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An integrated model for optimal selection of quality, maintenance, and production parameters with autocorrelated data

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Abstract. Statistical process monitoring, maintenance policy, and production have commonly been studied separately in the literature, whereas their integration can lead to more favorable conditions for the entire production system. Among all studies on integrated models, the underlying process is assumed to generate independent data. However, there are practical examples in which this assumption is violated because of the extraction of correlation patterns. Autocorrelation causes numerous false alarms when the process is in the in-control state or makes the traditional control charts react slowly to the detection of an out-of-control state. The Autoregressive Moving Average (ARMA) control chart is selected as an effective tool for monitoring autocorrelated data. Therefore, an integrated model subject to some constraints is proposed to determine the optimal decision variables of the ARMA control chart, economic production quantity, and maintenance policy in the presence of autocorrelated data. Due to the complexity of the model, a Particle Swarm Optimization (PSO) algorithm is applied to search for optimal decision variables. An industrial example and some comparisons are provided for more investigations. Moreover, sensitivity analysis is carried out to study the effects of model parameters on the solution of the economic-statistical design.

1. Introduction

The survival of producers in the world of competition with fast-changing markets and the spread of diversity in productions necessitates appropriate planning. In manufacturing systems, Statistical Process Monitoring (SPM) and Maintenance Policy (MP) are conventional tools to decrease the fraction of nonconforming items. There exist several studies in which these concepts are treated separately. For instance, the subjects of SPM, MP, and Economic Production Quantity (EPQ) were independently studied [1–3]. In recent years, the integration of production, maintenance, and quality has also attracted many researchers.

Control charts have gradually been approved in pioneer industries as effective tools used in statistical quality control to ensure quality and save manufacturing costs. It is mainly used to identify the change

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in the process before manufacturing nonconforming items in massive amounts. After introducing the basic theory of process monitoring by Shewhart, numerous control charts have been developed to achieve particular objectives under various assumptions [4]. One of the main assumptions of SPM is that the sampled observations at different time points must be independent. Nevertheless, the independence assumption is not realistic from two types of practical experiences: (1) Sampling in high frequency induces autocorrelation in some processes and (2) sampling from processes, such as chemical and environmental, that introduce inherent autocorrelation [5]. In some industrial/non-industrial processes, such as continuous manufacturing processes, financial processes, health care systems, environmental phenomena, and network monitoring, a correlation exists among adjacent observations [6]. The autocorrelation, if ignored, can lead to a significant influence on the statistical performance of traditional control charts [7].

This has led to the development of various control charts for autocorrelated data. Two model-based approaches can be used to treat the process when the serial correlation exists among observations. These approaches include the residual control charts and the modified control charts [8,9]. In the first approach, the control charts are applied to the residuals obtained after fitting a model to eliminate the correlated structure. The Special Cause Chart (SCC) was proposed as an initial study of this approach [10]. In the second approach, the correlated observations are directly used on the control charts in which the control limits are adjusted according to the autocorrelation structure. The modified control chart was initially proposed in [11]. In addition to the introduced approaches, neural network-based control charts can also be categorized as the third approach in which the data are processed without the necessity of identifying models or making adjustments [12]. Some recent studies using different approaches can be referred to in [13–15].

In SPM, we concentrate on modified control charts that benefit from plotting original observations and more straightforward interpretation by the operator. Of those, the Autoregressive Moving Average (ARMA) control chart is selected as a suitable one to monitor a sequence of autocorrelated data [16]. ARMA control chart considers the autocorrelation structure of the underlying process and utilizes the advantages of Exponentially Weighted Moving Average (EWMA) control chart for stationary processes, known as EWMAST control chart [17] and SCC. The sample to be monitored by this chart is based on an ARMA statistic. It benefits from allowing a more flexible choice of parameters. Thus, the performance of this chart is related to the autocorrelation structure of the statistic. As was shown for autocorrelated processes, if appropriate parameter values are chosen for an ARMA chart, its performance will be superior in comparison to SCC and EWMAST charts [16]. In this study, the underlying process is assumed to follow the ARMA (1,1) model for two reasons: (1) Being stationary, similar to most statistical process control systems, and (2) the existence of both autoregressive and moving average terms that make it possible to examine their effects separately [18]. Since we aim to monitor the autocorrelation type of ARMA processes, the ARMA control chart is used in this study to determine its design parameters via optimizing an integrated model.

Generally, the design of a control chart needs to optimally specify a set of variables that depends on the type of the considered chart and provided assumptions. The design of the ARMA chart requires specifying sample size (n), sampling interval (h), control limits width (l), moving average parameter (θ), and autoregressive parameter (ϕ). There can be found only five pieces of research on the design of the ARMA chart according to the best of our knowledge. Low and Lin [19] optimally determined the design parameters of the ARMA chart by minimizing Duncan's cost function. They used Weibull distribution to present the time that the process shifts. An Economical Design (ED) of the ARMA chart was presented in [20] to optimally determine its design parameters by minimizing the cost function of Lorenzen and Vance (L&V). Both [19] and [20] addressed only economic considerations and used a Genetic Algorithm (GA) for optimizations. However, EDs have been criticized for poor statistical properties, such as lower power in detecting shifts compared to Statistical Designs (SD) that merely aim to decrease the occurrences of Type-I or Type-II errors. Based on the idea of Economic-Statistical Design (ESD) using the L&V cost function, Costa and Fichera [21,22] optimally designed the ARMA chart with fixed parameters and Variable Sampling Intervals (VSI). They devised a modified type of differential evolution for optimization.

Heretofore, we discussed the design and modeling of control charts with autocorrelated data. However, since the concepts of SPM, MP, and EPQ can be defined in a unified model, separately modeling them may provide suboptimal solutions. By realizing such dependency, it was shown that implementing MP can result in lower quality costs and the probability of system failure, as well as increasing the production of conforming items [23]. Hadidi et al. [24] divided the literature on such models into two general categories: (1) Interrelated methods, in which a model is considered for one function bearing in mind the others, and (2) integrated models, in which two or more components are modeled simultaneously. In this paper, the second category is reviewed according to the proposed issue of monitoring autocorrelated data.

There are integrated models including double concepts of SPM, MP, and EPQ in several studies to amend the performance of manufacturing systems. Since SPM has been considered the main subject, only its integrations with MP and EPQ are referred to in double designs.

In traditional production planning, it is assumed that the economic value of production is determined without any defects and associated costs. However, under real conditions, the process may shift to the out-of-control state. This, while affecting the quality of products, imposes some costs on the system. This research gap has attracted the attention of many researchers to link quality and production issues. Among the double integrations of these issues, the reader can refer to [25–30].

On the other hand, improving product quality and reducing downtime and operating costs can be addressed through two critical issues quality control and maintenance policies their goals overlap a great deal. Only using the programs to improve the quality of products is not enough because the operating conditions of the processes, which are examined based on the maintenance policies, also affect the quality of the products. Thus, integrating quality control and maintenance has been focused on by some researchers [31–43]. Recently, Farahani and Tohidi [44] reviewed the literature on this issue.

Although simultaneously considering the triple concepts plays an essential role in reducing the costs of manufacturing systems, few scholars have already studied this subject. Among them, Ben-Daya and Makhdoum [45] introduced an integrated model to specify the EPQ and SPM parameters under several Preventive Maintenance (PM) rules. Lam and Rahim [46] presented an integrated model of EPQ, SPM, and MP for a deteriorating manufacturing system. Pan et al. [47] proposed a joint model of EPQ and MP for an imperfect production process based on a Shewhart control chart to minimize the cost function. Salmasnia et al. [48] modeled the integration of production run length, MP, and SPM in the presence of multiple assignable causes. Recently, new integrated models of triple concepts have been proposed, such as ESD under a Variable Parameter (VP) control chart for monitoring multivariate quality characteristics [49], under non-uniform sampling by considering the time value of money and the stochastic shift size [50], under adaptive control chart [51], and under an adaptive non-central chi-square control chart [52].

The properties of the existing studies in the literature are briefly listed in Table 1. It is observed that there are not any studies on triple integrated models in which the independence assumption for the data being monitored is violated. To bridge the existing gaps in the literature:

- This study presents a model by integrating the concepts of SPM, MP, and EPQ;
- In contrast to most of the studies in the literature that consider independence assumption for the underlying process, this study uses a control chart to monitor autocorrelated data of type ARMA and its special cases as autoregressive (AR) and Moving Average (MA) processes;
- The proposed method aims to optimally determine the decision variables by minimizing the constrained cost function.

The rest of this paper is organized as follows. In Section 2, the structure of the ARMA control chart to monitor an autocorrelated process is briefly described. The integrated model is described in detail in Section 3, and the proposed model is formalized according to a cost objective function subject to some constraints. In Section 4, the solution approach based on Particle Swarm Optimization (PSO) is explained. Then, in Section 5, an industrial example is provided to illustrate the solution procedure and indicate the applicability of the proposed model. Furthermore, some comparative studies and sensitivity analyses are presented in this section. Finally, conclusions and further perspectives are presented in Section 6.

2. ARMA control chart

The effectiveness of the ARMA control chart has been proved for monitoring autocorrelated data [16]. Suppose that the measured variable at time t, normally distributed with mean μ and variance σ_X^2, is mathematically expressed by:

$$X_t = uX_{t-1} + a_t - va_{t-1}, \quad a_t \in N(0, \sigma_a^2), \tag{1}$$

where the measurement at time $t-1$, and the noise factors at t and $t-1$, characterized by a_t and a_{t-1} respectively, are linearly combined to present the current measurement X_t. The constants u and v are the AR and the MA coefficients, respectively, with conditions $|u| < 1$ and $|v| < 1$. The variance of this process is shown as:

$$\sigma_X^2 = \frac{1 - 2uv + v^2}{1 - u^2}\sigma_a^2. \tag{2}$$

The sample statistic to be monitored by the ARMA control chart at time t is represented by:

$$Z_t = \phi Z_{t-1} + \theta_0 X_t - \theta X_{t-1}, \quad \theta_0 = 1 + \theta - \phi, \tag{3}$$

where ϕ and θ are the AR and the MA parameters, respectively. Note that the conditions $|\phi| < 1$ and $|\theta| < 1$ must be satisfied to guarantee the reversibility and stationary of the process being monitored. The mean of the sample statistic in Eq. (3) is μ, and the corresponding steady-state variance is as follows:

Table 1. Summarized literature review.

References	Integrated concepts			Design method		Structure of data	
	SPM	Inventory	Maintenance	ED	ESD	Independent	Correlated
Rahim [25]	√	√	–	√	–	√	–
Rahim and Ben–Daya [26]	√	√	–	√	–	√	–
Rahim and Ohta [27]	√	√	–	√	–	√	–
Cheng and Chou [28]	√	√	–	√	–	–	√
Pan et al. [29]	√	√	–	√	–	√	–
Gunay and Kula [30]	√	√	–	√	–	√	–
Tagaras [31]	√	–	√	√	–	√	–
Ben–Daya and Rahim [32]	√	–	√	√	–	√	–
Wu and Makis [33]	√	–	√	√	√	√	–
Zhou and Zhu [34]	√	–	√	√	–	√	–
Panagiotidou and Nenes [35]	√	–	√	√	–	√	–
Yin and Makis [36]	√	–	√	√	√	√	–
Ho and Quinino [37]	√	–	√	√	–	√	–
Xiang [38]	√	–	√	√	–	√	–
Yin et al. [39]	√	–	√	√	–	√	–
Abouei Ardakan et al. [40]	√	–	√	√	–	√	–
Rasay et al. [41]	√	–	√	√	–	√	–
Salmasnia et al. [42]	√	–	√	–	√	√	–
Salmasnia et al. [43]	√	–	√	–	√	√	–
Ben-Daya and Makhdoum [45]	√	√	√	√	–	√	–
Lam and Rahim [46]	√	√	√	√	–	√	–
Pan et al. [47]	√	√	√	–	√	√	–
Salmasnia et al. [48]	√	√	√	–	√	√	–
Salmasnia et al. [49]	√	√	√	–	√	√	–
Salmasnia et al. [50]	√	√	√	–	√	√	–
Salmasnia et al. [51]	√	√	√	–	√	√	–
Salmasnia et al. [52]	√	√	√	–	√	√	–
This paper	√	√	√	–	√	–	√

$$\sigma_Z^2 = \left[\frac{2(\theta-\phi)(1+\theta)}{1+\phi}+1\right]\sigma_X^2. \quad (4)$$

Accordingly, the upper and lower control limits of the ARMA chart are calculated by (l is the control limit coefficient):

$$[LCL, UCL] = [\mu \pm l\sigma_Z]. \quad (5)$$

In this research, the data monitored by this control chart follow the ARMA process. Since the existence of autocorrelation, the ARMA Simulation (AS) procedure is applied to approximately calculate Average Run Length (ARL) values when the process states are in-control and out-of-control, respectively indicated by ARL_0 and ARL_1. In each simulation run, M measurements in $S = 500$ columns are generated based on a set of design parameters (n, h, l, ϕ, θ). By considering $\mu_0 = 100$ and $\sigma_X^2 = 10$, the steps of AS procedure can be summarized below:

Step 1. Calculate σ_a from Eq. (2) by selecting u and v.

Step 2. Generate series $a_{i,j}$, $X_{i,j}$, and $X'_{i,j}$ of size M in S columns by (δ is the coefficient of the mean shift):

$$a_{i,j} \sim N(0,\sigma_a^2), \quad a_{0,j}=0, \quad X_{0,j}=\mu_0,$$

$$X_{i,j} = C + uX_{i-1,j} + a_{i,j},$$

$$X'_{i,j} = C + X_{i,j} + \delta\sigma_X, \quad \forall i = 1,...,M,$$

$$\forall j = 1,...,S. \quad (6)$$

Step 3. Set independent variables: n, h, l, ϕ, θ.

Step 4. Obtain the value of steady-state variance from Eq. (4) by replacing the estimation of σ_X.

Step 5. Calculate control limits using Eq. (5).

Step 6. Compute the Run Length (RL) values for $j = 1, \cdots, S$:

Step 6.1. Compute RL_{0j} until $LCL \leq Z_{i,j} \leq UCL$, where $Z_{i,j}$ is obtained by replacing $X_{i,j}$ in Eq. (3),

Step 6.2. Compute RL_{1j} until $LCL \leq Z_{i,j} \leq UCL$, where $Z_{i,j}$ is obtained by replacing $X'_{i,j}$ in Eq. (3).

Step 7. Calculate ARL_0 and ARL_1 by averaging them for $S = 500$ times from Step 6.

3. Model description

In real production environments, the process possibly deteriorates over time due to a variety of specific causes. Such so-called imperfect manufacturing systems are in contrast to previous classical perfect systems that assume faultless production. In this study, an imperfect production process including quality, maintenance, and production concepts investigated that operates in the in-control or out-of-control states. Accordingly, a model is proposed to optimize the cost function of the imperfect manufacturing system subject to some statistical constraints. Moreover, to be adapted to situations in which the independence assumption has deteriorated and the underlying process is autocorrelated, this study applies the ARMA control chart for the first time to notify the operators when the process shifts to the out-of-control state.

In the rest of this section, the main assumptions of modeling are first introduced. In the following subsection, different scenarios for the proposed model are defined. Then, the structure of the cost function is described. The proposed mathematical model is presented in the last subsection.

3.1. Notations and assumptions

The notations applied in this study can be seen in Table 2, which is categorized into five sets, including decision variables, indicators, time parameters, cost parameters, and process parameters. To simplify the mathematical modeling, some assumptions are listed to be held as follows:

1. The quality characteristic follows a normal distribution, and its autocorrelation structure is of the type ARMA, including the cases ARMA (1,1), AR (1), and MA (1).
2. The cycle always begins from the in-control state (i.e., $\mu = \mu_0$).
3. The in-control time of process follows a truncated Weibull distribution with scale parameter $\lambda > 0$ and shape parameter $\lambda > 0$ as (if $\lambda = 1$, it changes to truncated exponential distribution):

$$f(t|(k+1)h) = \frac{\lambda\gamma(\lambda t)^{\gamma-1}e^{-(\lambda t)^{\gamma}}}{1 - e^{-(\lambda(k+1)h)^{\gamma}}}. \quad (7)$$

4. Only one type of assignable cause exists. When it occurs, the process state shifts to the out-of-control as $\mu_1 = \mu_0 + \delta\sigma_X$. Note that the variance remains unchanged.
5. Two types of maintenance policies may happen, including PM and Reactive Maintenance (RM).
6. If after the kth sampling interval, no signal is detected due to falling a point outside the control limits, PM is performed when the $(k+1)$th interval is terminated.
7. When the process shifts to the out-of-control state during the sampling intervals between 0 and k, the search for the assignable cause begins. At that time, RM is performed to restitute it to the initial condition.
8. The production cycle terminates when either RM is performed after a true signal from the control chart or if the $(k + 1)$th sampling is implemented (each one occurs earlier).
9. The time required to run the reactive or the PM, detection of the false signal, and sampling is considered negligible compared to the time of the production cycle.

3.2. Scenarios description

The production process begins from the in-control state, and when an assignable cause occurs, it shifts to the out-of-control state. Figure 1 shows three possible scenarios that may occur in an imperfect production process [49].

Scenario 1 occurs when the state of the process stays in control for the whole time of the cycle. To ensure the reliability of the manufacturing process, PM is implemented when the production cycle is terminated. If the process shifts to an out-of-control state and then this deviation is detected before the end of the cycle, Scenario 2 occurs. In this case, RM is implemented to restore the process situation to the initial condition. Scenario 3 differs from Scenario 2 only in the disability of detecting the shift until the end of the cycle. In this case, PM is switched to RM when the shift is identified at the end. Here, each scenario is assumed to happen with probability [47]:

$$Pr(Sc_1) = 1 - F((k+1)h) = e^{-(\lambda(k+1)h)^{\gamma}}, \quad (8)$$

$$Pr(Sc_2) = F(kh)Pr(signaling|out\!-of\!-control\ state), \quad (9)$$

$$Pr(Sc_3) = F((k+1)h) - F(kh)$$
$$Pr(signaling|out\!-of\!-control\ state), \quad (10)$$

Table 2. Applied notations and abbreviations in the current paper.

Set	Notation	Description
Decision variables	n	Sample size
	h	Sampling interval
	l	Control limit parameter
	k	Number of samplings in the monitoring period
	ϕ	Autoregressive parameter of ARMA chart
	θ	Moving average parameter of ARMA chart
Index and indicator	i	Index of measurements in each column ($i = 1, \cdots, M$)
	j	Index of generated columns ($j = 1, \cdots, S$)
	r	Index of scenario ($r = 1, 2, 3$)
	t	Index of time
	y	Index of number of sampling intervals ($y = 1, \cdots, k$)
Time parameters	E	Time to sample and chart one item
	$E(T_{in}\|Sc_r)$	Expected value of in-control time under rth scenario
	$E(T_{out}\|Sc_r)$	Expected value of out-of-control time under rth scenario
	$f(t\|(k+1)h)$	The probability density function of truncated Weibull distribution
	$F(\mathrm{t})$	The cumulative function of truncated Weibull distribution
	T	Process cycle time
	T_1	Time to detect the assignable cause
	T_{in}	In-control time
	T_{out}	Out-of-control time
	γ	Shape parameter of truncated Weibull distribution
	λ	Scale parameter of truncated Weibull distribution
	τ	Expected time between an assignable cause and the next inspection
Cost parameters	A	Set up cost per production
	B	Inventory holding cost per unit per time unit
	C_f	Fixed cost of sampling
	C_{in}	Quality loss cost during in-control state
	C_{out}	Quality loss cost during out-of-control state
	C_{pm}	Preventive maintenance cost
	C_{rm}	Reactive maintenance cost
	C_v	Variable cost of sampling
	C_Y	Cost of false alarm inspection
	$E(C_M\|Sc_r)$	Expected maintenance cost for each scenario
	$E(C_Q\|Sc_r)$	Expected quality loss cost for each scenario
	$E(C_S\|Sc_r)$	Expected sampling cost per cycle time for each scenario
	$E(\mathrm{I})$	Expected summation of inventory holding and ordering costs
	$E(\mathrm{M})$	Expected maintenance cost
	$E(\mathrm{Q})$	Expected quality loss cost
	$E(\mathrm{S})$	Expected sampling cost
	ETC	Expected Total Cost
	IHC	Inventory Holding Cost
	SUC	Set Up Cost

Table 2. Applied notations and abbreviations in the current paper (continued).

Set	Notation	Description
	a_t	Noise factor (independent, identically distributed (iid) process)
	ARL_0	Average run length during the in-control period
	ARL_1	Average run length during out-of-control period
	d	Daily demand rate
	D	Annual demand rate
	LCL	Lower Control Limit of ARMA chart
	p	Production rate
	$Pr(Sc_r)$	Occurrence probability of rth scenario
	Q	Economic production quantity
	sr	Expected number of samplings in the in-control state under the rth scenario
	u	The autoregressive coefficient of the underlying process
Process parameters	UCL	Upper control limit of ARMA chart
	v	Moving average coefficient of the underlying process
	X_t	The variable of the underlying process
	Z_t	ARMA statistic corresponding to X_t
	α	Probability of false alarm or Type I error
	β	Probability of Type II error
	δ	The magnitude of a shift in process mean
	μ_0	Mean of ARMA sample statistic in state 0
	μ_1	Shifted mean value in state 1
	σ_a	The standard deviation of the noise factor
	σ_X	The standard deviation of the underlying process
	σ_Z	The standard deviation of ARMA sample statistic

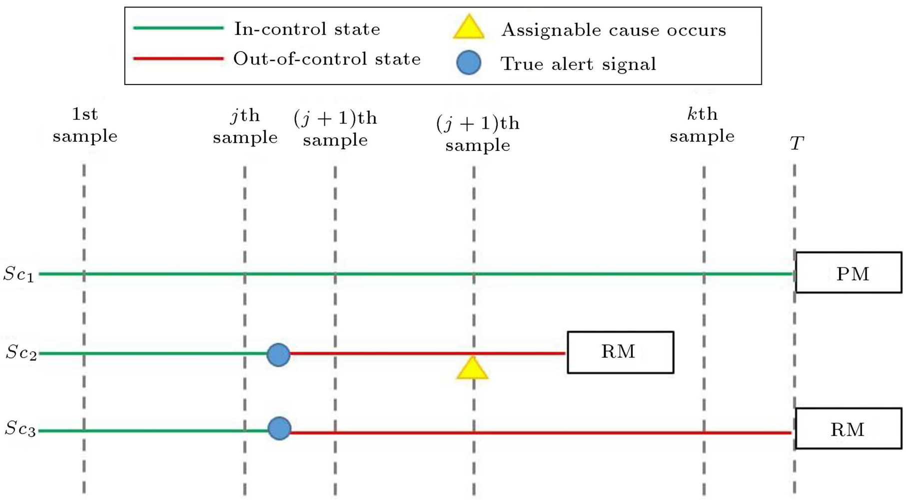

Figure 1. Graphical representation of scenarios.

where $F(.)$ is the cumulative function of the truncated Weibull distribution and Pr (*signaling|out-of-control state*) as the probability of triggering an alarm when the process shifts to out-of-control state, is expressed as (assuming that $\beta = 1 - (1/ARL_1)$):

$$Pr(signaling|out\text{–}of\text{–}control\ state) = 1 - \beta^{k-1}. \quad (11)$$

Moreover, the expected values of in-control time (T_{in}) and out-of-control time (T_{out}) in each scenario are calculated as follows [47]:

$$E(T_{in}|Sc_1) = (k+1)h, \qquad E(T_{out}|Sc_1) = 0, \quad (12)$$

$$E(T_{in}|Sc_2) = \int_0^{kh} t \times f(t|(k+1)h)\ dt,$$

$$E(T_{out}|Sc_2) = h \times ARL_1 - \tau + nE + T_1, \quad (13)$$

$$E(T_{in}|Sc_3) = \int_0^{(k+1)h} t \times f(t|(k+1)h)\ dt,$$

$$E(T_{out}|Sc_3) = (k+1)h - E(T_{in}|Sc_3), \quad (14)$$

where E indicates the time to sample and chart one item, T_1 is the time to detect the assignable cause, and τ, denoting the expected time between the assignable cause occurrence and the next inspection, is written as:

$$\tau = \int_0^{(k+1)h} t \times f(t|(k+1)h)\ dt - h\left(\sum_{y=1}^{k} e^{-(\lambda y h)^{\gamma}} - k e^{-(\lambda(k+1)h)^{\gamma}}\right). \quad (15)$$

3.3. Structure of cost function

At the beginning of this section, the expected values of the in-control time and the out-of-control time, as well as the occurrence probability of each scenario, were expressed. In this subsection, constituents of the cost function, including the quality loss cost, the sampling cost, the maintenance cost, and inventory-related costs, are described.

3.3.1. Quality loss cost

The expected quality loss cost is expressed by:

$$E(Q) = \sum_{r=1}^{3} E(C_Q|Sc_r)Pr(Sc_r). \quad (16)$$

The expected quality loss cost for each scenario is as follows:

$$E(C_Q|Sc_r) = \begin{cases} C_{in} \times E(T_{in}|Sc_r), & r = 1 \\ C_{in} \times E(T_{in}|Sc_r) + C_{out} \times E(T_{out}|Sc_r), & r = 2, 3 \end{cases} \quad (17)$$

where C_{in} indicates the quality loss cost in the in-control state, and C_{out} shows the quality loss cost in the out-of-control state.

3.3.2. Sampling cost

The expected sampling cost per cycle time is presented by the following formula:

$$E(S) = \sum_{r=1}^{3} E(C_S|Sc_r)Pr(Sc_r), \quad (18)$$

where the conditionally expected sampling cost per cycle time for each scenario is calculated as by (C_f and C_v are fixed and variable costs of sampling):

$$E(C_S|Sc_r) = \begin{cases} (C_f + C_v n)k, & r = 1, 3 \\ (C_f + C_v n)(E(T_{in}|Sc_r) + E(T_{out}|Sc_r))/h, & r = 2 \end{cases} \quad (19)$$

3.3.3. Maintenance cost

The expected maintenance cost per production cycle is computed as follows:

$$E(M) = \sum_{r=1}^{3} E(C_M|Sc_r)Pr(Sc_r), \quad (20)$$

where the expected maintenance cost per production for each scenario is as follows:

$$E(C_M|Sc_r) = \begin{cases} \frac{k \times C_Y}{ARL_0} + C_{pm}, & r = 1 \\ \frac{s_r \times C_Y}{ARL_0} + C_{rm}, & r = 2, 3 \end{cases} \quad (21)$$

where C_Y is the false alarm cost, C_{pm} and C_{rm} respectively represent PM and RM costs, and s indicates the expected number of samples obtained when the state of the process is in-control:

$$s_2 = \sum_{y=1}^{k-1} y \times [F_t((y+1)h) - F_t(yh) = \sum_{y=1}^{k-1} e^{-(\lambda y h)^{\gamma}} - (k-1)e^{-(\lambda k h)^{\gamma}}, \quad (22)$$

$$s_3 = E[number\ of\ samples\ taken\ while\ in\text{–}control\ |Sc_3] = \sum_{y=1}^{k} e^{-(\lambda y h)^{\gamma}} - k e^{-(\lambda(k+1)h)^{\gamma}}. \quad (23)$$

3.3.4. Inventory holding and setup costs

The Inventory Holding Cost (IHC) and setup cost (SUC) are respectively expressed by:

$$IHC = \frac{B \times Q}{2}\left(1 - \frac{d}{p}\right) = \frac{B \times T \times (p-d)}{2}, \quad (24)$$

$$SUC = \frac{D \times A}{p \times T}, \quad (25)$$

where B is the IHC per unit time, d is the daily demand, A is the SUC per production, and T characterizes the process cycle time. Since the process cycle operates for the $(k+1)h$ time units, the value of T is defined as equal to it [47]. Afterward, the summation of inventory costs is defined as $E(I) = IHC + SUC$.

According to the concepts mentioned above, the Expected Total Cost (ETC) is given by:

$$ETC = E(Q) + E(S) + E(M) + E(I). \tag{26}$$

Moreover, the EPQ can be computed by:

$$Q = p \times T. \tag{27}$$

3.4. Proposed model

To achieve an ESD, Eq. (26) is used as the objective function by employing some constraints. The proposed model, called the integrated-QIM model, is explained:

Min ETC,

s.t.:

$$ARL_0 \geq ARL_0^{\min}, \qquad ARL_1 \leq ARL_1^{\max},$$

$$kh \geq R_{Int}, \qquad nE \leq h,$$

$$n_{\min} \leq n \leq n_{\max}, \quad h_{\min} \leq h \leq h_{\max},$$

$$l_{\min} \leq l \leq l_{\max}, \quad k_{\min} \leq k \leq k_{\max},$$

$$\theta_{\min} \leq \theta \leq \theta_{\max}, \quad \phi_{\min} \leq \phi \leq \phi_{\max},$$

$$n, k \in N^+; \; h, l > 0; \; \theta, \phi \in R, \tag{28}$$

where a minimum value for ARL_0 is considered that maintains a reasonable ARL when a false alarm occurs. Moreover, a maximum value is dedicated to ARL_1 to provide an appropriate ARL for a shifted process. To ensure the continuity of the process, the interval for the PM policy is regulated by $kh \geq R_{Int}$. Constraint $nE \leq h$ guarantees the applicability of obtained solutions by declining sets of design parameters that the time required to take and chart samples of size n goes outside the sampling interval. Moreover, the design parameters n, h, l, k, θ, and ϕ are set between lower and upper bounds (note that n and k are discrete positive values and the others are continuous). These extreme values may be determined as desired bounds by the Decision-Maker (DM) or quality engineers. Briefly speaking, we altered the model in [49] by:

1. Using the ARMA control chart;
2. Considering ARL_0 and ARL_1 as statistical constraints;
3. Ensuring the continuity of process and applicability of solutions respectively by defining two constraints;
4. Adding constraints for the AR and the MA coefficients. Through optimization of the proposed model, six decision variables (i.e., n, h, l, k, θ, and ϕ) are determined.

As shown in the model, there exist four constraints. To transform the model into an unconstrained one, an appropriate penalty function can be defined to converge to the best solutions through optimization. Any violation of the constraints must be added to the objective function such that the solution is pushed back toward the feasible region. The penalized objective function is defined for solution S as follows:

$$fp(S) = ETC(S) \times (1 + viol_0(S) + viol_1(S) + viol_2(S) + viol_3(S)), \tag{29}$$

where $viol_0(S)$=max(0, $1-(ARL_0/ARL_0^{\min})$), $viol_1(S)$ = max(0, $(ARL_1/ARL_1^{\max}) - 1$), $viol_2(S)$=max(0, $1-(kh/R_{Int})$), and $viol_3(S)$=max(0, $(nE/h)-1$) are the violations from the corresponding constraints in Eq. (28). In the next section, an approach to optimize the model is provided.

4. Solution approach

By considering the proposed mathematical model in the previous section, a Non-Linear Programming (NLP) model under four constraints is solved. In the model, three decision variables, including l, θ, and ϕ, are only used for computing ARL values, and the objective function is indirectly affected by these variables. Moreover, solution space is non-convex because both continuous and discrete decision variables exist in the model. According to the mentioned reasons, exact methods cannot be helpful because the model is not solvable or its run requires much more time. Thus, meta-heuristic algorithms can be applied to obtain near-optimal solutions in a reasonable time for such complicated models. Among those, GA and PSO have already been used to solve similar models. Some applications of the GA to solve similar models can be found for ED of the ARMA control chart [20], ED of the VSI control chart [53], and robust ESD of the acceptance control chart [54].

Swarm Algorithms (SAs) are stochastic population-based meta-heuristic algorithms that utilize and imitate the processes of decentralized, self-organized systems. The most successfully applied SAs for solving the models of the control chart design is PSO [55]. It is a stochastic, population-based algorithm with unique searching ability by incorporating local and global searches. The optimization approach in PSO stems from the social behavior of birds, fishes, and so on. In addition to the powerful searching mechanism, some other advantages of its wide applications include computational efficiency and easy execution.

PSO has shown good performance in discontinuous space to solve the non-linear mathematical models of control chart design. For the ESD of the X-bar control chart, Chih et al. [56] optimized their proposed model by adapting PSO to cope with mixed continuous-discrete variables. Moreover, they found PSO more suitable and faster in convergence to the optimality compared to GA for designing the chart. Morabi et al. [57] solved the multi-objective model of this control chart using a hybrid epsilon constraint PSO. Salmasnia et al. [49] utilized PSO to optimize the cost function of integrating EPQ, MP, and VP-T^2 chart design subject to statistical constraints. In another study of integrating MP and control chart design for series systems, a robust optimization approach was presented utilizing PSO to minimize model costs under uncertain parameters [58]. Since the proposed model is also of non-linear type in the presence of both discrete and continuous variables, we employ PSO to optimize our proposed model according to the following steps:

Step 1. *Initialization*: Set the bounds on decision variables according to DM's considerations and the PSO parameters. Represent each solution as a particle by position $x_i^t = [n, h, l, k, \theta, \phi]$ and velocity V_i^t in the iteration t. Then, for each particle $i = 1, \cdots, N$:

- Based on Uniform distribution, generate the initial value of position for each particle using a random vector $x_i \sim U(b_l, b_u)$ and the initial value of velocity according to $V_i \sim U(-|b_u - b_l|, |b_u - b_l|)$, where b_l and b_u are indications of lower and upper limits of the search space, respectively;
- Initialize the *pbest* of each particle equal to its initial position as $pbest_i \rightarrow x_i$ (note that *pbest*, called personal best, is the best value experienced by the ith particle);
- By calculating the penalized objective function according to Eq. (29), if $fp(pbest_i) \leq fp(gbest)$, update $gbest \rightarrow pbest_i$ (the best solution found so far, called global best, is indicated by *gbest*).

Step 2. *Repetition*: Since the behavior of any particle is affected by the current velocity, the personal best, and the global best, it is necessary to update the velocity and the position in each iteration (note that we have $i = 1, \cdots, N$ for each particle (N indicates the population size) and $di = 1, \cdots, n_{di}$ for the dimension of each particle):

- Generate random numbers r_p and r_g from $U(b_l, b_u)$;
- Update the particle velocity by $V_i^t = wV_i^{t-1} + c_1 r_p(pbest_i^{t-1} - x_i^{t-1} + c_2 r_g(gbest_i^{t-1} - x_i^{t-1})$, where c_1 and c_2 are respectively cognition and social learning factors, and w is an inertia weight;
- Update the particle position by $x_i^t = x_i^{t-1} + V_i^t$;
- If $fp(x_i) \leq fp(pbest_i)$, update *pbest* of each particle;
- If $fp(pbest_i) \leq fp(gbest)$, update *gbest*.

Step 3. *Stopping*: If a predetermined number of iterations (m) is achieved or a solution with an acceptable objective function amount is attained, stop. The latest *gbest* holds the best solution achieved. Otherwise, go to Step 2.

Besides continuous decision variables in this study, there are discrete variables including n and k. To transform the discrete variables to the continuous, it is assumed that Rv_1 and Rv_2 are respectively two random digits that belong to the interval (0, 1) corresponding to n and k. Thus, the selected continuous values are transformed into discrete values by using the following formulas:

$$n = \min(n^{\min} + \left[Rv_1(n^{\max} - n^{\min} + 1)\right], n^{\max}), \quad (30)$$

$$k = \min(k^{\min} + \left[Rv_2(k^{\max} - k^{\min} + 1)\right], k^{\max}). \quad (31)$$

The values of cognition and social learning factors are usually considered so that their summation is equal to $c_1 + c_2 = 4$. Costa and Fichera [21] calibrated the factors of PSO for ESD of the ARMA control chart. We also tune the most suitable factors of PSO in Section 5 to better fit the problem at hand.

5. Experimental results

As pointed out earlier, it is aimed to optimize the proposed mathematical model with the cost function of the production cycle subject to some statistical constraints. To indicate the applicability and validate the effectiveness of the proposed model, several numerical examples are studied in this section. In Subsection 5.1, an industrial example is extended for the current study. Then, numerous comparisons are represented in Subsection 5.2 for performance evaluations. In Subsection 5.3, a sensitivity analysis is implemented to investigate the effects of some parameters on the solutions.

5.1. Numerical example

To illustrate the determination of decision variables via optimizing the proposed model, an industrial example is investigated. Consider a company with 125 working days/year, which sells a certain food product to a wholesaler in packages marked with a definite weight. Table 3 shows the nominal values of parameters adapted from [47] and those related

Table 3. Values of the parameters in the numerical example.

Parameter	μ	σ_X^2	u	v	δ	λ	γ
Value	100	10	0.475	0.00	2	0.01	1
Parameter	C_{in}	C_{out}	C_f	C_v	C_Y	C_{pm}	C_{rm}
Value	115	950	1	0.2	200	2400	5000
Parameter	E	T_1	p	d	D	A	B
Value	0.01	1	100	80	10000	60	10

to the ARMA control chart according to [21]. Since $u = 0.475$ and $v = 0.00$, the autocorrelation structure is ARMA (1,0) or AR (1). Moreover, the truncated exponential distribution is investigated as a special case of truncated Weibull by setting $\gamma = 1$. This example is simplified accordingly (the general assumptions are investigated in Subsections 5.2 and 5.3). The number of simulations is set to 500 to calculate $ARLs$ according to the AS procedure. In real applications, some priorities necessitate assigning limits on the decision variables and bounds on constraints to determine feasible space. Bearing in mind such circumstances, the integrated-QIM model is rewritten as follows:

Min ETC,

s.t.:

$$ARL_0 \geq 200, \qquad ARL_1 \leq 10,$$

$$kh \geq 5, \qquad nE \leq h,$$

$$1 \leq n \leq 20, \quad 0.01 \leq h \leq 6,$$

$$0.001 \leq l \leq 5, \quad 1 \leq k \leq 70,$$

$$0.001 \leq \theta \leq 0.999, \quad 0.001 \leq \phi \leq 0.999.$$

Once the PSO algorithm is implemented, an appropriate configuration of its influencing factors should be predetermined. We employed the L_9 orthogonal array experimental design to specify the factors of PSO for the current minimization problem, in which the characteristic of ETC is of type the smaller-the-better. As shown in Table 4, three levels of each factor must be planned. For the L_9 orthogonal array experimental design, nine distinct level configurations of assigning the factors are provided as individual trials according to Table 5. For the ith trial, three optimal objective functions, including ETC_{i1}, ETC_{i2}, and ETC_{i3}, are obtained using the PSO. Then, these outcomes are converted into a Signal-to-Noise (S/N) ratio via the following equation [56]:

$$\left(\frac{S}{N}\right)_i = -10 \log \left(\frac{1}{3}\sum_{j=1}^{3} ETC_{ij}^2\right), \quad i = 1, 2, \cdots, 9. \tag{32}$$

Table 4. Calibration of PSO: Plan of factors and levels adapted from Chih et al. (2011) [56].

Factor	**Level 1**	**Level 2**	**Level 3**
w	0.8	1	1.2
(c_1, c_2)	(1.5, 2.5)	(2.0, 2.0)	(2.5, 1.5)
N	20	50	80
m	50	100	150

Table 5. Experimental design of L_9 orthogonal array for PSO factors.

Trial	w	(c_1, c_2)	N	m	S/N
1	1	1	1	1	–74.0990
2	1	2	2	2	–73.9585
3	1	3	3	3	–73.9114
4	2	1	2	3	–73.9548
5	2	2	3	1	–73.9706
6	2	3	1	2	–74.0323
7	3	1	3	2	–73.9554
8	3	2	1	3	–73.9654
9	3	3	2	1	–73.9998

Table 6. S/N ratios for different levels of PSO factors (the best level for each factor was bolded).

Factor	**Level 1**	**Level 2**	**Level 3**
w	–73.99	–73.99	**–73.97**
(c_1, c_2)	–74.00	**–73.96**	–73.98
N	–74.03	–73.97	**–73.95**
m	–74.02	–73.98	**–73.94**

The results of calculating S/N ratios using Minitab 18 software are recorded in the last column of Table 5. For different levels of PSO factors, Table 6 is constructed by computing S/N ratios. The best level configuration of the factors is specified based on a maximal S/N ratio at each level for each factor since the characteristic of the S/N ratio is of type the-larger-the-better. Considering the main factors plot for S/N ratios from Figure 2 accompanied by the results of Table 6, the best level of each factor is set at $w = 1.2$, (c_1, c_2)=(2, 2), $N = 80$, and $m = 150$. Afterward, this configuration is used

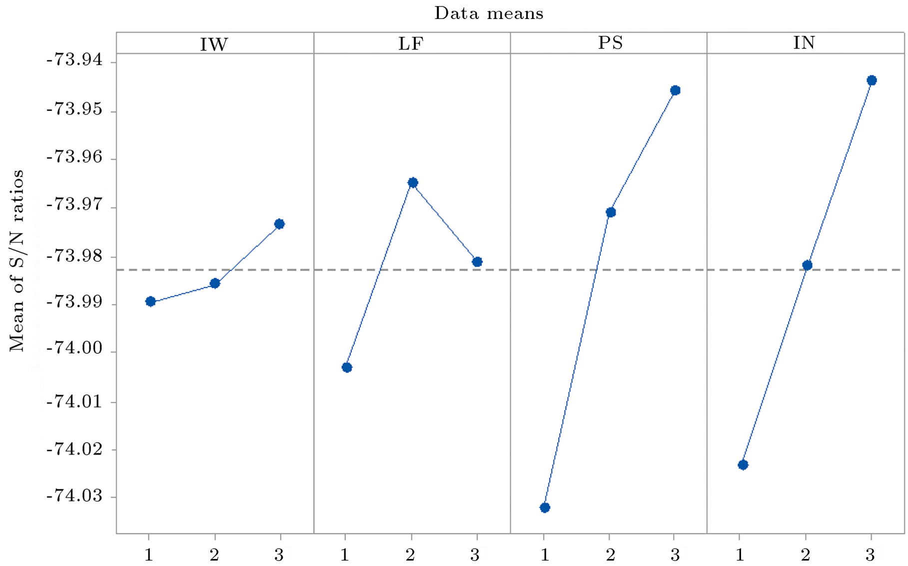

Figure 2. Main factors plot for S/N ratios.

Table 7. Comparison between integrated-QIM and joint-QIM models.

Model	$E(Q)$	$E(S)$	$E(M)$	$E(I)$	ETC	ARL_0	ARL_1
Integrated-QIM	690.63	26.28	2547.59	1669.63	4934.12	279.12	5.02
Joint-QIM	1236.75	5.31	2595.86	1549.19	5387.12	212.44	6.27

to optimally specify the values of decision variables by minimizing the model.

The proposed model, by considering the nominal values of parameters mentioned earlier and the best configuration of PSO factors, is solved by the PSO algorithm under a program coded using MATLAB (R2016b). The optimal solution is attained as follows:

$$\{n, h, l, k, \theta, \phi\} = \{1, 0.227, 2.738, 22, 0.000, 0.617\},$$

$$ETC^*_{QIM} = 4934.12, \quad EPQ = Q^* = 523.50.$$

Accordingly, it is suggested to set the control limits at 2.738. Moreover, a sample size of 1 should be inspected every 0.227 hours (i.e., 817 seconds). After inspecting 22 samples consecutively, PM should be employed if the process remains under control during the whole cycle. Also, ARL_0 and ARL_1 values are respectively obtained as 279.12 and 5.02 which indicate the proper performance of the ARMA chart. In this model, with an ETC of 4934.12, the EPQ of 523.50 is optimally obtained. The total demand for 10000 can be produced after about 20 production cycles (10000 divided by 523.50 equals 19.10).

5.2. Comparisons

To validate the effectiveness of our model for the problem of having the autocorrelation structure among data, extensive comparisons are presented here based on different aspects: (1) Comparison between models with and without integrating quality, inventory, and maintenance concepts, and (2) comparison the control charts for monitoring ARMA process including its special cases.

5.2.1. Comparison between integrated and joint models

This subsection is dedicated to the performance comparison of the integrated-QIM model with a model called joint-QIM. Quality, inventory, and maintenance decision variables are separately optimized according to this model. The procedure for calculating ETC for the joint-QIM is presented in the following steps:

1. The summation of inventory costs, i.e. IHC and SUC, is minimized to obtain an optimal value of process cycle time (T);
2. The decision variables of the ARMA chart, i.e. n, h, l, θ, and ϕ, are optimally attained by minimizing the summation of quality loss cost and sampling cost;
3. Using the values of T and h obtained previously, k is calculated as $k = (T/h) - 1$;
4. Finally, all six decision variables are embedded in the integrated objective function to calculate ETC for the joint-QIM.

Table 7 shows the obtained results of comparing the models. It can be seen that the value of ETC is

Table 8. Comparison between integrated and joint models by considering different ARMA parameters.

ARMA Parameters		ETC		
u	v	Joint-QIM model	Integrated-QIM model	Cost saving
0.00	0.00	5195.48	4909.58	285.90
	0.25	5179.58	4900.18	279.40
	0.50	5169.65	4896.91	272.74
	0.75	5159.44	4890.90	268.54
0.25	0.00	5208.04	4922.18	285.86
	0.25	5192.36	4912.05	280.31
	0.50	5177.88	4899.11	278.77
	0.75	5169.26	4894.53	274.73
0.50	0.00	5378.22	4936.36	441.86
	0.25	5368.91	4923.15	445.76
	0.50	5196.26	4909.78	286.48
	0.75	5176.31	4903.75	272.56
0.75	0.00	5388.57	5055.66	332.91
	0.25	5382.90	4957.23	425.67
	0.50	5372.82	4940.58	432.24
	0.75	5195.48	4910.47	285.01

decreased using the integrated model. Moreover, ARL values are improved in comparison to those of the joint-QIM. We also provided the details of the cost function. The total expected costs of quality loss and maintenance are decreased using the integrated model. Instead, the joint-QIM provides lower costs of sampling and inventory.

For more investigations, we compare these models by considering different AR and MA parameters for the numerical example presented previously. The underlying process includes AR (1) when $u \neq 0$ and $v = 0$, MA (1) when $u = 0$ and $v \neq 0$, and ARMA (1,1) when $u \neq 0$ and $v \neq 0$. Accordingly, 16 trials were performed. As shown in Table 8, the results of cost savings confirm that the integration of quality, inventory, and maintenance concepts leads to lower ETC than the joint-QIM. The lowest cost-saving, i.e. 272.56, is experienced in the 12th trial where ETC values by integrated and joint models are respectively 4903.75 and 5176.31. Investigating the effects of ARMA parameters indicates that increasing u and decreasing v lead to higher ETC values.

5.2.2. Comparison among monitoring techniques

The better statistical performance of the ARMA control chart for monitoring autocorrelated data was confirmed in [16] compared to EWMAST and SCC charts. However, it is necessary to investigate their performance in the framework of ESD. These monitoring techniques have different mechanisms, and their performance depends on the chosen values of the design parameters. We previously described how to design the ARMA control chart. Designing SCC needs to fit a model on a series of data to obtain uncorrelated ones. Then, the traditional monitoring technique under the independence assumption is applied to the uncorrelated data [10]. Using SCC in the framework of the integrated-QIM model, the decision variables include n, h, l, and k. The design of EWMAST requires setting an integer m for approximating the variance and a smoothing coefficient λ_E as well as n, h, l, and k. We set $m = 25$ and $\lambda_E = 0.2$ as suggested in [17].

Table 9. Comparison among various control charts using ETC values of the integrated-QIM.

ARMA parameters		Control charts		
u	v	SCC	EWMAST	ARMA
–0.950	0.000	4902.24	4888.26	**4884.49**
–0.475	0.000	4927.20	4902.41	**4896.43**
0.475	0.000	5031.38	4941.31	**4938.07**
0.950	0.000	5153.89	5118.12	**5076.99**
0.475	–0.900	5014.58	4956.94	**4952.35**
0.950	0.450	5186.57	5026.22	**4988.05**
0.950	–0.900	5802.70	5035.80	**4999.65**

To confirm the economic and statistical performance of our model, the results of applying ARMA, EWMAST, and SCC monitoring techniques are compared. We use the information presented in Table 3 by considering different combinations of ARMA coefficients from [16] to include AR (1) and ARMA (1,1). For the 6th and 7th combinations of $[u, v]$, infeasible solutions are obtained since the constraint of ARL_1 is not satisfied. Therefore, we relaxed the second constraint in the proposed model to get feasible solutions. Table 9 shows the results. The lowest ETC is bolded for different comparisons. It can be deduced

Table 10. Levels plan of factors for the sensitivity analysis.

Factor	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Notation	δ	u	v	λ	γ	C_{in}	C_{out}	C_f
Level 1	1	0.00	0.00	0.01	0.5	50	100	0.5
Level 2	4	0.25	0.25	0.03	1.0	115	950	1.0
Level 3	–	0.75	0.75	0.05	2.0	700	1500	4.0
Factor	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**
Notation	C_v	C_{pm}	C_{rm}	C_Y	E	T_1	A	B
Level 1	0.2	1000	2500	50	0.01	0.1	30	5
Level 2	0.5	2400	5000	200	0.05	1	60	10
Level 3	2.0	4000	7500	500	0.20	2	120	20

that using the ARMA monitoring technique in the proposed model generally results in the lowest ETC. For $[u, v]$=[0.95, 0.45], the optimal solutions of ARMA, EWMAST, and SCC control charts are respectively attained in detail as follows:

$$\{n, h, l, k, \theta, \phi\} = \{1, 0.11, 2.39, 46, 0, 0.01\},$$

$$ETC^*_{QIM} = 4988, \quad EPQ = Q^* = 517,$$

$$ARL_0 = 206, \quad ARL_1 = 10,$$

$$\{n, h, l, k, \lambda_E, m\} = \{1, 0.2, 2.01, 25, 0.2, 25\},$$

$$ETC^*_{QIM} = 5026, \quad EPQ = Q^* = 520,$$

$$ARL_0 = 202, \quad ARL_1 = 16,$$

$$\{n, h, l, k\} = \{1, 0.1, 2.83, 50\},$$

$$ETC^*_{QIM} = 5187, \quad EPQ = Q^* = 510,$$

$$ARL_0 = 206, \quad ARL_1 = 136.$$

5.3. Sensitivity analysis

The effects of various parameters on the solution of the proposed model are studied via sensitivity analysis. This is performed using the orthogonal-array Taguchi design and multiple regression. ETC is considered a dependent variable, and sixteen independent variables are treated as factors. Their level plans are shown in Table 10.

Table 11 shows how independent variables are assigned to the trials of the L_{54} array. For each trial, the PSO is used to obtain the optimal solutions of the model. We changed the upper bound of the second constraint from 10 to 40 in the proposed model to get feasible solutions in all trials. The output of the optimizations is recorded in Table 12. Minitab 18 software is used to analyze the results. Table 13 shows the output for ETC. Assuming a significance level of 0.1, the parameters δ, λ, γ, C_{in}, C_{pm}, C_{rm}, A, and B are significant. For each factor in Table 14, the difference between the two levels with the highest and lowest values is calculated and recorded in the delta row. These values indicate how much the change in the levels affects ETC values on average. Accordingly, the ranking is done from the largest delta value to the smallest. Among the significant factors, the greatest impact on ETC can be expected by changing C_{in} levels.

The effects of different levels of factors on ETC on average are also shown in Figure 3. Since ETC is of type the-smaller-the-better, the levels that result in the lowest mean of ETC are preferred. A larger δ generally reduces ETC because it can be easily detected by the control chart and thus can be fixed just in time. Similarly, a higher value of γ causes a reduction in ETC . In contrast, larger values of λ, C_{in}, C_{pm}, C_{rm}, A, and B lead to higher ETC values.

The adequacy of the model can be visually checked from Figure 4. Figure 4(a) is a graphical plot of normal probabilities versus residuals. It is seen that the points appropriately fit on the line. The p-value for the Anderson-Darling (AD) test of residuals is 0.177. This value is greater than the significance level of 0.05. Thus, there is no evidence to reject the normality assumption. It appears from Figure 4(b) that the residuals are randomly scattered around zero. In other words, there is no evidence of non-constant variance or missing terms in the model. However, residuals related to trials 16, 20, and 41 are outliers since their standardized values, respectively 2.52, 3.05, and -2.23, are out of the bounds of $[-2, +2]$ in the significance level of 0.05. We left it without more investigations. From Figure 4(c), the symmetry of the distribution is inferred to some extent. Figure 4(d) shows no evidence that the residuals are correlated with one another. Therefore, the adequacy of the model is confirmed.

Table 11. Levels of factors for generated trials with the Taguchi L_{54} design.

Trial	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
3	1	1	1	1	1	1	1	1	3	3	3	3	3	3	3	3
4	1	1	2	2	2	2	2	2	1	1	1	1	1	1	2	3
5	1	1	2	2	2	2	2	2	2	2	2	2	2	2	3	1
6	1	1	2	2	2	2	2	2	3	3	3	3	3	3	1	2
7	1	1	3	3	3	3	3	3	1	1	1	1	1	1	3	2
8	1	1	3	3	3	3	3	3	2	2	2	2	2	2	1	3
9	1	1	3	3	3	3	3	3	3	3	3	3	3	3	2	1
10	1	2	1	1	2	2	3	3	1	1	2	2	3	3	1	1
11	1	2	1	1	2	2	3	3	2	2	3	3	1	1	2	2
12	1	2	1	1	2	2	3	3	3	3	1	1	2	2	3	3
13	1	2	2	2	3	3	1	1	1	1	2	2	3	3	2	3
14	1	2	2	2	3	3	1	1	2	2	3	3	1	1	3	1
15	1	2	2	2	3	3	1	1	3	3	1	1	2	2	1	2
16	1	2	3	3	1	1	2	2	1	1	2	2	3	3	3	2
17	1	2	3	3	1	1	2	2	2	2	3	3	1	1	1	3
18	1	2	3	3	1	1	2	2	3	3	1	1	2	2	2	1
19	1	3	1	2	1	3	2	3	1	2	1	3	2	3	1	1
20	1	3	1	2	1	3	2	3	2	3	2	1	3	1	2	2
21	1	3	1	2	1	3	2	3	3	1	3	2	1	2	3	3
22	1	3	2	3	2	1	3	1	1	2	1	3	2	3	2	3
23	1	3	2	3	2	1	3	1	2	3	2	1	3	1	3	1
24	1	3	2	3	2	1	3	1	3	1	3	2	1	2	1	2
25	1	3	3	1	3	2	1	2	1	2	1	3	2	3	3	2
26	1	3	3	1	3	2	1	2	2	3	2	1	3	1	1	3
27	1	3	3	1	3	2	1	2	3	1	3	2	1	2	2	1
28	2	1	1	3	3	2	2	1	1	3	3	2	2	1	1	1
29	2	1	1	3	3	2	2	1	2	1	1	3	3	2	2	2
30	2	1	1	3	3	2	2	1	3	2	2	1	1	3	3	3
31	2	1	2	1	1	3	3	2	1	3	3	2	2	1	2	3
32	2	1	2	1	1	3	3	2	2	1	1	3	3	2	3	1
33	2	1	2	1	1	3	3	2	3	2	2	1	1	3	1	2
34	2	1	3	2	2	1	1	3	1	3	3	2	2	1	3	2
35	2	1	3	2	2	1	1	3	2	1	1	3	3	2	1	3
36	2	1	3	2	2	1	1	3	3	2	2	1	1	3	2	1
37	2	2	1	2	3	1	3	2	1	2	3	1	3	2	1	1
38	2	2	1	2	3	1	3	2	2	3	1	2	1	3	2	2
39	2	2	1	2	3	1	3	2	3	1	2	3	2	1	3	3
40	2	2	2	3	1	2	1	3	1	2	3	1	3	2	2	3
41	2	2	2	3	1	2	1	3	2	3	1	2	1	3	3	1
42	2	2	2	3	1	2	1	3	3	1	2	3	2	1	1	2
43	2	2	3	1	2	3	2	1	1	2	3	1	3	2	3	2
44	2	2	3	1	2	3	2	1	2	3	1	2	1	3	1	3
45	2	2	3	1	2	3	2	1	3	1	2	3	2	1	2	1

Table 11. Levels of factors for generated trials with the Taguchi L_{54} design (continued).

Trial	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q
46	2	3	1	3	2	3	1	2	1	3	2	3	1	2	1	1
47	2	3	1	3	2	3	1	2	2	1	3	1	2	3	2	2
48	2	3	1	3	2	3	1	2	3	2	1	2	3	1	3	3
49	2	3	2	1	3	1	2	3	1	3	2	3	1	2	2	3
50	2	3	2	1	3	1	2	3	2	1	3	1	2	3	3	1
51	2	3	2	1	3	1	2	3	3	2	1	2	3	1	1	2
52	2	3	3	2	1	2	3	1	1	3	2	3	1	2	3	2
53	2	3	3	2	1	2	3	1	2	1	3	1	2	3	1	3
54	2	3	3	2	1	2	3	1	3	2	1	2	3	1	2	1

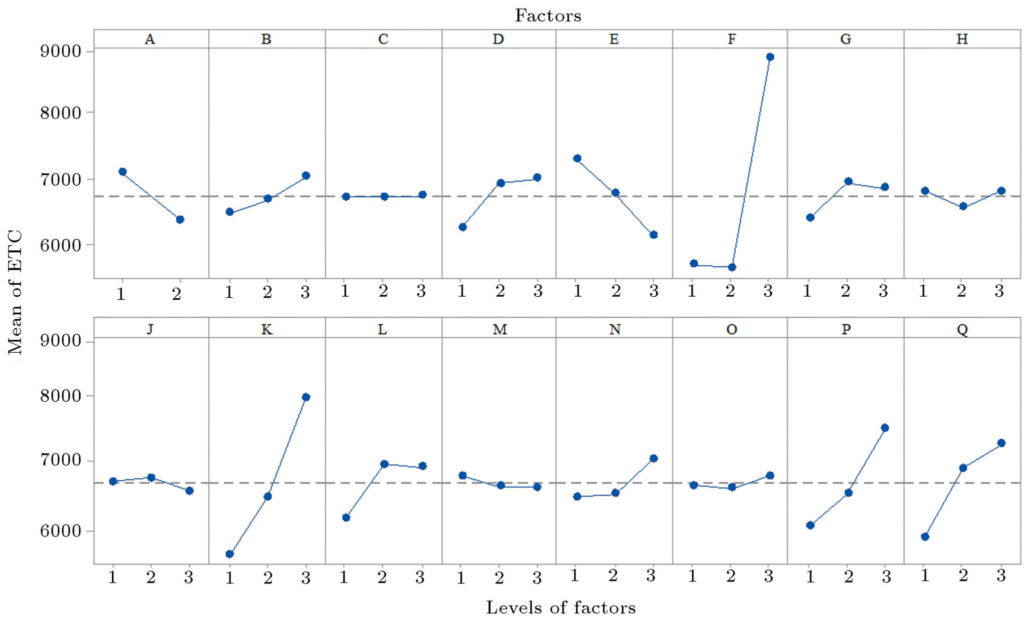

Figure 3. Main effects of sixteen independent factors on ETC .

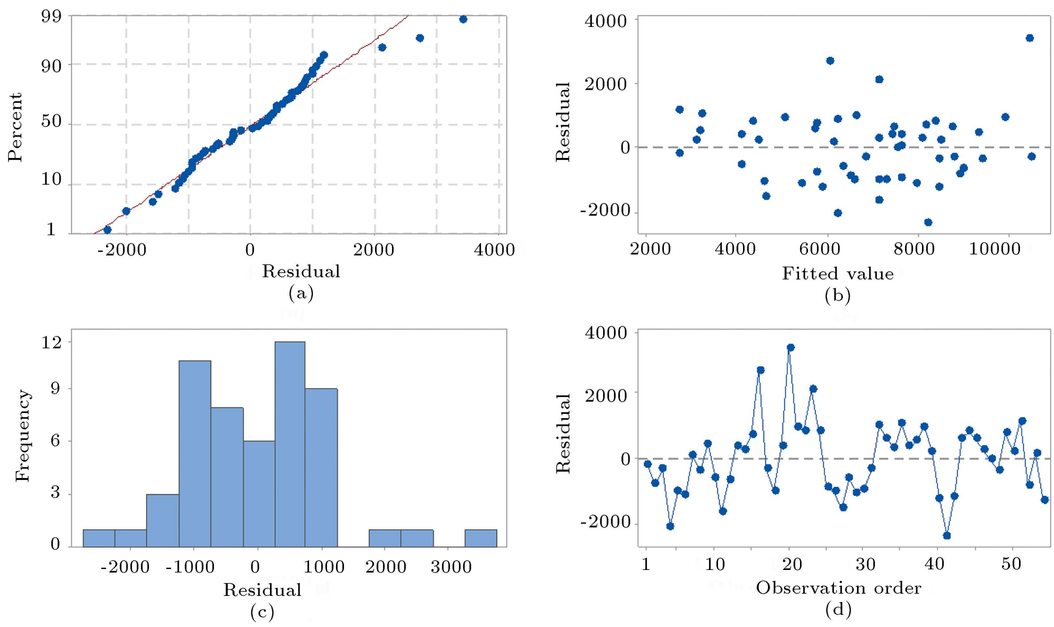

Figure 4. Residual plots: (a) Normal probability plot, (b) residuals versus fitted values, (c) histogram, and (d) residuals versus observation order

Table 12. Optimal values of functions and parameters for the generated trials with the Taguchi L_{54} design.

Trial	n	h	l	k	θ	ϕ	ETC	ARL_0	ARL_1
1	18	0.36	2.82	14	0.00	0.67	2504.11	263.43	9.75
2	3	0.71	2.81	7	0.09	0.83	4962.00	272.84	10.51
3	1	2.50	2.85	2	0.00	0.80	8425.27	362.29	9.41
4	1	0.16	2.82	31	0.03	0.80	4159.14	239.96	6.21
5	1	0.13	2.91	53	0.01	0.83	6120.72	371.11	6.47
6	1	0.25	2.90	20	0.14	0.77	6812.36	281.70	7.82
7	3	0.24	2.83	21	0.00	0.70	7675.94	223.35	3.13
8	1	0.10	3.52	50	0.00	0.91	8025.83	938.37	3.60
9	1	0.33	3.01	15	0.14	0.63	9778.77	381.78	6.67
10	10	2.51	3.27	2	0.08	0.56	3515.04	744.80	37.65
11	9	0.37	2.75	14	0.29	0.67	5454.65	207.03	16.53
12	1	0.12	2.73	46	0.03	0.86	8310.50	340.80	13.51
13	3	1.10	3.42	5	0.20	0.61	7966.69	850.76	37.11
14	1	0.10	3.68	50	0.10	0.85	8742.72	1076.67	18.75
15	2	0.32	2.89	16	0.38	0.76	8863.57	243.40	17.58
16	12	2.51	3.14	2	0.31	0.60	8709.08	555.68	20.34
17	1	0.10	3.20	50	0.00	0.89	6494.10	613.84	4.10
18	1	0.13	3.01	48	0.00	0.86	5589.80	384.61	3.81
19	1	0.10	2.08	50	0.07	0.95	7793.65	212.03	29.69
20	11	5.01	2.45	1	0.10	0.75	13801.74	201.48	39.69
21	1	0.10	2.25	50	0.12	0.92	10824.46	207.06	31.98
22	4	0.36	2.34	14	0.10	0.91	7074.87	232.68	28.41
23	3	0.63	2.60	8	0.15	0.81	9194.70	286.59	34.49
24	1	0.10	2.27	50	0.10	0.94	5154.64	213.40	27.55
25	9	2.54	3.33	2	0.24	0.75	5620.90	757.60	23.07
26	1	0.34	3.58	15	0.32	0.81	6287.75	1018.91	34.61
27	9	1.68	3.26	3	0.13	0.78	3089.80	692.93	15.28
28	1	0.10	3.61	50	0.00	0.22	5700.61	1006.24	1.44
29	1	0.28	3.14	18	0.95	0.63	3557.90	495.52	4.77
30	1	0.36	3.54	14	0.12	0.38	6689.60	956.43	1.71
31	1	0.10	3.43	50	0.00	1.00	10125.33	898.60	1.30
32	1	0.20	3.05	25	0.72	0.54	7619.16	410.48	3.14
33	1	0.36	2.91	14	0.16	0.28	8049.21	261.20	1.38
34	1	1.45	3.22	4	0.00	0.58	7434.74	618.33	1.02
35	1	5.00	4.46	1	0.28	0.27	4261.63	1200.19	3.92
36	1	1.00	2.80	5	0.36	0.39	4486.87	209.59	1.70
37	1	0.22	3.11	23	0.18	0.35	3706.19	488.52	1.74
38	1	1.01	2.93	5	0.03	0.00	5992.26	303.96	1.14
39	1	0.56	3.22	10	0.00	0.17	4708.67	627.35	1.29
40	1	0.31	2.84	16	0.49	0.39	7211.32	208.91	2.05
41	1	1.08	2.93	10	0.00	0.29	5817.19	287.44	1.15
42	1	0.33	3.33	15	0.53	0.66	4270.00	755.27	3.16
43	2	0.63	3.69	8	0.00	0.59	9316.07	1039.87	1.21
44	1	0.10	3.88	50	0.00	0.56	9170.05	1145.71	1.25
45	1	0.20	3.78	25	0.36	0.62	6265.97	1089.08	2.59

Table 12. Optimal values of functions and parameters for the generated trials with the Taguchi L_{54} design (continued).

Trial	n	h	l	k	θ	ϕ	ETC	ARL_0	ARL_1
46	5	0.28	3.34	18	0.68	0.56	8372.14	835.32	4.39
47	1	0.33	3.01	15	0.23	0.08	7495.27	450.49	1.47
48	1	0.26	3.21	19	0.46	0.44	9043.79	722.90	2.94
49	1	0.56	3.70	9	0.51	0.44	6522.14	1069.69	4.09
50	1	2.46	2.74	3	0.00	0.02	3314.77	213.47	1.05
51	1	1.67	3.21	3	0.37	0.51	3881.96	710.71	2.93
52	1	0.13	3.35	54	0.00	0.23	8081.50	749.39	1.28
53	1	0.10	2.99	50	0.00	0.30	6265.11	367.65	1.16
54	1	0.20	2.94	31	0.00	0.27	4610.97	334.92	1.18

Table 13. ANOVA for ETC.

Source	D.F.	Adj. S.S.	Adj. M.S.	F	P-value
Model*	8	206518199	25814775	18.48	0.000
A	1	6672799	6672799	4.78	0.034
D	1	5003314	5003314	3.58	0.065
E	1	12282596	12282596	8.79	0.005
F	1	88713102	88713102	63.52	0.000
K	1	51177437	51177437	36.64	0.000
L	1	5289052	5289052	3.79	0.058
P	1	19539063	19539063	13.99	0.001
Q	1	17840836	17840836	12.77	0.001
Residual error	45	62852438	1396721	–	–
Total	53	269370637	–	–	–

*ETC$=-0.974 - 703A + 373D - 584E + 1570F + 1192K + 383L + 737P + 704Q$

Table 14. Effects of independent parameters on ETC.

Factor	A	B	C	D	E	F	G	H
Level 1	7072	6466	6714	6246	7286	5690	6381	6808
Level 2	6369	6672	6717	6924	6758	5643	6929	6555
Level 3	–	7024	6731	6992	6118	8829	6852	6799
Delta	703	558	17	746	1168	3186	548	253
Rank	8	10	16	7	5	1	11	12
Factor	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**
Level 1	6749	5631	6197	6829	6516	6686	6063	5901
Level 2	6810	6516	7002	6675	6552	6644	6564	6952
Level 3	6603	8016	6964	6659	7094	6832	7536	7309
Delta	207	2385	805	170	579	188	1473	1408
Rank	13	2	6	15	9	14	3	4

6. Conclusions

In this study, we aimed to bridge the gap between traditional perfect production models and real production conditions. Therefore, we proposed a model for the imperfect production process by integrating the triple concepts of Statistical Process Monitoring (SPM), Maintenance Policy (MP), and Economic Production Quantity (EPQ). In some processes, the assumption that the data derived from the process are independent may not be true. The existence of autocorrelation among those data can lead to significant effects on the statistical performances of control charts if ignored.

Therefore, in contrast to most of the research in the literature, we considered the first-order Autoregressive Moving Average (ARMA) structure for the underlying process as well as its special cases, i.e. AR and MA. Since Jiang et al. [16] showed that the ARMA control chart, compared to EWMAST, and Special Cause Chart (SCC) charts, was more effective for monitoring a process with autocorrelated data, we also applied the ARMA chart in this study.

For optimizing the proposed integrated economic-statistical model, a Particle Swarm Optimization (PSO) algorithm was used to determine decision variables. Finally, this procedure was illustrated through an industrial example, some comparisons were made for validation, and the sensitivity analysis was finally implemented to distinguish the effects of the parameters on the objective function.

The results of comparative studies indicated that: (1) the integration of quality, inventory, and maintenance concepts leads to the significantly reduced Expected Total Cost (ETC) in comparison to the joint-QIM model, (2) using the ARMA chart as a monitoring method in the proposed model generally results in the lowest ETC in comparison to EWMAST and SCC control charts, and (3) increasing AR and decreasing MA constants of the underlying ARMA process have adverse effects on the values of ETC. Besides, the results of sensitivity analysis confirmed that the shift magnitude in the process mean and the shape parameter of the truncated Weibull distribution have inverse relationships with ETC, whereas larger values of the scale parameter of that distribution, the quality loss cost in the in-control state, the preventive maintenance cost, the reactive maintenance cost, the setup cost, and the inventory holding cost per unit time lead to the higher ETC values.

In future research, it is suggested to investigate other monitoring charts reviewed by Thaga and Sivasamy [9] for similar models. Designing attribute control charts with different sampling schemes, similar to [59], can be extended under the integrated model. Uncertainty in the model can be treated using a robust optimization approach applied in [54] and [60]. Recently, the ARMA control chart was applied to a ten-scenario model [61]. For the time being, we are trying to extend the Acceptance Control Chart (ACC) for autocorrelated processes.

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