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A real-time exhaustive search algorithm for the weapon-target assignment problem

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Weapon target assignment;
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Abstract. Weapon-Target Assignment (WTA) as an important part of the aerial defense cycle has long been studied. Challenges are usually finding fast-computing methods to search for an optimal or near-optimal solution in case of a large number of weapons and targets. This viewpoint gains significance in terms of mathematics; yet, practically, it has limited applicability in the mentioned context. In this paper, a real-time search algorithm was proposed which decomposed the WTA problem and provided a real-time exhaustive search algorithm by decreasing the size of solution space and deleting impossible solutions. Implementation of the algorithm for three typical scenarios exhibited excellent real-time performance and the possibility of finding exact solutions to large-scale problems.

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1. Introduction

In aerial defense cycle systems, Weapon-Target Assignment (WTA) as an important subsystem has been considered for decades. In a rough classification, there are large- and small-sized problem categories. Using exhaustive search, a small-sized static WTA problem is solved fast and the exact solution is obtained. In this case, all possible pairs of weapon targets are checked, the cost function is calculated, and the optimum solution is found. The nature of this problem can be characterized in such a way that if the size increases, the required computational burden significantly increases in finding an exact solution [1]. The literature on the WTA problem is classified into two study categories; mostly, some researchers have tried to use or propose new algorithms to solve the problem more effectively. Exhaustive search [1], Genetic Algorithm (GA) [1–4], Ant Colony together with GA [5], particle swarm optimization [6], large scale neighborhood [7,8] Fuzzy [9], game theory [10–12], and Markov decision process [13] are some examples of the research approaches to this problem. Subsequently, some studies have compared the efficiency of different algorithms such as Ant colony, GA, PSO, and maximum marginal return [14] as well as GA, tabu search, simulated annealing, and variable neighborhood search [15]. In those research interests, the target-based WTA problem or asset-based problems are solved. Furthermore, two types of static and dynamic WTAs have been studied in the literature. The problem is often considered as a single-objective case, while some studies have focused on multi-objective methods (such as Lotter et al. [16], Li et al. [17], Lotter and Van Vuuren [18], Zhou et al. [19], and Lotter [20]). The proposed algorithms are usually implemented for a static target-based form of the WTA problem.

Exact algorithms are proposed to solve the WTA problem for some special cases such as:

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1. When all the weapons are identical, or
2. When the targets can be reached by at most one weapon [21].

WTA problems are mostly NP-complete, discrete, stochastic, nonlinear, and usually large-sized; this is the reason why obtaining an optimal solution to the WTA problem is usually impossible [22]. In large-sized problems, if real-time solutions are required, heuristic algorithms are usually used. By using heuristic methods, near-optimal solutions may be found in a short time. The weapon assignment problem is also the case of a general multi-robot task allocation problem, wherein the objective is to optimally assign a set of robots to a set of tasks such that the overall system performance, subject to a set of constraints, is optimized [23]. The scheduling problem is another similar field of problems that contains two sub-problems: resource allocation and scheduling. In the resource allocation sub-problem, the computational burden is a function of the number of resources allocated [24]. Another branch of allocation problems is flexible job shop problem which includes routing (finding the best route to accomplish a job) and scheduling [25]. In the present study, a simple fast computing method is presented which tries to find the exact solution of a large-sized problem at a higher level (multi-weapon systems level) by simplifying the WTA problem. Concisely, our proposed algorithm tries to be simultaneously real-time and exact: two properties that generally are not collected in the previously proposed algorithms. The proposed algorithm decomposes the problem, distributes the targets to suitable defensive resources, executes favorite constraints, and finally generates a small-sized solution space in comparison with the common exhaustive search method which assumes all weapons engaging all targets.

2. WTA general considerations

In the following, some considerations about viewpoints involved in solving the WTA problem are presented.

2.1. The problem viewpoints

Traditional WTA problem equation: The traditional WTA problem in the static target-based form is a function of the form:

$$\min_{x_{i,k}\in\{0,1\}} F=\sum_{i=1}^{|T|} V_i \prod_{k=1}^{W}(1-P_{i,k})^{x_{i,k}}, \tag{1}$$

$$\text{subject to:} \quad \sum_{i=1}^{|T|} x_{i,k}=1, \qquad k=1,\cdots,|W|, \tag{2}$$

where the parameters are as follows:

T	A set of targets, $T=\{T_1,\cdots,T_N\}$
W	A set of weapons, $W=\{W_1,\cdots,W_M\}$
V_i	The value of target i
$P_{i,k}$	Kill probability for the pair, T_i, W_k
$x_{i,k}$	Decision variable. Its value is 1 if W_k is assigned to T_k and otherwise equals 0 [1]

It is desired to find an optimal solution that can minimize the probability of targets survival, in which the solution is a set of $\{T_i, W_k\}$ for all targets. Through Eq. (1), the formula considers all T^W solutions. It is assumed that all weapons can engage all targets without weapon inventory limitation. Eq. (1) can be modified to consider special problems using further constraints.

Problem sizes: In the WTA problem, challenges are usually finding fast-computing methods to examine optimal or near-optimal solutions in case of a large number of weapons and targets. This viewpoint is more considerable mathematically, but it has limited usage or practicality in the mentioned context. A usually ignored point is that in practical cases, all weapons rarely face a target. By removing never-occurring cases, the problem is simplified and consequently, its size decreases significantly.

Heuristic methods: In the process of solving large-sized WTA problems, the optimal solution is attained using the exhaustive search. While the computation time becomes long, heuristic methods are often developed and improved to solve the problem faster and find a solution more analogous to the optimal one. Heuristic methods have a special search algorithm and search only part of the possible solution space and the solution will be near-optimal.

Upon removing impossible solutions and using criteria in the possible solution space, based on decision-maker preferences, highly unacceptable solutions are removed. As the size of the search space significantly decreases, all the remaining solution space may be searchable and the exact best solution is obtained.

According to [26], the WTA solution space is reduced when solutions are deleted with a kill probability less than a minimum acceptable value. Through this filtering, the reduced problem is solved in less than a second for large-sized problems.

2.2. Simplified WTA

Usually, to find the WTA (near-optimal) solution in real time, a decrease in solution accuracy can be overlooked because of the large computation load associated with the problem. It is argued that in a real-world scenario, a very large-scale problem seldom occurs and the exact solution can be possibly attained in real time upon decreasing the size of the WTA problem. A realistic air defense scenario for a naval task group practicably consists of less than ten targets and ten weapon systems [1]. The WTA problem can be decomposed and solved in multiple stages. This causes the omission of some impossible solutions before starting the search process for finding the optimal solution [27]. The use of this solution makes it possible to find the exact solution to the WTA problem under single/multiple objectives through exhaustive search, even in large-sized WTA cases. Accordingly, an algorithm is designed to be practical and fast, and with some assumptions resulting in an exact solution.

The proposed algorithm can be compared with a Rule-based Heuristic Search (RHS) algorithm [28] as follows:

1. RHS uses some rules – which are extracted using methods such as data mining in solution space – to opt for a path to approach the proper solution among all possible cases; however, proposed algorithm first removes absolute impossible solutions from the search space and limits the problem and search space size while the required rules are predefined;
2. The starting point of the search process in RHS is estimated based on proper primary solution(s) with some computational burden; however, in PA, the starting point of search is selected based on the priorities of the tasks and no estimation is required;
3. In RHS, the solution space is searched with a predefined resolution which may be the reason for neglecting some of the possible solutions – a common way in heuristic methods. However, in PA, no possible solution is neglected in the search process.

3. The proposed WTA algorithm

The WTA is a part of the command-and-control process in an air defense process. After receiving targets' kinematic data and extracting their features, priorities are assigned. In the following, the WTA process is discussed in depth. Figure 1 presents the weapon assignment algorithm and, subsequently, Figure 2 depicts Weapon Assignment Decision Maker (WADM), inputs, geometric features, priorities, outputs pairs, and order of engagement. According to Figure 2, the processes in WADM are as follows:

Engageable targets: Based on the cross-parameter (Figure 3), for each Weapon-Target Pair (WTP), it can be calculated whether the predicted target path is within Weapon System (WS) engagement radius or not.

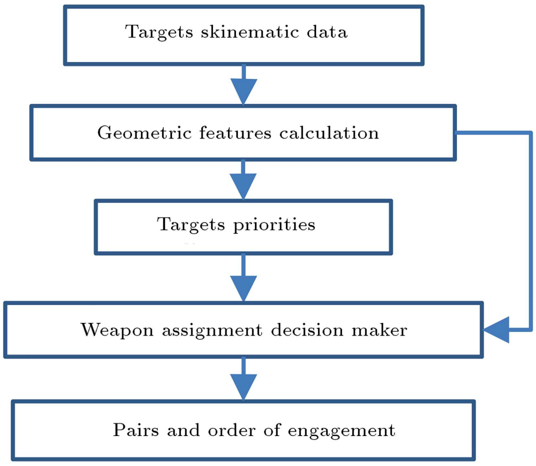

Figure 1. Weapon assignment algorithm.

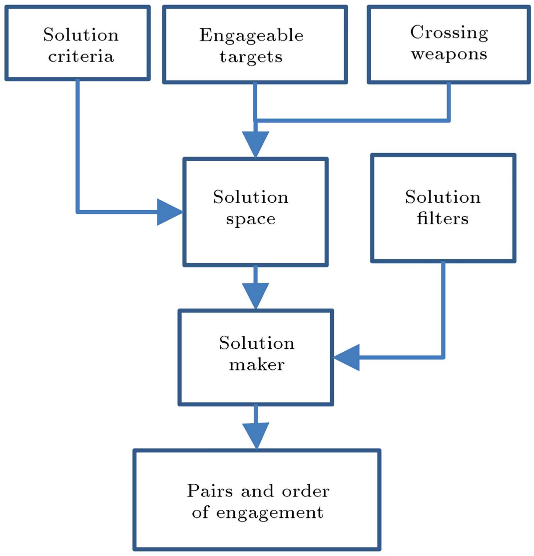

Figure 2. Weapon assignment decision-maker.

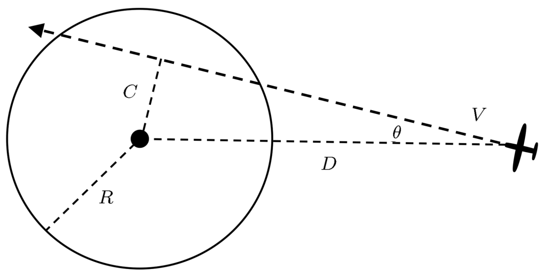

Figure 3. Target parameters (C: Cross, D: Distance).

Crossing weapons: Subsequently, for each target, Engageable Targets Matrix (ETM) is generated (Figure 4, (1)). ETM shows that each target could be sequentially engaged by which WSs. Crossing WSs are sorted in ETM in chronological order.

Engagement combinations: All possible engagement pairs are generated with ETM at hand. If the

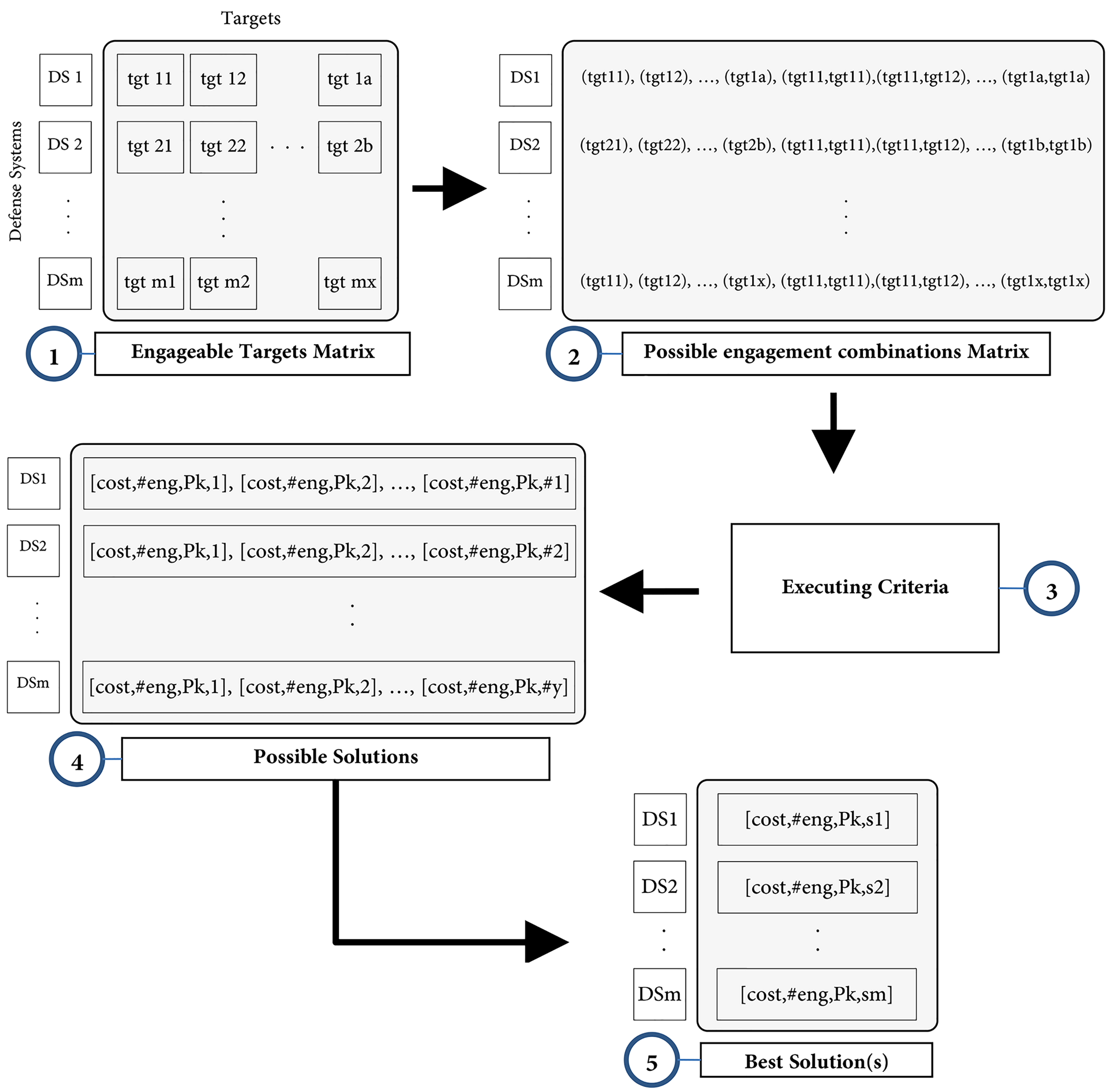

Figure 4. Computational process for the proposed Weapon-Target Assignment (WTA) algorithm.

maximum of two shots against each target is allowable and the predicted target path crosses m WSs, all combinations containing one or two shots is calculated as follows:

$$\text{number of combinations} = \binom{m}{1} + \binom{m}{2} + m$$
$$= \frac{m(m+3)}{2}. \qquad (3)$$

where $\binom{m}{1}$ is one shot, $\binom{m}{2}$ is two shots from different WSs, and m is two shots same WS.

For example, if a target's predicted path crosses two WSs, there will be 9 possible combinations using Eq. (3) for engaging the target, containing one-shot and two-shot combinations.

Solution criteria: There are decision factors that affect the WA problem solution. Some of them are listed in [16,20]. Before generating possible solutions, some criteria are defined. In this work, the probability of kill, the number of weapons used against each target, and engagement cost are applied to find the best solution (Figure 4: ③).

Furthermore, it is necessary to notice that among the solutions satisfying minimum acceptable kill probability condition, the solution that temporally occurs first is chosen.

Solution space: All possible solutions are generated for all WTPs according to the criteria. For each WTP, a vector is defined (Figure 4: ④) which contains information of the solution cost, number of engagements, probability of kill, and solution number (Eq. (4)):

$$Solution\ vector = [cost, eng, P_k, \#]. \tag{4}$$

where *cost* is engagement cost, *eng* is number of engagements, P_k is probability of kill, and # is solution number.

For example, Vector [8, 2, 0.9, 22] offers a solution that costs 8 units, and engagement is done by two shots against the target with the total probability of kill of 0.9. This vector belongs to the 23th generated solution for the target (counter starts from 0).

Solution filters: Having all possible solutions, the process of searching for the best solutions begins. Here, the decision-maker can filter the solutions based on decision criteria. For example, it can select solutions with the kill probability being greater than a margin, maximum limit on cost, or the number of engagements with each target.

Engagement strategy: In addition to criteria and filters, Engagement Strategy (ES) affects the solution space, as well. Each ES defines the rules on engagement and constraints for solution space and it resembles optimization strategies. The optimal solution based on each ES differs from others; some examples of ESs include engaging with the target as soon as possible, engaging with the highest probability of kill, and engaging with minimum cost. It is worth pointing out that each ES can be a combination of two or more criteria.

Solution maker: Solution-maker searches the solution space and applies the filter. Solution-maker attempts to find a set of the best weapon candidates to engage with targets and begins weapon assignment from the target with the highest priority (Figure 4: ⑤). If the weapon inventory is capacious enough, the first best solution is selected. Filters may omit all solutions for a specific target. For example, if there is no weapon or a combination of weapons (in case of multiple engagements with targets) to meet minimum acceptable P_k, another solution for the target will be chosen based on other criteria or solution with smaller P_k.

In the following, the problem is solved with the proposed algorithm through a limited or unlimited inventory condition. If the inventory is limited, the solution may not be optimal, while, the solution is optimal in other cases.

4. Case studies and simulation results

The performance of the proposed algorithm was tested in the context of a surface-based air defense system.

4.1. Engagement model

Test scenario: There are six long-range (L1–L6) and six medium-range (M1–M6) weapons with a limited inventory, positioned around a defended asset to protect it against six aerial targets approaching the asset. Weapons range and cost data are considered, as shown in Table 1. Six targets approach the defended asset. The future path is predicted according to the current position and velocity vector. Table 2 displays the threat priorities based on the degree of their threat to the defended asset. In the following, WSs inventory is shown in Table 3. If a target crosses a WS, it can be reached at a predictable distance and cross. Cross parameter is the minimum distance of the predicted target path to the center of the WS, as illustrated in Figure 3. Subsequently, crossing weapons for each target are shown in Table 4.

Table 1. Weapon systems range and cost data.

Weapon	Range	Cost
Long-range	100	7
Medium-range	50	4

Table 2. Targets priority.

Priority	1	2	3	4	5	6
Target	1	2	3	6	5	4

The probability of kill (P_k): It is assumed that kill probability for a specific resource is a function of the weapon-target distance and cross. For a particular weapon-target pair, the corresponding effect depends on the range of distances and crosses for the following reasons:

1. The shooting range of a weapon is limited;
2. The accuracy of a weapon depends on the range of targets [29].

While P_k increases, the target approaches WS, reaches a maximum value, and then decreases. Similar behavior was seen in [18,30]. Likewise, at a specific distance, as cross increases, P_k decreases (Figure 5). According

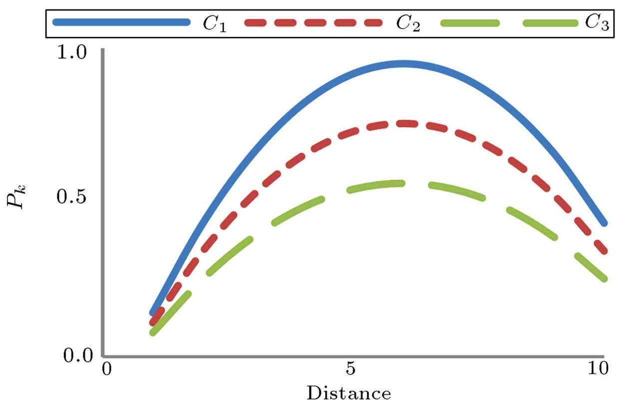

Figure 5. P_k behavior as a function of distance and cross ($C_1 < C_2 < C_3$).

Table 3. Weapons inventory.

L1	L2	L3	L4	L5	L6	M1	M2	M3	M4	M5	M6
9	7	8	7	8	7	9	9	7	7	7	9

Table 4. Crossing weapons.

Priority	Target	Crossing weapon systems							
1	1	1	7	2	8	9	10	3	4
2	2	5	4						
3	3	2	3	9	10	4			
4	6	5	6	1					
5	5	1	7	8	2	3			
6	4	2	1	8	7	11	5		

Table 5. The solution of the Weapon-Target Assignment (WTA) problem depicted in Figure 6 with the first Engagement Strategy (ES).

Priority	Target #	Weapon #	P_k max
1	1	L1	0.83
2	2	L5	0.76
3	3	M3	0.85
4	6	L6	0.88
5	5	L1	0.88
6	4	M5	0.88

to this behavior, a matrix is defined for P_k; whenever a target is processed, based on the distance and cross wrt WS, possible kill probabilities are predicted.

Engagement strategies: The WTA problem is solved for two different ESs:

1. Engaging with all targets with maximum P_k and one shot against each target;
2. Engaging with all targets with a minimum allowable P_k,

wherein two shots are allowable at maximum against each target with the aim of ensuring minimum cost.

4.2. Engagement scenario 1

To solve the problem based on the first ES, by considering target cross wrt each the crossing weapon, the best distance for engagement and the maximum P_k for each crossing weapon are calculated, respectively. The best solution based on the first ES is shown in Table 5. To solve the WAP for the second ES, all possible solutions containing possible single shots and a combination of all WSs for two shots are generated. In the next step, by using the minimum allowable P_k filter, the outlier solutions are ignored. Finally, the solution maker searches the remaining solution space for the best one. In the following, Figure 6 demonstrates the first simulation scenario, wherein the targets T1 to T6 are shown from their starting (detection) point and their direct predicted path along the defended space. For the illustrated problem in Figure 6, a number of possible solutions and final allowable solutions for each target are shown in Table 6. Furthermore, Table 7 contains outputs of the solution maker as the best results.

In the simulation, it is assumed that all targets move over a straight line with a random constant speed, except Target 1 which is hand-controlled.

4.3. Scenario 2

In the following, Figure 7 shows another typical scenario. The scenario contains five time-steps (shown in Figure 7 by ① to ⑤). The result of target prioritization and weapon assignment in the five time steps is presented in Figure 8 and Table 8, wherein Target 1 is not engageable in the time steps 1 and 3 because its predicted path is out of engagement zones of all WSs. Priority of target pairs (T2,T3), (T4,T5), and (T3,T4) vary in time-step transitions: ① to ②, ② to ③, and ④ to ⑤, respectively.

4.4. Engagement scenarios 3 and 4

Simulation was performed for both 12*12 (the scenario containing target starting point and predicted path, shown in Figure 9) and 12*18 (weapons * targets) problem instances, and the related results are presented in Tables 9 and 10.

Table 11 shows the computation time of solving the three problem instances in Sections 4.2 and 4.4 and demonstrates that the proposed algorithm solves them in real time.

4.5. Engagement scenario 5

The proposed algorithm was applied to 16 different scenarios based on four different numbers of WSs and targets. The results (Table 12) revealed that adding new targets had a greater impact on algorithm performance than adding new WSs. Figure 10 shows the computational time for an equal number of weapons and targets among the 16 scenarios. As a rough estimation, a problem of size 45*45 (weapons * targets) can be solved in less than one second (real-time) for a

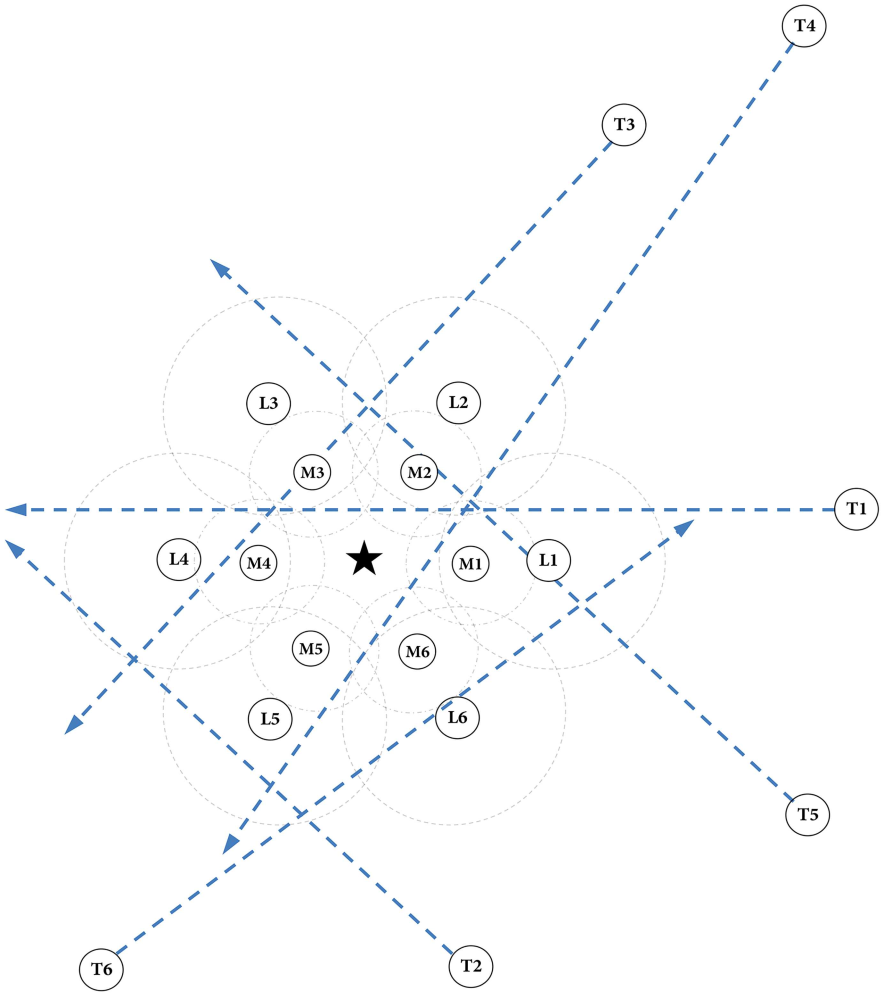

Figure 6. Weapon assignment test scenario (12 weapons * 6 targets).

Table 6. Possible solution combinations for the Weapon-Target Assignment (WTA) problem depicted in Figure 6 with the second Engagement Strategy (ES).

Priority	Target #	Crossing weapons #	Possible solutions	Allowable solutions
1	1	8	44	21
2	2	2	5	1
3	3	5	20	15
4	6	3	9	5
5	5	5	20	14
6	4	6	27	16

symmetric emplacement of defense system (similar to Figure 9).

Performance of the proposed algorithm highly depends on the defense system arrangement. In the worst case, i.e., when all targets face all weapons, the computational burden is similar to that of the exhaustive search algorithm. This problem may occur in particular cases. However, for many practical applications, the proposed algorithm can find a proper solution to a large-size problem in real time.

4.6. Discussion

In [1], the real-time condition assumes the solutions that are obtained in about one second. Hence, to

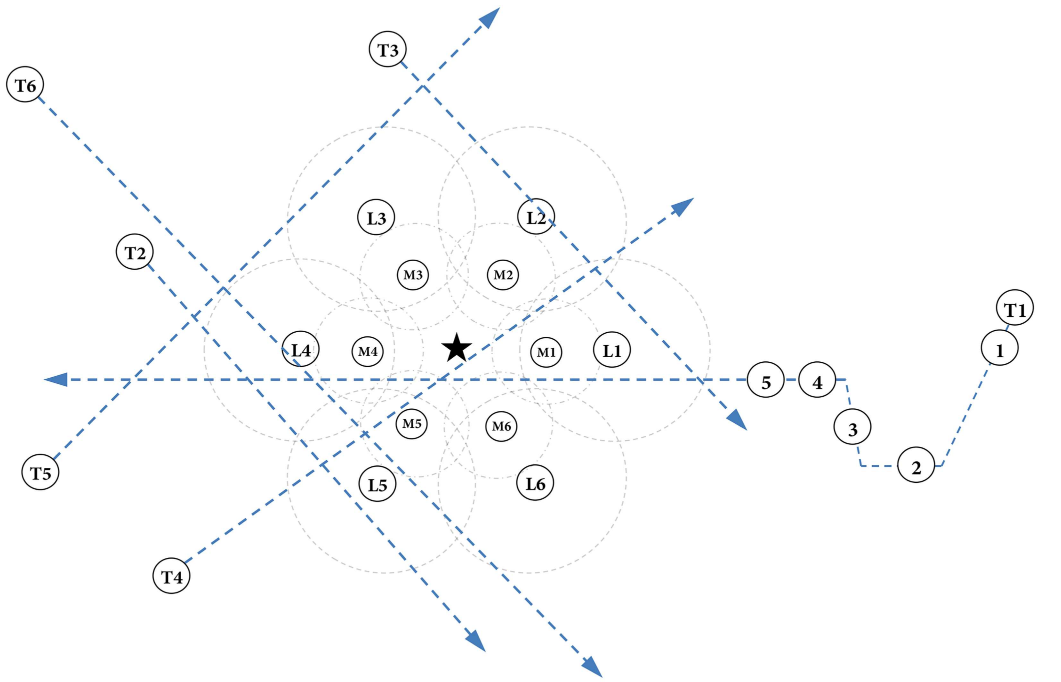

Figure 7. A typical scenario with five time steps.

Table 7. The solutions for the Weapon-Target Assignment (WTA) problem depicted in Figure 6 with the second Engagement Strategy (ES).

Priority	Target #	Weapon(s)	Pk	Cost
1	1	M1–	0.94	8
2	2	L5–	0.95	14
3	3	M3–	0.98	8
4	6	L5–L6	0.94	14
5	5	M1–M2	0.93	8
6	4	M2–	0.94	8

satisfy this condition, the size of a WTA problem which is solved in the static form using an exhaustive search algorithm should be smaller than 7*7 (weapons * targets). The proposed algorithm, based on the removal of impossible zones from the search space, could solve the static WTA problem of the size 24*24 (weapons * targets) in about 0.25 second. Therefore, the real-time condition is satisfied. Furthermore, a problem of size (12*6) in the form of Eq. (1) requires 6^{12} iterations, assuming that all weapons are used. Accordingly, the use of the exhaustive search is not suitable for real-time applications.

The problem with the proposed algorithm is decomposed and solved in multiple steps. First, 12*6 cases are checked to find crossing WSs. Next, the related criteria are applied and all possible solutions for each target are generated independently. In the presented defense arrangement, the maximum number of possible solutions for each target (with 12 weapons) is 44, given that the maximum number of crossing weapons for a target is 8. Given that each WS may fire multiple shots, the solution would be optimal if there was no weapon inventory limitation.

The computational burden of WTA problem can be reduced using two steps. Firstly, according to the arrangement of the defense system and linear predicted path of targets, the maximum number of crossing weapons for each target is limited. For the defensive arrangement presented in Figure 6, the maximum number of WS that meets a target is 8. Secondly, decision-makers' strategies for engagement (such as minimum acceptable kill probability) reduce the computations to a greater degree. Therefore, if there is no solution to exclude, assuming that all weapons do not meet all targets, the computational burden of the problem is reduced. In the presented case study, the weapon assignment problem was solved by reducing the large-sized, time-consuming (using exhaustive search), or imprecise (using heuristic search algorithm) problem to a simple fast-computing one, based on this fact that real-world problems are much simpler than basic WTA problem formulation (Eq. (1)). Finally, in Table 13, one can notice a general comparison between weapon assignment algorithms in terms of being real-time and optimality.

5. Conclusions

The Weapon-Target Assignment (WTA) problem was studied at the level of multiple weapon systems.

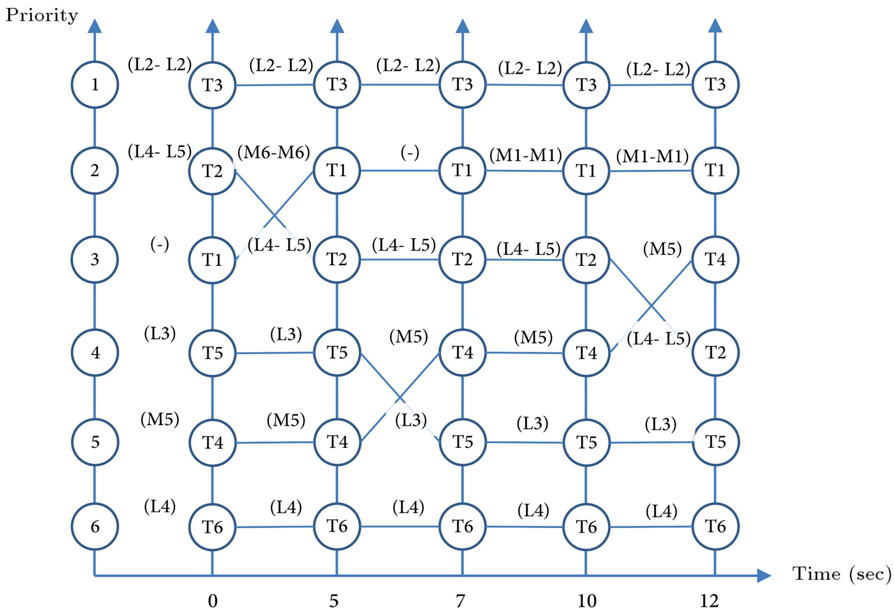

Figure 8. Targets priority and weapon assignment variations in the scenario shown in Figure 7.

Table 8. Prioritization and weapon assignment in five time steps.

Step 1: $T = 0$ (1)

Priority	Target #	Weapon(s)	P_k	Cost
1	3	L2–	0.98	14
2	2	L4–L5	0.93	14
3	1	—	—	—
4	5	L3	0.5	7
5	4	M5	0.9	4
6	6	L4	0.9	7

Step 2: $T = 5$ (2)

Priority	Target #	Weapon(s)	P_k	Cost
1	3	L2–	0.98	14
2	1	M6–	0.94	8
3	2	L4–L5	0.93	14
4	5	L3	0.5	7
5	4	M5	0.9	4
6	6	L4	0.9	7

Step 3: $T = 7$ (3)

Priority	Target #	Weapon(s)	P_k	Cost
1	3	L2–	0.98	14
2	1			
3	2	L4–L5	0.93	14
4	4	M5	0.9	4
5	5	L3	0.5	7
6	6	L4	0.9	7

Step 4: $T = 10$ (4)

Priority	Target #	Weapon(s)	P_k	Cost
1	3	L2–	0.98	14
2	1	M1–M	0.97	8
3	2	L4–L5	0.93	14
4	4	M5	0.9	4
5	5	L3	0.5	7
6	6	L4	0.9	7

Step 5: $T = 12$ (5)

Priority	Target #	Weapon(s)	P_k	Cost
1	3	L2–	0.98	14
2	1	M1–	0.97	8
3	4	M5	0.9	4
4	2	L4–L5	0.93	14
5	5	L3	0.5	7
6	6	L4	0.9	7

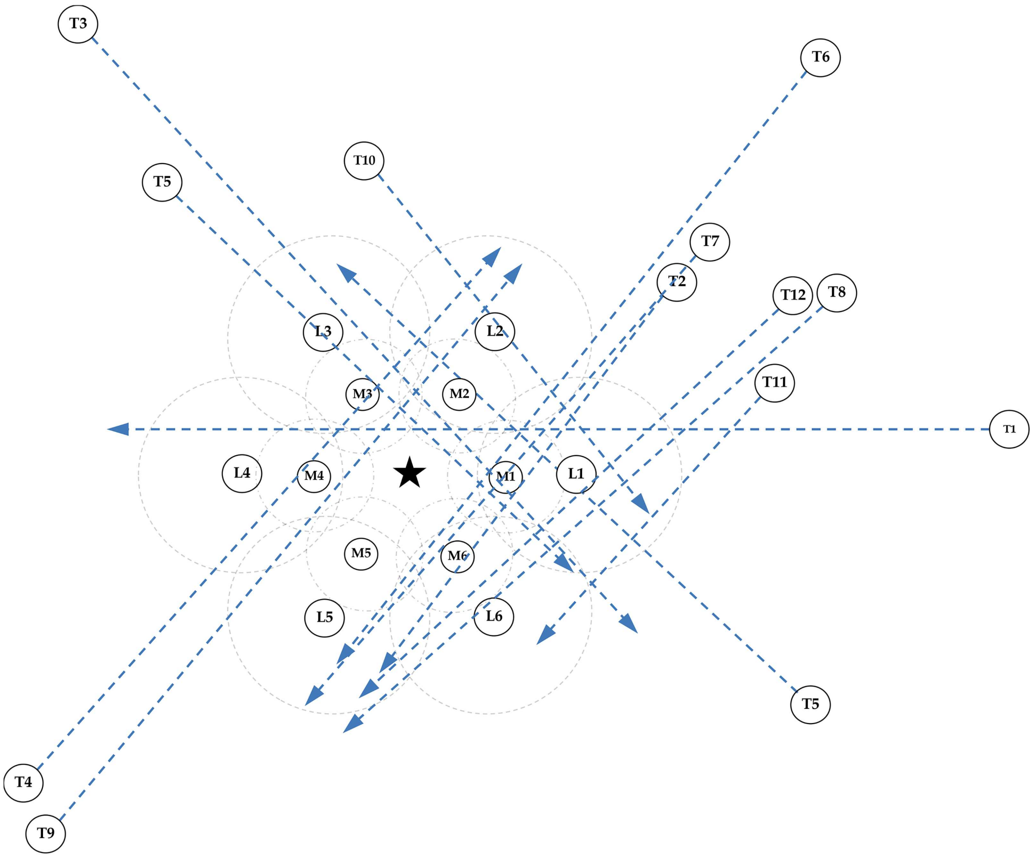

Figure 9. Weapon assignment test scenario (12 weapons * 12 targets).

Table 9. Solutions for the Weapon-Target Assignment (WTA) problem (12 weapons * 12 targets).

Priority	Target #	Crossing weapons #	Possible solutions	Allowable solutions	Weapon(s)	P_k	Cost
1	4	2	5	2	M2–M2	0.969	14
2	9	6	27	21	M3	0.90	7
3	1	6	27	18	L1–L1	0.978	8
4	3	8	44	21	L1–L1	0.938	8
5	5	2	5	4	M6	0.90	7
6	18	5	20	15	L1–L1	0.949	8
7	7	6	27	21	M2	0.90	7
8	14	4	14	9	M1	0.90	7
9	6	5	20	14	L3–L4	0.934	8
10	13	7	35	22	L1	0.90	4
11	16	7	35	20	L1	0.90	4
12	12	3	9	5	M1–M1	0.978	14

Table 10. Solutions to the Weapon-Target Assignment (WTA) problem (12 weapons * 18 targets).

Priority	Target #	Crossing weapons #	Possible solutions	Allowable solutions	Weapon(s)	P_k	Cost
1	4	6	27	18	L3–L3	0.978	8
2	9	4	14	10	M1	0.90	7
3	1	8	44	21	L1–L1	0.938	8
4	3	2	5	4	M5	0.90	7
5	5	6	27	18	L2	0.90	4
6	18	4	14	7	M1	0.90	7
7	7	6	27	21	M6	0.90	7
8	14	2	5	4	M6	0.90	7
9	6	7	35	21	M3	0.90	7
10	13	2	5	2	M3–M3	0.969	14
11	16	3	9	5	M4–M4	0.984	14
12	12	2	5	4	M3	0.90	7
13	2	6	27	21	M2	0.90	7
14	11	7	35	21	L1	0.90	4
15	8	1	2	1	M6–M6	0.938	14
16	10	1	2	2	M2	0.375	7
17	15	0	0	0	—	—	—
18	17	0	0	0	—	—	—

Table 11. Approximate computation time for the proposed algorithm.

Weapons	Targets	Time (milliseconds)
12	6	30
12	12	97
12	18	127

Table 12. Computational burden (in milliseconds) for 16 different scenarios.

	Targets			
Weapons	6	12	18	24
6	39	67	99	127
12	51	90	131	169
18	65	114	166	212
24	73	131	193	249

Accordingly, the assignment problem was solved in sequential steps. By removing the impossible/outlier solutions from solution search space in primary steps, the size of search space and required time for finding the optimal solution considerably decreased. Hence, instead of using imprecise heuristic search methods, an exact exhaustive method was used for finding a real-time solution to the WTA problem. In brief, the pros of the proposed algorithm are the possibility of offering a real-time solution, low computational burden, and providing an optimal solution (with some assumptions) for the multi-criteria WTA problem. To prove that it is as real-time and optimal as other algorithms, a general comparison between the different weapon assignment algorithms was presented.

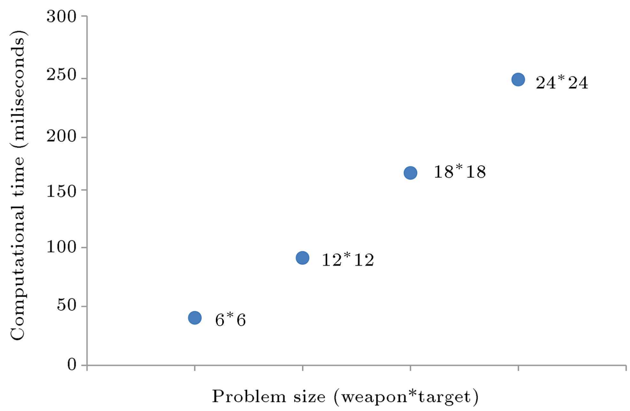

Figure 10. Computational time for an equal number of weapons and targets.

Table 13. A general comparison of weapon assignment algorithms.

Weapon assignment algorithms	Real-time	Optimal
Ant Co		
Genetic		
PSO		
VLSN	•	×
Tabu		
Simulated annealing		
MMR		
Neural networks	×	
Markov		
Game theory	×	×
Exhaustive search	×	•
Proposed algorithm	•	•

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