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Chance-constrained programming and robust optimization approaches for uncertain hub location problems in a cooperative competitive environment

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Abstract. In this paper, an integer programming model is offered for capacitated multi-allocation median hub location problems applicable to both cooperative and competitive environments among airlines. We divided the hubs into six independent categories by comparing the parameters of ticket price, travel time, and service quality for both the follower and leader airlines. The degrees of importance for the parameters of time and cost were determined by a multivariate Lagrange interpolation method, which could be of significant help in allocating travelers to the follower airline hubs. Then, with regard to the seasonal demand of travelers, travel demand was considered as an uncertain parameter. To identify the deterministic equivalent forms for the considered categories of hub location models, the robust optimization method and the chance-constrained programming model were employed. Finally, the developed model was tested for a case study. The results indicated that the coalition of follower airlines could absorb nearly 2% of the leader airline travelers with relatively lower travel cost and time.

1. Introduction

Since networks have a hierarchical structure, performing several travels from lower levels in demand spokes (origins) to lower levels in destination spokes is not cost-effective for companies due to the low demand level. In such cases, it is better to establish flows from other points, which are called hubs [1]. Hub location as an indirect transportation problem requires designing an efficient network. This applies when there is a need for economies of scale in transportation or communication.

Today, hub location can be adopted for transportation problems [2], postal service [3], order distribution [4], emergency services [5], and computer networks [6].

In the real world, transportation occurs via hubs in a competitive market, where several service providers compete with each other on gaining more market share. Accordingly, customers can choose one of them based on freight, service delivery time, type of offered services, and so on. Marianov et al. [7] proposed the first hub location problem in a competitive environment. In their model for the hub location of the follower player, the leader player had already selected its hub-and-spoke network. The proposed model used fewer variables and constraints than the previous location

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problems. The flow from origin to destination passed through only one path (including the maximum of two hubs). They showed that using more hubs would lead to lower costs. They also presented two models for the follower player. In the first model, if the cost of traveler movement by the follower player was lower than that by the leader player, then all of the travelers were attracted to the leader player hub. However, in the second model, three capture sets were presented based on the compared costs of travel by the follower and leader players, who attracted 50%, 75%, and 100% of travelers.

By changing the two models of Marianov et al. [7], Wagner [8] reduced the time needed to achieve an optimal solution to problems with more than 50 spokes and 5 hubs. He developed the capture sets to offer six cases based on the fact that flow from origin to destination passed through one or two paths.

Marianov et al. [7] and Wagner [8] used the values of 0.7, 0.9, and 1.1 as well as 50%, 75%, and 100% in their capture sets. They employed a multinomial Lagrange interpolation method in order to calculate exact values. The members of the capture sets of the follower airlines were determined by comparing the parameters of travel cost and time between the hubs of the leader and follower airline as well as service quality of the airports under their control. Moreover, the bounds of variables that belonged to these sets were determined by considering the degrees of importance of the parameters of travel time and travel cost from the viewpoint of travelers, the quality of airports and airlines, safety, and delays of airlines.

To identify the members of the capture sets, Marianov et al. [7] and Wagner [8] only compared the costs of moving a traveler by the follower and leader players. However, aside from the movement cost of a traveler, there are other parameters such as travel time and service quality of hubs (airports) in the real world, which can be a basis for the selection of players and hubs. To overcome the aforementioned shortcoming and determine the members of capture sets more accurately, in the present study, we compare travel times and the costs incurred by the travelers of each player. Additionally, comparing the service quality of hubs under the control of players is the cornerstone of our research.

In real-world situation, ignoring even one small uncertainty in data will result in an insignificant and meaningless (non-)optimal solution. Therefore, various approaches have been put forward in the literature to gain more realistic optimal solutions. Chance-constrained programming and robust optimization are the most important approaches in this area. Soyster [9], Ben-Tal and Nemirovski [10], and Bertsimas and Sim [11] have presented the main approaches for robust optimization, which are employed in the research on hub location under uncertainty. In regard to the possibility of controlling the conservativeness of robust solutions and linearity of the Bertsimas and Sim method [11], we adopt this approach and chance-constrained programming in order to demonstrate the uncertainty in travel demand for achieving more realistic solutions and then, compare the results.

The increase in the foreign air travelers to Iran [12] led us to the issue of organizing the travelers and determining the exact location of hubs. With this in mind, we developed the model of Marianov et al. [7] by defining a new type of capture sets under uncertainty for an alliance of airlines with a lower market share. In the proposed model, we present an optimal hub-and-spoke network for follower airlines in which their share of the travelers market increases in competition with other airlines through alliance. By comparing the parameters of time and cost of travel by the follower and leader airlines as well as service quality of the airports under their control, we divided hub sets of the follower airlines into six categories to generate the travelers' decision options and reduce the complexity of modeling. By adopting a multivariate Lagrange interpolation method, we determined the degrees of importance for the time and cost of travel from the perspective of the travelers. In the proposed model, spokes were demand points that could travel by using the hubs of the follower airlines. Finally, we examined a case study in which the Emirates airline was the leader airline and the follower airlines consisted of Mahan Airline, Iran Air, and Aseman Airline. The spokes were selected from a pool of cities, which passed the largest numbers of travelers from the European countries to the East Asian countries by the hubs of the Emirates airline. The results showed that the income of the follower airlines experienced a considerable growth compared to the Marianov et al. [7] model and could absorb nearly 2% of the travelers of the leader airline due to lower travel cost and time.

In the next section, the literature related to the area under study is reviewed. Section 3 presents a model for capacitated multiple-allocation median hub location in both cooperative and competitive environments. Then, to determine the degrees of importance for travel time and cost, a multivariate Lagrangian interpolation function is introduced. In Sections 4 and 5, the introduced model will be presented under uncertainty by employing robust optimization and chance-constrained programming approaches, separately. In Section 6, deterministic equivalent forms adopted from the robust optimization and chance-constrained programming approaches are tested in a case study. Also, the impact of change in the parameters of the problem on the value of the objective function will be investigated. Finally, some concluding remarks will be presented in the last section of the present paper.

2. Literature review

The most important research related to the subject of this paper is presented in the following. Sasaki and Fukushima [13] proposed a new model called "Stackelberg Hub Location" to explore the competition among a large company and several medium-size firms in which space solution was considered as a Continuous Space. They suggested that a leader company could lose a huge percentage of its travelers if they disregard the competition strategies. Based on the Stackelberg hub location model [13], Sasaki [14] designed a hub network in which two companies competed over maximizing their profits and their solution space was like a network. To avoid unprofitable services, he added a flow threshold constraint and considered the effect of the model given the abovementioned constraints and strategies of players. The results indicated that the flow threshold constraint was one of the most important factors in hub network design. Sasaki et al. [15] proposed a model for hub location in a competitive environment. Whereas Sasaki and Fukushima [13] and Sasaki [14] allowed using only one hub in the flow path from origin to destination, they studied more than one hub in their model. Eiselt and Marianov [16] proposed a hub location problem in a competitive environment in which customers based on the parameters of ticket price and flight time used the gravity-like function to select airlines. While in the previous models, the flow passing from origin to destination should only pass through one pat, the model offered by Eiselt and Marianov [16] allowed flows passing from origin to destination through different paths and hubs. They implemented their model for the follower player as a new entrant player to the Australian postal network.

Code Sharing Agreement is a commercial agreement in air transportation, in which two or more airlines share a common flight. In this paper, we consider the alliance between Mahan Airline, Iran Air, and Aseman Airline using Code Sharing Agreement. Lin [17] assessed the economic effects of alliance and cooperation between an international airline and a local airline. He suggested that two Stackelberg equilibrium points were achievable when a set of allied airlines was identified as the leader (or follower) player. Lin also demonstrated that in addition to an increase in the social welfare of travelers, the alliance might reduce international traveler excess and local direct travelers. Zou and Chen [18] investigated the effects of code sharing and global alliance on the performance of airlines, simultaneously. The results from a group of 81 airlines during the 2007-2012 period showed that the profit margin of an airline was positively associated with a few number of partners in code sharing. Yimga [19] investigated the relationship between alliances and on-time performance. He found evidence that code-sharing alliances would improve the on-time performance rate and allow for more efficient connections between flights. More efficient connections decrease the total travel time and thereby, increase the quality of a flight.

Adibi and Razmi [20] developed a two-stage stochastic programming for formulating stochastic uncapacitated multiple-allocation HLP in Iran. They considered three cases wherein: (1) flow was stochastic, (2) cost was stochastic, and (3) both flow and cost were stochastic. To evaluate their formulations, they used a case study based on the ten-node network of top cities of the air transportation network in Iran. Boukani et al. [21] developed two mathematical models for the capacitated single- and multiple-allocation p-hub median problems. They presented a robust optimization approach in order to consider uncertainty in the fixed cost of hub establishment and capacity of each hub. They showed that costs would increase when uncertainties were not considered in the model. An integrated hub location and revenue management problem was considered by Tikani et al. [22]. They presented a two-stage stochastic programming formulation to maximize the revenue made out of the transportation network and minimize hub installation costs. In the first stage, hub location, the link between the hub and the non-hub, and the protection level of tickets for different booking classes were determined. The booking limit of tickets could be obtained in the second stage. The demand was captured in a set of discrete scenarios under the average case. Robust optimization was proposed to handle the uncertainty in the demands of customer classes. The problem was demonstrated in the airline industry. Their results showed the efficiency of their methods for the instances with up to 25 nodes.

Nikoofal and Sadjadi [23] used the Bertimas and Sim's approach [11] to propose a robust model of the median hub location problem with uncertain travel costs. They compared the performance of the model with that of the min-max regret approach. Taking advantage of Bertsimas and Sim's approach [11], Ghaffari-Nasab et al. [24] proposed a robust model for the capacitated hub location problem (single allocation-multiple allocation) in which demand change was uncertain. They used uncertain demand only for capacity constraints and supposed that the objective function of demand was certain. Zetina et al. [25] proposed a robust counterpart for multiple-hub location problem in which (like Bertsimas and Sim's approach [11]) the level of conservativeness was controlled by using uncertainty budget. The problem was modeled in three modes: uncertain transportation cost, uncertain demand, and both simultaneously.

Chance-constrained programming was first introduced by Charnes and et al. [26]. Gao and Qin [27] developed a chance-constrained programming approach for p-hub center location problems under uncertain setting by characterizing travel time as an uncertain variable. Chance-constrained programming is often applicable to conditions where there is uncertainty in the data and parameters [28]. It is well known that when random input has a joint normal distribution, it can be reduced to a convex problem. Thus, it can be solved efficiently via convex programming techniques [29]. Accordingly, many problems in various areas can be formulated as chance-constrained programing. A series of applications has been reviewed in the literature [30].

3. Deterministic model formulation

In this section, we propose a new formulation for the capacitated multi-allocation median hub location problem in both cooperative and competitive environments. Then, sets, parameters, variables, objective function, and constraints of the model are presented. The notation used for the proposed deterministic model is presented in Table 1.

3.1. Objective function

The first term of the objective function Eq. (1) represents the total income from selling tickets to travelers passing from origin i (per each origin) to destination j (to all destinations) through hubs k of the follower airline. Accordingly, none of the hubs controlled by the follower airlines can be the origin or destination of travel. The second term of the objective function Eq. (1) represents the total income from selling tickets to travelers whose destinations are one of the hubs of the follower airlines. The third term of the objective function Eq. (1) represents the total income from selling tickets to travelers whose origins are one of the hubs of the follower airlines.

$$\begin{aligned} z = \max \sum_{k \in K} \sum_{i \in I-K} \sum_{j \neq i \in J-K} \sum_{s=1}^{2} \left(x_{kj}^{is} + y_{kj}^{is} + z_{kj}^{is} \right) \\ (\gamma_1 c_{ik} + \beta_1 c_{kj}) \\ + \sum_{i \in I-K} \sum_{j \in K} \sum_{s=1}^{2} \left(x_{jj}^{is} + y_{jj}^{is} + z_{jj}^{is} \right) (\gamma_2 c_{ij}) \\ + \sum_{k \in K} \sum_{j \in J-K} \sum_{s=1}^{2} \left(x_{kj}^{ks} + y_{kj}^{ks} + z_{kj}^{ks} \right) (\beta_2 c_{kj}) \\ (\beta_1 \leq \gamma_1 \leq \beta_2 \leq \gamma_2). \end{aligned} \tag{1}$$

Here is the reason why there is a relationship between ticket prices in the objective function Eq. (1) such that $\beta_1 \leq \gamma_1 \leq \beta_2 \leq \gamma_2$. When travelers use any other origin of the follower airline hubs to reach any other destination of the follower airline hubs, their profitability for the airlines will be higher and they deserve more discount. In other words, $\beta_1, \gamma_1 \leq \gamma_2$ and $\beta_1, \gamma_1 \leq \beta_2$. In this way, since travelers use a follower airline hub in the second part of their travel, airport costs are decreased. Also, because travelers from different origins aim to reach the same destination, taking advantage of a bigger airplane with lower ticket prices seems more logical; therefore $\beta_1 \leq \gamma_1$. When the origin or destination is one of the follower airline hubs, the amount of discount is lower due to shorter travel. Thus, when the origin is a follower airline hub, we have $\beta_2 \leq \gamma_2$. Because airport costs are lower, travelers from other origins are going to reach the aforementioned destination.

3.2. Constraints

Constraints (2) to (7) show that the flow is established when a hub exists:

$$\sum_{k \in N_{ij}^1} x_{kj}^{i1} \leq W_{ij} * p_1 \qquad \forall\, i \in I, \quad j \in J, \tag{2}$$

$$\sum_{k \in N_{ij}^2} x_{kj}^{i2} \leq W_{ij} * p_2 \qquad \forall\, i \in I, \quad j \in J, \tag{3}$$

$$\sum_{k \in M_{ij}^1} y_{kj}^{i1} \leq W_{ij} * q_1 \qquad \forall\, i \in I, \quad j \in J, \tag{4}$$

$$\sum_{k \in M_{ij}^2} y_{kj}^{i2} \leq W_{ij} * q_2 \qquad \forall\, i \in I, \quad j \in J, \tag{5}$$

$$\sum_{k \in P_{ij}^1} z_{kj}^{i1} \leq W_{ij} * r_1 \qquad \forall\, i \in I, \quad j \in J, \tag{6}$$

$$\sum_{k \in P_{ij}^2} z_{kj}^{i2} \leq W_{ij} * r_2 \qquad \forall\, i \in I, \quad j \in J. \tag{7}$$

According to Constraint (8), the maximum flow passing from all origins to all destinations through follower airline hubs equals $\left[W_{ij}\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)\right]$. Also, this constraint shows multiple allocations:

$$\sum_{s=1}^{2} \left(\sum_{k \in K} x_{kj}^{is} + \sum_{k \in K} y_{kj}^{is} + \sum_{k \in K} z_{kj}^{is} \right) \leq W_{ij} \left(\frac{U_f}{U_l}\right) \left(\frac{S_f}{S_l}\right) \left(\frac{t_l}{t_f}\right) \qquad \forall\, i, j. \tag{8}$$

Constraint (9) shows that the maximum incoming flow to hub K equals Γ_k:

$$\sum_{i \in I} \sum_{j \neq i \in J} \sum_{s=1}^{2} x_{kj}^{is} + y_{kj}^{is} + z_{kj}^{is} \leq \Gamma_k h_k \qquad \forall\, k \in K. \tag{9}$$

Constraint (10) shows that the minimum flow passing for using spoke k as a hub is equal to E_K:

Table 1. The notation used in the proposed deterministic model.

Sets:	
$N_{ij}^1 = \{ k \in K \vert c_{ik} + c_{kj} < C_{ij}, t_{ik} + t_k + t_{kj} \geq T_{ij}, V_k < V\}$	
$N_{ij}^2 = \{ k \in K \vert c_{ik} + c_{kj} < C_{ij}, t_{ik} + t_k + t_{kj} \geq T_{ij}, V_k \geq V\}$	
$M_{ij}^1 = \{ k \in K \vert c_{ik} + c_{kj} \geq C_{ij}, t_{ik} + t_k + t_{kj} < T_{ij}, V_k < V\}$	
$M_{ij}^2 = \{ k \in K \vert c_{ik} + c_{kj} \geq C_{ij}, t_{ik} + t_k + t_{kj} < T_{ij}, V_k \geq V\}$	
$P_{ij}^1 = \{ k \in K \vert c_{ik} + c_{kj} < C_{ij}, t_{ik} + t_k + t_{kj} < T_{ij}, V_k < V\}$	
$P_{ij}^2 = \{ k \in K \vert c_{ik} + c_{kj} < C_{ij}, t_{ik} + t_k + t_{kj} < T_{ij}, V_k \geq V\}$	
$K' = N_{ij}^1 \cup N_{ij}^2$	
$K'' = M_{ij}^1 \cup M_{ij}^2$	
$K''' = P_{ij}^1 \cup P_{ij}^2$	
$K = K' \cup K'' \cup K'''$	
Indices:	
I, i	Set and number of origin points
J, j	Set and number of destination points
K, k	Set and number of all hubs of the follower airlines
Parameters:	
c_{ik}	Ticket price for passing a traveler from origin i to hub k by the follower airlines
c_{kj}	Ticket price for passing a traveler from hub k to destination j by the follower airlines
C_{ij}	Total airfare of passing a traveler from origin i to destination j through the leader hub(s)
t_k	Average travel time delay for transferring travelers at the airport k
t_{ik}	Time needed to pass travelers from origin i to hub k by the follower airlines
t_{kj}	Time needed to pass travelers from hub k to destination j by the follower airlines
T_{ij}	Total time needed to pass from origin i to destination j by the leader hub(s)
W_{ij}	Total flow passing from origin i to destination j, which has already moved by the leader
Γ_k	Capacity of hub k
E_k	Minimum passing flow to consider spoke k as a hub
V_k	Service quality of airport k controlled by a follower airline
V	Average service quality of airports controlled by the leader airline
U_f	Quality of follower airline
U_l	Quality of leader airline
S_f	Safety of follower airline
S_l	Safety of leader airline
t_f	The average travel time delay to transfer travelers by the follower airlines
t_l	The average travel time delay to transfer travelers by the leader airline
f_c	Importance of cost for traveler
f_t	Importance of time for traveler
f_q	Importance of service quality of airport for traveler
M	Large positive number
$0 \leq p_1 < p_2 \leq 1$	Reduction factor of W_{ij} for sets N_{ij}^1, N_{ij}^2
$0 \leq q_1 < q_2 \leq 1$	Reduction factor of W_{ij} for sets M_{ij}^1, M_{ij}^2
$0 \leq r_1 < r_2 \leq 1$	Reduction factor of W_{ij} for sets P_{ij}^1, P_{ij}^2
$0 < \beta_1, \beta_2, \gamma_1, \gamma_2 \leq 1$	Discount factor for ticket price

Table 1. The notation used in the proposed deterministic model (continued).

Decision variables:	
h_k	Equals 1 if spoke k is selected as a hub, zero otherwise
x_{kj}^{i1}	Amount of passing flow by the follower from origin i to destination j through hub $k \in N_{ij}^1$
x_{kj}^{i2}	Amount of passing flow by the follower from origin i to destination j through hub $k \in N_{ij}^2$
y_{kj}^{i1}	Amount of passing flow by the follower from origin i to destination j through hub $k \in M_{ij}^1$
y_{kj}^{i2}	Amount of passing flow by the follower from origin i to destination j through hub $k \in M_{ij}^2$
z_{kj}^{i1}	Amount of passing flow by the follower from origin i to destination j through hub $k \in P_{ij}^1$
z_{kj}^{i2}	Amount of passing flow by the follower from origin i to destination j through hub $k \in P_{ij}^2$

$$E_k - \sum_{i\in I}\sum_{j\in J}\sum_{s=1}^{2}\left(x_{kj}^{is}+y_{kj}^{is}+z_{kj}^{is}\right) \le M(1-h_k)$$

$$\forall\ k \in K. \tag{10}$$

Constraint (11) guarantees that for each origin i and destination j, only variables x_{kj}^{is} related to airports k in the set K' can be non-zero and the remaining variables are equal to zero:

$$x_{kj}^{is} = 0$$

$$\forall\ i \in I, \quad j \in J; \quad \forall\ s = 1,2; \quad k \in K - K'. \tag{11}$$

Constraint (12) guarantees that for each origin i and destination j, only variables y_{kj}^{is} related to airports k in set K'' can be non-zero and the remaining variables are equal to zero:

$$y_{kj}^{is} = 0$$

$$\forall\ i \in I, \quad j \in J; \quad \forall\ s = 1,2; \quad k \in K - K''. \tag{12}$$

Constraint (13) guarantees that for each origin i and destination j, only variables z_{kj}^{is} related to airports k in set K''' can be non-zero and the remaining variables are equal to zero:

$$z_{kj}^{is} = 0$$

$$\forall\ i \in I, \quad j \in J; \quad \forall\ s = 1,2; \quad k \in K - K'''. \tag{13}$$

Constraints (14) and (15) show the domain of variables:

$$h_k \in \{0,1\} \qquad \forall\ k \in K, \tag{14}$$

$$x_{kj}^{i1}, x_{kj}^{i2}, y_{kj}^{i1}, y_{kj}^{i2}, z_{kj}^{i1}, z_{kj}^{i2} \in I^+ \cup \{0\}$$

$$\forall\ i \in I, \quad j \in J, \quad k \in K. \tag{15}$$

3.3. Determining the importance of travel time, travel cost, and service quality of hubs

f_c, f_t, and f_q represent the degrees of importance for the parameters of travel time, travel cost, and service quality of hubs for travelers. To determine them, we use data gathered by questionnaires from air travelers with different income levels, ages, and travel purposes. Also, we introduce the interpolation function of importance for time and cost in air travels by using a multivariate Lagrange interpolation method. Lagrange interpolation function with m variables and degree n is defined as Eq. (16) [31]:

$$f(X_1, X_2, \cdots, X_m) = \sum_{e_i.1\le n} \alpha_{e_i} X^{e_i}, \tag{16}$$

where $e_i = (e_{1i}, \cdots, e_{mi})$ and $\sum_{j=1}^{m} e_{ji} = 1$.

To uniquely determine f, we require:

$$\rho = \binom{n+m}{n},$$

independent points in Eq. (17):

$$(x_{1,i}, \cdots, x_{m,i}, f_i) \in R^{m+1}, \qquad 1 \le i \le \rho,$$

$$f_i = f(x_{1,i}, \cdots, x_{m,i}). \tag{17}$$

By calculating the degrees of importance for travel time, travel cost, and service quality of airport and by taking quality, safety, and travel time delay of the follower and leader airlines into account, parameters p_1, p_2, q_1, q_2, r_1, and r_2 are determined by Eq. (18):

$$p_1 = f_c * \left(\frac{U_f}{U_l}\right) * \left(\frac{S_f}{S_l}\right) * \left(\frac{t_l}{t_f}\right),$$

$$p_2 = (f_c + f_q) * \left(\frac{U_f}{U_l}\right) * \left(\frac{S_f}{S_l}\right) * \left(\frac{t_l}{t_f}\right),$$

$$q_1 = f_t * \left(\frac{U_f}{U_l}\right) * \left(\frac{S_f}{S_l}\right) * \left(\frac{t_l}{t_f}\right),$$

$$q_2 = (f_t + f_q) * \left(\frac{U_f}{U_l}\right) * \left(\frac{S_f}{S_l}\right) * \left(\frac{t_l}{t_f}\right),$$

$$r_1 = (f_c + f_t) * \left(\frac{U_f}{U_l}\right) * \left(\frac{S_f}{S_l}\right) * \left(\frac{t_l}{t_f}\right),$$

$$r_2 = (f_c + f_t + f_q) * \left(\frac{U_f}{U_l}\right) * \left(\frac{S_f}{S_l}\right) * \left(\frac{t_l}{t_f}\right). \tag{18}$$

4. Robust optimization

In this section, we present the robust counterpart of the model proposed in the previous section by employing the Bertsimas and Sim's approach [11]. Given the impact of seasonal changes (e.g., travel in high season) and weather conditions on the number of travelers, we consider w_{ij} as uncertain. The values of the uncertain parameter $\tilde{w}_{ij}$ are selected based on a symmetric distribution in the interval $[w_{ij} - \widehat{w}_{ij}, w_{ij} + \widehat{w}_{ij}]$ with an average of w_{ij}.

The notation used in the proposed robust model is presented in Table 2.

4.1. Robust model formulation

Given that uncertain parameters w_{ij} are included in Constraints (2) to (8), we employ the Bertsimas and Sim's approach [11] and by defining the variable H, we show the abovementioned constraints in the form of Constraints (19) to (26):

$$\sum_{k\in N_{ij}^1} x_{kj}^{i1} - W_{ij}(f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H \le 0 \qquad \forall\, i\in I,\quad j\in J, \tag{19}$$

$$\sum_{k\in N_{ij}^2} x_{kj}^{i2} - W_{ij}(f_c+f_q)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H \le 0 \qquad \forall\, i\in I,\quad j\in J, \tag{20}$$

$$\sum_{k\in M_{ij}^1} y_{kj}^{i1} - W_{ij}(f_t)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H \le 0 \qquad \forall\, i\in I,\quad j\in J, \tag{21}$$

$$\sum_{k\in M_{ij}^2} y_{kj}^{i2} - W_{ij}(f_t+f_q)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H \le 0 \qquad \forall\, i\in I,\quad j\in J, \tag{22}$$

$$\sum_{k\in P_{ij}^1} z_{kj}^{i1} - W_{ij}(f_c+f_t)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H \le 0 \qquad \forall\, i\in I,\quad j\in J, \tag{23}$$

$$\sum_{k\in P_{ij}^2} z_{kj}^{i2} \le W_{ij}(f_c+f_t+f_q)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H \le 0 \qquad \forall\, i\in I,\quad j\in J, \tag{24}$$

$$\sum_{s=1}^{2}\left(\sum_{k\in K} x_{kj}^{is} + \sum_{k\in K} y_{kj}^{is} + \sum_{k\in K} z_{kj}^{is}\right) - W_{ij}\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H \le 0 \qquad \forall\, i\in I,\quad j\in J, \tag{25}$$

$$1 \le H \le 1. \tag{26}$$

Given that Constraints (19) to (26) are defined for each i and each j, they are themselves subject to the maximum $i\times j$ constraint. Moreover, since each of these constraints has only one w_{ij}, we have $\Gamma_{ij}^{const} \in [0, |F^{const}|]$, where $F^{const} \subseteq \{0,1\}$. Based on the research conducted by Bertsimas and Sim [11], Constraint (19) can turn into Constraint (27):

$$\begin{aligned}&\sum_{k\in N_{ij}^1} x_{kj}^{i1} - W_{ij}(f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H \\ &+ \max_{\forall\,(i,j)\in F^1}\left\{\lfloor\Gamma_{ij}^1\rfloor\, \widehat{w}_{ij}(f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H \right.\\ &\left. + \left(\Gamma_{ij}^1 - \lfloor\Gamma_{ij}^1\rfloor\right)\widehat{w}_{ij}(f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H\right\} \\ &\le 0 \qquad \forall\, i\in I,\quad j\in J.\end{aligned} \tag{27}$$

Similarly, Constraints (20) to (26) are modified.

Table 2. The notation used in the proposed robust model.

Sets:	
$F^{const} = \left\{(i,j) \mid i\in I, j\in J, W_{ij} \text{ in constraint } const \text{ has noise } (\widehat{W}_{ij} > 0)\right\}$, $const \in \{1,\cdots,7\}$	
$\bigcup_{const=1}^{7} \left\|F^{const}\right\| \le \|I\| \times \|J\|$	
Parameters:	
$\widehat{W}_{ij}$	Deviation from W_{ij}
Γ_{ij}^{const}	Number of uncertain parameters w_{ij} for the constraint of $const$ ($const = 1,\cdots,7$)
Decision variables:	
$P_{ijf^{const}}^{const}$, z_{ij}^{const}	Dual auxiliary variables for constraint $const$ ($const = 1,\cdots,7$)
H	A variable defined to include the right-hand side vector b as a column in the technical matrix A

Then, by proving Theorem 1, we can show that Constraint (27) is equal to a linear optimization problem. The way of proving the theorem for other similar constraints is the same.

Theorem 1. For providing H^*, Function (28):

$$\beta_1(H^*,\Gamma^1_{ij}) = \max_{\forall\ (i,j)\in F^1}\left\{\lfloor\Gamma^1_{ij}\rfloor\,\widehat{w}_{ij}(f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H^* + (\Gamma^1_{ij}-\lfloor\Gamma^1_{ij}\rfloor)\,\widehat{w}_{ij}(f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H^*\right\}, \tag{28}$$

is equal to the linear programming problem below:

$$\beta_1(H^*,\Gamma^1_{ij})=\max\widehat{w}_{ij}(f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H^* z_{ij},$$

$$\text{s.t.}\qquad z_{ij}\le\Gamma^1_{ij}$$

$$0\le z_{ij}\le 1\qquad\forall\,(i,j)\in F^1. \tag{29}$$

Proof. We prove the theorem in two cases:

Case 1: Γ^1_{ij} is an integer. Therefore, $\Gamma^1_{ij}\in[0,1]$; then, $\Gamma^1_{ij}=1$. In this case, $\beta_1(H^*,\Gamma^1_{ij})$ is equal to:

$$\beta_1(H^*,\Gamma^1_{ij}) = \max_{\forall\ (i,j)\in F^1}\left\{\widehat{w}_{ij}(f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H^*\right\} = \widehat{w}_{ij}(f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H^*. \tag{30}$$

Because the optimization problem is a maximization one, we consider $z_{ij}=\Gamma^1_{ij}=1$. Therefore, the problem can become feasible and the optimal value for the objective function is equal to:

$$\widehat{w}_{ij}(f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H^*. \tag{31}$$

Hence, in this case, Eq. (29) is correct.

Case 2: Γ^1_{ij} is non-integer. Therefore, $\Gamma^1_{ij}\in[0,1]$; then, $\lfloor\Gamma^1_{ij}\rfloor=0$. In this case, $\beta_1(H^*,\Gamma^1_{ij})$ is equal to:

$$\beta_1(H^*,\Gamma^1_{ij}) = \max_{\forall(i,j)\in F^1}\left\{(\Gamma^1_{ij}-0)\widehat{w}_{ij}(f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H^*\right\} = \Gamma^1_{ij}\widehat{w}_{ij}(f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H^*. \tag{32}$$

Because the optimization problem is a maximization one, we consider $z_{ij}=\Gamma^1_{ij}$. Therefore, the problem can become feasible and the optimal value for objective function is equal to:

$$\Gamma^1_{ij}\widehat{w}_{ij}(f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H^*. \tag{33}$$

Hence, in this case, Eq. (29) is correct. ■

By using Theorem 1 and taking advantage of the strong duality theorem, it can be shown that the robust counterpart of the robust model presented in Section 3 is the following:

$$(1,10-15)$$

$$\sum_{k\in N^1_{ij}} x^{i1}_{kj} - W_{ij}(f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H + z^1_{ij}\Gamma^1_{ij} + \sum_{f^1\in F^1}P^1_{ijf^1}\le 0\qquad\forall\, i\in I,\quad j\in J,$$

$$\sum_{k\in N^2_{ij}} x^{i2}_{kj} - W_{ij}(f_c+f_q)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H + z^2_{ij}\Gamma^2_{ij} + \sum_{f^2\in F^2}P^2_{ijf^2}\le 0\qquad\forall\, i\in I,\quad j\in J,$$

$$\sum_{k\in M^1_{ij}} y^{i1}_{kj} - W_{ij}(f_t)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H + z^3_{ij}\Gamma^3_{ij} + \sum_{f^3\in F^3}P^3_{ijf^3}\le 0\qquad\forall\, i\in I,\quad j\in J,$$

$$\sum_{k\in M^2_{ij}} y^{i2}_{kj} - W_{ij}(f_t+f_q)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H + z^4_{ij}\Gamma^4_{ij} + \sum_{f^4\in F^4}P^4_{ijf^4}\le 0\qquad\forall\, i\in I,\quad j\in J,$$

$$\sum_{k\in P^1_{ij}} z^{i1}_{kj} - W_{ij}\,(f_c+f_t)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H + z^5_{ij}\Gamma^5_{ij} + \sum_{f^5\in F^5}P^5_{ijf^5}\le 0\qquad\forall\, i\in I,\quad j\in J,$$

$$\sum_{k\in P^2_{ij}} z^{i2}_{kj} - W_{ij}(f_c+f_t+f_q)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H + z^6_{ij}\Gamma^6_{ij} + \sum_{f^6\in F^6}P^6_{ijf^6}\le 0\qquad\forall\, i\in I,\quad j\in J,$$

$$\sum_{s=1}^{2}\left(\sum_{k\in K} x_{kj}^{is}+\sum_{k\in K} y_{kj}^{is}+\sum_{k\in K} z_{kj}^{is}\right)$$
$$- W_{ij}\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)H + z_{ij}^{7}\Gamma_{ij}^{7}$$
$$+\sum_{f^7\in F^7} P_{ijf^7}^{7}\leq 0 \qquad \forall\, i,j,$$

$$\widehat{W}_{ij}\left((f_c)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)\right)H\leq z_{ij}^{1}+P_{ijf^1}^{1}$$
$$\forall\, i\in I,\quad j\in J,\quad f^1\in F^1,$$

$$\widehat{W}_{ij}\left((f_c+f_q)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)\right)H\leq z_{ij}^{2}+P_{ijf^2}^{2}$$
$$\forall\, i\in I,\quad j\in J,\quad f^2\in F^2,$$

$$\widehat{W}_{ij}\left((f_t)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)\right)H\leq z_{ij}^{3}+P_{ijf^3}^{3}$$
$$\forall\, i\in I,\quad j\in J,\quad f^3\in F^3,$$

$$\widehat{W}_{ij}\left((f_t+f_q)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)\right)H\leq z_{ij}^{4}+P_{ijf^4}^{4}$$
$$\forall\, i\in I,\quad j\in J,\quad f^4\in F^4,$$

$$\widehat{W}_{ij}\left((f_c+f_t)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)\right)H\leq z_{ij}^{5}+P_{ijf^5}^{5}$$
$$\forall\, i\in I,\quad j\in J,\quad f^5\in F^5,$$

$$\widehat{W}_{ij}\left((f_c+f_t+f_q)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)\right)H\leq z_{ij}^{6}+P_{ijf^6}^{6}$$
$$\forall\, i\in I,\quad j\in J,\quad f^6\in F^6,$$

$$\widehat{W}_{ij}\left(\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right)\right)H\leq z_{ij}^{7}+P_{ijf^7}^{7}$$
$$\forall\, i\in I,\quad j\in J,\quad f^7\in F^7$$

$$1\leq H\leq 1,$$

$$z_{ij}^{const},P_{ijf^{const}}^{const}\geq 0$$
$$\forall\, i\in I,\quad j\in J,\quad f^{const}\in F^{const},$$
$$const\in\{1,2,\cdots,7\}. \tag{34}$$

5. Chance-constrained programming

In this section, taking into account the uncertain travel demand, we present the deterministic equivalent of the model presented in Section 3 using the chance-constrained programming approach. Suppose we have a probable scheduling problem as follows:

$$\max z=\sum_{i=1}^{n} c_i x_i,$$
$$\text{s.t.:}\qquad Ax\leq d,$$
$$P\left(\sum_{i=1}^{n} a_i x_i\leq\tilde{b}\right)\geq 1-\alpha,$$
$$x_i\geq 0\qquad i=1,\cdots,n, \tag{35}$$

where $\tilde{b}$ is an independent random variable and has a normal distribution. It can be shown that the probable constraint $P(\sum_{i=1}^{n} a_i x_i\leq\tilde{b})\geq 1-\alpha$ is equivalent to Constraint (36) [32]:

$$\sum_{i=1}^{n} a_i x_i\leq E\left(\tilde{b}\right)-Z_{1-\alpha}\sqrt{Var\left(\tilde{b}\right)}, \tag{36}$$

in which $Z_{1-\alpha}$ represents the point of the standard normal distribution so that $P(Z>Z_{1-\alpha})=\alpha$.

Assuming that the parameter W_{ij} of the random variable has normal distribution, it can be deduced from Constraint (36) that Constraints (2) to (8) change in Constraints (37) through (43).

$$\sum_{k\in N_{ij}^{1}} x_{kj}^{i1}\leq E\left(\tilde{W}_{ij}\right)*p_1-Z_{1-\alpha}p_1*\sqrt{Var\left(\tilde{W}_{ij}\right)}$$
$$\forall\, i\in I,\quad j\in J, \tag{37}$$

$$\sum_{k\in N_{ij}^{2}} x_{kj}^{i2}\leq E\left(\tilde{W}_{ij}\right)*p_2-Z_{1-\alpha}p_2*\sqrt{Var\left(\tilde{W}_{ij}\right)}$$
$$\forall\, i\in I,\quad j\in J, \tag{38}$$

$$\sum_{k\in M_{ij}^{1}} y_{kj}^{i1}\leq E\left(\tilde{W}_{ij}\right)*q_1-Z_{1-\alpha}q_1*\sqrt{Var\left(\tilde{W}_{ij}\right)}$$
$$\forall\, i\in I,\quad j\in J, \tag{39}$$

$$\sum_{k\in M_{ij}^{2}} y_{kj}^{i2}\leq E\left(\tilde{W}_{ij}\right)*q_2-Z_{1-\alpha}q_2*\sqrt{Var\left(\tilde{W}_{ij}\right)}$$
$$\forall\, i\in I,\quad j\in J, \tag{40}$$

$$\sum_{k\in P_{ij}^1} z_{kj}^{i1} \le E\left(\tilde{W}_{ij}\right) * r_1 - Z_{1-\alpha} r_1 * \sqrt{Var\left(\tilde{W}_{ij}\right)}$$

$$\forall\, i \in I, \quad j \in J, \tag{41}$$

$$\sum_{k\in P_{ij}^2} z_{kj}^{i2} \le W_{ij} * r_2 - Z_{1-\alpha} r_2 * \sqrt{Var\left(\tilde{W}_{ij}\right)}$$

$$\forall\, i \in I, \quad j \in J, \tag{42}$$

$$\sum_{s=1}^{2}\left(\sum_{k\in K} x_{kj}^{is} + \sum_{k\in K} y_{kj}^{is} + \sum_{k\in K} z_{kj}^{is}\right) \le E\left(\tilde{W}_{ij}\right)\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right) - Z_{1-\alpha}\left(\frac{U_f}{U_l}\right)\left(\frac{S_f}{S_l}\right)\left(\frac{t_l}{t_f}\right) * \sqrt{Var\left(\tilde{W}_{ij}\right)} \quad \forall\, i,j. \tag{43}$$

6. Case study

In this paper, we study a case to examine the proposed model in designing an optimal hub-and-spoke network for a follower airline, in which the Emirates airline is the leader and an alliance consisting of Mahan Airline, Iran Air, and Aseman Airline can play the role of the follower.

6.1. Determining the importance of travel time, travel cost, and service quality of hubs

To determine f_c, f_t, and f_q, we distributed 100 questionnaires to collect opinions of travelers at Imam Khomeini International Airport (with different income levels, ages, and travel purposes) about the importance of travel cost and travel time.

Age and income levels in the developed questionnaire are indicated in Table 3.

The averages of opinions for travelers with the same income, age, and travel purpose about the degrees of importance of travel time and cost by the type of travel (business, educational, or recreational) are summarized in Tables 4, 5, and 6, respectively.

Using the values in Tables 4 to 6 as well as multivariate Lagrange interpolation method, the quadratic interpolation functions for travel time and cost in air travels are presented as Eqs. (44) [31]:

$$f_{tw}(x,y) = -0.016x^2 + 0.0004y^2 + 0.014xy + 1.49x - 0.17y + 20.87,$$

Table 3. Age and income levels of travelers.

Age levels	[13,20)	[20,30)	[30,50)	[50,75)	More than 75
Income levels ($)	Less than 500	[500,1250]	(1250,3750]	More than 3750	—

Table 4. Degrees of the importance for travel time and cost in business travel.

Number of travelers	**Income level ($)**	**Age level**	**Degree of the importance of travel time (%)**	**Degree of the importance of travel cost (%)**
7	950	25	50	55
8	6000	40	80	25
6	1000	43	60	50
6	3000	45	70	30
8	7000	45	90	20
9	3200	60	75	35

Table 5. Degrees of the importance for travel time and cost in educational travel.

Number of travelers	**Income level ($)**	**Age level**	**Degree of the importance of travel time (%)**	**Degree of the importance of travel cost (%)**
3	980	18	25	75
2	480	29	20	85
4	1000	27	30	80
1	1700	23	35	60
3	900	33	30	85
3	2020	40	40	50

Table 6. Degrees of the importance for travel time and cost in recreational travel.

Number of travelers	Income level ($)	Age level	Degree of the importance of travel time (%)	Degree of the importance of travel cost (%)
6	900	17	35	55
9	1000	25	65	40
5	1100	48	50	50
7	450	35	70	20
5	8000	60	90	10
8	3000	77	85	30

$$\begin{aligned} f_{cw}(x,y) =& \ 0.039x^2 + 0.013y^2 - 0.0341xy \\ & - 2.57x - 0.051y + 102.34, \\ f_{te}(x,y) =& - 0.0157x^2 - 0.087y^2 + 0.0042xy \\ & + 1.187x + 3.23y - 15.25, \\ f_{ce}(x,y) =& - 0.0007x^2 - 0.164y^2 - 0.036xy \\ & + 0.97x + 2.73y + 53.07, \\ f_{th}(x,y) =& - 0.18x^2 - 0.148y^2 + 0.456xy + 7.67x \\ & - 12.19y + 8.58, \\ f_{ch}(x,y) =& \ 0.11x^2 + 0.064y^2 - 0.327xy - 4.24x \\ & + 12.5y + 27.67, \end{aligned} \tag{44}$$

where f_{tw}, f_{te}, and f_{th} represent the importance of time and f_{cw}, f_{ce}, and f_{ch} denote the importance of cost in business, educational, and recreational travels, respectively.

We use the coefficient of determination (R^2), which is presented as Eq. (45), not only to demonstrate the accuracy of the quadratic Lagrange interpolation functions in Eqs. (44), but also to show that how many opinions about the degrees of importance of travel time and cost are covered by those functions [33]:

$$R^2 = 1 - \frac{\sum_{i=1}^{N} \left(y_i - \widehat{y}_i\right)^2}{\sum_{i=1}^{N} \left(y_i - \bar{y}\right)^2}, \tag{45}$$

where N is the number of total observations, y is a dependent variable, $\bar{y}$ is the average value of y, and $\widehat{y}$ represents the values predicted by the interpolation functions. Using Eq. (45), it can be seen that the more the value of R^2, the higher the accuracy of the interpolation model will be.

The coefficients of determination for Functions (44) are derived by substituting their values into Eq. (46), which include:

$$\begin{aligned} & R^2_{f_{tw}}=0.9994, \quad R^2_{f_{cw}}=0.995, \quad R^2_{f_{te}}=0.9996, \\ & R^2_{f_{ce}}=0.9999, \quad R^2_{f_{th}}=0.995, \quad R^2_{f_{ch}}=0.9994. \end{aligned} \tag{46}$$

Given the values for coefficients of determination for Functions (44), it can be stated that these functions interpolate the points in Tables 4, 5, and 6 very accurately. Additionally, one may use them as a basis for calculating the degrees of importance of time and cost for travelers with different ages, income levels, and purposes.

Eq. (47) calculates the average value of Function (45) in their domain of definition:

$$\int_0^{10000} \int_{13}^{100} f(x,y)dxdy + \frac{\int_0^{10000} \int_{13}^{100} |f(x,y)| \, dxdy - \int_0^{10000} \int_{13}^{100} f(x,y)dxdy}{2}. \tag{47}$$

In Eq. (47), it is assumed that negative values are substituted by zero. The second term represents this.

Using Eq. (47), the average values of Function (37) are presented as Eqs. (48) and (49):

$$M_{f_{tw}}=67.44, \quad M_{f_{te}}=16.01, \quad M_{f_{th}}=56.91, \tag{48}$$

$$M_{f_{cw}}=39.31, \quad M_{f_{ce}}=13.36, \quad M_{f_{ch}}=37.89. \tag{49}$$

The average values in Eqs. (48) and (49) are equal to 47% and 30%, respectively. Therefore, it can be concluded that travel time has a high priority for nearly 47% of travelers. Even so, only 30% of travelers consider travel cost as an important factor. Without loss of generality, one may assume that the remaining 23% is related to a factor such as quality of airport and its service and others (f_q other than time and cost). Accordingly, we can conclude that the importance of time (f_t), cost (f_c), and service quality (f_q) for travelers is equal to Eq. (50):

$$f_t = 47\%, \qquad f_c = 30\%, \qquad f_q = 23\%. \tag{50}$$

According to Eq. (51) and assuming that $\frac{U_f}{U_l}=0.4$, $\frac{S_f}{S_l}=0.2$, and $\frac{t_l}{t_f}=0.25$, the values of parameters p_1, p_2, q_1, q_2, r_1, and r_2 are presented as Eqs. (43) by substituting the abovementioned values into Eq. (19):

$$p_1 = f_c * \left(\frac{U_f}{U_l}\right) * \left(\frac{S_f}{S_l}\right) * \left(\frac{t_l}{t_f}\right)$$

$$= 0.3 * (0.4) * (0.2) * (0.25) = 0.006,$$

$$p_2 = (f_c + f_q) * \left(\frac{U_f}{U_l}\right) * \left(\frac{S_f}{S_l}\right) * \left(\frac{t_l}{t_f}\right)$$

$$= (0.3 + 0.23) * (0.4) * (0.2) * (0.25) = 0.011,$$

$$q_1 = f_t * \left(\frac{U_f}{U_l}\right) * \left(\frac{S_f}{S_l}\right) * \left(\frac{t_l}{t_f}\right)$$

$$= 0.47 * (0.4) * (0.2) * (0.25) = 0.009,$$

$$q_2 = (f_t + f_q) * \left(\frac{U_f}{U_l}\right) * \left(\frac{S_f}{S_l}\right) * \left(\frac{t_l}{t_f}\right)$$

$$= (0.47 + 0.23) * (0.4) * (0.2) * (0.25) = 0.014,$$

$$r_1 = (f_c + f_t) * \left(\frac{U_f}{U_l}\right) * \left(\frac{S_f}{S_l}\right) * \left(\frac{t_l}{t_f}\right)$$

$$= (0.3 + 0.47) * (0.4) * (0.2) * (0.25) = 0.015,$$

$$r_2 = (f_c + f_t + f_q) * \left(\frac{U_f}{U_l}\right) * \left(\frac{S_f}{S_l}\right) * \left(\frac{t_l}{t_f}\right)$$

$$= (0.3+0.47+0.23) * (0.4) * (0.2) * (0.25) = 0.02. \qquad (51)$$

6.2. Data needed for implementation of the model

Origins, destinations, and hub candidates of the follower airlines are presented in Tables 7 and 8.

The service quality of airports, the capacity of hub candidates, and the minimum number of passing travelers to consider a point as a hub are presented in Table 9.

Based on the first row of Table 9, it is obvious that $V_k < V (\forall\ k)$.

The information summarized in Table 9 has been adjusted for a particular period (October 19–30, 2017).

The weekly average numbers of travelers transferred by the Emirates airport in the first six months of the year 2017 are presented in Table 10.

The other necessary input data for the model include:

- The time and cost of passing travelers between an origin and destination through the Dubai international airport by the Emirates Airline;
- The cost of passing one traveler between an origin and hub candidate and between a hub candidate and destination by the follower airlines;

Table 7. Origins and destinations.

Origin	**Label**	i_1	i_2	i_3	i_4	i_5
	Airport	Hamburg	London	Stockholm	Paris	Rome
Destination	**Label**	j_1	j_2	j_3	j_4	j_5
	Airport	Beijing	Bangkok	Kuala Lumpur	Delhi	Karachi

Table 8. Hub candidates for the follower airlines.

Label	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8
Airport	Imam Khomeini	Mashhad	Tabriz	Isfahan	Shiraz	Bandar Abbas	Yazd	Zahedan

Table 9. Service quality, capacity, and the minimum number of passing travelers to consider an airport as a hub.

Airport	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	**Dubai**[a]
Service quality (*)	3	3	3	3	3	1	1	1	4
Capacity	125,136	110,469	33,721	57,813	61,018	34,009	21,111	18,989	—
Minimum number	2,700	2,400	700	1,300	1,400	700	500	400	—

[a]: Currently, Dubai international airport is the only hub of the Emirates airline.

Table 10. Number of travelers.

Number of travelers	j_1	j_2	j_3	j_4	j_5
i_1	4020	3350	1340	4020	4020
i_2	4020	2680	1340	4020	3350
i_3	4020	4020	2010	4020	3350
i_4	2010	2010	670	2010	2010
i_5	4020	4020	1340	4020	4020

- The time of passing travelers between an origin and hub candidate and between a hub candidate and destination by the follower airlines.

Furthermore, according to the information provided by Iran Airports and Air Navigation Company, it is assumed that the average travel time delay for the transfer of travelers in the airport k of a follower airline is 60 minutes.

6.3. Implementation of the uncertainty model

Based on the abovementioned information, we implement a robust model and chance-constrained programming as follows.

6.3.1. Implementation of the robust model

We implement a robust model in a situation where all parameters w_{ij} and Γ_k are uncertain. By uncertainty of all parameters w_{ij}, we mean that:

$$\Gamma_{ij}^{const} = 1,$$

$$\forall\, i \in I, \quad j \in J, \quad const \in \{1, 2, \cdots, 7\}. \tag{52}$$

Also, we suppose that:

$$\widehat{w}_{ij} = 0.1 * w_{ij} \qquad \forall\, i \in I, \quad j \in J. \tag{53}$$

By using the GAMS software and through the Baron Solving method, it can be shown that the optimal value of the objective function for $\beta_1 = 0.8$ and $\gamma_1 = 0.9$ is $Z^* = \$702,464$, whereas the optimal value in the Marianov et al. [7] model is $Z^* = \$670,263$. Therefore, it can be concluded that the location of a hub in the present paper leads to a significant increase in the income of the follower airlines.

Table 11 shows the share of each hub of the follower airlines from the leader airline travelers.

According to Table 11, Imam Khomeini and Mashhad airports with 499 and 258 travelers have the largest international traffic among Iranian airports, respectively.

Given the results, it is clear that the Iranian airlines account for 2% of the travelers of the Emirates Airline (i.e., 1510 travelers).

Figure 1(a) and (b) show the path of passing travelers through hubs of the follower.

Table 11. Number of travelers passing the follower airports.

Number of travelers (airport)	j_1	j_2	j_3	j_4	j_5
i_1	$38(k_2)$ $43(k_7)$	$32(k_1)$ $36(k_5)$	$34(k_8)$	$38(k_1)$ $43(k_7)$	$81(k_1)$
i_2	$24(k_2)$ $57(k_5)$	$25(k_1)$ $29(k_2)$	$13(k_7)$ $14(k_4)$	$38(k_4)$ $43(k_8)$	$36(k_1)$ $32(k_2)$
i_3	$43(k_5)$ $38(k_1)$	$38(k_2)$ $43(k_6)$	$19(k_2)$ $21(k_6)$	$38(k_1)$ $43(k_7)$	$47(k_1)$
i_4	$28(k_5)$	$19(k_1)$ $21(k_6)$	$7(k_6)$	$19(k_4)$ $21(k_7)$	$21(k_1)$ $19(k_2)$
i_5	$31(k_2)$ $27(k_5)$ $43(k_8)$	$38(k_1)$ $43(k_6)$	$13(k_7)$ $14(k_6)$	$43(k_1)$ $38(k_8)$	$43(k_1)$ $38(k_2)$

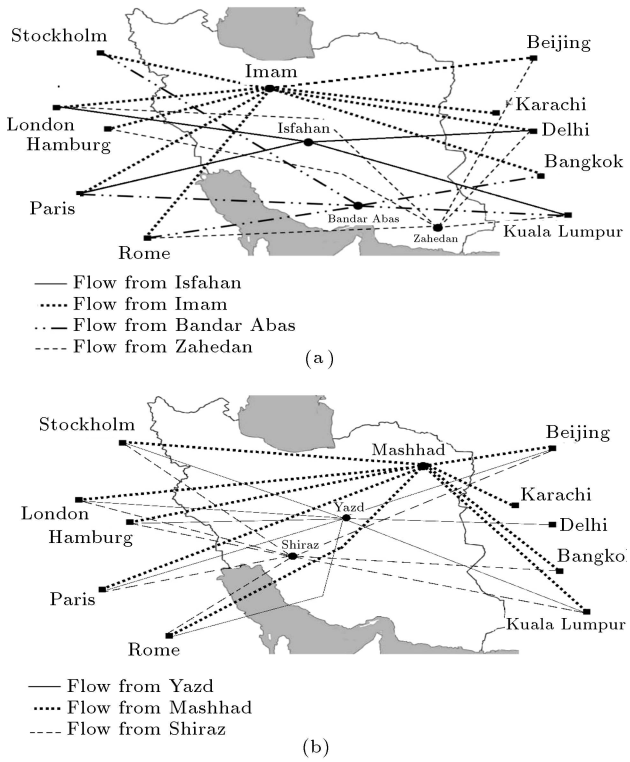

Figure 1. Hub-and-spoke network of the follower airlines (uncertainty mode).

6.3.2. Implementation of the chance-constrained programming

First, we show that the random variable W_{ij} has a normal distribution so that the distribution functions of "P-value" and "AD" can be calculated using the Mini tab software. These values are given in Table 12.

Table 12. Distribution functions.

Row	Distribution	AD	*P*-value
1	Normal	1.765	< 0.005
2	Logistic	1.653	< 0.005
3	Box-cox transformation	2.619	< 0.005
4	Gamma	4.566	< 0.005
5	Log logistic	4.69	< 0.005
6	Log normal	4.877	< 0.005
7	Exponential	9.495	< 0.003

Table 13. Average travel demand.

Number of travelers	j_1	j_2	j_3	j_4	j_5
i_1	3618	3015	1206	3618	3618
i_2	3618	2412	1206	3618	3015
i_3	3618	3618	1809	3618	3015
i_4	1809	1809	1206	1809	1809
i_5	3618	3618	1206	3618	3618

Table 14. Travel standard deviation.

Number of travelers	j_1	j_2	j_3	j_4	j_5
i_1	21	19	12	20	20
i_2	20	17	14	20	19
i_3	19	20	15	20	19
i_4	15	17	11	16	14
i_5	20	19	13	20	20

Given the values of "AD" and "*P*-value" in Table 12, it is concluded that the best distribution for displaying the parameters is the normal distribution. Therefore, taking into account the matrices of $E(\tilde{W}_{ij})$ and $\sqrt{Var(\tilde{W}_{ij})}$ through Tables 13 and 14 and placing them in Constraints (38) to (44), the optimal value of the objective function with $Z_{0.95} = 1.645$ is equal to $Z^* = \$695,082$.

Therefore, it can be concluded that if random constraints are possible with a probability of 95%, the value of the objective function will be $695,082.

Table 15 shows the share of each hub of the follower airlines from the leader airline travelers.

Since it is not possible to determine with certainty which chance-constrained programming and robust optimization methods provide an accurate approximation of the optimal problem [34], considering that the value of the objective function in the robust optimization method is greater than that in the random constrained programming, in the following, the result of the robust optimization method is considered as the basis for decision-making and analysis of the results.

Table 15. Number of travelers passing the follower airports.

Number of travelers (airport)	j_1	j_2	j_3	j_4	j_5
i_1	$38(k_2)$ $43(k_7)$	$32(k_1)$ $36(k_5)$	$14(k_8)$	$38(k_1)$ $43(k_7)$	$80(k_1)$
i_2	$24(k_2)$ $57(k_5)$	$25(k_1)$ $29(k_2)$	$12(k_7)$ $14(k_4)$	$38(k_4)$ $43(k_8)$	$35(k_1)$ $31(k_2)$
i_3	$43(k_5)$ $38(k_1)$	$38(k_2)$ $43(k_6)$	$19(k_2)$ $21(k_6)$	$38(k_1)$ $43(k_7)$	$67(k_1)$
i_4	$28(k_5)$	$19(k_1)$ $21(k_6)$	$7(k_6)$	$19(k_4)$ $21(k_7)$	$21(k_1)$ $19(k_2)$
i_5	$31(k_2)$ $27(k_5)$ $43(k_8)$	$38(k_1)$ $43(k_6)$	$13(k_7)$ $14(k_6)$	$43(k_1)$ $38(k_8)$	$43(k_1)$ $38(k_2)$

6.3.3. Comparison of costs and travel times

Based on the information gathered about the Emirates airline, it can be stated that the cost of passing the existing travelers is $1,252,088. Therefore, we can conclude that the alliance of the Iranian airlines and using the competitive edge they gain not only leads to an increase in their market share by $702,464, but also decreases the ticket costs for travelers by $549,624. The value of 549,624 is achieved by the cost of the Emirates airline (1,252,088) minus the cost of alliance among the Iranian airlines (702,464) for passing 1,510 travelers. If $\beta_1 = 0.8$ and $\gamma_1 = 0.9$, on average, there will be about $364 reduction in ticket price per traveler. Furthermore, it can be said that the time needed to transport all the existing travelers (in Table 11) by the Emirates airline is about 21,000 h, while this time for the alliance of the Iranian airlines is about 19,516. When we consider the alliance of the Iranian airlines and the hubs under their control, travel time per traveler decreases by about 59 minutes on average. This value is gained by the time of the Emirates airline (21,000 h) minus the time of the alliance among the Iranian airlines (19,516 h) for passing 1,510 travelers.

6.3.4. Sensitivity analysis

To assess the impact of the quality and safety of the follower airlines and the number of hubs under their control and examine changes in the discount factor (β_1, γ_1) for the value of z, we implement the model with different values of β_1, γ_1, U_f, U_l, S_f, S_l and $|K|$. The results are presented in Tables 12 to 14.

Table 16 shows the impact of discount factors β_1

Table 16. Sensitivity analysis of discount factors.

Discount factors	$\beta_1 = 1$, $\gamma_1 = 1$	$\beta_1 = 0$, $\gamma_1 = 1$	$\beta_1 = 0.9$, $\gamma_1 = 0.9$
z	822585	746016	740397

Table 17. Sensitivity analysis of the number of airports.

$\|K\|$	4	5	6	7
z	665843	688690	693449	702464

Table 18. Sensitivity analysis of quality and safety.

$\frac{U_f}{U_l}$, $\frac{S_f}{S_l}$	**0.5, 0.2**	**0.4, 0.3**	**0.5, 0.3**
z	878080	1053696	1317120

and γ_1 for the follower airlines on the value of the objective function.

Based on the last column of Table 16, it can be stated that a 0.1% increase in the discount factor β_1 leads to an increase in the value of z by $37,933. On the other hand, based on the second column of Table 16, a 0.1% increase in the discount factor γ_1 results in an increase in the value of z by $43,552. Therefore, we conclude that the discount factor γ_1 has a higher impact on profitability than the discount factor β_1. Also, based on the first column of Table 16 and the cost of passing travelers by the Emirates airline ($1,252,088), it can be concluded that a decrease in the ticket prices for travelers ($429,503) is possible when the follower airlines do not use the discount factor in passing the travelers.

Table 17 indicates the impact of the number of airports controlled by the follower airlines on the value of the objective function.

As shown in Table 17, an increase in the number of hub airports of the follower airlines leads to an increase in the value of the objective function. By simple calculations, it can be shown that adding one airport to the airports controlled by the follower airlines leads to a 1% increase in their gained revenue.

Table 18 shows the impact of the quality and safety of the follower airlines on the value of the objective function.

The first column of Table 18 shows that a 10% increase in the quality of the follower airlines leads to a 25% increase in the value of the objective function, while a 10% increase in the safety of the follower airlines (second column of Table 18) leads to a 50% increase in revenue of the follower airlines. Therefore, it can be concluded that the safety of the airlines is more effective in traveler attraction and revenue increase. The third column of Table 18 shows that a 10% increase in each of the parameters of quality and safety for the follower airlines leads to a nearly 88% increase in the value of the objective function.

Table 19. The impact of the uncertain parameters of travel demand.

Γ_{ij}^{const}	$\widehat{w}_{ij}$	z^*
10	$0.1(w_{ij})$	658474
	$0.2(w_{ij})$	632802
	$0.3(w_{ij})$	602738
15	$0.1(w_{ij})$	642389
	$0.2(w_{ij})$	600211
	$0.3(w_{ij})$	553851
20	$0.1(w_{ij})$	634977
	$0.2(w_{ij})$	585367
	$0.3(w_{ij})$	531586
25	$0.1(w_{ij})$	619656
	$0.2(w_{ij})$	554344
	$0.3(w_{ij})$	485050

Table 20. The impact of the number of uncertain parameters (travel demand).

Number of uncertain parameters	**Percentage of change in the objective function**
10	6
15	9
20	10
25	13

To examine the impact of the number of uncertain parameters, we consider Table 19.

According to Table 19, the higher the number of uncertain parameters in the model, the worse the value of the objective function will be. For example, when all the 33 w_{ij} parameters are uncertain with the following deviation value:

$$\widehat{w}_{ij} = 0.3 * w_{ij}, \tag{54}$$

the value of the objective function is equal to $485050, which is $217414 lower than the optimal value of a certain model ($702464).

Also, it is clear that as the number of uncertain parameters increases, the value of the objective function decreases accordingly and becomes more realistic. Table 20 shows the change percentage in the value of the objective function in a robust model in comparison with a certain model in various modes (the number of travelers has a 10% deviation from the real value).

7. Conclusion

In this paper, in order to organize the travelers and determine the exact location of hubs, we developed the model of Marianov et al. [7] by defining a new type of capture sets under uncertainty in which the follower airlines attracted a percentage of the leader airline travelers. We divided the nominated airports for the follower airline hubs into six independent categories by comparing the parameters of travel time, travel cost, and service quality of the airports of the follower and leader airlines. By employing a multivariate Lagrange interpolation function, we determined the degrees of importance for the time and cost of travel from the travelers' perspective. By using the degrees of importance for the time and cost of travel as well as comparing the parameters of quality, safety, and delays between the leader and follower airlines, we allocated travelers to the determined six sets. Based on the seasonal demand of the local travelers, we considered the travel demand of hubs as an uncertain parameter. To identify the deterministic equivalent forms for these categories of hub location models, the robust optimization method and chance-constrained programming were employed. The results showed that the income of the follower airlines would experience a considerable growth compared to the Marianov et al. [7] model and could absorb nearly 2% of the travelers of the leader airline due to lower travel costs and times. By examining the change in the parameters of the model, we observed that improvement in the quality and safety of the follower airlines and increase in the number of hubs under their control would have a positive impact on their gained revenue.

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