



Sharif University of Technology

Scientia Iranica

Transactions D: Computer Science & Engineering and Electrical Engineering

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Guest editorial: Special issue on collective behavior of nonlinear dynamical networks

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Collective behaviors of dynamical networks are the focus of intense research in various fields of science [1,2]. Dynamical networks can be considered as populations of interacting nonlinear systems from which complex spatiotemporal patterns can emerge [3–5]. For instance, one of such emerging patterns is synchronization, which refers to the strongest form of network cooperative dynamics [6,7]. Each individual in the network tends to share common rhythms and the same dynamical behavior in the synchronization state [8]. The emergence of the traveling and propagating waves, especially spiral waves, is another example of fascinating collective behaviors of dynamical networks [9]. The other important examples are associated with the simultaneous coexistence of both incoherent and coherent states in networks, which is called chimera state [10,11].

Various studies in the literature have investigated the mechanism of appearance of collective behavior in dynamical networks, numerically and analytically [12–15]. Generally, they have pointed out three main factors in the emergence of collective behaviors [16,17]: 1) the dynamics of the individual system in each node, 2) the coupling type and strength, and 3) the topology of the network. Different types of complex nonlinear systems can be located in each network node, such as systems expressed by ordinary differential (or difference) equations [18] and fractional-

order systems [19–21]. Finding the proper coupling strength that regulates the interactions of dynamical networks is another important point in this field of research [22,23]. The structure of a network can also affect the functions of emerging collective behaviors [24]. The dynamical networks can be identical or non-identical [25], weighted or unweighted [26], directed or undirected [27], time-varying [28,29] or fixed in different types of topologies such as regular, random [30], scale-free [31,32], small-world [33], etc. Many recent hot topics are related to collective behavior of nonlinear dynamical networks. Some examples are “resilience indicators of complex networks” [34], “multilayer and multiplex networks” [35,36], “coherence resonance” [37,38], “explosive synchronization” [39,40], “cluster synchronization” [41,42], “chimera states in networks” [43,44], “wave propagation in networks” [45], and “spiral waves in networks” [46,47].

This mini special issue reviews the current state of the art in the research on collective behaviors of the dynamical networks, which is the key factor in reaching more accurate network models as well as enriching our knowledge about the function of the natural networks. In [48], the authors investigated the slowness in ischemic stroke patients. A Trier Social Stress Test (TSST) is used to reveal the slowness of the biological system. The slowness of dynamics is calculated for the ECG of healthy individuals and

patients with ischemic stroke. Ten healthy individuals and nine ischemic stroke patients are studied. Six early warning indicators based on slowness and variability are used in this study. The indicators are applied to the RR interval of individuals in four stages. Heart rate variations are studied as another measure of slowness for the dynamics. The results reveal that there is no significant difference in the slowness of healthy and patient cases.

In [49], the authors considered the network of Lorenz systems with time-varying links to study synchronization and chimera patterns. It is assumed that the non-local connections of the network are switched on/off with a specified period, while the local links are fixed. The ratio of time of on to off links is called the discontinuity rate. They investigated the network for different periods and discontinuity rates analytically and numerically. They reported a new pattern called intermittent transient chimera in which the chimera and the synchronization changed alternatively in time. The results show that when the continuous links change to switching, the coupling strength needed for synchronization increases. Furthermore, as the discontinuity rate decreases, the region for observing chimeras is enlarged.

The authors in [50] introduced an adaptive coupling for the network of randomly coupled Kuramoto-Sakaguchi oscillators. The adaptive coupling allows for incorporation of the dynamics of the oscillators in the strength of connections. They reported the emergence of several synchronized, cluster synchronized, and partial synchronized patterns relying on the coupling. The effect of the random delusion of the links was also investigated. The results indicated that as the number of links decreased, the incoherency increased in the network. However, the authors found some sparse topologies which point to stable in-phase synchronization.

A modified Fitzhugh and Rinzel neuron model was proposed in [51] upon introducing the magnetic flux variable. The authors analyzed the stability of the novel model by calculating the equilibrium points in the presence and absence of electromagnetic induction. Furthermore, the bifurcation diagrams and the Lyapunov exponents were derived and it was shown that the model was multi-stable and could exhibit diverse firing patterns. The authors also investigated the effects of coupling strength as well as the frequency and amplitude of the external stimuli on the behaviors of a two-dimensional network of the proposed model. They compared wave propagation in the network with and without electromagnetic induction.

Thus, this special issue provides a brief perspective of current research on the collective behavior of dynamical networks and we hope that the related researchers in this field find it useful. We wish to

express our appreciation to the authors of all the papers in this special issue for the excellent contributions as well as the reviewers for their high-quality work on reviewing the manuscripts.

References

1. Li, Z., Duan, Z., Chen, G., and Huang, L. "Consensus of multiagent systems and synchronization of complex networks: A unified viewpoint", *IEEE Trans. Circuits Syst. I Regul. Pap.*, **57**, pp. 213–224 (2009).
2. Perc, M., Gómez-Gardenes, J., Szolnoki, A., Flórida, L.M., and Moreno, Y. "Evolutionary dynamics of group interactions on structured populations: a review", *J. R. Soc. Interface*, **10**, p. 20120997 (2013).
3. Boccaletti, S., Latora, V., Moreno, Y., Chavez, M., and Hwang, D.-U. "Complex networks: Structure and dynamics", *Phys. Rep.*, **424**, pp. 175–308 (2006).
4. Estrada, E. "Introduction to complex networks: structure and dynamics", in: *Evol. Equations Appl. Nat. Sci.*, Springer, pp. 93–131 (2015).
5. Gao, Z.-K., Small, M., and Kurths, J. "Complex network analysis of time series", *EPL (Europhys. Lett.)*, **116**, p. 50001 (2017).
6. Arenas, A., Díaz-Guilera, A., Kurths, J., Moreno, Y., and Zhou, C. "Synchronization in complex networks", *Phys. Rep.*, **469**, pp. 93–153 (2008).
7. Pikovsky, A., Kurths, J., Rosenblum, M., and Kurths, J. "Synchronization: a universal concept in nonlinear sciences", Cambridge university press (2003).
8. Boccaletti, S., Kurths, J., Osipov, G., Valladares, D., and Zhou, C. "The synchronization of chaotic systems", *Phys. Rep.*, **366**, pp. 1–101 (2002).
9. Huang, X., Xu, W., Liang, J., Takagaki, K., Gao, X., and Wu, J.-Y. "Spiral wave dynamics in neocortex", *Neuron*, **68**, pp. 978–990 (2010).
10. Parastesh, F., Jafari, S., Azarnoush, H., Shahriari, Z., Wang, Z., Boccaletti, S., and Perc, M., *Chimeras*, *Phys. Rep.*, **898**, pp. 1–114 (2020).
11. Abrams, D.M. and Strogatz, S.H. "Chimera states for coupled oscillators", *Phys. Rev. Lett.*, **93**, p. 174102 (2004).
12. Lu, J. and Chen, G. "A time-varying complex dynamical network model and its controlled synchronization criteria", *IEEE Trans. Autom. Control*, **50**, pp. 841–846 (2005).
13. Mobayen, S., Kingni, S.T., Pham, V.-T., Nazarimehr, F., and Jafari, S. "Analysis, synchronisation and circuit design of a new highly nonlinear chaotic system", *Int. J. Syst. Sci.*, **49**, pp. 617–630 (2018).
14. Rajagopal, K., Khalaf, A.J.M., Parastesh, F., Moroz, I., Karthikeyan, A., and Jafari, S. "Dynamical behavior and network analysis of an extended Hindmarsh-Rose neuron model", *Nonlinear Dyn.*, **98**, pp. 477–487 (2019).

15. Tavazoei, M.S. and Haeri, M. "Synchronization of chaotic fractional-order systems via active sliding mode controller", *Physica A*, **387**, pp. 57–70 (2008).
16. Belykh, I., Hasler, M., Lauret, M., and Nijmeijer, H. "Synchronization and graph topology", *Int. J. Bifurcation Chaos*, **15**, pp. 3423–3433 (2005).
17. Pecora, L.M. and Carroll, T.L. "Synchronization in chaotic systems", *Phys. Rev. Lett.*, **64**, p. 821 (1990).
18. Panahi, S., Nazarimehr, F., Jafari, S., Sprott, J.C., Perc, M., and Repnik, R. "Optimal synchronization of circulant and non-circulant oscillators", *Appl. Math. Comput.*, **394**, p. 125830 (2021).
19. Asheghan, M.M., Beheshti, M.T.H., and Tavazoei, M.S. "Robust synchronization of perturbed Chen's fractional-order chaotic systems", *Commun. Nonlinear Sci. Numer. Simul.*, **16**, pp. 1044–1051 (2011).
20. Asheghan, M.M., Míguez, J., Hamidi-Beheshti, M.T., and Tavazoei, M.S. "Robust outer synchronization between two complex networks with fractional order dynamics", *Chaos*, **21**, p. 033121 (2011).
21. Bao, H.-B. and Cao, J.-D. "Projective synchronization of fractional-order memristor-based neural networks", *Neural Networks*, **63**, pp. 1–9 (2015).
22. Pecora, L.M. and Carroll, T.L. "Master stability functions for synchronized coupled systems", *Phys. Rev. Lett.*, **80**, p. 2109 (1998).
23. Nazarimehr, F., Panahi, S., Jalili, M., Perc, M., Jafari, S., and Ferčec, B. "Multivariable coupling and synchronization in complex networks", *Appl. Math. Comput.*, **372**, p. 124996 (2020).
24. Belykh, I., De Lange, E., and Hasler, M. "Synchronization of bursting neurons: What matters in the network topology", *Phys. Rev. Lett.*, **94**, p. 188101 (2005).
25. Osipov, G.V., Pikovsky, A.S., Rosenblum, M.G., and Kurths, J. "Phase synchronization effects in a lattice of nonidentical Rössler oscillators", *Phys. Rev. E*, **55**, p. 2353 (1997).
26. Chavez, M., Hwang, D.-U., Amann, A., Hentschel, H., and Boccaletti, S. "Synchronization is enhanced in weighted complex networks", *Phys. Rev. Lett.*, **94**, p. 218701 (2005).
27. Rosenblum, M.G. and Pikovsky, A.S. "Detecting direction of coupling in interacting oscillators", *Phys. Rev. E*, **64**, p. 045202 (2001).
28. Belykh, I.V., Belykh, V.N., and Hasler, M. "Blinking model and synchronization in small-world networks with a time-varying coupling", *Physica D*, **195**, pp. 188–206 (2004).
29. Sorrentino, F. and Ott, E. "Adaptive synchronization of dynamics on evolving complex networks", *Phys. Rev. Lett.*, **100**, p. 114101 (2008).
30. Porfiri, M., Stilwell, D.J., and Boltt, E.M. "Synchronization in random weighted directed networks", *IEEE Trans. Circuits Syst. I Regul. Pap.*, **55**, p. 3170–3177 (2008).
31. Wang, Q., Chen, G., and Perc, M. "Synchronous bursts on scale-free neuronal networks with attractive and repulsive coupling", *PLoS One*, **6**, p. e15851 (2011).
32. Wang, Q., Perc, M., Duan, Z., and Chen, G. "Synchronization transitions on scale-free neuronal networks due to finite information transmission delays", *Phys. Rev. E*, **80**, p. 026206 (2009).
33. Lu, J., Yu, X., Chen, G., and Cheng, D. "Characterizing the synchronizability of small-world dynamical networks", *IEEE Trans. Circuits Syst. I Regul. Pap.*, **51**, pp. 787–796 (2004).
34. Reggiani, A. "Accessibility, connectivity and resilience in complex networks", in *Accessibility Analysis and Transport Planning*, Edward Elgar Publishing (2012).
35. Boccaletti, S., Bianconi, G., Criado, R., Del Genio, C.I., Gómez-Gardenes, J., Romance, M., Sendina-Nadal, I., Wang, Z., and Zanin, M. "The structure and dynamics of multilayer networks", *Phys. Rep.*, **544**, pp. 1–122 (2014).
36. Rakshit, S., Majhi, S., Bera, B.K., Sinha, S., and Ghosh, D. "Time-varying multiplex network: Intralayer and interlayer synchronization", *Phys. Rev. E*, **96**, p. 062308 (2017).
37. Pikovsky, A.S. and Kurths, J. "Coherence resonance in a noise-driven excitable system", *Phys. Rev. Lett.*, **78**, p. 775 (1997).
38. Zakharova, A., Vadivasova, T., Anishchenko, V., Koseska, A., and Kurths, J. "Stochastic bifurcations and coherencelike resonance in a self-sustained bistable noisy oscillator", *Phys. Rev. E*, **81**, p. 011106 (2010).
39. Zhang, X., Boccaletti, S., Guan, S., and Liu, Z. "Explosive synchronization in adaptive and multilayer networks", *Phys. Rev. Lett.*, **114**, p. 038701 (2015).
40. Zou, Y., Pereira, T., Small, M., Liu, Z., and Kurths, J. "Basin of attraction determines hysteresis in explosive synchronization", *Phys. Rev. Lett.*, **112**, p. 114102 (2014).
41. Dahms, T., Lehnert, J., and Schöll, E. "Cluster and group synchronization in delay-coupled networks", *Phys. Rev. E*, **86**, p. 016202 (2012).
42. Pecora, L.M., Sorrentino, F., Hagerstrom, A.M., Murphy, T.E., and Roy, R. "Cluster synchronization and isolated desynchronization in complex networks with symmetries", *Nature Commun.*, **5**, pp. 1–8 (2014).
43. Majhi, S., Bera, B.K., Ghosh, D., and Perc, M. "Chimera states in neuronal networks: A review", *Phys. Life Rev.*, **28**, pp. 100–121 (2019).
44. Rakshit, S., Faghani, Z., Parastesh, F., Panahi, S., Jafari, S., Ghosh, D., and Perc, M. "Transitions from chimeras to coherence: An analytical approach by means of the coherent stability function", *Phys. Rev. E*, **100**, p. 012315 (2019).
45. Fu, X., Small, M., and Chen, G., *Propagation Dynamics on Complex Networks: Models, Methods and Stability Analysis*, John Wiley & Sons (2013).

46. Ma, J., Hu, B., Wang, C., and Jin, W. “Simulating the formation of spiral wave in the neuronal system”, *Nonlinear Dyn.*, **73**, pp. 73–83 (2013).
47. Rajagopal, K., Parastesh, F., Azarnoush, H., Hatef, B., Jafari, S., and Berec, V. “Spiral waves in externally excited neuronal network: Solvable model with a monotonically differentiable magnetic flux”, *Chaos*, **29**, p. 043109 (2019).
48. Nazarimehr, F., Shahbodaghy, F., Hatef, B., and Rajagopal, K. “How can we measure the slowing down in healthy and ischemic stroke individuals?”, *Sci. Iran.*, **28**(3) (2021).
49. Wang, Z., Hussain, I., Pham, V.T., and Kapitaniak, T. “Discontinuous coupling and transition from synchronization to an intermittent transient chimera state”, *Sci. Iran.*, **28**(3) (2021).
50. Vock, S., Berner, R., Yanchuk, S., and Schöll, E. “Effect of diluted connectivities on cluster synchronization of adaptively coupled oscillator networks”, *Sci. Iran.*, **28**(3) (2021).
51. Wang, Z., Zhang, P.Z., Moroz, I., and Karthikeyan, A. “Complex dynamics of a Fitzhugh-Rinzel neuron model considering the effect of electromagnetic induction”, *Sci. Iran.*, **28**(3) (2021).