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An availability evaluation method for desalination process with three-state equipment under a specific repair queuing policy

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Abstract. Oil waste is one of the most important pollutants in the oil and gas industry. Since the oil flowing in the wells contains significant amount of saltwater, the effluent amount rises upon increasing the oil reservoir extraction. Separating the saltwater from the extracted oil before starting the refinery process plays an essential role in reducing the oil costs and benefiting from the transfer capacity as well. This paper presents a new Chapman-Kolmogorov Equation-Based (CKEB) method to evaluate the availability of a desalination system with three-state equipment and weighted-k-out-of-n configuration. In this system, the equipment is repairable, and each repair facility can repair all equipment types of different sub-systems (pump stations). All failures and repairs were considered to have a constant rate (with exponential distribution) and use the Chapman-Kolmogorov equations to drive the system availability. Then, the presented method was validated using a simulation technique. Finally, the elapsed times of solving both techniques were calculated and compared. The results confirmed the superiority of the CKEB technique in terms of computational time. Compared with the simulation technique, the computational time ratio for the CKEB method was in the range of $0.0002\%-0.0058\%$ for the small-size problems, $0.05\%-0.94\%$ for the medium-size problems, and 1.31%-5.39% for the large-size problems.

1. Introduction

Oil drainage that comes out of the reservoir during oil extraction is one of the most important pollutants in the oil and gas industry. With an increase in the extraction of oil reservoirs, the amount of effluent will increase. Over time, upon decreasing the amount of oil in the well and replacing it with saltwater, the amount of saltwater increases, which requires separation from the oil. For this purpose, the extracted oil in the wells is sent to desalination units for separation of saline waste after decontamination in the units. Assume that the waste materials are not separated from the oil before delivering to the refineries. In this case, corrosion of pipelines and reduction of the transfer debit are unwanted consequences. In addition, disruptions would occur during distillation and corrosion in the steam boilers and other devices. The presence of these waste materials in the exported oil reduces the oil cost as well as the useful transportation capacity. Therefore, the reliability (availability) of the separation systems of this wastewater from oil and injection is the priority of oil companies.

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Sewage sludge contains significant amounts of heavy metals and different types of salts, and these materials in nature are one of the major environmental pollutants. Since the sewage sludge is injected into the wells, it is essential to separate the environmental pollutant before injection. The presence of these pollutant materials in the sewage sludge is a consequence of incomplete separation in the desalination units. For this reason, it is recommended for wastewater treatment systems to separate oil and particulate matter from the wastewater and prepare it for injection. The desalination plants require a variety of equipment, and many of them are three-state equipment. For example, given the number of the required healthy blades, the water pumps have three functional states: working, failed, and low-performance working statuses.

This study uses Chapman-Kolmogorov equation to evaluate the availability of a desalination system with three-state equipment. The system equipment is considered repairable, and all repair facilities in this system can repair all the equipment of different plant sections. Initially, the states of the system based on the numbers of sub-systems (number of equipment), redundant equipment, under-repair equipment in all sub-systems, and repair facilities are determined. Then, the total system availability is calculated using the Chapman-Kolmogorov equation. To validate the proposed method, the system availability is calculated using a simulation technique. Finally, based on the numbers of sub-systems (equipment) and redundant components as well as the mission time of the system, condition was defined where the Chapman-Kolmogorov Equation-Based (CKEB) method had a shorter computational time than the simulation method.

This paper consists of seven parts. Section 2 discusses the literature review. Section 3 deals with the system definitions. Section 4 presents the problem definitions. Section 5 deals with solving methods. Section six gives some numerical examples and finally, the last section concludes the study and suggests further studies.

2. Literature review

The current study presents a new availability evaluation for a desalination system with three-state components. The literature review is presented in three directions. The first direction is concerned with Multi-State Systems (MSS) where the general reliability (availability) evaluation of the configuration of different MSS systems is assessed. The second direction is dedicated to the evaluation of reliability (availability) of the systems for Redundancy Allocation Problem (RAP). The last direction is involved in the reliability evaluation of desalination systems.

Given the MSS and according to the classical reliability models, most of the systems, as well as their components, have two states: working and failed states. A particular system component can operate at any level of functionality from 0% to 100% with a given probability. These systems are called the MSS. Upon increasing the number of the components in an MSS, the system states as well as the computational complexities will rapidly increase; in other words, calculating the reliability of the MSS using basic reliability methods, if not impossible, is too complicated. Therefore, in order to reduce its complexity, Ushakov [1] introduced Universal Generating Function (UGF) method. The UGF is a well-known convenient method to calculate the MSS reliability with series, parallel, and series-parallel configurations. However, upon increasing the system components, the computational time of UGF will remarkably increase.

Wu and Chen [2] proposed a recursive algorithm to estimate the reliability of the weighted k-out-of-n system. It is another well-known technique for calculating the reliability of the MMSs. Higashiyama [3] gave some instructions to evaluate the reliability of the binary-state weighted k-out-of-n system in a shorter time than that of the proposed recursive method. Sharifi et al. [4] worked on a network consisting of two elements with incremental and constant failure rates in real-time situations. Later, they studied the real-time reliability of a k-out-of-n load-sharing system with n identical components [5].

Lisnianski and Ding [6] studied the redundancy of a repairable MSS using a combinatorial method of statistical processes and UGF analysis. Sharifi et al. [7] proposed an algorithm to evaluate the real-time reliability of a k-out-of-n systems with identical components and fuzzy failure rates. Levitin et al. [8] considered the optimal standby element sequencing problem for k-out-of-n: G heterogeneous cold-standby systems. They optimized the expected system mission cost by selecting the initial sequence of the system components. Guilani et al. [9] proposed a new practical approach to estimate the reliability of unrepairable three-state systems. They first presented a proper definition of the system states and then, provided differential equations using the Markov process and Chapman-Kolmogorov equation. Through solving these differential equations which calculated the probabilities of the states and system reliability, the processing time was significantly reduced and compared with other available techniques. Lu et al. [10] examined the reliability of a large MSS with repairable components. They reported that some Phased-Mission Systems (PMS) might contain a large number of steps and repairable components in many engineering applications. Levitin et al. [11] optimized the 1-out-of-N: G cold standby systems consisting of non-repairable components with different productivity or load levels. They first suggested analysis of the system reliability and expected mission cost and then, formulated and solved the optimal dynamic load distribution of the completed work-dependent component load. Sharifi and Taghipour [12] considered a k-out-of-n system with non-identical components to optimize the system inspection interval. In their model, the system was an MSS operating based on the number of working components during each inspection interval.

Many real-case problems like the RAP deal with calculating the system reliability. Different system configurations were incorporated in the RAP, and the reliability of systems was calculated before optimizing the redundancy of systems. The general mathematical model for the RAP was presented by Fyffe, Hines et al. [13]. The presented model aims to maximize the system reliability under the cost and weight constraints.

Later, many researchers have optimized the RAP considering different MSS configurations. For example, Ramirez-Marquez and Coit [14] considered a series-parallel MSS with capacitated binary components which could provide different MSS performance levels and calculated the system reliability using UGF. Tian and Zou [15] optimized a multi-objective RAP for general series-parallel MSS. Later, they presented an optimization model for a series-parallel MSS, which jointly determined the optimal component state distribution and optimal redundancy for each stage. The relationship between the component state distribution and component cost was discussed based on an assumption on the treatment of the components [16]. Ouzineb et al. [17] worked on an RAP with series-parallel MSS and used an efficient Tabu Search (TS) algorithm to solve the presented problem. They remarked that the performance levels of the system possessed a continuous range between the perfect functioning and complete failure. Mousavi et al. [18] investigated a bi-objective RAP for a series-parallel MSS with non-repairable components. They reported that both performance rates and availability of the components could be considered fuzzy due to the uncertainties. They used a fuzzy UGF to evaluate the availability of their system.

Wang et al. [19] presented a RAP for cold-standby systems with degrading components and approximated the objective function of their models. They used Genetic Algorithm (GA) to solve the presented RAP. Lai and Yeh [20] proposed a swarm-based approach called two-stage simplified swarm optimization to solve a RAP for a bridge MSS. Shahriari, Sharifi et al. [21] modeled the system reliability considering the time-dependent continuous performance rate of the MSS. They stated that the system components might have a continuous performance rate between zero and the maximum performance rate. George-Williams and Patelli [22] evaluated the reliability of the MSS with a flow simulation approach and hybrid events. They reported that the complexity of the systems and multi-state structural features made it challenging to evaluate their reliability and availability. Despite the emergence of diverse techniques for analyzing complex MSSs, simulation seems to be the only proper and doable approach to real systems. Attar et al. [23] studied the optimization method approach based on simulation for RAP and availability of the repairable MSS with any statistical distribution.

Some other studies on RAP considering MSS are presented in Table 1 (the studies were conducted between 2018–2021).

With regard to evaluating the reliability (availability) of desalination systems, El-Nashar [39] developed a method that incorporated equipment reliability considerations into the optimal design of cogeneration systems for power and desalination. Hosseini et al. [40] studied the effects of the equipment reliability on analyzing the thermo-economic of the combined power and multi-stage-flash water desalination plant using the Markov process. They expanded their method by considering a multi-objective optimization model and designed a combined gas turbine and multi-stage flash desalination plant [41]. Zhou et al. [42] addressed three essential reliability aspects, namely the incompleteness of the system boundary, unrepresentativeness of the database, and omission of uncertainty analysis that drive uncertainty in the life cycle assessment of the desalination plants. Ailliot et al. [43] used the stochastic (Markov-switching auto-regressive model) weather generators for the optimal design and reliability evaluation of the hybrid Photovoltaic/Wind-Generator systems providing energy to desalination plants. Wang et al. [44] recommended the utilization of decision-support tools to incorporate uncertainties, seasonal demand forecasts, and system operational constraints of the desalination plants to assist decision-makers in designing more reliable systems using a hidden Markov chain. In all these studies, the effect of considering the system reliability on the desalination process has been investigated. In this research, the system availability was calculated for a system containing three-state repairable components through the CKEB method. To this end, the procedure presented by Guilani et al. [9] was taken into consideration.

A common assumption made to simplify the systems with several sub-systems is that every repair facility may only work on the components of one sub-system. This assumption dramatically reduces the number of the system states. However, in real-case problems, the repair facilities can operate and repair all equipment of different sections (sub-systems) of a plant. This assumption, however, was violated in this study, and an assumption was added to the model that would allow every repair facility to work on all the sub-system components. In this regard, the Chapman-Kolmogorov equation and simulation method were

Table 1. Some recent research (between 2018–2020) related to RAP considering MSS.

Name of the researcher (s)	Year	Single or multi -objective (s) ?	Repairable components ?	The used technique for evaluating the system's availability (reliability)
Essadqi et al. [24]	2018	Multi	×	UGF
Tavana et al. [25]	2018	Multi	√	Markov process
Sharifi et al. [26]	2019	Multi	×	UGF
Sharifi et al. [27]	2019	Single	×	Technical & Organizational Activities (TOA)
Sun et al. [28]	2019	Multi	×	Considering the availability's upper and lower bond
Hadipour et al. [29]	2019	Multi	√	–
Sharifi et al. [30]	2020	Single	×	Recursive algorithm
Xu et al. [31]	2020	Single	×	Interval-valued UGF
Borhani-Alamdar and Sharifi [32]	2020	Single	√	UGF
Xiahou et al. [33]	2020	Multi	×	Evidential network
Sharifi et al. [34]	2021	Single	×	TOA
Zaretalab et al. [35]	2020	Single	√	UGF
Sharifi and Taghipour [36]	2021	Single	×	UGF-based method
Du and Li [37]	2020	Single	×	UGF
Sharifi et al. [38]	2022	Single	×	–

employed to evaluate the system availability. Using the Chapman-Kolmogorov equation to evaluate system availability, we presented eight different rules. Then, we calculated the transition probabilities between the system states, as well as the transition matrix using these eight rules. The results (availability) obtained from the viewpoint of computational time based on both methods were compared at the end. The novelties of the current paper can be summarized as follows:

- Developing a CKEB method to measure the availability of systems with multi-tasking repair facilities;
- Presenting a closed-form set of differential equations to calculate the system availability;
- Comparing the performance of the given model based on the simulation technique.

The problem raised in this paper is an availability problem. This system possesses a series-parallel configuration and components with three states of performance. These components are repairable, and every repair facility can offer service to all sub-systems. The mathematical model and solution offered for the series-parallel system are based on the assumption that redundancy policy is of active type. The failure and repair rates of the available components are constant. The problem objective is to calculate the availability of the system by using differential equations and state diagrams. Given the problem conditions, the system under study is a state-dependent one and for this reason, Markov analysis gains significance in examining this problem.

3. System definitions

In this section, we present some descriptions for a real-world desalination system.

3.1. The recovery treatment process at a desalination plant

Figure 1 shows the recovery treatment process at a desalination plant. Figure 1 shows that after adding bactericide materials and reverse demulsifies to the wastewater, it moves to the skimmer reservoir. After a

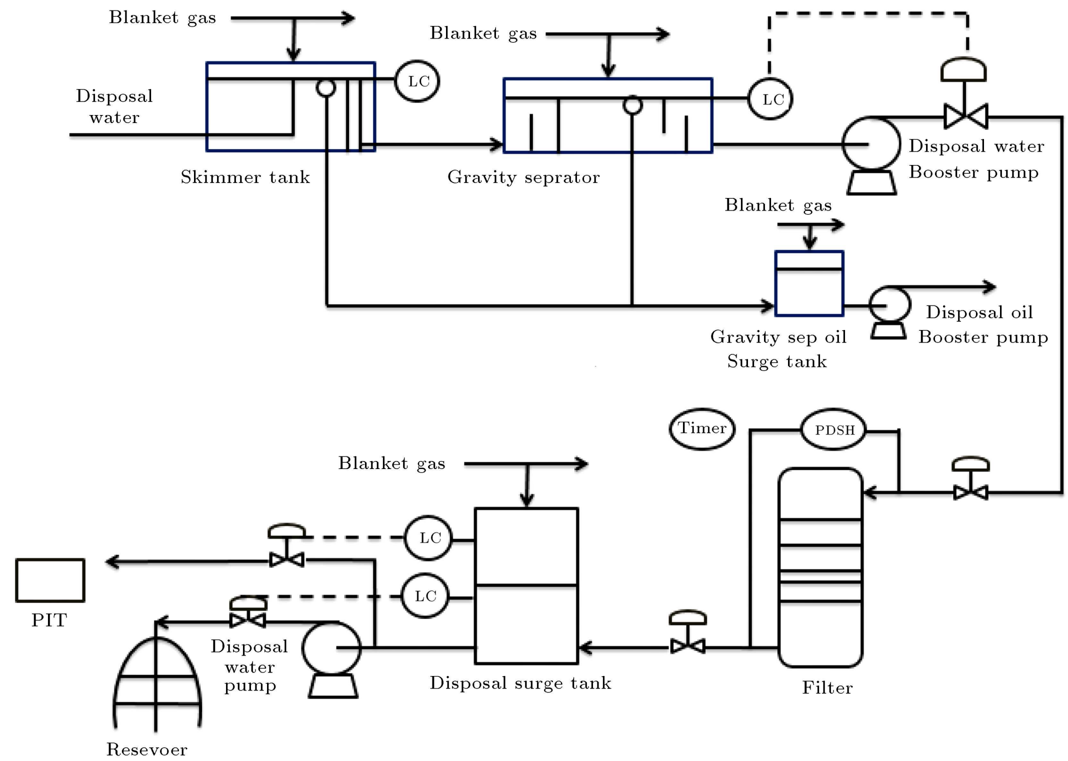

Figure 1. Recovery treatment process at a desalination plant.

while, a portion of the oil droplets comes to the water surface where it is collected by a pipe and sent to the reservoir to store contaminated oil. Then, it enters the gravity separator and again, a portion of the oil drops is collected and the remaining will be sent to the contaminated oil reservoir. The water is then pumped to under pressure filters that are designed to collect the solid particles with the dimensions of more than 10 microns, and the dimension of the remaining oil drops to below 25 ppm. Eventually, the wastewater is stored in a volatile reservoir and injected into the wells by pressure pumps. At the time of any technical problem in the plant, the wastewater will be sent to the pit, hence environmental pollution.

3.2. Components of the studied filtration and injection system

3.2.1. Skimmer tank

The dimensions of the skimmer tank are 10 m in diameter and 5 m in height. The reservoir is equipped with a cochlear plate, which allows the oil to remain on the surface with a lifespan, and it can be removed by a cut tube that may change. Water is also ejected out of the tube through a pipe.

3.2.2. Gravity separator

The weighing separator consists of two cavity piers with a width of 2.5 m, a length of 3.15 m, and a depth of 1.5 m covered by a steel sheet. The oil floating on the water surface is collected using a rotating tube, while the water is drained from the bottom.

3.2.3. Filter

The utilized sandwich filters are 1.6–2.6 m in diameter and 2 m in height; these filters are designed for solid particles up to 10 microns and oil reduction of around 25 ppm.

3.2.4. The dirty oil surge tank

This tank received the oil separated in a skimmer tank and pumped the remaining water to the desalination plant. This tank is 3 m in diameter and 2 m in height with a design pressure of 0.02 bar.

3.2.5. Disposal water pump

A wastewater pump is a single-stage pump of OH6 designed for working in harsh conditions. This pump receives the wastewater from the disposal water surge tank and injects it into the well. In case when the required injection pressure is greater than 1200 psi, pre-pressure pumps are used.

4. Problem definition

As shown in Figure 1, five different types of pumps are functioning at the desalination plant. Failure of each pump will disrupt the desalination process. In this system, considering the number of healthy blades, each pump can be considered three-state equipment. In case all the blades are in good conditions, the pump will

Table 2. Indexes, parameters, and decision variables.

Indexes	
i	Counter of pump stations
Parameters	
S	Number of the system pump stations
n_i	Number of pumps of the pump station i (redundant pump)
λ_{i1}	Semi-failure rate of a working pump in the pump station i
λ_{i2}	The complete-failure rate of a working pump in the pump station i
λ_{i3}	The complete-failure rate of a semi-working pump of the pump station i
μ_i	Repair rate of a failed pump of the pump station i
M	Total number of repair facilities
w_i	Number of functional pumps in the pump station i
s_i	Number of semi-working pumps in sub-system i
m_i	The number of repair facilities which are working in the pump station i
Po_i	Working power of the pump station i
k_i	The minimum required working power for pump station i to be considered as an operational pump station

continue to work with its maximum performance. In case more than one blade has been damaged, the pump would be deactivated; however, if only a blade fails, the pump will keep working with lower performance. In this paper, the reduced performance of the pumps is considered to be half of that of the new pump. Moreover, all repair facilities can repair all the existing pumps (of any type).

The system under study is a weighted k-out-of-n system with a multi-state component. Since each pump has three states, it should be considered an MSS. It is noted that for each sub-system (pump-station), when a specific number (k_i) of the pumps are available, the pump station is considered as an operational pump station. For example, if $k_i = 1.5$, the pump station is working with equal or more than one fully-working and one semi-working pump or with more than two semi-working pumps. One of the ways to enhance the availability of such systems is to use redundant equipment. In case the available budget and space for the component redundancy are limited, the main problem here is to find the best solution to assign the redundant pumps. The first step for optimal allocation is to find a way to measure the system availability. The two following methods, i.e., CKEB and simulation technique, were developed to calculate the such system availability.

4.1. The general procedure of calculating the system's availability

Guilani et al. [9] calculated the reliability of a three-state system using UGF, recursive algorithm, and differential equations (such as Chapman-Kolmogorov equations). They reported that the computational time of the differential equations model was significantly less than that of the other two techniques, especially for large-scale systems. The present research employed the method of differential equations to calculate the system availability. The general procedure of the model solution in this paper is as follows:

- Step 1: Determine the system states;
- Step 2: Obtain a general state diagram for this system;
- Step 3: Determine the relations between the system states;
- Step 4: Establish a set of differential equations between the system states and calculate the system transition matrix using the Chapman-Kolmogorov equations;
- Step 5: Calculate the system availability (which is the main objective of the current study);
- Step 6: Validate the model using the simulation technique.

4.2. Nomenclatures

The notation (indexes and parameters) used in this paper is presented in Table 2.

4.3. System configuration

The general diagram of an RAP is presented in Figure 2. The system consists of S serially connected pump

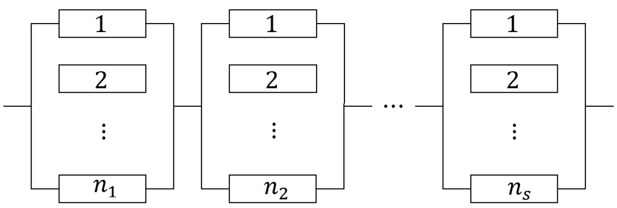

Figure 2. System configuration.

stations where the ith ($i = 1, \cdots, S$) pump station has n_i redundant pumps.

Three operational states can be considered for each pump: working, semi-working (or partially failed), and failed. For the pump station i, the number of the working pumps is shown by w_i, the number of the semi-working pumps by h_i, and the number of the repair facilities which are working on the pump stations pumps by m_i. Consider that a repair facility can fix a pump when it is completely failed. In this case, the pump station condition can be determined by (w_i, h_i, m_i). Based on the presented notation, the condition of the system can be represented by $\{(w_1, h_1, m_1), \cdots, (w_s, h_s, m_s)\}$. The system states are determined based on the values of w_i, h, and m_i. For example, the first state is $\{(n_1, 0, 0), \cdots, (n_s, 0, 0)\}$, indicating that the pumps of all pump stations are fully working, and no semi-working pump is working in the pump stations. Moreover, all repair facilities are available and none of them are working. In each particular state, the working performance of the pump station can be evaluated using Eq. (1):

$$Po_i = w_j + h_j/2; \quad i = 1, ..., h. \tag{1}$$

The system is working for all pump stations if $Po_i \geq k_i$. Since the repair facilities can serve the pumps of all pump stations, determination of the system state becomes very complicated, especially when the number of pump stations increases. The number of the possible states can dramatically rise. A particular state is reachable from other different states, as shown in Figure 3, according to which all scenarios may happen for a particular system state: the partial failure of the functional pump, complete failure of the functional pump, complete failure of a partially failed pump, and repair of a failed pump. In order to draw a state-space diagram of the system, only the states with $Po_i \geq k_i$ should be taken into account.

In general, when a failure happens, two scenarios are desirable: (a) a repair facility is free that starts to repair the failed pump immediately after the failure, or (b) there is no available repair facility in the system and the failed pump must stay on the waiting repair queue. Likewise, two scenarios are desirable after repairing a failed pump: (a) a failed pump in the repair queue requires a free repair facility, or (b) no pump is on the repair queue.

In Figure 3, state 0 is a general state of the system, and states 1 to 8 are all possible states among which state 0 is reachable. Figure 3 can be divided into eight different rules which may happen to this system. In other words, all possible system states can be expressed by the eight following rules. A general condition for all rules is presented in Eq. (2):

$$0 \leq w_j + h_j \leq n_j; \qquad \forall j \neq i. \tag{2}$$

4.3.1. Rule 1

In Figure 3, the correspondence state of Rule 1 is State 1. Figure 4 shows the semi-failure of a working pump in a pump station. In the pump station i, there is (w_i+1) working pumps (h_i-1) that are partially failed pumps and m_i busy repair facilities. The number of

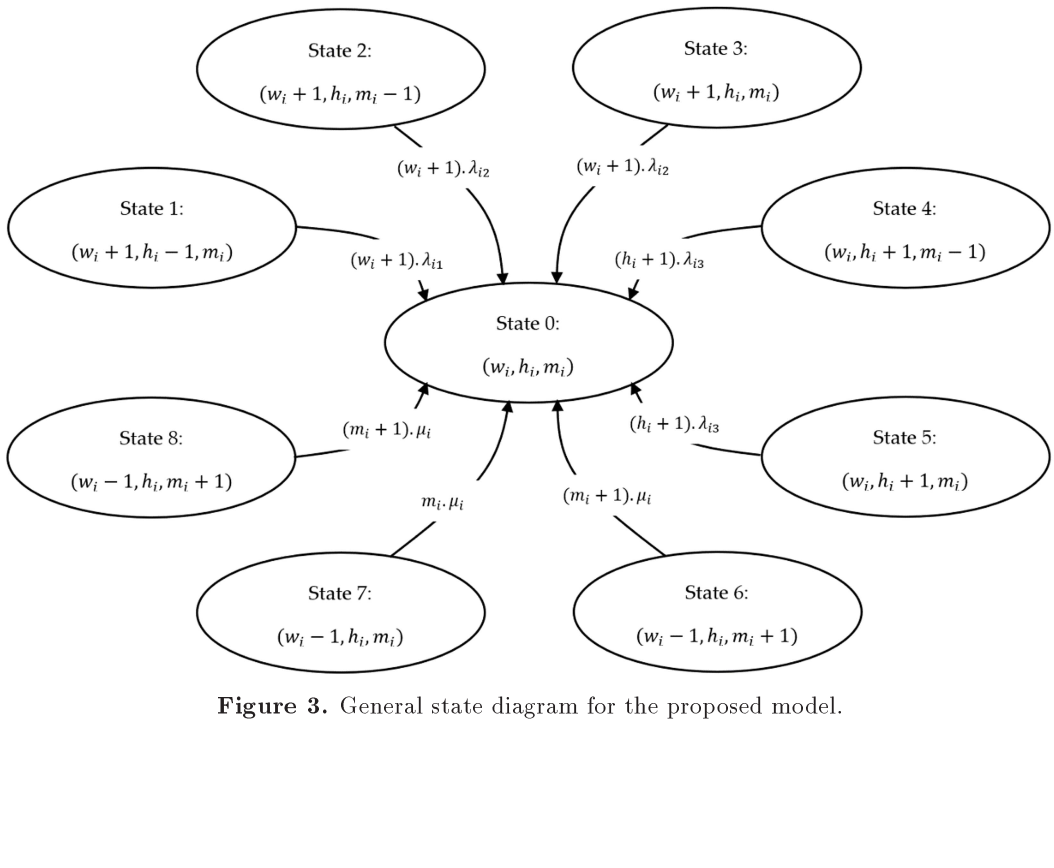

Figure 3. General state diagram for the proposed model.

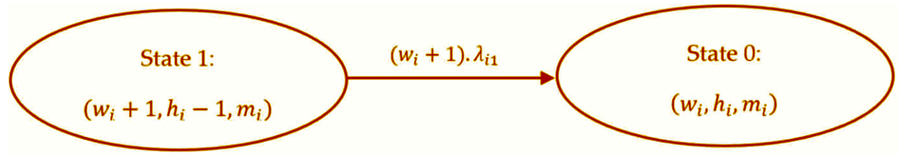

Figure 4. Diagram of the state related to Rule 1.

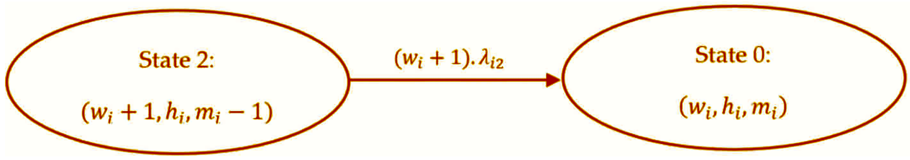

Figure 5. Diagram of the state related to Rule 2.

working pumps reduces by one unit upon partial failure of one pump in this pump station, and the number of semi-working pumps adds one unit. Since the state has $(w_i + 1)$ working pumps and the partial failure rate of each pump is $\lambda_i 1$, the transition rate from this state to state 0 is equal to $(w_i + 1)\lambda_{i1}$. The conditions for using Rule 1 are presented in Eqs. (3)–(6):

$$0 \leq m_i \leq n_i - w_i - h_i, \tag{3}$$

$$0 \leq m_j \leq n_j - w_j - h_j; \qquad \forall j \neq i, \tag{4}$$

$$m_i + \sum_{j \neq i} m_j \leq M, \tag{5}$$

$$1 \leq w_i + h_i \leq n_i \quad \& \quad 1 \leq h_i \leq n_i \quad \&$$

$$0 \leq w_i \leq n_i - 1. \tag{6}$$

4.3.2. Rule 2

In Figure 3, the corresponding state of Rule 2 is state 2. Figure 5 illustrates the full-failure of a working pump in a pump station. In the pump station i, there are $(w_i + 1)$ working pumps, h_i partially-failed pumps, and $(m_i - 1)$ busy repair facilities. Followed by full failure of one pump in this pump station, the number of the working pumps decreases one unit and the number of busy repair facilities increases by one unit. Since the state has $(w_i + 1)$ working pumps and the full-failure rate of each pump is λ_{i2}, the transition rate from this state to state 0 is equal to $(w_i + 1)\lambda_{i2}$. The conditions for using Rule 2 are presented in Eqs. (7) to (10):

$$1 \leq m_i \leq n_i - w_i - h_i, \tag{7}$$

$$0 \leq m_j \leq n_j - w_j - h_j; \quad \forall j \neq i, \tag{8}$$

$$(m_i - 1) + \sum_{j \neq i} m_j \leq M, \tag{9}$$

$$0 \leq w_i + h_i \leq n_i - 1 \quad \& \quad 0 \leq w_i \leq n_i - 1. \tag{10}$$

4.3.3. Rule 3

In Figure 3, the correspondence state of Rule 3 is state 3. Similar to Rule 2, Figure 6 illustrates the full failure of a working pump in the pump station. In this rule, however, there is no available repair facility. Therefore, after the full failure of one pump, the number of working pumps decreases by one unit, but the number of busy repair facilities does not change. Since the state has $(w_i + 1)$ working pumps and the full failure rate of each pump is λ_{i2}, the transition rate from this state to state 0 is equal to $(w_i + 1)\lambda_{i2}$. Eqs. (11) to (14) elaborate the conditions for using Rule 4:

$$0 \leq m_i \leq n_i - w_i - h_i, \tag{11}$$

$$0 \leq m_j \leq n_j - w_j - h_j; \quad \forall j \neq i, \tag{12}$$

$$m_i + \sum_{j \neq i} m_j = M, \tag{13}$$

$$0 \leq w_i + h_i \leq n_i - 1 \quad \& \quad 0 \leq w_i \leq n_i - 1. \tag{14}$$

4.3.4. Rule 4

In Figure 3, the correspondence state of Rule 4 isstate 4. Figure 7 illustrates the full-failure of a

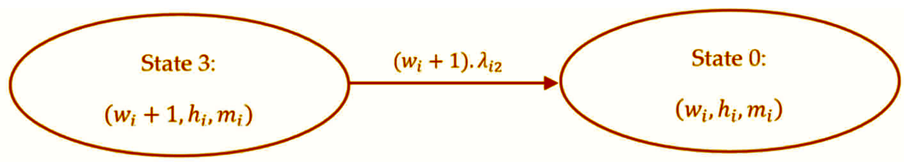

Figure 6. Diagram of the state related to Rule 3.

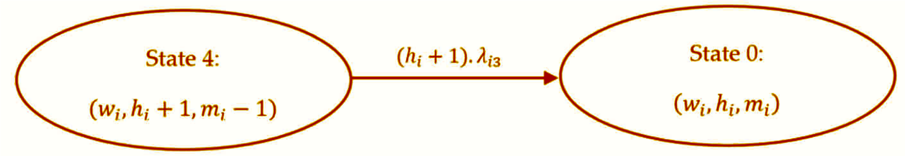

Figure 7. Diagram of the state related to Rule 4.

semi-failed pump in a pump station. In the pump station i, there are w_i working pumps, $(h_i + 1)$ partially failed pumps, and $(m_i - 1)$ busy repair facilities. Followed by full failure of one semi-failed pump, the number of the semi-failed pumps will be reduced by one unit and the number of the busy repair facilities increases by one unit. Since the state has $(h_i + 1)$ semi-failed pumps and fully failure rate of each pump is λ_{i3}, the transition rate from this state to state 0 is equal to $(h_i + 1)\lambda_{i3}$. The conditions for using Rule 4 are presented in Eqs. (15) to (18):

$$1 \leq m_i \leq n_i - w_i - h_i, \tag{15}$$

$$0 \leq m_j \leq n_j - w_j - h_j; \quad \forall j \neq i, \tag{16}$$

$$(m_i - 1) + \sum_{j \neq i} m_j < M, \tag{17}$$

$$0 \leq w_i + h_i \leq n_i - 1 \quad \& \quad 0 \leq w_i \leq n_i - 1. \tag{18}$$

4.3.5. Rule 5

In Figure 3, the correspondence state of Rule 5 is state 5. Figure 8 illustrates the full failure of a semi-failed pump in a pump station. According to this rule, there is no available repair facility. Therefore, after the full failure of one semi-failed pump, the number of semi-failed pumps decreases by one unit, but the number of busy repair facilities does not change. Since the state has $(h_i + 1)$ semi-working pumps and the full failure rate of each pump is λ_{i4}, the transition rate from this state to state 0 is equal to $(h_i + 1)\lambda_{i4}$. The conditions for using Rule 4 are presented in Eqs. (19) to (22):

$$0 \leq m_i \leq n_i - w_i - h_i, \tag{19}$$

$$0 \leq m_j \leq n_j - w_j - h_j; \quad \forall j \neq i, \tag{20}$$

$$m_i + \sum_{j \neq i} m_j = M, \tag{21}$$

$$0 \leq w_i + h_i \leq n_i - 1 \quad \& \quad 0 \leq h_i \leq n_i - 1. \tag{22}$$

4.3.6. Rule 6

Rules 6, 7, and 8 are related to pump repair. In Figure 3, the correspondence state of Rule 6 is state 6. Rule 6, given in Figure 9, indicates that the repair of one pump is finished, and no failed pump is in the repair queue. In the pump station i, there are $(w_i - 1)$ working pumps, h_i partially failed pumps, and $(m_i + 1)$ busy repair facilities. Once the repaired pump starts working, the number of working pumps increases by one unit and the number of under repair pumps decreases by one unit. Since there are $(m_i + 1)$ under repair pumps and the repair rate of each pump is equal to μ_i, the transition rate from this state to state 0 is equal to $(m_i + 1).\ \mu_i$. The conditions for using Rule 6 are presented in Eqs. (23) to (26):

$$m_i = n_i - w_i - h_i, \tag{23}$$

$$m_j = n_j - w_j - h_j; \quad \forall j \neq i, \tag{24}$$

$$m_i + \sum_{j \neq i} m_j < M, \tag{25}$$

$$1 \leq w_i + h_i \leq n_i. \tag{26}$$

4.3.7. Rule 7

In Figure 3, the correspondence state of Rule 7 is

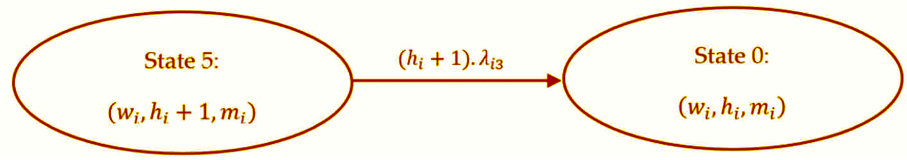

Figure 8. Diagram of the state related to Rule 5.

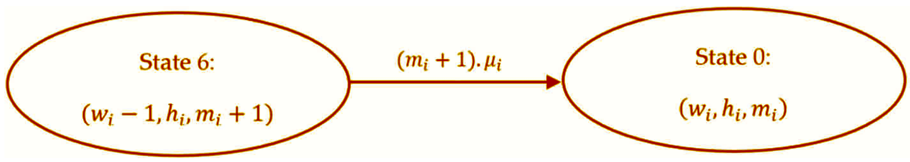

Figure 9. Diagram of the state related to Rule 6.

Figure 10. Diagram of the state related to Rule 7.

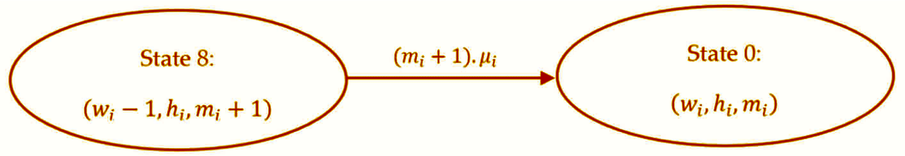

Figure 11. Diagram of the state related to Rule 8.

state 7. Rule 7, given in Figure 10, indicates that the repair of one pump is finished, and the pump station has at least one failed pump on the repair queue. In the pump station i, there is $(w_i - 1)$ working pumps, h_i partially-failed pumps, and m_i busy repair facilities. Based on the repair policy, which will be later discussed in Section 4.4, the pump station i has the priority of repair; thus, the repair facility is allocated to one of the failed pump(s) of the pump station i. In this case, the number of the working pumps of the pump station i increases by one unit, but the number of the under repair pumps does not change. Since there are m_i under-repair pumps and the repair rate of each pump is equal to μ_i, the transition rate from this state to state 0 is equal to $m_i\mu_i$. The conditions for using Rule 7 are presented in Eqs. (27) to (33):

$$L = \{j \,|n_j - w_j - h_j > m_j; \;\; j \neq i\}, \tag{27}$$

$$w_L = \min_{j \in L} (w_L), \tag{28}$$

$$l = \min_{j \in L, w_j = w_L} (j), \tag{29}$$

$$\text{if} \left\{ \begin{array}{l} (L \neq \emptyset \;\&\; w_i < w_l \;\&\; n_i - w_i - h_i \geq m_i) \\ \text{or} \\ (L \neq \emptyset \;\&\; w_i = w_l \;\&\; n_i - w_i - h_i \\ \quad \geq m_i \;\&\; i < l) \\ \text{or} \\ (L = \emptyset \;\&\; n_i - w_i - h_i \geq m_i) \end{array} \right\}$$

$$\text{then } m_i + \sum_{j \neq i} m_j = M, \tag{30}$$

$$1 \leq m_i \leq n_i - w_i - h_i, \tag{31}$$

$$0 \leq m_j \leq n_j - w_j - h_j; \quad \forall j \neq i, \tag{32}$$

$$1 \leq w_i + h_i \leq n_i - 1. \tag{33}$$

4.3.8. Rule 8

In Figure 3, the correspondence state of Rule 8 is state 8. Rule 8, presented in Figure 11, indicates that the one pump's repair is finished and the pump station has at least one failed pump on the repair queue. In the pump station i, there are $(w_i - 1)$ working pumps, h_i partially failed pumps, and $(m_i + 1)$ busy repair facilities. Based on the repair policy, which will be later presented in Section 4.4, the pump station i has no priority for repair; therefore, the repair facility is allocated to one of the failed pump(s) of another pump station. In this case, the number of the working pumps of the pump station i increases by one unit, and the number of pumps under repair decreases by one unit. Since there are $(m_i + 1)$ under repair pumps and the repair rate of each pump is equal to μ_i, the transition rate from this state to state 0 is equal $(m_i + 1)\mu_i$. The conditions for using Rule 8 are presented in Eqs. (34) to (40):

$$L = \{j \,|n_j - w_j - h_j > m_j; \quad j \neq i\}, \tag{34}$$

$$w_L = \min_{j \in L} (w_L), \tag{35}$$

$$l = \min_{j \in L, w_j = w_L} (j), \tag{36}$$

$$\text{if} \left\{ \begin{array}{l} (L \neq \phi \;\&\; w_i < w_l \;\&\; n_i - w_i - h_i = m_i) \\ \text{or} \\ \big(L \neq \phi \;\&\; w_i = w_l \;\&\; \big\{ (n_i - w_i - h_i = m_i) \\ \quad \text{or} \, (n_i - w_i - h_i > m_i \;\&\; i > l) \big\} \big) \\ \text{or} \\ (L \neq \phi \;\&\; w_i > w_l) \end{array} \right\}$$

$$\text{then} \quad m_i + \sum_{j \neq i} m_j = M, \tag{37}$$

$$0 \leq m_i \leq n_i - w_i - h_i, \tag{38}$$

$$0 \leq m_j \leq n_j - w_j - h_j; \quad \forall j \neq i, \tag{39}$$

$$1 \leq w_i + m_i \leq n_i. \tag{40}$$

4.4. Repair queuing policy

Rules 7 and 8 are the same as Rule 6. However, some pumps in Rules 7 and 8 are on the repair queue, and a released repair facility must work on one of the failed pumps in the repair queue. In this condition, an important question arises: Which failed pump should be allocated to the available repair facility? To answer this question and solve this problem, a policy should be implemented that determines the repair priority of the pumps in the repair queue. This policy is defined in four steps:

Step 1: Determine the R set of pump stations with at least one pump on the repair queue.

Step 2: Determine the working pumps of the pump station(s) of R.

Step 3: Allocate the repair facility to the failed pump of the pump station with the lowest working pump in R.

Step 4: If there is more than one pump station with the lowest working pump in R, allocate the repair facility to the pumps of the pump station with a lower pump station index (lower value of i).

According to the eight rules mentioned above, the state diagram can be determined for any system with any number of pumps and pump stations. In any situation, one of the above rules will comply. For example, consider a system with three pump stations. Each pump station consists of four pumps, and three repair facilities are available. Consider the state {(2,1,1),(1,0,1),(2,1,0)}. In this state and for the first pump station, two fully working and one semi-working pump are available, and one pump is under repair; therefore, this station has no pump on the repair queue. One fully working and no semi-working pump are available in the second pump station, and one pump is under repair; therefore, this station has two pumps on the repair queue. In the third pump station, two fully working and one semi-working pump are available, while no pump is under repair; therefore, this station has one pump on the repair queue. For this state, we have $R = \{2, 3\}$ which is the index of the pump stations with at least one pump on the repair queue. The working power of the second pump station is equal to $Po_2 = 1 \times w_2 + 0.5 \times h_2 = 1 \times 1 + 0.5 \times 0 = 1$, and the power of the third pump station is equal to $Po_3 = 1 \times w_3 + 0.5 \times h_3 = 1 \times 2 + 0.5 \times 1 = 2.5$. Given that one repair facility is available, this repair facility should be allocated to the pump station with lower working power than others, i.e., the second pump station. For the state {(1,2,1),(1,2,1),(2,2,0)}, however, the working power of the first and second pump stations is equal to ($Po_1 = Po_2 = 2$); therefore, the available repair facility is allocated to the pump station with the lower index of i, i.e., the first pump station.

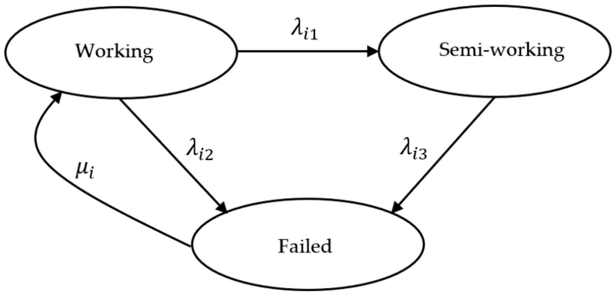

Figure 12. State-space diagram of a system with one repairable member.

$$\begin{array}{r} \\ Working\,(w) \\ Semi-working\,(h) \\ Failed\,(f) \end{array} \begin{array}{c} \begin{array}{ccc} w & h & f \end{array} \\ \begin{bmatrix} -(\lambda_{i1}+\lambda_{i2}) & 0 & \mu_i \\ \lambda_{i1} & -\lambda_{i3} & 0 \\ \lambda_{i2} & \lambda_{i3} & -\mu_i \end{bmatrix} \end{array}$$

Figure 13. Transition matrix related to Figure 12.

4.5. Transition matrix

The transition matrix is a matrix whose elements are the transition probabilities between different system states. The state-space diagram of a system with one repairable pump is presented in Figure 12, and the correspondence transition matrix is shown in Figure 13.

4.6. Differential equations

In memoryless models and based on the Markov process and Chapman-Kolmogorov equations, we can obtain differential equations. Here, $P_n(\text{t})$ represents the probability that the system is in state n at time t. As a result, $P_n(\text{t})$ is obtained from the differential equation according to Eq. (41):

$$P'_n(t) + \left(\sum_{i\in output\ fellows\ from\ state\ n} \lambda_i\right) . P_n(t) = \sum_{j\in input\ fellows\ to\ state\ n} \{\lambda_i . P_j(t)\}. \qquad (41)$$

By solving the set of the differential equation, the probability of each system state at time t is obtained. The sum of these probabilities is the system availability. To do the calculations, all coding is done in MATLAB R2019b software.

4.7. Simulation

One of the most common tools used for analyzing the models and real-world systems is simulation [22,23,28,30,31]. Particularly, in models and systems that are random in nature, this procedure is very efficient. To validate our proposed method, a simulation technique was employed. The simulation flowchart of the repairable three-state system is presented in Figure 14.

In different periods, the event times are randomly created concerning their failure rates. Considering that the failure and repair rates are constant, these times are calculated using Eq. (42):

$$t=\frac{-Ln\left\{Rand\left(R'\right)\right\}}{\lambda}. \quad (42)$$

Although the simulation technique in this paper is used to validate the proposed model, this method could notably be used to obtain repairable three-state systems with any distribution.

5. Numerical example

In this section, the instances at three different levels are solved for different purposes. First, a very small-sized instance is addressed to illustrate how the system states are determined and the set of differential equations between the system states is calculated; a very large-size instance is solved to compare the results of two solving methodologies in detail and to highlight the complexity of the presented method. Finally, different 15 instances are taken into account to illustrate the superiority of the CKEB technique to the simulation technique.

Due to some restrictions (the company's policy) on reporting real data, some of the model parameters are estimated by different forecasting techniques using on-hand information. These estimated data are the model parameters including the transition rates. Data-driven concepts may be used (i.e., statistical and data mining techniques).

The first instance is a very small-sized system with two pump stations, while each pump station has two repairable pumps. Of note, one repair facility is available to repair the pumps when they entirely fail. The minimum required working performance for each pump station is considered as $k_i=1$, implying that each pump station is considered operational if at least one pump is in fully working conditions or two pumps are in semi-working conditions. The system has 43 states that are divided into two categories: 17 working states and 26 failed states. All system states for this instant are numbered in Table 3.

For the instance mentioned above, based on Eq. (41), the set of differential equations is presented in Eqs. (43) to (60):

$$p_1'(t)+2.(\lambda_{11}+\lambda_{12}+\lambda_{21}+\lambda_{21}).$$

$$p_1(t)=\mu_2.p_4(t)+\mu_1.p_8(t), \quad (43)$$

$$p_2'(t)+(\lambda_{11}+\lambda_{12}+\lambda_{13}+2.\lambda_{21}+2.\lambda_{22}).$$

$$p_2(t)=2.\lambda_{11}.p_1(t)+\mu_2.p_9(t), \quad (44)$$

$$p_3'(t)+(2.\lambda_{11}+2.\lambda_{12}+\lambda_{21}+\lambda_{22}+\lambda_{23}).$$

$$p_3(t)=2.\lambda_{21}.p_1(t)+\mu_1.p_{12}(t), \quad (45)$$

$$p_4'(t)+(2.\lambda_{11}+2.\lambda_{12}+\lambda_{21}+\lambda_{22}+\mu_2).$$

$$p_4(t)=2.\lambda_{22}.p_1(t)+\lambda_{23}.$$

$$p_3(t)+\mu_1.p_{13}(t), \quad (46)$$

$$p_5'(t)+2.(\lambda_{11}+\lambda_{12}+\lambda_{23}).$$

$$p_5(t)=\lambda_{21}.p_3(t)+\mu_1.p_{15}(t), \quad (47)$$

$$p_6'(t)+(\lambda_{11}+\lambda_{12}+\lambda_{13}+\lambda_{21}+\lambda_{22}+\lambda_{23}).$$

$$p_6(t)=2.\lambda_{21}.p_2(t)+2.\lambda_{11}.p_3(t), \quad (48)$$

$$p_7'(t)+2.(\lambda_{13}+\lambda_{21}+\lambda_{22}).$$

$$p_7(t)=\lambda_{11}.p_2(t)+\mu_2.p_{16}(t), \quad (49)$$

$$p_8'(t)+(\lambda_{11}+\lambda_{12}+2.\lambda_{21}+2.\lambda_{22}+\mu_1).$$

$$p_8(t)=2.\lambda_{12}.$$

$$p_1(t)+\lambda_{13}.$$

$$p_2(t)+\mu_2.p_{14}(t), \quad (50)$$

$$p_9'(t)+(\lambda_{11}+\lambda_{12}+\lambda_{13}+\lambda_{21}+\lambda_{22}+\mu_2).$$

$$p_9(t)=2.\lambda_{22}.p_2(t)+2.\lambda_{11}.p_4(t)+\lambda_{23}.p_6(t), \quad (51)$$

$$p_{10}'(t)+(\lambda_{11}+\lambda_{12}+\lambda_{13}+2.\lambda_{23}).$$

$$p_{10}(t)=2.\lambda_{11}.p_5(t)+2.\lambda_{21}.p_7(t), \quad (52)$$

$$p_{11}'(t)+(2.\lambda_{13}+\lambda_{21}+\lambda_{22}+\lambda_{23}).$$

$$p_{11}(t)=\lambda_{11}.p_6(t)+2.\lambda_{21}.p_7(t), \quad (53)$$

$$p_{12}'(t)+(\lambda_{11}+\lambda_{12}+\mu_1+\lambda_{21}+\lambda_{22}+\lambda_{23}).$$

$$p_{12}(t)=2.\lambda_{12}.p_3(t)+\lambda_{13}.p_6(t)+2.\lambda_{22}.p_8(t), \quad (54)$$

$$p_{13}'(t)+(\lambda_{11}+\lambda_{12}+\mu_1+\lambda_{21}+\lambda_{22}).$$

$$p_{13}(t)=2.\lambda_{22}.p_8(t)+2.\lambda_{23}.p_{12}(t), \quad (55)$$

$$p_{14}'(t)+(\lambda_{11}+\lambda_{12}+\lambda_{21}+\lambda_{22}+\mu_2).$$

$$p_{14}(t)=2.\lambda_{12}.p_4(t)+\lambda_{13}.p_9(t), \quad (56)$$

$$p_{15}'(t)+(\lambda_{11}+\lambda_{12}+\mu_1+2.\lambda_{23}).$$

$$p_{15}(t)=2.\lambda_{12}.p_5(t)+\lambda_{13}.p_{10}(t)+\lambda_{21}.p_{12}(t), \quad (57)$$

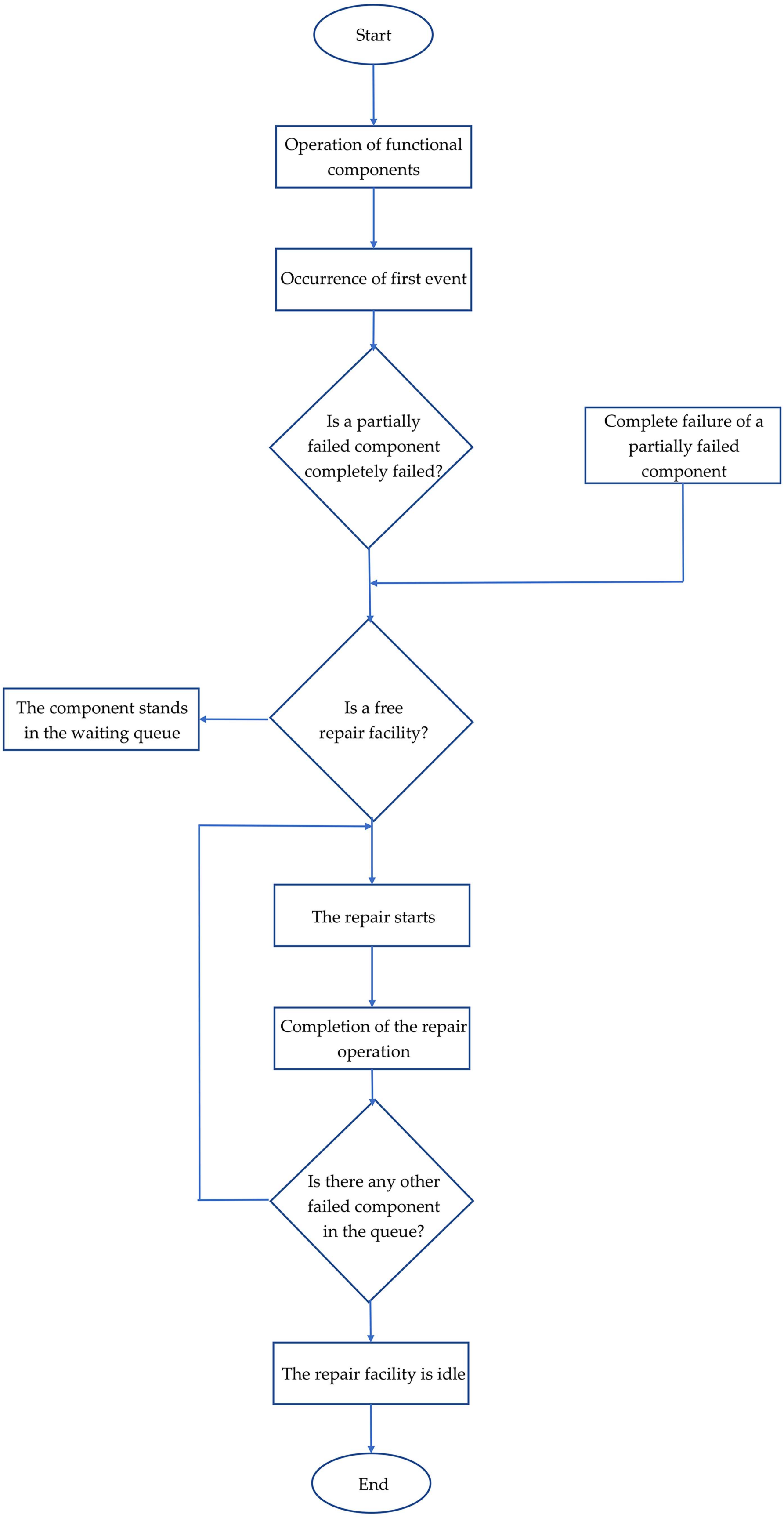

Figure 14. Simulation flowchart of repairable three-state system.

Table 3. State definition of the presented instant.

	Total working power of the system ($Po_T = Po_1 + Po_2$)	**State**
System is working	$Po_T = 4.0$	$1: \{(2,0,0),(2,0,0)\}$,
	$Po_T = 3.5$	$2: \{(1,1,0),(2,0,0)\}$,
		$3: \{(2,0,0),(1,1,0)\}$,
	$Po_T = 3.0$	$4: \{(2,0,0),(1,0,1)\}$,
		$5: \{(2,0,0),(0,2,0)\}$,
		$6: \{(1,1,0),(1,1,0)\}$,
		$7: \{(0,2,0),(2,0,0)\}$,
		$8: \{(1,0,1),(2,0,0)\}$,
	$Po_T = 2.5$	$9: \{(1,1,0),(1,0,1)\}$,
		$10: \{(1,1,0),(0,2,0)\}$,
		$11: \{(0,2,0),(1,1,0)\}$,
		$12: \{(1,0,1),(1,1,0)\}$,
	$Po_T = 2.0$	$13: \{(1,0,1),(1,0,0)\}$,
		$14: \{(1,0,0),(1,0,1)\}$,
		$15: \{(1,0,1),(0,2,0)\}$,
		$16: \{(0,2,0),(1,0,1)\}$,
		$17: \{(0,2,0),(0,2,0)\}$,
System is working	$Po_T = 2.5$	$\{(2,0,0),(0,1,1)\}$, $\{(0,1,1),(2,0,0)\}$,
	$Po_T = 2.0$	$\{(2,0,0),(0,0,1)\}$, $\{(1,1,0),(0,1,1)\}$, $\{(0,1,1),(1,1,0)\}$, $\{(0,0,1),(2,0,0)\}$,
	$Po_T = 1.5$	$\{(1,1,0),(0,0,1)\}$, $\{(1,0,0),(0,1,1)\}$, $\{(1,0,1),(0,1,0)\}$, $\{(0,2,0),(0,1,1)\}$,
		$\{(0,0,1),(1,1,0)\}$, $\{(0,1,1),(1,0,0)\}$, $\{(0,1,0),(1,0,1)\}$, $\{(0,1,1),(0,2,0)\}$,
	$Po_T = 1.0$	$\{(1,0,0),(0,0,1)\}$, $\{(1,0,1),(0,0,0)\}$, $\{(0,1,0),(0,1,1)\}$,
		$\{(0,0,1),(1,0,0)\}$, $\{(0,0,0),(1,0,1)\}$, $\{(0,1,1),(0,1,0)\}$,
	$Po_T = 0.5$	$\{(0,1,0),(0,0,1)\}$, $\{(0,1,1),(0,0,0)\}$, $\{(0,0,0),(0,1,1)\}$, $\{(0,0,1),(0,1,0)\}$,
	$Po_T = 0.0$	$\{(0,0,1),(0,0,0)\}$, $\{(0,0,0),(0,0,1)\}$.

$$p'_{16}(t) + (2.\lambda_{13} + \lambda_{21} + \lambda_{22} + \mu_2).$$

$$p_{16}(t) = 2.\lambda_{22}.p_7(t) + \lambda_{11}.p_9(t) + \lambda_{23}.p_{11}(t), \qquad (58)$$

$$p'_{17}(t) + (2.\lambda_{13} + 2.\lambda_{23}).$$

$$p_{17}(t) = \lambda_{11}.p_{10}(t) + \lambda_{21}.p_{11}(t), \qquad (59)$$

$$p'_f(t) = \lambda_{12}.p_2(t) + \lambda_{22}.p_3(t) + (\lambda_{21}+$$

$$\lambda_{22}).p_4(t) + 2.\lambda_{23}.p_5(t) + (\lambda_{12}$$

$$+ \lambda_{22}).p_6(t) + 2.\lambda_{13}.p_7(t) + (\lambda_{11}$$

$$+ \lambda_{12}).p_8(t) + (\lambda_{12} + \lambda_{21}$$

$$+ \lambda_{22}).p_9(t) + (\lambda_{12} + 2.\lambda_{23}).p_{10}(t)$$

$$+ (2.\lambda_{13} + \lambda_{22}).$$

$$p_{11}(t)+(\lambda_{11}+\lambda_{12}+\lambda_{22}).$$
$$p_{12}(t)+(\lambda_{11}+\lambda_{12}+\lambda_{21}+\lambda_{22}).$$
$$p_{13}(t)+(\lambda_{11}+\lambda_{12}+\lambda_{21}$$
$$+\lambda_{22}).p_{14}(t)+(\lambda_{11}+\lambda_{12}$$
$$+2.\lambda_{23}).p_{15}(t)+(2.\lambda_{13}+\lambda_{21}+\lambda_{22}).$$
$$p_{16}(t)+(2.\lambda_{13}+2.\lambda_{23}).p_{17}(t). \quad (60)$$

The system availability is the sum of the probabilities of $p_i(t)$ for all system working states. The estimated availability for this example is equal to $A(100) = 0.95601$ for a 100-hour system mission time.

To evaluate the complexity of the computational methods, a new large-sized problem is solved using both methods. Different mission times of the system from 100 to 450 hours were taken into account. The system consists of four pump stations with five repair facilities. The first, second, third, and fourth pump stations have four, two, three, and one pumps, respectively. The minimum required working performances for all pump stations are considered as $k_i = 0.5$. The presented instance has 2575 working states and one failed state. The failure rate of the pumps in these four $\{(\lambda_{11} = 0.045,\ \lambda_{12} = 0.028,\ \lambda_1 = 0.035), (\lambda_{21} = 0.015,\ \lambda_{22} = 0.043,\ \lambda_{23} = 0.020), (\lambda_{31} = 0.025,\ \lambda_{32} = 0.035,\ \lambda_{33} = .037), (\lambda_{41} = 0.014,\ \lambda_{42} = 0.044,\ \lambda_{43} = 0.026)\}$, respectively, and the repair rate for all pumps is equal to $\mu_i = 0.195$; $i = 1, 2, 3, 4$. The number of the simulation technique runs is ten times, and the number of simulation iterations for each run is 1,000,000. The results of solving the presented instance using both methods are presented in Table 4. In this table, when the system operation time is considered 450 hours, the values of the system availability calculated through both methods are equal, even when the calculation times vary significantly. Figure 15 shows the system availability calculated by both of the methods, and Figure 16 presents the calculation time of both methods.

As shown in Table 4, with an increase in the mission time of the system, the computational time of the CKEB method does not significantly change. In contrast, the computational time of the simulation method dramatically increases, thus confirming the efficiency of the CKEB method in terms of computational time.

As shown in Figure 15, upon increasing the mission time of the system, the availability of both techniques converges to the same values mainly because upon increasing the mission time of the system, the simulation method approaches its steady-state condition and the availability of the simulation method achieves the availability of the real system, which is the same as calculated using the CKEB method.

According to Figure 16, with an increase in the mission time of the system, the computational time the CKEB method will not significantly change. However, the computational time of the simulation method will significantly increase. Based on Figures 15 and 16, the following essential points can be concluded:

- The availability obtained from both methods is equal, which is the reason behind the proper functioning of the CKEB method;
- The computational time of the CKEB method is significantly less than that of the simulation method,

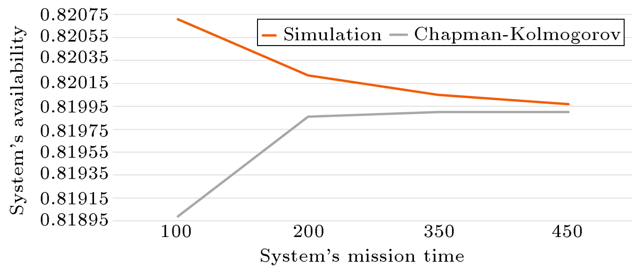

Figure 15. System availability for different system operation time obtained with both methods.

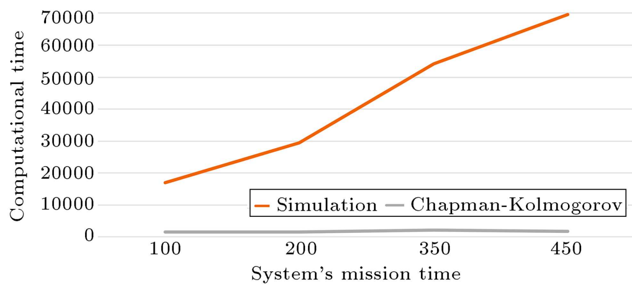

Figure 16. Computational time of both techniques.

Table 4. Availability and calculation time of two methods for different system's mission time.

System's mission time (hours)	100	200	350	450
Availability$_{sim}$	0.82071	0.82022	0.82005	0.81997
Availability$_{diff}$	0.81899	0.81986	0.81990	0.81990
Elapsed time$_{sim}$	16992.00	29373.38	54134.43	69562.19
Elapsed time$_{diff}$	1591.10	1583.63	2128.11	1663.66

Table 5. The results for 15 sample instants.

Problem number	System's configuration	Number of the repair facilities	Number of the system's states	System's availability after 200 hours: Proposed model	Simulation: Average of 10 runs	Simulation: The standard deviation of 10 runs	Elapsed time (seconds): Proposed model	Simulation: Average of 10 runs	Simulation: The standard deviation of 10 runs
1	[1,1]	1	8	0.74967	0.74990	0.00067	0.02	7926.05	117.25
2	[1,2]	1	17	0.80665	0.80631	0.00213	0.07	10298.10	235.13
3	[1,2]	2	17	0.84450	0.84461	0.00143	0.46	7892.44	211.63
4	[2,2]	3	35	0.96027	0.96039	0.00086	0.34	9894.69	223.35
5	[2,2]	4	35	0.96027	0.96030	0.00121	0.30	10272.08	271.85
6	[1,2,3]	1	229	0.74543	0.74595	0.00325	10.85	22020.66	370.02
7	[2,2,3]	2	535	0.93804	0.93802	0.00125	56.23	23884.42	460.44
8	[2,2,3]	5	340	0.95702	0.95722	0.00083	23.24	18198.22	213.19
9	[3,3,3]	5	1054	0.99046	0.99057	0.00058	229.00	24487.54	323.54
10	[2,2,2,2]	5	1125	0.91653	0.91735	0.00144	353.31	26755.38	424.01
11	[4,2,3,1]	5	2576	0.81986	0.82022	0.00099	1583.63	29373.38	446.53
12	[2,1,3,2]	3	1101	0.81795	0.81781	0.00112	331.12	25265.04	219.00
13	[2,2,1,4]	5	1380	0.81765	0.81790	0.00188	570.06	27372.54	453.53
14	[3,2,3,1]	7	1557	0.81857	0.81903	0.00066	720.97	26510.92	603.98
15	[2,2,1,2,1]	4	1300	0.72271	0.72212	0.00225	582.96	22370.16	313.13

which is the reason behind the superiority of the CKEB method.

5.1. Managerial insights

To better compare the solving methodologies, we solved 15 different instances on small, medium, and large scales. These instances are designed based on the configurations of the real pumps in different desalination plants under study. In these instances, the number of pump stations varies from two to five and the number of repair facilities from one to seven. Therefore, the number of the system states varies from 8 to 2576. Other system parameters are generated randomly. The results of these 15 instances are presented in Table 5.

As observed in Table 5, the simulation runs ten times for each instance, the number of the simulation iterations for each run equals 1,000,000, and the system mission time equals 200 hours. In this table, for all system configurations, the computational time of the CKEB method is significantly less than that of the simulation technique. A T-test was used for comparing the availability and computational time of both methods using Minitab 17. The results of the availability and computational time of the systems are presented in Table 6.

As presented in Table 6, both methods obtained equal availability, confirming the accuracy of the CKEB method.

Given the workplace constraint in a desalination plant, it is essential to optimize the configuration of the pumps to keep the plant functional. In addition, allocating the proper number of multi-tasking maintenance facilities increases the reliability of the pump stations. For this reason, different techniques like the RAP can be efficiently implemented. The RAP is an NP-hard problem in nature. Therefore, lack of an appropriate model to calculate the objective function of the problem adds up to the problem complexity. In this paper, a CKEB technique is employed to calculate the availability of a system that consists of the series sub-systems. The components of each sub-system are considered repairable components, while some multi-tasking maintenance facilities repaired the failed components of all sub-systems.

The computational time ratio for the CKEB method to the simulation technique is between 0.0002% and 0.0058% for the small-sized problems, between 0.05% and 0.94% for the medium-sized problems, and between 1.31% and 5.39% for the large-sized problems. Here, the presented model can reduce the complexity of the optimization methods used to optimize the configuration of the components in such systems.

Table 6. T-values and P-values for comparing two solving methodologies.

Problem number	System's availability after 200 hours		Elapsed time (seconds)	
	T-value	P-value	T-value	P-value
1	1.32	0.219	213.77	0.000
2	–0.50	0.626	138.50	0.000
3	0.24	0.813	117.93	0.000
4	0.44	0.669	140.09	0.000
5	0.08	0.939	119.49	0.000
6	0.51	0.625	188.10	0.000
7	–0.05	0.961	163.65	0.000
8	0.76	0.466	269.59	0.000
9	0.60	0.563	237.10	0.000
10	1.80	0.105	196.91	0.000
11	1.15	0.280	196.80	0.000
12	–0.40	0.702	360.04	0.000
13	0.42	0.684	186.88	0.000
14	2.20	0.055	135.03	0.000
15	–0.83	0.428	220.03	0.000

6. Conclusion and further studies

In this paper, a new method based on the Chapman-Kolmogorov equations was presented for evaluating the availability of a desalination system's pump stations with three-state repairable pumps. At the same time, each repair facility can service the pumps of all pump stations. The differential equations were compiled by using the Chapman-Kolmogorov equations. Then, by solving the differential equations, the availability of the system was calculated. Finally, to validate the proposed model, we used the simulation method. The results showed that the Chapman-Kolmogorov Equations-Based (CKEB) technique could measure the system availability which has a significantly shorter computational time. The computational time ratio for the CKEB method on simulation technique was between 0.0002% and 0.0058% for the small-sized problems; between 0.05% and 0.94% for the medium-sized problems; and between 1.31% and 5.39% for the large-sized problems. On average, the computational time of the CKEB method for calculating the system availability was 1.12% of the computational time of the simulation technique, which is the only other technique used for calculating such system availability. Moreover, considering the proposed CKEB method steps, the complexity of the proposed method in practice is less than the simulation technique.

For expanding the results of the current research, some directions for future studies are given as follows: optimizing the configuration of components using the results of the current research; solving the model by considering cold or warm standby policy; drawing the problem closer to the real-case condition; reviewing and assessment of the reliability (availability) of repairable continuous-state problems; solving the model with a different repair policy for under-repair equipment; solving the model by considering the repair event for partially failed components; and solving the model with a different repair policy for under-repair equipment.

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