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The analysis of bullwhip effect in supply chain based on hedging strategy compared with optimal order quantity strategy

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Abstract. Bullwhip effect is one of the most important problems in the supply chain management. It can cause a supply chain to experience significant inefficiency. Despite the considerable scope of research about bullwhip effect, few studies have investigated this phenomenon, which is caused by product price fluctuation. This study considers a two-period supply chain consisting of one supplier, one wholesaler, and one retailer. The wholesale price may increase greatly in the beginning of the second period. In this case, a large number of end customers will purchase the product from a retailer. In response to the demands of the end customers in the second period, two ordering strategies available to the retailer are considered: optimal order quantity strategy and hedging strategy with call option. For each strategy, we calculate the bullwhip effect ratio for two periods and compare the results. We found that the lower exercise price in hedging strategy compared with the wholesale price in the optimal order quantity strategy must not contribute to extra product purchase. The research provides new insights into how hedging strategy can reduce bullwhip effect.

1. Introduction

A supply chain is defined as a system of suppliers, manufacturers, distributors, retailers, and customers in which material, financial, and information flows connect participants in both directions [1]. In the supply chain system, there are various forms of uncertainty. The “bullwhip effect” is a short-hand term for a dynamic uncertainty phenomenon in supply chains [2]. It is one of the most studied phenomena in supply chain management [3]. It is defined as “the amplification of demand (or order) variance, from customer to factory, as demand information passes back through the supply chain” [4]. The bullwhip effect can cause supply chains to experience significant inefficiency, e.g., providing poor service to customers, weak demand forecasting, loss of income and customers, and extra inventory capital in the entire chain of warehouses [5]. In order to control or eliminate the bullwhip effect, we must first understand its causes [6]. Lee et al. [7] introduced four basic causes of this phenomenon:

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1. Wrong demand forecasting;
2. Grouping of orders into batches;
3. Fluctuation in the products prices;
4. Corporate policies regarding shortage.

In this paper, we focus on the bullwhip effect arising from price fluctuations.

The law of demand is one of the most fundamental concepts in economics. The quantity demanded rises as the price of products falls [8]. The demand law projects an inverse relationship between price and quantity demanded; all things are to be equal. Yet, there are some exceptions, e.g., the quantity demanded increases with price [9]. These exceptions include Giffen goods, speculative goods, conspicuous goods, conspicuous necessities, future expectations about prices, demand for necessaries, and customer's irrational behavior [10]. The supply chain members might experience panic buying by these exceptions. People usually buy a large number of products, known as panic buying or consumer hoarding, to avoid future shortage [11] so that they can feel a sense of security, comfort, and momentary escape from or alleviation of stress [12] despite the constant increase in prices. These patterns are, in fact, behavioral and emotional responses to scarcity [13]. In a realistic market, consumers make purchasing decisions with respect to price in not only the current period but also past and future periods [14]. Considerable price fluctuations were driven by either huge shortages or surpluses in capacity. The shortages were exacerbated by panic buying and over-ordering, followed by a sudden drop in demand [15]. Any panic buying could lead to problems such as bullwhip effect [16].

Panic buying has been frequently observed. For example, fears of a sharp rise in the price of toilet paper prompted panic buying in Taiwan when toilet paper manufactures were expecting a 10 to 30% price rise [17]. In a short while in September 2008, the price of different types of rice in Iran rose suddenly while being on high demand [18]. Another instance is the panic buying caused by the price fluctuation of cotton market in October 2015 [19]; cooking oil and flour in Perak in January 2008 [20]; wheat in Middle Eastern and North African countries in August 2007 [21]; and everything from electronics to wine in Russia in December 2014 [22].

Recently, financial hedging has been receiving considerable attention in the literature of operations management [23]. Hedging is the act of protecting oneself against futures loss [24]. By definition, it "involves taking counterbalancing actions so that, loosely speaking, the future value varies less over the possible states of nature. These counterbalancing actions involve trading financial instruments, including short-selling, futures, options, and other financial derivatives" [25]. The wise use of derivatives for hedging purposes allows for an effective reduction of price risk exposure [26]. Hedging is known as price insurance, risk shifting, or risk transference function [24]. In order to ensure protection against various risks derived from production, demand, and price, option contracts have been extensively used in many industries such as fashion apparel industry, food processing industry, and automobile industry [27]. According to a survey of large US nonfinancial firms, approximately 40% of responding firms routinely purchase options or futures contracts in order to hedge price risks [28].

We know the price fluctuation caused by panic buying and, then, the bullwhip effect. Panic buying is now a frequent occurrence in many countries, especially after the recent COVID-19 pandemic. Also, we are witnessing an increase in the number of panic buying cases for different reasons in Iran. The number of articles related to panic buying emphasizes the need to consider consumer behavior under extreme conditions. Financial hedging is an approach to the management of price fluctuations. Many researchers have worked on bullwhip effect and hedging independently. The motivation of this study is to propose a new approach based on hedging to control the bullwhip effect. In this paper, the bullwhip effect is analyzed based on hedging strategy through call option contracts compared with optimal order quantity strategy.

This paper is organized as follows: Section 2 reviews the related literature. Section 3 illustrates the problem description. Section 4 presents the proposed model and elaborates on the details. Section 5 addresses the bullwhip effect measures and Section 6 compares them. Section 7 shows numerical analysis. Finally, Section 8 discusses conclusions and managerial insights.

2. Literature review

There have been numerous studies addressing the bullwhip effect in recent years. However, a few have investigated the bullwhip effect due to price fluctuation [29]. Also, the use of financial hedging continues to be increasing over the years. In this research, we consider the hedging role in managing the bullwhip effect. Hence, in the following lines, we will only review the literature relevant to bullwhip effect and financial hedging.

2.1. Bullwhip effect

Lee et al. [7] demonstrated that price fluctuation could give rise to the bullwhip effect. Moyaux and McBurney [30] found that some of kinds of speculators could stabilize the price in a market and reduce price fluctuations caused by the bullwhip effect. Ozelkan and

Cakanyildirim [31] analyzed the impact of procurement price variability in the upstream of a supply chain on the downstream retail prices. Due to the reverse direction of the price variability propagation (compared to the direction of the bullwhip effect in order variability), they referred to this behavior as the Reverse Bullwhip effect in Pricing (RBP).

Rong et al. [16] studied how pricing strategies could affect the variability of customer orders. According to their findings, when customer behavior is sufficiently strategic, the customer order process under a one-Period Correction (1PC) pricing strategy gets more volatile than the capacity process.

Bolarin et al. [32] evaluated the impact of price fluctuations on the variability of orders along a traditional multilevel supply chain. They found when the bullwhip effect would emerge. Su and Geunes [33] examined the results of bullwhip effect from price fluctuations in a two-echelon supply chain with deterministic and price-sensitive demand. They provided numerical evidence to demonstrate that increased system profit could coexist with the bullwhip effect as a result of price increases.

Ma et al. [34] presented a price-sensitive demand model and first-order autoregressive pricing process. Their findings showed that the retailer should share their customer demand and price information with their upstream businesses. Also, the wholesaler should adopt end-demand and order information, especially when the product price sensitivity coefficient is large or the demand shocks are low.

Wang et al. [35] investigated the bullwhip effect in terms of consumer behavior. They developed insights into the influence of consumer price forecasting behavior on the bullwhip effect. Their results demonstrated that consumer price forecasting behavior could reduce the bullwhip effect, especially when the consumer sensitivity to price changes is medium.

Ma et al. [36] attained insights into how the bullwhip effect in two parallel supply chains with interacting price-sensitive demands was affected in contrast to a single-product condition in a serial supply chain.

Ma and Xie [37] focused on the dynamic pricing game of the duopoly air conditioner market with disturbance in demand. Their results indicated that the bullwhip effect between the order quantity and the actual demand was weakened gradually along with the price adjustment.

Gao et al. [38] investigated the difference in bullwhip effects in online and offline retail supply chains and offered insights into how frequent price discounts in e-commerce could affect the bullwhip effect in the online retail supply chain. Tai et al. [39] found that bullwhip effect could be, under conditions, stronger or weaker than the case where the price was not considered.

Gamasaee and Fazel Zarandi [40] analyzed the impact of joint demand, orders, lead time, and pricing decisions on reducing bullwhip effect. Their results point out a significant reduction in bullwhip effect.

Adnan and Ozelkan [41] investigated the behavior of the bullwhip effect with respect to the price adjustment speed and historical price discount sensitivity. Their results showed that controlling price discount sensitivity was useful for supply chain companies.

Zanddizari et al. [42] modeled the concept of Distance to Loss (DL) by bullwhip effect. This concept is a function of the retailer's selling price, the manufacturer's wholesale price, salvage value of the end item, the retailer's expected demand, and the retailer's variance of demand.

Feng et al. [43] studied the customers' order variability and the firms' profit under several representative heuristic pricing strategies. They realized that the bullwhip effect or reverse bullwhip effect could occur as a consequence of supply dramatic shock and adjusting the prices simultaneously.

Qu and Raff [44] found that a decentralized supply chain might be more resilient to demand shocks than a vertically integrated supply chain. Their results indicate that adjusting the wholesale price is valuable when the bullwhip effect is most likely to occur and potentially most harmful for manufacturers.

2.2. Financial hedging

Some empirical studies such as Allayannis et al. [45], Carter et al. [46], Bannai et al. [47], Chen and Lin [48], Treanor et al. [49], Brusset and Bertrand [50], Luo et al. [51], Swidan and Merkert [52], and Merkert and Swidan [53] demonstrated that financial derivatives enhanced firms' financial performance. Alam and Gupta [54] found that firms engaged in hedging compared to non-hedgers had less volatility in the firm's value.

Kallapur and Eldenburg [55] examined that operational hedging policies including strategies such enhancing business operation's flexibility, diversifying production lines, and varying the combination of variable and fixed costs. Borensztein et al. [56] employed a dynamic optimization model to quantify the potential welfare gains of hedging against commodity price risk for commodity-exporting countries. They found that hedging enhanced domestic welfare by reducing export income volatility and decreasing the country's need to hold precautionary reserves.

Liu et al. [57] showed particular conditions where supply chain coordination could be reached. They provided practical insights into the manufacturer and retailer. Tauser and Cajka [58] focused on selected aspects of risk management in agricultural business and compared different hedging methods relevant to managing the commodity risk associated with agricultural production.

Turcic et al. [28] greatly deepened the understanding of why and how individual firms should hedge. Yang et al. [59] introduced three coordinating option contracts led by the supplier to reduce the retailer's risk, where the call option contract, the put option contract, and bidirectional option contract could reduce the shortage risk, the inventory risk, and bilateral risk, respectively.

Park et al. [60] examined a firm's production planning, pricing, and financial hedging decisions under exchange-rate and demand uncertainty.

Kouvelis et al. [61] studied the hedging of cash-flow risks in a bilateral supply chain of a supplier and manufacturer. They characterized the interaction of hedging decisions of the supply chain partners and the associated effects of market conditions, production efficiencies, and cash-flow correlation.

Kouvelis et al. [62] considered a firm purchasing a storable commodity from a spot market with price fluctuations and access to an associated financial derivatives market. In this circumstance, they surveyed two types of hedging instruments and compared their performances.

Liu and Wang [63] presented a network equilibrium model for supply chain networks with strategic financial hedging. They considered multiple competing firms. The firms were exposed to commodity price risk and exchange rate risk and they used future contracts to hedge the risks.

Hu et al. [64] built a simple theoretical model to compare the implications of fuel financial hedge and operational fuel efficiency on the expected profit of airlines. They found that financial hedge was more efficient in reducing airlines' profit volatility/risk exposure, while operational improvement would generate a higher expected profit level when its effectiveness was sufficiently high. Hainaut [65] studied hedging strategies of crop harvest incomes with futures and options on indexes of cumulated average temperatures.

March et al. [66] investigated a supply chain in which the vendor could adopt two financial approaches as means of hedging stocks in order to reduce the commodity risk related to the high price fluctuations.

Although the number of studies on supply chain management as well as financial fields has been increasing in recent years, none of the published articles has examined the effect of hedging on the bullwhip effect. In fact, as part of its novelty, the present study identifies what will happen to bullwhip effect ratio if the hedging strategy is applied. The current research focuses on hedging strategy compared with optimal order quantity strategy for calculating bullwhip effect ratio in a two-echelon supply chain.

3. Problem description

This study considered a two-period supply chain [11,67] consisting of one supplier, one wholesaler, and one retailer. The supplier manufactures a single product sold to the wholesaler. The wholesaler sells the product to the retailer and then, the retailer sells it to end consumers. We assume that there is a large population of end consumers in the market. Also, we presume that the retailer will receive the order at the beginning of each period and the lead time is zero. In the first period, the product price is constant and at the beginning of the second period, the product price may increase significantly, which is reasonable in many situations. In each period, the price is independent and identically distributed (i.i.d.) from a normal distribution with average μ and variance σ^2. If the product price increases greatly, a large number of end customers will purchase the product from the retailer. This is contrary to the law of demand and the reasons of this event were mentioned in the introduction section. Therefore, demand is a dependent variable on the price of the product.

We assume that in the first period, the initial inventory level is zero and the retailer orders the optimal order quantity from the wholesaler. At the end of the first period, the leftover products are carried over to the second period for sales and incur a holding cost. For managing the demands of the end customers in the second period, there are two ordering strategies available to the retailer, which are optimal order quantity strategy and hedging strategy. The retailer uses the call option contract for long hedging strategy. This contract is concluded between the wholesaler and the retailer. We suppose that shortage is not allowed and in the second period, the retailer can buy additional units from an emergency source at a higher price.

This study aimed to address the following research question:

- "What are the results of hedging on the bullwhip effect ratio?"

According to the conditions listed above, for each period, we will calculate the retailer's optimal order using optimal order quantity strategy and hedging strategy; for these strategies, the retailer's bullwhip effect is measured by the ratio of the order quantity variance, encountered by the wholesaler, to the demand variance faced by retailer. The ratio values are compared to each other. This ratio has been employed by many researchers [39,68–71]. We also consider the retailer to be risk neutral. When the retailer is risk neutral, they choose to maximize their own expected profit [57].

4. The proposed model

In this section, the retailer's optimal order quantities are determined by the optimal order quantity and the hedging strategy.

4.1. Notations

To develop the model, notations are summarized as follows:

- **Sets:**

$t = \{1, 2\}$	Time periods ($t = 1$ and $t = 2$ show the first and second periods, respectively)
$i = \{1, 2\}$	Types of price changes; ($i = 1$ shows the product price as constant or the small change price per unit occurs; $i = 2$ shows significant increase in price per unit), (for $t = 1$, $i \neq 2$)
$j = \{1, 2\}$	Types of retailer's ordering decisions; ($j = 1$ shows that the retailer only uses an optimal order quantity strategy and $j = 2$ shows the retailer uses hedging strategy), (for $t = 1$, $j \neq 2$).

- **Decision variables:**

q_t^{ij}	The retailer's order quantity in the period t under decision j and price change i
$(q_t^{ij})^*$	The retailer's optimal order quantity in the period t under decision j and price change i

Also, by assumptions explained in the text, q_1^{12}, q_1^{21}, and q_1^{22} are not defined.

- **Parameters:**

p_t	The spot price per unit in the period t
p_2^k	The exercise price per unit in the second period
θ_2	The significant increase in the wholesale price per unit in the second period ($\theta_2 > 0$)
ε_2	The small change in the wholesale price per unit in the second period, (ε_2 can be positive or negative or zero)
w_t	The wholesale price per unit in non-hedging in the period t
φ	The difference between the wholesale price and the exercise price per unit in the second period
m_t	The retailer's fixed percentage profit margin in the period t ($m_t > 0$)
n_2	The emergency purchasing price per unit by the retailer in the second period
c_{o2}	The option price per unit in the second period
h_t	The holding cost per unit in the period t
c_t	The order cost per unit in the period t
μ	The average of the product price in the period t
σ^2	The variance of the product price in the period t
σ	The standard deviation of the product price in the period t
d_1	The product demand in the first period
μ_{d_1}	The end customer's average demand in the first period
$\sigma_{d_1}^2$	The variance of the end customer's demand in the first period
σ_{d_1}	The standard deviation of the end customer's demand in the first period
d_2^i	The product demand in the second period under price change i
$\mu_{d_2^i}$	The end customer's average demand in the second period under price change i
$\sigma_{d_2^i}^2$	The variance of the end customer's demand in the second period under price change i
$\sigma_{d_2^i}$	The standard deviation of the end customer's demand in the second period under price change i
r	Consumer sensitivity to price increases in the second period
a	Basic market demand
b	The demand curve slope
$f(x)$	The probability distribution function of the end customer demand to the retailer
$F(x)$	The cumulated distribution function of the end customer demand to retailer
$S(q_t^{ij})$	The retailer's expected sales in the period t under decision j and price change i
$I(q_t^{ij})$	The expected leftover inventory in the period t under decision j and price change i
$H(q_t^{ij})$	The expected order quantity to the emergency source in the period t under decision j and price change i
$\pi(q_t^{ij})$	The retailer's expected profit in the period t under decision j and price change i
$\bar{q}_{q_1^{ij},q_2^{ij}}$	Total average retailer's order
$BWE_{q_1^{ij},q_2^{ij}}$	Bullwhip effect on the retailer's optimal order quantities under decision j and price change i, ($i \neq 1$)

$p_2^k < w_2 < n_2 < p_2$

4.2. The relation between the wholesale price and the retail price

The wholesaler is selling a product to the retailer at w_t and the retailer is using a fixed percentage profit margin ($m_t > 0$) to identify p_t [72]. The relation between p_t and w_t is given in Eq. (1):

$$p_t = (1 + m_t)w_t. \tag{1}$$

4.3. Types of price changes

In the first period, w_1 is fixed. We have Eq. (2):

$$p_1 = (1 + m_1)w_1. \tag{2}$$

At the beginning of the second period, the product price is constant or the small change price per unit (ε_2) or significant increase (θ_2) in price per unit occurs. Therefore, the relation between w_1 and w_2 is given in Eqs. (3) and (4), respectively:

$$w_2 = w_1 + \varepsilon_2, \tag{3}$$

$$w_2 = w_1 + \theta_2. \tag{4}$$

By substituting Eqs. (3) and (4) into Eq. (1), we will have Eqs. (5) and (6) for the second period:

$$p_2 = (1 + m_2)(w_1 + \varepsilon_2), \tag{5}$$

$$p_2 = (1 + m_2)(w_1 + \theta_2). \tag{6}$$

4.4. Types of demand model

In this paper, the demand of end customers is considered as the product price function and is shown with linear function. In the first period, the linear demand model is considered as Eq. (7):

$$d_1(p_1) = a - bp_1. \tag{7}$$

For the second period with constant product price or small change price per unit, we consider the linear demand model as Eq. (8):

$$d_2^1(p_2) = a - bp_2. \tag{8}$$

For the second period, when a significant increase in price per unit occurs, the linear demand model can be written as Eq. (9) [35]:

$$d_2^2(p_2) = (a - bp_2) + rb(p_2 - p_1), \qquad r > 1. \tag{9}$$

In Eq. (9), the first term on the right-hand side of the equation expresses the underlying demand and is a decreasing function of p_2, while the second term represents the impact of price behavior on the demand. p_2 is higher than p_1; therefore, the customers buy more to reduce their future needs.

4.5. The retailer's first period order quantity

The retailer's first period expected profit is given in Eq. (10):

$$\pi\left(q_1^{11}\right) = p_1 S\left(q_1^{11}\right) - w_1 q_1^{11} - h_1 q_1^{11} - c_1 q_1^{11}. \tag{10}$$

The linear demand model is given in Eq. (11):

$$d_1 = a - bp_1. \tag{11}$$

The inverse demand equation will be given in Eq. (12):

$$p_1 = \frac{1}{b}(a - d_1). \tag{12}$$

Proposition 1. We substitute Eq. (12) in Eq. (10) and solve it for q_1^{11}. The retailer's first period optimal order quantity is given by:

$$\left(q_1^{11}\right)^* = \mu_{d_1} + \sqrt{2\pi}\sigma_{d_1}\left[\frac{1}{2} - \frac{w_1 + h_1 + c_1}{\frac{1}{b}(a - d_1)}\right]. \tag{13}$$

Proof. See Appendix A.

4.6. The retailer's second period order quantity

At the beginning of the second period, to purchase products from the wholesaler, the retailer will face one of two options about the product price:

- Constant price or small change price per unit (ε_2);
- Significant increase in price per unit (θ_2).

The retailer can use the optimal order quantity strategy or hedging strategy. Figure 1 shows product price changes and the retailer ordering decisions in the second period. Therefore, there are four scenarios, a given in Table 1. In the second period, the retailer's ordering process is shown in Figure 2.

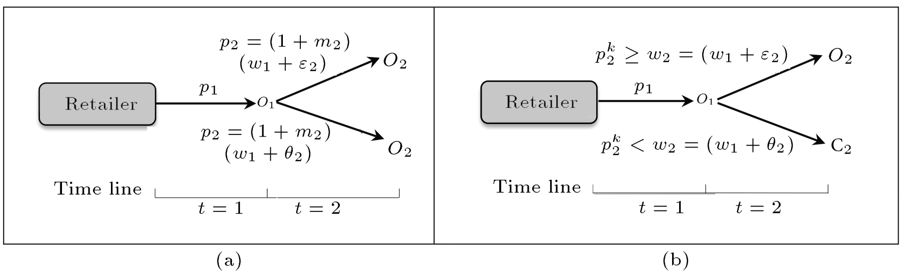

Figure 1. Retailer's ordering decisions in the second period: (a) Retailer using an optimal order quantity strategy (O_2) and (b) Retailer using hedging strategy (C_2).

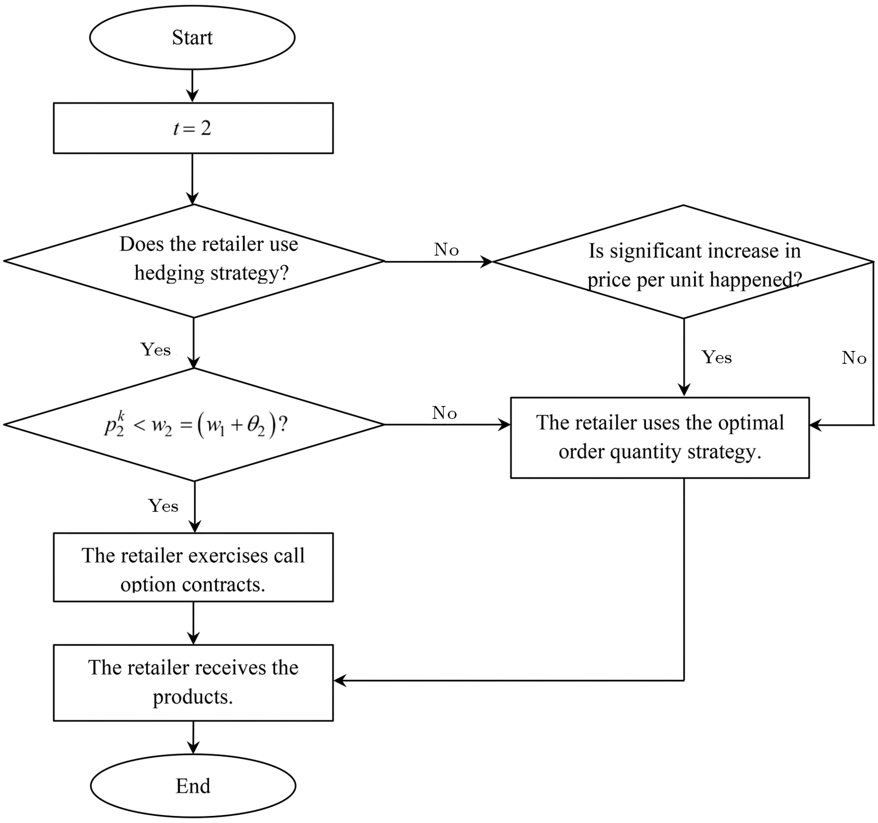

Figure 2. Retailer's ordering process in the second period.

Table 1. Description of scenarios.

Scenarios	Ordering strategy	Price changes
1	Optimal order quantity	$w_2 = w_1 + \varepsilon_2$
2		$w_2 = w_1 + \theta_2$
3	Hedging	$w_2 = w_1 + \varepsilon_2$
4		$w_2 = w_1 + \theta_2$

For each scenario, we calculate the retailer's optimal order quantity.

- **Scenario 1.** The retailer's expected profit is given in Eq. (14):

$$\pi\left(q_2^{11}\right) = p_2 S\left(q_2^{11}\right) - w_2 q_2^{11} - c_2 q_2^{11} - h_2\left(q_2^{11} + I\left(q_1^{11}\right)\right) - n_2 H\left(q_2^{11}\right). \quad (14)$$

The linear demand model is given in Eq. (15):

$$d_2^1 = a - bp_2. \quad (15)$$

The inverse demand equation will be as Eq. (16):

$$p_2 = \frac{1}{b}(a - d_2^1). \quad (16)$$

Proposition 2. We substitute Eq. (16) in Eq. (14) and solve it for q_2^{11}. The retailer's optimal order quantity is given by:

$$\left(q_2^{11}\right)^* = \mu_{d_2^1} + \sqrt{2\pi}\sigma_{d_2^1}\left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\frac{1}{b}(a - d_2^1) + n_2}\right]. \quad (17)$$

Proof. See Appendix B.

- **Scenario 2.** The retailer's expected profit is given in Eq. (18):

$$\pi\left(q_2^{21}\right) = p_2 S\left(q_2^{21}\right) - w_2 q_2^{21} - c_2 q_2^{21} - h_2\left(q_2^{21} + I\left(q_1^{11}\right)\right) - n_2 H\left(q_2^{21}\right). \quad (18)$$

The linear demand model is given in Eq. (19):

$$d_2^2 = (a - bp_2) + rb(p_2 - p_1). \quad (19)$$

By substituting Eqs. (2) and (6) into Eq. (19), we have Eq. (20):

$$d_2^2 = (a - bp_2) + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb. \quad (20)$$

The inverse demand equation will be as Eq. (21):

$$p_2 = \frac{1}{b}\left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2\right). \quad (21)$$

Proposition 3. We substitute Eq. (21) for p_2 in Eq. (18) and solve it for q_2^{21}. The retailer's optimal order quantity is given by Eq. (22) as shown in Box I.

Proof. See Appendix C.

$$\left(q_2^{21}\right)^* = \mu_{d_2^2} + \sqrt{2\pi}\sigma_{d_2^2}\left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\left[\frac{1}{b}(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2)\right] + n_2}\right]. \quad (22)$$

Box I

- **Scenario 3.** The retailer's expected profit is as Eq. (23):

$$\pi\left(q_2^{12}\right) = p_2 S\left(q_2^{12}\right) - w_2 q_2^{12} - c_2 q_2^{12} - c_{o2} q_2^{12} - h_2\left(q_2^{12} + I\left(q_1^{11}\right)\right) - n_2 H\left(q_2^{12}\right). \quad (23)$$

The linear demand model is given in Eq. (24):

$$d_2^1 = a - bp_2. \quad (24)$$

The inverse demand equation is given in Eq. (25):

$$p_2 = \frac{1}{b}(a - d_2^1). \quad (25)$$

Proposition 4. We substitute Eq. (25) for p_2 in Eq. (23) and solve it for q_2^{12}. The retailer's optimal order quantity is given by:

$$\left(q_2^{12}\right)^* = \mu_{d_2^1} + \sqrt{2\pi}\sigma_{d_2^1}\left[\frac{1}{2} - \frac{w_2 + c_2 + c_{o2} + h_2}{\frac{1}{b}(a - d_2^1) + n_2}\right]. \quad (26)$$

Proof. See Appendix D.

- **Scenario 4.** The retailer's expected profit is measured in Eq. (27):

$$\pi(q_2^{22}) = p_2 S(q_2^{22}) - p_2^k q_2^{22} - c_{o2} q_2^{22} - h_2(q_2^{22} + I(q_1^{11})) - n_2 H(q_2^{22}). \quad (27)$$

The linear demand model is given in Eq. (28):

$$d_2^2 = (a - bp_2) + rb(p_2 - p_1). \quad (28)$$

By substituting Eqs. (2) and (6) into Eq. (28), we will have Eq. (29):

$$d_2^2 = (a - bp_2) + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb. \quad (29)$$

The inverse demand equation is given in Eq. (30):

$$p_2 = \frac{1}{b}(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2). \quad (30)$$

Also, the relation between p_2^k and w_2 will appear as in Eq. (31):

$$w_2 = p_2^k + \varphi. \quad (31)$$

Proposition 5. We substitute Eq. (30) for p_2 in Eq. (27) and solve it for q_2^{22}. The retailer's optimal order quantity is given by:

$$\left(q_2^{22}\right)^* = \mu_{d_2^2} + \sqrt{2\pi}\sigma_{d_2^2}\left[\frac{1}{2} - \frac{p_2^k + c_{o2} + h_2}{\frac{1}{b}(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2) + n_2}\right]. \quad (32)$$

Proof. See Appendix E.

Table 2 shows the retailer's optimal quantities for each scenario in the second period.

5. Bullwhip effect measures

In the previous section, the retailer's optimal order quantity was calculated using the optimal order quantity strategy and hedging strategy. This section develops expressions for bullwhip effect using the two strategies. The bullwhip effect ratio is calculated

Table 2. The retailer's optimal order quantity in the second period under decision j and price change i.

Scenario	The retailer's optimal order quantity
1	$\left(q_2^{11}\right)^* = \mu_{d_2^1} + \sqrt{2\pi}\sigma_{d_2^1}\left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\frac{1}{b}(a - d_2^1) + n_2}\right]$
2	$\left(q_2^{21}\right)^* = \mu_{d_2^2} + \sqrt{2\pi}\sigma_{d_2^2}\left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\left[\frac{1}{b}(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2)\right] + n_2}\right]$
3	$\left(q_2^{12}\right)^* = \mu_{d_2^1} + \sqrt{2\pi}\sigma_{d_2^1}\left[\frac{1}{2} - \frac{w_2 + c_2 + c_{o2} + h_2}{\frac{1}{b}(a - d_2^1) + n_2}\right]$
4	$\left(q_2^{22}\right)^* = \mu_{d_2^2} + \sqrt{2\pi}\sigma_{d_2^2}\left[\frac{1}{2} - \frac{p_2^k + c_{o2} + h_2}{\frac{1}{b}(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2) + n_2}\right]$

according to q_1^{11} and q_2^{21}. We then repeat the process for q_1^{11} and q_2^{22}. Next, the results are compared to each other. It should be noted that for the second period, we consider only Scenarios 2 and 4 because the product price increase occurs in these scenarios.

To quantify the bullwhip effect, we can use Eq. (33) where σ_o^2 shows the variance of retailer order quantity and σ_D^2 is the variance of end customer demand:

$$BWE = \frac{\sigma_o^2}{\sigma_D^2}. \tag{33}$$

Based on the preceding assumption, we can conclude that:

$$Cov(d_1, d_2^i) = 0. \tag{34}$$

Before calculating the bullwhip effect ratio, we have Eqs. (35) and (36) as follows:

$$\bar{q}_{q_1^{ij}, q_2^{ij}} = \frac{1}{2}\left(q_1^{ij} + q_2^{ij}\right), \tag{35}$$

$$\sigma_o^2 = \frac{1}{T-1}\sum_{t=1}^{T}\left(q_t^{ij} - \bar{q}_{q_1^{ij}, q_2^{ij}}\right)^2. \tag{36}$$

The variance of the market demand during the two periods can be written as Eq. (37):

$$\sigma_D^2 = Var(d_1, d_2^2) = \sigma_{d_1}^2 + \sigma_{d_2^2}^2 + 2Cov(d_1, d_2^2)$$
$$= \sigma_{d_1}^2 + \sigma_{d_2^2}^2. \tag{37}$$

According Eq. (11), we have Eqs. (38) and (39) as follows:

$$\sigma_{d_1}^2 = b^2\sigma^2, \tag{38}$$

$$\sigma_{d_1} = b\sigma. \tag{39}$$

According Eqs. (19) and (28), we have Eqs. (40) and (41) as follows:

$$\sigma_{d_2^2}^2 = (b^2 + 2r^2b^2)\sigma^2, \tag{40}$$

$$\sigma_{d_2^2} = \sqrt{(b^2 + 2r^2b^2)}\sigma. \tag{41}$$

Therefore, according to Eqs. (38) and (40), we have Eq. (42) as follows:

$$\sigma_D^2 = (2b^2 + 2r^2b^2)\sigma^2. \tag{42}$$

Also, through Eqs. (11), (15), (20), and (29), we can reach Eqs. (43) and (44) as follows:

$$\mu_{d_1} = \mu_{d_2^1} = a - b\mu, \tag{43}$$

$$\mu_{d_2^2} = a - b\mu + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb. \tag{44}$$

5.1. Bullwhip effect ratio for optimal order quantity strategy

With substituting Eqs. (35) and (36) in Eq. (33), we have Eq. (45) as follows:

$$BWE_{q_1^{11}, q_2^{21}} = \frac{\sigma_o^2}{\sigma_D^2} = \frac{\left(q_1^{11} - \bar{q}_{q_1^{11}, q_2^{21}}\right)^2 + \left(q_2^{21} - \bar{q}_{q_1^{11}, q_2^{21}}\right)^2}{\sigma_D^2}. \tag{45}$$

Theorem 1. The bullwhip effect for optimal order quantity strategy obtained by Eq. (46) as shown in Box II.

Proof. See Appendix F.

Proposition 6. Bullwhip effect exists (i.e., $BWE_{q_1^{11}, q_2^{21}} > 1$) if Inequality (47) holds:

$$[(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb + 1.25\left[\sqrt{b^2 + 2r^2b^2}\sigma\right] + 2.5[b\sigma]\left[\frac{w_1 + h_1 + c_1}{\frac{1}{b}(a - d_1)}\right] > 2.5\left[\sqrt{b^2 + 2r^2b^2}\sigma\right]\left[\frac{w_2 + h_2 + c_2}{\left[\frac{1}{b}(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2)\right] + n_2}\right] + 1.25[b\sigma] + \sqrt{2(2b^2 + 2r^2b^2)}\sigma. \tag{47}$$

Proof. See Appendix G.

5.2. Bullwhip effect ratio for hedging strategy

According to Eqs. (35) and (36), we have Eq. (48) as follows:

$$BWE_{q_1^{11}, q_2^{21}} = \frac{\frac{1}{2}\left[\begin{array}{l}[(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb + \sqrt{2\pi}\left[\sqrt{b^2 + 2r^2b^2}\sigma\right]\left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\left[\frac{1}{b}(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2)\right] + n_2}\right] \\ - \sqrt{2\pi}[b\sigma]\left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{\frac{1}{b}(a - d_1)}\right]\end{array}\right]^2}{(2b^2 + 2r^2b^2)\sigma^2}. \tag{46}$$

Box II

$$BWE_{q_1^{11},q_2^{22}}=\frac{\frac{1}{2}\left[\begin{array}{l}[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\left[\frac{1}{2}-\frac{p_2^k+c_{o2}+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]\\ \quad -\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\end{array}\right]^2}{(2b^2+2r^2b^2)\sigma^2}. \tag{49}$$

Box III

$$BWE_{q_1^{11},q_2^{22}}=\frac{\sigma_o^2}{\sigma_D^2}=\frac{\left(q_1^{11}-\bar{q}_{q_1^{11},q_2^{22}}\right)^2+\left(q_2^{22}-\bar{q}_{q_1^{11},q_2^{22}}\right)^2}{\sigma_D^2}. \tag{48}$$

Theorem 2. The bullwhip effect for hedging strategy obtained by Eq. (49) as shown in Box III.

Proof. See Appendix H.

Proposition 7. Bullwhip effect exists (i.e., $BWE_{q_1^{11},q_2^{22}}>1$) if Inequality (50) holds:

$$[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+1.25\left[\sqrt{b^2+2r^2b^2}\sigma\right]$$
$$+2.5[b\sigma]\left[\frac{w_1+h_1+c_1}{\frac{1}{b}(a-d_1)}\right]>2.5\left[\sqrt{b^2+2r^2b^2}\sigma\right]$$
$$\left[\frac{p_2^k+h_2+c_{o2}}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]$$
$$+1.25[b\sigma]+\sqrt{2(2b^2+2r^2b^2)}\sigma. \tag{50}$$

Proof. See Appendix I.

6. Comparison of the bullwhip effect ratios using different strategies

In this section, the bullwhip effect ratios for the optimal order quantity strategy and hedging strategy are compared. To compare the bullwhip effect under the two retailer's ordering decisions in the second period, we deduce the following theorem:

Theorem 3. Let $BWE_{q_1^{11},q_2^{22}}$ ($BWE_{q_1^{11},q_2^{21}}$) be the bullwhip effect using hedging strategy (optimal order quantity strategy), assuming that the product price for two periods is i.i.d. from normal distribution. If we have $c_{o2}>c_2+\varphi$, then:

$$BWE_{q_1^{11},q_2^{22}}<BWE_{q_1^{11},q_2^{21}}. \tag{51}$$

Proof. See Appendix J.

From Theorem 3, we know that Inequality (51) depends on the following three parameters: the option price, c_{o2}; the order cost in the second period, c_2; and the difference between the wholesale price and the exercise price in the second period, φ. We can explain Theorem 3 as follows. If the retailer expects that the wholesale price will increase greatly in the beginning of the second period and this, in turn, puts people in the rush to buy the product, the retailer could hedge against the price fluctuations with call option. However, the lower exercise price (p_2^k) compared with the wholesale price (w_2) must not contribute to extra product purchasing by retailer. According to $c_{o2}>c_2+\varphi$, Eqs. (22) and (32), the retailer's optimal order with hedging strategy will be less than the retailer's optimal order using optimal order quantity strategy. From the theoretical perspective, it has been pointed out that to ensure the validity of Inequality (51), as $c_2+\varphi$ increases, c_{o2} must increase. Totally, under the problem description in Section 3, when the retailer's optimal order using hedging strategy is less than the retailer's optimal order under optimal order quantity strategy, the bullwhip effect using hedging strategy ($BWE_{q_1^{11},q_2^{22}}$) is less than the bullwhip effect using the optimal order quantity strategy ($BWE_{q_1^{11},q_2^{21}}$).

7. Numerical analysis

In the preceding sections, the retailer's optimal orders and bullwhip effect measures under the problem description were calculated in Section 3 and then, the bullwhip effect ratios for the optimal order quantity strategy and hedging strategy were compared. This section provides numerical experiments to show the results and illustrate the impact of changing the value of parameters on the bullwhip effect measures. This section consists of two parts. First, in Section 7.1, we compare d_1, d_2^1, and d_2^2 and show why we considered $r>1$. Subsequently, in Section 7.2, we contrast $BWE_{q_1^{11},q_2^{22}}$ with $BWE_{q_1^{11},q_2^{21}}$. We survey the impacts of changing option price (c_{o2}), difference between the wholesale price and the exercise price (φ), customer sensitivity to price increase (r), demand curve slope (b), the significant increase in the wholesale price (θ_2), and the standard deviation of the product price (σ) on $BWE_{q_1^{11},q_2^{21}}$ and $BWE_{q_1^{11},q_2^{22}}$ in Subsections 7.2.1–

7.2.6, respectively. Also, we fixed $a = 200$, $m_1 = 0.25$, $m_2 = 0.3$, $w_1 = 30$, $p_2^k = 29$, $c_1 = 3$, $c_2 = 4$, $h_1 = 1.8$, $h_2 = 2$, $n_2 = 43$, and $\mu = 30$.

7.1. Comparison between d_1, d_2^1, and d_2^2

The end customers' demands in the first period (d_1) and the second periods (d_2^1 and d_2^2) are shown in Figure 3. The parameters r and b were changed to the following values $r \in \{0, 0.1, 0.2, 0.3, 0.4, \cdots, 2\}$ and $b \in \{1, 2, 3, 4, 5, 6\}$. The corresponding demands were computed using Eqs. (11), (15); (24), (19); or (28). d_2^1 (second period demand in Scenarios 1 and 3) is slightly smaller than d_1 because the product price changes were not significant; however, based on Eq. (5), the retailer's fixed percentage profit margin in period 2 (m_2) is greater than in period 1 (m_1).

For $r > 1$, d_2^2 (demands in Scenarios 2 and 4) is bigger than d_1 and d_2^1, which shows that the end customer rushes to buy due to the significant increase in product price. For $0 < r < 1$, d_2^2 is smaller than d_1 and d_2^1. For $r = 1$, d_2^2 is equal to d_2^1. As a result, we only consider $r > 1$.

7.2. Comparison between $\mathrm{BWE}_{q_1^{11},q_2^{21}}$ and $\mathrm{BWE}_{q_1^{11},q_2^{22}}$

From Appendix J, we know $\Delta BWE = BWE_{q_1^{11},q_2^{22}} - BWE_{q_1^{11},q_2^{21}}$. This section surveys the effect of parameter value changes on ΔBWE.

7.2.1. Option price (c_{o2})

Figure 4 illustrates how ΔBWE changes with c_{o2} at different values of σ when $b = 4$, $r = 1.5$, $\theta_2 = 6$. From Figure 4, it can be observed that for $c_{o2} = 11$, we have $\Delta BWE = 0$. By increasing the option price, c_{o2}, from 11 to 20, ΔBWE will decrease because ΔBWE is negatively correlated with c_{o2}. Also, for $c_{o2} \in \{11, 12, 13, \cdots, 20\}$, when the standard deviation of the price rises from 0.5 to 1.5, ΔBWE continues to be negative, but its value rises. For example, for $c_{o2} = 18$ and $\sigma = 0.5, 1, 1.5$, the value of ΔBWE is -2.0216, -1.0408, and -0.7138, respectively. This means that when the standard deviation of product price is lower, the hedging strategy outperforms the optimal order strategy in attenuating the bullwhip effect.

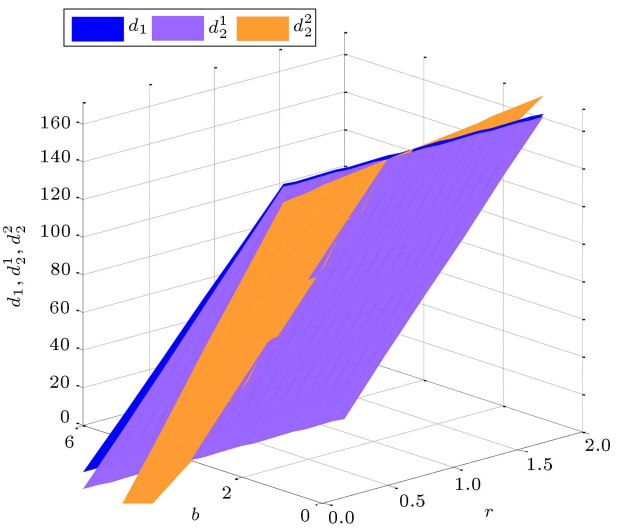

Figure 3. The influence of r and b on d_1, d_2^1, and d_2^2 when $\theta_2 = 6$.

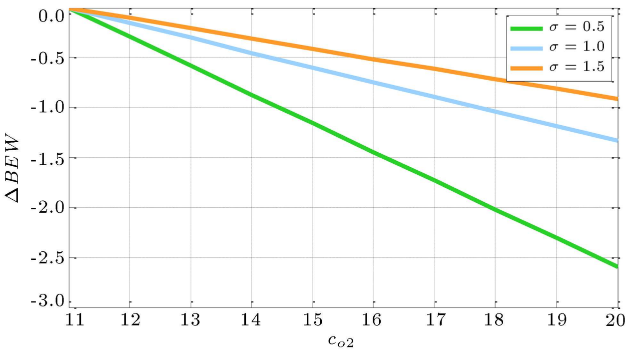

Figure 4. The influence of c_{o2} on ΔBWE when $b = 4$, $r = 1.5$, $\theta_2 = 6$.

7.2.2. Difference between the wholesale price and the exercise price (φ)

Figure 5 illustrates how ΔBWE changes with φ at different values of σ when $b = 4$, $r = 1.5$, $\theta_2 = 6$, and $c_{o2} = 20$. We consider $p_2^k \in \{21, 22, 23, 24, \cdots, 29\}$; therefore, according Eq. (31), we have $\varphi \in \{7, 8, \cdots, 15\}$. For this value, ΔBWE is negative, but it increases while keeping c_{o2} constant. Because as φ increases, the value of $c_{o2} - (c + \varphi)$ decrease.

Also, for $\varphi \in \{7, 8, \cdots, 15\}$, when the standard deviation of the price rises from 0.5 to 1.5, ΔBWE continues to be negative and its value rises. This means that the standard deviation of product price is lower, the hedging strategy outperforms the optimal order strategy in attenuating the bullwhip effect.

7.2.3. Consumer sensitivity to price increases (r)

Figure 6 indicates how ΔBWE changes with r at different values of σ when $b = 4$, $\theta_2 = 6$, and $c_{o2} = 15$. We consider $r \in \{1, 1.1, 1.2, \cdots, 2\}$. By increasing the

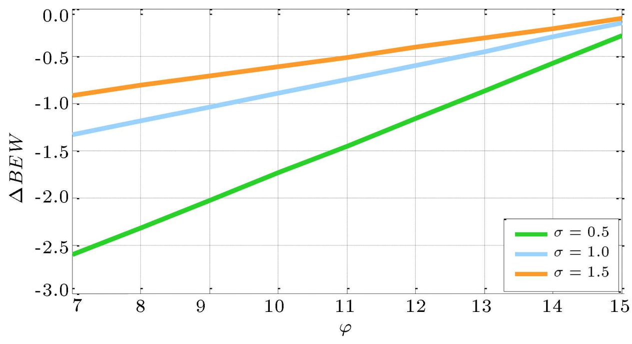

Figure 5. The influence of φ on ΔBWE when $b = 4$, $r = 1.5$, $\theta_2 = 6$, $c_{o2} = 20$.

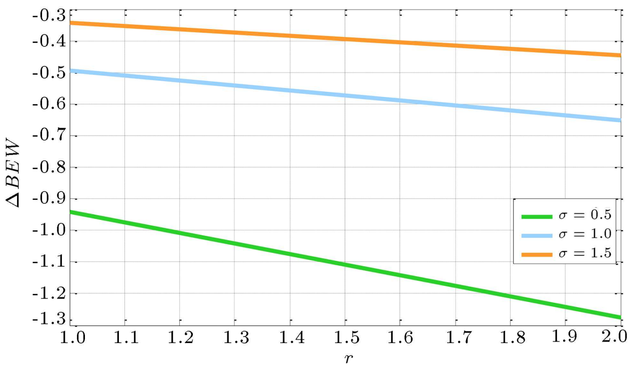

Figure 6. The influence of r on ΔBWE when $b = 4$, $\theta_2 = 6$, $c_{o2} = 15$.

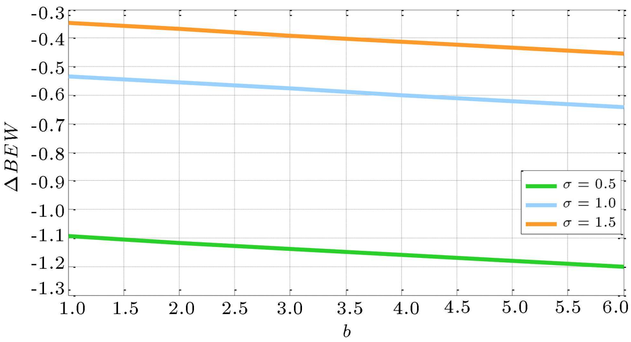

Figure 7. The influence of b on ΔBWE when $c_{o2} = 15$, $\theta_2 = 6$, $r = 1.5$.

value of r, the value of ΔBWE decreases. This means that when consumer sensitivity to price increases, the hedging strategy outperforms optimal order quantity strategy in attenuating the bullwhip effect. By increasing standard deviation of product price, σ, and consumer sensitivity to price, r, ΔBWE continues to be negative, but its value rises. By increasing the product price fluctuation and the rush the end customers feel, the hedging strategy outperforms the optimal order quantity strategy, but it will be less effective.

7.2.4. The demand curve slope (b)

Figure 7 illustrates how ΔBWE changes with b at different values of σ when $r = 1.5$, $\theta_2 = 6$, and $c_{o2} = 15$. We consider $b \in \{1, 2, 3, \cdots, 6\}$. Because from Eqs. (12), (21), and (30), we know that the end customer demand is correlated with the slope of demand curve, b, negatively. This means that as b increases, the volume of end customer demand decreases. By increasing the value of b, the value of ΔBWE decreases. Also, by raising the standard deviation, σ, and the slope of demand curve, b, the value of ΔBWE becomes negative, but its value increases. Therefore, it can be stated that the hedging strategy is better than optimal order strategy in attenuating the bullwhip effect.

7.2.5. Significant increase in the wholesale price (θ_2)

Figure 8 illustrates how ΔBWE changes with θ_2 at different values of σ when $b = 4$, $r = 1.5$, and $c_{o2} = 15$. We consider $\theta_2 \in \{1, 2, 3, \cdots, 10\}$. w_2 varies directly similar to θ_2 according to Eq. (4). When θ_2 increases, w_2 increases and φ rises. While keeping c_{o2} constant and based on Theorem 3, the value of $c_{o2} - (\theta_2 + \varphi)$ decreases and the difference between $BWE_{q_1^{11}, q_2^{21}}$ and $BWE_{q_1^{11}, q_2^{22}}$ is reduced. As a result, the value of ΔBWE rises. At $\theta_2 = 10$, we have $c_{o2} = (\theta_2 + \varphi)$, then $\Delta BWE = 0$, as shown in Figure 8. Also, upon increasing the standard deviation, σ, the value of ΔBWE is negative, but its values increase.

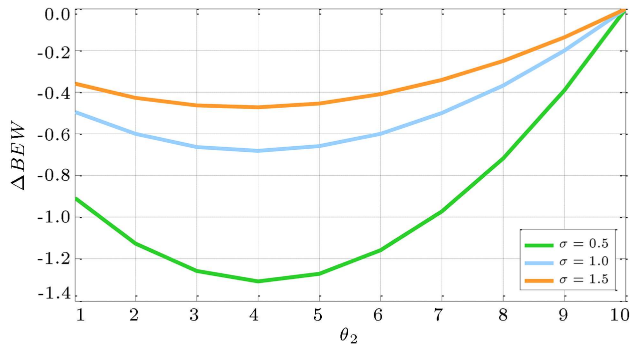

Figure 8. The influence of θ_2 on ΔBWE when $b = 4$, $r = 1.5$, $c_{o2} = 15$.

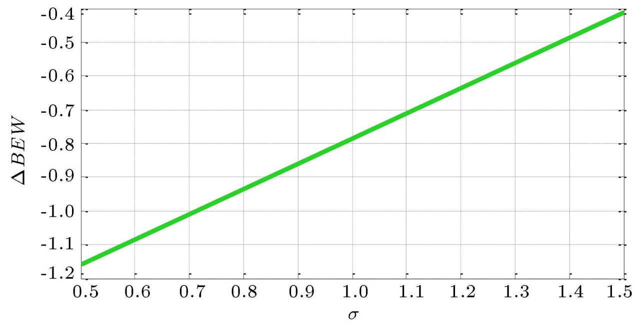

Figure 9. The influence of σ on ΔBWE when $b = 4$, $r = 1.5$, $\theta_2 = 6$, $c_{o2} = 15$.

7.2.6. The standard deviation of the product price (σ)

Figure 9 illustrates how ΔBWE changes with σ when $b = 4$, $r = 1.5$, $\theta_2 = 6$, and $c_{o2} = 15$. We consider $\sigma \in \{0.5, 0.6, \cdots, 1.5\}$. By increasing σ, the value of ΔBWE increases. This means that by raising the product price standard deviation, the hedging strategy outperforms optimal order quantity strategy in reducing bullwhip effect, but it will be less effective.

8. Conclusions

This paper introduced the hedging strategy for controlling bullwhip effect and compared it to optimal order quantity strategy. Analytical expressions were derived for the bullwhip effect ratio using two strategies, the hedging strategy and the optimal order quantity strategy. In the following section, the results are given. These results provide some useful managerial insights on the implementation of these strategies:

1. When the product price fluctuations cause panic buying and they make the bullwhip effect, the hedging strategy can help to control it. The retailer may use a long hedge to fix the good price and manage the bullwhip effect;
2. If the option price (c_o) is greater than the sum of the order costs and the difference between the wholesale price and the exercise price ($c + \varphi$), the hedging strategy is better than the optimal order quantity strategy in controlling the bullwhip effect. Thus, in this case, purchasing through a hedging strategy will be more expensive than buying through an optimal order quantity strategy. It prevents the retailer from buying too much. If the retailer buying spree for a cheap item exceeds a certain threshold and the intensity of customer demand decreases, the unsold products are kept to be dealt with over the next periods and this, in turn, will increase the bullwhip effect;
3. The retailer's ordering behavior is important when the product price is subject to volatility. In this circumstance, the retailers buy products via call option contract cheaper than other methods and they should be careful about the order quantities. The lower exercise price in hedging strategy compared with the wholesale price in the optimal order quantity strategy must not contribute to excessive product purchasing. A large number of products may protect the retailers against high fluctuations in the demand of end customers, but increases the bullwhip effect ratio. Therefore, it is important to determine the ordering strategy and the order quantities when the product price undergoes fluctuations, or we expect it to be;
4. The product price is one of the important factors that the end customers pay attention to. Also, the retailers consider the price as the criterion for the sales strategy. Accurate price forecasting and predicting end customer behavior can help retailers to choose the right ordering strategy. If the retailer correctly forecasts price increasing, the use of hedging strategy could help to control the bullwhip effect considerably;
5. The price standard deviation is a statistical expression that indicates price fluctuations in the market. High price fluctuations can lead to unstable markets and emotional decisions on the part of the end customers, driven mostly by fear and greed. The high price standard deviation means high price volatility. In this situation, while ($c_{o2} > c + \varphi$) is established, for controlling the bullwhip effect, the hedging strategy is better than the optimal order quantity strategy, but its effectiveness is low;
6. The bullwhip effect is not completely eliminated by hedging strategy. A summary of our findings indicates that the reduction of bullwhip effect is important when there are price fluctuations in markets and companies can use hedging strategy to attenuate the bullwhip effect.

This paper recommends several future directions to add to our understanding of the influence of hedging strategy on the bullwhip effect. First, our model considers only linear demand function; the other demand functions require further study. Second, this paper assesses only the optimal order quantity strategy compared to the hedging strategy while other ordering strategies can be considered, as well. Finally, extending the two-period supply chain to multi-period chains would be another contribution for the future studies.

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Appendix A: Proof of Proposition 1

We substitute Eq. (12) for p_1 in Eq. (10). Thus, we have:

$$\pi\left(q_1^{11}\right)=\left[\frac{1}{b}(a-d_1)\right]S\left(q_1^{11}\right)-w_1q_1^{11}-h_1q_1^{11}-c_1q_1^{11}. \tag{A.1}$$

The expected sales in the first period will be in Eq. (A.2):

$$S\left(q_1^{11}\right)=\min\left(d_1,q_1^{11}\right)=q_1^{11}-\int_0^{q_1^{11}}F(x)dx. \tag{A.2}$$

The expected leftover will be given in Eq. (A.3):

$$I\left(q_1^{11}\right)=E\left(q_1^{11}-d_1\right)^+=q_1^{11}-S\left(q_1^{11}\right). \tag{A.3}$$

Regarding Eq. (A.1) and taking the first derivative with respect to q_1^{11}, we obtain Eq. (A.4):

$$\frac{\partial\pi\left(q_1^{11}\right)}{\partial q_1^{11}}=\left[\frac{1}{b}(a-d_1)\right]\frac{\partial S\left(q_1^{11}\right)}{\partial q_1^{11}}-w_1-h_1-c_1. \tag{A.4}$$

Result of differentiating $S(q_1^{11})$ is as follows:

$$\frac{\partial S\left(q_1^{11}\right)}{\partial q_1^{11}}=1-F\left(q_1^{11}\right). \tag{A.5}$$

Regarding Eqs. (A.2) and (A.5) and taking the second derivative with respect to q_1^{11}, we obtain Eq. (A.6):

$$\frac{\partial^2 S\left(q_1^{11}\right)}{\partial\left(q_1^{11}\right)^2}=-f\left(q_1^{11}\right). \tag{A.6}$$

Regarding Eq. (A.1) and taking the second derivative with respect to q_1^{11}, we obtain Eq. (A.7):

$$\frac{\partial^2\pi\left(q_1^{11}\right)}{\partial\left(q_1^{11}\right)^2}=-\left[\frac{1}{b}(a-d_1)\right]f\left(q_1^{11}\right)<0. \tag{A.7}$$

To solve Eq. (A.1), we consider Eq. (A.4):

$$\frac{\partial\pi\left(q_1^{11}\right)}{\partial q_1^{11}}=0. \tag{A.8}$$

The retailer's optimal order quantity is given by:

$$\left(q_1^{11}\right)^*=F^{-1}\left(1-\frac{w_1+h_1+c_1}{\frac{1}{b}(a-d_1)}\right). \tag{A.9}$$

We consider $x\sim N(\mu,\sigma^2)$ and, in turn, will have Eqs. (A.10)–(A.13):

$$F^{-1}(x)=\mu+\left(\sqrt{2}\sigma\right)erf^{-1}(2x-1), \tag{A.10}$$

$$erf^{-1}(x)=\sqrt{\pi}\left(\frac{1}{2}x+\frac{1}{24}\pi x^3+\frac{7}{960}\pi^2x^5+\cdots\right), \tag{A.11}$$

$$erf^{-1}(x)\sim\frac{\sqrt{\pi}}{2}x, \tag{A.12}$$

$$erf^{-1}(2x-1)=\frac{\sqrt{\pi}}{2}(2x-1). \tag{A.13}$$

By substituting Eq. (A.13) into Eq. (A.10), we will have Eq. (A.14):

$$F^{-1}(x)=\mu+\frac{\sqrt{2\pi}\sigma}{2}(2x-1). \tag{A.14}$$

By considering Eqs. (A.9) and (A.14), we will have Eq. (A.15):

$$\left(q_1^{11}\right)^*=\mu_{d_1}+\sqrt{2\pi}\sigma_{d_1}\left[\frac{1}{2}-\frac{w_1+h_1+c_1}{\frac{1}{b}(a-d_1)}\right]. \tag{A.15}$$

This completes the proof. □

Appendix B: Proof of Proposition 2

We substitute Eq. (16) for p_2 in Eq. (14). Thus, we have:

$$\pi\left(q_2^{11}\right)=\left[\frac{1}{b}(a-d_2^1)\right]S\left(q_2^{11}\right)-w_2q_2^{11}-c_2q_2^{11}-h_2\left(q_2^{11}+I\left(q_1^{11}\right)\right)-n_2H\left(q_2^{11}\right). \tag{B.1}$$

The expected sales will be given in Eq. (B.2):

$$S\left(q_2^{11}\right)=\min\left(d_2^1,q_2^{11}\right)=q_2^{11}-\int_0^{q_2^{11}}F(x)dx. \tag{B.2}$$

The expected order quantity to the emergency source will be given in Eq. (B.3):

$$H\left(q_2^{11}\right)=H\left(d_2^1,q_2^{11}+I\left(q_1^{11}\right)\right)=E\left[d_2^1-q_2^{11}-I\left(q_1^{11}\right)\right]^+$$
$$=\mu_{d_2^1}-S\left(q_2^{11}\right)-I\left(q_1^{11}\right). \tag{B.3}$$

Regarding Eq. (B.1) and taking the first derivative with respect to q_1^{11}, we obtain Eq. (B.4):

$$\frac{\partial\pi\left(q_2^{11}\right)}{\partial q_2^{11}}=\left[\frac{1}{b}(a-d_2^1)\right]\frac{\partial S\left(q_2^{11}\right)}{\partial q_2^{11}}-w_2-c_2$$
$$-h_2-n_2\frac{\partial H\left(q_2^{11}\right)}{\partial q_2^{11}}. \tag{B.4}$$

Result of differentiating $S(q_2^{11})$ is as follows:

$$\frac{\partial S\left(q_2^{11}\right)}{\partial q_2^{11}}=1-F\left(q_2^{11}\right). \tag{B.5}$$

Result of differentiating $H(q_2^{11})$ is as follows:

$$\frac{\partial H\left(q_2^{11}\right)}{\partial q_2^{11}}=-\frac{\partial S\left(q_2^{11}\right)}{\partial q_2^{11}}=F\left(q_2^{11}\right)-1. \tag{B.6}$$

Regarding Eqs. (B.5) and (B.6) and taking the second derivative with respect to q_2^{11}, we obtain Eqs. (B.7) and (B.8):

$$\frac{\partial^2 S\left(q_2^{11}\right)}{\partial {q_2^{11}}^2}=-f\left(q_2^{11}\right), \tag{B.7}$$

$$\frac{\partial^2 H\left(q_2^{11}\right)}{\partial {q_2^{11}}^2}=f\left(q_2^{11}\right). \tag{B.8}$$

Regarding Eq. (B.1) and taking the second derivative with respect to q_2^{11}, we obtain Eq. (B.9):

$$\frac{\partial^2\pi\left(q_2^{11}\right)}{\partial {q_2^{11}}^2}=-\left[\frac{1}{b}(a-d_2^1)+n_2\right]f\left(q_2^{11}\right)<0. \tag{B.9}$$

Therefore, it is clear that $\pi(q_2^{11})$ is concave. To solve Eq. (B.1), we consider Eq. (B.4):

$$\frac{\partial\pi\left(q_2^{11}\right)}{\partial q_2^{11}}=0. \tag{B.10}$$

The retailer's optimal order quantity is given by:

$$\left(q_2^{11}\right)^*=F^{-1}\left(1-\frac{w_2+c_2+h_2}{\frac{1}{b}(a-d_2^1)+n_2}\right). \tag{B.11}$$

By considering Eqs. (A.14) and (B.11), we will reach Eq. (B.12):

$$\left(q_2^{11}\right)^*=\mu_{d_2^1}+\sqrt{2\pi}\sigma_{d_2^1}\left[\frac{1}{2}-\frac{w_2+c_2+h_2}{\frac{1}{b}(a-d_2^1)+n_2}\right]. \tag{B.12}$$

Also, according to Eq. (16) and ($n_2<p_2$) mentioned in Subsection 4.1, the denominator of the fraction of Eq. (B.12) is not zero. Therefore, the boundary conditions are established.

This completes the proof. □

Appendix C: Proof of Proposition 3

We substitute Eq. (21) for p_2 in Eq. (18). Thus, we have:

$$\pi\left(q_2^{21}\right)=\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]$$
$$S\left(q_2^{21}\right)-w_2q_2^{21}-c_2q_2^{21}-h_2\left(q_2^{21}+I\left(q_1^{11}\right)\right)$$
$$-n_2H\left(q_2^{21}\right). \tag{C.1}$$

The expected sales will be given in Eq. (C.2):

$$S\left(q_2^{21}\right)=\min\left(d_2^2,q_2^{21}\right)=q_2^{21}-\int_0^{q_2^{21}}F(x)dx. \tag{C.2}$$

The expected order quantity to the emergency source will be given in Eq. (C.3):

$$H\left(q_2^{21}\right)=H\left(d_2^2,q_2^{21}+I\left(q_1^{11}\right)\right)=E\left[d_2^1-q_2^{21}-I\left(q_1^{11}\right)\right]^+$$
$$=\mu_{d_2^2}-S\left(q_2^{21}\right)-I\left(q_1^{11}\right). \tag{C.3}$$

Regarding Eq. (C.1) and taking the first derivative with respect to q_2^{21}, we obtain Eq. (C.4):

$$\frac{\partial\pi\left(q_2^{21}\right)}{\partial q_2^{21}}=\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]$$
$$\frac{\partial S\left(q_2^{21}\right)}{\partial q_2^{21}}-w_2-c_2-h_2-n_2\frac{\partial H\left(q_2^{21}\right)}{\partial q_2^{21}}. \tag{C.4}$$

Result of differentiating $S(q_2^{21})$ is as follows:

$$\frac{\partial S\left(q_2^{21}\right)}{\partial q_2^{21}}=1-F\left(q_2^{21}\right). \tag{C.5}$$

Result of differentiating $H(q_2^{21})$ is as follows:

$$\frac{\partial H\left(q_2^{21}\right)}{\partial q_2^{21}}=-\frac{\partial S\left(q_2^{21}\right)}{\partial q_2^{21}}=F\left(q_2^{21}\right)-1. \tag{C.6}$$

Regarding Eqs. (C.5) and (C.6) and taking the second derivative with respect to q_2^{21}, we obtain Eqs. (C.7) and (C.8):

$$\frac{\partial^2 S\left(q_2^{21}\right)}{\partial {q_2^{21}}^2}=-f\left(q_2^{21}\right), \tag{C.7}$$

$$\frac{\partial^2 H\left(q_2^{21}\right)}{\partial q_2^{21^2}} = f\left(q_2^{21}\right). \tag{C.8}$$

Regarding Eq. (C.1) and taking the second derivative with respect to q_2^{21}, we obtain Eq. (C.9):

$$\frac{\partial^2 \pi\left(q_2^{21}\right)}{\partial q_2^{21^2}} = -\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb - d_2^2) + n_2\right] f\left(q_2^{21}\right) < 0. \tag{C.9}$$

Therefore, it is clear that $\pi(q_2^{21})$ is concave.

To solve Eq. (C.1), we consider Eq. (C.4):

$$\frac{\partial \pi\left(q_2^{21}\right)}{\partial q_2^{21}} = 0. \tag{C.10}$$

The retailer's optimal order quantity is given by:

$$\left(q_2^{21}\right)^* = F^{-1}\left(1 - \frac{w_2+c_2+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right). \tag{C.11}$$

By considering Eqs. (A.14) and (C.11), we have Eq. (C.12):

$$\left(q_2^{21}\right)^* = \mu_{d_2^2} + \sqrt{2\pi}\sigma_{d_2^2}\left[\frac{1}{2} - \frac{w_2+c_2+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]. \tag{C.12}$$

This completes the proof. □

Appendix D: Proof of Proposition 4

We substitute Eq. (25) for p_2 in Eq. (23). Thus, we have:

$$\pi\left(q_2^{12}\right) = \left[\frac{1}{b}(a-d_2^1)\right] S\left(q_2^{12}\right) - w_2 q_2^{12} - c_2 q_2^{12} - c_{o2} q_2^{12} - h_2\left(q_2^{12} + I\left(q_1^{11}\right)\right) - n_2 H\left(q_2^{12}\right). \tag{D.1}$$

The expected sales will be given in Eq. (D.2):

$$S\left(q_2^{12}\right) = \min\left(d_2^1, q_2^{12}\right) = q_2^{12} - \int_0^{q_2^{12}} F(x)dx. \tag{D.2}$$

The expected order quantity to the emergency source will be given in Eq. (D.3):

$$H\left(q_2^{12}\right) = H\left(d_2^1, q_2^{12} + I\left(q_1^{11}\right)\right) = E\left[d_2^1 - q_2^{12} - I\left(q_1^{11}\right)\right]^+ = \mu_{d_2^1} - S\left(q_2^{12}\right) - I\left(q_1^{11}\right). \tag{D.3}$$

Regarding Eq. (D.1) and taking the first derivative with respect to q_2^{12}, we obtain Eq. (D.4):

$$\frac{\partial \pi\left(q_2^{12}\right)}{\partial q_2^{12}} = \left[\frac{1}{b}(a-d_2^1)\right]\frac{\partial S\left(q_2^{12}\right)}{\partial q_2^{12}} - w_2 - c_2 - c_{o2} - h_2 - n_2\frac{\partial H\left(q_2^{12}\right)}{\partial q_2^{12}}. \tag{D.4}$$

Result of differentiating $S(q_2^{12})$ is as follows:

$$\frac{\partial S\left(q_2^{12}\right)}{\partial q_2^{12}} = 1 - F\left(q_2^{12}\right). \tag{D.5}$$

Result of differentiating $H(q_2^{12})$ is as follows:

$$\frac{\partial H\left(q_2^{12}\right)}{\partial q_2^{12}} = -\frac{\partial S\left(q_2^{12}\right)}{\partial q_2^{12}} = F\left(q_2^{12}\right) - 1. \tag{D.6}$$

Regarding Eqs. (D.5) and (D.6) and taking the second derivative with respect to q_2^{12}, we obtain Eqs. (D.7) and (D.8):

$$\frac{\partial^2 S\left(q_2^{12}\right)}{\partial q_2^{12^2}} = -f\left(q_2^{12}\right), \tag{D.7}$$

$$\frac{\partial^2 H\left(q_2^{12}\right)}{\partial q_2^{12^2}} = f\left(q_2^{12}\right). \tag{D.8}$$

Regarding Eq. (D.1) and taking the second derivative with respect to q_2^{12}, we obtain Eq. (D.9):

$$\frac{\partial^2 \pi\left(q_2^{12}\right)}{\partial q_2^{12^2}} = -\left[\frac{1}{b}(a-d_2^1) + n_2\right] f\left(q_2^{12}\right) < 0. \tag{D.9}$$

Therefore, it is clear that $\pi(q_2^{12})$ is concave.

To solve Eq. (D.1), we consider Eq. (D.4):

$$\frac{\partial \pi\left(q_2^{12}\right)}{\partial q_2^{12}} = 0. \tag{D.10}$$

The retailer's optimal order quantity is given by:

$$\left(q_2^{12}\right)^* = F^{-1}\left(1 - \frac{w_2+c_2+c_{o2}+h_2}{\frac{1}{b}(a-d_2^1)+n_2}\right). \tag{D.11}$$

By considering Eqs. (A.14) and (D.11), we have Eq. (D.12):

$$\left(q_2^{12}\right)^* = \mu_{d_2^1} + \sqrt{2\pi}\sigma_{d_2^1}\left[\frac{1}{2} - \frac{w_2+c_2+c_{o2}+h_2}{\frac{1}{b}(a-d_2^1)+n_2}\right]. \tag{D.12}$$

Also, according to Eq. (25) and ($n_2 < p_2$) mentioned in Subsection 4.1, the denominator of the fraction of Eq. (D.12) is not zero. Therefore, the boundary conditions are established.

This completes the proof. □

Appendix E: Proof of Proposition 5

We substitute Eq. (30) for p_2 in Eq. (27). Thus, we have:

$$\pi\left(q_2^{22}\right)=\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right] S\left(q_2^{22}\right)-p_2^k q_2^{22}-c_{o2}q_2^{22}-h_2\left(q_2^{22}+I\left(q_1^{11}\right)\right) - n_2 H\left(q_2^{22}\right). \tag{E.1}$$

The expected sales will be given in Eq. (E.2):

$$S(q_2^{22})=\min\left(d_2^2,q_2^{22}\right)=q_2^{22}-\int_0^{q_2^{22}} F(x)dx. \tag{E.2}$$

The expected order quantity to the emergency source will be given in Eq. (E.3):

$$H\left(q_2^{22}\right)=H\left(d_2^2,q_2^{22}+I\left(q_1^{11}\right)\right)=E\left[d_2^2-q_2^{22}-I\left(q_1^{11}\right)\right]^+ =\mu_{d_2^2}-S\left(q_2^{22}\right)-I\left(q_1^{11}\right). \tag{E.3}$$

Regarding Eq. (E.1) and taking the first derivative with respect to q_1^{11}, we obtain Eq. (E.4):

$$\frac{\partial\pi\left(q_2^{22}\right)}{\partial q_2^{22}}=\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb - d_2^2)\right]\frac{\partial S(q_2^{22})}{\partial q_2^{22}}-p_2^k-c_{o2}-h_2-n_2\frac{\partial H(q_2^{22})}{\partial q_2^{22}}. \tag{E.4}$$

Result of differentiating $S(q_2^{22})$ is as follows:

$$\frac{\partial S\left(q_2^{22}\right)}{\partial q_2^{22}}=1-F\left(q_2^{22}\right). \tag{E.5}$$

Result of differentiating $H(q_2^{22})$ is as follows:

$$\frac{\partial H\left(q_2^{22}\right)}{\partial q_2^{22}}=-\frac{\partial S\left(q_2^{22}\right)}{q_2^{22}}=F\left(q_2^{22}\right)-1. \tag{E.6}$$

Regarding Eqs. (E.5) and (E.6) and taking the second derivative with respect to q_2^{22}, we obtain Eqs. (E.7) and (E.8):

$$\frac{\partial^2 S\left(q_2^{22}\right)}{\partial {q_2^{22}}^2}=-f\left(q_2^{22}\right), \tag{E.7}$$

$$\frac{\partial^2 H\left(q_2^{22}\right)}{\partial {q_2^{22}}^2}=f\left(q_2^{22}\right). \tag{E.8}$$

Regarding Eq. (E.1) and taking the second derivative with respect to q_2^{22}, we obtain Eq. (E.9):

$$\frac{\partial^2\pi\left(q_2^{22}\right)}{\partial {q_2^{22}}^2}=-\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb - d_2^2)+n_2\right]f(q_2^{22})<0. \tag{E.9}$$

Therefore, it is clear that $\pi(q_2^{22})$ is concave.

To solve Eq. (E.1), we consider Eq. (E.4):

$$\frac{\partial\pi\left(q_2^{22}\right)}{\partial q_2^{22}}=0. \tag{E.10}$$

The retailer's optimal order quantity is given by:

$$\left(q_2^{22}\right)^*=F^{-1}\left(1 - \frac{p_2^k+c_{o2}+h_2}{\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)+n_2}\right). \tag{E.11}$$

By considering Eqs. (A.14) and (E.11), we have Eq. (E.12):

$$\left(q_2^{22}\right)^*=\mu_{d_2^2}+\sqrt{2\pi}\sigma_{d_2^2}\left[\frac{1}{2} - \frac{p_2^k+c_{o2}+h_2}{\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)+n_2}\right]. \tag{E.12}$$

This completes the proof. □

Appendix F. Proof of Theorem 1

$\bar{q}_{q_1^{11},q_2^{21}}$ is calculated as follows:

$$\bar{q}_{q_1^{11},q_2^{21}}=\frac{1}{2}\left(q_1^{11}+q_2^{21}\right)=\frac{1}{2}\left[\left(\mu_{d_1}+\sqrt{2\pi}\sigma_{d_1}\left[\frac{1}{2} - \frac{w_1+h_1+c_1}{\frac{1}{b}(a-d_1)}\right]\right)+\mu_{d_2^2}+\sqrt{2\pi}\sigma_{d_2^2}\left[\frac{1}{2} - \frac{w_2+c_2+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]\right]. \tag{F.1}$$

With substituting Eqs. (42) and (F.1) in Eq. (45), we obtain Eq. (F.2) is shown in Box F.I. By substituting Eqs. (39), (41), (43), and (44) in Eq. (F.2), we reach Eq. (F.3) as shown in Box F.II.

This completes the proof. □

Appendix G. Proof of Proposition 6

Bullwhip effect exists if Inequality (G.1) holds:

$$BWE_{q_1^{11},q_2^{21}}>1. \tag{G.1}$$

By substituting Eq. (46) in Inequality (G.1), we have

$$BWE_{q_1^{11},q_2^{21}}=\frac{\frac{1}{2}\left[\left(\mu_{d_2^2}+\sqrt{2\pi}\sigma_{d_2^2}\left[\frac{1}{2}-\frac{w_2+c_2+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]\right)-\left(\mu_{d_1}+\sqrt{2\pi}\sigma_{d_1}\left[\frac{1}{2}-\frac{w_1+h_1+c_1}{\frac{1}{b}(a-d_1)}\right]\right)\right]^2}{(2b^2+2r^2b^2)\sigma^2}. \tag{F.2}$$

Box F.I

$$BWE_{q_1^{11},q_2^{21}}=\frac{\frac{1}{2}\left[\begin{matrix}[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\left[\frac{1}{2}-\frac{w_2+c_2+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]\\ -\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\end{matrix}\right]^2}{(2b^2+2r^2b^2)\sigma^2}. \tag{F.3}$$

Box F.II

Inequalities (G.2)–(G.5) are shown in Box G.I. We know if $x^2>y^2$, then $x>y$ and $-x<-y$. Also, We consider $\frac{\sqrt{2\pi}}{2}\simeq 1.25$, $\sqrt{2\pi}\simeq 2.5$. Therefore, we have Inequalities (G.6) and (G.7):

$$\begin{aligned}&[(m_2-m_1)w_1+(1+m_2)\theta_2]rb\\&\quad+1.25\left[\sqrt{b^2+2r^2b^2}\sigma\right]-2.5\left[\sqrt{b^2+2r^2b^2}\sigma\right]\\&\quad\left[\frac{w_2+c_2+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]\\&\quad-1.25[b\sigma]+2.5[b\sigma]\left[\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\\&\quad>\sqrt{2(2b^2+2r^2b^2)}\sigma,\end{aligned} \tag{G.6}$$

$$\begin{aligned}&-\Bigg[[(m_2-m_1)w_1+(1+m_2)\theta_2]rb\\&\quad+1.25\left[\sqrt{b^2+2r^2b^2}\sigma\right]-2.5\left[\sqrt{b^2+2r^2b^2}\sigma\right]\\&\quad\left[\frac{w_2+c_2+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]\\&\quad-1.25[b\sigma]+2.5[b\sigma]\left[\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\Bigg]\\&\quad<-\left[\sqrt{2(2b^2+2r^2b^2)}\sigma\right].\end{aligned} \tag{G.7}$$

After simplification, we have Inequality (G.8):

$$\begin{aligned}&[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+1.25\left[\sqrt{b^2+2r^2b^2}\sigma\right]\\&\quad+2.5[b\sigma]\left[\frac{w_1+c_1+h_1}{\left[\frac{1}{b}(a-d_1)\right]}\right]>2.5\left[\sqrt{b^2+2r^2b^2}\sigma\right]\\&\quad\left[\frac{w_2+c_2+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]\\&\quad+1.25[b\sigma]+\sqrt{2(2b^2+2r^2b^2)}\sigma.\end{aligned} \tag{G.8}$$

Bullwhip effect exists (i.e., $BWE_{q_1^{11},q_2^{21}}>1$) if Inequality (G.8) holds.

This completes the proof. □

Appendix H. Proof of Theorem 2

$\bar{q}_{q_1^{11},q_2^{22}}$ is calculated as follows:

$$\begin{aligned}\bar{q}_{q_1^{11},q_2^{22}}&=\frac{1}{2}\left(q_1^{11}+q_2^{22}\right)\\&=\frac{1}{2}\Bigg[\left(\mu_{d_1}+\sqrt{2\pi}\sigma_{d_1}\left[\frac{1}{2}-\frac{w_1+h_1+c_1}{\frac{1}{b}(a-d_1)}\right]\right)\\&\quad+\mu_{d_2^2}+\sqrt{2\pi}\sigma_{d_2^2}\Bigg[\frac{1}{2}\\&\quad-\frac{p_2^k+c_{o2}+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\Bigg]\Bigg].\end{aligned} \tag{H.1}$$

With substituting Eq. (42) and (H.1) in Eq. (48), we obtain Eq. (H.2) is shown in Box H.I. Upon

$$\frac{\frac{1}{2}\left[\begin{array}{l}[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\left[\frac{1}{2}-\frac{w_2+c_2+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]}\right]\\ \quad -\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\end{array}\right]^2}{(2b^2+2r^2b^2)\sigma^2}>1, \quad \text{(G.2)}$$

$$\frac{1}{2(2b^2+2r^2b^2)\sigma^2}\Bigg[[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\Bigg[\frac{1}{2}$$

$$-\frac{w_2+c_2+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\Bigg]-\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\Bigg]^2>1, \quad \text{(G.3)}$$

$$\Bigg[[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\left[\frac{1}{2}-\frac{w_2+c_2+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]$$

$$-\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\Bigg]^2>2(2b^2+2r^2b^2)\sigma^2, \quad \text{(G.4)}$$

$$\Bigg[[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\left[\frac{1}{2}-\frac{w_2+c_2+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]$$

$$-\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\Bigg]^2-2(2b^2+2r^2b^2)\sigma^2>0. \quad \text{(G.5)}$$

Box G.I

$$BWE_{q_1^{11},q_2^{22}}=\frac{\frac{1}{2}\left[\left(\mu_{d_2^2}+\sqrt{2\pi}\sigma_{d_2^2}\left[\frac{1}{2}-\frac{p_2^k+c_{o2}+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]\right)-\left(\mu_{d_1}+\sqrt{2\pi}\sigma_{d_1}\left[\frac{1}{2}-\frac{w_1+h_1+c_1}{\frac{1}{b}(a-d_1)}\right]\right)\right]^2}{(2b^2+2r^2b^2)\sigma^2}. \quad \text{(H.2)}$$

Box H.I

substituting Eqs. (39), (41), (43), and (44) in Eq. (H.2), we reach Eq. (H.3) as shown in Box H.II.

This completes the proof. □

Appendix I. Proof of Proposition 7

Bullwhip effect exists if Inequality (I.1) holds:

$$BWE_{q_1^{11},q_2^{22}}>1. \quad \text{(I.1)}$$

Upon substituting Eq. (49) in Eq. (I.1), we have Inequalities (I.2)–(I.5) are shown in Box I.I. We know that if $x^2>y^2$, then $x>y$ and $-x<-y$. Also, we consider $\frac{\sqrt{2\pi}}{2}\simeq 1.25$, $\sqrt{2\pi}\simeq 2.5$.

Therefore, we have Inequalities (I.6) and (I.7):

$$[(m_2-m_1)w_1+(1+m_2)\theta_2]rb$$

$$+1.25\left[\sqrt{b^2+2r^2b^2}\sigma\right]-2.5\left[\sqrt{b^2+2r^2b^2}\sigma\right]$$

$$\left[\frac{p_2^k+c_{o2}+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]$$

$$-1.25[b\sigma]+2.5[b\sigma]\left[\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]$$

$$>\sqrt{2(2b^2+2r^2b^2)}\sigma, \quad \text{(I.6)}$$

$$BWE_{q_1^{11},q_2^{22}}=\frac{\frac{1}{2}\left[\begin{array}{l}[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\left[\frac{1}{2}-\frac{p_2^k+c_{o2}+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]\\ \quad -\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\end{array}\right]^2}{(2b^2+2r^2b^2)\sigma^2}. \tag{H.3}$$

Box H.II

$$\frac{\frac{1}{2}\left[\begin{array}{l}[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\left[\frac{1}{2}-\frac{p_2^k+c_{o2}+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]\\ \quad -\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\end{array}\right]^2}{(2b^2+2r^2b^2)\sigma^2}>1, \tag{I.2}$$

$$\frac{1}{2(2b^2+2r^2b^2)\sigma^2}\left[[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\left[\frac{1}{2}\right.\right.$$

$$\left.\left.-\frac{p_2^k+c_{o2}+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]-\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\right]^2>1, \tag{I.3}$$

$$\left[[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\left[\frac{1}{2}-\frac{p_2^k+c_{o2}+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]\right.$$

$$\left.-\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\right]^2>2(2b^2+2r^2b^2)\sigma^2, \tag{I.4}$$

$$\left[[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\left[\frac{1}{2}-\frac{p_2^k+c_{o2}+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]\right.$$

$$\left.-\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\right]^2-2(2b^2+2r^2b^2)\sigma^2>0. \tag{I.5}$$

Box I.I

$$-\left[[(m_2-m_1)w_1+(1+m_2)\theta_2]rb\right.$$

$$+1.25\left[\sqrt{b^2+2r^2b^2}\sigma\right]-2.5\left[\sqrt{b^2+2r^2b^2}\sigma\right]$$

$$\left[\frac{p_2^k+c_{o2}+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]$$

$$\left.-1.25[b\sigma]+2.5[b\sigma]\left[\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\right]$$

$$<-\left[\sqrt{2(2b^2+2r^2b^2)}\sigma\right]. \tag{I.7}$$

After simplification, we have Inequality (I.8):

$$[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+1.25\left[\sqrt{b^2+2r^2b^2}\sigma\right]$$

$$+2.5[b\sigma]\left[\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]>2.5\left[\sqrt{b^2+2r^2b^2}\sigma\right]$$

$$\left[\frac{p_2^k+c_{o2}+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]$$

$$+1.25[b\sigma]+\sqrt{2(2b^2+2r^2b^2)}\sigma. \qquad \text{(I.8)}$$

Bullwhip effect exists (i.e., $BWE_{q_1^{11},q_2^{22}}>1$) if Inequality (I.8) holds.

This completes the proof. □

Appendix J. Proof of Theorem 3

From Eqs. (46) and (49), we obtain Eq. (J.1) is shown in Box J.I. To prove Theorem 3, we need to indicate that $\Delta BWE<0$.

According to Eqs. (13), (22), (32), (39), and (41), we reach Inequalities (J.2) and (J.3):

$$\frac{1}{\sqrt{2(2b^2+2r^2b^2)\sigma}}\Bigg[[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\Bigg[\frac{1}{2}-\frac{p_2^k+h_2+c_{o2}}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\Bigg]-\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+h_1+c_1}{\frac{1}{b}(a-d_1)}\right]\Bigg]>0, \qquad \text{(J.2)}$$

$$\frac{1}{\sqrt{2(2b^2+2r^2b^2)\sigma}}\Bigg[[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\Bigg[\frac{1}{2}-\frac{w_2+h_2+c_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\Bigg]-\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+h_1+c_1}{\frac{1}{b}(a-d_1)}\right]\Bigg]>0. \qquad \text{(J.3)}$$

We know that if $x^2-y^2<0$ and $x,y>0$, then $x<y$. Therefore, we have Inequality (J.4):

$$\frac{1}{\sqrt{2(2b^2+2r^2b^2)\sigma}}\Bigg[[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\Bigg[\frac{1}{2}-\frac{p_2^k+h_2+c_{o2}}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\Bigg]-\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+h_1+c_1}{\frac{1}{b}(a-d_1)}\right]\Bigg]$$
$$<\frac{1}{\sqrt{2(2b^2+2r^2b^2)\sigma}}\Bigg[[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\Bigg[\frac{1}{2}-\frac{w_2+h_2+c_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\Bigg]-\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+h_1+c_1}{\frac{1}{b}(a-d_1)}\right]\Bigg]. \qquad \text{(J.4)}$$

After simplification, Inequality (J.4) is reduced to Inequality (J.5) and, then, Inequality (J.6):

$$\Delta BWE=BWE_{q_1^{11},q_2^{22}}-BWE_{q_1^{11},q_2^{21}}$$
$$=\frac{\frac{1}{2}\left[[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\left[\frac{1}{2}-\frac{p_2^k+c_{o2}+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]-\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\right]^2}{(2b^2+2r^2b^2)\sigma^2}$$
$$-\frac{\frac{1}{2}\left[[(m_2-m_1)w_1+(1+m_2)\theta_2]rb+\sqrt{2\pi}\left[\sqrt{b^2+2r^2b^2}\sigma\right]\left[\frac{1}{2}-\frac{w_2+c_2+h_2}{\left[\frac{1}{b}(a+[(m_2-m_1)w_1+(1+m_2)\theta_2]rb-d_2^2)\right]+n_2}\right]-\sqrt{2\pi}[b\sigma]\left[\frac{1}{2}-\frac{w_1+c_1+h_1}{\frac{1}{b}(a-d_1)}\right]\right]^2}{(2b^2+2r^2b^2)\sigma^2}. \qquad \text{(J.1)}$$

Box J.I

$$\frac{w_2+c_2+h_2}{\left[\frac{1}{b}\left(a+\left[\left(m_2-m_1\right)w_1+(1+m_2)\theta_2\right]rb-d_2^2\right)\right]+n_2}$$

$$<\frac{p_2^k+c_{o2}+h_2}{\left[\frac{1}{b}\left(a+\left[\left(m_2-m_1\right)w_1+(1+m_2)\theta_2\right]rb-d_2^2\right)\right]+n_2}, \quad (J.5)$$

$$w_2+c_2+h_2<p_2^k+c_{o2}+h_2. \quad (J.6)$$

As a result, according to Eq. (31) and Inequality (J.6), we obtain Inequality (J.7):

$$c_{o2}>c_2+\varphi. \quad (J.7)$$

Upon considering Inequality (J.7), we can prove Theorem 3.

This completes the proof. □

Biographies

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