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Research Note

Development of a new variable repetitive group sampling plan based on EWMA yield index

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Abstract. The present study aims to develop a new variable repetitive group sampling plan using the Exponentially Weighted Moving Average (EWMA) statistic based on the yield index for the submitted lot. The optimal parameters of the proposed plan were determined under three scenarios based on the Average Sample Number (ASN). ASN should be minimized to decrease the inspection time and cost using the optimization problem for the required quality levels and sundry combinations of producer's and consumer's risks. A comparison study was conducted to determine the efficiency of the proposed plan. Furthermore, the proposed plan was presented with an example elaborating its applicability in the industry. The proposed plan was compared with the single sampling plan and repetitive group sampling plan based on the yield index. The upshots were tabulated for different quality levels. The obtained results demonstrated that with respect to performance, the proposed sampling plan was more lucrative than the existing sampling plans in terms of ASN.

1. Introduction

Quality control and improvement is nowadays a significant business strategy for many industries, manufacturers, and distributors. Quality is itself a competitive advantage. A trade that can satisfy customers through quality control and improvement can achieve success in the global market. As a result, companies compete for quality improvement of their products using a variety of statistical techniques and tools. Inspection of the final product is always done based on acceptance sampling plans which are important tools for promoting product quality in factories. Generally, there are three procedures for a submitted lot:

1. The lot is accepted with no inspection;
2. 100% inspection is carried out, i.e., every item in the lot is inspected and all defective units are removed (faulty products are returned to the supplier, reworked, and replaced with known good items or discarded);
3. Acceptance sampling plans [1] are taken into account.

Control charts and acceptance sampling plans are two statistical tools that are widely used in the industries. The evaluation process is completed through the control charts, and the inspection of the products is carried out using the acceptance sampling plans. There are several different ways to classify acceptance sampling plans. One major classification is by data

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type, i.e., based on variables and attributes. Variables are quality characteristics that are measured on a numerical scale. Attributes are quality characteristics that are introduced on a "go, no-go" basis. The primary advantage of variable sampling plan is that the same Operating Characteristic (OC) curve can be obtained with a sample size smaller than that required by an attribute sampling plan [1]. The variable sampling plan is more informative than the attribute sampling plan. Therefore, the variable sampling plan is usually used when the inspection is destructive or expensive. More details about acceptance sampling plans can be observed in the studies conducted by Jennett and Welch [2], Pearn and Wu [3], Pearn and Wu [4], Yen and Chang [5], Wu et al. [6], Fallah Nezhad and Nesaee [7], Arizono et al. [8], Wu and Liu [9], Vangjeli [10], Fallah Nezhad et al. [11], Fallah Nezhad and Zahmatkesh Saredorahi [12], and Fallah Nezhad and Golbafian [13].

Many types of acceptance sampling plans have been introduced in the literature so far. The sampling plan presented in this study is the Repetitive Group Sampling plan (RGS) using the Exponentially Weighted Moving Average (EWMA) statistic based on yield index for the submitted lot. An RGS design, which is an extension of the single sampling scheme, is employed when sampling is destructive and costly [14]. Sherman [15] put forward the RGS scheme. A variable RGS plan based on the process capability index was developed by Wu [16] using the concept of Taguchi loss function. Yen et al. [17] developed a variable RGS plan based on one-sided process capability indices for the one-sided specification. Wang [18] developed a single sampling plan based on an EWMA model for linear profiles. Yan et al. [19] extended the variable RGS plan based on the coefficient of variation. Fallah Nezhad et al. [20] represented an optimization problem for the acceptance sampling plan based on the Maxima Nomination Sampling (MNS) method. Wu and Liu [21] discussed the concept of RGS to develop a new variable sampling plan for lot sentencing on the basis of process fraction nonconforming. Wang and Tamirat [22] designed a sampling plan based on the EWMA model with a yield index of lot sentencing for autocorrelation among polynomial profiles. Nesaee and Fallah Nezhad [23] investigated variable sampling plans based on the yield index S_{pk}.

A majority of the accessible acceptance sampling plans in the literature have not utilized the obtained information from the past and made a decision about the acceptance or rejection of products based on the already available information. This type of sampling plans is called "without memory plans" [24]. Generally, EWMA statistics have been widely used in control charts to detect small shifts in the competition of the traditional Shewhart control chart. Yen et al. [25] employed EWMA statistics based on the yield index to develop a sampling plan that took into consideration the connection between the process performance and manufacturing specifications. Aslam et al. [26] presented an improved acceptance sampling plan based on EWMA statistics. Azam et al. [27] offered a repetitive acceptance sampling plan based on EWMA statistics using the regression estimator. An acceptance sampling plan using the modified EWMA statistic was presented by Khan et al. [28].

In this study, the plan proposed by Yen et al. [25] was extended. In other words, the RGS plan was designed for lot sentencing using the EWMA statistics based on the yield index. In order to obtain the required parameters of the proposed sampling plan, the optimization problem was employed, taking into account a number of smoothing constant values. Moreover, Variable Repetitive Group Sampling (VRGS) plan based on the EWMA yield index was compared with both VRGS plan and conventional single sampling plan on the basis of yield index. The rest of this study is organized as follows. The process yield, S_{pk}, and $\hat{S}_{pk}^{EWMA_i}$ are introduced in Section 2. The mathematical model and required parameters of the proposed plan are presented in Section 3. A comparative study and the obtained results are demonstrated in Section 4. An application example is given in Section 5. Finally, conclusion remarks are presented in Section 6.

2. Process capability index

Kane [29] proposed the simplest process capability index C_p at the end of the twentieth century. Kotz and Johnson [30] and Wu et al. [31] developed advanced capability indices used for evaluating the process performance from different aspects. Three common capability indices are expressed as follows:

$$C_a = 1 - \frac{|\mu - M|}{d}, \tag{1}$$

$$C_p = \frac{USL - LSL}{6\sigma}, \tag{2}$$

$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\}, \tag{3}$$

where $d = \left(\frac{USL-LSL}{2}\right)$, USL is the upper specification limit, LSL the lower specification limit, and $M = \left(\frac{USL+LSL}{2}\right)$ the midpoint between two specification limits. The degree of process centering (the ability to cluster around the midpoint) is measured by the index C_a. The index C_p measures the process precision with respect to two-sided specification limits. The index C_{pk} considers process variation magnitude and departure from the midpoint; however, it only provides an approximate measure of the actual process yield. This significant observation is of significance in developing a new yield index S_{pk} which can be regarded as a smooth version of C_{pk} [21]. Boyles [32] proposed the yield index S_{pk} that could provide an exact measurement of the process yield for normally distributed processes. The index S_{pk} is defined as follows:

$$S_{pk} = \frac{1}{3}\phi^{-1}\left\{\frac{1}{2}\phi\left(\frac{USL-\mu}{\sigma}\right)+\frac{1}{2}\phi\left(\frac{\mu-LSL}{\sigma}\right)\right\}, \tag{4}$$

where $\phi^{-1}(\cdot)$ is the inverse function of standard normal CDF $\phi(\cdot)$ [9].

In addition, S_{pk} establishes a relationship between manufacturing specifications and process performance which can precisely measure the process yield. Table 1 demonstrates the corresponding process yields as well as nonconformities in Parts Per Million (PPM) for S_{pk}. In practice, since the process parameters μ and σ are unknown, they should be estimated from the collected sample data. To estimate the yield measure, $\hat{S}_{pk}$ is expressed as follows [9]:

$$\hat{S}_{pk} = \frac{1}{3}\phi^{-1}\left\{\frac{1}{2}\phi\left(\frac{USL-\bar{x}}{s}\right)+\frac{1}{2}\phi\left(\frac{\bar{x}-LSL}{s}\right)\right\}$$

$$= \frac{1}{3}\phi^{-1}\left\{\frac{1}{2}\phi\left(3\hat{C}_p\hat{C}_a\right)+\frac{1}{2}\phi\left(3\hat{C}_p\left(2-\hat{C}_p\right)\right)\right\}, \tag{5}$$

where $\bar{x} = \sum_{i=1}^{n} x_i/n$ is the sample mean and $s = \left[\sum_{i=1}^{n}(x_i-\bar{x})^2/(n-1)\right]^{\frac{1}{2}}$ is the sample standard deviation. Since the exact distribution of $\hat{S}_{pk}$ is mathematically intractable even under the normal distribution, Lee et al. [33] provided a normal approximation for the distribution of $\hat{S}_{pk}$ using the Taylor expansion technique. The estimator $\hat{S}_{pk}$ can be approximately presented as follows:

Table 1. S_{pk} values and the corresponding nonconformities [17].

Yield	PPM	S_{pk}
0.997300204	2699.796	1.00
0.999033152	966.848	1.10
0.999681783	318.217	1.20
0.999903807	96.193	1.30
0.999933927	66.073	1.33
0.999973309	26.691	1.40
0.999993205	6.795	1.50
0.999998413	1.587	1.60
0.999999456	0.544	1.67
0.999999660	0.340	1.70
0.999999933	0.067	1.80
0.999999988	0.012	1.90
0.999999998	0.002	2.00

$$\hat{S}_{pk} \approx S_{pk} + \frac{1}{\sigma\sqrt{n}}\frac{W}{\phi(3S_{pk})}, \tag{6}$$

where:

$$W = \begin{cases} \sqrt{\frac{n}{2}}\left[\frac{a(s^2-\sigma^2)}{\sigma}\right] - \sqrt{n}\frac{b(\bar{x}-\mu)}{\sigma} & \text{for } \mu < M \\ \sqrt{\frac{n}{2}}\left[\frac{a(s^2-\sigma^2)}{\sigma}\right] + \sqrt{n}\frac{b(\bar{x}-\mu)}{\sigma} & \text{for } \mu > M \end{cases} \tag{7}$$

$$a = \frac{1}{\sqrt{2}}\left\{\frac{USL-\mu}{\sigma}\phi\left(\frac{USL-\mu}{\sigma}\right)\right.$$

$$\left.+\frac{\mu-LSL}{\sigma}\phi\left(\frac{\mu-LSL}{\sigma}\right)\right\}$$

$$= \frac{1}{\sqrt{2}}\{3C_P(2-C_a)\phi(3C_P(2-C_a))$$

$$+3C_PC_a\phi(3C_PC)\}, \tag{8}$$

$$b = \phi\left(\frac{USL-\mu}{\sigma}\right) - \phi\left(\frac{\mu-LSL}{\sigma}\right)$$

$$= \phi\left(\frac{d-(\mu-M)}{\sigma}\right) - \phi\left(\frac{d+(\mu-M)}{\sigma}\right)$$

$$= \phi\{3C_P(2-C_a)\} - \phi\{3C_PC_a\}. \tag{9}$$

Therefore, the estimator $\hat{S}_{pk}$ is roughly distributed as $N(S_{pk}, [a^2+b^2]\{36n[\phi(3S_{pk})^2]\}^{-1})$. The PDF of can be expressed as [9]:

$$f_{\hat{S}_{pk}}(x) = \sqrt{\frac{18n}{\pi}}\frac{\phi(3S_{pk})}{\sqrt{a^2+b^2}}\exp\left[-\frac{18n(\phi(3S_{pk}))^2}{a^2+b^2}\right.$$

$$\left.\times(x-3S_{pk})^2\right], \qquad -\infty < x < +\infty. \tag{10}$$

Table 2 shows S_{pk} values as well as the corresponding C_p and C_a. Hence, we can calculate the required parameters (a and b) to obtain the critical values and the required sample sizes of sampling plans.

2.1. The $\hat{S}_{pk}^{\mathrm{EWMA}_i}$ index

Since the asymptotic sampling distribution of $\hat{S}_{pk}$ is

Table 2. S_{pk} values and the corresponding C_p and C_a [17].

S_{pk}	C_p	C_a
1.0	1.1	0.845651
1.33	1.4	0.912325
1.5	1.6	0.906850
1.67	1.7	0.960124
2.0	2.1	0.934484

normally distributed with a mean S_{pk} and a variance $[a^2 + b^2]\{36n[\phi(3S_{pk})^2]\}^{-1}$, that of $\hat{S}_{pk}^{EWMA_i}$ can be obtained as a normal distribution with the mean $[1-(1-\lambda)^i S_{pk}]$ and variance $[\lambda/2-\lambda][1-(1-\lambda)^{2i}][a^2 + b^2]\{36n[\phi^2(3S_{pk})]\}^{-1}$. Note that $\hat{S}_{pk}^{EWMA_i}$ will follow the normal distribution with the mean S_{pk} and variance $[\lambda/2-\lambda][a^2 + b^2]\{36n[\phi^2(3S_{pk})]\}^{-1}$ when (i) is large. Therefore, the probability of accepting the lot can be expressed as [25]:

$$P\left(\hat{S}_{pk}^{EWMA_i} \geq k|S_{pk}\right)$$

$$=1-P\left(\frac{\hat{S}_{pk}^{EWMA_i}-S_{pk}}{\sqrt{[\lambda/2-\lambda][a^2+b^2]\{36n[\phi^2(3S_{pk})]\}^{-1}}} \leq \frac{k-S_{pk}}{\sqrt{[\lambda/2-\lambda][a^2+b^2]\{36n[\phi^2(3S_{pk})]\}^{-1}}}\right)$$

$$=1-\phi\left(\frac{k-S_{pk}}{\sqrt{[\lambda/2-\lambda][a^2+b^2]\{36n[\phi^2(3S_{pk})]\}^{-1}}}\right). \tag{11}$$

3. Developing a new variable RGS plan based on EWMA yield index

In this section, a VRGS plan is investigated using EWMA statistics based on the yield index. In order to introduce this approach, the requirements and contracts between the producer and consumer are accomplished by acceptable and rejectable quality levels. Therefore, the probability of lot acceptance must be more than $(1-\alpha)$ when the quality level of the submitted lot is at AQL (Acceptable Quality Level), and the probability of lot acceptance must be no more than β when the quality level of the submitted lot is at RQL (Rejectable Quality Level). In this regard, to design the proposed sampling plan, the OC curve should pass through two specified points $(AQL, 1-\alpha)$ and (RQL, β). For specified $(\alpha, \beta, S_{AQL}, S_{RQL})$, $P_a(p)$, and $P_r(p)$ are the probabilities of accepting and rejecting the entire lot at the quality level (p), respectively.

$$P_a(p) = P\left(\hat{S}_{pk}^{EWMA_i} \geq k_a|p\right), \tag{12}$$

$$P_r(p) = P\left(\hat{S}_{pk}^{EWMA_i} < k_r|p\right). \tag{13}$$

Finally, the OC function of the variable RGS plan can be expressed as follows:

$$\pi_A(p) = \frac{P_a(p)}{P_a(p)+P_r(p)}. \tag{14}$$

Eq. (14) can be rewritten as follows:

$$\pi_A(S_{pk}) = \frac{\Pr\left(\hat{S}_{pk}^{EWMA_i} \geq k_a\right)}{\Pr\left(\hat{S}_{pk}^{EWMA_i} \geq k_a\right)+1-\Pr\left(\hat{S}_{pk}^{EWMA_i} \geq k_r\right)}. \tag{15}$$

Therefore, the required sample size n and critical values (k_a, k_r) of the proposed sampling plan can be determined through the model proposed by Balamurali and Jun [14].

$$S = S_{AQL},$$

$$\pi_A(S_{AQL}) = \frac{\Pr\left(\hat{S}_{pk}^{EWMA_i} \geq k_a\right)}{\Pr\left(\hat{S}_{pk}^{EWMA_i} \geq k_a\right)+1-\Pr\left(\hat{S}_{pk}^{EWMA_i} \geq k_r\right)} \geq 1-\alpha, \tag{16}$$

$$S = S_{RQL},$$

$$\pi_A(S_{RQL}) = \frac{\Pr\left(\hat{S}_{pk}^{EWMA_i} \geq k_a\right)}{\Pr\left(\hat{S}_{pk}^{EWMA_i} \geq k_a\right)+1-\Pr\left(\hat{S}_{pk}^{EWMA_i} \geq k_r\right)} \leq \beta. \tag{17}$$

Optimum parameters of VRGS plan can be obtained by solving two nonlinear simultaneous equations with the minimal objective function. In fact, the best combinations of $(n,\ k_a,\ k_r)$ were found to minimize the Average Sample Number (ASN) by satisfying two constraints. Therefore, the ASN is defined as the average number of items in each used lot for decision-making. An appropriate sampling plan is the one that requires a minimal ASN inspected from the lot while providing identical protection for both producer and consumer (Balamurali and Jun [34], Wu [16]). The ASN of the proposed RGS plan at the quality level p can be designated as follows:

$$ASN(p) = \frac{n}{P_a(p)+P_r(p)}. \tag{18}$$

The optimization model can be expressed as follows:

$$\min \quad ASN,$$

subject to:

$$S = S_{AQL} \frac{\Pr\left(\hat{S}_{pk}^{EWMA_i} \geq k_a\right)}{\Pr\left(\hat{S}_{pk}^{EWMA_i} \geq k_a\right)+1-\Pr\left(\hat{S}_{pk}^{EWMA_i} \geq k_r\right)} \geq 1-\alpha, \tag{19}$$

$$S=S_{RQL}\frac{\Pr\left(\hat{S}_{pk}^{EWMA_i}\geq k_a\right)}{\Pr\left(\hat{S}_{pk}^{EWMA_i}\geq k_a\right)+1-\Pr\left(\hat{S}_{pk}^{EWMA_i}\geq k_r\right)}\leq \beta. \tag{20}$$

Finally, the procedure of the VRGS plan based on $\hat{S}_{pk}^{EWMA_i}$ can be elaborated as follows:

Step 1: Choose the producer's risk (α), consumer's risk (β), and quality requirements (i.e., S_{AQL} and S_{RQL}).

Step 2: Take a random sample with n observations from the current lot at the time i and compute $\bar{x}$ and S. Then, calculate the value of $\hat{S}_{pk}^{EWMA_i}$ via the given λ.

$$\hat{S}_{pk}^{EWMA_i}=\lambda\hat{S}_{pki}+(1-\lambda)\hat{S}_{pk}^{EWMA_{i-1}}, \tag{21}$$

where $0<\lambda\leq 1$ is called the smoothing constant. In addition, $\hat{S}_{pk}^{EWMA_{i-1}}$ is obtained from the preceding lots and $\hat{S}_{pki}$ is from the ith lot.

Step 3: Accept the lot if $\hat{S}_{pk}^{EWMA_i}\geq k_a$, and reject the lot if $\hat{S}_{pk}^{EWMA_i}<k_r$. If $k_r\leq\hat{S}_{pk}^{EWMA_i}<k_a$, repeat Steps 2 and 3 ($k_a>k_r$, k_r, and k_a are critical values).

In order to determine the plan parameters, we consider three scenarios as follows:

1. In Scenario 1, the ASN function is minimized at S_{AQL};
2. Scenario 2 is implemented by minimizing the ASN function at the quality level S_{RQL};
3. In Scenario 3, the objective function is evaluated to minimize the average value of $ASN(S_{AQL})$ and $ASN(S_{RQL})$.

As mentioned earlier, the plan parameters can be obtained by an optimization problem whose objective function is to minimize the value of the ASN. Moreover, constraints are regulated by satisfying the itemized quality levels and risks.

Scenario 1. Scenario 1 illustrates the ASN function of RGS plan based on the EWMA yield index at the quality level S_{AQL}. Plan parameters are specified by minimizing the ASN:

$$\min\ ASN(S_{AQL})=\frac{n}{P_a\left(S_{AQL}\right)+P_r\left(S_{AQL}\right)}=\frac{n}{\Pr\left(\hat{S}_{pk}^{EWMA_i}<k_r\right)+\Pr\left(\hat{S}_{pk}^{EWMA_i}\geq k_a\right)}. \tag{22}$$

Scenario 2. ASN function is minimized at the quality level S_{RQL} in Scenario 2. Hence, the objective function can be formulated to obtain the plan parameters as follows:

$$\min\ ASN(S_{RQL})=\frac{n}{P_a\left(S_{RQL}\right)+P_r\left(S_{RQL}\right)}=\frac{n}{\Pr\left(\hat{S}_{pk}^{EWMA_i}<k_r\right)+\Pr\left(\hat{S}_{pk}^{EWMA_i}\geq k_a\right)}. \tag{23}$$

Scenario 3. The given objective function in Eq. (24) is investigated to determine the plan parameters based on Scenario 3:

$$\min\ \frac{1}{2}\left(ASN(S_{AQL})+ASN(S_{RQL})\right). \tag{24}$$

There are several combinations of producer's and consumer's risks (α, β) and different values of λ used for solving two nonlinear simultaneous equations. The optimization problems are solved using a grid search method. In other words, plan parameters are obtained by searching in an organized multidimensional grid as $n=3(1)1000$, $k_a=0.6(0.001)2.2$, and $k_r=0.6(0.001)2.2$. Optimization problems in three scenarios are implemented in MATLAB R2017a through a grid search procedure. Tables 3–5 show three parameters (n,k_a,k_r) for different producers' and consumers' risks (α, β) and diverse values of λ under three scenarios. For instance, in Scenario 1, if $(\alpha,\beta)=(0.075,0.025)$, $\lambda=0.3$ and quality levels are set to $S_{AQL}=1.67$ and $S_{RQL}=1.5$, the best values for plan parameters used for minimizing the ASN will be $(n,k_a,k_r)=(34,1.662,1.524)$. This combination indicates that based on 34 inspected items, the entire lot will be accepted if $\hat{S}_{pk}^{EWMA_i}\geq 1.662$; otherwise, it will be rejected if $\hat{S}_{pk}^{EWMA_i}<1.524$. On the contrary, if $1.524\leq\hat{S}_{pk}^{EWMA_i}<1.662$, the procedure of the proposed sampling plan is repeated.

According to Tables 3–5, upon increasing the value of smoothing constant, the required sample size would also decrease. As a result, smaller values of λ are preferred. For example, based on the combination $(\alpha,\beta)=(0.05,0.01)$ and Scenario 3, the required sample sizes for $\lambda=0.1$, 0.3, 0.6, and 1 are obtained at 14, 47, 114, and 266, respectively.

Tables 6–11 represent an increasing trend in ASN value when the smoothing constant increases. In fact, $\lambda=0.1$ presents the smallest values of ASN in Scenarios 1–3. According to Tables 6–11, we can observe that the ASN of the proposed plan depends on the quality levels of the submitted lot under the three scenarios. Therefore, the quality levels have a remarkable influence on the ASN values. As mentioned earlier, Scenario 1 is calculated based on the AQL, and it presents the smallest ASN on the basis of AQL in contrast with other scenarios. Therefore,

Table 3. The plan parameters (n, k_a, k_r) for Scenario 1 under $(S_{AQL}, S_{RQL}) = (1.67, 1.5)$.

		$S_{AQL} = 1.67$, $S_{RQL} = 1.5$											
		$\lambda = 0.1$			$\lambda = 0.3$			$\lambda = 0.6$			$\lambda = 1$		
α	β	n	k_r	k_a	n	k_r	k_a	n	k_r	k_a	n	k_r	k_a
0.1	0.1	6	1.499	1.657	19	1.491	1.665	44	1.484	1.672	106	1.489	1.667
	0.075	6	1.492	1.675	22	1.505	1.662	52	1.501	1.666	121	1.501	1.666
	0.05	8	1.522	1.658	24	1.508	1.672	59	1.510	1.670	142	1.514	1.666
	0.025	10	1.537	1.661	31	1.528	1.669	81	1.537	1.661	189	1.537	1.661
	0.01	13	1.554	1.661	44	1.555	1.660	94	1.542	1.673	232	1.548	1.667
0.075	0.1	6	1.479	1.666	22	1.492	1.653	52	1.488	1.657	122	1.489	1.656
	0.075	7	1.495	1.661	24	1.498	1.658	60	1.502	1.654	141	1.503	1.653
	0.05	8	1.505	1.664	28	1.511	1.658	69	1.513	1.656	157	1.510	1.659
	0.025	10	1.523	1.664	34	1.524	1.662	85	1.528	1.659	186	1.520	1.666
	0.01	13	1.541	1.663	43	1.540	1.664	111	1.546	1.658	239	1.538	1.666
0.05	0.1	7	1.477	1.655	24	1.480	1.652	57	1.477	1.655	132	1.476	1.656
	0.075	8	1.490	1.653	28	1.496	1.647	60	1.479	1.663	156	1.494	1.649
	0.05	9	1.500	1.656	28	1.490	1.665	69	1.492	1.663	169	1.498	1.657
	0.025	11	1.516	1.658	40	1.525	1.649	90	1.516	1.657	206	1.514	1.659
	0.01	14	1.534	1.658	47	1.534	1.658	114	1.534	1.658	271	1.535	1.656
0.025	0.1	9	1.477	1.638	29	1.472	1.643	73	1.477	1.638	166	1.474	1.641
	0.075	9	1.474	1.651	31	1.478	1.647	74	1.476	1.649	174	1.477	1.648
	0.05	10	1.484	1.654	34	1.485	1.652	89	1.495	1.643	211	1.497	1.641
	0.025	12	1.500	1.655	41	1.502	1.653	96	1.498	1.657	230	1.501	1.654
	0.01	16	1.525	1.649	52	1.522	1.652	129	1.524	1.650	289	1.520	1.654
0.01	0.1	10	1.457	1.642	35	1.463	1.636	81	1.456	1.642	186	1.454	1.644
	0.075	11	1.467	1.641	37	1.468	1.640	90	1.468	1.640	213	1.470	1.638
	0.05	13	1.485	1.635	42	1.481	1.639	102	1.481	1.639	236	1.480	1.640
	0.025	15	1.497	1.640	50	1.497	1.640	124	1.499	1.638	278	1.495	1.642
	0.01	18	1.512	1.643	59	1.510	1.645	139	1.507	1.648	324	1.507	1.648

minimizing ASN under the ideal condition (S_{AQL}) can be a motivation for improving product quality. For instance, if $\lambda = 0.1$, $(\alpha, \beta) = (0.01, 0.01)$ and $(S_{AQL}, S_{RQL}) = (2, 1.67)$, the ASNs of VRGS plan based on EWMA yield index are calculated as 9.388, 11.092, and 10.342 for Scenarios 1–3, respectively.

Tables 9–11 present the results of the proposed sampling plan based on $(S_{AQL}, S_{RQL}) = (1.67, 1.33)$. In these tables, the obtained ASNs based in Scenario 1 are smaller than those in Scenarios 2 and 3. In this regard, Scenario 1 outperforms the other two scenarios. Moreover, smaller values of the smoothing constant are more prestigious than larger values.

4. Application example

The applicability of the VRGS plan based on the EWMA yield index is illustrated by a particular model of Multi-Crystalline Silicon (MCS) suggested by Wu and Liu [9]. Solar cell products manufactured by crystalline silicon wafers account for more than 90% of all solar cells produced worldwide. There are two major

Table 4. The plan parameters (n, k_a, k_r) for Scenario 2 under $(S_{AQL}, S_{RQL}) = (1.67, 1.5)$.

		$S_{AQL} = 1.67$, $S_{RQL} = 1.5$											
		$\lambda = 0.1$			$\lambda = 0.3$			$\lambda = 0.6$			$\lambda = 1$		
α	β	n	k_r	k_a	n	k_r	k_a	n	k_r	k_a	n	k_r	k_a
0.1	0.1	6	1.499	1.657	23	1.516	1.641	54	1.512	1.645	128	1.514	1.643
	0.075	7	1.513	1.654	22	1.505	1.662	57	1.513	1.654	133	1.513	1.654
	0.05	8	1.522	1.658	26	1.518	1.662	64	1.520	1.660	148	1.519	1.661
	0.025	9	1.525	1.673	32	1.532	1.666	73	1.525	1.673	173	1.527	1.671
	0.01	10	1.525	1.690	36	1.534	1.681	86	1.532	1.683	204	1.534	1.681
0.075	0.1	8	1.516	1.630	27	1.517	1.629	65	1.516	1.630	149	1.514	1.632
	0.075	9	1.525	1.632	26	1.508	1.648	63	1.508	1.648	147	1.508	1.648
	0.05	9	1.519	1.650	30	1.519	1.650	69	1.513	1.656	161	1.513	1.656
	0.025	10	1.523	1.664	35	1.528	1.659	82	1.524	1.663	193	1.525	1.662
	0.01	12	1.533	1.672	41	1.535	1.669	94	1.529	1.676	210	1.524	1.681
0.05	0.1	9	1.508	1.625	31	1.511	1.622	73	1.508	1.625	172	1.509	1.624
	0.075	10	1.516	1.628	33	1.515	1.629	84	1.520	1.624	196	1.520	1.624
	0.05	11	1.522	1.634	34	1.514	1.642	89	1.522	1.635	198	1.517	1.639
	0.025	11	1.516	1.658	40	1.525	1.649	99	1.527	1.647	231	1.527	1.647
	0.01	13	1.526	1.666	44	1.527	1.665	114	1.534	1.658	251	1.528	1.664
0.025	0.1	12	1.510	1.607	41	1.512	1.606	106	1.518	1.600	225	1.509	1.608
	0.075	13	1.516	1.611	44	1.517	1.611	109	1.519	1.609	241	1.514	1.613
	0.05	15	1.527	1.613	45	1.517	1.622	107	1.515	1.624	255	1.517	1.622
	0.025	14	1.517	1.639	51	1.525	1.631	115	1.518	1.638	271	1.519	1.637
	0.01	16	1.525	1.649	52	1.522	1.652	133	1.527	1.647	313	1.528	1.646
0.01	0.1	17	1.515	1.588	59	1.518	1.585	140	1.516	1.587	334	1.518	1.585
	0.075	18	1.519	1.593	61	1.520	1.592	148	1.520	1.592	345	1.520	1.592
	0.05	18	1.518	1.605	61	1.519	1.604	148	1.519	1.604	349	1.520	1.603
	0.025	20	1.525	1.614	66	1.524	1.615	164	1.526	1.613	374	1.524	1.615
	0.01	21	1.527	1.629	69	1.525	1.631	169	1.526	1.630	590	1.525	1.631

types of crystalline silicon: monocrystalline silicon and MCS. Since the thickness of the MCS wafer has a significant influence on the electric conductivity, the manufacturer usually considers the thickness as the critical quality characteristic. In this study, a special model of MCS wafer with 6-inch square (15.6*15.6 mm) was employed. The specification limits of thickness are (LSL = 160 μm, T = 190 μm, USL = 220 μm) (Wu and Liu [9]). According to the contract, assume that the values of (S_{AQL}, S_{RQL}) are set to (1.67, 1.5) and the producer's and consumer's risks are regulated to $\alpha = 0.075$ and $\beta = 0.05$. The thickness of the collected sample data is illustrated in Table 12.

Based on the specified values in the contract, plan parameters can be obtained from Table 3. In the case of using the proposed plan with $\lambda = 1$, the sample size and critical values can be calculated as $n = 157$, $k_a = 1.659$, and $k_r = 1.510$. Therefore, 157 samples should be randomly taken from the submitted lot. Based on these 157 samples, the sample mean,

Table 5. The plan parameters (n, k_a, k_r) for Scenario 3 under $(S_{AQL}, S_{RQL}) = (1.67, 1.5)$.

		$S_{AQL}=1.67, S_{RQL}=1.5$											
		$\lambda=0.1$			$\lambda=0.3$			$\lambda=0.6$			$\lambda=1$		
α	β	n	k_r	k_a	n	k_r	k_a	n	k_r	k_a	n	k_r	k_a
0.1	0.1	6	1.499	1.657	20	1.498	1.658	50	1.502	1.654	113	1.498	1.658
	0.075	7	1.513	1.654	22	1.505	1.662	57	1.513	1.654	121	1.501	1.666
	0.05	8	1.522	1.658	26	1.518	1.662	64	1.520	1.660	142	1.514	1.666
	0.025	9	1.525	1.673	31	1.528	1.669	73	1.525	1.673	176	1.529	1.669
	0.01	12	1.546	1.669	38	1.540	1.675	94	1.542	1.673	204	1.534	1.681
0.075	0.1	7	1.500	1.645	22	1.492	1.653	56	1.498	1.647	135	1.502	1.643
	0.075	7	1.495	1.661	26	1.508	1.648	63	1.508	1.648	141	1.503	1.653
	0.05	9	1.519	1.650	28	1.511	1.658	69	1.513	1.656	161	1.513	1.656
	0.025	10	1.523	1.664	35	1.528	1.659	82	1.524	1.663	193	1.525	1.662
	0.01	13	1.541	1.663	41	1.535	1.669	96	1.531	1.673	239	1.538	1.666
0.05	0.1	8	1.494	1.638	29	1.504	1.629	71	1.505	1.628	163	1.503	1.630
	0.075	9	1.504	1.639	30	1.504	1.639	69	1.498	1.645	161	1.498	1.645
	0.05	10	1.512	1.644	34	1.514	1.642	81	1.512	1.644	184	1.509	1.647
	0.025	11	1.516	1.658	40	1.525	1.649	99	1.527	1.647	207	1.515	1.659
	0.01	14	1.534	1.658	47	1.534	1.658	114	1.534	1.658	266	1.534	1.658
0.025	0.1	11	1.501	1.616	34	1.492	1.624	83	1.493	1.623	192	1.492	1.624
	0.075	11	1.499	1.627	38	1.502	1.624	88	1.497	1.629	205	1.497	1.629
	0.05	12	1.505	1633	42	1.510	1.629	100	1.508	1.631	238	1.510	1.629
	0.025	14	1.517	1.639	46	1.515	1.641	115	1.518	1.638	271	1.519	1.637
	0.01	16	1.525	1.649	52	1.522	1.652	129	1.524	1.650	313	1.528	1.646
0.01	0.1	14	1.497	1.604	47	1.497	1.604	114	1.497	1.604	274	1.500	1.601
	0.075	15	1.502	1.608	50	1.502	1.608	120	1.501	1.609	280	1.501	1.609
	0.05	16	1.507	1.615	54	1.508	1.614	121	1.500	1.621	309	1.509	1.613
	0.025	17	1.510	1.628	59	1.514	1.624	139	1.511	1.627	321	1.510	1.628
	0.01	19	1.517	1.638	66	1.521	1.635	160	1.521	1.635	358	1.517	1.639

Table 6. The Average Sample Number (ASN) values for Scenario 1 under $(S_{AQL}, S_{RQL}) = (2, 1.67)$.

α	β	λ									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.01	0.01	9.388	19.738	31.268	44.251	59.062	75.860	95.302	118.089	144.875	177.006
	0.05	6.735	14.145	22.476	31.835	42.434	54.546	68.453	84.868	104.168	127.257
	0.1	5.690	11.904	18.910	26.783	35.689	45.880	57.628	71.378	87.571	107.067
0.05	0.05	5.317	11.216	17.792	25.215	33.621	43.049	54.226	67.169	82.329	100.702
	0.1	4.321	8.896	14.131	19.995	26.660	34.296	43.051	53.320	65.440	79.980
0.1	0.05	4.588	9.508	15.095	21.406	28.524	36.643	46.084	57.047	70.055	85.500
	0.1	3.798	7.440	11.613	16.371	21.827	28.034	35.152	43.584	53.454	65.448

Table 7. The Average Sample Number (ASN) values for Scenario 2 under $(S_{AQL}, S_{RQL}) = (2, 1.67)$.

α	β	λ									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0.01	0.01	11.092	23.406	37.134	52.536	69.998	89.998	112.941	139.771	171.677	209.843
	0.05	10.195	21.569	34.277	48.527	64.576	83.014	104.423	129.152	158.630	193.698
	0.1	9.565	20.128	31.993	45.349	60.384	77.512	97.359	120.587	147.978	180.810
0.05	0.05	6.195	13.105	20.765	29.426	39.253	50.326	63.232	78.482	96.015	117.589
	0.1	5.611	11.742	18.612	26.367	35.227	45.214	56.799	70.538	86.420	105.469
0.1	0.05	4.610	9.724	15.433	21.895	29.172	37.440	49.995	58.344	71.577	87.449
	0.1	4.067	8.428	13.430	18.956	25.274	35.550	40.806	50.548	62.091	75.822

Table 8. The Average Sample Number (ASN) values for Scenario 3 under $(S_{AQL}, S_{RQL}) = (2, 1.67)$.

α	β	λ									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0.01	0.01	10.342	21.837	34.741	49.125	65.501	84.052	105.699	130.990	160.425	196.186
	0.05	8.845	18.658	29.642	41.953	55.958	71.868	90.390	111.915	137.566	167.712
	0.1	8.141	17.091	27.061	38.417	38.623	51.115	82.694	102.231	125.613	153.346
0.05	0.05	5.854	12.285	19.481	27.599	36.821	47.300	59.438	73.514	90.303	110.392
	0.1	5.008	10.582	16.766	23.768	31.691	40.643	51.039	63.364	77.631	94.938
0.1	0.05	4.599	9.668	15.352	21.710	28.999	37.199	46.719	57.966	71.030	86.715
	0.1	3.933	7.934	12.637	17.875	23.801	30.617	38.377	47.580	58.269	71.407

Table 9. The Average Sample Number (ASN) values for Scenario 1 under $(S_{AQL}, S_{RQL}) = (1.67, 1.33)$.

α	β	λ									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0.01	0.01	5.761	12.06	19.132	27.103	36.154	46.454	58.352	72.308	88.656	108.351
	0.05	4.214	8.821	13.976	19.794	26.399	33.954	42.613	52.717	64.711	79.175
	0.1	3.749	7.486	11.893	16.830	22.495	28.860	36.199	44.859	54.994	67.253
0.05	0.05	3.668	6.853	10.928	15.419	20.558	26.388	33.173	41.117	50.442	61.572
	0.1	3.324	5.525	8.746	12.414	16.559	21.282	26.690	33.007	40.504	49.530
0.1	0.05	3.387	5.789	9.171	12.976	17.363	22.245	27.975	34.705	42.486	51.905
	0.1	3.090	4.559	7.086	10.066	13.390	17.214	21.612	26.759	32.832	40.127

Table 10. The Average Sample Number (ASN) values for Scenario 2 under $(S_{AQL}, S_{RQL}) = (1.67, 1.33)$.

α	β	λ									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0.01	0.01	7.115	15.023	23.775	33.755	44.953	57.687	72.611	89.839	110.300	134.667
	0.05	6.629	13.896	22.095	31.320	41.689	53.700	67.342	83.379	102.384	125.068
	0.1	6.223	13.044	20.677	29.283	39.044	50.200	63.176	78.051	95.854	117.133
0.05	0.05	4.022	8.397	13.294	18.836	25.110	32.287	40.494	50.217	61.681	75.331
	0.1	3.709	7.585	12.006	17.037	22.748	29.113	36.584	45.384	55.640	68.059
0.1	0.05	3.347	6.166	9.803	13.872	18.497	23.746	29.913	36.993	45.464	55.408
	0.1	3.117	5.451	8.613	12.141	16.201	20.779	26.142	32.403	39.600	48.462

Table 11. The Average Sample Number (ASN) values for Scenario 3 under $(S_{AQL}, S_{RQL}) = (1.67, 1.33)$.

α	β	λ									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0.01	0.01	6.622	13.868	21.914	31.110	41.451	53.366	67.078	82.815	101.697	124.177
	0.05	5.693	12.044	19.074	27.041	36.095	46.414	58.350	72.191	88.469	108.069
	0.1	5.382	11.128	17.666	25.027	33.217	42.744	53.727	66.433	73.270	89.549
0.05	0.05	3.845	7.770	12.320	17.460	23.311	29.934	37.639	46.451	57.140	69.838
	0.1	3.517	6.740	10.732	15.165	20.220	26.013	32.660	40.440	49.586	60.659
0.1	0.05	3.367	6.052	10.176	13.546	18.099	23.222	29.180	36.151	44.393	54.182
	0.1	3.103	5.005	7.975	11.277	15.014	19.331	24.288	30.028	36.843	45.042

Table 12. The thickness of the collected sample data (unit: μm) [9].

187	201	188	177	184	193	189	195	193	188	195	189	191	190	190	196	175	180	183	191
207	186	210	199	208	196	187	177	191	180	188	186	190	187	175	188	189	174	200	186
200	196	189	191	182	184	181	203	195	190	178	190	201	179	184	187	191	181	186	189
203	192	188	200	180	198	177	196	187	203	177	170	182	191	182	177	183	192	204	182
187	186	195	184	171	183	188	175	186	186	185	188	184	173	187	196	182	205	190	195
178	180	189	182	183	195	191	192	190	196	176	187	195	179	192	183	184	203	195	171
193	182	194	183	188	186	180	190	189	184	176	195	174	190	197	186	195	189	191	210
176	183	168	195	201	185	193	179	190	188	197	199	182	189	189	167	192			

sample standard deviation, and $\hat{S}_{pki}$ can be computed as $\bar{x} = 188.1019$, $s = 8.5028$, and $\hat{S}_{pki} = 1.14965$, respectively. Based on the decision rule, the entire lot will be accepted if $\hat{S}_{pk}^{EWMA_i} \geq 1.659$. If $\hat{S}_{pk}^{EWMA_i} < 1.510$, the submitted lot will be rejected; otherwise, if $1.510 \leq \hat{S}_{pk}^{EWMA_i} < 1.659$, the procedure of the proposed plan will be repeated. Here, assume that $\hat{S}_{pk}^{EWMA_{i-1}} = 1.1052$ and $\hat{S}_{pk}^{EWMA_i}$ based on Eq. (21) is calculated as 1.14965. Therefore, the lot will be rejected ($\hat{S}_{pk}^{EWMA_i} < k_r$).

5. Comparison study

In this section, in order to examine the efficiency of the proposed VRGS plan based on EWMA yield index, a comparison study is performed. Therefore, the variable single sampling plan proposed by Wu and Liu [9] and VRGS plan based on the yield index suggested by Wu and Liu [21] are compared with the VRGS plan using the EWMA statistics based on the yield index under a number of combinations of producer's and consumer's risks and different quality levels. Tables 13 and 14 demonstrate the results of the comparison study based on Scenario 1.

As observed in Tables 13 and 14, the proposed plan yielded better outcomes than Variable Single Sampling (VSS) and VRGS plans based on the yield index. In fact, a plan with smaller ASN could considerably reduce the inspection cost and time. The rate of reduction was computed based on $\lambda = 0.8$. The obtained results demonstrated reduction rates of over 55% and 51% in ASN values of the proposed plan, compared with the VSS plan in Tables 13 and 14, respectively. Similarly, the proposed plan showed a reduction of over 32% in the ASN values compared to the VRGS plan based on the yield index in Tables 13 and 14.

Furthermore, the proposed VRGS plan based on the EWMA yield index showed a considerable reduction in ASN values. Consequently, the proposed sampling plan presents the desired protection by decreasing the inspection cost. For instance, the ASN values for VSS, VRGS plan based on S_{pk}, and VRGS plan based on the EWMA yield index were obtained as 425, 247.360, and 62.076 for $(\alpha, \beta) = (0.03, 0.05)$ and quality levels of $(S_{AQL}, S_{RQL}) = (1.5, 1.33)$ in terms of $\lambda = 0.4$, respectively. According to the results, when the quality levels alter to $(S_{AQL}, S_{RQL}) = (1.5, 1.33)$, the required

Table 13. The results of a comparison study for different sampling plans under $(S_{AQL}, S_{RQL}) = (1.33, 1)$.

$S_{AQL} = 1.33$, $S_{RQL} = 1$										
α	β	The rate of reduction (%)	VSS	VRGS plan based on EWMA yield index λ					VRGS plan process yield	The rate of reduction (%)
				0.1	0.4	0.6	0.8	1		
0.01	0.01	66.38	133	3.755	16.766	28.740	44.706	67.023	67.019	33.29
	0.03	67.18	112	3.408	13.803	23.600	36.757	55.066	55.067	33.25
	0.05	67.39	102	3.281	12.469	21.362	33.260	49.698	49.704	33.08
	0.07	67.44	95	3.210	11.621	19.885	30.927	46.341	46.339	33.26
	0.09	67.13	89	3.163	10.944	18.761	29.255	43.735	43.729	33.10
0.03	0.01	61.89	105	3.587	15.012	25.737	40.018	59.989	59.976	33.28
	0.03	63.43	87	3.280	11.916	20.427	31.814	47.664	47.654	33.24
	0.05	63.90	78	3.167	10.581	18.078	28.155	42.215	42.213	33.30
	0.07	64.08	72	3.107	9.708	16.610	25.861	38.637	38.632	33.06
	0.09	63.96	67	3.067	9.078	15.524	24.148	36.200	36.117	33.14
0.05	0.01	59.33	92	3.485	14.049	24.139	37.412	56.121	56.133	33.35
	0.03	60.72	75	3.194	10.945	18.719	29.463	43.592	43.583	32.40
	0.05	61.93	67	3.088	9.575	16.397	25.507	38.154	38.156	33.15
	0.07	62.07	61	3.029	8.673	14.902	23.139	34.685	34.678	33.27
	0.09	62.45	57	3.003	8.066	13.778	21.405	32.110	32.098	33.31
0.07	0.01	57.19	83	3.406	13.330	22.852	35.532	53.210	53.206	33.22
	0.03	59.48	67	3.117	10.221	17.452	27.148	40.615	40.618	33.16
	0.05	60.21	59	3.020	8.829	15.111	23.476	35.211	35.215	33.33
	0.07	60.79	54	3.006	7.987	13.675	21.175	31.751	31.761	33.33
	0.09	61.10	50	3.002	7.317	12.503	19.450	29.212	29.195	33.38
0.09	0.01	55.49	76	3.324	12.693	21.759	33.825	50.734	50.726	33.32
	0.03	58.21	61	3.046	9.577	16.387	25.490	38.236	38.229	33.32
	0.05	59.40	54	3.007	8.270	14.097	21.923	32.879	32.879	33.32
	0.07	59.12	48	3.003	7.358	12.616	19.622	29.432	29.427	33.32
	0.09	59.18	44	3.000	6.750	11.551	17.963	26.953	26.950	33.35

sample sizes and ASN values are obtained more than $(S_{AQL}, S_{RQL}) = (1.33, 1)$. Therefore, sampling plans depend on the quality levels.

6. Conclusions

The present study aimed to develop the variable repetitive group sampling plan using the Exponentially Weighted Moving Average (EWMA) statistics based on the yield index. Several tables were employed to determine the plan parameters of the proposed plan under different combinations of quality levels and producer's and consumer's risks. To this end, three different scenarios were employed to assess the Average Sample Number (ASN). The scenarios were executed by minimizing an objective function and satisfying two restrictions based on the risks that producers and consumer face. In addition, an appropriate sampling plan was designed based on the smallest value of ASN. According to the findings, the Variable Repetitive

Table 14. The results of a comparison study for different sampling plans under $(S_{AQL}, S_{RQL}) = (1.5, 1.33)$.

$S_{AQL} = 1.5$, $S_{RQL} = 1.33$										
α	β	The rate of reduction (%)	VSS	VRGS plan based on EWMA yield index λ					VRGS plan process yield	The rate of reduction (%)
				0.1	0.4	0.6	0.8	1		
0.01	0.01	63.77	740	21.286	100.764	172.414	268.097	402.144	401.944	33.30
	0.03	65.09	611	16.886	80.155	144.250	213.280	319.919	319.924	33.33
	0.05	65.55	550	15.012	71.023	121.882	189.474	284.092	284.090	33.30
	0.07	65.60	504	13.698	65.330	111.601	173.380	260.114	260.094	33.34
	0.09	65.90	475	12.867	60.794	104.191	161.984	242.976	242.945	33.32
0.03	0.01	58.70	598	19.509	92.669	159.185	246.962	370.189	370.203	33.29
	0.03	60.62	484	15.085	71.429	122.253	190.582	284.816	284.802	33.08
	0.05	61.11	425	13.090	62.076	106.222	165.265	247.402	247.360	33.19
	0.07	61.92	392	11.834	56.125	95.994	149.261	223.538	223.663	33.27
	0.09	62.00	362	10.862	51.588	88.447	137.567	206.281	206.284	33.31
0.05	0.01	55.44	528	18.653	88.290	151.258	235.290	352.685	352.634	33.28
	0.03	58.02	422	14.022	66.476	114.013	177.171	265.547	265.584	33.29
	0.05	58.87	370	12.048	57.088	97.803	152.188	228.149	228.163	33.30
	0.07	59.15	333	10.788	51.068	87.467	136.029	204.139	204.100	33.35
	0.09	59.50	307	9.817	46.761	80.030	124.350	186.225	186.184	33.21
0.07	0.01	53.03	480	17.856	84.622	144.998	225.465	339.108	338.996	33.49
	0.03	55.59	378	13.273	62.949	107.830	167.863	251.331	251.295	33.20
	0.05	56.89	330	11.341	53.518	91.503	142.253	213.926	214.034	33.53
	0.07	57.54	298	10.027	47.571	81.550	126.540	189.416	189.318	33.16
	0.09	58.00	273	9.033	43.063	73.731	114.673	172.039	172.148	33.39
0.09	0.01	51.03	445	17.309	81.907	140.108	217.919	326.258	326.263	33.21
	0.03	53.86	345	12.638	59.778	102.591	159.187	238.951	239.086	33.42
	0.05	55.19	301	10.662	50.637	86.484	134.865	201.530	201.484	33.06
	0.07	55.90	270	9.393	44.494	76.447	119.073	177.554	177.515	32.92
	0.09	56.49	246	8.476	40.164	68.869	107.029	160.309	160.329	33.24

Group Sampling (VRGS) plan based on the EWMA yield index had a minimum value of the ASN compared with other plans. As a result, the proposed plan was more efficient than Variable Single Sampling (VSS) and VRGS plans based on the yield index. For future researches, the proposed plan can be developed for non-normal distribution and compared with other plans.

Nomenclature

Abbreviations

EWMA	Exponentially Weighted Moving Average
OC	Operating Characteristic
VRGS	Variables Repetitive Group Sampling
RGS	Repetitive Group Sampling
AQL	Acceptable Quality Level
RQL	Rejectable Quality Level
ASN	Average Sample Number
VSS	Variables Single Sampling

Parameters

α	Producer's risk
β	Consumer's risk
λ	Smoothing constant
S_{AQL}	Quality level
S_{RQL}	Quality level

Decision variables

n	Sample size

k_a Critical value for acceptance

k_r Critical value for rejection

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