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Applying SVSSI sampling scheme to np-chart to decrease the time of detecting shifts using Markov chain approach and Monte Carlo simulation

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Abstract. One of the main criteria for judging the power of control charts is their ability to perform fast detection of deviations and shifts in the process. Average Time to Signal (ATS) and Adjusted Average Time to Signal (AATS) are among such criteria calculated under a certain state and assumption. Several studies have shown that based on the idea of variable design for control charts and by demarcating the limits between safe and unsafe regions, quick discovery of shifts is facilitated and sensitivity to small changes increases. In this paper, a new variable sampling scheme with three sample sizes and two different sampling intervals, called SVSSI, is developed to increase the efficiency of the control chart np. Through various numerical examples, the performance of this scheme is evaluated by calculating ATS and AATS values through the application of Markov chain method. Monte Carlo simulation method is used to validate the results of Markov chain method of SVSSI sampling scheme. In comparison with other schemes, SVSSI exhibits better performance in all conditions.

1. Introduction

Control chart is the most important tool to control processes during production. Average Run Length (ARL), Average Time to Signal (ATS), and Adjusted Average Time to Signal (AATS) are well-known criteria to measure the performance of control charts. The ARL criterion is applied only in cases where the sampling interval is supposed to be fixed during the process; otherwise, ATS or AATS criterion must be used. Based on the ARL, several studies have been conducted on the Variable Sample Size (VSS) method in which only the sample size varies. Upon comparing VSS method with the fixed sampling method by Teoh et al. [1], Muhammad et al. [2], Annadi et al. [3], Amiri et al. [4], Aparisi [5], and Yeong et al. [6], different

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states for various control charts have been carried out; accordingly, positive results have been achieved using VSS. Due to the performance of the VSS sampling method, design of variable control charts has attracted much attention. Further, studies have mostly been conducted to shorten the time of signal by ATS and AATS criteria on other variable sampling methods such as VSI (with Variable Sampling Interval) and VSSI (with Variable Sample Size and Sampling Interval); (see e.g. [7–10]). ATS and AATS are comprehensive and well-known criteria that can be used to evaluate any type of sampling methods for control charts. Ultimately, Costa and Rahim [11] presented the Full Adaptive (FA) or Variable Parameter (VP) sampling scheme. The main difference between the FA sampling scheme and other sampling schemes is that in the case of the former, in addition to the variable values of the sample size and sampling interval, the control and warning limits are also considered variable. To study the details of designing this sampling scheme, (see [12–18]).

ATS, which represents the mean time of detecting the change from the moment of the occurrence of the shift in the process mean to the moment that control chart signals, was applied by Prabhu et al. [17] to evaluate the VSSI sampling method for control chart X. This criterion, calculated based on the concepts of the Markov chain and the transfer between different states, was also used by Aparisi and Haro [18] to measure the performance of the VSI sampling method for the T^2 Hotelling chart. In their study, for different shifts, the ATS values for VSI and fixed sampling methods were calculated and compared. Moreover, taking into account the same assumptions about the ATS calculation, Aparisi and Haro [19] evaluated VSSI and other sampling methods for the T^2 Hotelling chart based on the lowest amount of ATS. In order to monitor simple linear profiles, Kazemzadeh et al. [20] provided a complete study on the reduction of ATS and other statistical indices by fixed, VSS, and VSI sampling methods with different shift values and input parameters. Luo and Wu [21] as well as Wu and Luo [22] explored in separate studies the effects of variable sampling methods on the np control chart by numerical examples and considered ATS in different states. They provided a statistical model aimed at reducing ATS to assess VSS, VSI, VSSI and fix sampling methods. Similar researches including Abdella et al. [23], Chen and Hsieh [24], Lee and Khoo [25], Zhang et al. [26], Chen et al. [27] and Castagliola et al. [28] performed ATS-based evaluation for the results.

ATS criterion is based on the assumption that the process starts from out-of-control state. However, Costa [29] proposed another approach to analyze the results assuming that the process begins from the in-control state, and the difference between the duration of a production cycle and in-control time indicates the duration of change detection, i.e., AATS. They calculated AATS in different states for the variable X control chart. Faraz et al. [30] presented the differences between ATS and AATS in their research and then, conducted their assessments for VSI-T^2 control chart based on AATS. Accordingly, although the application of ATS simplifies and reduces the volume of calculations, AATS can be more realistic for assessments because it is usually in control before initiation of any process. Lin and Chou [31] considered both AATS and ATS for assessing the results. They analyzed the influence of using VSS, VSI, and VSSI sampling methods on the X control chart under normal and non-normal conditions based on AATS and ATS. Taking into account the AATS criteria, Katebi and Moghadam [13] implemented a comprehensive study on the performances of VSS, VSI, VSSI, FA, and fixed sampling schemes. According to the obtained results, the relative superiority of the FA scheme was evident compared to other schemes. In recent years, AATS has been of greater interest to researchers (e.g., refer to [32–34]).

In all the mentioned researches and other similar works such as Faraz and Saniga [35], Mahadik [36], Khaw et al. [37], and Saha et al. [38], the application of VSSI and FA sampling methods to designing control charts indicates better performance. An important point to notice is whether the performance of control charts can be further improved or not. Faraz and Parsian [39] proposed the idea of using Double Warning Lines (DWL) for the T^2 control chart. The DWL sampling method is similar to the VSSI method, except for using two separate warning lines for sample sizes and sampling intervals to monitor the process. Despite being more complicated, it has a better performance than VSSI. Since then, the economic and economic-statistical performances of using this sampling method were investigated by Faraz et al. [40] and Faraz and Saniga [41].

Mahadik and Shirke [42,43] evaluated the SVSSI sampling method for the X and T^2 control charts in separate studies. Their proposed method, which is simpler and more practical than DWL, improved the performance of the X and T^2 control charts and led to better results than VSSI and FA schemes. Furthermore, in terms of costs and by taking into account the statistical criteria, Katebi et al. [44] assessed SVSSI sampling method for the T^2 control chart. Their results were satisfactory and costs were reduced by using SVSSI method in comparison to other methods.

Despite the widespread use of np control chart in the literature, the current research introduces some contributions in the following paragraphs.

For the characteristics of np control charts, mostly two variables have been considered for their simplicity.

However, in this paper, the scheme SVSSI presented for the np control chart provides three different sample sizes and two different sampling intervals depending on the process situations. Besides being easy to understand and implement, this scheme is superior to the other sampling schemes in terms of performance.

Since the ARL criterion is not applicable because of time variability, two important criteria for checking the time of signal, namely AATS and ATS, are presented and fully discussed in this paper. Comprehensive evaluations based on ATS and AATS are provided. Accordingly, by improving these criteria through the proposed sampling algorithm, better performance of the control chart, reduction of detection time, avoiding the production of further defective items, and decrease in costs are expected.

In Section 2, the method of sampling is introduced. Statistical criteria are reviewed in Section 3. By providing various numerical examples in Section 4, variable sampling methods are evaluated and compared. Finally, conclusions are made along with suggestions for the future.

2. Designing SVSSI-np control chart

In the SVSSI sampling method, three sample sizes n_1, n_2, and n_3 and two sampling intervals h_1 and h_2 are used, assuming that $n_1 < n_2 < n_3$ and $h_1 < h_2$. To determine the time of change in sample size and sampling interval of subgroups, two warning limits W_i^1 and W_i^2 and one control limit K_i are used. Depending on the size of the sample, the values of these limits vary at different i values. The SVSSI sampling method is briefly described as follows:

$$\begin{cases} n_1, h_1 & 0 \le np < W_i^1 \\ n_2, h_2 & W_i^1 \le np < W_i^2 \\ n_3, h_2 & W_i^2 \le np < K_i \end{cases}$$

Therefore, if the value of the statistic for the $(i-1)$th subgroup be in the first region $(0 \le np < W_i^1)$, there is no reason for a change in the process to occur. Thus, the ith subgroup with small sample size n_1 and longer sampling interval h_1 is plotted on the chart. If the value of the statistic be in the second region $(W_i^1 \le np < W_i^2)$, then it is likely that a change in the mean will occur. Therefore, it is possible to signal a warning by the chart in the next subgroup. To have more control over the process, the ith subgroup with the size of n_2 is taken after the interval of h_2. Finally, if the statistic be in the third region $(W_i^2 \le np < K_i)$, then it is more likely that a change in the mean of the process will occur. Hence, the next sample is taken with the increased size of n_3 after the interval of h_2. In Figure 1, the SVSSI-np control chart is shown based on the description given above.

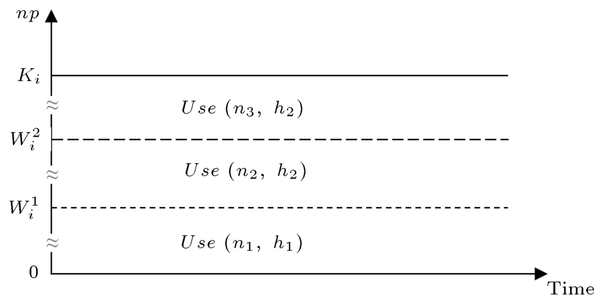

Figure 1. SVSSI–np control chart.

The limits of the warning and control of the SVSSI-np chart are calculated as $n_i p_0 + r(n_i p_0(1 - p_0))^{0.5}$, where r is the coefficient of control and warning limit and p_0 represents the mean of the process in the in-control state (the parameters are defined in Appendix A). At each sampling time, the occurrence probability for one of the following scenarios exists:

1. If $0 \le np < W_i^1$, the next sample with a size of n_1 is taken after the interval of h_1;
2. If $W_i^1 \le np < W_i^2$, the next sample with a size of n_2 is taken after the interval of h_2; and
3. If $W_i^2 \le np < K_i$, the next sample with a size of n_3 is taken after the interval of h_2.

Furthermore, the occurrence probability for the fourth state should be considered in which $np \ge K_i$. In this state, the control chart signals that the process is out of control. Accordingly, the process is stopped for further investigations. If the process is identified to be in control, this signal is considered incorrect. Otherwise, corrective actions are taken to find the cause of change, fix it, and thus, restore the process. Since the design of the control chart SVSSI-np is based on the concepts of the Markov chain, according to Fallahnezhad et al. [45] and Faraz and Saniga [35], this state can be regarded as the absorbing state in Markov chains.

3. Calculation of efficiency

In this section, statistical well-known criteria that can be used to evaluate any types of sampling schemes for control charts are reviewed.

3.1. AATS

As stated, the AATS criterion is based on the assumption that the process begins from the in-control state. According to the scenarios described in Section 2, all of them can be in-control or out-of-control. Thus, the occurrence of eight states is probable at each sampling of the process. The change of state or the probability of transform between different states using the properties of the Markov chain is shown in this section with p_{ij}. According to Faraz and Saniga [35], p_{ij} values can be defined as follows:

$$p_{11} = Pr\left(1 \to 1\right) = Pr\left(n_1, h_1, 0 \leq np < W_1^1\right)$$
$$= F\left(n_1, W_1^1, p_0\right) \times \exp\left(-\lambda h_1\right),$$
$$p_{16} = Pr\left(1 \to 6\right) = Pr\left(n_1, h_1, W_1^1 \leq np < W_1^2\right)$$
$$= \left(F\left(n_1, W_1^2, p_1\right) - F\left(n_1, W_1^1, p_1\right)\right)$$
$$\times \left(1 - \exp\left(-\lambda h_1\right)\right),$$
$$p_{28} = Pr\left(2 \to 8\right) = Pr\left(n_2, h_2, K_2 \leq np\right)$$
$$= \left(1 - F\left(n_2, K_2, p_1\right)\right) \times \left(1 - \exp\left(-\lambda h_2\right)\right),$$
$$p_{32} = Pr\left(3 \to 2\right) = Pr\left(n_3, h_2, W_3^1 \leq np < W_3^2\right)$$
$$= \left(F\left(n_3, W_3^2, p_0\right) - F\left(n_3, W_3^1, p_0\right)\right)$$
$$\times \exp\left(-\lambda h_2\right).$$

The np control chart based on the Binomial distribution plots the number of defective items in the sample. Therefore, F represents the cumulative distribution function of Binomial in calculating each p_{ij}. One of the main assumptions of this research in obtaining AATS, which is based on Costa [29] and Faraz et al. [30], is to consider the occurrence of the deviation according to Poisson distribution at a rate of λ per time unit. Thus, the time interval between consecutive occurrences of deviations follows exponential distributions with parameter λ. As another main assumption, only one assignable cause leads to shifting the mean of the process from p_0 to p_1. The value of p_1 for the shift size of r is obtained as follows:

$$p_1 = p_0 + r\sqrt{p_0(1-p_0)}. \tag{1}$$

Accordingly, the calculations of all p_{ij}'s for a Markov chain with seven transition states and one absorbing state are given below (the probability of transforming from an out-of-control state into in-control states and the probability of transforming from an absorbing state into other states is supposed to be zero):

$$p_{11} = F\left(n_1, W_1^1, p_0\right) \times \exp\left(-\lambda h_1\right),$$
$$p_{12} = \left(F\left(n_1, W_1^2, p_0\right) - F\left(n_1, W_1^1, p_0\right)\right)$$
$$\times \exp\left(-\lambda h_1\right),$$
$$p_{13} = \left(F\left(n_1, K_1, p_0\right) - F\left(n_1, W_1^2, p_0\right)\right)$$
$$\times \exp\left(-\lambda h_1\right),$$
$$p_{14} = \left(1 - F\left(n_1, K_1, p_0\right)\right) \times \exp\left(-\lambda h_1\right),$$
$$p_{15} = F\left(n_1, W_1^1, p_1\right) \times \left(1 - \exp\left(-\lambda h_1\right)\right),$$
$$p_{16} = \left(F\left(n_1, W_1^2, p_1\right) - F\left(n_1, W_1^1, p_1\right)\right)$$
$$\times \left(1 - \exp\left(-\lambda h_1\right)\right),$$
$$p_{17} = \left(F\left(n_1, K_1, p_1\right) - F\left(n_1, W_1^2, p_1\right)\right)$$
$$\times \left(1 - \exp\left(-\lambda h_1\right)\right),$$
$$p_{18} = \left(1 - F\left(n_1, K_1, p_1\right)\right) \times \left(1 - \exp\left(-\lambda h_1\right)\right),$$
$$p_{21} = F\left(n_2, W_2^1, p_0\right) \times \exp\left(-\lambda h_2\right),$$
$$p_{22} = \left(F\left(n_2, W_2^2, p_0\right) - F\left(n_2, W_2^1, p_0\right)\right)$$
$$\times \exp\left(-\lambda h_2\right),$$
$$p_{23} = \left(F\left(n_2, K_2, p_0\right) - F\left(n_2, W_2^2, p_0\right)\right)$$
$$\times \exp\left(-\lambda h_2\right),$$
$$p_{24} = \left(1 - F\left(n_2, K_2, p_0\right)\right) \times \exp\left(-\lambda h_2\right),$$
$$p_{25} = F\left(n_2, W_2^1, p_1\right) \times \left(1 - \exp\left(-\lambda h_2\right)\right),$$
$$p_{26} = \left(F\left(n_2, W_2^2, p_1\right) - F\left(n_2, W_2^1, p_1\right)\right)$$
$$\times \left(1 - \exp\left(-\lambda h_2\right)\right),$$
$$p_{27} = \left(F\left(n_2, K_2, p_1\right) - F\left(n_2, W_2^2, p_1\right)\right)$$
$$\times \left(1 - \exp\left(-\lambda h_2\right)\right),$$
$$p_{28} = \left(1 - F\left(n_2, K_2, p_1\right)\right) \times \left(1 - \exp\left(-\lambda h_2\right)\right),$$
$$p_{41} = p_{31} = F\left(n_3, W_3^1, p_0\right) \times \exp\left(-\lambda h_2\right),$$
$$p_{42} = p_{32} = \left(F\left(n_3, W_3^2, p_0\right) - F\left(n_3, W_3^1, p_0\right)\right)$$
$$\times \exp\left(-\lambda h_2\right),$$
$$p_{43} = p_{33} = \left(F\left(n_3, K_3, p_0\right) - F\left(n_3, W_3^2, p_0\right)\right)$$
$$\times \exp\left(-\lambda h_2\right),$$
$$p_{44} = p_{34} = \left(1 - F\left(n_3, K_3, p_0\right)\right) \times \exp\left(-\lambda h_2\right),$$
$$p_{45} = p_{35} = F\left(n_3, W_3^1, p_1\right) \times \left(1 - \exp\left(-\lambda h_2\right)\right),$$
$$p_{46} = p_{36} = \left(F\left(n_3, W_3^2, p_1\right) - F\left(n_3, W_3^1, p_1\right)\right)$$
$$\times \left(1 - \exp\left(-\lambda h_2\right)\right),$$
$$p_{47} = p_{37} = \left(F\left(n_3, K_3, p_1\right) - F\left(n_3, W_3^2, p_1\right)\right)$$
$$\times \left(1 - \exp\left(-\lambda h_2\right)\right),$$
$$p_{48} = p_{38} = \left(1 - F\left(n_3, K_3, p_1\right)\right)$$
$$\times \left(1 - \exp\left(-\lambda h_2\right)\right),$$

$$p_{55} = F\left(n_1, W_1^1, p_1\right),$$

$$p_{56} = F\left(n_1, W_1^2, p_1\right) - F\left(n_1, W_1^1, p_1\right),$$

$$p_{57} = F\left(n_1, K_1, p_1\right) - F\left(n_1, W_1^2, p_1\right),$$

$$p_{58} = 1 - F\left(n_1, K_1, p_1\right),$$

$$p_{65} = F\left(n_2, W_2^1, p_1\right),$$

$$p_{66} = F\left(n_2, W_2^2, p_1\right) - F\left(n_2, W_2^1, p_1\right),$$

$$p_{67} = F\left(n_2, K_2, p_1\right) - F\left(n_2, W_2^2, p_1\right),$$

$$p_{68} = 1 - F\left(n_2, K_2, p_1\right),$$

$$p_{75} = F\left(n_3, W_3^1, p_1\right),$$

$$p_{76} = F\left(n_3, W_3^2, p_1\right) - F\left(n_3, W_3^1, p_1\right),$$

$$p_{77} = F\left(n_3, K_3, p_1\right) - F\left(n_3, W_3^2, p_1\right),$$

$$p_{78} = 1 - F\left(n_3, K_3, p_1\right),$$

$$p_{88} = 1.$$

For more information on calculating transmission states for other variable control charts such as VSS, VSI, VSSI, and FA, the reader(s) can refer to Luo and Wu [21], Fallahnezhad et al. [45], Katebi and Moghadam [13], Faraz et al. [30], and Faraz and Saniga [35].

Therefore, the AATS based on the Markov chain approach is calculated as follows:

$$AATS = B \times (I - Q)^{-1} \times (h_1, h_2, h_2, h_2, h_1, h_2, h_2)' - \frac{1}{\lambda}, \tag{2}$$

where $1/\lambda$ is the mean of exponential distribution, which indicates the duration of in-control state for the process. B is a vector of initial probabilities, which is recommended to avoid potential problems at the start, apply more control over the process, and start the sampling of the process from the state between the last warning threshold and the control limit (see [44–46]). Therefore, the vector B is considered as (0,0,1,0,0,0,0). Moreover, I is an identity matrix of degree 7 and Q is calculated as follows:

$$Q = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} & p_{17} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} & p_{27} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} & p_{37} \\ p_{41} & p_{42} & p_{43} & p_{44} & p_{45} & p_{46} & p_{47} \\ 0 & 0 & 0 & 0 & p_{55} & p_{56} & p_{57} \\ 0 & 0 & 0 & 0 & p_{65} & p_{66} & p_{67} \\ 0 & 0 & 0 & 0 & p_{75} & p_{76} & p_{77} \end{bmatrix}.$$

As can be seen, the matrix Q contains all states of transitions except for the transition to absorbing state.

3.2. ATS

The major difference in calculation of ATS compared to AATS is in assumption of the process beginning 'in-control' or 'out-of-control' conditions. In the calculation of this criterion, according to Faraz et al. [30], it is assumed that the process starts from the out-of-control state ($d > 0$). Accordingly, only the occurrence probability for four states in the out-of-control state can be defined for the process. In this section, the probability of transition between different states, shown by p_{ij}^d, is obtained as follows:

$$p_{11}^d = p_{55}, \quad p_{21}^d = p_{65}, \quad p_{31}^d = p_{75},$$

$$p_{12}^d = p_{56}, \quad p_{22}^d = p_{66}, \quad p_{32}^d = p_{76},$$

$$p_{13}^d = p_{57}, \quad p_{23}^d = p_{67}, \quad p_{33}^d = p_{77},$$

$$p_{14}^d = p_{58}, \quad p_{24}^d = p_{68}, \quad p_{34}^d = p_{78}, \quad p_{44}^d = p_{88},$$

and:

$$ATS = B^d \times \left(I^d - Q^d\right)^{-1} \times (h_1, h_2, h_2)', \tag{3}$$

where B^d is the initial probability vector when beginning the process from the out-of-control state and is equal to (0,0,1). Here, the start of the process is also considered to be between the last warning threshold and the control limit. I^d is an identity matrix of degree 3 and Q^d is described as follows:

$$Q^d = \begin{bmatrix} p_{11}^d & p_{12}^d & p_{13}^d \\ p_{21}^d & p_{22}^d & p_{23}^d \\ p_{31}^d & p_{32}^d & p_{33}^d \end{bmatrix}.$$

4. Numerical examples and comparisons

In this section, the performance of different sampling methods is studied based on both AATS and ATS. For each criterion, the results are shown in three separate tables based on different values of the fixed sample size (n_0) and the fixed sampling interval (h_0). In each table, the studies are performed based on five different values of 0.03, 0.05, 0.08, 0.12, and 0.18 for p_0 and six different values of 0.05, 0.10, 0.30, 0.50, 0.70, and 0.90 for d, respectively. Given that: (1) The shifts are usually considered up to 8 hours and (2) The time intervals less than 0.1 in practice can be problematic because the process should be allowed to produce n unit within a short time h_2; the values of sampling intervals range from 0.1 to 8. Moreover, the range of sample sizes is considered between 1 and 50. In Table 1, the approach to determining the values of sample sizes and sample intervals is given for the SVSSI-np control chart.

In this research, the optimal values of AATS and ATS for different states are obtained by searching among different values of sample sizes and sampling intervals in the space of the desired charts. In other

Table 1. The ranges of sample size and sampling interval for SVSSI–np control chart.

	Min	Max	Step
n_1	1	$n_0 - 1$	1
n_2	$n_1 + 1$	$n_3 - 1$	1
n_3	$n_0 + 1$	50	1
h_1	h_0	8	0.1
h_2	0.1	$h_0 - 0.1$	0.1

words, the initial parameters for which the given criterion has the lowest value have been obtained. Then, at these points and for the obtained initial parameters, the optimal values of the given criterion are calculated in each step so that the comparison between different sampling methods can be possible. The results of the optimal values of AATS and ATS in all the calculations presented are based on the level parameter (λ) of 0.05 and the coefficients (r) of 1, 2, and 3, which are considered for K_i, W_i^2, and W_i^1, respectively. Therefore, the optimal AATS and ATS values of different sampling methods along with the optimal parameters of the SVSSI sampling method are presented in Tables 2–4 and Tables 5–7, respectively. More information on the computation of AATS and ATS can be found in Appendix B.

As shown, comprehensive investigations of different sampling methods were provided based on AATS and ATS. According to the results in Tables 2–7, use of VSS, VSI, and VSSI sampling schemes improved the performance of the *np* control chart in detecting changes, among which the VSSI sampling scheme exhibits better performance. Changing the fixed control limits of the VSSI scheme to the variable control limits of the FA scheme significantly reduced AATS and ATS values. This is compatible with the results previously published by Katabi and Moghaddam [13]. Although it brings greater complexity when $K_1 \neq K_2$ changes in the FA scheme, the obtained results of AATS and ATS show a decrease in both.

Furthermore, it is obvious that SVSSI sampling method, compared to the FA scheme, detects the shifts in the process in a shorter time. Generally, comparison of the SVSSI sampling method with other sampling methods indicates its superiority in most cases. Therefore, use of three sample sizes and two sampling intervals in the SVSSI scheme along with variable warning and control limits has greater impact on improving the detection speed of shifts in the *np* control chart than other schemes.

For more investigations, Monte Carlo simulation method is used to evaluate the results of Markov method of SVSSI sampling scheme. The simulation method is applied to approximately calculate ATS values by generating random data from Binomial distribution in 10,000 iterations. In this evaluation, the parameters are set according to Tables 7–9. As depicted in Figures 2–4, the results of simulation are almost the same as those obtained in Tables 7–9 using the Markov chain method.

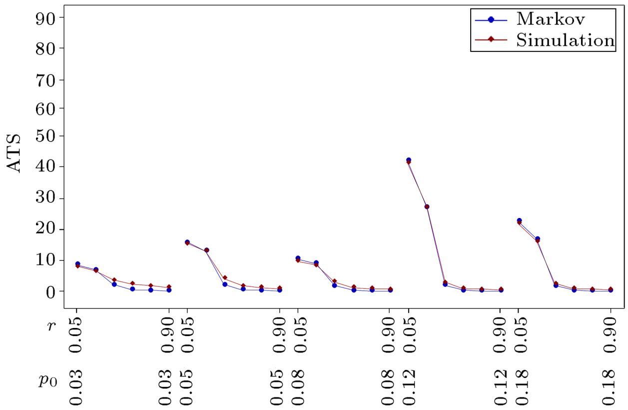

Figure 2. Comparing the optimal results of ATS obtained by Markov and simulation methods for SVSSI–np control chart when $n_0 = 4$ and $h_0 = 1$.

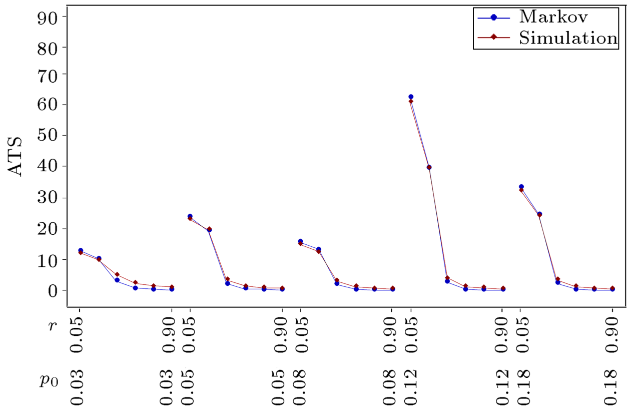

Figure 3. Comparing the optimal results of ATS obtained by Markov and simulation methods for SVSSI–np control chart when $n_0 = 6$ and $h_0 = 1.5$.

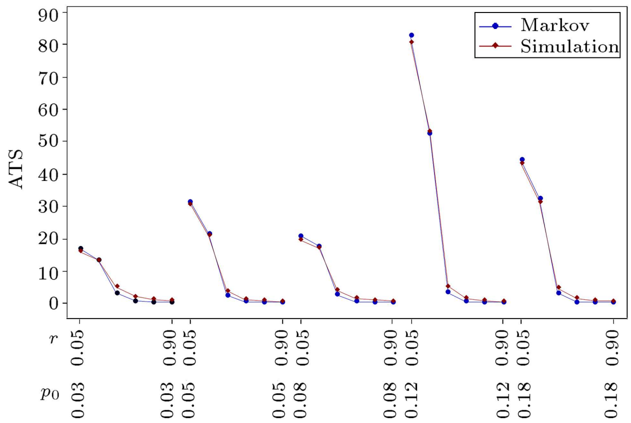

Figure 4. Comparing the optimal results of ATS obtained by Markov and simulation methods for the SVSSI-np control chart when $n_0 = 8$ and $h_0 = 2$.

Table 2. Comparing optimal AATS values of np control charts when $n_0 = 4$ and $h_0 = 1$.

		SVSSI design parameters					AATS					
p_0	r	n_1	n_2	n_3	h_1	h_2	SVSSI	FA	VSSI	VSS	VSI	Shewhart
0.03	0.05	3	9	10	1	0.1	8.5	8.5	12.69	12.69	103.17	117.76
	0.1	3	10	33	1	0.2	6.93	6.93	10.33	10.34	67.94	79.71
	0.3	3	47	48	1	0.8	3.83	3.87	5.88	5.88	21.35	27.76
	0.5	3	47	48	1	0.9	2.54	2.59	3.75	4.06	9.97	14.2
	0.7	3	47	48	1	0.9	1.91	1.94	2.36	3.08	5.65	8.7
	0.9	3	47	48	1	0.9	1.54	1.56	1.81	2.46	3.6	5.91
0.05	0.05	1	5	6	1	0.2	15.92	15.92	55.06	62.7	48.31	48.32
	0.1	1	6	48	1	0.4	13.42	13.43	37.85	43.87	35.15	35.16
	0.3	3	47	48	1	0.1	4.56	6.86	8.34	10.71	14.18	14.18
	0.5	3	47	48	1	0.1	2.18	2.66	2.85	3.84	7.73	7.74
	0.7	3	47	48	1	0.1	1.6	1.68	1.73	2.46	4.91	4.92
	0.9	3	47	48	1	0.1	1.3	1.32	1.35	2.02	3.41	3.42
0.08	0.05	1	4	8	1	0.1	10.19	10.19	40.06	40.05	21.14	21.15
	0.1	1	4	50	1	0.1	8.84	8.84	30.73	30.72	16.32	16.34
	0.3	3	49	50	1	0.1	3.35	5.58	8.33	11	7.51	7.52
	0.5	3	49	50	1	0.1	1.58	2.62	2.71	3.66	4.36	4.38
	0.7	3	49	50	1	0.1	1.19	1.75	1.78	2.43	2.88	2.89
	0.9	3	49	50	1	0.1	0.99	1.4	1.42	2.04	2.05	2.06
0.12	0.05	2	5	8	1	0.1	42.24	53.36	285.66	297.61	101.36	109.6
	0.1	2	40	45	1	0.1	27.59	42.47	176.07	201.8	71.92	79.1
	0.3	3	45	50	1	0.1	3.17	11.94	11.94	14.52	23.99	28.54
	0.5	3	47	50	1	0.1	1.26	3.78	3.78	4.61	10.45	13.58
	0.7	3	47	50	1	0.1	0.96	2.5	2.5	3.08	5.32	7.56
	0.9	3	47	50	1	0.1	0.82	2	2	2.58	3.02	4.66
0.18	0.05	2	6	31	1	0.1	22.29	24.7	128.41	150.23	620.01	634.49
	0.1	2	49	50	1	0.1	16.62	20.47	97.47	115.55	426	438.88
	0.3	2	49	50	1	0.1	2.78	9.57	9.57	12.84	122.49	131.09
	0.5	2	49	50	1	0.1	1.26	2.68	2.68	3.59	45.59	51.67
	0.7	2	49	50	1	0.1	0.98	1.82	1.82	2.4	19.7	24.12
	0.9	2	46	50	1	0.1	0.82	1.42	1.44	2.04	9.34	12.59

The performance of each control chart is highly dependent on the variations of sample size and sampling interval. Therefore, the proper design of the control chart is of great importance. For this reason, another assumption is considered and another method called $VSSI_n$ is examined. In this method, the next sampling is performed after the interval h_1 whenever the statistic is placed in the state of $W_i^1 \leq np < W_i^2$. Therefore, only p_{2j} values change as follows (other p_{ij} values are calculated in the same way as SVSSI method):

$$p_{21} = F\left(n_2, W_2^1, p_0\right) \times \exp\left(-\lambda h_1\right),$$

$$p_{22} = \left(F\left(n_2, W_2^2, p_0\right) - F\left(n_2, W_2^1, p_0\right)\right) \times \exp\left(-\lambda h_1\right),$$

$$p_{23} = \left(F\left(n_2, K_2, p_0\right) - F\left(n_2, W_2^2, p_0\right)\right) \times \exp\left(-\lambda h_1\right),$$

$$p_{24} = \left(1 - F\left(n_2, K_2, p_0\right)\right) \times \exp\left(-\lambda h_1\right),$$

Table 3. Comparing optimal AATS values of np control charts when $n_0 = 6$ and $h_0 = 1.5$.

		SVSSI design parameters					AATS					
p_0	r	n_1	n_2	n_3	h_1	h_2	SVSSI	FA	VSSI	VSS	VSI	Shewhart
0.03	0.05	3	9	10	1.5	0.1	12.75	12.75	19.01	19.02	73.96	73.97
	0.1	3	10	33	1.5	0.2	10.39	10.4	15.49	15.49	50.5	50.52
	0.3	3	47	48	1.5	1.2	5.71	5.79	8.8	8.8	18.15	18.17
	0.5	5	47	48	1.5	0.1	2.88	3.26	3.54	5.08	9.55	9.56
	0.7	5	47	48	1.5	0.1	2.05	2.14	2.23	3.37	6	6.01
	0.9	5	47	48	1.5	0.1	1.66	1.68	1.73	2.78	4.17	4.19
0.05	0.05	1	6	20	1.5	0.3	23.88	23.88	44.97	44.97	31.02	31.04
	0.1	1	6	48	1.5	0.5	20.12	20.14	32.88	32.88	22.8	22.82
	0.3	5	48	49	1.5	0.1	4.06	10.2	12.38	13.49	9.53	9.55
	0.5	5	48	49	1.5	0.1	2.02	3.95	4.22	5.7	5.37	5.39
	0.7	5	48	49	1.5	0.1	1.52	2.49	2.55	3.64	3.51	3.53
	0.9	5	48	49	1.5	0.1	1.27	1.95	1.98	2.98	2.52	2.54
0.08	0.05	1	4	8	1.5	0.1	15.29	15.29	60.06	60.05	103.61	113.03
	0.1	1	4	50	1.5	0.1	13.26	13.26	46.06	46.06	69.56	77.57
	0.3	4	49	50	1.5	0.1	3.67	8.33	12.34	16.41	20.71	25.43
	0.5	4	49	50	1.5	0.1	1.81	3.88	4.01	5.42	8.75	11.86
	0.7	4	49	50	1.5	0.1	1.39	2.6	2.63	3.59	4.52	6.68
	0.9	4	49	50	1.5	0.1	1.17	2.08	2.1	3.02	2.7	4.23
0.12	0.05	2	5	8	1.5	0.1	62.81	72.76	139.17	149.78	353.44	364.56
	0.1	2	40	45	1.5	0.1	40.84	53.09	97.59	106.55	229.29	238.93
	0.3	3	45	50	1.5	0.1	4.6	14.41	17.76	21.64	58.62	64.65
	0.5	3	47	50	1.5	0.1	1.84	4.7	4.95	6.38	21.32	25.38
	0.7	3	47	50	1.5	0.1	1.4	2.82	2.87	3.94	9.49	12.34
	0.9	3	47	50	1.5	0.1	1.2	2.05	2.09	3.15	4.85	6.88
0.18	0.05	2	6	31	1.5	0.1	33.04	37.06	191.41	208.87	82.17	88.98
	0.1	2	49	50	1.5	0.1	24.51	30.7	135.77	144.52	57.67	63.65
	0.3	2	49	50	1.5	0.1	4.04	14.16	14.16	19.17	17.94	21.73
	0.5	2	49	50	1.5	0.1	1.85	3.97	3.97	5.32	7.25	9.76
	0.7	5	49	50	1.5	0.1	1.34	2.69	2.69	3.54	3.5	5.18
	0.9	5	49	50	1.5	0.1	1.09	2.04	2.08	3.02	1.96	3.07

$$p_{25} = F\left(n_2, W_2^1, p_1\right) \times \left(1 - \exp\left(-\lambda h_1\right)\right),$$

$$p_{26} = \left(F\left(n_2, W_2^2, p_1\right) - F\left(n_2, W_2^1, p_1\right)\right) \times \left(1 - \exp\left(-\lambda h_1\right)\right),$$

$$p_{27} = \left(F\left(n_2, K_2, p_1\right) - F\left(n_2, W_2^2, p_1\right)\right) \times \left(1 - \exp\left(-\lambda h_1\right)\right),$$

$$p_{28} = \left(1 - F\left(n_2, K_2, p_1\right)\right) \times \left(1 - \exp\left(-\lambda h_1\right)\right).$$

Besides, the AATS and ATS values are equal to:

$$AATS = B \times (I - Q)^{-1} \times (h_1, h_1, h_2, h_2, h_1, h_1, h_2)' - \frac{1}{\lambda}, \tag{4}$$

$$ATS = B^d \times \left(I^d - Q^d\right)^{-1} \times (h_1, h_1, h_2)'. \tag{5}$$

Similar to the previous investigations, the $VSSI_n$ sampling method is evaluated based on AATS and ATS. The results of comparing this method with the SVSSI sampling method are presented in Tables 8–10.

In each table, the values of AATS and ATS metrics for $VSSI_n$ and $SVSSI$ sampling methods are

Table 4. Comparing optimal AATS values of np control charts when $n_0 = 8$ and $h_0 = 2$.

		SVSSI design parameters					AATS					
p_0	r	n_1	n_2	n_3	h_1	h_2	SVSSI	FA	VSSI	VSS	VSI	Shewhart
0.03	0.05	3	9	10	2	0.1	17	17	25.34	25.34	55.16	55.19
	0.1	3	10	33	2	0.3	13.86	13.87	20.64	20.64	37.96	37.99
	0.3	7	47	48	2	0.1	5.81	7.69	11.72	11.72	14.03	14.06
	0.5	7	48	50	2	0.1	2.81	4.32	4.68	6.69	7.55	7.59
	0.7	7	48	50	2	0.1	2.04	2.83	2.94	4.44	4.85	4.88
	0.9	7	48	50	2	0.1	1.68	2.22	2.28	3.65	3.46	3.49
0.05	0.05	1	6	20	2	0.4	31.84	31.85	59.94	59.94	184.99	198.49
	0.1	6	48	49	2	0.1	21.91	26.85	43.82	43.83	114.87	126.02
	0.3	6	48	49	2	0.1	4.46	12.4	16.42	17.98	29.17	35.35
	0.5	7	47	48	2	0.1	2.12	5.24	5.59	7.51	11.63	15.58
	0.7	7	47	48	2	0.1	1.61	3.3	3.38	4.78	5.9	8.6
	0.9	7	47	48	2	0.1	1.38	2.58	2.62	3.91	3.52	5.43
0.08	0.05	1	4	12	2	0.1	20.39	20.39	79.2	80.04	61.48	61.51
	0.1	1	4	50	2	0.2	17.68	17.69	53.27	60.15	42.92	42.95
	0.3	4	49	50	2	0.1	4.83	10.39	15.37	19.9	14.97	15
	0.5	4	49	50	2	0.1	2.39	3.85	4.25	6.4	7.37	7.4
	0.7	4	49	50	2	0.1	1.84	2.41	2.5	4.04	4.37	4.4
	0.9	4	49	50	2	0.1	1.56	1.83	1.89	3.34	2.91	2.95
0.12	0.05	2	5	40	2	0.1	83.38	97.01	185.31	199.66	121.58	130.07
	0.1	2	40	45	2	0.1	54.09	70.77	129.91	142.03	80.18	87.48
	0.3	3	45	50	2	0.1	6.03	19.15	23.58	28.69	21.77	26.14
	0.5	7	47	50	2	0.1	2.39	6.18	6.56	8.39	8.45	11.25
	0.7	7	47	50	2	0.1	1.7	3.43	3.51	5.01	4.12	5.96
	0.9	7	40	47	2	0.1	1.39	2.35	2.4	3.9	2.4	3.61
0.18	0.05	2	6	31	2	0.1	43.8	49.42	254.4	278.43	186.16	194.59
	0.1	2	49	50	2	0.1	32.39	40.94	180.82	192.63	122.33	129.72
	0.3	2	49	50	2	0.1	5.3	14.61	18.76	25.45	31.22	35.81
	0.5	2	49	50	2	0.1	2.45	4.54	4.89	7	11.02	14
	0.7	5	49	50	2	0.1	1.77	2.67	2.73	4.22	4.82	6.76
	0.9	5	46	50	2	0.1	1.44	1.89	1.95	3.42	2.54	3.77

compared. Moreover, the percentage difference between the optimal values of each criterion is calculated. It is clear that a small change in the sampling method leads to weaker results and attenuated performance of the np control chart in most cases. Therefore, choosing the correct sampling interval and sample size in each state of the process is very important to achieve better results.

5. Concluding remarks and future research

By introducing variable sampling methods, a significant improvement in the performance of control charts has been achieved in terms of faster identification of shifts in the processes. After reviewing Variable Sample Size (VSS), Variable Sampling Interval (VSI), Variable Sample Size and Sampling Interval (VSSI), and Full

Table 5. Comparing optimal ATS values of np control charts when $n_0 = 4$ and $h_0 = 1$.

		SVSSI design parameters					AATS					
p_0	r	n_1	n_2	n_3	h_1	h_2	SVSSI	FA	VSSI	VSS	VSI	Shewhart
0.03	0.05	3	9	10	1	0.1	8.5	8.6	11.86	12.76	102.78	118.26
	0.1	3	10	33	1	0.1	6.75	6.92	9.52	10.42	67.55	80.2
	0.3	3	47	48	1	0.1	2.01	2.62	5.08	5.98	20.95	28.26
	0.5	3	47	48	1	0.1	0.47	0.82	1.04	2.4	9.58	14.7
	0.7	3	47	48	1	0.1	0.17	0.27	0.29	1.33	5.25	9.2
	0.9	3	47	48	1	0.1	0.11	0.14	0.14	1.08	3.2	6.41
0.05	0.05	1	5	6	1	0.1	15.59	15.75	53.07	61.53	47.92	48.82
	0.1	1	6	48	1	0.1	12.83	13.14	36.04	42.83	34.76	35.66
	0.3	3	47	48	1	0.1	2.12	4.72	5.64	8.5	13.78	14.68
	0.5	3	47	48	1	0.1	0.31	0.87	0.92	2.28	7.34	8.24
	0.7	3	47	48	1	0.1	0.13	0.23	0.23	1.27	4.22	5.41
	0.9	3	47	48	1	0.1	0.1	0.12	0.12	1.05	3.02	3.92
0.08	0.05	1	4	8	1	0.1	10.37	10.29	38.83	39.73	20.74	21.64
	0.1	1	4	50	1	0.1	8.84	8.88	29.52	30.42	15.93	16.83
	0.3	3	49	50	1	0.1	1.69	3.89	5.51	8.72	7.12	8.02
	0.5	3	49	50	1	0.1	0.23	0.64	0.65	2.01	3.97	4.87
	0.7	3	49	50	1	0.1	0.12	0.16	0.17	1.16	2.84	3.38
	0.9	3	49	50	1	0.1	0.1	0.11	0.11	1.02	1.66	2.56
0.12	0.05	2	5	40	1	0.1	41.94	52.87	280.47	293.45	100.96	110.09
	0.1	2	40	45	1	0.1	26.8	41.49	170.11	196.73	71.52	79.6
	0.3	3	45	50	1	0.1	1.81	7.91	7.91	11.11	23.6	29.04
	0.5	3	47	50	1	0.1	0.22	0.79	0.79	2.1	10.05	14.07
	0.7	3	49	50	1	0.1	0.11	0.16	0.16	1.13	4.92	8.06
	0.9	3	49	50	1	0.1	0.1	0.1	0.1	1.01	2.62	5.16
0.18	0.05	2	6	31	1	0.1	22.52	24.85	123.99	146.67	619.61	634.98
	0.1	2	45	49	1	0.1	16.7	20.46	93.49	112.35	425.6	439.38
	0.3	2	45	49	1	0.1	1.65	6.74	6.74	10.6	122.09	131.58
	0.5	2	45	49	1	0.1	0.2	0.54	0.54	1.92	45.19	52.17
	0.7	2	48	49	1	0.1	0.11	0.13	0.13	1.09	19.3	24.62
	0.9	2	48	49	1	0.1	0.1	0.1	0.1	1.01	8.94	13.09

Adaptive (FA) sampling methods, this study proposed a new SVSSI sampling method for the np control chart in which three sample sizes and two sampling intervals are used to design. After introduction, designing of this method based on the concept of the Markov chain was considered. To evaluate the SVSSI method and compare it to other sampling methods, the minimum Adjusted Average Time to Signal (minAATS) and the minimum Average Time to Signal (minATS) were considered as criteria. Moreover, comparisons were

Table 6. Comparing optimal ATS values of np control charts when $n_0 = 6$ and $h_0 = 1.5$.

		SVSSI design parameters					AATS					
p_0	r	n_1	n_2	n_3	h_1	h_2	SVSSI	FA	VSSI	VSS	VSI	Shewhart
0.03	0.05	3	9	10	1.5	0.1	12.67	12.86	17.74	19.14	73.31	74.71
	0.1	3	10	33	1.5	0.1	10.05	10.32	14.22	15.62	49.86	51.26
	0.3	3	47	48	1.5	0.1	2.93	3.87	5.83	8.97	17.51	18.91
	0.5	5	47	48	1.5	0.1	0.51	0.98	1.05	3.16	8.9	10.3
	0.7	5	47	48	1.5	0.1	0.17	0.29	0.29	1.92	5.35	6.75
	0.9	5	47	48	1.5	0.1	0.11	0.14	0.14	1.61	3.53	4.93
0.05	0.05	1	6	20	1.5	0.1	23.56	23.57	43.71	45.11	30.38	31.78
	0.1	1	6	48	1.5	0.1	19.16	19.66	31.64	33.04	22.16	23.56
	0.3	5	47	48	1.5	0.1	2.04	6.94	8.31	12.75	8.89	10.29
	0.5	5	47	48	1.5	0.1	0.31	1.23	1.3	3.42	4.73	6.13
	0.7	5	47	48	1.5	0.1	0.13	0.28	0.28	1.9	2.87	4.28
	0.9	5	47	48	1.5	0.1	0.11	0.13	0.13	1.58	1.88	3.28
0.08	0.05	1	4	8	1.5	0.1	15.5	15.39	58.2	59.6	102.96	113.77
	0.1	1	4	50	1.5	0.1	13.17	13.27	44.23	45.63	68.91	78.31
	0.3	4	49	50	1.5	0.1	1.9	5.76	8.09	13.08	20.05	26.17
	0.5	4	49	50	1.5	0.1	0.25	0.88	0.9	3.02	8.1	12.6
	0.7	4	49	50	1.5	0.1	0.12	0.19	0.19	1.74	3.87	7.42
	0.9	4	49	50	1.5	0.1	0.1	0.11	0.11	1.53	2.05	4.97
0.12	0.05	2	5	40	1.5	0.1	62.3	72.27	137.18	149.15	352.79	365.3
	0.1	2	40	45	1.5	0.1	39.6	51.95	95.71	106	228.62	239.68
	0.3	3	45	50	1.5	0.1	2.54	9.58	11.68	16.66	57.97	65.39
	0.5	3	47	50	1.5	0.1	0.26	0.97	1.01	3.04	20.66	26.12
	0.7	3	49	50	1.5	0.1	0.11	0.17	0.17	1.68	8.83	13.08
	0.9	3	49	50	1.5	0.1	0.1	0.1	0.1	1.52	4.19	7.62
0.18	0.05	2	6	31	1.5	0.1	33.34	37.22	184.73	207.68	81.52	88.72
	0.1	2	45	49	1.5	0.1	24.54	30.64	133.12	143.4	57.01	64.39
	0.3	2	45	49	1.5	0.1	2.3	9.9	9.9	15.89	17.29	22.47
	0.5	5	45	49	1.5	0.1	0.22	0.74	0.74	2.88	6.6	10.5
	0.7	5	48	49	1.5	0.1	0.11	0.14	0.14	1.64	2.84	5.92
	0.9	5	48	49	1.5	0.1	0.1	0.1	0.1	1.51	1.31	3.81

made based on different values of p_0, d, n_0, and h_0 to evaluate the sampling methods from different angles. Optimal values of the criteria and parameters of the control chart were obtained through grid search and exact method.

The results of different numerical comparisons indicate the superiority of SVSSI sampling method. Besides, by making a change in the design of the method, $VSSI_n$ method was introduced. The results of $VSSI_n$ comparison proved the superiority of SVSSI again. Therefore, by choosing the proper sampling method and parameters, the duration of the out-

Table 7. Comparing optimal ATS values of np control charts when $n_0 = 8$ and $h_0 = 2$.

		SVSSI design parameters					AATS					
p_0	r	n_1	n_2	n_3	h_1	h_2	SVSSI	FA	VSSI	VSS	VSI	Shewhart
0.03	0.05	3	9	10	2	0.1	16.85	17.11	23.62	25.52	54.27	56.17
	0.1	3	10	33	2	0.1	13.35	13.72	18.93	20.83	37.08	38.98
	0.3	7	47	48	2	0.1	3	5.12	7.68	11.96	13.14	15.04
	0.5	7	47	48	2	0.1	0.53	1.26	1.34	4.21	6.67	8.57
	0.7	7	47	48	2	0.1	0.18	0.34	0.35	2.56	3.97	5.87
	0.9	7	47	48	2	0.1	0.12	0.15	0.15	2.15	2.57	4.47
0.05	0.05	1	6	20	2	0.1	31.36	31.39	58.25	60.15	184.08	199.48
	0.1	6	20	48	2	0.1	21.36	26.18	42.15	44.05	113.97	127
	0.3	7	47	48	2	0.1	2.28	8.64	10.97	17	28.27	36.33
	0.5	7	47	48	2	0.1	0.33	1.6	1.68	4.57	10.72	16.56
	0.7	7	47	48	2	0.1	0.13	0.33	0.34	2.54	4.99	9.58
	0.9	7	47	48	2	0.1	0.11	0.14	0.14	2.11	2.62	6.41
0.08	0.05	1	4	12	2	0.1	20.72	24.49	77.57	79.47	60.59	62.49
	0.1	1	4	50	2	0.1	17.51	17.66	51.9	60.49	42.03	43.93
	0.3	4	49	50	2	0.1	2.43	7.04	10.06	16.83	14.09	15.99
	0.5	4	49	50	2	0.1	0.29	0.91	0.98	3.85	6.49	8.39
	0.7	4	49	50	2	0.1	0.12	0.19	0.19	2.3	3.48	5.38
	0.9	4	49	50	2	0.1	0.1	0.11	0.11	2.04	2.03	3.93
0.12	0.05	2	5	40	2	0.1	82.66	96.32	182.62	198.87	120.67	131.05
	0.1	2	40	45	2	0.1	52.4	69.22	127.37	141.33	79.27	88.47
	0.3	3	45	50	2	0.1	3.27	12.68	15.46	22.21	20.87	27.12
	0.5	7	47	50	2	0.1	0.3	1.23	1.29	4.05	7.54	12.23
	0.7	7	49	50	2	0.1	0.12	0.19	0.19	2.23	3.21	6.94
	0.9	7	49	50	2	0.1	0.1	0.11	0.11	2.02	1.5	4.6
0.18	0.05	2	6	31	2	0.1	44.15	49.59	245.47	276.91	185.25	195.58
	0.1	2	45	49	2	0.1	32.39	40.82	177.24	191.21	121.42	130.7
	0.3	2	45	49	2	0.1	2.95	10.32	13.05	21.19	30.31	36.79
	0.5	5	49	50	2	0.1	0.24	0.84	0.88	3.79	10.11	14.98
	0.7	5	48	49	2	0.1	0.11	0.14	0.14	2.17	3.91	7.74
	0.9	5	48	49	2	0.1	0.1	0.1	0.1	2.01	1.63	4.75

of-control state was reduced and the changes that occurred in the process mean could be detected quickly. As a result, generating more defective items than those from other methods can be avoided. Thus, from a theoretical point of view and taking into account practical considerations, it can be admitted that the application of three sample sizes and two sampling intervals, as shown in Figure 1, is a superior and more efficient method than other in detecting the changes.

The np control chart is used to decide whether the observations match the technical specifications and pre-specified requirements or not. Thus, several qualitative

Table 8. Comparing optimal AATS and ATS values of sampling methods $VSSI_n$ and SVSSI when $n_0 = 4$ and $h_0 = 1$.

		AATS			ATS		
p_0	r	$\mathbf{VSSI}_n$	SVSSI	Δ_{AATS} (%)	$\mathbf{VSSI}_n$	SVSSI	Δ_{ATS} (%)
0.03	0.05	8.5013	8.4971	0.05	8.7896	8.4952	3.35
	0.1	6.9334	6.9272	0.09	7.0473	6.754	4.16
	0.3	3.8312	3.8266	0.12	2.3251	2.0114	13.49
	0.5	2.5413	2.5409	0.02	0.5978	0.4659	22.06
	0.7	1.911	1.9117	–0.04	0.2096	0.1689	19.42
	0.9	1.5413	1.542	–0.05	0.1243	0.1137	8.53
0.05	0.05	15.9262	15.919	0.05	15.9077	15.5929	1.98
	0.1	13.4314	13.4159	0.12	13.2502	12.8262	3.2
	0.3	5.3062	4.557	14.12	3.0394	2.1169	30.35
	0.5	2.32	2.1823	5.94	0.5317	0.3076	42.15
	0.7	1.6022	1.5982	0.25	0.1815	0.13	28.37
	0.9	1.2799	1.3006	-1.62	0.1152	0.1049	8.94
0.08	0.05	10.1952	10.1935	0.02	10.4754	10.3715	0.99
	0.1	8.8402	8.8374	0.03	8.9986	8.8367	1.8
	0.3	4.847	3.3472	30.94	2.923	1.6902	42.18
	0.5	2.1268	1.5804	25.69	0.3978	0.2322	41.63
	0.7	1.599	1.1882	25.69	0.1379	0.1162	15.74
	0.9	1.3343	0.9856	26.13	0.1044	0.1021	2.2
0.12	0.05	52.2464	42.2385	19.16	51.8923	41.9391	19.18
	0.1	36.4141	27.5882	24.24	35.5929	26.8033	24.69
	0.3	4.8657	3.1654	34.94	3.1696	1.8134	42.79
	0.5	1.7806	1.2646	28.98	0.3785	0.2238	40.87
	0.7	1.3086	0.9575	26.83	0.1278	0.1118	12.52
	0.9	1.1198	0.8173	27.01	0.1021	0.101	1.08
0.18	0.05	28.8104	22.2859	22.65	29.0045	22.521	22.35
	0.1	23.4137	16.6226	29	23.3607	16.6961	28.53
	0.3	4.5333	2.7774	38.73	3.1359	1.6543	47.25
	0.5	1.7627	1.2556	28.77	0.3427	0.195	43.1
	0.7	1.3426	0.976	27.31	0.1177	0.1074	8.75
	0.9	1.1494	0.8241	28.3	0.1009	0.1004	0.5

characteristics can be examined simultaneously in this control chart. Based on such features, the chart control np is widely applied in practice. This chart is a useful tool to control and evaluate the behavior of a process from low-cost operations to mass production over time. Thus, practitioners can identify and stop problems before experiencing great losses. The main purpose of this paper was to investigate this issue by evaluating AATS and ATS criteria for different sampling schemes in order to present the best design in terms of reducing

Table 9. Comparing optimal AATS and ATS values of sampling methods $VSSI_n$ and $SVSSI$ when $n_0 = 6$ and $h_0 = 1.5$.

		AATS			ATS		
p_0	r	VSSI$_n$	SVSSI	Δ_{AATS} (%)	VSSI$_n$	SVSSI	Δ_{ATS} (%)
0.03	0.05	12.7536	12.7483	0.04	13.1341	12.6747	3.5
	0.1	10.4022	10.3922	0.1	10.5133	10.0515	4.39
	0.3	5.7177	5.7095	0.14	3.4204	2.9325	14.26
	0.5	3.0163	2.8848	4.36	0.7305	0.508	30.46
	0.7	2.0646	2.0529	0.57	0.2366	0.1738	26.54
	0.9	1.6457	1.6597	–0.85	0.1299	0.1146	11.78
0.05	0.05	23.8918	23.8822	0.04	23.9837	23.5588	1.77
	0.1	20.1496	20.1205	0.14	19.8199	19.1602	3.33
	0.3	6.2904	4.0606	35.45	3.9829	2.0388	48.81
	0.5	2.8966	2.0213	30.22	0.7203	0.3133	56.5
	0.7	2.0862	1.5169	27.29	0.2183	0.1313	39.85
	0.9	1.7101	1.2669	25.92	0.2108	0.1051	50.14
0.08	0.05	15.2945	15.2924	0.01	15.6583	15.4978	1.03
	0.1	13.2626	13.2589	0.03	13.4414	13.1742	1.99
	0.3	5.759	3.6676	36.32	3.6801	1.9008	48.35
	0.5	2.5532	1.8064	29.25	0.5024	0.2513	49.98
	0.7	1.8969	1.3874	26.86	0.1514	0.1177	22.26
	0.9	1.5755	1.1742	25.47	0.1058	0.1022	3.4
0.12	0.05	73.7149	62.809	14.79	73.3425	62.2976	15.06
	0.1	50.1509	40.8394	18.57	48.9955	39.5995	19.18
	0.3	7.2481	4.5974	36.57	4.6493	2.5397	45.37
	0.5	2.6563	1.8447	30.55	0.5033	0.2619	47.96
	0.7	1.9551	1.4046	28.16	0.1388	0.1139	17.94
	0.9	1.6721	1.198	28.35	0.1028	0.1011	1.65
0.18	0.05	43.1859	33.0408	23.49	43.4258	33.3357	23.24
	0.1	35.0879	24.5061	30.16	34.9532	24.5435	29.78
	0.3	6.7612	4.0397	40.25	4.6031	2.3004	50.02
	0.5	2.5605	1.8513	27.7	0.4331	0.2199	49.23
	0.7	1.7901	1.3406	25.11	0.1225	0.108	11.84
	0.9	1.4402	1.0884	24.43	0.101	0.1004	0.59

the time of signal. Therefore, the practitioners can use the SVSSI scheme as an efficient scheme to reduce the time of signal, leading to a reduction in the production of defective items. For this purpose, after considering the parameters of the control chart and evaluating its performance, they can decide whether to reduce or increase the sample size and sampling interval.

For future research, one can consider cases in which the distribution of the failure or the duration of in-control time is not exponential. Moreover, the

Table 10. Comparing optimal AATS and ATS values of sampling methods $VSSI_n$ and $SVSSI$ when $n_0 = 8$ and $h_0 = 2$.

		AATS			ATS		
p_0	r	VSSI$_n$	SVSSI	Δ_{AATS} (%)	VSSI$_n$	SVSSI	Δ_{ATS} (%)
0.03	0.05	17.0079	17.0017	0.04	17.4782	16.8542	3.57
	0.1	13.8732	13.8584	0.11	13.9788	13.3489	4.51
	0.3	7.5865	5.8079	23.44	4.5157	2.9951	33.67
	0.5	3.9793	2.8055	29.5	0.9301	0.5265	43.39
	0.7	2.7303	2.0385	25.34	0.2744	0.1788	34.84
	0.9	2.1779	1.6849	22.64	0.1372	0.1158	15.6
0.05	0.05	31.8596	31.8447	0.05	31.945	31.3639	1.82
	0.1	26.8694	21.9112	18.45	26.3896	21.3569	19.07
	0.3	7.2704	4.4564	38.7	4.7945	2.2782	52.48
	0.5	3.2422	2.1247	34.47	0.877	0.3334	61.98
	0.7	2.2409	1.6059	28.34	0.2499	0.134	46.38
	0.9	1.8509	1.3823	25.32	0.1249	0.1055	15.53
0.08	0.05	20.396	20.3942	0.01	20.8412	20.7221	0.57
	0.1	17.6871	17.682	0.03	17.8843	17.5118	2.08
	0.3	7.6622	4.8259	37.02	4.8429	2.4282	49.86
	0.5	3.4046	2.3904	29.79	0.6148	0.2851	53.63
	0.7	2.3458	1.8421	21.47	0.1637	0.12	26.7
	0.9	1.795	1.5611	13.03	0.1071	0.1023	4.48
0.12	0.05	98.2828	83.3819	15.16	97.746	82.656	15.44
	0.1	66.8477	54.093	19.08	65.2661	52.3957	19.72
	0.3	9.3683	6.0316	35.62	5.8789	3.2659	44.45
	0.5	3.3331	2.3928	28.21	0.6056	0.2977	50.84
	0.7	2.2666	1.7003	24.98	0.1479	0.1154	21.97
	0.9	1.8029	1.3909	22.85	0.1034	0.1011	2.22
0.18	0.05	57.5619	43.798	23.91	57.847	44.1504	23.68
	0.1	46.7633	32.392	30.73	46.5457	32.3909	30.41
	0.3	8.9931	5.3045	41.02	6.0703	2.9465	51.46
	0.5	3.3994	2.4492	27.95	0.5307	0.2447	53.89
	0.7	2.3808	1.7697	25.67	0.1278	0.1086	15.02
	0.9	1.8612	1.4368	22.8	0.1012	0.1004	0.79

SVSSI-np control chart can be evaluated for further research from different economic and economic-statistical points of view (see e.g. [47]). The improvement of the np control chart by designing other sampling methods necessitates more investigations. For the time being, we are trying to extend this study for an fraction defective chart expanded in [48]. The occurrence of an assignable cause in the process was one of the main assumptions of this study. Assuming the occurrence of multiple assignable causes is another suggestion for future research to pursue.

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Appendix A

Abbreviations and symbols used in designing SVSSI sampling scheme

ATS	Average Time to Signal
AATS	Adjusted Average Time to Signal
SVSSI	Three sample sizes and two sampling intervals scheme
ARL	Average Run Length
VSS	Variable Sample Size
VSI	Variable Sampling Interval
VSSI	Variable Sample Size and Sampling Interval
FA	Full Adaptive
DWL	Double Warning Lines
n_i	Sample size ($i = 1, 2, 3$)
h_i	Sampling interval ($i = 1, 2$)
W_i^1, W_i^2	Lower and higher warning limits ($i = 1, 2, 3$)
K_i	Higher control limit ($i = 1, 2, 3$)
p_0	The in-control defective ratio
p_1	The out-of-control defective ratio
r	The coefficient of control and warning limits

p_{ij}	The transition probability between the states i and j
d	The level of the shift in process
$F(.)$	The binomial cumulative distribution function
λ	The exponential distribution parameter
B	The initial probabilities vector
I	The identity matrix
Q	The identity matrix
$VSSI_n$	Three sample sizes and two different sampling intervals scheme (with different methods and assumptions with SVSSI scheme)

Appendix B

Procedure of computing AATS and ATS for SVSSI sampling scheme

1. Input p_0, n_0, h_0, and d
   1.1. Calculate p_1 using Eq. (1)
2. Set values of $AATS = +\infty$ or $ATS = +\infty$
3. Loop n_1^* from $\{1, 2, ..., n_0 - 1\}$
   3.1. Calculate $W_1^{1*}, W_1^{2*}, K_1^*$
4. Loop n_3^* from $\{n_0 + 1, ..., 50\}$
   4.1. Calculate $W_3^{1*}, W_3^{2*}, K_3^*$
5. Loop n_2^* from $\{n_1 + 1, ..., n_3 - 1\}$
   5.1. Calculate $W_2^{1*}, W_2^{2*}, K_2^*$.
6. Similarly, loop h_2^* and h_1^* from $\{0.1, 0.2, ..., h_0 - 0.1\}$ and $\{h_0, ..., 8\}$ respectively
7. Calculate $AATS^*$ or ATS^* using Eq. (2) and Eq. (3).
   7.1. If $AATS^* < AATS$ or $ATS^* < ATS$, then
      7.1.1. $AATS = AATS^*$ or $ATS = ATS^*$
      7.1.2. $n_1 = n_1^*, n_3 = n_3^*, n_2 = n_2^*, h_2 = h_2^*, h_1 = h_1^*$
      7.1.3. $W_1^1 = W_1^{1*}, W_1^2 = W_1^{2*}, K_1 = K_1^*$
      7.1.4. $W_3^1 = W_3^{1*}, W_3^2 = W_3^{2*}, K_3 = K_3^*$, and
      7.1.5. $W_2^1 = W_2^{1*}, W_2^2 = W_2^{2*}, K_2 = K_2^*$

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