Scientia Iranica A (2016) 23(3), 907–917

SCIENTIA IRANICA

Sharif University of Technology

Scientia Iranica

Transactions A: Civil Engineering

www.scientiairanica.com

Research Note

Bed load sediment transport in sewers at limit of deposition

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Received 15 September 2014; received in revised form 17 June 2015; accepted 5 September 2015

KEYWORDS

Bed load;
Group Method of Data Handling (GMDH);
Limit on deposition;
Sediment transport;
Sensitivity analysis;
Sewer.

Abstract. Sedimentation is a problematic issue concerning sewer system design. In order to reduce sediment deposition in sewer systems, two new equations are presented with a smoothing function and Group Method of Data Handling (GMDH) to estimate minimum flow velocity. For this purpose, dimensional analysis is used to determine the factors affecting sediment transport at limit of deposition. These factors are categorized in five different groups: transport, transport mode, flow resistance, sediment, and motion. Six different models are presented for predicting the densimetric Froude number (Fr) using the smoothing function and GMDH. The models presented with these two methods are compared with existing equations. The results indicate that the equations proposed using the smoothing function (MAPE = 5.05, RMSE = 0.24, and AIC = −43.04) and GMDH (MAPE = 5.39, RMSE = 0.3, and AIC = 72.78) are more accurate than existing models. Furthermore, sensitivity analysis is performed to examine the efficacy of each of the dimensionless parameters presented by the best model in estimating Fr.

1. Introduction

Wastewater systems need to be designed in such a way that maximum flow can be transported in sewers. For these systems, self-cleansing through dry weather flow is required as well. Besides, combined systems must be designed for transporting collected wastewater and surface runoff. Due to low discharge rates, the probability of sectional sedimentation increases in the dry season. Therefore, such systems always encounter sedimentation problems. Sewer hydraulic capacity can be affected by sediments in two ways: Sediments on the pipe bed cause flow cross section reduction and increased hydraulic roughness due to the present sediments in flow [1]. Ackers et al. [2] showed that sediment deposition on the bed channel may increase the roughness height by up to 10% of the pipe diameter and decrease the pipe flow capacity by up to 20%.

Traditional methods employed to prevent sedimentation based on minimum shear stress or velocity were comprehensively addressed by Vongvisessomjai et al. [3]. These methods cannot precisely predict sediment conditions under different hydraulic circumstances, thus leading to under or overdesign [1,4,5]. Therefore, methods are still required to determine the sewer gradient by considering the factors influencing sediment transport in sewers with regards to discharge.

Numerous research works have been presented in the field of sediment transport at limit of deposition in the form of dimensional analyses [6] and semi-experimental equations [7]. Nalluri et al. [6] carried out an extensive experimental investigation on sediment transport in channels and developed empirical

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equations with high correlation coefficients. To apply a semi-experimental equation for sediment transport at deposition limit, May et al. [7] considered the forces affecting particles. They used a separate equation for the impact of initial velocity and expressed it as a parameter in the proposed equation. To overcome difficulties with the accurate and optimal design of stormwater sewer systems, Almedeij and Almohsen [8] made some remarks regarding Camp's criterion and recommended a method that necessitates more efficient stormwater sewer system development with lower flow strength limits. Vongvisessomjai et al. [3] studied sediment transport at limit of deposition for suspended and bed load conditions and deduced an equation for sediment transport at deposition limit. Also, based on sediment transport, a process of self-cleansing in rectangular sewers was proposed by Almedeij [9]. Ota and Perrusquìa [10] presented a semi-experimental equation for sediment transport at deposition limit in sewers by considering sediment particles and measuring the velocity in two different channels. A new equation incorporating sediment deposit thickness was proposed by Bong et al. [11] who also confirmed the existing equations for incipient motion in a rigid rectangular channel.

Recently, Soft Computing (SC) methods such as the Artificial Neural Network (ANN) [5], evolutionary algorithm [12], and Adaptive Neuro Fuzzy Inference System (ANFIS) [13,14] have been utilized to solve nonlinear problems. Ebtehaj and Bonakdari [1] evaluated the performance of ANFIS in forecasting sediment transport. Ebtehaj and Bonakdari [15] optimized the multilayer perceptron (MLP) network with three different kinds of training in terms of ability to estimate sediment transport in a clean pipe. Ebtehaj and Bonakdari [16] used evolutionary algorithms to optimize the weights of different layers in order to minimize the target function of ANN to estimate the densimetric Froude number. Among the most powerful SC techniques is the Group Method of Data Handling (GMDH) that is extensively used in different science fields, such as caloric and feed efficiency, air pollution, cyclone separators, discharge coefficients in side orifices, scour depth in clear-water and live-bed conditions, cohesive soils, downstream of a ski-jump bucket spillway, and scour around bridge piers and vertical piles [17-23].

The main objective of this article is to present equations that use the smoothing function and Group Method of Data Handling (GMDH) to achieve results superior to the existing equations. To achieve this objective, the parameters influencing sediment transport at limit of deposition are first examined and then different equations are presented by categorizing them into dimensionless groups. Three sets of experimental data comprising a vast range of parameters affecting sediment transport at limit of deposition are used to examine the accuracy of the equations. The best equation is selected and suggested. Subsequently, the accuracy of the selected equation is compared with that of the existing equations. Moreover, the effect of each dimensionless parameter on Fr prediction is studied.

2. Sediment transport equations

To calculate the limiting velocity at limit of deposition, sediment transport equations for pipe sewers have been proposed for dimensional analysis to survey the effective dimensionless parameters and semi-experimental equations. Non-cohesive sediment equations are divided into three groups through dimensional analysis. These parameters are characterized by the densimetric Froude number (Fr). The equations containing the transport parameter (φ) and flow parameter (Ψ) are in the first group and are related to each other in the form of $\Psi = a\varphi^b$ by the Darcy-Weisbach resistance equation ($S_0 = \lambda_s V^2/8gR$). Novak and Nalluri's [24] equation is among the equations proposed in the first group:

$$\varphi = \frac{C_V V R}{\sqrt{(s-1)gd^3}}, \tag{1}$$

$$\Psi = \frac{(s-1)d}{RS_0}, \tag{2}$$

$$\mathrm{Fr} = \frac{V}{\sqrt{g(s-1)d}} = 1.77 C_V^{1/3} \left(\frac{d}{R}\right)^{-1/3} \lambda_s^{-2/3}, \tag{3}$$

where R is the hydraulic radius, S_0 is the channel slope, V is the flow velocity, λ_s is the overall sediment friction factor, d is the median diameter of particles, s is the specific gravity of sediment, and C_V is the volumetric sediment concentration.

The equations in the second group are similar to those in the first group in terms of volumetric sediment concentration (C_V), median diameter of particles (d), and the overall sediment friction factor (λ_s) considered. The dimensionless particle number D_{gr} $\left(= d(g(s-1)/v^2)^{1/3}\right)$ is what differentiates these two equation groups. Azamathulla et al. [13] used these parameters and presented the following equation (Eq. (4)):

$$\mathrm{Fr} = \frac{V}{\sqrt{g(s-1)d}} = 0.22 C_V^{0.16} D_{gr}^{-0.14} \left(\frac{d}{R}\right)^{-0.29} \lambda_s^{-0.51}, \tag{4}$$

λ_s is calculated in Nalluri and Kithsiri's [25] equation (Eq. (5)), which is achieved using Kithsiri [26] and Mayerle's [27] experimental results as follows:

$$\lambda_s = 0.851 \lambda_c^{0.86} C_V^{0.04} D_{gr}^{0.03}, \tag{5}$$

where λ_c is the clear water friction factor of the channel.

Equations containing volumetric sediment concentration (C_V) and relative flow depth (d/R) to estimate the densimetric Froude number are applied in the dimensional analysis and comprise the third group of equations. Unlike the previous groups, the sediment friction factor is not used in these equations. Vongvisessomjai et al. [3] presented an equation as follows:

$$\mathrm{Fr} = \frac{V}{\sqrt{g(s-1)d}} = 4.31 C_V^{0.226} \left(\frac{d}{R}\right)^{-0.616}. \tag{6}$$

Semi-experimental equations are based on forces affecting particles in a balanced condition. A verification of the existing sediment transport equations at limit of deposition using 7 different data sets of May et al. [7] (presented by Ackers et al. [2]) yielded the following equations:

$$C_V = 3.03 \times 10^{-2} \left(\frac{D^2}{A}\right) \left(\frac{d}{D}\right)^{0.6} \left(\frac{V^2}{g(s-1)D}\right)^{1.5} \left(1 - \frac{V_t}{V}\right)^4, \tag{7}$$

$$V_t = 0.125 \left[g(s-1)d\right]^{0.5} \left[\frac{y}{d}\right]^{0.47}, \tag{8}$$

where V is the flow velocity, D is the pipe diameter, y is the flow depth, d is the median diameter of particles, A is the cross-sectional flow area, C_V is the volumetric sediment concentration, and V_t is the velocity of sediment incipient motion (Eq. (8)).

3. Data used

In this study, Ab Ghani [28], Ota et al. [29], and Vongvisessomjai et al.'s experimental results [3] are employed. Bed conditions at non-deposition and deposition limits were studied by Ab Ghani [28]. Experiments were done on three pipes with 154, 305, and 450 mm of diameter and 20.5 m of length. The data ranges in Ab Ghani's experiments [28] for non-deposition are as follows:

$$0.072 < d(mm) < 8.3, \; 0.153 < y/D < 0.8,$$

$$0.24 < V(\mathrm{m/s}) < 1.216, \; 0.033 < R(\mathrm{m}) < 0.136,$$

$$0.0007 < S_0 < 0.0056, \text{ and } 1 < C_V(\mathrm{ppm}) < 145.$$

To survey the deposition limit, Ota et al. [29] applied 6 different dimensions of the median diameter of particles (ranging from 0.71 to 5.61 mm). The authors carried out 20 different tests with 5 sediment ranges, while $d = 2$ mm was constant, to study the granulation effect on sediment transport at limit of deposition. The ranges of data applied in the experiments are as follows:

$$0.39 < y/D < 0.84, \; 0.515 < V < 0.736,$$

$$0.05 < R < 0.086, \text{ and } 16 < C_V < 59.$$

Vongvisessomjai et al. [3] carried out experiments using pipes with 100 and 150 mm of diameter and 16 m of length. Two sections 6 m apart from each other, 5.5 m from downstream, and 4.5 m from upstream were used to measure the flow velocity. The data ranges in Vongvisessomjai et al.'s experiments [3] are as follows:

$$4 < C_V(\mathrm{ppm}) < 90, \; 0.032 < R(\mathrm{m}) < 0.012,$$

$$0.002 < S_0 < 0.006, \; 0.2 < d(\mathrm{mm}) < 0.43,$$

$$0.2 < y/D < 0.4, \text{ and } 0.237 < V(\mathrm{m/s}) < 0.626.$$

Using three sets of data presented in this study, 218 different samples were placed in two groups (training and testing) through random sampling without replacement. 80% of the data (174 samples) was used for model prediction and 20% (44 samples) was used to test the model.

4. Methodology

In order to survey sediment transport at limit of deposition and present an equation to estimate the minimum velocity to prevent sedimentation (limiting velocity), the effective parameters on sediment particle movement should be identified. Previous experimental studies [6,7,27,28] have shown that the most influential parameters on predicting sediment transport in pipe channels are pipe diameter (D), specific gravity of sediment (s), dimensionless particle number (D_{gr}), volumetric sediment concentration (C_V), hydraulic radius (R), median diameter of particles (d), flow depth (y), overall sediment friction factor (λ_s), cross-sectional flow area (A), and flow velocity (V). To consider the effect of each parameter, 5 different groups (transport, transport mode, sediment, flow resistance, and motion) with different proposed dimensionless parameters are shown in Table 1. To study the impact of each parameter from the different dimensionless groups, the dimensionless parameters used to estimate Fr are presented in Table 2.

To achieve a model with an appropriate function, the smoothing function is applied in statistic and science research [30] as points that are closer to central process data cause greater weight and more points cause less weight. This function serves two purposes: first, a greater amount of extraction data

Table 1. Different groups with dimensionless parameters related to the prediction of sediment transport in sewer pipes.

Dimensionless groups	Type of parameters
Motion	$\mathrm{Fr} = V/\sqrt{gd(s-1)}$
Transport	C_V
Sediment	$D_{gr}, d/D, s$
Transport mode	$d/R, D^2/A, R/D$
Flow resistance	λ_s

Table 2. Dependent parameters in predicting Fr by considering the effects of dimensionless parameters.

Model number	Dependent parameters	Independent parameters
1	$\mathrm{Fr} = V\sqrt{gd(s-1)}$	$C_V, D_{gr}, d/R, \lambda_s$
2	$\mathrm{Fr} = V\sqrt{gd(s-1)}$	$C_V, D_{gr}, D^2/A, \lambda_s$
3	$\mathrm{Fr} = V\sqrt{gd(s-1)}$	$C_V, D_{gr}, R/D, \lambda_s$
4	$\mathrm{Fr} = V\sqrt{gd(s-1)}$	$C_V, d/D, d/R, \lambda_s$
5	$\mathrm{Fr} = V\sqrt{gd(s-1)}$	$C_V, d/D, D^2/A, \lambda_s$
6	$\mathrm{Fr} = V\sqrt{gd(s-1)}$	$C_V, d/D, R/D, \lambda_s$

helps data analysis process and second, it becomes stronger and more flexible. Different algorithms are used for smoothing, but the histogram method is the simplest and most appropriate. In this study, the modified Fejer smoothing function is used. Due to a periodical and average zero error, the Cos function is applied as the smoother. An objection may be that the Sin function shows the same characteristic. According to that, the Sin and Cos functions show an interval of $\pi/2$ and based on the obtained results, applying Cos is more practical.

5. Overview of GMDH

The Group Method of Data Handling (GMDH) is a self-organized method [31,32] presented by Ivakhnenko [33]. This method has different layers, each with different neurons. All neurons in this network have a similar structure with 2 inputs and 1 output. Also, each neuron map between inputs and output with 5 weights and 1 bias has the following equation:

$$y^*_{ik} = N(x_{i\alpha}, x_{i\beta}) = b^k + w_1^k x_{i\alpha} + w_2^k x_{i\beta} + w_1^k x_{i\alpha}^2 + w_2^k x_{i\beta}^2 + w_5^k x_{i\alpha} x_{i\beta},$$
$$i = 1, 2, ..., N, \tag{9}$$

where N is the input data, and $k = 1, 2, ..., C_m^2$ and $\alpha, \beta \in \{1, 2, ..., m\}$ where m is the number of neurons in the previous layer. In this method, the weight is calculated based on the least squares error method; then, it is given to each neuron as a certain constant value. The main feature of GMDH is that the neurons from the previous layer generate $C_m^2 = (m(m-1))/2$ neurons for the next layer. A number of generated neurons are eliminated to avoid network divergence [34]. The criterion for selecting and removing a set of neurons in a layer is the relative total square error (r_j^2) between the actual output (y_i) and the jth output neuron calculated as follows:

$$r_j^2 = \left(\sum_{i=1}^{N} (y_i - y^*_{ij})^2 / \sum_{i=1}^{N} y_i^2\right), \quad j \in \{1, 2, 3, ..., C_m^2\}, \tag{10}$$

where m is the number of selected methods in the previous layers.

The mapping between input and output variables in GMDH-type neural networks is done with the Volterra nonlinear function as follows:

$$\hat{y} = a_0 + \sum_{i=1}^{m} a_i x_i + \sum_{i=1}^{m}\sum_{j=1}^{m} a_{ij} x_i x_j + \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m} a_{ijk} x_i x_j x_k + ... \tag{11}$$

The structure is summarized as two-variable quadratic equations as follows:

$$y_i = f(x_{ip}, x_{iq}) = a_0 + a_1 x_{ip} + a_2 x_{iq} + a_3 x_{ip} x_{iq} + a_4 x_{ip}^2 + a_5 x_{iq}^2. \tag{12}$$

The function f has six unknown factors, which, for all pairs of dependent variables in a system, $\{(x_{ip}, x_{iq}), i = 1, 2, ..., N\}$, estimate the desired output $\{(y_i), i = 1, 2, ..., N\}$.

The following expression is minimized based on the minimum square error criterion:

$$E = \sum_{k=1}^{N} \left[(f(x_{ki}, x_{kj}) - y_i)^2\right] \rightarrow \mathrm{Min}. \tag{13}$$

Therefore, Eq. (14) with six unknown variables must be solved:

$$\begin{cases} y_1 = a_0 + a_1 x_{1p} + a_2 x_{1q} + a_3 x_{1p} x_{1q} + a_4 x_{1p}^2 + a_5 x_{1q}^2 \\ y_2 = a_0 + a_1 x_{2p} + a_2 x_{2q} + a_3 x_{2p} x_{2q} + a_4 x_{2p}^2 + a_5 x_{2q}^2 \\ \cdots \\ \cdots \\ y_N = a_0 + a_N x_{2p} + a_N x_{2q} + a_3 x_{Np} x_{2q} + a_4 x_{NP}^2 + a_5 x_{Nq}^2 \end{cases} \tag{14}$$

Eq. (14) can be rewritten as the following matrix:

$$Y = Aa, \tag{15}$$

where:

$$Y = \{y_1, y_2, y_3, ..., y_N\}, \tag{16}$$

$$a = \{a_0, a_1, a_2, a_3, a_4, a_5\}^T, \tag{17}$$

$$A = \begin{bmatrix} 1 & x_{1p} & x_{1q} & x_{1p}x_{1q} & x_{1p}^2 & x_{1q}^2 \\ 1 & x_{2p} & x_{2q} & x_{2p}x_{2q} & x_{2p}^2 & x_{2q}^2 \\ ... & & & & & \\ ... & & & & & \\ 1 & x_{Np} & x_{Nq} & x_{Np}x_{Nq} & x_{Np}^2 & x_{Nq}^2 \end{bmatrix} \tag{18}$$

Therefore, to solve this equation, it is necessary to calculate the reverse-like of a non-square matrix.

A significant issue in neural network design is determining the number of layers and output structures such as weight numbers, their initializing values, and a trigger function of each neuron to achieve proper mapping between input and output data.

Due to the high capability of evolutionary methods to find the global optimum in different calculation spaces such as the non-differentiable space, they are widely utilized in neural network design [35,36]. In this study, the Genetic Algorithm (GA) is used to design the GMDH structure. By categorizing the data into training and testing data, the errors of training and testing serve as two objective functions. With the input data, the GA calculates the objective function and offers the optimum structure of a GMDH-type neural network. In the hybrid GMDH, based on GA (GMDH-GA), shorter neurons can be mutated from several layers and be combined with longer neurons; therefore, in GMDH, the connections of neurons are not only limited to adjacent layers [37].

6. Results and discussion

The results of the comparison between previous equations and the equation proposed in this study are presented herein using the criteria of Root Mean Square Error (RMSE) and Mean Average Percentage Error (MAPE) defined below:

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\mathrm{Fr}_{\mathrm{Observed}} - \mathrm{Fr}_{\mathrm{Equation}})^2}, \tag{19}$$

$$\mathrm{MAPE} = \left(\frac{100}{n}\right)\sum_{i=1}^{n}\left(\frac{|\mathrm{Fr}_{\mathrm{Observed}} - \mathrm{Fr}_{\mathrm{Equation}}|}{\mathrm{Fr}_{\mathrm{Observed}}}\right). \tag{20}$$

The Root Mean Squared Error (RMSE) is a criterion of mean error, which has no upper limit and its lowest possible value is zero, representing the best estimation by the model. The Mean Average Percentage Error (MAPE) expresses the estimated value in relation to the observed value. MAPE is a non-negative index which has no upper limit. The considered model performs the best when this index value is zero.

The above indices present the estimated values as predicted mean errors and provide no information on the distribution of the equations' predicted error. Therefore, the presented models must be evaluated using other indices, such as Average Absolute Relative Error (AARE) and Threshold Statistics (TS). The index TSx shows the error distribution of the values predicted by each model for x% of predictions. This parameter is determined for different values of AARE. The value of the index TS is determined for x% of predictions as:

$$\mathrm{TS}_x = \frac{Y_x}{n} \times 100, \tag{21}$$

$$\mathrm{AARE} = \left(\frac{1}{n}\right)\sum_{i=1}^{n}\left(\frac{\mathrm{Fr}_{\mathrm{Observed}} - \mathrm{Fr}_{\mathrm{Equation}}}{\mathrm{Fr}_{\mathrm{Observerd}}}\right), \tag{22}$$

where Y_x is the number of predicted values among all data for each value of AARE smaller than x% in the above equation.

Moreover, since the above-mentioned criteria do not consider the variance and average of each model, simultaneously, the Akaike Information Criterion (AIC) is utilized to evaluate the proposed and existing equations. The model in this method was selected based on the following norm; the lower the value of the criterion, the better the model:

$$\mathrm{AIC} = n \times \log\left(\frac{1}{n}\sum_{i=1}^{n}(\mathrm{Fr}_{\mathrm{Observed}} - \mathrm{Fr}_{\mathrm{Equation}})^2\right) + 2 \times k, \tag{23}$$

where n and k are the number of samples and coefficients, and the exponents that are used in each equation, respectively. The Akaike criterion is a relative goodness measure of fit statistical model. The AIC based on the information entropy concept is grounded. When a given model to describe reality is used, it suggests a relative measure of lost information. Therefore, the AIC is used to describe the trade-off between bias and variance in model construction and complexity [4,38,39]. To introduce an equation that produces good results under different hydraulic conditions, a number of equations have been presented in the general form below using the models presented in Table 2, Cos (V) as smoothing function and nonlinear regression analysis in Minitab:

$$\mathrm{Fr} = \alpha C_v^{\beta}\left(D_{gr}, \frac{d}{D}\right)^{\chi}\left(\frac{d}{R}, \frac{D^2}{A}, \frac{R}{D}\right)^{\delta}\lambda_s^{\varepsilon}\,(\mathrm{Cos}(V))^{\varphi}. \tag{24}$$

Table 3 presents the results of Fr prediction by the models given in Table 2 using the statistical indices.

Table 3. Examining the accuracy of the presented model using statistical indices.

Diameter	Index	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
$D = 100$ mm - smooth bed	MAPE	2.80	19.88	11.42	**2.55**	15.24	2.80
	RMSE	0.16	1.04	0.59	**0.14**	1.14	0.16
	AIC	-14.45	6.46	0.12	**-15.82**	7.52	-14.45
$D = 150$ mm - smooth bed	MAPE	3.56	15.21	10.93	**3.31**	14.84	3.56
	RMSE	**0.20**	1.16	0.89	**0.20**	1.02	**0.20**
	AIC	**-13.32**	7.79	4.59	**-13.32**	6.20	**-13.32**
$D = 154$ mm - smooth bed	MAPE	7.57	54.31	53.61	**7.13**	8.84	7.57
	RMSE	0.33	1.87	1.83	**0.32**	0.43	0.33
	AIC	-11.18	15.76	15.45	**-11.92**	-7.06	-11.18
$D = 225$ mm - smooth bed	MAPE	2.35	10.67	11.05	**2.14**	4.73	2.35
	RMSE	**0.09**	0.47	0.41	**0.09**	0.23	**0.09**
	AIC	-75.73	-19.75	-23.84	**-76.47**	-43.65	-75.73
$D = 305$ mm - smooth bed	MAPE	7.04	20.24	18.72	**6.71**	5.34	**6.71**
	RMSE	0.28	1.35	0.69	**0.27**	0.96	**0.27**
	AIC	-43.24	17.42	-8.03	**-44.75**	4.25	**-44.75**
$D = 305$ mm - rough bed	MAPE	4.37	11.96	12.44	**4.30**	5.40	**4.30**
	RMSE	**0.20**	0.44	0.44	**0.20**	0.20	**0.20**
	AIC	-85.47	-40.23	-40.51	**-86.24**	-86.25	**-86.24**
$D = 450$ mm - smooth bed	MAPE	**4.46**	4.98	4.85	4.47	4.76	4.47
	RMSE	0.42	**0.41**	**0.41**	0.42	**0.41**	0.42
	AIC	**6.24**	6.22	6.23	**6.24**	6.22	**6.24**
All data	MAPE	4.59	19.61	17.57	**4.37**	8.45	4.54
	RMSE	0.24	0.96	0.75	**0.23**	0.63	0.24
	AIC	-33.88	-0.90	-6.57	**-34.61**	-16.11	-34.20

Models 2, 3, and 5 do not yield good results as they have relatively large RMSE and MAPE compared with other equations. It can generally be stated that models 1, 4, and 6 offer relatively better results and have nearly the same level of accuracy in approximately all states. Therefore, it can be stated that using the sediment parameter in both forms of D_{gr} and d/D and also using the transport mode in the forms of R/D and d/R will lead to relatively similar results. Besides, models 2 and 3 were used, which do not present good results according to the table. Model 4 provides the best results in almost all states. As mentioned earlier, the AIC index will be used to select the best model. This index has no minimum or maximum limit, and due to the presence of the log function in Eq. (23), it may even become negative. The smaller the value of this index, the stronger the accuracy of the model appears to be [4]. According to Table 3, the value of this index is AIC $= -36.61$ for all data, which is smaller than that of the other models. The equation obtained from model 4 is:

$$\mathrm{Fr} = 1.575 C_V^{0.178} \left(\frac{d}{D}\right)^{-0.021} \left(\frac{d}{R}\right)^{-0.594} \lambda_s^{0.171} \left(\mathrm{Cos}(V)\right)^{-0.395}. \tag{25}$$

Figure 1 compares the results obtained for predicting Fr using the model presented in this study (Eq. (25)) with the Fr values obtained from the experiments. The figure generally indicates that almost all values predicted by Eq. (25) are fairly consistent with the

Table 4. Examining the accuracy of the presented model (Eq. (25)), GMDH, and the existing equations using statistical indices.

	Training data			Testing data		
	MAPE	**RMSE**	**AIC**	**MAPE**	**RMSE**	**AIC**
Novak and Nalluri [24] - Eq. (3)	39.20	2.33	136.01	35.33	2.09	36.18
May et al. [7] - Eq. (7)	8.06	1.45	64.21	8.57	1.34	25.23
Vongvisessomjai et al. [3] - Eq. (6)	5.60	0.33	-162.10	8.56	0.38	-31.09
Azamathulla et al. [13] - Eq. (4)	20.28	1.14	29.46	18.89	1.00	10.12
Proposed equation - Eq. (25)	**4.32**	**0.23**	**-206.90**	**5.05**	**0.24**	**-43.04**
GMDH - Eq. (26)	4.89	0.27	-2.94	5.39	0.30	72.78

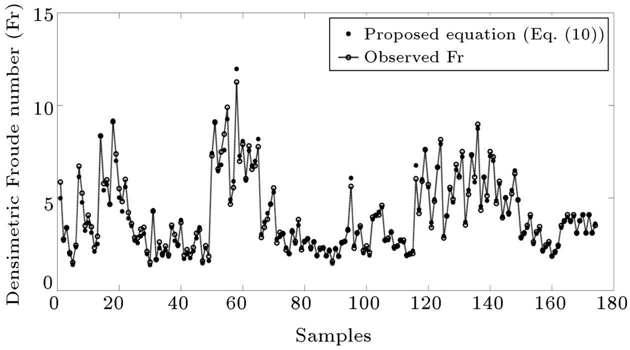

Figure 1. Comparing the performances of the presented model in predicting Fr with the experimental values (training).

actual values. The results of the statistical indices in Table 4 for the training state (MAPE= 4.32 and RMSE= 0.023) signify the accuracy of the predictions made by this equation.

In addition to Eq. (25) obtained with the most influential parameters on predicting Fr as model 4 (Table 2) and Cos (V) as the smoothing function, an equation presented by the GMDH-type neural network is as follows:

$$\mathrm{Fr} = 0.291 + 0.246Y_4 + 0.599Y_5 - 0.114Y_4^2 - 0.0683Y_5^2 + 0.202Y_4 \times Y_5, \tag{26a}$$

$$Y_1 = 10^3 \times \left[0.00354 - 0.0968d/R + 0.202\lambda_s + 1.386(d/R)^2 - 1.988\lambda_s^2 - 2.792d/R \times \lambda_s\right], \tag{26b}$$

$$Y_2 = 10^5 \times \left[0.000068 + 0.546C_V - 0.00203d/R - 0.00174C_V^2 + 0.0198(d/R)^2 - 8.565C_V \times d/R\right], \tag{26c}$$

$$Y_3 = 10^4 \times \left[0.000847 - 0.0196d/R - 0.0116d/D - 0.0169(d/R)^2 - 0.994(d/D)^2 + 1.0865d/R \times d/D\right], \tag{26d}$$

$$Y_4 = 10^2 \times \left[0.068 - 0.00295Y_1 - 0.732d/R + 0.000789Y_1^2 + 6.409(d/R)^2 - 0.151Y_1 \times d/R\right], \tag{26e}$$

$$Y_5 = 2.594 + 0.125Y_2 - 0.427Y_3 - 0.316Y_2^2 - 0.368Y_3^2 + 0.839Y_2 \times Y_3. \tag{26f}$$

Figure 2 compares the results of the existing equations, Eqs. (25) and (26), with the experimental results

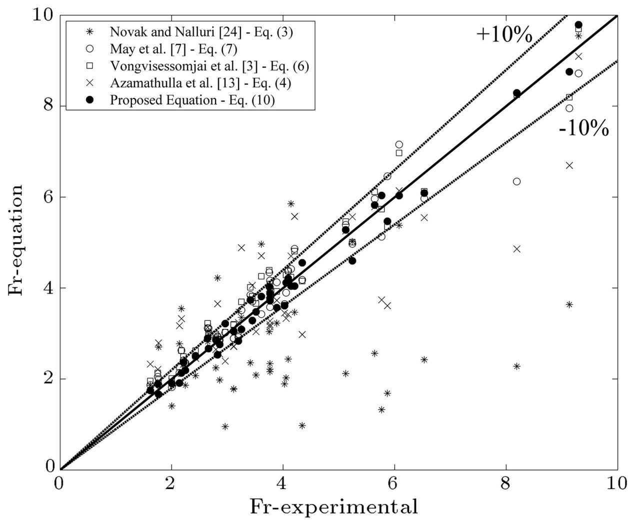

Figure 2. Comparing the Fr results predicted by the equations presented in this study and the existing limit of deposition equations with actual Fr (testing).

that were obtained from testing the model selected through random sampling without replacement. The figure indicates that Eq. (25) and GMDH (Eq. (26)) predict Fr values with a relative error of less than 10% for most samples. Also, according to Table 4, these equations predict well with MAPEs of 5.05 and 5.39, respectively. Therefore, equations 25 and 26 do not display a significant decrease in prediction accuracy when the data used for Fr prediction differs from the data used to train the model compared with the training state, since the values of the RMSE and MAPE statistical indices only slightly differ from each other for these equations. Novak and Nalluri [24] and Azamathulla et al.'s equations [13] are not very accurate. Figure 2 shows that Novak and Nalluri's equation [24] often predicts values below the actual values. Therefore, it leads to solid matter deposition in the channel. Table 4 indicates MAPE of 35.53 for this equation, which is 7 times larger than that in Eq. (25). As a result, this equation cannot be used with confidence. Azamathulla et al.'s equation [13] makes predictions that are either greater or lower than actual values, which leads to uneconomical design and sediment deposition, respectively. Table 4 presents the statistical index values for this equation as MAPE = 18.89 and RMSE = 1.00, which support the suggested equation's higher accuracy to predict Fr. May et al. [7] and Vongvisessomjai et al.'s equations [3] yield better results than the equations of Novak and Nalluri [24] and Azamathulla et al. [13]. By considering Table 4 and the presented statistical indices, the suggested equation is more accurate than the latter two equations mentioned.

Figure 3 presents the error distribution of different equations and signifies that the error distribution of Eqs. (25) and (26) shows better Fr prediction than existing equations. It is clear that approximately 90% of predictions made by Eqs. (25) and (26) have relative errors of less than 10% while Vongvisessomjai et al. [3], Azamathulla et al. [13], May et al. [7], and Novak and Nalluri's [24] equations present about 25%, 78%, 72%, and 13% (respectively) of the predictions with a relative error of less than 10%. Some of the existing equations also predict with high error percentage, whereby the greatest relative errors of Eqs. (4), (6), (7), and (3) are 68%, 22%, 36%, and 80%, respectively, while the largest relative errors of Eqs. (25) and (26) are 14% and 16%, respectively.

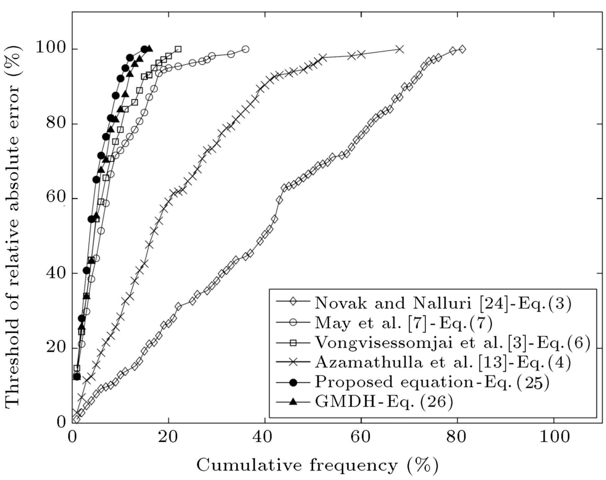

Figure 3. Error distribution of Eq. (25), GMDH, and the existing equations for training and testing.

However, the equations containing the smoothing function (Eq. (25)) and GMDH (Eq. (26)) can highly accurately predict Fr in both training and testing stages. Thus, the number of coefficients in each equation is important for easy calculation in practical engineering. AIC is the next index presented in this study to select the best model by considering accuracy and the number of coefficients in a specific equation [4]. This equation consists of two parts: The first part considers the logarithmic subtraction of predicted results from the actual results, and the second part considers the effects of the coefficients used in the model. Since this equation contains log, it is possible for the value of this index to be negative as well. Therefore, the smallest value presented by this equation (considering the positive and negative signs) seems to be the best answer. Considering the value of this index that is related to the existing and the proposed Eqs. (25) and (26), Eq. (25) has the smallest values with AIC = -43 and -206.9 for testing and training data, respectively.

Table 5 examines the effect of not considering each of the dimensionless parameters on the selected model (model 4). It can be seen that not considering each of the parameters related to the sediment, flow resistance, transport, and transport mode dimensionless groups in Table 2 leads to a decline in Fr prediction accuracy such that in the first mode, where the parameters of all four groups are considered in predicting Fr, which is related to the "motion" group, all three indices presented yield the best result. Not ignoring the parameters of the "flow resistance" and "sediment" groups (models 2 and 4) has no significant effect on prediction accuracy; the predictions made by these two modes have a relative error of approximately 7%, while not using the

Table 5. Sensitivity analysis of the dimensionless parameters presented in Eq. (25).

Models	MAPE	RMSE	AIC
1- Fr $= f(C_V, d/D, d/R, \lambda_s)$	**4.47**	**0.24**	**-261.94**
2- Fr $= f(C_V, d/D, d/R)$	7.21	0.65	-222.00
3- Fr $= f(C_V, d/D, \lambda_s)$	12.09	0.61	-83.89
4- Fr $= f(C_V, d/R, \lambda_s)$	7.03	0.60	-243.39
5- Fr $= f(d/D, d/R, \lambda_s)$	11.74	0.72	-50.90

parameters of the "transport" and "transport mode" groups (models 3 and 5) has significant impact on the results and the predicted values are less accurate than those of other models.

7. Conclusion

One of the crucial subjects related to the transport of flow passing through wastewater networks is sediment transport. In this study, the parameters affecting sediment transport at limit of deposition were categorized in five different groups, including flow resistance, transport, transport mode, transport, and sediment. The aim of this study was to predict the parameter Fr. In order to propose an equation for Fr prediction, three different sets of data were used, including a vast range of different parameters and two methods, i.e. the smoothing function and GMDH. The proposed equation considers parameters that produce results superior to other parameters; thus, Eq. (25) was used to compute Fr. The conducted study indicates that the presented equation is fairly accurate for different pipe diameters as well as rough and smooth beds. Comparing Eq. (25) and GMDH with the existing equations shows that Eq. (25) (MAPE = 5.05, RMSE = 0.24, and AIC = -43.04) and GMDH (MAPE = 5.39, RMSE = 0.3, and AIC = 72.78) are more accurate than the existing equations. The results also show that the equation presented with the smoothing function (Eq. (25)) has a lower AIC than GMDH, because Eq. (25) requires a lower coefficient than GMDH to predict Fr. Therefore, Eq. (25) was selected as the best among all equations. In addition, the effect of each dimensionless parameter presented in the model on predicting Fr was examined through sensitivity analysis. According to the results, using all parameters from the four groups, simultaneously, will yield the best outcome.

Nomenclature

A	Cross-sectional flow area
C_V	Volumetric sediment concentration
D	Pipe diameter
D_{gr}	Dimensionless particle number ($= d(g(s-1)/v^2)^{1/3}$)
d	Median diameter of particles
Fr	Densimetric Froude number
M	Number of neurons in the previous layer (Eq. (9))
m	Number of selected methods in the previous layers (Eq. (10))
N	Input data (Eq. (9))
R	Hydraulic radius
r_j^2	Relative total square error between the actual output and the jth output neuron (Eq. (10))
s	Specific gravity of sediment
S_0	Channel slope
V	Flow velocity
V_t	Velocity of the incipient motion of sediment (Eq. (8))
y	Flow depth
y_i	Actual output
y_{ij}^*	jth output neuron
φ	Transport parameter c
Ψ	Flow parameter (Eq. (2))
λ_c	Clear water friction factor of the channel
λ_s	Overall sediment friction factor

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