Scientia Iranica E (2023) 30(3), 1204–1223

Sharif University of Technology
Scientia Iranica
Transactions E: Industrial Engineering
http://scientiairanica.sharif.edu

A sustainable production-inventory model joint with preventive maintenance and multiple shipments for imperfect quality items

A. Fallahi[a], M. Azimi-Dastgerdi[b], and H. Mokhtari[b,*]

a. *Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran.*
b. *Department of Industrial Engineering, Faculty of Engineering, University of Kashan, Kashan, Iran.*

Received 9 May 2020; received in revised form 11 January 2021; accepted 15 March 2021

KEYWORDS
Sustainable inventory;
Economic production quantity;
Preventive maintenance;
Multiple shipments;
Imperfect quality items.

Abstract. The quality of products, maintenance activities, and transportation policies are primary concerns of managers in inventory and production planning problems. Environmental issues and regulations are growing increasingly and they have attracted much attention to achieve sustainable production. Previous authors have conducted a wide range of studies on these problems separately. Regarding the gap of an integrated framework, a sustainable economic production quantity model is formulated by considering preventive maintenance and multiple shipments policy where a portion of produced items is defective. Two particular cases are studied. In Case I, the production period's demand has been satisfied by the items produced in the previous cycle. In Case II, simultaneous production and consumption during the production period are considered and mathematically formulated. An analytical method is presented for solving the models, and a numerical example is discussed for both cases. The comparison of two cases proves that Case II is more beneficial and the overall cost of the inventory system is reduced. Sensitivity analysis of the models is performed, and some insights are derived by changing some of the parameters.

1. Introduction

Inventory is defined as holding items and materials for the purpose of satisfying feature demand. There are various reasons for maintaining an inventory, e.g., dealing with market volatility, avoiding shortage, and balancing the production processes. These reasons have drawn the attention of operations research experts to design and optimize inventory systems. As a result, Whitmann [1] presented the first classic inventory model, namely Economic Order Quantity (EOQ). The objective was to minimize the overall cost of the system including procurement cost, ordering cost, and holding cost. Although the development of EOQ was an important advancement in inventory systems, the model was not consistent with items in some production settings. In this regard, the Economic Production Quantity (EPQ) model was developed [2]. The primary difference between EOQ and EPQ was relaxing the assumption about the instant receiving of orders. Afterwards, several researchers have investigated more advanced inventory systems and tried to consider real-world situations.

*. *Corresponding author. Tel.: +98 3155912476*
E-mail addresses: ali.fallahi@ie.sharif.edu (A. Fallahi); azimi9413@gmail.com (M. Azimi-Dastgerdi); mokhtari_ie@kashanu.ac.ir (H. Mokhtari)

doi: 10.24200/sci.2021.55927.4475

According to the Intergovernmental Panel on Climate Change (IPCC) reports, the global warming problem poses a severe threat to the world. Greenhouse gases emission, particularly carbon, can be realized as the principal cause of global warming. The latest statistics have confirmed that a considerable portion of emission is related to the industrial and transportation sectors [3]. Based on the above concern, the concept of sustainable production has emerged. Sustainable production can be defined as producing the desired items by responsible production processes that attempt to take environmental anxieties into account. One of the most effective methods for curb emission is making operational adjustments and optimizing the production and inventory system variables, considering the environmental initiatives.

The inventory system managers are interested in reducing the system cost as much as possible. Nevertheless, attaining this goal is not possible without paying attention to the quality of products. Quality becomes one of the substantial factors that can affect business success. At first, the formulation of classic inventory models was performed without quality criteria. However, in recent years, the indicated rationales have directed researchers' concentration to this crucial factor.

A wide range of activities and parameters can affect the quality of a production system. Maintenance plans are one of these high-impact activities. A maintenance plan commonly compromises different technical activities such as checking, service, repair or, if needed, replacing the equipment and parts. Maintenance plans can fall into different categories. Preventive Maintenance (PM) is one of the first and effective maintenance plans. PM deals with a group of prescheduled activities that should be performed to keep the production state at the desired level. Production plants need inventory models that can fulfill the need for an appropriate maintenance plan.

The classic inventory models have another unrealistic assumption for simplicity, which is about the continuous delivery policy. In real-world states, vehicles are used for item delivery, and a continuous replenishment procedure is usually unenforceable. Several parameters are included in multiple shipment delivery, such as the vehicle capacity, transportation time to demand point, etc. Consideration of these parameters can significantly change the performance of an inventory system.

As will be seen in the literature review, no work presents PM, multiple shipments policy, and carbon emission in an integrated production-inventory framework. Therefore, we try to cover this gap by presenting a joint sustainable EPQ model with imperfect quality items under PM and multiple shipments assumptions.

The rest of this paper is given as follows. Section 2 presents an attempt to review the previously published works. Section 3 develops two cases for a sustainable EPQ model with imperfect quality items, multiple deliveries of goods, and PM policy. A numerical example is solved for the presented models and sensitivity analysis for some parameters of models is provided in Section 4. Section 5 explains the conclusion and feature directions of the current research.

2. Literature review

After the classic inventory models, researchers have tried to present the extensions by considering the real-world situations [4–8]. In this section, a literature review is presented with a focus on the features of this work.

2.1. Inventory systems with carbon emission consideration

Hua et al. [9], for the first time, examined a market-based carbon emission trading mechanism in the EOQ model. A fixed carbon emission cap was considered for the system and the shortage or surplus quota could be bought or sold through, respectively. Bouchery et al. [10] tried to integrate the economic, social, and environmental factors in the classic EOQ by developing a multi-objective replenishment model. Chen et al. [11] addressed EOQ models with four carbon emission policies: strict cap, cap-and-offset, cap-and-price, and carbon tax. A new variant of the sustainable EOQ model was studied by Battini et al. [12] in which the Life Cycle Analysis (LCA) and direct accounting method were used to attain the carbon emission factors. Hovelaque and Bironneau [13] modeled a replenishment system for which price and carbon emission depended on the demand. An inventory EOQ model for growing items with an emphasis on environmental concerns was developed by Zhang et al. [14]. Kazemi et al. [15] worked on the EOQ model with low-quality items and extended this model by considering the warehousing and disposal emission cost. Taleizadeh et al. [16] formulated four EPQ models according to various shortage scenarios. The direct accounting approach was used to consider the carbon footprints. The possibility of investment in carbon emission reduction was studied by Lee [17]. Mishra et al. [18] suggested a sustainable inventory model for non-instantaneous deteriorating items where ordering cost, carbon emission, and deterioration rate depended on the investment.

2.2. Inventory systems with imperfect quality items

As an important extension of inventory models with defective items, Salameh and Jaber [19] supposed that all items in an EOQ system did not have a perfect quality. Chang [20] proposed two EOQ models by considering demand and defective rate of system as fuzzy parameters. The contribution of Rezaei [21] paper was considering the backorder shortage in the EOQ model with imperfect items. Lo et al. [22] suggested a deteriorating production-inventory model where the deterioration rate of products was a random variable. Partial backorder, inflation, and both retailer and manufacturer viewpoints were other aspects of this work. Chung et al. [23] studied the imperfect quality items in the two-warehouse inventory model for the first time. An EPQ model with stochastic demand considering inflation and defective items was presented by Sarkar and Moon [24]. Konstantaras et al. [25] analyzed the effect of learning during the inspection process in two cases of the EOQ model with faulty items. Wee et al. [26] considered some inspection constraints in their EPQ model with defective items. Unlike the many previous assumptions, the inspection rate can be less than or equal to the production rate. Chang et al. [27] presented an EOQ model with imperfect items where the inspection error and delay in payment were taken into account in the inspection operation. A production-inventory system was developed by Mokhtari [28]. In this work, the raw materials for a production system were supplied by order issues, and the final goods were produced by implementing the production process. In Mokhtari's paper [28], the imperfect quality was considered for raw material and final products. Khakzad and Gholamian [29] presented a deteriorating inventory model in which the average deterioration rate depended on the inspection process. Stock-dependent demand production-inventory model with imperfect quality goods and permissible delay in payment was formulated by Dhaka et al. [30].

2.3. Inventory systems with multiple shipments policy

The first appearance of multiple shipments policy in inventory systems literature was in the work of Chiu et al. [31]. In addition to the multiple shipment concept, they incorporated the need for reworking of defective items in their model. CáRdenas-BarróN et al. [32] relaxed the assumption of Chiu et al. [31] model, which was about the fixed number of shipments. Ritha and Martin [33] proposed the packaging and switching cost in the replenishment model under multiple shipments policy. Taleizadeh et al. [34] modeled a production-inventory system considering multiple shipments policy where Lot-size, the selling price, and the number of shipments were the decision variables of the model. Taleizadeh et al. [35] worked on an EPQ model with imperfect items for joint optimization of selling price and lot-size considering the multiple shipments assumptions. An inventory system with capital investment-dependent setup cost, defective items, and rework process was studied by Priyan and Uthayakumar [36]. Multiple deliveries policy was one of the components of the formulated model. The EPQ model for deteriorating items, where the delivery of products abides by multiple shipments policy, was suggested by Kalantari and Taleizadeh [37].

2.4. Inventory systems with maintenance plans

Rezg et al. [38] investigated a production system with maintenance plans wherein there was a minimum available constraint in the proposed model. Liao et al. [39] proposed a deteriorating production system. Rework and maintenance can be either perfect or imperfect in their system. Repair time and machine breakdown were assumed as random variables in the article of Widyadana and Wee [40] for a deteriorating production-inventory model. An inventory model with stochastic maintenance and rework time was the result of Wee and Widyadana [41]. An extension of EPQ models with maintenance activities was presented by Liao [42], where the warranty program was considered in the mathematical model. Jafari and Makis [43] formulated an EPQ model with PM and condition monitoring assumptions. They formulated the problem by using Semi-Markov decision. Peng and van Houtum [44] tried to optimize condition-based maintenance and lot-sizing policies simultaneously. An imperfect inventory control system was proposed by Nasr et al. [45] to find optimal maintenance scheduling and lot-size policies where the quality of items would correlate with each other. Taleizadeh [46] studied a multi-item manufacturing model with a partial backorder shortage. PM can be performed whether the stock level is positive or negative. Lai et al. [47] presented hybrid planning of PM and emergency maintenance in a defective EPQ model. Tsao et al. [48] considered the predictive maintenance plan for EPQ with defective items and reworks. An EPQ model with PM activities was analyzed by Mokhtari and Asadkhani [49] where the production of defective items was considered. Table 1 represents a summary of our literature review results.

We conclude that there is a research opportunity for developing a sustainable EPQ-based model that integrates PM, quality of items, and multiple shipments policy. Due to everyday real-world situations, two cases are presented for this structure.

3. Problem description and formulation

This section explains the detail of the proposed inventory system and, then, presents a mathematical formulation. The main components of the current study can be summerized as follows:

- Production of defective items occurs with a known probability;

Table 1. The features of the related inventory systems in the literature.

Paper	Year	Model type		Features			
		EOQ	EPQ	Maintenance plan	Imperfect quality	Multiple shipments	Carbon emission
Salameh and Jaber [19]	2000	×			×		
Chang [20]	2004	×			×		
Rezaei [21]	2005	×			×		
Lo et al. [22]	2007	×	×		×		
Rezg et al. [38]	2008		×	×			
Chung et al. [23]	2009	×			×		
Liao et al. [39]	2009		×	×	×		
Hua et al. [9]	2011	×					×
Chiu et al. [31]	2011		×		×	×	
Widyadana and Wee [40]	2011		×	×			
Sarkar and Moon [24]	2011		×		×		
Bouchery et al. [10]	2012	×					×
Konstantaras et al. [25]	2012	×			×		
CáRdenas-BarróN et al. [32]	2012		×		×	×	
Chen et al. [11]	2013	×					×
Wee et al. [26]	2013		×		×		
Ritha and Martin [33]	2013		×		×	×	
Wee and Widyadana [41]	2013		×	×	×		
Battini et al. [12]	2014	×				×	×
Taleizadeh et al. [34]	2015		×		×	×	
Liao [42]	2015		×	×	×		
Jafari and Makis [43]	2015		×	×			
Chang et al. [27]	2016	×			×		
Taleizadeh et al. [35]	2016		×		×	×	
Peng and van Houtum [44]	2016		×	×			
Zhang et al. [14]	2016	×					×
Priyan and Uthayakumar [36]	2017		×		×	×	
Nasr et al. [45]	2017		×	×	×		
Kazemi et al. [15]	2018	×			×		×
Taleizadeh [46]	2018		×	×	×		
Kalantari and Taleizadeh [37]	2018		×			×	
Taleizadeh et al. [16]	2018		×				×
Mokhtari [28]	2019	×	×		×		
Lai et al. [47]	2019		×	×	×		
Lee [17]	2019	×					×
Tsao et al. [48]	2019		×	×	×		
Mokhtari and Asadkhani [49]	2019		×	×	×		
Khakzad and Gholamian [29]	2020	×			×		
Mishra et al. [18]	2020	×					×
Dhaka et al. [30]	2020	×			×		

- A 100% inspection policy is performed to recognize the defective items;
- A disposal policy is considered for the identified defective items;
- The delivery policy is multiple shipments;
- PM plan is a part of the inventory system;
- Carbon emission is formulated by the direct accounting method.

Assume that the lead time parameter is zero in this system and an order is placed at the beginning of the cycle. The shortage of items is not permitted. When the PM activities begin, the production operation is suspended and then, continue as planned. The production period consists of sequential s-cycles: a production sub-cycle t_p is initiated, and after the production of items of size Q_{sc}, maintenance activities are implemented during a maintenance sub-cycle t_m. x is a decision variable that represents the number of maintenance activities implemented. PM activities impose a cost on the system. We define K_m as the maintenance cost per activity. There are two setup costs in this model: K as the setup cost for the overall cycle and K_s as the setup cost for a production sub-cycle. It is assumed that a fixed θ percentage of Q size orders are defective. These defective items are discovered through 100% inspection at the end of the production period. The items are disposed of at the cost of C_s. The number of shipments is considered as a fixed parameter n. The delivery is done by one vehicle with travel time t_T, and the capacity for this vehicle is Q_T. K_T is the system delivery cost per shipment. According to the tangibility of the direct accounting approach [16], carbon emission is formulated by this method. The carbon emissions from various activities of the system are translated to an evident cost parameter. The parameters C_{ep}, C_{ei}, C_{et}, and C_{mt} show the production emission cost, the inventory holding emission cost, the transportation emission cost, and the maintenance emission cost, respectively.

We investigate two cases for the proposed structure:

- Case I: There is no demand satisfaction during the production period from the produced items of the same period. The produced items in one production period are used in the demand period of the same cycle and production period of the next cycle;
- Case II: There is demand satisfaction during the production cycle. The produced items in one cycle are used in the demand period and the production period of the same cycle.

Notations: The notations of the proposed models can be summarized as follows:

Parameters

P	The production rate per unit of time
D	The demand rate per unit of time
K	The setup cost per unit of cycle
I_{i1}	The inventory level of Case II before demand satisfaction in PM time
I_{i2}	The inventory level of Case II after demand satisfaction in PM time
K_s	The setup cost per unit of production sub-cycle
K_m	The PM cost per activity
h	The holding cost per unit of item per unit of time
C	The procurement cost per unit of item
C_s	The disposal cost per unit of a defective item
K_T	The delivery cost per unit of shipment
C_{ep}	The production emission cost per unit of item
C_{ei}	The inventory holding emission cost per unit of item per unit of time
C_{et}	The transportation emission cost per unit of shipment
C_{em}	The PM emission cost per unit of activity.
t_p	The production time of the machine in one sub-cycle
t_m	The PM activity time in one sub-cycle
θ	The percentage of defective items per batch
n	Number of shipments to deliver the batch to customers

Variables

Q	The production quantity in one cycle
Q_{sc}	The production quantity in one sub-cycle
Q_T	The capacity of the vehicle to carry items
T_P	The production period in one cycle
T_D	The demand period in one cycle
x	The number of maintenance activities implemented in one cycle
t_T	The travel time of the vehicle

Abbreviations

OC	The total setup cost
PMC	The total PM cost
PC	The total procurement cost
DC	The total disposal cost
TrC	The total transportation cost

HC	The total holding cost
PCEC	The total production emission cost
HCEC	The total inventory holding emission cost
TCEC	The total transportation emission cost
MCEC	The total PM emission cost

3.1. Case I: Asynchronous production and consumption

The produced items in the production period of a cycle are used for demand satisfaction in the demand period of the same cycle and the production period of the next cycle. In this case, the goal is to determine the EPQ while considering the above assumption to minimize the total cost of the inventory system. The production period is divided into serial production and maintenance sub-cycles. At first, production is performed to produce Q_{sc} during time t_p and after that, the maintenance activities are performed during t_m. After the end of the last sub-cycle, $Q(1-\theta)$ defective items are recognized by a 100% inspection process and disposed of at a cost of C_s. The number of shipments is a prespecified parameter n according to company policy. The travel time of the vehicle is indicated by t_T. Also, system management should decide the vehicle capacity Q_T. Figure 1 depicts the inventory level for Case I.

A total cycle is composed of a production cycle and demand cycle due to Eq. (1):

$$T = T_P + T_D. \tag{1}$$

Regarding the line slope Eq. (2), we have:

$$\tan\alpha = p = \frac{Q_{sc}}{t_p}. \tag{2}$$

Thus, the production quantity in one production sub-cycle can be calculated as follows:

$$Q_{sc} = Pt_p. \tag{3}$$

The number of production and maintenance activities can be denoted by dividing the overall order quantity by the batch size in one sub-cycle:

$$x = \frac{Q}{Q_{sc}} = \frac{Q}{Pt_p}. \tag{4}$$

The production period in one cycle is given as follows:

$$T_P = x\,(t_p + t_m) = \frac{Q}{Pt_p}\,(t_p + t_m). \tag{5}$$

Regarding the line slope equation, we have:

$$\tan\beta = D = \frac{(1-\theta)\,Q}{(T_D + T_P)}. \tag{6}$$

By simplifying Eq. (6), we arrive at:

$$T_D = \frac{(1-\theta)\,Q}{D} - T_P. \tag{7}$$

Since the production period is calculated based on the model parameters in Eq. (5), this equation is substituted into Eq. (7) to calculate the demand period as follows:

$$T_D = \frac{(1-\theta)\,Q}{D} - \frac{Q}{Pt_p}\,(t_p + t_m). \tag{8}$$

The transportation time of the vehicle is calculated as:

$$t_T = \frac{T_D}{n} = \frac{1}{n}\left[\frac{(1-\theta)\,Q}{D} - \frac{Q}{Pt_p}\,(t_p + t_m)\right]. \tag{9}$$

Finally, the vehicle capacity can be expressed as:

$$Q_T = \frac{(1-\theta)\,Q}{n}. \tag{10}$$

There are two types of setup costs in this model. One is related to the complete cycle and another is for operation runs in each sub-cycle. Eq. (11) shows the total setup cost of the system for one cycle of Case I:

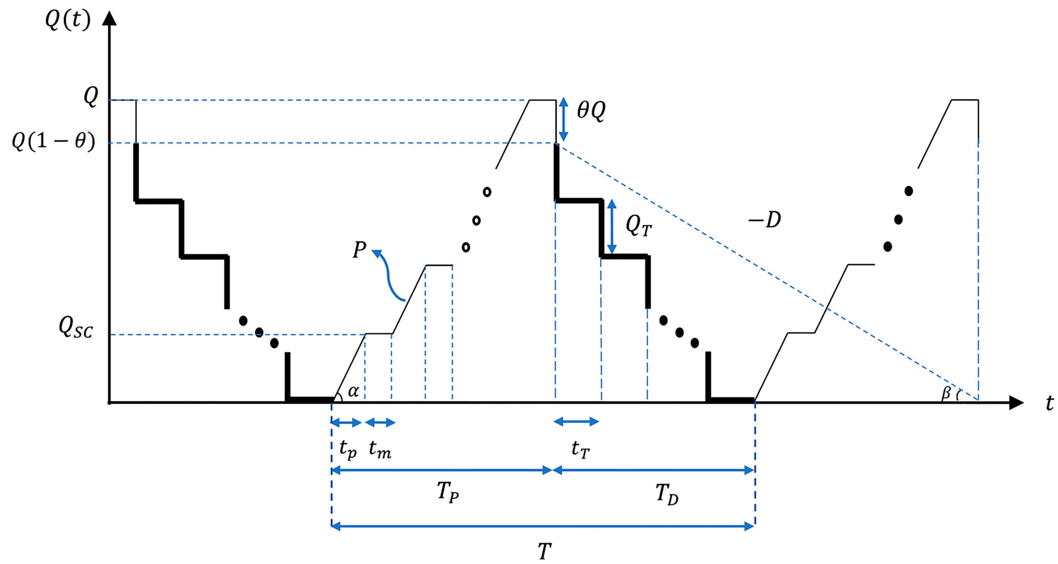

Figure 1. Inventory level of Case I.

$$OC = K + xK_s = K + \frac{Q}{Q_{sc}}K_s = K + \frac{Q}{Pt_p}K_s. \quad (11)$$

The PM cost for a particular cycle can be obtained as:

$$PMC = xK_m = \frac{Q}{Q_{sc}}K_m = \frac{Q}{Pt_p}K_m. \quad (12)$$

Procurement of a batch with size Q incurs the purchasing cost for inventory system as:

$$PC = CQ. \quad (13)$$

The disposal of produced defective items by any method or policy imposes a disposal cost on the system, which can be indicated as:

$$DC = C_s(\theta Q). \quad (14)$$

The number of shipments is assumed to be the only parameter that can influence the transportation cost of non-defective items. Therefore, we computed the transportation cost of the system in one cycle by Eq. (15):

$$TrC = nK_T. \quad (15)$$

We separate the holding cost of the inventory system. The inventory holding cost in the production period (T_P) can be written as follows:

$$HC_1 = h\left[\frac{Qt_p}{2} + Qt_m + \frac{Q(x-1)(t_p+t_m)}{2}\right]$$

$$= h\left[\frac{Qt_p}{2} + Qt_m + \frac{Q\left(\frac{Q}{Pt_p}-1\right)(t_p+t_m)}{2}\right]$$

$$= \frac{h}{2}\left[Qt_m + \frac{Q^2(t_p+t_m)}{Pt_p}\right]. \quad (16)$$

A detailed description of the calculation procedure is presented in Appendix A.

The holding cost of items during the demand period (T_D) can be achieved as follows:

$$HC_2 = h\left(\frac{n-1}{2n}\right)(1-\theta)QT_D$$

$$= h\left(\frac{n-1}{2n}\right)\left[\left(\frac{(1-\theta)^2Q^2}{D}\right) - \left(\frac{Q^2}{pt_p}(t_p+t_m)(1-\theta)\right)\right]. \quad (17)$$

We also explain the extraction of the average inventory level during the demand period through Appendix B.

The production emission cost is calculated as:

$$PCEC = C_{ep}D. \quad (18)$$

The inventory holding emission cost is formulated as follows:

$$HCEC = C_{ei}\left[\frac{Qt_m}{2} + \frac{Q^2(t_p+t_m)}{2Pt_p} + \left(\frac{n-1}{2n}\right)\left[\left(\frac{(1-\theta)^2Q^2}{D}\right) - \left(\frac{Q^2}{pt_p}(t_p+t_m)(1-\theta)\right)\right]\right]. \quad (19)$$

In addition, the transportation carbon emission is:

$$TCEC = C_{et}n. \quad (20)$$

We model the PM emission cost as follows:

$$MCEC = C_{em}x = C_{em}\frac{Q}{Pt_p}. \quad (21)$$

The total cost objective function per unit of a cycle for Case I is determined as:

$$TC(Q) = K + CQ + \frac{Q}{Pt_p}K_s + C_s\theta Q + \frac{Q}{Pt_p}K_m + nK_T + \frac{h}{2}\left[Qt_m + \frac{Q^2(t_p+t_m)}{Pt_p}\right] + h\left(\frac{n-1}{2n}\right)\left[\left(\frac{(1-\theta)^2Q^2}{D}\right) - \left(\frac{Q^2(t_p+t_m)(1-\theta)}{pt_p}\right)\right] + C_{ep}D + C_{ei}\left[\frac{Qt_m}{2} + \frac{Q^2(t_p+t_m)}{2Pt_p} + \left(\frac{n-1}{2n}\right)\left[\left(\frac{(1-\theta)^2Q^2}{D}\right) - \left(\frac{Q^2}{pt_p}(t_p+t_m)(1-\theta)\right)\right]\right] + C_{et}n + C_{em}\frac{Q}{Pt_p}. \quad (22)$$

For computing the total cost during the planning horizon, we multiply $\frac{1}{T}$ by $TC(Q)$ and simplify the obtained formula, thus leading to Eq. (23) as follows:

$$TCU(Q) = TC(Q) \times \frac{1}{T} = \frac{KD}{(1-\theta)Q} + \frac{CD}{(1-\theta)} + \frac{DK_s}{Pt_p(1-\theta)} + \frac{C_s\theta D}{(1-\theta)} + \frac{DK_m}{Pt_p(1-\theta)} + \frac{nK_TD}{(1-\theta)Q} + \frac{h}{2}\left[\frac{t_mD}{(1-\theta)} + \frac{Q(t_p+t_m)D}{Pt_p(1-\theta)}\right] + h\left(\frac{n-1}{2n}\right)\left[(1-\theta)Q - \left(\frac{QD(t_p+t_m)}{Pt_p}\right)\right] + \frac{C_{ep}D^2}{(1-\theta)Q} + C_{ei}\left[\frac{t_mD}{2(1-\theta)}\right.$$

$$Q^* = \sqrt{\frac{KD+nK_TD+C_{ep}D^2+nC_{et}D}{\frac{hD(t_p+t_m)}{2Pt_p}+h\left(\frac{n-1}{2n}\right)\left[(1-\theta)^2-\left(\frac{D(t_p+t_m)(1-\theta)}{pt_p}\right)\right]C_{ei}\left[\frac{D(t_p+t_m)}{2Pt_p}+\left(\frac{n-1}{2n}\right)\left[(1-\theta)^2-\left(\frac{D(t_p+t_m)(1-\theta)}{pt_p}\right)\right]\right]}}. \quad (27)$$

Box I

$$+\frac{Q\left(t_p+t_m\right)D}{2Pt_p\left(1-\theta\right)}+\left(\frac{n-1}{2n}\right)$$
$$\left[\left(1-\theta\right)Q-\left(\frac{QD\left(t_p+t_m\right)}{Pt_p}\right)\right]\Bigg]$$
$$+\frac{nC_{et}D}{\left(1-\theta\right)Q}+\frac{DC_{em}}{Pt_p\left(1-\theta\right)}. \quad (23)$$

We optimize the obtained function by an analytical method as follows. In the following, it is proven that the present function is convex and there is a global minimum for this function. By Differentiation from the total cost function, Eq. (24) is elicited as follows:

$$\frac{d\left(TCU\left(Q\right)\right)}{dQ}=\frac{-KD}{\left(1-\theta\right)Q^2}+\frac{-nK_TD}{\left(1-\theta\right)Q^2}$$
$$+\frac{hD\left(t_p+t_m\right)}{2Pt_p\left(1-\theta\right)}+h\left(\frac{n-1}{2n}\right)$$
$$\left[\left(1-\theta\right)-\left(\frac{D\left(t_p+t_m\right)}{pt_p}\right)\right]$$
$$-\frac{C_{ep}D^2}{\left(1-\theta\right)Q^2}+C_{ei}\left[\frac{D\left(t_p+t_m\right)}{2Pt_p\left(1-\theta\right)}\right.$$
$$+\left(\frac{n-1}{2n}\right)\Bigg[\left(1-\theta\right)$$
$$-\left(\frac{D\left(t_p+t_m\right)}{pt_p}\right)\Bigg]\Bigg]-\frac{nC_{et}D}{\left(1-\theta\right)Q^2}. \quad (24)$$

The second derivation of this function is attained as:

$$\frac{d^2\left(TCU\left(Q\right)\right)}{d^2Q}=\frac{KD}{\left(1-\theta\right)Q^3}+\frac{nK_TD}{\left(1-\theta\right)Q^3}$$
$$+\frac{C_{ep}D^2}{\left(1-\theta\right)Q^3}+\frac{nC_{et}D}{\left(1-\theta\right)Q^3}. \quad (25)$$

The second derivation of the objective function is always positive; therefore, the convexity of the total cost function is approved and we can calculate the EPQ by setting the first derivation equal to zero:

$$\frac{d\left(TCU\left(Q\right)\right)}{dQ}=\frac{-KD}{\left(1-\theta\right)Q^2}+\frac{-nK_TD}{\left(1-\theta\right)Q^2}$$
$$+\frac{hD\left(t_p+t_m\right)}{2Pt_p\left(1-\theta\right)}+h\left(\frac{n-1}{2n}\right)$$
$$\left[\left(1-\theta\right)-\left(\frac{D\left(t_p+t_m\right)}{pt_p}\right)\right]$$
$$-\frac{C_{ep}D^2}{\left(1-\theta\right)Q^2}+C_{ei}\left[\frac{D\left(t_p+t_m\right)}{2Pt_p\left(1-\theta\right)}\right.$$
$$+\left(\frac{n-1}{2n}\right)\Bigg[\left(1-\theta\right)$$
$$-\left(\frac{D\left(t_p+t_m\right)}{pt_p}\right)\Bigg]\Bigg]-\frac{nC_{et}D}{\left(1-\theta\right)Q^2}=0. \quad (26)$$

By simplifying Eq. (26), we obtain Q^*, which is then shown in Box I.

3.2. Case II: Simultaneous production and consumption

All of the explained assumptions for Case I are valid in this case, except that the produced items in a particular cycle are consumed in the same cycle, and a portion of produced items are consumed in the production period. Figure 2 shows the inventory level of Case II.

There is the assumption about the imperfect produced items in the system wherein they are accumulated during the production period T_P and finally, disposed of after the end of the production process. Regarding this situation, an extras hypothesis is needed to prevent the shortage during the production and maintenance time. If θQ_s is defective in each production sub-cycle, then the demand during maintenance and production is satisfied by consuming $(1-\theta)Q_s$ and the following equation in each sub-cycle should be valid:

$$\left(1-\theta\right)Q_s-Dt_m>0. \quad (28)$$

A complete cycle is composed of the production period and demand period:

$$T=T_P+T_D.$$

By using the line slope formula:

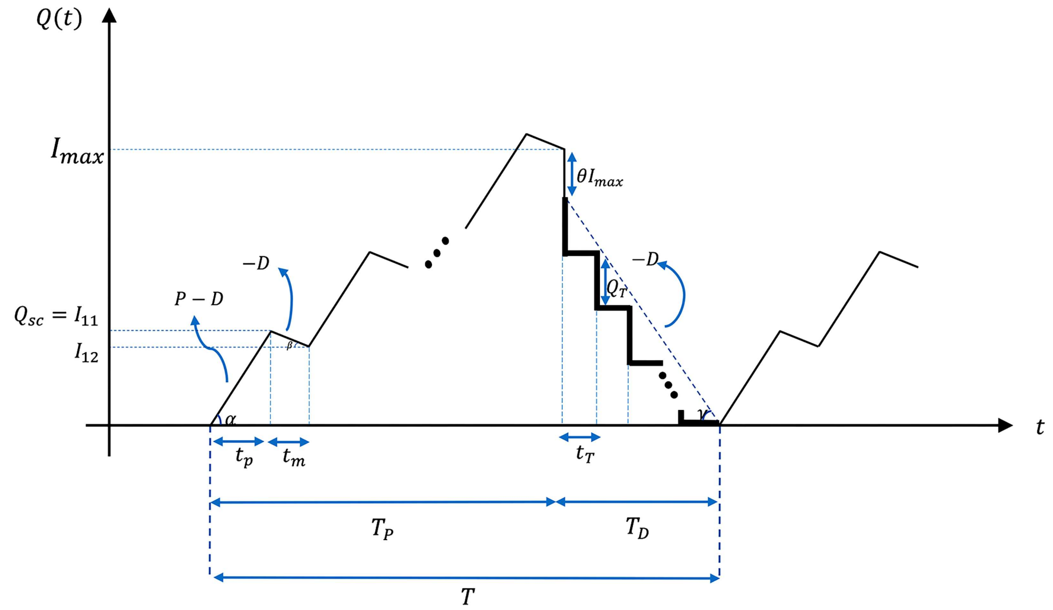

Figure 2. Inventory level of Case II.

$$\tan\alpha = P - D = \frac{Q_{sc}}{t_p}. \tag{29}$$

the Q_{SC} value is calculated as follows:

$$Q_{sc} = (P - D)\, t_p. \tag{30}$$

Similar to Case II, the number of PM activities is calculated as:

$$x = \frac{Q}{Q_{sc}} = \frac{Q}{(P - D)\, t_p}. \tag{31}$$

The production period of Case II is:

$$T_P = x\,(t_p + t_m) = \frac{Q}{(P - D)\, t_p}\,(t_p + t_m)\,. \tag{32}$$

By considering the line slope equation:

$$\tan\gamma = D = \frac{(1-\theta)\, I_{\max}}{T_D}. \tag{33}$$

The demand period can be calculated as follows:

$$T_D = \frac{(1-\theta)\, I_{\max}}{D} = \frac{x\,[(1-\theta)\,(P - D)\, t_p - (1-\theta)\, D t_m]}{D}. \tag{34}$$

The transportation time can be expressed as follows:

$$t_T = \frac{T_D}{n} = \frac{1}{n}\left[\frac{x\,[(1-\theta)\,(P - D)\, t_p - (1-\theta)\, D t_m]}{D}\right]. \tag{35}$$

We can calculate the vehicle's capacity as:

$$Q_T = \frac{(1-\theta)\, I_{\max}}{n} = \frac{(1-\theta)\,[x\,(P - D)\, t_p - xDt_m]}{n}. \tag{36}$$

The setup cost is imposed on the inventory system as:

$$K + xK_s = K + \frac{Q}{Q_{sc}} K_s = K + \frac{Q}{(P - D)\, t_p} K_s. \tag{37}$$

The PM cost is expressed as:

$$xK_m = \frac{Q}{Q_{sc}} K_m = \frac{Q}{(P - D)\, t_p} K_m. \tag{38}$$

The purchasing cost for a batch with the size of Q can be calculated by multiplying the purchasing cost of an item and the batch size as:

$$PC = CQ. \tag{39}$$

As mentioned earlier, the defective items are accumulated during the production period and then, their disposal should be done after the production operation. If C_s be the disposal cost per unit of an item, then the total disposal cost for a particular cycle can be formulated as:

$$DC = C_s \theta I_{\max} = C_s \theta \frac{Q}{(P-D)\, t_p}\,[(p-D)\, t_p - Dt_m]\,. \tag{40}$$

The transportation cost of Case II is calculated as follows:

$$TrC = nK_T. \tag{41}$$

To formulate the holding cost, similar to Case I, the inventory holding costs for the production period and the demand period are obtained separately. In Case II and its production period, production and demand satisfaction occurred at the same time. To prevent the shortage, $(1-\theta)\, Q_{sc} - dt_m > 0$ should be valid. In the following, the inventory level before the implementation of maintenance activities in sub-cycle i is defined as I_{i1} and after the end of this activity as I_{i2} where:

$$I_{i1} = i\,(P-D)\,t_p - (i-1)\,Dt_m, \quad \forall i = 1,\ldots,x, \tag{42}$$

$$I_{i2} = i\,(P-D)\,t_p - (i-1)\,Dt_m, \quad \forall i = 1,\ldots,x. \tag{43}$$

It is obvious that $I_{02} = 0$. By using the above equations, the average stock level in the production period is as follows:

$$A_1 = \sum_{i=1}^{x}\left[\left(\frac{I_{i1}+I_{(i-1)2}}{2}\right)t_p + \left(\frac{I_{i1}+I_{i2}}{2}\right)t_m\right]. \tag{44}$$

By expanding Eq. (44):

$$A_1 = \frac{1}{4}\left[\frac{2Q^2}{(P-D)} - \frac{2QDt_m\,(Q-(P-D)\,t_p)}{(P-D)^2 t_p} + \frac{2Qt_p t_m\,(Q+(P-D)\,t_p)}{(P-D)\,t_p} - \frac{2Q^2Dt_m^2}{(P-D)^2 t_p^2}\right]. \tag{45}$$

Thus, Eq. (46) shows the inventory holding cost in the production period:

$$HC_1 = h\left[\frac{2Q^2}{(P-D)} - \frac{2QDt_m\,(Q-(P-D)\,t_p)}{(P-D)^2 t_p} + \frac{2Qt_m\,(Q+(P-D)\,t_p)}{(P-D)\,t_p} - \frac{2Q^2Dt_m^2}{(P-D)^2 t_p^2}\right]. \tag{46}$$

The inventory holding cost during the demand period is obtained similar to Case I according to Eq. (47):

$$HC_2 = h\left(\frac{n-1}{2n}\right)\left(\frac{(1-\theta)\left[Q-\frac{Q}{(P-D)t_p}Dt_m\right]^2}{D}\right). \tag{47}$$

The detailed procedure is clarified in Appendix C.

The carbon emission cost for Case II can be formulated as follows:

$$PCEC = C_{ep}D. \tag{48}$$

The inventory holding emission cost is:

$$C_{ei}\left(\frac{Q^2}{2\,(P-D)} - \frac{QDt_m\,(Q-(P-D)\,t_p)}{2(P-D)^2 t_p} + \frac{Qt_m\,(Q+(P-D)\,t_p)}{2\,(P-D)\,t_p} - \frac{Q^2Dt_m^2}{2(P-D)^2 t_p^2} + \left(\frac{n-1}{2n}\right)\left(\frac{(1-\theta)\left[Q-\frac{Q}{(P-D)t_p}Dt_m\right]^2}{D}\right)\right). \tag{49}$$

The transportation emission cost is as follows:

$$PCEC = C_{et}n. \tag{50}$$

The PM emission cost can be written as:

$$MCEC = C_{em}x = C_{em}\frac{Q}{(P-D)\,t_p}. \tag{51}$$

The total cost of Case II per unit of the cycle is given below:

$$\begin{aligned} TC\,(Q) = {} & K + \frac{Q}{(P-D)\,t_p}K_s + \frac{Q}{(P-D)\,t_p}K_m \\ & + CQ + C_s\theta\frac{Q}{(P-D)\,t_p}\left[(p-D)\,t_p - Dt_m\right] \\ & + nK_T + h\left[\frac{Q^2}{2\,(P-D)} - \frac{QDt_m\,(Q-(P-D)\,t_p)}{2(P-D)^2 t_p} + \frac{Qt_m\,(Q+(P-D)\,t_p)}{2\,(P-D)\,t_p} - \frac{Q^2Dt_m^2}{2(P-D)^2 t_p^2}\right] \\ & + h\left(\frac{n-1}{2n}\right)\left(\frac{(1-\theta)\left[Q-\frac{Q}{(P-D)t_p}Dt_m\right]^2}{D}\right) \\ & + C_{ep}D + C_{ei}\left(\frac{Q^2}{2\,(P-D)} - \frac{QDt_m\,(Q-(P-D)\,t_p)}{2(P-D)^2 t_p} + \frac{Qt_m\,(Q+(P-D)\,t_p)}{2\,(P-D)\,t_p} - \frac{Q^2Dt_m^2}{2(P-D)^2 t_p^2} \right. \\ & \left. + \left(\frac{n-1}{2n}\right)\left(\frac{(1-\theta)\left[Q-\frac{Q}{(P-D)t_p}Dt_m\right]^2}{D}\right)\right). \end{aligned} \tag{52}$$

Finally, to obtain the total cost over the planning horizon, we should multiply $\frac{1}{T}$ by the total cost of the model as:

$$\begin{aligned} TCU\,(Q) & = TC\,(Q) \times \frac{1}{T} \\ & = \frac{1}{[pt_p - \theta pt_p + \theta Dt_p + \theta Dt_m]} \\ & \left\{\frac{KDt_p\,(P-D)}{Q} + K_sD + K_mD \right. \\ & + CDt_p\,(P-D) + \frac{nK_TDt_p\,(P-D)}{Q} \\ & + C_s\theta Dt_p\,(P-D)\left(1-\frac{Dt_m}{t_p\,(P-D)}\right) \end{aligned}$$

$$+\frac{nC_{et}Dt_p\left(P-D\right)}{Q}+C_{em}D$$
$$+h\left[\frac{QDt_p}{2}-\frac{D^2t_m\left(Q-\left(P-D\right)t_p\right)}{2\left(P-D\right)}\right.$$
$$\left.+\frac{Dt_m\left(Q+\left(P-D\right)t_p\right)}{2}-\frac{QD^2{t_m}^2}{2t_p\left(P-D\right)}\right]$$
$$+\frac{C_{ep}D^2t_p\left(P-D\right)}{Q}+h\left(\frac{n-1}{2n}\right)\left(1-\theta\right)$$
$$\left(P-D\right)Qt_p\left(1-\frac{Dt_m}{t_p\left(P-D\right)}\right)^2$$
$$+C_{ei}\left[\frac{QDt_p}{2}-\frac{D^2t_m\left(Q-\left(P-D\right)t_p\right)}{2\left(P-D\right)}\right.$$
$$+\frac{Dt_m\left(Q+\left(P-D\right)t_p\right)}{2}-\frac{QD^2{t_m}^2}{2t_p\left(P-D\right)}$$
$$+\left(\frac{n-1}{2n}\right)\left(1-\theta\right)\left(P-D\right)$$
$$\left.\left.Qt_p\left(1-\frac{Dt_m}{t_p\left(P-D\right)}\right)^2\right]\right\}. \tag{53}$$

It is confirmed that the presented total cost Eq. (53) is convex and there is a global minimum for this function. Similar to Case I, the first derivation of the objective function is calculated as follows:

$$\frac{d\left(TCU\left(Q\right)\right)}{dQ}=\frac{1}{\left[pt_p-\theta pt_p+\theta Dt_p+\theta Dt_m\right]}$$
$$\left\{\frac{-KDt_p\left(P-D\right)}{Q^2}-\frac{nK_TDt_p\left(P-D\right)}{Q^2}\right.$$
$$-\frac{C_{ep}D^2t_p\left(P-D\right)}{Q^2}$$
$$+h\left[\frac{Dt_p}{2}-\frac{D^2t_m}{2\left(P-D\right)}+\frac{Dt_m}{2}\right.$$
$$\left.-\frac{D^2{t_m}^2}{2t_p\left(P-D\right)}\right]+h\left(\frac{n-1}{2n}\right)\left(1-\theta\right)$$
$$\left(P-D\right)t_p\left(1-\frac{Dt_m}{t_p\left(P-D\right)}\right)^2$$
$$+C_{ei}\left[\frac{Dt_p}{2}-\frac{D^2t_m}{2\left(P-D\right)}+\frac{Dt_m}{2}\right.$$
$$-\frac{D^2{t_m}^2}{2t_p\left(P-D\right)}+\left(\frac{n-1}{2n}\right)\left(1-\theta\right)$$
$$\left.\left(P-D\right)t_p\left(1-\frac{Dt_m}{t_p\left(P-D\right)}\right)^2\right]$$
$$\left.-\frac{nC_{et}Dt_p\left(P-D\right)}{Q^2}\right\}. \tag{54}$$

Moreover, the second derivation is as follows:

$$\frac{d^2\left(TCU\left(Q\right)\right)}{d^2Q}=\frac{1}{Q^3\left[pt_p-\theta pt_p+\theta Dt_p+\theta Dt_m\right]}$$
$$\left\{2KDt_p\left(P-D\right)+2nK_TDt_p\right.$$
$$\left(P-D\right)+2nC_{ep}D^2t_p\left(P-D\right)$$
$$\left.+2nC_{et}Dt_p\left(P-D\right)\right\}. \tag{55}$$

Since Eq. (55) is always positive, the convexity of the objective function for Case II is approved, and the EPQ is obtained by setting the first derivative equal to zero, as shown in Box II and Eq. (56). The optimal values of the dependent variables like x, Q_T, etc. are calculated regarding Q^*. There is an important note about the value of x^* which indicates the number of maintenance activities during the production period. It is possible that this variable does not get an integer value, and this is not a feasible solution. A simple algorithm is presented for calculating the optimal number of maintenance activities when there is an infeasible solution. The algorithm includes eight steps below and can be used for both cases:

1. Find x^* regarding Q^*;
2. If x^* is an integer, set $x^*=x^*$ and stop the algorithm;
3. If x^* is not integer, set $x_l=[x^*]$ and $x_u=[x^*]+1$;
4. Regarding x_l and Q_{sc}, calculate Q_l by Eqs. (4) and (31) for Cases I and II, respectively;
5. Calculate TCU_l for Cases I and II by Eqs. (23) and (53), respectively;

$$Q^*=\sqrt{\frac{Dt_p\left(P-D\right)\left[K+nK_TD+C_{ep}D+nC_{et}\right]}{\left(C_{ei}+h\right)\left[\left(\frac{n-1}{2n}\right)\left(1-\theta\right)\left(P-D\right)t_p\left(1-\frac{Dt_m}{t_p\left(P-D\right)}\right)^2\right]+\left(C_{ei}+h\right)\left[\frac{Dt_p}{2}-\frac{D^2t_m}{2\left(P-D\right)}+\frac{Dt_m}{2}-\frac{D^2{t_m}^2}{2t_p\left(P-D\right)}\right]}}. \tag{56}$$

Box II

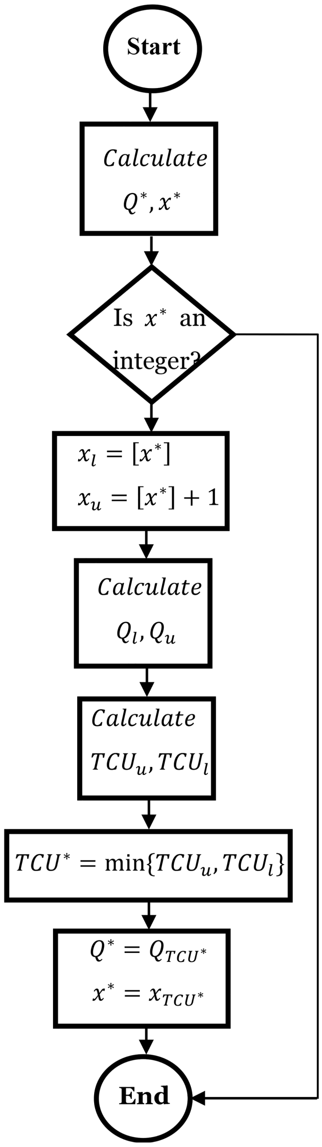

Figure 3. The flowchart of the proposed algorithm.

6. Regarding x_l and Q_{sc}, Calculate Q_u by Eq. (4) for Case I and by Eq. (31) for Case II;
7. Calculate TCU_u for Case I by Eq. (23) and for Case II by Eq. (53);
8. Find the minimum total cost as $TCU^* = \min\{TCU_l, TCU_u\}$ and set the related Q as Q^* and the related x as x^*.

Figure 3 shows the flowchart of the proposed algorithm.

4. Computational experiments

To investigate the performance of the proposed models, a numerical example is presented in this section. After that, a sensitivity analysis is implemented to see how the model may be affected by changing the key parameters.

4.1. Numerical example

An inventory system is considered with $P = 12000$, $D = 6000$, $C = 50$, $C_s = 3$, $K = 1500$, $K_s = 3$, $K_m = 200$, $h = 0.15$, $K_T = 500$, $t_p = 0.5$, $t_m = 0.02$, $\theta = 0.07$, $n = 5$, $C_{ep} = 30$, $C_{ei} = 10$, $C_{et} = 40$, and $C_{em} = 25$. Figures 4 and 5 show the convex behavior of objective function for Cases I and II, respectively. The computational results are presented in Table 2. The better performance of Case II is validated according to this table. However, the number of maintenance activities is infeasible and we, thus, need to execute the algorithm. The following results in Table 3 are achieved following the implementation of the algorithm:

The better performance of Case II is also confirmed after the implementation of the algorithm, based on the data in Table 3. Accordingly, when there is demand satisfaction during the overall cycle (Case II), the total cost of the inventory system would be lower (nearly 46%) than when there is demand satisfaction just in the demand period (Case I).

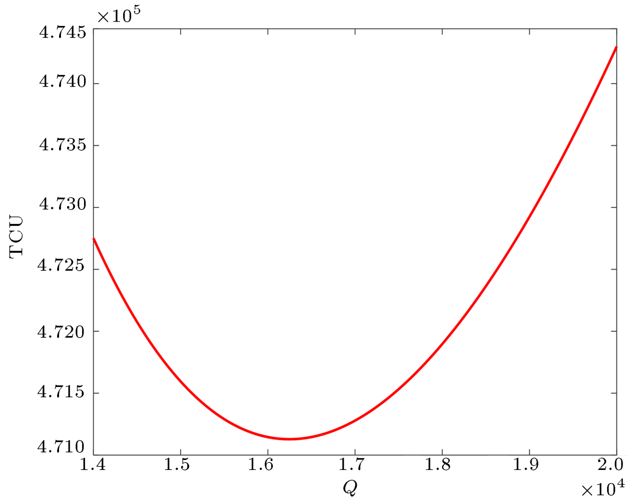

Figure 4. Total cost function behavior of Case I.

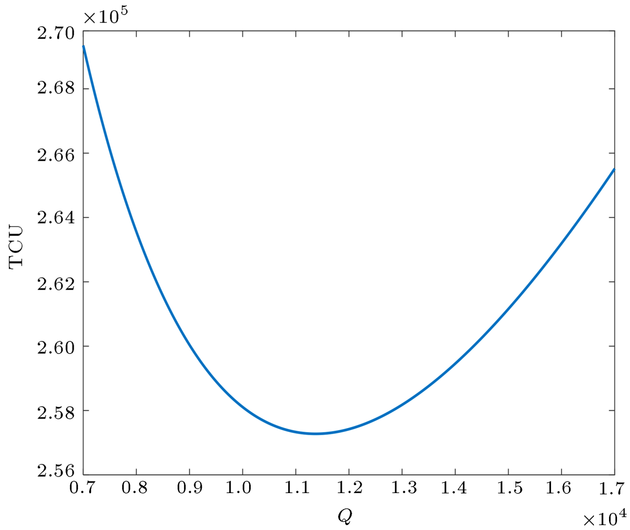

Figure 5. Total cost function behavior of Case II.

Table 2. The computational results of the numerical example.

	Q^*	TCU^*	Q^*_{sc}	Q^*_T	T^*_P	T^*_D	T^*	t^*_T	x^*
Case I	16246.69	471128.30	6000.00	3021.88	1.41	1.11	2.52	0.22	2.71
Case II	11371.62	257275.59	3000.00	2030.52	1.97	1.69	3.66	0.34	3.79

Table 3. The computational after the execution of the algorithm.

	Q^*	TCU^*	Q^*_{sc}	Q^*_T	T^*_P	T^*_D	T^*	t^*_T	x^*
Case I	18000.00	471897.21	6000.00	3348.00	1.56	1.23	2.79	0.25	3.00
Case II	12000.00	257421.09	3000.00	2142.72	2.08	1.79	3.87	0.77	4.00

Table 4. Sensitivity analysis of cases with respect to the portion of defective items.

		Optimal value				Change percentage			
		Q		TCU		Q		TCU	
Parameter	**Change**	**Case I**	**Case II**	**Case I**	**Case II**	**Case I**	**Case II**	**Case I**	**Case II**
	–30%	16026.14	11319.70	462296.20	254890.46	–1.36	–0.46	–1.87	–0.93
	–20%	16099.41	11336.93	465192.79	255680.24	–0.91	–0.31	–1.26	–0.62
	–10%	16172.93	11354.23	468136.46	256475.27	–0.45	–0.15	–0.64	–0.31
θ	0%	16246.69	11371.62	471128.30	257275.59	0.00	0.00	0.00	0.00
	+10%	16320.69	11389.08	474169.49	258081.26	+0.46	+0.15	+0.65	+0.31
	+20%	16394.92	11406.63	477261.19	258892.33	+0.91	+0.31	+1.30	+0.63
	+30%	16469.37	11424.26	480404.64	259708.85	+1.37	+0.46	+1.97	+0.95

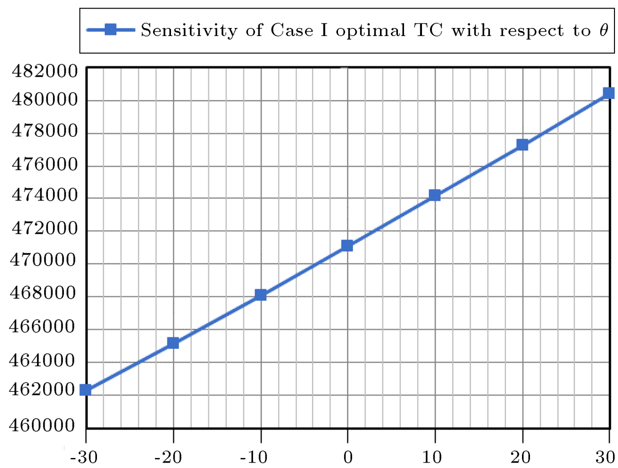

Figure 6. Sensitivity analysis of Case I total cost to θ.

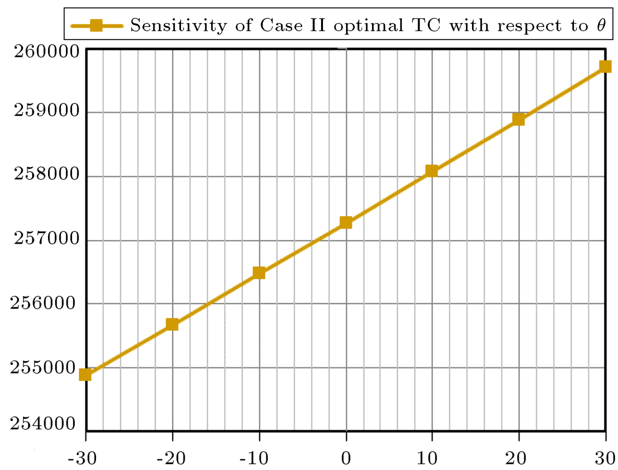

Figure 7. Sensitivity analysis of Case II total cost to θ.

4.2. Sensitivity analysis

The example's parameters may not always be fixed and various factors can influence them. The parameters vary in the range of -30 to $+30$ percentage, and both of the presented models are solved to see how the optimal solutions are affected.

Figure 6 and Figure 7 are drawn to show that as the number of defective items increases, greater disposal cost is imposed on the system, which raises the overall cost of the system. We can conclude from Table 4 that the systems with a higher defective rate should place large order sizes to prevent the shortage.

Table 5. Sensitivity analysis of cases with respect to the maintenance activities time.

		Optimal value				Change percentage			
		Q		***TCU***		***Q***		***TCU***	
Parameter	**Change**	**Case I**	**Case II**	**Case I**	**Case II**	**Case I**	**Case II**	**Case I**	**Case II**
	–30%	16261.84	11311.11	470795.61	257744.33	+0.09	–0.53	–0.07	+0.18
	–20%	16256.78	11331.15	470906.52	257588.60	+0.06	–0.36	–0.05	+0.12
	–10%	16251.73	11351.32	471017.42	257432.35	+0.03	–0.18	–0.02	+0.06
t_m	0%	16246.69	11371.62	471128.30	257275.59	0.00	0.00	0.00	0.00
	+10%	16241.65	11392.06	471239.17	257118.30	–0.03	+0.18	+0.02	–0.06
	+20%	16236.62	11412.64	471350.03	256960.49	–0.06	+0.36	+0.05	–0.12
	+30%	16231.59	11433.36	471460.87	256802.15	–0.09	+0.54	+0.07	–0.18

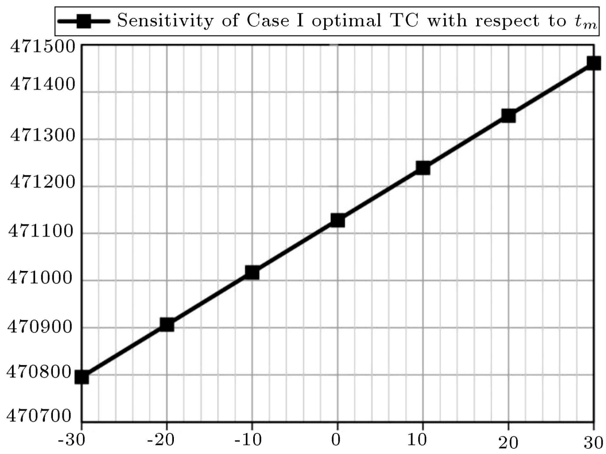

Figure 8. Sensitivity analysis of Case I total cost to t_m.

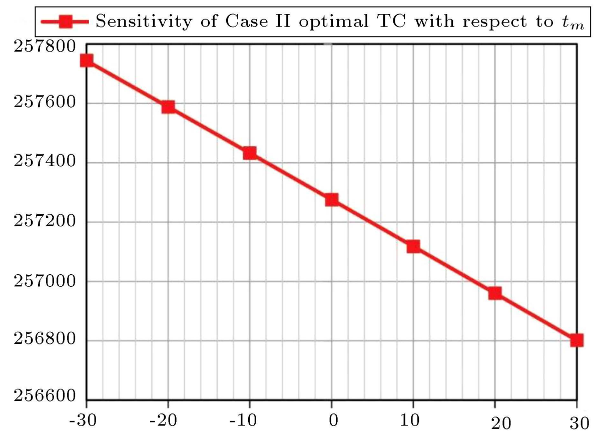

Figure 9. Sensitivity analysis of Case II total cost to t_m.

Table 5 illustrates that when the maintenance time is increased, the optimal order quantity of Case I and the optimal total cost of Case II are reduced. Interestingly, increasing the time of maintenance activities results in an increasing trend in Case I total cost and a decreasing trend in Case II total cost. The adverse behavior of cost functions is depicted in Figures 8 and 9.

It was predictable that with an increase in the delivery cost, the system needs to decrease total transportation costs by delivering more items to the demand point in one shipment. Due to the correlation between Q and Q_T, the batch size should be increased logically. We proved these hypotheses by increasing delivery cost, solving the related models, and presenting the results in Table 6. It is obvious that K_T is a cost parameter,

Table 6. Sensitivity analysis of cases with respect to the transportation cost of items.

		Optimal value				Change percentage			
		Q		***TCU***		***Q***		***TCU***	
Parameter	**Change**	**Case I**	**Case II**	**Case I**	**Case II**	**Case I**	**Case II**	**Case I**	**Case II**
	–30%	16213.58	11348.44	470830.17	257070.64	–0.20	–0.20	–0.06	–0.08
	–20%	16224.63	11356.17	470929.62	257139.00	–0.14	–0.14	–0.04	–0.05
	–10%	16235.66	11363.90	471028.99	257207.32	–0.07	–0.07	–0.02	–0.03
K_T	0%	16246.69	11371.62	471128.30	257275.59	0.00	0.00	0.00	0.00
	+10%	16257.71	11379.33	471227.55	257343.81	+0.07	+0.07	+0.02	+0.03
	+20%	16268.73	11387.04	471326.72	257411.99	+0.14	+0.14	+0.04	+0.05
	+30%	16279.73	11394.75	471425.83	257480.12	+0.20	+0.20	+0.06	+0.08

Table 7. Sensitivity analysis of cases with respect to the PM cost.

		Optimal value				Change percentage			
		Q		*TCU*		*Q*		*TCU*	
Parameter	**Change**	**Case I**	**Case II**	**Case I**	**Case II**	**Case I**	**Case II**	**Case I**	**Case II**
K_m	–30%	16246.69	11371.62	471063.79	257213.50	0.00	0.00	–0.01	–0.02
	–20%	16246.69	11371.62	471085.29	257234.20	0.00	0.00	–0.01	–0.02
	–10%	16246.69	11371.62	471106.80	257254.89	0.00	0.00	–0.00	–0.01
	0%	16246.69	11371.62	471128.30	257275.59	0.00	0.00	0.00	0.00
	+10%	16246.69	11371.62	471149.81	257296.28	0.00	0.00	+0.00	+0.01
	+20%	16246.69	11371.62	471171.31	257316.98	0.00	0.00	+0.01	+0.02
	+30%	16246.69	11371.62	471192.82	257337.67	0.00	0.00	+0.01	+0.02

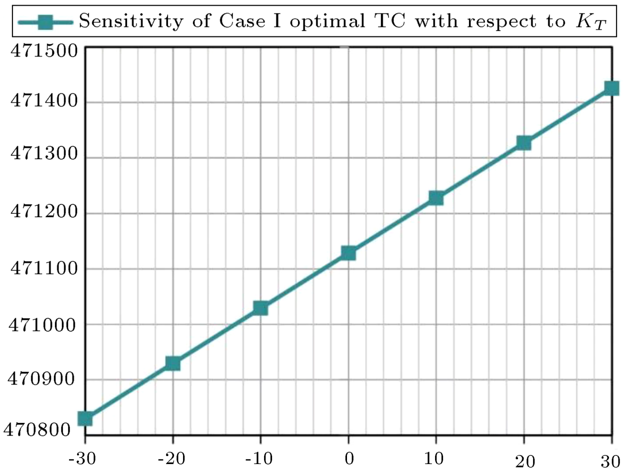

Figure 10. Sensitivity analysis of Case I total cost to K_T.

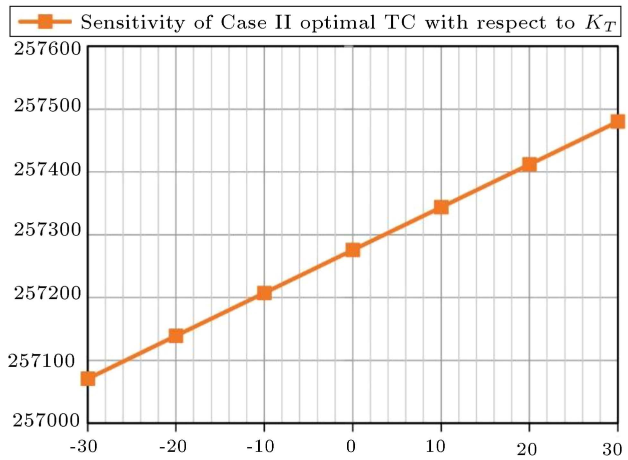

Figure 11. Sensitivity analysis of Case II total cost to K_T.

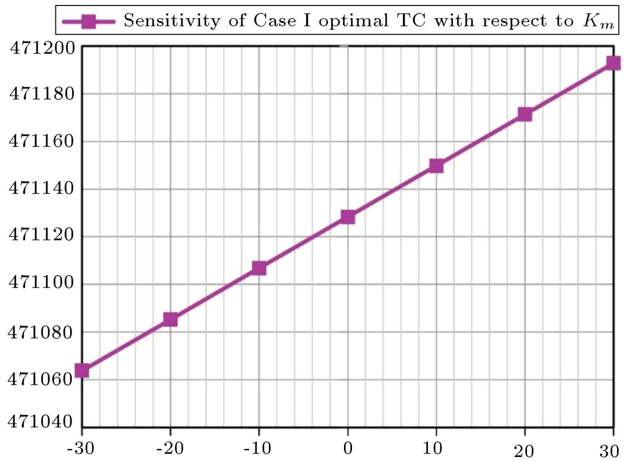

Figure 12. Sensitivity analysis of Case I total cost to K_m.

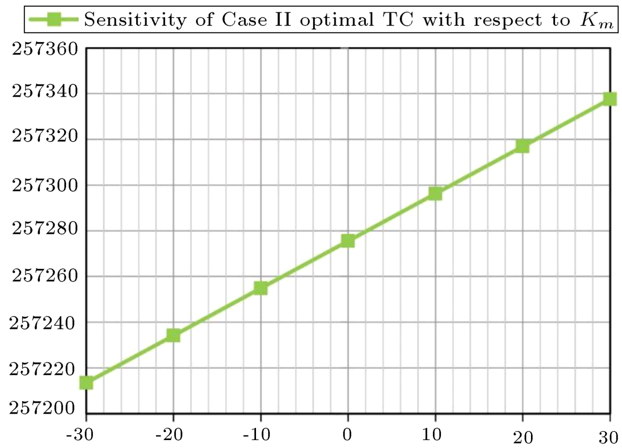

Figure 13. Sensitivity analysis of Case II total cost to K_m.

and increasing the cost parameter will raise the total cost of cases. The trends of change in cost functions are shown in Figures 10 and 11.

We analyzed the impact of maintenance activities cost on the performance of the cases. From the results given in Table 7, we found that K_m would not affect the optimal batch size. The total costs in Cases I and II increase following the rise of the maintenance cost. These results are presented in Figures 12 and 13.

The considered cost parameters for the emission of the system may not be stable due to some reasons such as regulation change. By increasing the production emission cost parameter, we concluded that the management should attempt to compensate the

Table 8. Sensitivity analysis of cases with respect to the production carbon emission cost.

		Optimal value				Change percentage			
		Q		***TCU***		***Q***		***TCU***	
Parameter	**Change**	**Case I**	**Case II**	**Case I**	**Case II**	**Case I**	**Case II**	**Case I**	**Case II**
	–30%	13659.21	9560.55	447829.42	241258.86	–15.93	–15.93	–4.95	–6.23
	–20%	14572.84	10200.03	456056.17	246914.31	–10.30	–10.30	–3.20	0.03
	–10%	15432.48	10801.72	463796.73	252235.53	–5.01	–5.01	–1.56	–1.96
C_{ep}	0%	16246.69	11371.62	471128.30	257275.59	0.00	0.00	0.00	0.00
	+10%	17022.00	11914.29	478109.58	262074.84	+4.77	+4.77	+1.48	+1.87
	+20%	17763.51	12433.29	484786.45	266664.82	+9.34	+9.34	+2.90	+3.65
	+30%	18475.28	12931.48	491195.55	271070.74	+13.72	+13.72	+4.26	+5.36

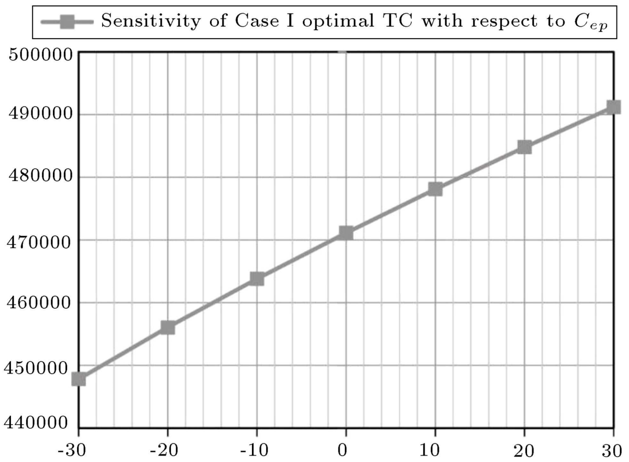

Figure 14. Sensitivity analysis of Case I total cost to C_{ep}.

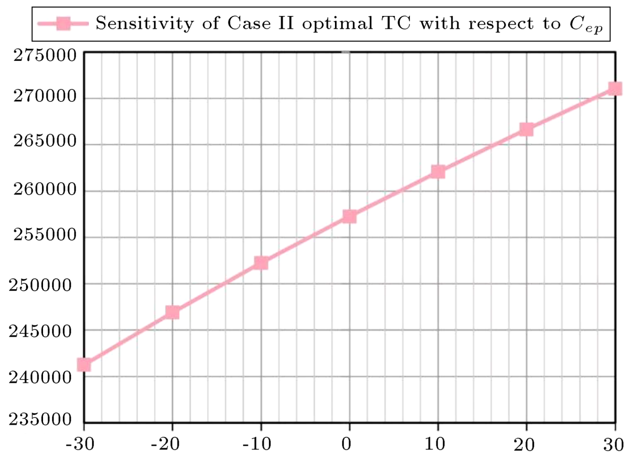

Figure 15. Sensitivity analysis of Case II total cost to C_{ep}.

increase by producing fewer items. The increasing trend of total costs is inevitable in this state. The results are presented in Table 8, Figures 14, and 15. As presented below, optimal order size and total cost of Case I are more sensitive to change in this parameter than Case II ones.

5. Conclusions and feature directions

Equipment maintenance and related activities, transportation policies, and quality aspects are among the competitive business environment challenges. On the other hand, global warming problems cause different pressure from governments on companies to consider these concerns in their operations. We found that no paper has studied the sustainable production-inventory model that would consider the imperfect quality of items, Preventive Maintenance (PM) plan, and multiple shipments policy. Two cases were studied and mathematically formulated. By proving the convexity of function, the optimization was done by derivation methods. We compared the numerical example of cases and concluded that simultaneous production and consumption (Case II) imposed lower cost (less than half) on the system than asynchronous production and consumption (Case I). Moreover, the Economic Production Quantity (EPQ) was lower in Case II. Extensive sensitivity analyses were performed to get insights into the performances of models.

Some related real-world assumptions can make the model more realistic. Other maintenance categories such as condition-based maintenance can be incorporated in the models, in addition to preventive ones. We assumed that there was only one vehicle for the transformation of items where this might not be valid in practice. Formulating the problem by considering the percentage of defective items as a random variable and the rework process can be another extension.

References

1. Harris, F.W. "How many parts to make at once", *The Magazine of Management*, **38**(6), pp. 947–950 (1990).

2. Taft, E. "The most economical production lot", *Iron Age*, **101**(18), pp. 1410–1412 (1918).

3. Tiwari, S., Daryanto, Y., and Wee, H.M. "Sustainable inventory management with deteriorating and imper-

fect quality items considering carbon emission", *Journal of Cleaner Production*, **192**, pp. 281–292 (2018).

4. Mokhtari, H., Naimi-Sadigh, A., and Salmasnia, A. "A computational approach to economic production quantity model for perishable products with backordering shortage and stock-dependent demand", *Scientia Iranica. Transaction E, Industrial Engineering*, **24**(4), pp. 2138–2151 (2017).

5. Hemmati, M., Fatemi Ghomi, S., and Sajadieh, M.S. "Inventory of complementary products with stock-dependent demand under vendor-managed inventory with consignment policy", *Scientia Iranica*, **25**(4), pp. 2347–2360 (2018).

6. Mokhtari, H. and Asadkhani, J. "Economic order quantity for imperfect quality items under inspection errors, batch replacement and multiple sales of returned items", *Scientia Iranica*, **28**(5), pp. 2890–2909 (2022).

7. Mokhtari, H. and Fallahi, A. "The economic production quantity model considering production quantity (EPQ) model considering inflation, time value of money and investment variable in production capacity", *Sharif Journal of Industrial Engineering & Management*, **35.1**(2.2), pp. 53–67 (2020). DOI: 10.24200/j65.2019.51273.1902

8. Mokhtari, H. "Joint ordering and reuse policy for reusable items inventory management", *Sustainable Production and Consumption*, **15**, pp. 163–172 (2018).

9. Hua, G., Cheng, T., and Wang, S. "Managing carbon footprints in inventory management", *International Journal of Production Economics*, **132**(2), pp. 178–185 (2011).

10. Bouchery, Y., Ghaffari, A., Jemai, Z., et al. "Including sustainability criteria into inventory models", *European Journal of Operational Research*, **222**(2), pp. 229–240 (2012).

11. Chen, X., Benjaafar, S., and Elomri, A. "The carbon-constrained EOQ", *Operations Research Letters*, **41**(2), pp. 172–179 (2013).

12. Battini, D., Persona, A., and Sgarbossa, F. "A sustainable EOQ model: Theoretical formulation and applications", *International Journal of Production Economics,* **149**, pp. 145–153 (2014).

13. Hovelaque, V. and Bironneau, L. "The carbon-constrained EOQ model with carbon emission dependent demand", *International Journal of Production Economics,* **164**, pp. 285–291 (2015).

14. Zhang, Y., Li, L.-Y., Tian, X.-Q., et al. "Inventory management research for growing items with carbon-constrained", in *2016 35th Chinese Control Conference (CCC)*, IEEE, pp. 9588–9593 (2016).

15. Kazemi, N., Abdul-Rashid, S.H., Ghazilla, R.A.R., et al. "Economic order quantity models for items with imperfect quality and emission considerations", *International Journal of Systems Science: Operations & Logistics*, **5**(2), pp. 99–115 (2018).

16. Taleizadeh, A.A., Soleymanfar, V.R., and Govindan, K. "Sustainable economic production quantity models for inventory systems with shortage", *Journal of Cleaner Production*, **174**, pp. 1011–1020 (2018).

17. Lee, J.-Y. "Investing in carbon emissions reduction in the EOQ model", *Journal of the Operational Research Society*, **71**(8), pp. 1289–1300 (2020).

18. Mishra, U., Wu, J.-Z., Tsao, Y.-C., et al. "Sustainable inventory system with controllable non-instantaneous deterioration and environmental emission rates", *Journal of Cleaner Production*, **244**, 118807 (2020).

19. Salameh, M. and Jaber, M. "Economic production quantity model for items with imperfect quality", *International Journal of Production Economics*, **64**(1–3), pp. 59–64 (2000).

20. Chang, H.-C. "An application of fuzzy sets theory to the EOQ model with imperfect quality items", *Computers & Operations Research*, **31**(12), pp. 2079–2092 (2004).

21. Rezaei, J. "Economic order quantity model with backorder for imperfect quality items", In *Proceedings. 2005 IEEE International Engineering Management Conference*, **2**, IEEE, pp. 466–470 (2005).

22. Lo, S.-T., Wee, H.-M., and Huang, W.-C. "An integrated production-inventory model with imperfect production processes and Weibull distribution deterioration under inflation", *International Journal of Production Economics*, **106**(1), pp. 248–260 (2007).

23. Chung, K.-J., Her, C.-C., and Lin, S.-D. "A two-warehouse inventory model with imperfect quality production processes", *Computers & Industrial Engineering*, **56**(1), pp. 193–197 (2009).

24. Sarkar, B. and Moon, I. "An EPQ model with inflation in an imperfect production system", *Applied Mathematics and Computation,* **217**(13), pp. 6159–6167 (2011).

25. Konstantaras, I., Skouri, K., and Jaber, M. "Inventory models for imperfect quality items with shortages and learning in inspection", *Applied Mathematical Modelling*, **36**(11), pp. 5334–5343 (2012).

26. Wee, H.-M., Wang, W.-T., and Yang, P.-C. "A production quantity model for imperfect quality items with shortage and screening constraint", *International Journal of Production Research*, **51**(6), pp. 1869–1884 (2013).

27. Chang, C.-T., Cheng, M.-C., and Soong, P.-Y. "Impacts of inspection errors and trade credits on the economic order quantity model for items with imperfect quality", *International Journal of Systems Science: Operations & Logistics*, **3**(1), pp. 34–48 (2016).

28. Mokhtari, H. "Optimal lot size in a manufacturing system with imperfect raw materials and defective

finished products", *Scientia Iranica*, **26**(4), pp. 2561–2578 (2019).

29. Khakzad, A. and Gholamian, M.R. "The effect of inspection on deterioration rate: An inventory model for deteriorating items with advanced payment", *Journal of Cleaner Production*, **254**, 120117 (2020).

30. Dhaka, V., Pareek, S., and Mittal, M. "Stock-dependent inventory model for imperfect items under permissible delay in payments", In *Optimization and Inventory Management*, Springer, pp. 181–194 (2020).

31. Chiu, Y.-S.P., Liu, S.-C., Chiu, C.-L., et al. "Mathematical modeling for determining the replenishment policy for EMQ model with rework and multiple shipments", *Mathematical and Computer Modelling*, **54**(9–10), pp. 2165–2174 (2011).

32. CáRdenas-BarróN, L.E., Taleizadeh, A.A., and TreviñO-Garza, G. "An improved solution to replenishment lot size problem with discontinuous issuing policy and rework, and the multi-delivery policy into economic production lot size problem with partial rework", *Expert Systems with Applications*, **39**(18), pp. 13540–13546 (2012).

33. Ritha, W. and Martin, N. "Replenishment policy for EMQ model with rework, multiple shipments, switching and packaging", *Int. J. Eng. Res. Technol*, **2**(1), pp. 1–8 (2013).

34. Taleizadeh, A.A., Kalantari, S.S., and Cárdenas-Barrón, L.E. "Determining optimal price, replenishment lot size and number of shipments for an EPQ model with rework and multiple shipments", *Journal of Industrial and Management Optimization*, **11**(4), pp. 1059–1071 (2015).

35. Taleizadeh, A.A., Kalantari, S.S., and Cárdenas-Barrón, L.E. "Pricing and lot sizing for an EPQ inventory model with rework and multiple shipments", *Top*, **24**(1), pp. 143–155 (2016).

36. Priyan, S. and Uthayakumar, R. "Setup cost reduction EMQ inventory system with probabilistic defective and rework in multiple shipments management", *International Journal of System Assurance Engineering and Management*, **8**(2), pp. 223–241 (2017).

37. Kalantari, S.S. and Taleizadeh, A.A. "Mathematical modelling for determining the replenishment policy for deteriorating items in an EPQ model with multiple shipments", *International Journal of Systems Science: Operations & Logistics*, **7**(2), pp. 164–171 (2020).

38. Rezg, N., Dellagi, S., and Chelbi, A. "Joint optimal inventory control and preventive maintenance policy", *International Journal of Production Research*, **46**(19), pp. 5349–5365 (2008).

39. Liao, G.-L., Chen, Y.H., and Sheu, S.-H. "Optimal economic production quantity policy for imperfect process with imperfect repair and maintenance", *European Journal of Operational Research*, **195**(2), pp. 348–357 (2009).

40. Widyadana, G.A. and Wee, H.M. "Optimal deteriorating items production inventory models with random machine breakdown and stochastic repair time", *Applied Mathematical Modelling*, **35**(7), pp. 3495–3508 (2011).

41. Wee, H.M. and Widyadana, G.A. "A production model for deteriorating items with stochastic preventive maintenance time and rework process with FIFO rule", *Omega*, **41**(6), pp. 941–954 (2013).

42. Liao, G.-L. "Production and maintenance policies for an epq model with perfect repair, rework, free-repair warranty, and preventive maintenance", *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, **46**(8), pp. 1129–1139 (2015).

43. Jafari, L. and Makis, V. "Joint optimal lot sizing and preventive maintenance policy for a production facility subject to condition monitoring", *International Journal of Production Economics*, **169**, pp. 156–168 (2015).

44. Peng, H. and van Houtum, G.-J. "Joint optimization of condition-based maintenance and production lot-sizing", *European Journal of Operational Research*, **253**(1), pp. 94–107 (2016).

45. Nasr, W.W., Salameh, M., and Moussawi-Haidar, L. "Economic production quantity with maintenance interruptions under random and correlated yields", *International Journal of Production Research*, **55**(16), pp. 4544–4556 (2017).

46. Taleizadeh, A.A. "A constrained integrated imperfect manufacturing-inventory system with preventive maintenance and partial backordering", *Annals of Operations Research*, **261**(1–2), pp. 303–337 (2018).

47. Lai, X., Chen, Z., and Bidanda, B. "Optimal decision of an economic production quantity model for imperfect manufacturing under hybrid maintenance policy with shortages and partial backlogging", *International Journal of Production Research*, **57**(19), pp. 6061–6085 (2019).

48. Tsao, Y.-C., Lee, P.-L., Liao, L.-W., et al. "Imperfect economic production quantity models under predictive maintenance and reworking", *International Journal of Systems Science: Operations & Logistics*, **7**(4), pp. 347–360 (2019).

49. Mokhtari, H. and Asadkhani, J. "Extended economic production quantity models with preventive maintenance", *Scientia Iranica*, **27**(6), pp. 3253–3264 (2020).

Appendix A: Derivation of inventory holding cost during production period for Case I. If $x = 1$:

$$HC_1 = h\left[1\left(\frac{Qt_p}{2} + Qt_m\right)\right]. \tag{A.1}$$

If $x = 2$:

$$HC_1 = h\left[2\left(\frac{Qt_p}{4} + \frac{Qt_m}{4}\right) + 1\left(\frac{Q}{2}\left(t_p + t_m\right)\right)\right]. \tag{A.2}$$

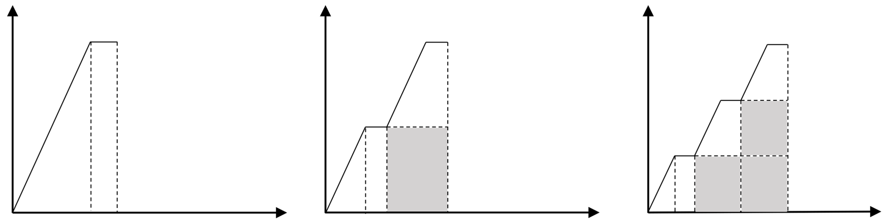

Figure A.1. The production period in Case I.

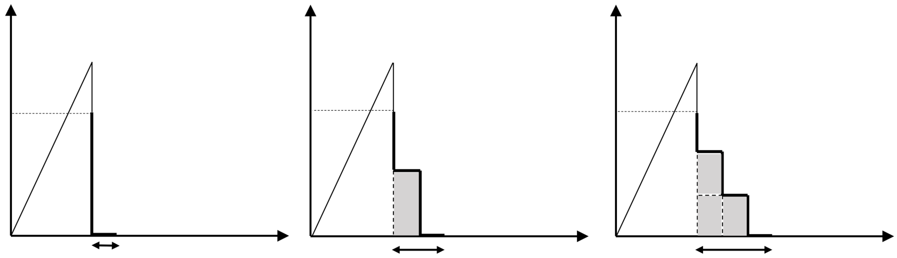

Figure B.1. The demand period in Case I.

If $x = 3$:

$$HC_1 = h\left[3\left(\frac{Qt_p}{6} + \frac{Qt_m}{3}\right) + (1+2)\left(\frac{Q}{3}(t_p + t_m)\right)\right]. \quad (A.3)$$

If $x = x$:

$$HC_1 = h\left[x\left(\frac{Qt_p}{2x} + \frac{Qt_m}{x}\right) + \left(\frac{x(x-1)}{2}\right)\left(\frac{Q}{x}(t_p + t_m)\right)\right] = h\left[\left(\frac{Qt_p}{2} + Qt_m\right) + \left(\frac{Q(x-1)(t_p + t_m)}{2}\right)\right]. \quad (A.4)$$

The production period in Case I for different values of x is represented in Figure A.1.

Appendix B: Derivation of inventory holding cost during demand period for Case I If $n = 1$:

$$HC_2 = 0. \quad (B.1)$$

If $n = 2$:

$$HC_2 = h\left(\frac{(1-\theta)Q}{2} \times \frac{T_D}{2}\right) = h\left(\frac{1}{2^2}\right)(1-\theta)QT_D. \quad (B.2)$$

If $n = 3$:

$$HC_2 = h\left(\frac{2(1-\theta)Q}{3} \times \frac{T_D}{3} + \frac{(1-\theta)Q}{3} \times \frac{T_D}{3}\right) = h\left(\frac{1+2}{3^2}\right)(1-\theta)QT_D. \quad (B.3)$$

If $n = n$:

$$HC_2 = h\left(\frac{\sum_{i=1}^{n-1} i}{n^2}\right)(1-\theta)QT_D = h\left(\frac{\frac{n(n-1)}{2}}{n^2}\right)(1-\theta)QT_D = h\left(\frac{n-1}{2n}\right)(1-\theta)QT_D. \quad (B.4)$$

The demand period in Case I for different values of n is represented in Figure B.1.

Appendix C: Derivation of inventory holding cost during demand period for Case II. If $n = 1$:

$$HC_2 = 0. \quad (C.1)$$

If $n = 2$:

$$HC_2 = h\left(\frac{(1-\theta)I_{\max}}{2} \times \frac{T_D}{2}\right) = h\left(\frac{1}{2^2}\right)(1-\theta)I_{\max}T_D. \quad (C.2)$$

If $n = 3$:

$$HC_2 = h\left(\frac{2(1-\theta)I_{\max}}{3} \times \frac{T_D}{3} + \frac{(1-\theta)I_{\max}}{3} \times \frac{T_D}{3}\right) = h\left(\frac{1+2}{3^2}\right)(1-\theta)I_{\max}T_D. \quad (C.3)$$

If $n = n$:

$$HC_2 = h\left(\frac{\sum_{i=1}^{n-1} i}{n^2}\right)(1-\theta)I_{\max}T_D = h\left(\frac{\frac{n(n-1)}{2}}{n^2}\right)(1-\theta)I_{\max}T_D$$

$$HC_2 = h\left(\frac{n-1}{2n}\right)\frac{(1-\theta)\,I_{\max}^2}{D} = h\left(\frac{n-1}{2n}\right)\left(\frac{(1-\theta)\left[x\,(P-D)\,t_p - xDt_m\right]^2}{D}\right) = h\left(\frac{n-1}{2n}\right)$$
$$\left(\frac{(1-\theta)\left[\frac{Q}{(P-D)t_p}\,(P-D)\,t_p - \frac{Q}{(P-D)t_p}Dt_m\right]^2}{D}\right). \tag{C.5}$$

Box C.I

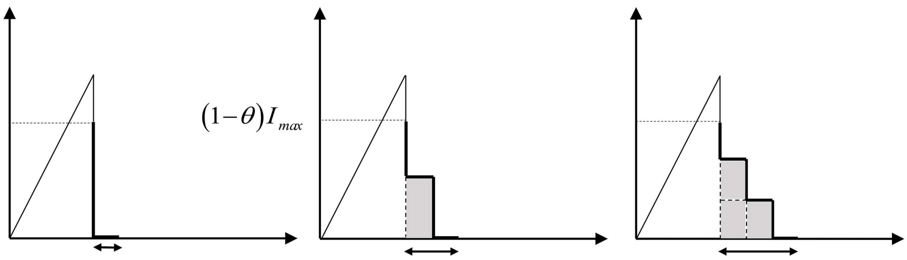

Figure C.1. The demand period in Case II.

$$= h\left(\frac{n-1}{2n}\right)(1-\theta)\,I_{\max}T_D. \tag{C.4}$$

We show that $T_D = \frac{I_{\max}}{D}$, $\mathrm{I}_{\max} = x\,(P-D)\,t_p - xDt_m$ and $x = \frac{Q}{Q_{sc}} = \frac{Q}{(P-D)t_p}$. So we have Eq. (C.5), which is shown in Box C.I. The demand period in Case II for different values of n is represented in Figure C.1.

Biographies

Ali Fallahi received his MSc degree in Industrial Engineering from the Department of Industrial Engineering, Sharif University of Technology in 2022. His research interests include radiation therapy treatment planning, inventory control, supply chain management, and optimization in healthcare. He has published several articles in reputable international journals, such as Computers & Industrial Engineering, Expert Systems with Applications, Applied Soft Computing, and Journal of Cleaner Production.

Mohammad Azimi-Dastgerdi received his BSc degree in Industrial Engineering from University of Kashan, Iran in 2019. His research interests include the applications of operations research to the inventory control problems.

Hadi Mokhtari is currently an Associate Professor of Industrial Engineering at University of Kashan, Iran. His current research interests include the applications of operations research and artificial intelligence techniques to the areas of project scheduling, production scheduling, manufacturing supply chains, and engineering economic problems. He has also published several papers in international journals such as Computers and Operations Research, International Journal of Production Research, Applied Soft Computing, Journal of Cleaner Production, Neurocomputing, International Journal of Advanced Manufacturing Technology, IEEE Transactions on Engineering Management, and Expert Systems with Applications.