2\delta-\eta)\alpha\gamma\mu[\Phi_3 - 2L(\alpha-\beta)] \\ -2(c_I - L\tau)(\delta+\eta)[(\delta-\eta)(2\delta-\eta)\big(2\lambda(\alpha-\beta)(2\alpha-\beta)-\alpha\gamma^2\big) - 2(\alpha-\beta)(2\alpha-\beta)\delta\mu^2] \end{pmatrix}}{4(\delta+\eta)[2(\alpha-\beta)(\delta-\eta)(2\alpha-\beta)(2\delta-\eta)\lambda - \big((\alpha-\beta)(2\alpha-\beta)\delta\mu^2 + (\delta-\eta)(2\delta-\eta)\alpha\gamma^2\big)]}, \tag{45}$$

$$\theta^N = \frac{\alpha\gamma(\delta-\eta)(2\delta-\eta)\Phi_3 + \delta\mu(\alpha-\beta)(2\alpha-\beta)\Phi_2 - 2L(\alpha-\beta)(\delta-\eta)\big(\alpha\gamma(2\delta-\eta) + \delta\mu\tau(2\alpha-\beta)\big)}{2[2(\alpha-\beta)(\delta-\eta)(2\alpha-\beta)(2\delta-\eta)\lambda - \big((\alpha-\beta)(2\alpha-\beta)\delta\mu^2 + (\delta-\eta)(2\delta-\eta)\alpha\gamma^2\big)]}. \tag{46}$$

$\Theta_1 = (\delta-\eta)(2\delta-\eta)[4\beta\lambda(2\alpha-\beta)+\alpha\gamma^2]-2\beta\delta\mu^2(2\alpha-\beta)$, $\Theta_2 = \alpha[(\delta-\eta)(2\delta-\eta)[4\lambda(2\alpha-\beta)-\gamma^2]-2\delta\mu^2(2\alpha-\beta)]$, $\Theta_3 = \delta[2(2\delta-\eta)[2\lambda(\alpha-\beta)(2\alpha-\beta)-\alpha\gamma^2]-\mu^2(\alpha-\beta)(2\alpha-\beta)]$, $\Theta_4 = 2\eta(2\delta-\eta)[2\lambda(\alpha-\beta)(2\alpha-\beta)-\alpha\gamma^2]+\delta\mu^2(\alpha-\beta)(2\alpha-\beta)$.

Now, from $\tau R^N - D^N = 0$, we get
$L = \frac{\alpha\Phi_3[(\alpha-\beta)\left(2\lambda(\delta-\eta)(2\delta-\eta)-\delta\mu^2\right)+\tau\gamma\delta\mu(\delta-\eta)]-\delta\Phi_2[\tau(\delta-\eta)\left(2\lambda(\alpha-\beta)(2\alpha-\beta)-\alpha\gamma^2\right)+\alpha\gamma\mu(\alpha-\beta)]}{\Psi_2}$

Putting this value of L in the above equations, the optimal values of the decision variables can be obtained as given in Proposition 2.

Appendix C

Proof of Proposition 4

On simplification, we have

$$\begin{aligned}
\theta^N - \theta^J &= \frac{\alpha\delta\Phi_1\big((\alpha-\beta)\Phi_2 + (\delta-\eta)\tau\Phi_3\big)}{\Psi_2} - \frac{\alpha\Phi_1\big((\alpha-\beta)\Phi_2 + (\delta-\eta)\tau\Phi_3\big)}{\Psi_3} \\
&= \frac{4\alpha^2\eta\lambda(\alpha-\beta)^2(\delta-\eta)\Phi_1\big((\alpha-\beta)\Phi_2 + (\delta-\eta)\tau\Phi_3\big)}{\Psi_2\Psi_3} > 0 \\
p_i^J - p_i^N &= \frac{\alpha^2\eta\lambda(\alpha-\beta)(\delta-\eta)[(\alpha-\beta)\Phi_2 + (\delta-\eta)\tau\Phi_3]}{2(\alpha+\beta)(2\alpha+\beta)\Psi_2\Psi_3} \times \big[\lambda\tau(\alpha-\beta)(\delta-\eta) - \gamma\Phi_1\big] \\
&> 0, \quad \text{if } \lambda > \frac{\gamma\Phi_1}{\tau(\alpha-\beta)(\delta-\eta)}. \\
w_i^J - w_i^N &= \frac{\alpha\eta\lambda(\alpha-\beta)(\delta-\eta)[(\alpha-\beta)\Phi_2 + (\delta-\eta)\tau\Phi_3]}{2(\alpha+\beta)\Psi_2\Psi_3} \times \big[\tau\lambda(\delta-\eta)(2\alpha-\beta)(\alpha-\beta) - \alpha\gamma\Phi_1\big] \\
&> 0, \quad \text{as } \frac{\alpha}{2\alpha-\beta} < 1. \\
m_i^J - m_i^N &= \frac{\alpha\eta\lambda(\alpha-\beta)[(\alpha-\beta)\Phi_2 + (\delta-\eta)\tau\Phi_3]}{2(\delta+\eta)\Psi_2\Psi_3} \times \big[\alpha(\alpha-\beta)^2\mu^2 + \tau^2(\delta-\eta)^2\big(2\lambda(2\alpha-\beta)(\alpha-\beta) - \alpha\gamma^2\big)\big] \\
&> 0, \quad \text{if } \lambda > \frac{\alpha[\gamma^2\tau^2(\delta-\eta)^2 - \mu^2(\alpha-\beta)^2]}{2\tau^2(2\alpha-\beta)(\alpha-\beta)(\delta-\eta)^2}.
\end{aligned}$$

Appendix D

Proof of Proposition 5

$\Pi_{RM}^T \geq \Pi_{RM}^N$

$\Rightarrow (w_1^T - c)D_1^C + (w_2^T - c)D_2^C - (m_1^T + c_I)R_1^C - (m_2^T + c_I)R_2^C - \lambda(\theta^C)^2 + F_{RM} \geq (w_1^N - c)D_1^N + (w_2^N - c)D_2^N - (m_1^N + c_I)R_1^N - (m_2^N + c_I)R_2^N - \lambda(\theta^N)^2$

$\Rightarrow F_{RM} \geq (w_1^N - c)D_1^N + (w_2^N - c)D_2^N - (m_1^N + c_I)R_1^N - (m_2^N + c_I)R_2^N - \lambda(\theta^N)^2 - (w_1^T - c)D_1^C - (w_2^T - c)D_2^C + (m_1^T + c_I)R_1^C + (m_2^T + c_I)R_2^C + \lambda(\theta^C)^2 (= F_{RM}^{min})$.

$\Pi_{Ri}^T \geq \Pi_{Ri}^N$

$\Rightarrow (p_i^C - w_i^T)D_i^C - F_{Ri} \geq (p_i^N - w_i^N)D_i^N$

$\Rightarrow F_{Ri} \leq (p_i^C - w_i^T)D_i^C - (p_i^N - w_i^N)D_i^N (= F_{Ri}^{max})$.

$\Pi_{Ci}^T \geq \Pi_{Ci}^N$

$\Rightarrow (m_i^T - a_i^C)R_i^C - F_{Ci} \geq (m_i^N - a_i^N)R_i^N$

$\Rightarrow F_{Ci} \leq (m_i^T - a_i^C)R_i^C - (m_i^N - a_i^N)R_i^N (= F_{Ci}^{max})$.

Additional symbols

Symbols related to Proposition 1

$\Phi_1 = \gamma\tau(\delta-\eta) - \mu(\alpha-\beta), \Phi_2 = R_{01} + R_{02} - 2c_I(\delta-\eta), \Phi_3 = D_{01} + D_{02} - 2c(\alpha-\beta)$,

$\Phi_4 = (\alpha-\beta)(3\alpha+\beta)\big(2\lambda(\delta-\eta) - \mu^2\big) + 4\alpha\gamma\mu\tau(\delta-\eta) + (\delta-\eta)^2\tau^2(4\alpha\lambda - \gamma^2)$,

$\Phi_5 = (\alpha-\beta)(\alpha+3\beta)\big(2\lambda(\delta-\eta) - \mu^2\big) + 4\beta\gamma\mu\tau(\delta-\eta) + (\delta-\eta)^2\tau^2(4\beta\lambda + \gamma^2)$,

$\Phi_6 = (\delta-\eta)(3\delta+\eta)\tau^2\big(2\lambda(\alpha-\beta)-\gamma^2\big) + 4\delta\gamma\mu\tau(\alpha-\beta) + (\alpha-\beta)^2(4\delta\lambda-\mu^2)$,

$\Phi_7 = (\delta-\eta)(\delta+3\eta)\tau^2\big(2\lambda(\alpha-\beta)-\gamma^2\big) + 4\eta\gamma\mu\tau(\alpha-\beta) + (\alpha-\beta)^2(4\eta\lambda+\mu^2)$,

$\Psi_1 = 4\lambda(\alpha-\beta)(\delta-\eta)\big((\alpha-\beta)+(\delta-\eta)\tau^2\big) - 2\Phi_1^2$.

Symbols related to Proposition 2

$\Xi_1 = [(\alpha-\beta)(3\alpha+\beta)\big(2\lambda(\delta-\eta)(2\delta-\eta)-\delta\mu^2\big) + 4\alpha\gamma\delta\mu\tau(\delta-\eta) + \delta(\delta-\eta)^2\tau^2(4\lambda(2\alpha-\beta)-\gamma^2)]$,

$\Xi_2 = [\alpha(\alpha-\beta)(\alpha+3\beta)\big(2\lambda(\delta-\eta)(2\delta-\eta)-\delta\mu^2\big) + 4\alpha\beta\gamma\mu\tau(\delta-\eta) + \delta(\delta-\eta)^2\tau^2(4\lambda(2\alpha-\beta)+\alpha\gamma^2)]$,

$\Xi_3 = [(\delta-\eta)(3\delta+\eta)\tau^2\big(2\lambda(\alpha-\beta)(2\alpha-\beta)-\alpha\gamma^2\big) + 4\alpha\delta\gamma\mu\tau(\alpha-\beta) + \alpha(\alpha-\beta)^2(4\lambda(2\delta-\eta)-\mu^2)]$,

$\Xi_4 = [\delta(\delta-\eta)(\delta+3\eta)\tau^2\big(2\lambda(\alpha-\beta)(2\alpha-\beta)-\alpha\gamma^2\big) + 4\alpha\gamma\delta\eta\mu\tau(\alpha-\beta) + \alpha(\alpha-\beta)^2(4\eta\lambda(2\delta-\eta)+\delta\mu^2)]$,

$\Xi_5 = [(\alpha-\beta)(7\alpha+5\beta)\big(2\lambda(\delta-\eta)(2\delta-\eta)-\delta\mu^2\big) + 2\gamma\delta\mu\tau(\delta-\eta)(5\alpha^2+2\alpha\beta-\beta^2) + \delta(\delta-\eta)^2\tau^2(12\lambda(2\alpha^2-\beta^2) - \gamma^2(3\alpha+2\beta))]$,

$\Xi_6 = [\alpha(\alpha-\beta)(\alpha^2+7\alpha\beta+4\beta^2)\big(2\lambda(\delta-\eta)(2\delta-\eta)-\delta\mu^2\big) - 2\alpha\gamma\delta\mu\tau(\delta-\eta)(\alpha^2-4\alpha\beta-3\beta^2) + \delta(\delta-\eta)^2\tau^2(4\beta\lambda(5\alpha^2 - 2\beta^2) + \alpha(3\alpha+2\beta)\gamma^2)]$,

$\Xi_7 = [(\delta-\eta)(7\delta+5\eta)\tau^2\big(2\lambda(\alpha-\beta)(2\alpha-\beta)-\alpha\gamma^2\big) + 2\alpha\gamma\mu\tau(\alpha-\beta)(5\delta^2+2\delta\eta-\eta^2) + \alpha(\alpha-\beta)^2(12\lambda(2\delta^2-\eta^2) - \mu^2(3\delta+2\eta))]$,

$\Xi_8 = [\delta(\delta-\eta)(\delta^2+7\delta\eta+4\eta^2)\tau^2\big(2\lambda(\alpha-\beta)(2\alpha-\beta)-\alpha\gamma^2\big) + 2\alpha\gamma\delta\mu\tau(\alpha-\beta)(-\delta^2+4\delta\eta+3\eta^2) + \alpha(\alpha-\beta)^2(4\eta\lambda(5\delta^2 - 2\eta^2) + \delta(3\delta+2\eta)\mu^2)]$,

$\Psi_2 = 4\lambda(\alpha-\beta)(\delta-\eta)\big(\alpha(\alpha-\beta)(2\delta-\eta) + (\delta-\eta)(2\alpha-\beta)\delta\tau^2\big) - 2\alpha\delta\Phi_1^2$.

Symbols related to Proposition 3

$\Delta_1 = [(\alpha-\beta)(3\alpha+\beta)\big(4\lambda(\delta-\eta)-\mu^2\big) + 4\alpha\gamma\mu\tau(\delta-\eta) + (\delta-\eta)^2\tau^2(4\lambda(2\alpha-\beta)-\gamma^2)]$,

$\Delta_2 = [\alpha(\alpha-\beta)(\alpha+3\beta)\big(4\lambda(\delta-\eta)-\mu^2\big) + 4\beta\gamma\mu\tau(\delta-\eta) + (\delta-\eta)^2\tau^2(4\beta\lambda(2\alpha-\beta)+\alpha\gamma^2)]$,

$\Delta_3 = [(\delta-\eta)(3\delta+\eta)\tau^2\big(2\lambda(\alpha-\beta)(2\alpha-\beta)-\alpha\gamma^2\big) + 4\alpha\delta\gamma\mu\tau(\alpha-\beta) + \alpha(\alpha-\beta)^2(8\delta\lambda-\mu^2)]$,

$\Delta_4 = [(\delta-\eta)(\delta+3\eta)\tau^2\big(2\lambda(\alpha-\beta)(2\alpha-\beta)-\alpha\gamma^2\big) + 4\alpha\gamma\eta\mu\tau(\alpha-\beta) + \alpha(\alpha-\beta)^2(8\eta\lambda+\mu^2)]$,

$\Delta_5 = [(\alpha-\beta)(7\alpha+5\beta)\big(4\lambda(\delta-\eta)-\mu^2\big) + 2\gamma\mu\tau(\delta-\eta)(5\alpha^2+2\alpha\beta-\beta^2) + (\delta-\eta)^2\tau^2(12\lambda(2\alpha^2-\beta^2)-\gamma^2(3\alpha+2\beta))]$,

$\Delta_6 = [\alpha(\alpha-\beta)(\alpha^2+7\alpha\beta+4\beta^2)\big(4\lambda(\delta-\eta)-\mu^2\big) - 2\alpha\gamma\mu\tau(\delta-\eta)(\alpha^2-4\alpha\beta-3\beta^2) + (\delta-\eta)^2\tau^2(4\beta\lambda(5\alpha^2-2\beta^2) + \alpha(3\alpha+2\beta)\gamma^2)]$,

$\Delta_7 = [(\delta-\eta)(7\delta+\eta)\tau^2\big(2\lambda(\alpha-\beta)(2\alpha-\beta)-\alpha\gamma^2\big) + 2\alpha\gamma\mu\tau(\alpha-\beta)(5\delta-\eta) + 3\alpha(\alpha-\beta)^2(8\delta\lambda-\mu^2)]$,

$\Delta_8 = [(\delta-\eta)(\delta+7\delta)\tau^2\big(2\lambda(\alpha-\beta)(2\alpha-\beta)-\alpha\gamma^2\big) - 2\alpha\gamma\mu\tau(\alpha-\beta)(\delta-5\eta) + 3\alpha(\alpha-\beta)^2(8\eta\lambda+\mu^2)]$,

$\Psi_3 = 4\lambda(\alpha-\beta)(\delta-\eta)\big(2\alpha(\alpha-\beta) + (\delta-\eta)(2\alpha-\beta)\tau^2\big) - 2\alpha\Phi_1^2$.

Tables

Table 1. Parameter-values

Example	D_{01}	D_{02}	R_{01}	R_{02}	α	β	γ	δ	η	μ	c	c_I	τ	λ
1	100	80	15	12	1.5	0.15	1.0	4.0	2.5	0.65	2.0	0.7	0.7	0.3
2	80	70	12	10	0.25	0.18	0.15	0.5	0.2	0.1	100	15	0.9	15
3	150	100	20	15	2.0	0.25	1.0	6.0	2.5	0.65	6.0	2.0	0.9	5.0

Table 2. Optimal results of the proposed models

Optimal	Example 1				Example 2				Example 3			
results	C-Model	N-Model	J-Model	T-Model	C-Model	N-Model	J-Model	T-Model	C-Model	N-Model	J-Model	T-Model
w_1	-	56.68	59.38	45.97	-	714.81	737.40	576.67	-	55.08	56.91	31.87
w_2	-	50.62	53.32	46.57	-	703.18	725.77	585.04	-	43.97	45.80	33.26
p_1	61.58	66.11	67.18	61.58	695.49	798.88	816.46	695.49	56.99	68.66	69.60	56.99
p_2	55.52	56.87	57.94	55.52	683.85	779.90	779.48	683.85	45.88	51.67	52.61	45.88
θ	7.47	4.77	3.78	7.47	3.21	2.37	2.23	3.21	2.76	1.55	1.42	2.76
m_1	-	8.35	10.28	18.43	-	82.51	102.96	125.91	-	5.83	7.50	14.55
m_2	-	8.58	10.51	18.29	-	83.94	104.39	125.34	-	6.12	7.79	14.42
a_1	11.67	4.08	1.34	11.67	64.45	37.43	32.80	64.45	7.35	1.81	1.23	7.35
a_2	11.90	4.45	1.69	11.90	65.88	39.70	34.95	65.88	7.64	2.27	1.67	7.64
D	36.84	23.52	18.64	36.84	54.42	40.20	37.69	54.42	75.50	42.53	38.99	75.50
R	52.64	33.60	26.63	52.64	60.46	44.66	42.88	60.46	83.88	47.26	43.33	83.88
Π_{RM}	-	914.44	727.99	607.76 + F_{RM}	-	20019.3	18772.3	17485.7 + F_{RM}	-	1527.91	1406.06	566.86 + F_{RM}
Π_{R1}	-	133.32	91.33	365.83 - F_{R1}	-	1766.99	1562.65	3529.58 - F_{R1}	-	368.56	322.01	1262.36 - F_{R1}
Π_{R2}	-	58.65	32.13	120.16 - F_{R2}	-	1471.43	1285.50	2441.38 - F_{R2}	-	118.35	92.65	318.70 - F_{R2}
Π_{C1}	-	72.98	122.39	183.22 - F_{C1}	-	1016.12	1486.64	1888.41 - F_{C1}	-	97.18	139.57	310.91 - F_{C1}
Π_{C2}	-	68.18	114.19	163.48 - F_{C2}	-	978.90	1436.78	1767.50 - F_{C2}	-	89.04	128.64	275.96 - F_{C2}
Π	1440.45	1247.57	1088.03	1440.45	27112.6	25252.7	24543.9	27112.6	2734.79	2201.04	2088.94	2734.79

Table 3. Optimal results of the proposed models for 100% recycling

Optimal results	C-Model	N-Model	J-Model	T-Model
w_1	-	58.78	60.54	47.44
w_2	-	52.72	54.48	48.04
p_1	70.23	71.59	71.54	70.23
p_2	64.17	62.36	62.31	64.17
θ	29.91	17.26	14.47	29.91
m_1	-	13.80	15.99	30.79
m_2	-	14.03	16.22	30.64
a_1	23.30	9.52	6.51	23.30
a_2	23.54	9.89	6.86	23.54
Π_{RM}	-	1243.19	1045.03	564.69 + F_{RM}
Π_{R1}	-	246.28	181.59	779.25 - F_{R1}
Π_{R2}	-	139.36	91.92	390.06 - F_{R2}
Π_{C1}	-	73.32	137.34	224.08 - F_{C1}
Π_{C2}	-	68.50	128.65	202.19 - F_{C2}
Π	2160.29	1770.65	1584.53	2160.29

#Note: optimal values are calculated by considering the data sets of Example 1.

Figures

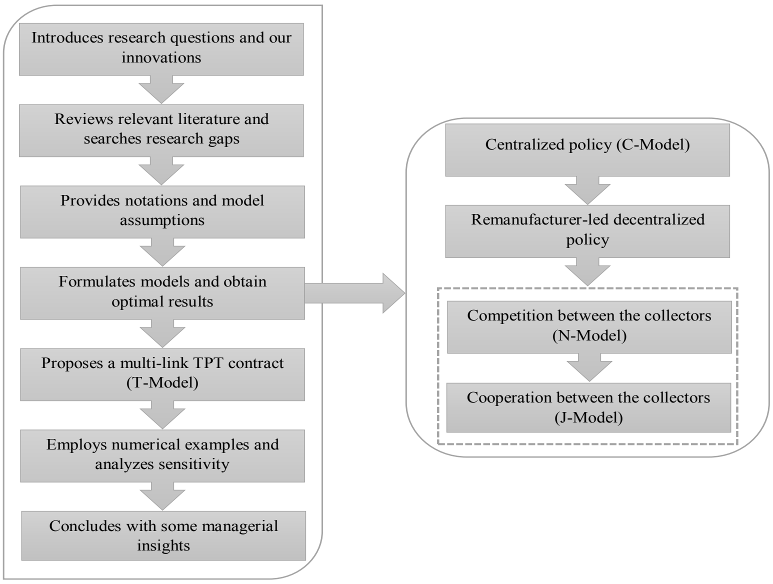
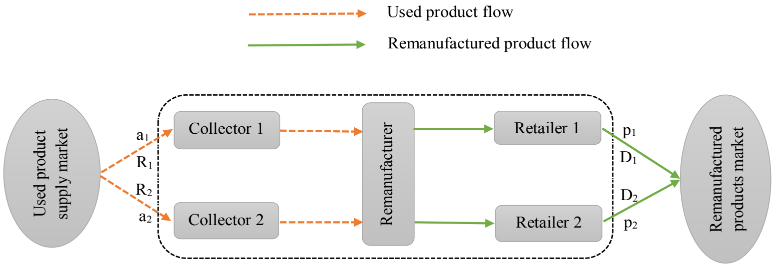
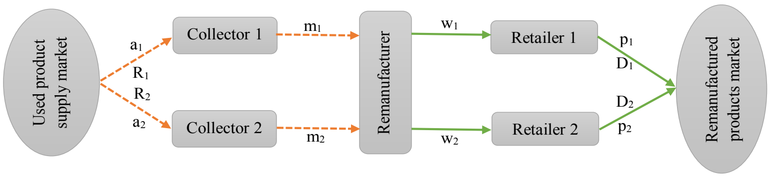


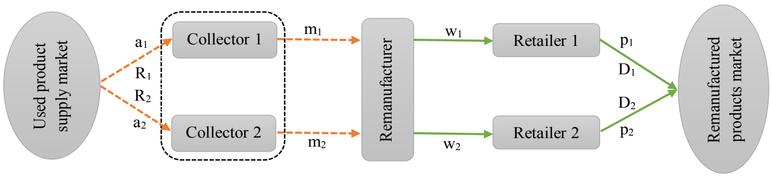
Figure 1. The framework of the study.



(a) C-Model

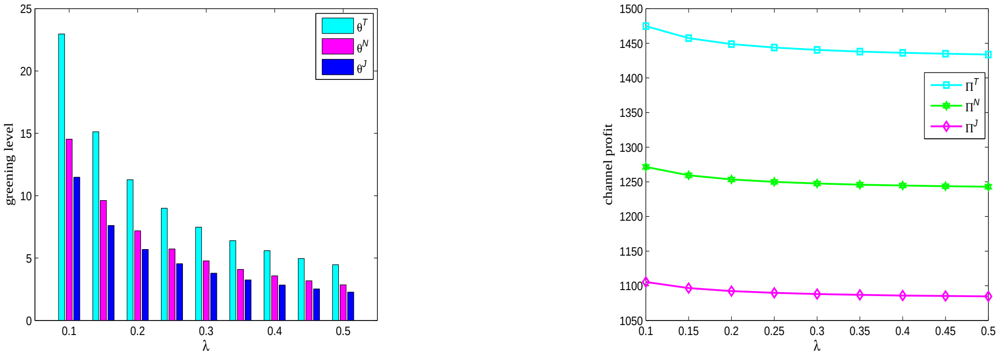


(b) N-Model



(c) J-Model

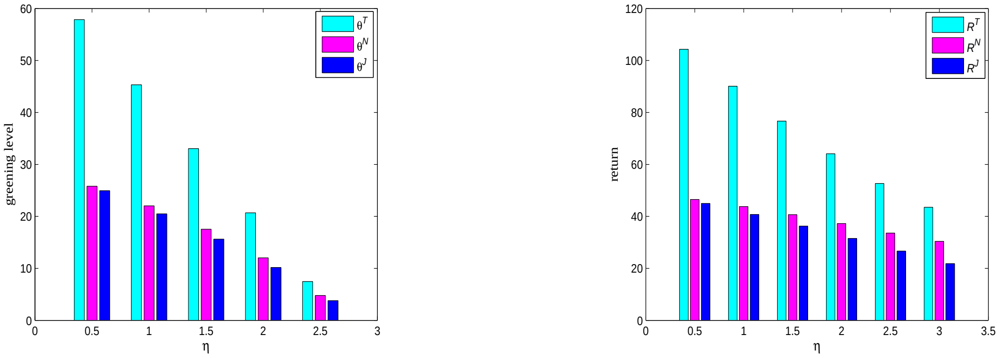
Figure 2. Proposed supply chain models.



(a) λ vs greening level.

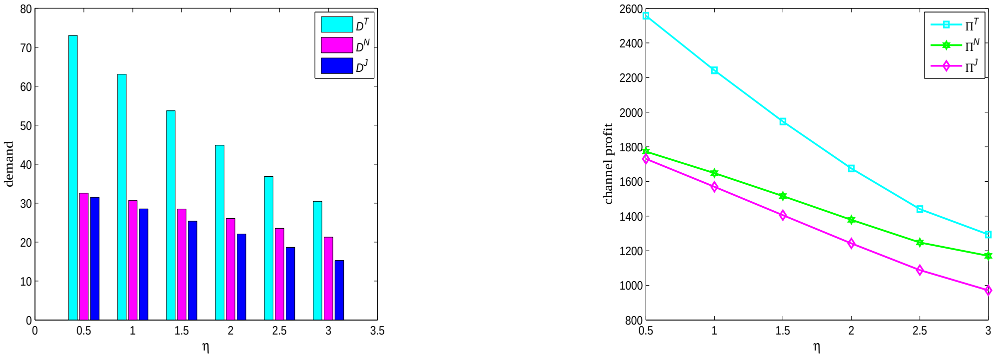
(b) λ vs channel profit.

Figure 3. Sensitivity of greening level and channel profit w.r.t. λ.



(a) η vs greening level.

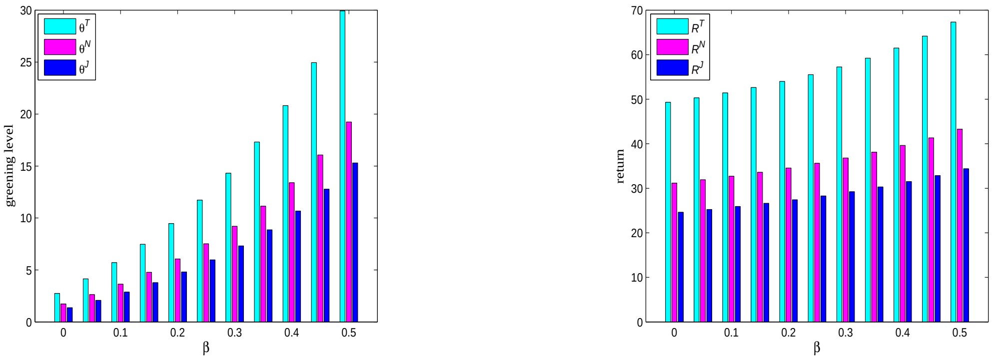
(b) η vs return.



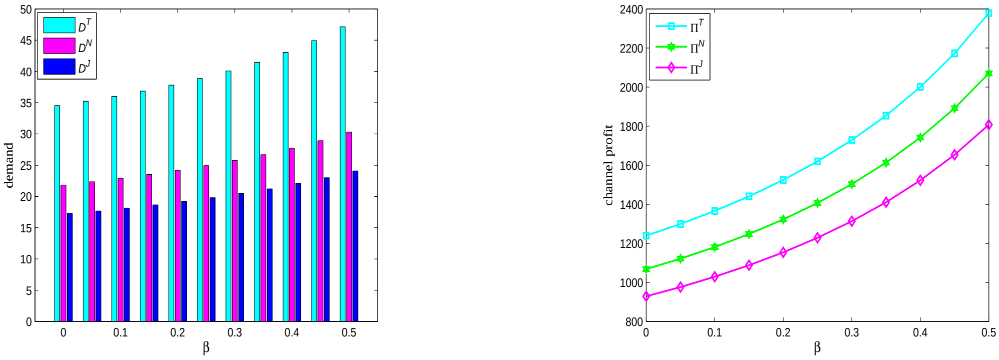
(c) η vs demand.

(d) η vs profit.

Figure 4. Sensitivity of optimal results w.r.t. η.

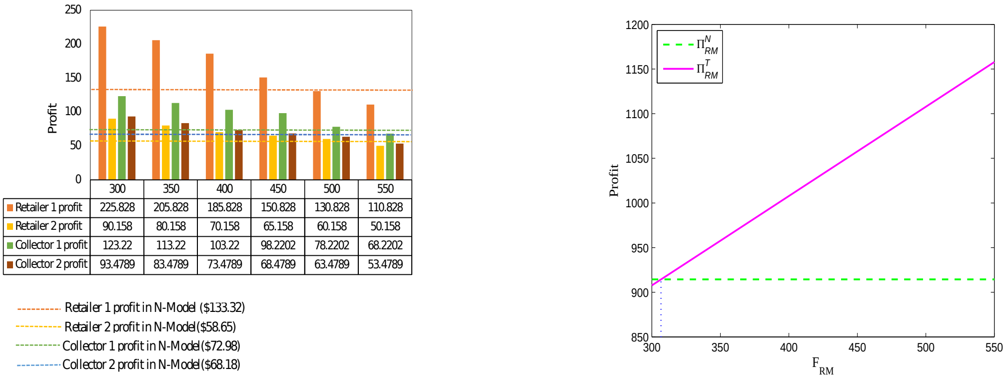


(a) β vs greening level. (b) β vs return.



(c) β vs demand. (d) β vs profit.

Figure 5. Sensitivity of optimal results w.r.t. β.



	300	350	400	450	500	550
Retailer 1 profit	225.828	205.828	185.828	150.828	130.828	110.828
Retailer 2 profit	90.158	80.158	70.158	65.158	60.158	50.158
Collector 1 profit	123.22	113.22	103.22	98.2202	78.2202	68.2202
Collector 2 profit	93.4789	83.4789	73.4789	68.4789	63.4789	53.4789

(a) retailers and collectors' profit w.r.t. F_{RM}. (b) remanufacturer's profit w.r.t. F_{RM}.

Figure 6. Sensitivity of channel members' profit w.r.t. F_{RM}.

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