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A three-level servicing contract with an integrated maintenance plan, warranty policy, technology level, and pricing model that considers the time value of money

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Technology level;
Time value of money.

Abstract. A common way to address customer concerns in the post-warranty period is to provide an extended warranty. Although sometimes the manufacturer is reluctant to offer an extended warranty, an agent takes on this task to maintain market share. For this purpose, the warranty policy is presented as a three-level servicing contract with the objectives of maximizing the manufacturer profit, the agent profit, and customer satisfaction. The model considers two approaches to controlling the number of product failures: (1) using the technology level used in manufacturing as an effective factor in product reliability, and (2) using non-periodic maintenance activities to maintain the product reliability at an acceptable level. To calculate the costs imposed on each side of the contract more accurately, the time value of money is considered in the calculation of financial flows. To illustrate the effectiveness of the approach, three comparative studies are provided. The first one shows the impact of the presence of the agent and the provision of an extended warranty period, while the second one proves the importance of preventive maintenance to reduce costs and increase the interests of each side. The results of the last one show the effect of considering the time value of money in calculating cash flows.

1. Introduction

Nowadays, business models have undergone many changes to meet the needs of customers, and organizations have to revise business processes and management strategies to maintain their markets. One of the tools that manufacturers use to increase market share and customer satisfaction is providing after-sale services during the warranty period, which leads to an assurance in customers minds. In most models, the warranty period is presented with a time limit, and the customer consumption rate is not considered. These models not only do not differentiate customers from each other but also bear high costs due to the high consumption of some customers. To address this weakness, the warranty period is thought to be two-dimensional in the sense that two-dimensional warranties take into account both age and consumption, as well as the potential combination of the two. Singpurwalla and Wilson [1] modeled the satisfaction of the customer and the manufacturer in terms of the number of product failures in the two-dimensional warranty period using game theory.

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Huang et al. [2] developed a two-dimensional warranty policy by considering a bivariate Weibull process to analyze the breakdown process of a repairable product simultaneously in terms of time and usage. Tong et al. [3] aim to determine two-dimensional warranty limits under one-dimensional preventive maintenance. Ye and Murthy [4] suggested a strategy that offers buyers a two-dimensional warranty menu to provide buyer-friendly warranties. Gertsbakh and Kordonsky [5], Chun and Tang [6], and Baik et al. [7] provided additional information on two-dimensional warranties.

Today, extended warranties are used as a solution to enhance customer assurance by covering possible product failures after the end of the basic warranty period. Wu and Longhurst [8] investigated the manufacturer costs during both basic and extended warranty periods.

In some circumstances, the manufacturer is not able to provide an extended warranty for some reasons, including restrictions on the circulating currency. To maintain customer assurance, the manufacturer allows the agent to take advantage of this period. In this regard, Esmaeili et al. [9] introduced the agent to support the manufacturer product during the post-warranty period under a three-level servicing contract. Asgharizadeh and Murthy [10] developed a stochastic model to study the impact of equipment reliability and the number of buyers being serviced on the agent optimal strategies using a game-theoretic formulation. Readers interested in extended warranties can study Bouguerra et al. [11], Chang and Lin [12], Lam and Lam [13], and Hartman and Laksana [14].

Performing maintenance activities and choosing the appropriate level of technology are known as two effective factors to reduce the cost of product failure. Maintenance policies can be divided into periodic and non-periodic categories. In the periodic *pm* policy, the time interval between each two consecutive *pm* activities is the same, while non-periodic maintenance policies aim to maintain the reliability of the product at an appropriate level. For more information on periodic *pm* policies, see Chien (2008), Salmasnia and Yazdekhasti [15], and Kim et al. [16], while non-periodic maintenance policies can be found in Park et al. [17], Su and Wang [18], and Huang et al. [19].

In spite of what is given in the previous paragraph, in many articles, maintenance is the only factor to reduce failures, while the fact that the product technology level also has a significant effect on the number of failures and the attraction of customer attention is ignored. In other words, when introducing a new product to the market, key technical variables such as design and reliability should be taken into account. In this regard, Darghouth et al. [20] developed a model to obtain optimal design, warranty, and price for products sold with a maintenance service contract. DeCroix [21] proposed a game theory-based model for companies that must set warranty policies, reliability parameters, and prices for their products.

Considering the time value of money is very important to calculate financial flows more accurately, especially in processes where the costs are imposed at different times. Articles by Luciano and Peccati [22], Van der Laan [23], Disney et al. [24], and Lin et al. [25] highlight the importance of the time value of money. Therefore, taking the net present value into account is necessary in the analysis of manufacturer, agent, and customer costs. However, according to our knowledge, the only reference in the warranty literature is Teng [26], who calculated the manufacturer cost under an extended warranty taking into account the time value of money. Table 1 summarizes and briefly introduces them to better illustrate the existing research gaps.

In order to fill the research gaps and overcome the above problems, a three-level model including the manufacturer profit, agent profit, and customer satisfaction under a two-dimensional warranty policy with the capability of being extended is developed. In addition, the appropriate technology level used in manufacturing the product and the implementation of non-periodic maintenance activities are taken as two approaches to reduce product failure. On the other hand, the time value of money is considered in the calculation of financial flows to improve the accuracy of calculating the costs imposed on the contract parties over time.

The rest of this study is as follows: In the next section, the problem definition and its assumptions are fully explained. In Section 3, a mathematical model is introduced, considering the level of technology used in manufacturing, a two-dimensional extended warranty, and the application of a non-periodic maintenance policy in a three-level service contract. In Section 4, the solution approach is illustrated. Section 5 is called "experimental results," which includes three parts: (1) a numerical example; (2) a sensitivity analysis; and (3) a comparative study. Eventually, the conclusions are expressed in Section 6.

2. Problem definition

In a competitive world, manufacturers shifted from providing warranties only in the time dimension to two-dimensional warranties in terms of both time and usage, so that each dimension that reaches its end point first ends the warranty, and this is a trick to reduce the warranty service provider cost. On the other hand, it is important to address the concerns of customers about the cost of the post-warranty period. To tackle this problem, the extended warranty has attracted the attention of many manufacturers as an attractive policy for customers. However, sometimes

Table 1. Literature review.

Ref.	Point of view			Non-periodic preventive	Time value of money	Tech.[b] level	2-D[c] warranty	Extended warranty	Optimization		Pricing	
	Cust.[a]	Agent	Manu-facturer						Single objective	Multiple objective	Product	Extended warranty
[21]			✓			✓			✓			
[6]	✓		✓						✓			
[13]	✓		✓					✓				
[7]			✓				✓					
[16]	✓		✓						✓			
[26]			✓		✓			✓	✓			
[34]	✓		✓						✓			
[35]			✓						✓			
[36]			✓				✓					
[14]	✓							✓				✓
[12]			✓						✓			
[11]	✓		✓					✓		✓		✓
[37]			✓						✓			
[9]	✓	✓	✓						✓		✓	✓
[18]				✓					✓			
[2]	✓		✓				✓		✓			
[19]			✓	✓			✓		✓			
[25]			✓			✓						
[17]			✓	✓					✓			
[15]	✓		✓							✓		
[20]			✓			✓			✓		✓	
[38]	✓		✓	✓			✓	✓	✓			
Present research	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓

a. Cust.: Custumer; b. Tech.: Technology; and c. 2-D: Two-dimensional.

the manufacturer is not willing to provide an extended warranty, and an external unit will be responsible for this task. According to what is stated, a two-dimensional warranty contract based on age and usage is provided, with three sides: manufacturer, customer, and agent.

As we know, the level of technology used in manufacturing the product has a significant effect on the cost of manufacturing, the number of failures, and reliability. Consequently, in this study, the technology level is considered a decision variable. Furthermore, it is assumed that the time-to-failure follows a Weibull distribution, which describes the proper behavior of the failure of mechanical products. According to what was previously mentioned, the number of product failures during its lifetime is affected by three factors: time, usage rate, and technology level. As a result, the failure rate can be expressed as in Figure 1.

Providing customers with a warranty imposes a considerable cost on the warranty service provider. An effective way to reduce the cost of providing services is to carry out maintenance activities. These activities are effective when they respond to the failure rate trend and are run to maintain product reliability at

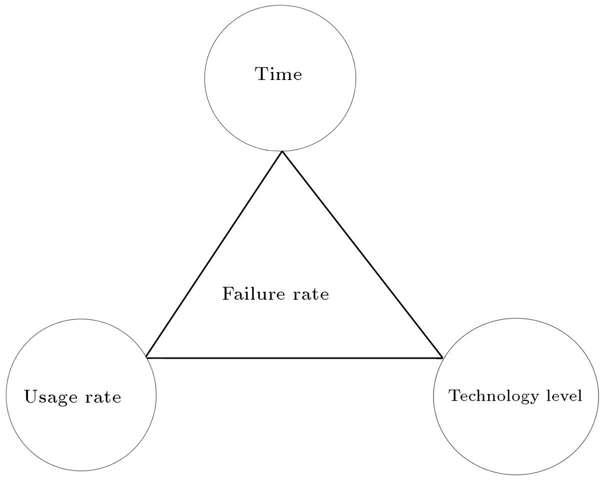

Figure 1. Factors affecting the failure rate.

an acceptable level during flexible intervals. As can be seen in Figure 2, non-periodic maintenance policy is presented in which the cost of activity is incurred by the manufacturer during the basic warranty period, while it is incurred by the agent during the extended warranty period. Also, with regard to the lengthy periods of basic and extended warranty, taking the time value of money into account is important for a more accurate calculation of the costs imposed on the manufacturer and the agent. As a result, Net Present Value (NPV) is used to improve the accuracy of calculating maintenance and repair costs.

Since the manufacturer profit and the agent profit are respectively related to product sales and extended warranty sales, on the other hand, customer satisfaction is reduced due to the increase in product purchase price and extended warranty. The model considers the product price and the extended warranty price as two decision variables to achieve a balance that maximizes the three objective functions.

2.1. Notations

Table 2 shows the notations used in the problem formulation prior to developing the presented mathematical model. Notations are classified into three types, as shown in Table 2: indices, decision variables, and parameters.

2.2. Problem assumptions

The assumptions in this problem are outlined as follows:

- The product can be repaired, and depreciation is caused by age and usage. In the absence of maintenance operations, the product failure rate increases.
- Product depreciation behavior can be explained by the Weibull process.
- All product failures during the warranty period are fixed with minimal repairs. This means that the reliability of the product after repairs is the same as that before the failure.
- Maintenance strategies are implemented by the manufacturer and the agent during the basic warranty and extended warranty, respectively.
- Preventive maintenance activities are imperfect, which means the repaired product is better than before the failure and worse than the new product.
- All failures during the warranty period are rectified by a minimal repair, and the customer does not pay any fees.
- The time required to repair a defective product, like the time required for preventive maintenance, is insignificant and can be ignored.
- Because of market conditions, the price and warranty period remain constant throughout the product life cycle.
- The extended warranty period runs from the end of the basic warranty period to the end of the product useful lifetime.

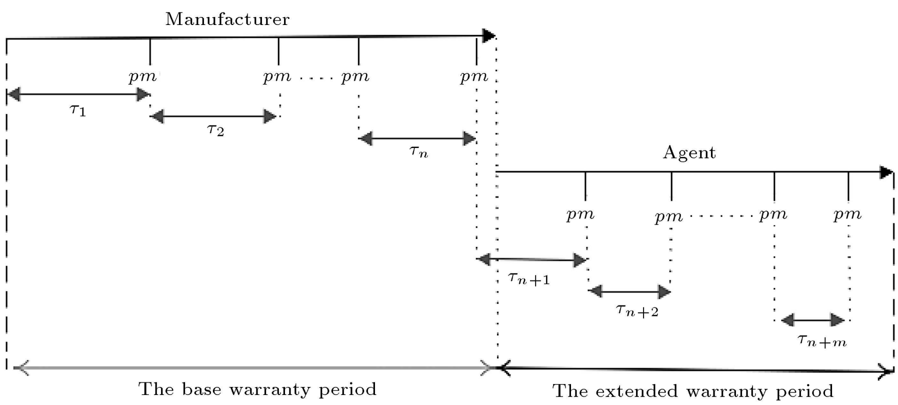

Figure 2. Illustration of the relationship between the manufacturer and the agent in the provision of warranty service and non-periodic maintenance policy.

Table 2. Notations.

Notations	Descriptions
Index:	
j	Index of preventative activity
Decision variables:	
m	Preventive maintenance level, $0 \leq m < M$, and an integer
τ_1	Time of the first non-periodic maintenance activity
θ	Technology level
PW_E	Extended warranty price
P_P	Sale price of the product
Parameters:	
L, K	Lifetime and lifetime use of the product
U, W	Age and usage limits of a two-dimensional warranty
$v(t)$	Virtual age after implementing a pm activity
$\delta(m)$	pm age reduction factor with level m
ir	Interested rate
$f(t\|r,\theta)$	Time to failure density function
$F(t\|r,\theta)$	Time to failure cumulative distribution function
$\beta(r,\theta)$	Scale parameter for Weibull distribution as a function of r and θ
$\lambda(t,\|r,\theta)$	Conditional failure rate
$E[N_Y\|r]$	The expected number of failures per unit item conditional on $R = r$
$E[CW_Y\|r]$	The expected warranty servicing cost per unit item conditional on $R = r$
RSC_X	The average revenue of the service contract under option X
$Cp_j(m)$	The cost of the jth pm action at level m
C_p	Production cost
r	usage rate ($\underline{u}$)
Π_l	Manufacturer and agent ($l = M, A$) profit
Π_c	Customer satisfaction

3. Model description

In this section, firstly, the failure rate function is introduced, and then the cost of each preventive maintenance item is calculated. Afterwards, the costs imposed on the manufacturer and agent during the two-dimensional basic and extended warranty periods are obtained. The values of decision variables, including product price P_P, the price of an extended warranty PW_E, the design variable θ, time of the first non-periodic maintenance activity τ_1, and maintenance activity level m, as well as the profit functions of manufacturer, agent, and customer satisfaction, are then obtained.

3.1. Modeling product failure based on technology level

Since the usage rate r is a non-negative random variable and varies from one customer to the next, a uniform distribution function $G(r), g(r)$ is considered for this variable. Also, since modeling is done for mechanical products, it is assumed that the time to failure follows a Weibull distribution function. In other words, the number of product failures follows a Non-Homogeneous Poisson Process (NHPP), which means the product failure rate is an ascending function of time and customer usage rate. In addition, it is a known fact that the failure rate is a function of the technology level used to produce the product. Therefore, the scale parameter of the Weibull distribution is considered a function of the technology level θ and the usage rate of r. This concept is inspired by Darghouth et al. [20] and is explained as follows:

$$\beta(r,\theta) = \frac{\gamma_0}{\gamma_1 r + \gamma_2 \theta}, \tag{1}$$

where γ_0, γ_1, and γ_2 parameters have positive values, which can be estimated by means of historical data from recorded warranty information in the factory. Finally, the failure rate can be expressed as follows:

$$\lambda(t) = \frac{f(t)}{1 - F(t)},$$

$$\lambda(t\,|r,\theta) = \frac{f(t\,|r,\theta)}{1 - F(t\,|r,\theta)} = \frac{\alpha}{\beta(r,\theta)}\left(\frac{t}{\beta(r,\theta)}\right)^{\alpha-1}. \tag{2}$$

3.2. Modeling the imperfect maintenance strategy

The concept of "virtual age" is used in this study to illustrate the effect of activities, which was first introduced by Kijima et al. [27] and then developed by Kijima [28]. On the other hand, in order to maintain the reliability of the product at a predetermined level, a non-periodic maintenance policy is used in which the number of failures of the product during the pm intervals remains constant. Thus, the time for performing the jth pm is as follows:

$$\int_{\tau_{j-1}}^{\tau_j} \lambda(v_{j-1}+t-\tau_{j-1})dt$$

$$=\int_0^{\tau_1}\lambda(t)dt\int_{\tau_{j-1}}^{\tau_j}\frac{\alpha}{\beta}\left(\frac{v_{j-1}+t-\tau_{j-1}}{\beta}\right)^{(\alpha-1)}dt$$

$$=\int_0^{\tau_1}\frac{\alpha}{\beta}\left(\frac{t}{\beta}\right)^{(\alpha-1)}dt\frac{\alpha}{\beta}\left(\frac{1}{\beta}\right)^{(\alpha-1)}$$

$$\int_{\tau_{j-1}}^{\tau_j}(v_{j-1}+t-\tau_{j-1})^{(\alpha-1)}dt$$

$$=\frac{\alpha}{\beta}\left(\frac{1}{\beta}\right)^{(\alpha-1)}\int_0^{\tau_1}(t)^{(\alpha-1)}dt$$

$$[(v_{j-1}+t-\tau_{j-1})^{\alpha}]_{\tau_{j-1}}^{\tau_j}=[t^{\alpha}]_0^{\tau_1}$$

$$\Rightarrow (v_{j-1}+\tau_j-\tau_{j-1})^{\alpha}-{v_{j-1}}^{\alpha}={\tau_1}^{\alpha}$$

$$\Rightarrow v_{j-1}+\tau_j-\tau_{j-1}=({\tau_1}^{\alpha}+{v_{j-1}}^{\alpha})^{\frac{1}{\alpha}}, \quad (3)$$

where $v_{j-1}+t-\tau_{j-1}$ represents the virtual age and τ_{j-1} denotes the actual age of the product in $(j-1)$th maintenance activity. And finally:

$$\tau_j=({\tau_1}^{\alpha}+{v_{j-1}}^{\alpha})^{\frac{1}{\alpha}}-v_{j-1}+\tau_{j-1}. \quad (4)$$

As a result, for a given pm level, the product's virtual age after jth PM can be calculated as follows:

$$\nu_1=\delta(m).\tau_1$$

$$\nu_j=\nu_{j-1}+\delta(m)(\tau_j-\tau_{j-1}) \quad j\geq 2, 0\leq m\leq M, \quad (5)$$

where $\delta(m)$ denotes the age-reduction coefficient and is a descending function of m as $\delta(m)=(1+m)e^{-m}$. So, $\delta(m)$ can vary in the interval $[0,1]$, and a higher value of m signifies a lower $\delta(m)$. In particular, for an infinite maintenance effort M (impossible to achieve in practice), the product is restored to as good as it was following the last pm $\delta(\infty)=0$. Hence, this model does not allow pm to be perfect. If $m=0$, then $\delta(0)=1$, $\nu_j=\tau_j, j\geq 1$, and practically PM activity has been rendered ineffective. In general, if $0<m<M$, the item is partially repaired and the failure rate is reduced.

In this research, it is assumed that PM level is fixed during the warranty period. In this situation, Eq. (5) can be rewritten recursively as follows.

$$\nu_j=\nu_{j-1}+\delta(m)(\tau_j-\tau_{j-1})$$

$$=\nu_{j-2}+\delta(m)(\tau_j-\tau_{j-2})$$

$$=...$$

$$=\nu_0+\delta(m)(\tau_j-\tau_0). \quad (6)$$

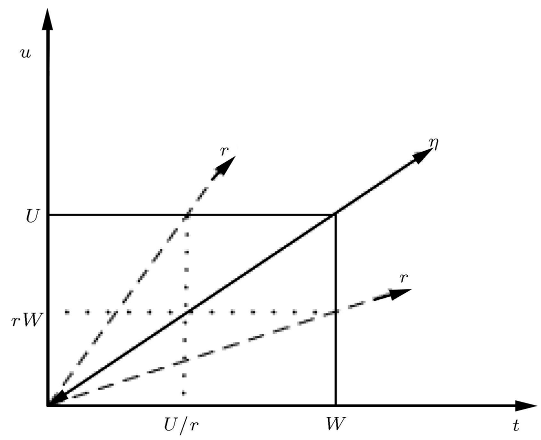

Figure 3. Two-dimensional warranty policy.

3.3. Warranty period ranges

In this paper, a rectangular two-dimensional warranty with $[0,W]\times[0,U]$ limits, as can be seen in Figure 3, is used. In other words, the warranty expires when the product true age reaches W, or its true usage reaches U (whichever occurs first). Consequently, two modes, $r\leq\eta$ and $r>\eta$, are selected according to the customer usage rate.

Based on Figure 3, if the usage rate is $r>\eta$, usage limit comes sooner than the time limit, while if the usage rate is $r\leq\eta$, the time limit comes sooner than the usage limit, and the warranty period finishes.

3.4. Preventive maintenance cost

With regard to Darghouth et al. [20], for a given maintenance activity at level m, the PM cost is calculated by means of Eq. (7):

$$Cp_j(m)=a+b(m)+c_j, \quad j=1,2,... . \quad (7)$$

where a denotes the fixed cost and $b(m)$ denotes the contribution of the improvement level m to the pm cost, obtained as follows:

$$b(m)=\frac{\beta'.m}{1-e^{(-\phi[M-m])}}\to\beta'>0,$$

$$j=1,2,..., \quad \beta'>0, \quad \phi>0, \quad 0\leq m\leq M, \quad (8)$$

where β' and ϕ are two positive constants. According to Eq. (8), if $m=0$, no improvement has been made, and it results in $b(m)=0$. And if $m\to M$ then $b(m)\to\infty$, implying that the product is almost a new one. c_j denotes the cost of gaining knowledge and practical skills necessary for doing maintenance activities. By introducing this parameter, the effect of learning on calculating the cost of each pm activity is actually reflected. In fact, with each PM activity, the costs of PM providers will reduce due to the increase in experience. According to Wright [29], these costs can be expressed in the form of a learning curve:

$$c_j = c_1.j^{\xi} \rightarrow \xi = \frac{\log^{0.8}}{\log^{2}}, \qquad j = 1, 2, \dots , \tag{9}$$

where c_1 is a fixed value denoting the learning cost for the first pm activity. It should be noted that parameters c_j and a are not affected by the level.

Finally, non-periodic PM during the basic warranty is calculated using Eq. (10), which takes into account the time value of money:

$$n_{pm_1} = \max\left\{ j \left| \tau_j < W, j \geq 0 \right.\right\} r \leq \eta,$$

$$n_{pm_2} = \max\left\{ j \left| \tau_j < \frac{U}{r}, j \geq 0 \right.\right\} r > \eta,$$

$$C_{pm_x}(m) = \sum_{j=1}^{n_{pm_x}} Cp_j(m) \times e^{-ir\tau_j}, \qquad x = 1, 2. \tag{10}$$

3.5. Non-periodic preventive maintenance strategy in a two-dimensional warranty space

To reduce the number of failures during the warranty period, a non-periodic preventive maintenance policy is implemented.

According to the equation $u = rt$ and warranty limits, the reference usage rate can be defined as $\eta = \frac{U}{W}$, which results in two modes:

1. $r \leq \eta$: As shown in Figure 4, the warranty period for this option ends when the actual age and usage reach W and rW, respectively.

   In this option, the number of PM activities in the warranty period is:

   $$n_{pm_1} = \max\left\{ j \left| \tau_j < W, j \geq 0 \right.\right\}. \tag{11}$$

   In other words, maintenance activities are carried out n_{pm_1} times in the warranty period, and after doing n_{pm_1}th non-periodic activities at the end of the warranty period, the warranty provider does no extra PM.

   As mentioned before, it is assumed that time-to-failure follows a Weibull distribution. Using Eqs. (1) and (5), the expected number of failures for a constant usage rate under the condition $r \leq \eta$, during the warranty period, can be calculated by:

   $$E\left[N(\Omega) \left| r \leq \eta \right.\right] = \sum_{j=1}^{n_{pm_1}-1} \int_{\nu_{j-1}}^{\nu_j} \lambda\left(t \left| r, \theta \right.\right) dt + \int_{\nu_{n_{pm}}}^{\nu_{n_{pm}}+W-\nu_{n_{pm}}} \lambda\left(t \left| r, \theta \right.\right) dt. \tag{12}$$

   Given the time value of money, the expected cost of warranty service for a fixed usage rate on the condition of $r \leq n$ is obtained by the following equation:

   $$E\left[C_{WB} \left| r \leq \eta \right.\right] = C_r \sum_{j=1}^{n_{pm_1}} \int_{\nu_{j-1}}^{\nu_j} \lambda\left(t \left| r, \theta \right.\right) e^{-irt} dt + C_r \int_{\nu_{n_{pm_1}}}^{\nu_{n_{pm_1}}+W-\tau_{pm_1}} \lambda\left(t \left| r, \theta \right.\right) e^{-irt} dt + C_{pm_1}(m). \tag{13}$$

2. $r > \eta$: According to Figure (5), the time and usage of warranty termination for this option are $\frac{U}{r}$ and U, respectively.

In addition, Eq. (14) is used to calculate the number of non-periodic preventive maintenance activities performed during the warranty period.

$$n_{pm_2} = \max\left\{ j \left| \tau_j < \frac{U}{r}, j \geq 0 \right.\right\}. \tag{14}$$

Therefore, the expected number of failures for a fixed usage rate during the warranty period is based on

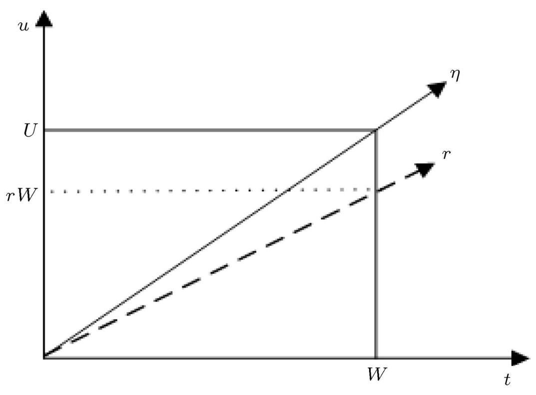

Figure 4. Warranty period when $r \leq \eta$.

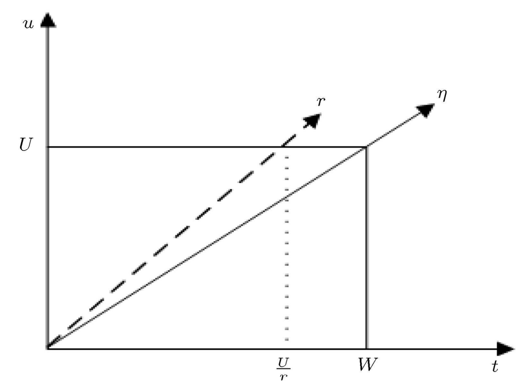

Figure 5. Warranty period when $r > \eta$.

the condition that $r > \eta$, which is obtained by:

$$E\left[N_{WB}\left|r>\eta\right.\right]=\sum_{j=1}^{n_{pm_2}}\int_{\nu_{j-1}}^{\nu_j}\lambda\left(t\left|r,\theta\right.\right)dt+\int_{\nu_{n_{pm_2}}}^{\nu_{n_{pm_2}}+\frac{U}{r}-\tau_{pm_2}}\lambda\left(t\left|r,\theta\right.\right)dt. \tag{15}$$

The expected cost of the warranty service for a fixed usage rate on the condition that $r > \eta$ and based on the time value of money for this option is equal to:

$$E\left[C_{WB}\left|r>\eta\right.\right]=C_r\sum_{j=1}^{n_{pm_2}}\int_{\nu_{j-1}}^{\nu_j}\lambda\left(t\left|r,\theta\right.\right)e^{-irt}dt+C_r\int_{\nu_{n_{pm_2}}}^{\nu_{n_{pm_2}}+\frac{U}{r}-\tau_{pm_2}}\lambda\left(t\left|r,\theta\right.\right)e^{-irt}dt+C_{pm_2}(m), \tag{16}$$

$$E\left[C_{WB}\right]=\int_0^{\eta}E\left[C_{WB}\left|r\le\eta\right.\right]g(r)dr+\int_{\eta}^{\infty}E\left[C_{WB}\left|r>\eta\right.\right]g(r)dr. \tag{17}$$

Ultimately, the total expected cost of services is calculated using Eq. (17).

3.6. Mathematical modeling of the agent

As mentioned earlier, for the cost of extended warranty services in the proposed model, the agent is in charge. To calculate this cost, first, the warranty cost for the entire lifetime $[0, W + W_E] \times [0, U + U_E]$ of the product is obtained using Eqs. (11) to (17), then the cost of the basic warranty period is subtracted from it.

$$E\left[C_{WL}\right]=\begin{cases}E\left[C_{WL}\right] & r\le\eta\\ E\left[C_{WL}\right] & r>\eta\end{cases},\qquad \eta=\frac{L}{K}$$

$$E\left[C_{WE}\right]=E\left[C_{WL}\right]-E\left[C_{WB}\right]. \tag{18}$$

Agent revenue is derived from the sale of an extended warranty, which is a decision variable and is expressed as follows:

$$RSC_A=PW_E. \tag{19}$$

The time of warranty termination is equal to W and $\frac{U}{r}$ in the modes $r \le \eta$ and $r > \eta$, respectively. In other words, in accordance with Figure 6, the warranty period ends when the production age reaches $\min\left\{W, \frac{U}{r}\right\}$. The lifetime of the product is similarly formulated as $\min\left\{W_{life}, \frac{U_{life}}{r}\right\}$ when W_{life} and U_{life}

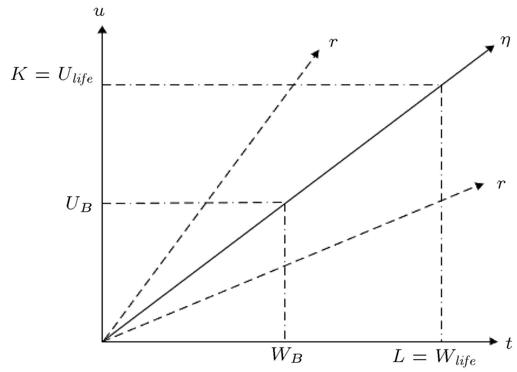

Figure 6. Two-dimensional warranty and lifetime policy.

as can be seen in the figure denote product lifecycle limits in two dimensions of age and usage, respectively.

When the extended warranty is not provided during the post-base warranty period, the costs imposed on the customer in the absence of the agent are calculated by Eq. (20):

$$E\left[C_{N.P}\right]=C_r\left(1+\alpha_e\right)\int_{r_{\min}}^{r_{\max}}\int_{\min\left\{W,\frac{U}{r}\right\}}^{\min\left\{W_{life},\frac{U_{life}}{r}\right\}}\lambda\left(t\left|r,\theta\right.\right)g(r)dtdr. \tag{20}$$

According to these equations, agent income must comply with Constraint (21):

$$RSC_A<COSTNOPM. \tag{21}$$

The expected agent benefit is equal to the subtraction of the extended warranty costs from the revenue resulting from selling the extended warranty. It is modeled as follows:

$$\prod_A{}'=RSC_A-E\left[C_{WE}\right]$$

$$\prod_A=(1-\rho)\times\prod_A{}'\qquad 0\le\rho<1, \tag{22}$$

where ρ is the percentage of the agent profit retained by the manufacturer as a concession for providing the warranty.

3.7. Manufacturer mathematical modeling

As previously stated, the manufacturer income per product is determined by two factors of the product price and the agent profit as follows:

$$RSC_M=P_P+\rho\times\prod_A{}'\qquad 0\le\rho<1. \tag{23}$$

The cost of manufacturing the product is proportional to its technology level, so the higher the technology level used in manufacturing, the higher the cost of manufacturing.

$$C_P = \psi_0 + \psi_1 e^{\left(\frac{\theta_{\max}-\theta}{\theta-\theta_{\min}}\right)}, \tag{24}$$

where $\theta_{\min}$ and $\theta_{\max}$ are design variable limits and parameters ψ_0 and ψ_1, respectively, represent the product fixed manufacturing cost and sensitivity coefficient of the design variable.

The cost of the manufacturer warranty services is calculated using Eqs. (11) to (17):

$$E\left[C_{WB}\right] = \begin{cases} E\left[C_{WB}\right] & r \le \eta \\ E\left[C_{WB}\right] & r > \eta \end{cases}, \qquad \eta = \frac{U_B}{W_B}. \tag{25}$$

The manufacturer expected profit can be obtained by subtracting the cost of manufacturing and the provision of the basic warranty from the income from selling the product and giving the concession to the agent. It is modeled as follows:

$$\prod_M = RSC_M - E\left[C_{WB}\right] - C_P. \tag{26}$$

3.8. Mathematical modeling for customer

Customer satisfaction can be considered a function of the design variable, which means that the higher technology level leads to more customer revenue based on the equation $R = \varphi.\frac{1}{\theta}$, where φ is the technology level sensitivity coefficient of the product and its multiplication by the level of product manufacturing technology ($\frac{1}{\theta}$) results in the customer revenue per time unit. By multiplying this number by the average product availability time for the customer, the customer revenue over the product life time is calculated, and because the repair times are assumed to be zero or very close to zero, the availability time can be considered to be equal to the product life time. It is mathematically expressed as follows:

$$\omega(r) = \min\left\{\frac{U_B + U_E}{r}, W_B + W_E\right\},$$

$$L = \int_{r_{\min}}^{r_{\max}} \omega(r).g(r)dr, \tag{27}$$

$$RSC_C = R.L.$$

The expected customer satisfaction is equal to the subtraction of the cost of purchasing the product and the extended warranty from the customer revenue. This is modeled as follows:

$$\prod_C = RSC_C - Pp - PW_E. \tag{28}$$

3.9. The aggregation approach based on desirability concept

Given that the goal of this problem is to maximize the manufacturer profit, the agent profit, and customer satisfaction all at the same time, the desirability function approach proposed by Salmasnia et al. [30] is used. In this approach, each objective function is converted to a scale-less value in the interval in the form of $d(\prod_i)$, $i = M, A, C$, which is called desirability. It increases as the corresponding objective value increases.

$$\mathrm{d}\left(\prod_i\right) = \begin{cases} 1 & Z_i \ge Z_i^{\max} \\ \frac{Z_i - Z_i^{\min}}{Z_i^{\max} - Z_i^{\min}} & Z_i^{\min} < Z_i < Z_i^{\max} \\ 0 & Z_i \le Z_i^{\min} \end{cases}$$

$$i = M, A, C. \tag{29}$$

In Eq. (29), $Z_i^{\max}$ and $Z_i^{\min}$ denote the acceptable maximum and minimum values for the ith objective function, respectively. In this approach, $Z_i^{\max}$ is calculated solely by maximizing the objective function, and $Z_i^{\min} = \psi_i \times Z_i^{\max}$, $i = M, A, C$ is considered, where ψ_i is a coefficient in the interval and is selected based on the opinion of the decision maker.

To integrate three desirability functions into a single phrase, the geometric mean (Eq. (30)) is used. Clearly, D can vary in the interval $[0, 1]$, and D increases as the balance of the objective functions becomes more favorable. The other important feature of Eq. (30) is that if any $d\left(\prod_i\right) = 0$ (that is, if one of the objective functions is unacceptable), then $D = 0$.

$$D = \mathrm{Max}\left[d\left(\prod_M\right) \times d\left(\prod_A\right) \times d\left(\prod_C\right)\right]. \tag{30}$$

4. Solution approach

The model presented in Section 3, due to the fact that decision variables in the objective function are in integral bounds, has a complexity that does not allow it to be solved by means of exact methods. Meta-heuristic algorithms are one of the best methods for solving problems of high complexity because they can achieve suitable solutions in reasonable times.

In this paper, in order to achieve optimal values of decision variables, Particle Swarm Optimization (PSO) is implemented because of its good performance in optimizing non-linear models, unique searching mechanism, simplicity in concept, computational efficiency, and ease of implementation. It is one of the most popular meta-heuristic algorithms that recently has been widely applied by some researchers such as Salmasnia et al. [31,32].

The solution representation is a key factor in developing the PSO algorithm, which could be a string of both integers and real numbers. The solution representation for the proposed model includes five-dimensional vectors, where each dimension refers to a certain decision variable. The preventive maintenance level (m) in the presented model is an integer, while the decision variables, including product price P_P, price of extended warranty PW_E, design variable θ, and time of the first non-periodic maintenance activity τ_1 are real numbers. For generating the initial value of each continuous decision variable, a uniformly distributed random value is generated between the lower and upper limits of the considered decision variable. Furthermore, in order to generate an initial value for discrete decision variables, a random value from a uniform distribution in the interval [0, 1] is generated. The values of discrete variables, i.e., the preventive maintenance level (m), are obtained according to Eq. (31):

$$m = \min(n_{\min} + \text{floor}((m_{\max} - m_{\min} + 1) \times R), m_{\max}), \tag{31}$$

where $m_{\min}$ and $m_{\max}$ are the lower and upper limits of m, respectively. Furthermore, R is a random number in the interval [0, 1] drawn from a uniform distribution.

5. Experimental results

The aim of this study is to maximize the manufacturer profit, the agent profit, and customer satisfaction under a three-level servicing contract. In this section, a numerical example is presented to demonstrate the applicability of the proposed model. The rest of this section is divided into three parts as follows. Section 5.1 presents the mathematical programming and the assigned values to model parameters. Then, the PSO algorithm is used to optimize the model. In Section 5.2, the effects of parameters on the objective functions are analyzed. Section 5.3 includes three comparisons to demonstrate the effectiveness of the proposed model.

5.1. A numerical example

Assume a product for which the manufacturer offers a two-dimensional basic warranty and the agent provides a two-dimensional extended warranty. To control the number of failures, on the one hand, a non-periodic maintenance policy is used, and on the other, the technology level used in manufacturing the product is considered. The values of parameters that are extracted from Esmaeili et al. [9] and Darghouth et al. [20] are shown in Table 3.

In terms of model complexity, this problem is solved in MATLAB 2017 using the PSO algorithm, and the optimized results are shown in Table 4.

Based on the obtained results, the design variable used in manufacturing is suggested to be $\theta = 0.2442$, and the first maintenance activity is implemented at time $\tau_1 = 0.2124$ with level $m = 4$. In addition, the price at which the product is sold must be $P_P = 9.7731$, and the price at which the extended warranty is sold must be $PW_E = 2.5976$, to maximize the objective functions.

5.2. Sensitivity analysis

Sensitivity analysis is very common in the domain of optimization. In general, it is the task of understanding the behavior of the final solution to an optimization problem because of changes in the input parameters [33]. In this section, the effect of the parameters C_r, β', U_B, W_B, U_{life}, and W_{life} on three objective functions is investigated. In this regard, Taguchi experimental design L_{27} is used as a tool for generating problems by making targeted changes in the model parameters. For example, C_r takes three values of 0.25, 0.3, and 0.35. As can be seen in Table 5, the parameters take each of their levels into account in 9 different problems generated by Taguchi L_{27}. In problem sets (1, 2, 3, 16, 17, 17, 18, 22, 23, 23, 24), (4, 5, 6, 10, 10, 11, 11, 12, 25, 26, 26, 27) and (7, 8, 9, 9, 13, 14, 14, 15, 19, 19, 20, 21), for example, the U_B parameter has low, medium, and high levels. It is noteworthy that the values of the

Table 3. The values of the parameters in the numerical example.

parameter	α	γ_0	γ_1	γ_2	a	ϕ	c_1
Value	2	0.23	0.8	0.5	0.002	0.1	0.005
Parameter	ξ	α_e	ir	ψ_0	ψ_1	$\theta_{\min}$	$\theta_{\max}$
Value	-0.35	0.28	0.15	1.5	1.5	0.1	0.4
Parameter	φ	$Z_M^{\max}$	$Z_A^{\max}$	$Z_C^{\max}$	$Z_M^{\min}$	$Z_A^{\min}$	$Z_C^{\min}$
Value	0.85	10.6641	4	39.022	1.5	0.5	1
Parameter	C_r	β'	U_B	W_B	U_{life}	W_{life}	
Value	0.3	0.005	3	3	6	6	

Table 4. Values of decision variables and objective functions.

Decision variable	P_P	PW_E	θ	τ_1	m
Value	9.7731	2.5976	0.2442	0.2124	4
Objective function	Π_M		Π_A		Π_C
Value	3.6031		2.6022		3.1012

Table 5. Problems generated by Taguchi design L_{27}.

Instance	C_r	β'	U_B	W_B	U_{life}	W_{life}	Value of objective functions		
							Agent profit	Manufacturer profit	Customer satisfaction
1	0.25	0.002	2	2	4.5	4.5	1.1768	2.5747	1.1789
2	0.25	0.002	2	2	6.0	6.0	2.6867	4.0881	2.6854
3	0.25	0.002	2	2	7.5	7.5	3.9912	5.3885	3.994
4	0.25	0.005	3	3	4.5	4.5	1.3744	2.7723	1.3763
5	0.25	0.005	3	3	6.0	6.0	2.6429	4.0443	2.6415
6	0.25	0.005	3	3	7.5	7.5	3.9538	5.351	3.9565
7	0.25	0.008	4	4	4.5	4.5	1.0092	2.9939	1.5207
8	0.25	0.008	4	4	6.0	6.0	2.5283	3.9283	2.5285
9	0.25	0.008	4	4	7.5	7.5	3.9166	5.3138	3.9192
10	0.3	0.002	3	4	4.5	6.0	1.7962	3.1977	1.7947
11	0.3	0.002	3	4	6.0	7.5	3.2097	4.6065	3.2127
12	0.3	0.002	3	4	7.5	4.5	2.112	3.5097	2.1143
13	0.3	0.005	4	2	4.5	6.0	1.5712	2.9691	1.5732
14	0.3	0.005	4	2	6.0	7.5	3.168	4.566	3.1716
15	0.3	0.005	4	2	7.5	4.5	1.8241	3.2249	1.8232
16	0.3	0.008	2	3	4.5	6.0	1.5188	2.919	1.5186
17	0.3	0.008	2	3	6.0	7.5	3.1292	4.5237	3.1303
18	0.3	0.008	2	3	7.5	4.5	2.2497	3.6469	2.2525
19	0.35	0.002	4	3	4.5	7.5	2.2651	3.664	2.2662
20	0.35	0.002	4	3	6.0	4.5	1.8028	3.2014	1.8042
21	0.35	0.002	4	3	7.5	6.0	3.2579	4.568	3.2579
22	0.35	0.005	2	4	4.5	7.5	2.2191	3.618	2.2201
23	0.35	0.005	2	4	6.0	4.5	1.92	3.3204	1.9196
24	0.35	0.005	2	4	7.5	6.0	3.2171	4.6171	3.217
25	0.35	0.008	3	2	4.5	7.5	2.1968	3.5952	2.1986
26	0.35	0.008	3	2	6.0	4.5	1.4601	2.86	1.4602
27	0.35	0.008	3	2	7.5	6.0	3.1763	4.5764	3.1764

other parameters in each of the twenty-seven problems remain constant according to Table 3.

Tables 6 through 8 show the average of the obtained values for agent profit, manufacturer profit, and customer satisfaction in the twenty-seven instances for each level of each parameter. For example, the number 3.866511 in Table 6 is the average value of the manufacturer profit in instances 1, 2, 3, 10, 11, 12, 19, 20, and 21, in which the parameter β' is at its first level. Δ is the difference between the obtained maximum and minimum values for three levels of each parameter. Finally, the Δ values are ranked, and the rank line reports the effect of the parameters on the objective function in ascending order.

According to Table 6, when product lifetime W_{life} increases, three objective functions increase as well. Because the increase in parameter W_{life} directly affects customer satisfaction in accordance with Eq. (28), an increase in customer satisfaction to achieve a balance in three objective functions leads to an increase in

Table 6. Sensitivity analysis of the parameters on the agent profit.

Level	C_r	β'	U_B	W_B	U_{life}	W_{life}
1	2.586656	2.4776	2.456511	2.361244	1.658789	1.658789
2	2.286544	2.432289	2.4358	2.466067	2.488378	2.488378
3	2.390578	2.353889	2.371467	2.436467	3.077633	3.116611
Δ	0.300111	0.123711	0.085044	0.104822	1.418844	1.457822
Rank	3	4	6	5	2	1

Table 7. Sensitivity analysis of the parameters on the manufacturer profit.

Level	C_r	β'	U_B	W_B	U_{life}	W_{life}
1	4.050544	3.866511	3.855156	3.760322	3.122689	3.122689
2	3.684833	3.831456	3.834789	3.854511	3.878667	3.878667
3	3.780056	3.817467	3.825489	3.9006	4.466256	4.514078
Δ	0.365711	0.049044	0.029667	0.140278	1.343567	1.391389
Rank	3	5	6	4	2	1

Table 8. Customer satisfaction parameter sensitivity analysis.

Level	C_r	β'	U_B	W_B	U_{life}	W_{life}
1	2.644556	2.4787	2.457378	2.362389	1.716656	1.716656
2	2.2879	2.555167	2.4368	2.467111	2.488133	2.488133
3	2.391133	2.411667	2.429411	2.494089	3.079	3.1188
Δ	0.356656	0.1435	0.027967	0.1317	1.362344	1.402144
Rank	3	4	6	5	2	1

product sales price and the extended warranty price, and eventually the agent and manufacturer profit are also affected and increased in accordance with Eqs. (22) and (26), respectively. The rank lines in Tables 6, 7, and 8 confirm that, when compared to the other parameters, W_{life} has the greatest impact and U_B has the least effect on the three objectives of manufacturer profit, agent profit, and customer satisfaction over a lifetime.

5.3. Comparative study

In this section, three comparative studies to illustrate the effectiveness of the suggested model are presented. In the first comparative study, the effect of the presence of an agent and the provision of an extended warranty on the profit of the manufacturer and the customer is investigated. In the second comparative study, the effect of carrying out preventive maintenance activities is analyzed. Finally, in the final study, the effect of accounting for the time value of money on the calculation of the costs imposed on each side of the contract is investigated.

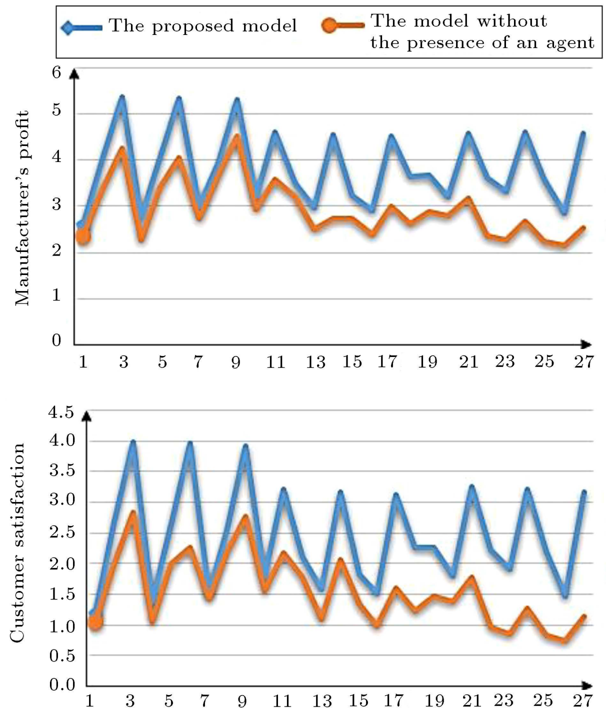

Figure 7. Graphical representation of the improvement percentage in Comparative study 1.

Comparison study 1: Agent presence in extended warranties

One of customers concerns is the cost of the post-warranty period. When the product age increases, the number of failures increases as well, and as a result, more repair costs are imposed on the customer. In this case, the extended warranty is a customer-friendly offer to reduce customer concerns. However, in some circumstances, the manufacturer is not able to provide this period because of the limited circulation of funds. As a result, the presence of an agent is planned to reduce concerns. This comparative study examines the importance of the presence of an agent in warranty contracts. In this regard, the proposed model is contrasted with the model without an agent.

As it was expected, based on Figure 7, the proposed mathematical model has better performance in all three objective functions compared to the model without the agent. Table 9 shows the situation in which the agent is not present for the extended warranty and maintenance activities are not performed during the post-warranty period. This leads to an increase in the number of product failures. To strike a balance

Table 9. Comparison of the proposed model with the model in which the agent is absent.

Model	**Instance**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
The proposed model	Manufacturer profit	2.57	4.08	5.38	2.77	4.04	5.35	2.99	3.92	5.31
	Customer satisfaction	1.17	2.68	3.99	1.37	2.64	3.95	1.52	2.52	3.91
The model without the presence of an agent	Manufacturer profit	2.35	3.39	4.24	2.25	3.41	4.057	2.73	3.67	4.53
	Customer satisfaction	1.04	1.99	2.84	1.05	2.00	2.25	1.41	2.16	2.77
Improvement percentage of the manufacturer profit		9.56	20.27	26.97	23.13	18.57	31.87	9.57	6.82	17.11
Improvement percentage of the customer satisfaction		12.38	34.478	40.292	30.85	31.52	75.190	7.530	16.649	41.003
Model	**Instance**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
The proposed model	Manufacturer profit	3.19	4.6	3.5	2.96	4.56	3.22	2.91	4.52	3.64
	Customer satisfaction	1.79	3.21	2.11	1.57	3.17	1.82	1.51	3.13	2.25
The model without the presence of an agent	Manufacturer profit	2.94	3.57	3.19	2.5	2.73	2.73	2.39	3.01	2.62
	Customer satisfaction	1.55	2.17	1.78	1.09	2.05	1.33	0.99	1.61	1.22
Improvement percentage of the manufacturer profit		8.499	28.964	9.9116	18.76	66.86	17.851	22.118	50.254	38.82904
Improvement percentage of the customer satisfaction		15.281	47.676	18.561	43.57	5 4.02	36.959	53.611	94.332	84.042
Model	**Instance**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
The proposed model	Manufacturer profit	3.66	3.20	4.56	3.61	3.32	4.61	3.59	2.86	4.57
	Customer satisfaction	2.26	1.8	3.25	2.22	1.91	3.21	2.19	1.46	3.17
The model without the presence of an agent	Manufacturer profit	2.87	2.77	3.18	2.35	2.25	2.66	2.23	2.13	2.53
	Customer satisfaction	1.47	1.37	1.78	0.95	0.85	1.26	0.83	0.73	1.14
Improvement percentage of the manufacturer profit		27.29	15.18	43.52	53.44	46.98	73.42	60.95	33.98	80.31
Improvement percentage of the customer satisfaction		53.34	31.18	82.89	131.3	123.9	154.77	164.00	99.91	179.56

Note: In all instances, the average, maximum, and minimum improvement percentages at the manufacturer profit in all of the instances were: 31.8558, 80.3081, and 6.8227, and for customer satisfaction were 65.1489, 179.5935, and 7.5307.

between the two objective functions of manufacturer profit and customer satisfaction, since lack of implementation of maintenance activities in the post-warranty period entails a lot of costs for the customer, the manufacturer is forced to lower the selling price of the product, which results in a reduction in the manufacturer profit. According to the results in Table 9, for example, in instance 27, where repair cost and lifetime are at their highest levels, the highest percentages of improvement in both manufacturer profit and customer satisfaction are obtained among the 27 instances.

Comparison study 2: Maintenance activities implementation

In this comparative study, the proposed model is compared in 27 instances (presented in Table 5) with the same model with the difference that maintenance activities are not implemented in order to demonstrate the effectiveness of implementing maintenance activities on the objective functions.

Based on Figure 8, for 27 instances, the proposed model for all three objective functions – manufacturer profit, agent profit, and customer satisfaction – has had better performance in comparison to the model without pm activities. In this regard, according to Table 10, the most improvement is related to the agent profit, with the minimum, maximum, and average of 18.2553, 599.1795, and 79.5093 percent in 27 instances. The greatest improvement is related to instance 3, because the extended warranty period has the longest duration compared to the other instances, which increases the costs of the agent, and the effect

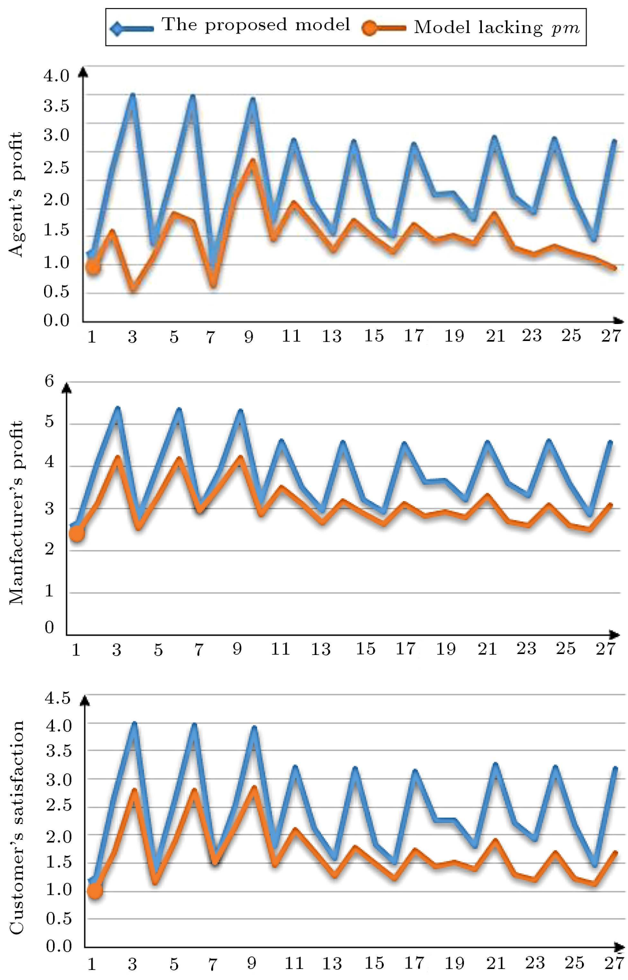

Figure 8. Graphical representation of the improvement percentage in Comparative study (2).

Table 10. Comparison of the proposed model with the model without preventive maintenance (pm).

Model	Instance	1	2	3	4	5	6	7	8	9
The proposed model	Agent profit	1.17	2.68	3.99	1.37	2.64	3.95	1.01	2.53	3.92
	Manufacturer profit	2.57	4.09	5.39	2.77	4.04	5.35	2.99	3.92	5.31
	Customer satisfaction	1.19	2.68	3.99	1.38	2.64	3.96	1.52	2.53	3.92
The model lacking pm	Agent profit	0.98	1.59	0.57	1.14	1.91	1.75	0.64	2.14	2.83
	Manufacturer profit	2.39	3.11	4.20	2.54	3.31	4.18	2.97	3.54	4.23
	Customer satisfaction	1.003	1.68	2.80	1.14	1.90	2.79	1.50	2.13	2.83
Improvement percentage of the agent profit		20.43	68.21	599.7	20.29	38.35	125.2	56.22	18.25	38.40
Improvement percentage of the manufacturer profit		7.41	31.27	28.23	8.88	22.08	27.97	0.94	10.99	25.68
Improvement percentage of the customer satisfaction		17.54	59.12	42.58	20.28	38.46	41.80	0.74	18.33	38.39
Model	**Instance**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
The proposed model	Agent profit	1.79	3.21	2.11	1.57	3.16	1.82	1.51	3.13	2.24
	Manufacturer profit	3.19	4.61	3.51	2.97	4.57	3.23	2.92	4.52	3.65
	Customer satisfaction	1.79	3.21	2.11	1.57	3.17	1.82	1.52	3.13	2.25
The model lacking pm	Agent profit	1.45	2.09	1.70	1.26	1.78	1.47	1.22	1.71	1.43
	Manufacturer profit	2.85	3.49	3.10	2.66	3.18	2.88	2.62	3.11	2.83
	Customer satisfaction	1.45	2.09	1.70	1.26	1.78	1.47	1.22	1.72	1.42
Improvement percentage of the agent profit		23.79	53.19	23.82	24.20	77.73	23.41	23.90	82.20	57.27
Improvement percentage of the manufacturer profit		12.23	31.65	13.00	11.32	43.40	11.99	11.22	45.23	28.72
Improvement percentage of the customer satisfaction		23.55	53.57	23.95	24.58	78. 08	23.45	23.77	81.95	57.79
Model	**Instance**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
The proposed model	Agent profit	2.26	1.80	3.25	2.21	1.92	3.21	2.19	1.46	3.17
	Manufacturer profit	3.66	3.20	4.56	3.61	3.32	4.61	3.59	2.86	4.57
	Customer satisfaction	2.27	1.80	3.26	2.22	1.91	3.21	2.19	1.46	3.17
The model lacking pm	Agent profit	1.52	1.38	1.89	1.29	1.19	1.33	1.21	1.11	0.94
	Manufacturer profit	2.92	2.78	3.29	2.69	2.59	3.07	2.61	2.51	3.09
	Customer satisfaction	1.51	1.38	1.89	1.29	1.19	1.68	1.21	1.11	1.69
Improvement percentage of the agent profit		48.99	29.76	71.84	71.24	61.23	140.8	81.19	30.82	236.1
Improvement percentage of the manufacturer profit		25.41	14.79	38.46	34.09	28.19	50.02	37.68	13.62	48.05
Improvement percentage of the customer satisfaction		49.18	29.83	72.13	71.54	61.16	91.48	81.12	30.85	87.84

Note: In all instances the average, maximum, and minimum improvement percentages at the agent profit were: 79.5093, 599.1795, and 18.2553; at the manufacturer profit were 24.5417, 50.0227, and 0.9406; and for customer satisfaction were 46.0421, 91.4767, and 0.7352, respectively.

of performing pm activities in reducing costs is well visible. In other words, the agent profit in the proposed model is 3.9912, while not implementing maintenance activities results in an increase in the number of failures, leading to an increase in failure costs and consequently a decrease in the agent profit of 0.5704. The average improvements in customer satisfaction and manufacturer profit are 46.0421% and 24.5417%, respectively. Improving customer satisfaction with an average of 46.0421% means that when maintenance activities are not implemented, the number of failures increases, resulting in an increase in the basic and extended warranty costs. As a result, to compensate for the problem, the manufacturer and the agent raise the price of the product and the price of the extended warranty, lowering customer satisfaction.

Comparison study 3: Considering the time value of money

In this comparative study, the presented model is compared to the same model without considering the time value of money in 27 instances given in Table 5 to demonstrate the impact of considering the time value of money on objective functions.

It should be remembered that in the proposed model, the time value of money is considered by the NPV method. In other words, the cost of providing the base warranty and the extended warranty (minimal repairs and maintenance activities) imposed on the manufacturer and the agent is returned to its present value. In contrast, the manufacturer profit and the agent profit depend on the product price and the extended warranty price, respectively, that are received at the beginning of the period. Consequently, when the NPV method is used, the cost of providing the warranty service is less sensible. In this regard, to create a balance among the three objective functions, the product price and the extended warranty price are reduced. Finally, according to Figure 9, the NPV method leads to an increase in manufacturer profit, agent profit, and customer satisfaction. According to Table 11, the average improvement for the manufacturer profit, agent profit, and customer satisfaction

Table 11. Comparison of the proposed model to the model that does not take into account the time value of money.

Model	Instance	1	2	3	4	5	6	7	8	9
The proposed model	Agent profit	1.17	2.68	3.99	1.37	2.64	3.95	1.00	2.52	3.91
	Manufacturer profit	2.57	4.08	5.38	2.77	4.04	5.35	2.99	3.92	5.31
	Customer satisfaction	1.17	2.68	3.99	1.37	2.64	3.95	1.52	2.52	3.91
The model that does not take into account the time value of money	Agent profit	1.10	2.67	3.73	1.18	2.49	3.90	0.82	2.33	3.83
	Manufacturer profit	2.50	3.82	4.95	2.58	3.82	4.90	2.75	3.50	4.87
	Customer satisfaction	1.11	2.43	3.43	1.16	2.3	3.52	1.24	2.03	3.49
Improvement percentage of the agent profit		6.22	0.26	6.80	15.7	5.94	1.16	22.8	8.32	2.12
Improvement percentage of the manufacturer profit		2.86	6.75	8.71	7.44	5.84	9.08	8.71	12.1	9.05
Improvement percentage of the customer satisfaction		6.15	10.2	16.1	17.7	13.8	12.39	22.24	24.48	12.01
Model	**Instance**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
The proposed model	Agent profit	1.79	3.20	2.11	1.57	3.1	1.82	1.51	3.12	2.24
	Manufacturer profit	3.19	4.60	3.50	2.96	4.5	3.22	2.91	4.52	3.64
	Customer satisfaction	1.79	3.21	2.11	1.57	3.1	1.82	1.51	3.13	2.25
The model that does not take into account the time value of money	Agent profit	1.76	2.93	1.61	1.20	3.0	1.70	1.13	2.94	2.00
	Manufacturer profit	2.76	4.18	2.95	2.55	4.4	2.56	2.48	4.11	3.22
	Customer satisfaction	1.39	2.69	1.61	1.20	3.10	1.19	1.13	2.59	1.24
Improvement percentage of the agent profit		1.73	9.45	30.5	30.34	2.24	6.84	33.76	6.10	12.1
Improvement percentage of the manufacturer profit		15.63	9.95	18.65	16.04	1.58	25.7	17.49	9.91	13.18
Improvement percentage of the customer satisfaction		28.57	19.11	30.65	30.32	2.15	53.21	33.91	20.62	80.98
Model	**Instance**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
The proposed model	Agent profit	2.26	1.80	3.25	2.21	1.92	3.21	2.19	1.46	3.17
	Manufacturer profit	3.66	3.20	4.56	3.61	3.32	4.61	3.59	2.86	4.57
	Customer satisfaction	2.26	1.80	3.25	2.22	1.91	3.21	2.19	1.46	3.17
The model that does not take into account the time value of money	Agent profit	1.82	1.59	3.20	2.17	1.78	3.20	2.12	0.94	3.16
	Manufacturer profit	3.31	2.53	4.43	3.38	3.18	4.24	3.51	2.23	4.19
	Customer satisfaction	2.07	1.80	3.19	1.99	1.78	2.72	2.12	0.94	2.64
Improvement percentage of the agent profit		24.37	13.09	1.73	2.03	7.56	0.45	3.582	54.78	0.46
Improvement percentage of the manufacturer profit		10.54	26.27	3.11	6.74	4.40	8.78	2.253	27.68	8.99
Improvement percentage of the customer satisfaction		9.310722	0.121922	1.847822	11.09412	7.52242	17.85832	3.461322	53.73222	19.88733

Note: In all instances average, maximum, and minimum improvement percentages at the agent profit were 11.5039, 54.7839, and 0.2605; at the manufacturer profit were 11.0229, 27.6818, and 1.5813; for customer satisfaction were 20.7283, 80.9856, and 0.1219, respectively.

in the 27 solved instances is 11.5, 11.27, and 20.72%, respectively.

6. Conclusion

To fill the gaps in the warranty literature, a two-dimensional warranty model with the possibility of extension under a three-level service contract was developed. It aimed to maximize the three objective functions of manufacturer profit, agent profit, and customer satisfaction simultaneously. Furthermore, the manufacturing technology level and pm level were identified as two important factors influencing the product failure rate. Following that, sensitivity analysis was performed on six parameters $(C_r, \beta', U_B, W_B, U_{life}, W_{life})$. The results showed that the parameter W_{life} has the highest effect, while the parameter U_B has the least effect on the three objective functions. Moreover, to prove the effectiveness of the proposed model, three comparative studies were conducted with the following objectives. In the first comparative study, the provision of the extended warranty by the agent was investigated. The results, as expected, showed that the reduction in the number of failures due to implementing the maintenance activities in the post-warranty period resulted in a reduction of costs and an increase in customer satisfaction, which means that this factor leads to an increase in the manufacturer profit due to an increase in the price of the product. In the second comparison, the importance and necessity of implementing maintenance activities were investigated in terms of saving costs and increasing profits. The results of the comparison showed that the implementation of maintenance activities had the strongest effect on agent profit, with an improvement of 79.5093% on average. In the last comparison, the effect of taking the time value of money into account was examined. This comparison showed that not considering the time value of money creates

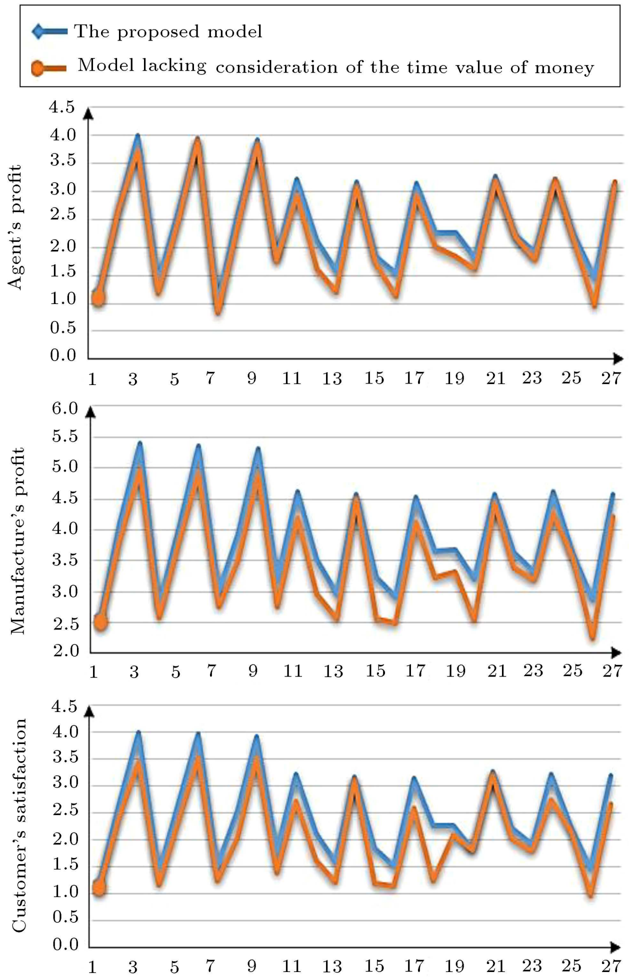

Figure 9. Graphical representation of the improvement percentage in Comparative study (3).

deceptive satisfaction and benefits for the customer, the agent, and the manufacturer, which is far from reality.

We propose expanding on this paper in two ways for future research. First, by implementing a pro-rata warranty policy and sharing maintenance and repair costs between the manufacturer and the customer during the warranty period, and second, by offering services to customers as options, using appropriate tools such as game theory.

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