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Fault detection in cracked structures under moving load through a recurrent-neural-networks-based approach

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Abstract. The present study is based on the development of an inverse approach in the domain of Recurrent Neural Networks (RNNs) to identify and quantify multiple cracks on a cantilever beam structure subjected to transit mass. First, the responses of the multi-crack structure subjected to transit load were determined using fourth-order Runge-Kutta numerical method and Finite Element Analysis (FEA) executed using ANSYS software to authenticate the employed numerical method. The existence and positions of cracks were identified from the measured dynamic excitation of the structure. The crack severities were found as a forward problem through FEA. The modified Elman's Recurrent Neural Networks (ERNNs) approach was implemented to predict the locations and severity of cracks in the structure as an inverse problem by applying Levenberg-Marquardt (L-M) back propagation algorithm. The analogy was carried out in a supervised manner to check the convergence of the proposed algorithm. The results of the proposed ERNNs method were in good agreement with the theoretical and FEA results.

1. Introduction

Damage identification and assessment in the structures using vibration data have drawn the attention of the researchers since several decades ago. Effective structural damage detection is the key to structural health monitoring and condition assessment of structures. Several techniques have been developed and applied to detecting and quantifying the severity of damages as forward and inverse problems.

Chaudhari and Maiti [1] employed the Frobenius method to analyse the transverse vibration of a slender beam in and off the presence of cracks. Chinchalkar [2] developed a numerical method to determine the crack location in a stepped beam using the lowest three natural frequencies of the structure. Valoor et al. [3] developed a self-adapting vibration control method for a composite beam structure using Diagonal Recurrent Neural Network (DRNN) and Feed Forward Neural Network (FFNN). Lee et al. [4] developed a damage detection method for bridge structures under vehicle loading using the ambient vibration data by Finite Element Analysis (FEA) followed by experimental verifications. They employed the neural network techniques in damage assessment of the structure as an inverse problem. Kao and Hung [5] presented a neural-network-based method for structural damage detection. They formulated the method in two steps of structural system identification and damage detection. Seker et al. [6] applied Elman's Recurrent Neural Network (ERNN) to the diagnosis and condition monitoring of a nuclear power plant structure with rotating machinery.

Using the changes in natural frequencies of the

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structure, Kim and Stubbs [7] proposed a crack detection method to locate and quantify the severity of cracks. Law and Zhu [8] presented a damage detection technique using the changes in the nonlinear characteristics of a damaged reinforced concrete beam under moving vehicle. Nahvi and Jabbari [9] developed a crack identification method for cantilever beam structures using experimental modal data and finite element model. The approach was based on measurement of natural frequencies and mode shapes of the structure. Chasalevris and Papadopoulos [10] investigated the multiple-crack detection method for beam-like structures under vibrating conditions. Schafer and Zimmermann [11] presented the Recurrent Neural Networks (RNNs) as universal approximators in the state space model. They also extended the capabilities of the RNN and normalized it for error correction. Zhu and Law [12] established a damage detection method for a simply supported concrete bridge in time domain. They used the interaction forces between the bridge and traversing vehicles as the excitation forces on the damaged structure.

Li and Yang [13] developed a damage identification method using the Artificial Neural Networks (ANNs) technique based on statistical properties of structural dynamic responses. Talebi et al. [14] adopted the RNN in fault identification and isolation with application to satellite altitude control subsystem. Sayyad and Kumar [15] studied a crack detection method for a simply supported beam with single crack by the measurement of natural frequencies. They also developed the relationship among the natural frequencies, crack location, and crack size. Perez and Gonzalez [16] proposed a neural-network-based damage identification method to localize and quantify the damage extent using modal data. Based on ANNs technique, Shu et al. [17] presented a damage detection method using the statistical properties of structural dynamic responses as damage indices for input. By applying the concept of probability distribution function, Asnaashari and Sinha [18] developed a crack identification analogy in time domain approach. A novel method was developed by Oshima et al. [19] for the condition monitoring of a bridge structure based upon mode shape analysis of the response of a moving vehicle.

Hakim et al. [20] developed an ANNs-based approach for localizing the position and quantifying the severities of cracks in an I-beam structure. They considered the first five natural frequencies and mode shapes of the structure as input to the network model. Kourehli [21] presented a feed forward Back Propagation Neural Network (BPNN) technique to quantify damage and estimate its location. He used incomplete modal data for the training of the ANNs model. Vosoughi [22] developed a hybrid method to identify cracks in a beam-like structure using the Euler-Bernoulli and fracture mechanics theories. Aydin and Kisi [23] proposed a damage diagnosis method for beam-like structures using ANNs. Multi-Layer Perceptron (MLP) and Radial Basis Neural Networks (RBNNs) were employed to identify the location and severities of cracks. Jena and Parhi [24] determined the responses of different types of beam structures subjected to moving load under variable damage conditions of structure.

Koç et al. [25] combined the finite element and neural networks method to predict the end deflection of a barrel and investigated the consequences of an accelerating projectile. Back propagation algorithm was implemented in the model. He and Zhu [26] developed a closed-form solution for the dynamic response of a damaged simply supported structure under a transit load and investigated the effects of the loss of local stiffness. The aim of their work was damage localization based on moving load-induced response of the structure. Limongelli et al. [27] presented an experimental method for the early detection of damage in deteriorated bridge structures. Amezquita-Sanchez et al. [28] conducted a literature survey of the implementation of ANNs in the area of civil engineering for structural system credentials problem. Jena and Parhi [29,30] carried out numerical along with FEA and experimental studies to determine the responses of different types of beam structures subjected to moving load. Yeang et al. [31] developed an algorithm for damage localization in a structure subjected to moving vehicle. Obrien et al. [32] used the response of vehicle axle force as information to detect the existence of damage in a bridge structure. Toloue et al. [33] carried out an experimental work to develop a damage detection procedure by using noisy accelerometers and damage load vectors in a three-dimensional framed structure. He et al. [34] applied the mode shape curvature concepts as damage localizing method for vibrating structures. The mode shapes were extracted for a structure subjected to a moving vehicle in this methodology. Using the generalized S-transformation approach, Tehrani et al. [35] developed a damage localization method for the flexural members of structures. They validated this method with numerical examples followed by experimental studies. Zhang et al. [36] adopted the concept of contact-point response of a transit vehicle for fault detection in bridge structures.

The majority of damage detection methods involve the use of measured structural responses under dynamic excitation as forward problem and application of ANNs as inverse problem to predict structural damages. RNNs are superior in performance to FFNNs as they provide explicit model memory and are able to identify inter-temporal dependencies. The dynamic memory is supplied by means of feedback connections in RNNs. In the present article, a numerical method followed by FEA verification is proposed as forward method to identify the locations and quantify the severities of cracks by the dynamic excitation of the structure. The modified ERNNs approach is employed as an inverse method to quantify the cracks and predict their possible locations in the structure. An analogy will be carried out in a supervised manner.

2. Problem formulation

The schematic view of a damaged cantilever beam with multiple cracks subjected to transit mass is shown in Figure 1. A mass 'M' is moving across the beam from the fixed end to the free end of the damaged cantilever beam with a speed of 'v'. Including the effects of inertial, centrifugal, and Coriolis forces and ignoring the damping effects and longitudinal vibrations of the beam, the equation of motion for a beam under transit mass at no loading condition considering Euler-Bernoulli's beam theory is given as:

$$EI\frac{\partial^4 y}{\partial x^4}+m\frac{\partial^2 y}{\partial t^2}=P(t)\delta(x-\eta), \tag{1}$$

where EI is flexural rigidity, m beam mass per unit length, δ Dirac delta function, x beam deflection at the considered point 'Z', $\eta=vt$ position of the transit mass at any time 't,' and v speed of the transit mass. Also, relative crack depth is $\alpha=\frac{d}{H}$, $\alpha_{1,2,3}=\frac{d_{1,2,3}}{H}$, and relative crack positions is $\beta_{1,2,3}=\frac{L_{1,2,3}}{L}$.

Finally, $P(t)$ indicates the force induced due to the transit mass 'M' $=Mg-M\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial \eta}\right)^2 y(\eta,t)$.

Substituting the value of $P(t)$ in Eq. (1), we have:

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+\rho A\frac{\partial^2 y(x,t)}{\partial t^2}=\left[Mg-M\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial \eta}\right)^2 y(\eta,t)\right]\delta(x-\eta). \tag{2}$$

The solution to Eq. (2) can be written in series form, i.e.:

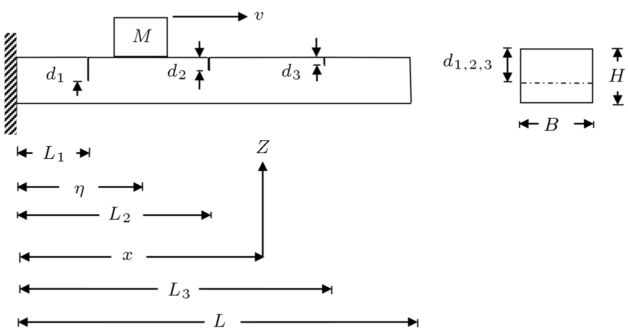


Figure 1. Schematic view of the damaged cantilever beam with multiple cracks under transit mass.

$$y(x,t)=\sum_{n=1}^{\infty}\phi_n(x)T_n(t), \tag{3}$$

where $y(x,t)$ is transverse deflection of the beam, $\phi_n(x)$ shape function of the beam, $T_n(t)$ amplitude function to be calculated, and n number of modes of vibration.

For calculating $\phi_n(x)$, Eq. (3) can be written as:

$${\phi_n}^{iv}(x)-{\lambda_n}^4\phi_n(x)=0. \tag{4}$$

Here, $\lambda_n^4=\rho A\frac{\omega_n^2}{EI}$ and ω_n is natural frequency of the beam.

Substituting Eq. (3) in the right part of Eq. (2) and doing the simplifications give:

$$\left[Mg-M\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial \eta}\right)^2\sum_{n=1}^{\infty}\phi_n(\eta)T_n(t)\right]\delta(x-\beta)=\sum_{n=1}^{\infty}\phi_n(x)T_n(t). \tag{5}$$

We can simplify Eq. (5) and reach the final solution equation, as presented earlier by Jena and Parhi [29,30], as follows:

$$EI\lambda_n^4 T_n(t)+\rho A T_{n,tt}(t)-\left(\frac{M}{V_n}\right)\left[g-\sum_{q=1}^{\infty}\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial \eta}\right)^2\phi_q(\eta)T_q(t)\right]\phi_n(\eta)=0. \tag{6}$$

The response of the vibrating structure is determined by solving Eq. (6) using Runge-Kutta fourth-order rule [29,30]. The response of the structure due to the interaction of moving load is calculated by the solution to Eq. (6). It is done by using Runge-Kutta method through developing a MATLAB code.

3. Finite Element Analysis (FEA) of cracked structures under transit mass using ANSYS

FEA of the cracked beam structure under transit mass is carried out by employing transient dynamic analysis method in ANSYS Workbench 2015. The responses of the structure are calculated in different damage scenarios. The numerical method inbuilt in ANSYS is Newmark-β integration method.

The equation of motion of a structure under travelling mass in FEA (transient dynamic analysis) can be articulated as:

$$M[\ddot{x}_t]+C[\dot{x}_t]+K[x_t]=F(t), \tag{7}$$

where x is the displacement of the structure $\dot{x}$ and $\ddot{x}$ are velocity and acceleration of the transit mass,

Table 1. Frequency ratios of the damaged cantilever structure.

Mode no.	$\boldsymbol{\alpha_{1,2,3} = 0.25, 0.5, 0.35}$ $\boldsymbol{L_{1,2,3} = 0.384, 0.48, 0.64}$	$\boldsymbol{\alpha_{1,2,3} = 0.4, 0.6, 0.45}$ $\boldsymbol{L_{1,2,3} = 0.44, 0.56, 0.768}$	$\boldsymbol{\alpha_{1,2,3} = 0.25, 0.5, 0.35}$ $\boldsymbol{L_{1,2,3} = 0.44, 0.56, 0.768}$	$\boldsymbol{\alpha_{1,2,3} = 0.4, 0.6, 0.45}$ $\boldsymbol{L_{1,2,3} = 0.384, 0.48, 0.64}$
1	0.9901	0.9931	0.9703	0.9875
2	0.9631	0.9808	0.9885	0.9703
3	0.9891	0.9789	0.9702	0.9891

$$\text{Percentage of deviation} = \frac{\text{(FEA values - Theoretical values)}}{\text{FEA values}} \times 100,$$

$$\text{Average percentage of deviation} = \frac{\text{Sum of the percentage deviations}}{\text{Total number of observations}},$$

$$\text{Total percentage of deviation} = \frac{\text{Sum of the average percentage of deviation}}{\text{Total number of average percentage of deviations}}.$$

Box I

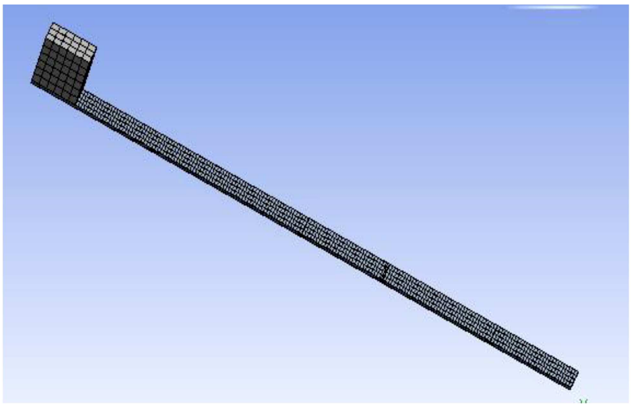


Figure 2. Transit mass-structure interaction of the cracked cantilever beam for $\alpha_{1,2,3} = 0.25, 0.5, 0.35$, $\beta_{1,2,3} = 0.384, 0.48, 0.64$, and $M = 2.5$ kg.

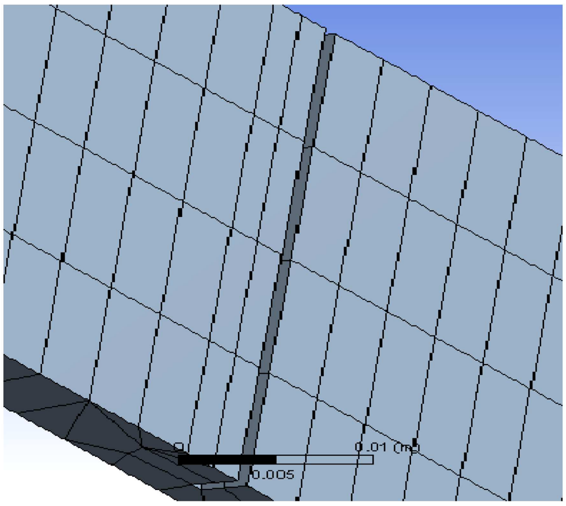


Figure 3. Magnified view of the crack zone for $\alpha = 0.5$.

respectively. Also, $F(t)$ is applied force, $K[x_t]$ stiffness force, $C[\dot{x}_t]$ damping force, and $M[\ddot{x}_t]$ inertial force.

Initially, modal analyses for up to five modes of vibration are carried out. In the present analysis, Newmark-β integration method is adopted under zero damping, unconditional stability, and constant average acceleration conditions to find out the responses of the structure in ANSYS Workbench 2015. In ANSYS, the responses of the structure at different locations of the transit mass and the particular location of the structure are calculated. The dynamic interaction of the moving mass and the cantilever structure is shown in Figure 2. The magnified view of a crack is shown in Figure 3. The dimensions of the cantilever structure are same as those in the numerical formulation with the same damage configurations, traversing mass, and speed. The crack is represented in an enhanced view (Figure 3). The transient structural dynamics analysis of the cracked cantilever beam in ANSYS Workbench 2015 is shown in Figure 4. The frequency ratios of the cantilever beam in various damage configurations are represented in Table 1. Dimensions of the structures are same as those in the experimental model with the same damage configurations, traversing mass, and speed. The expression for percentage of results between the FEA and theoretical values are represented in Box I.

4. Forward problem formulation

The results obtained from the numerical method are

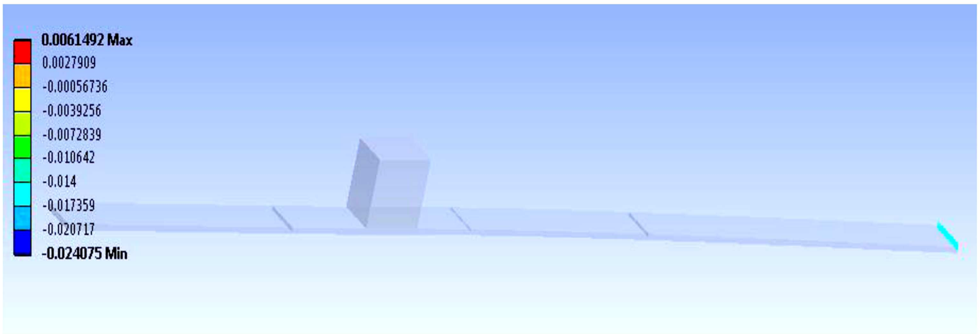


Figure 4. Schematic view of transient structural model for the cracked cantilever structure.

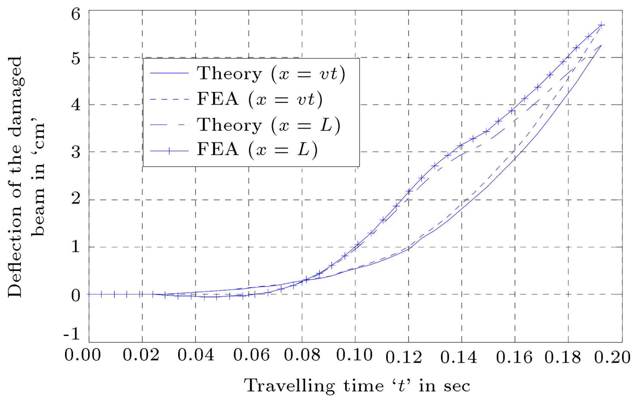


Figure 5. Deflection vs. travelling time for $M = 2.5$ kg, $v = 6.5$ m/s, $\alpha_{1,2,3} = 0.25, 0.52, 0.35$, and $\beta = 0.384, 0.48, 0.64$.

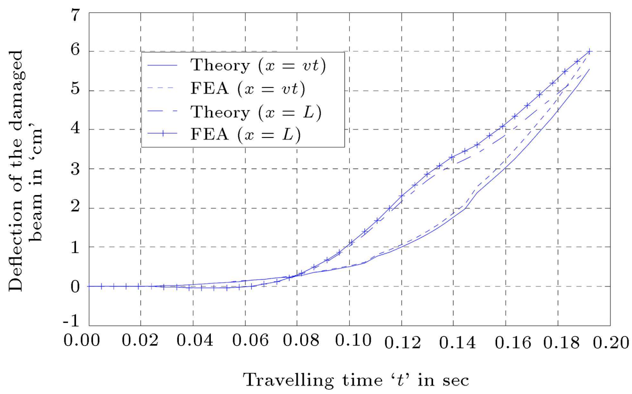
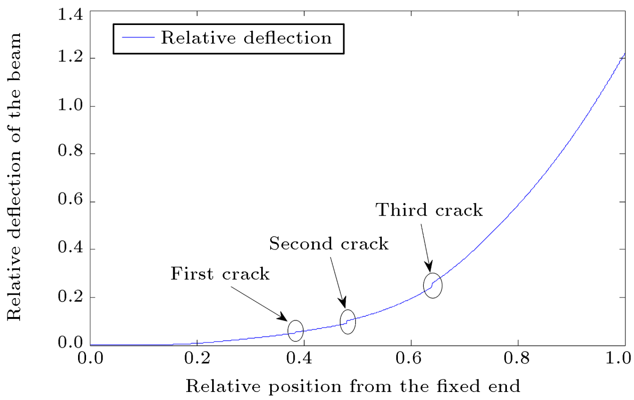


Figure 6. Deflection vs. travelling time for $M = 2.5$ kg, $v = 6.5$ m/s, $\alpha_{1,2,3} = 0.42, 0.61, 0.45$, and $\beta_{1,2,3} = 0.44, 0.56, 0.768$.

verified by FEA. To evaluate the forward problem for determining the response of the damaged structure due to the moving mass, a numerical example is formulated for a damaged cantilever beam made up of mild steel with the dimensions of 125 cm × 6 cm × 0.5 cm, speed of 6.5 m/s, and moving mass of 2.5 kg.

Relative crack depth is $\alpha_{1,2,3} = {}^{d_{1,2,3}}/_{H}$ and relative crack location is $\beta_{1,2,3} = {}^{L_{1,2,3}}/_{L}$. The subscripts 1, 2, and 3 stand for the first, second, and third positions, respectively.

The numerical analysis and FEA of the responses of the cracked cantilever structure under transit load are illustrated in Figures 5 and 6. The deflections at the free end ($x = L$) and at any location ($x = vt$) of the cracked structure under transit load are determined (Figures 5 and 6). The probable existence and locations of cracks are estimated from the measured dynamic response of the vibrated cracked structure under transit load. The forward problem analysis of the existence and locations of cracks is given in Figures 7(a) and 7(b). After detecting and localizing the cracks on the structure, the severity of cracks is determined from the natural frequencies and mode shape analyses of the structures by FEA [10] using ANSYS Workbench 2015. It is observed that the results of FEA agree well with the



(a)

Figure 7a. Crack detection of the beam for $\beta_{1,2,3} = 0.384, 0.48, 0.64$.

theoretical results. The details are elaborated on in the section devoted to the analysis of results in this study.

5. Modified Elman's Recurrent Neural Networks (ERNNs) approach to damage detection in a structure subject to transit mass as an inverse problem

The ERNNs are partial RNNs which identify patterns

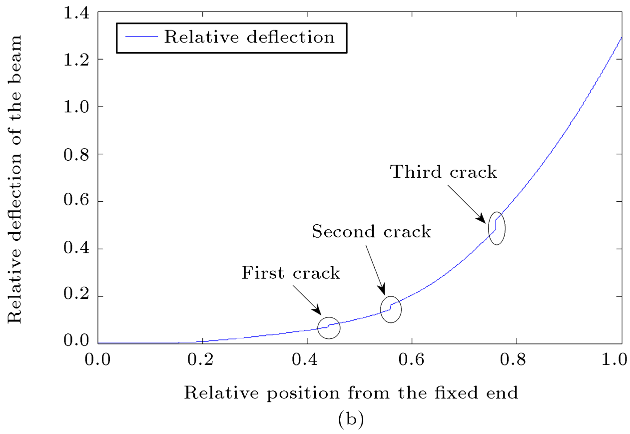


Figure 7b. Crack detection of the beam for $\beta_{1,2,3} = 0.44, 0.56, 0.768$.

in a sequence of values by implementing back propagation analysis through the mechanism of time learning. The ERNNs which was named after the researcher Elman which include the conception between the feed forward and recurrent network [37]. There are four layers in ERNNs, namely input, output, hidden, and context. The context layer is structured on the feedback connections from the hidden layer. The context layer provides the network with dynamic memory. This paper introduces the approach of modified ERNNs for fault detection in a damaged structure under transit mass. The modified structural architecture of ERNNs is shown in Figure 8. The present ERNNs model includes one input and output, three hidden, and two context layers. There are six neurons in each of the input and output layers, while those in each context

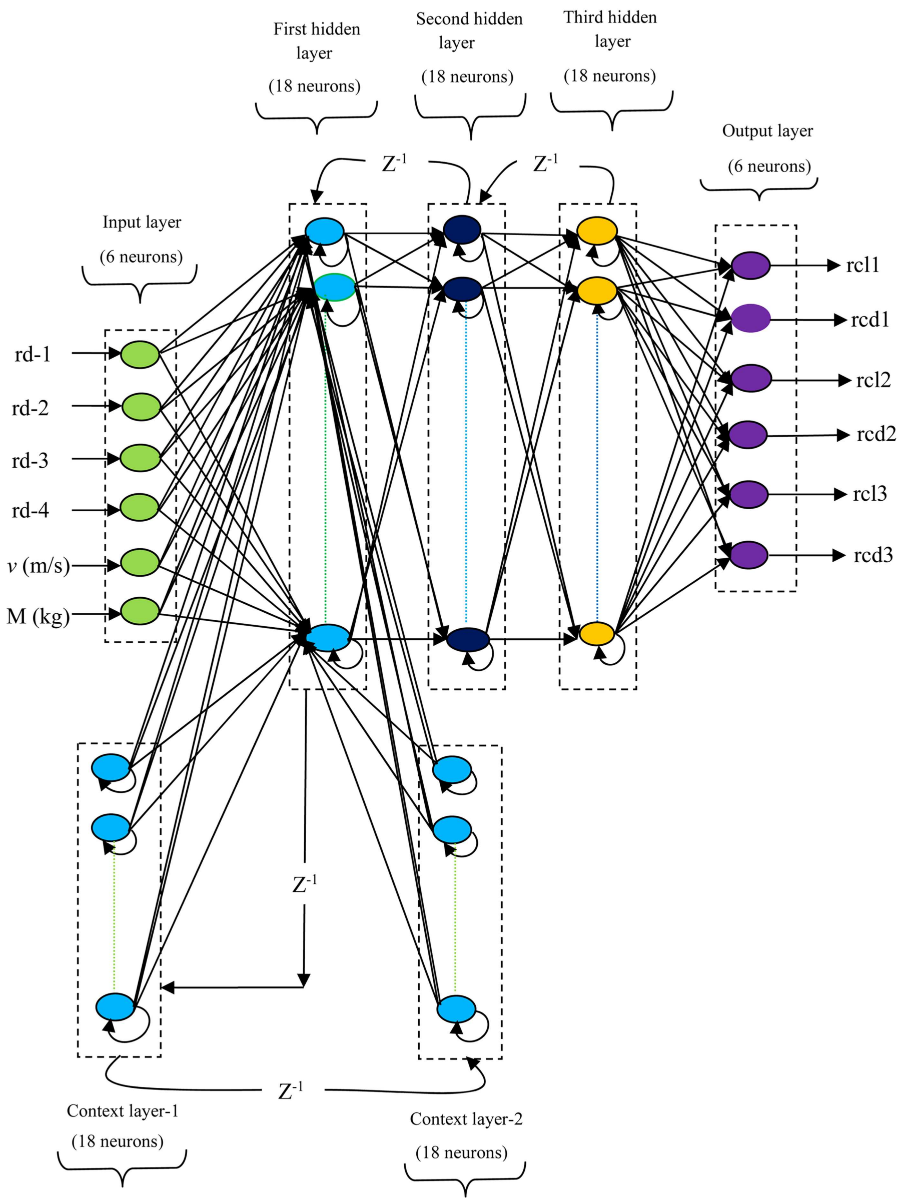


Figure 8. Modified Elman's Recurrent Neural Networks (ERNNs) architectural model.

and hidden layers are 18. The numbers of neurons in the hidden layer are the same as those in the context layer, because the context layer can copy or accumulate all the exact data or information and reuse it later. The first hidden layer gives information to context layer 1 through feedback links and again, collects information from context layer 1 as output. Context layer 1 also supplies feedback signals to context layer 2 and the first hidden layer gets the information as output from the nodes of context layer 2. Likewise, dynamic memories are provided for the network model using feedback connection from context layers 1 and 2. The feedback links are also supplied from the nodes in a hidden layer to those in the corresponding preceding hidden layer. The feedback and self-recurrent connections have one time delay unit. Apart from input and output layers, all the nodes in the context and hidden layers have self-recurrent links. Due to the existence of self-recurrent links, the nodes in the hidden layers supply extra generalities to the network structure for recognition of non-linear systems.

6. Use of Levenberg-Marquardt (L-M) back propagation method in the Recurrent Neural Networks (RNNs)

L-M back propagation algorithm, which is fast and stable, is implemented in the present RNNs analysis. It uses the steepest descent method and Gauss-Newton method in combination. This algorithm allows for the high speed of the Gauss-Newton and stability of the steepest descent analyses. The mechanism of the proposed algorithm is such that it transforms into the steepest descent analysis to make a quadratic estimation and then, transforms into the Gauss-Newton analysis to enhance the convergence of the algorithm throughout the training procedure.

The fundamental equation of the L-M back propagation algorithm [37] is given by:

$$\theta_{k+1} = \theta_k - ({J_k}^T J_k + \chi I)^{-1} J_k e_k, \tag{8}$$

where θ is the weight of connection or synaptic weights of the neuron, J the Jacobian matrix, which is evaluated by the Gauss-Newton method, I the identity matrix, and χ the combination coefficient. When the value of χ approaches zero, Eq. (8) will perform as a Gauss-Newton method and when χ is very large, Eq. (8) performs as the steepest descent method. According to the update rule of the L-M algorithm, if the predicted error is smaller than the previous error, then the value of χ should be reduced to decrease the implication of gradient descent method. On the other hand, if the calculated error is more than the previous error, it is required to increase the value of χ.

We have $\chi = (1/v)$ here and ν performs as the training constant or step size. Also, the error vector is $e = \psi_{desired} - \psi_{actual}$, where $\psi_{desired}$ is the calculated output vector and ψ_{actual} the real output vector.

$$\varepsilon = \text{Error function} = \frac{1}{2} \sum_{\substack{all\ training \\ patterns}} \sum_{all\ outputs} e^2. \tag{9}$$

The execution of the L-M back propagation algorithm depends on the value of J and the iterative training performance in weight updating. During the training procedure, back propagation recurs for every output value to accomplish the consecutive rows of the Jacobin matrix. The values of the error back propagating units are also analysed for each neuron of the hidden and output layers, separately, in the calculation of both forward and backward values. After determination of the Jacobian Matrix, the training procedure of the network starts.

During the training and operation steps of the network model, the training patterns are fed forward to include the following components:

- $i = 1, 2...N$, where N is the number of nodes in the input layer;
- $j_1 = j_2 = j_3 = 1, 2, ..S$, where S is the number of nodes in each of the hidden layers;
- $l_1, = l_2 = 1, 2, ..T$, where T is the number of nodes in each of the context layers 1 and 2;
- $k = 1, 2, ..O$, where O is the number of nodes in the output layer;
- X_{1-6} and V_{1-6}, are the values of context nodes in the context layers 1 and 2, respectively;
- W, total input values in the input layer;
- rd, relative deflection of the structure under moving mass = deflection of damaged beam to undamaged beam at a specified instant of time;
- $rd-1$, $rd-2$, $rd-3$, and $rd-4$, relative deflections of the structure under moving mass at the specified instants of time '$t/4$,' '$t/2$,' '$3t/4$,' and 't,' respectively;
- W, the values of the input layers;
- $W_1 = rd-1, W_2 = rd-2, W_3 = rd-3$, and $W_4 = rd-4$;
- W_5, traversing speed of the mass (v);
- W_6, weight of the moving mass (M);
- t, total travelling time of the traversing mass to cross the beam;
- ψ_1, ψ_3, and ψ_5, the first ($rcl1$), second ($rcl2$), and third ($rcl3$) relative crack locations, respectively;
- ψ_2, ψ_4, and ψ_6, the first ($rcd1$), second ($rcd2$), and third (rcd3) relative crack depths, respectively;

- γ, the value of self-recurrent links in each node of the layers (context layers 1 and 2; first, second, and third hidden layers);
- θ, weights of connection or synaptic weights of neurons;
- ψ_{l1}^{t-1} and ψ_{l2}^{t-1}, the net output values of the nodes at time $t-1$ of the context layers 1 and 2, respectively;
- ψ_{l1}^{t} and ψ_{l2}^{t}, the net output values of the nodes at time t of the context layers 1 and 2, respectively;
- ψ_{j1}^{t-1}, ψ_{j2}^{t-1} and ψ_{j3}^{t-1}, the net output values of the nodes at time $t-1$ of the first, second, and third hidden layers, respectively;
- ψ_{j1}^{t}, ψ_{j2}^{t}, and ψ_{j3}^{t}, the net output values of the nodes at time index t of the first, second, and third hidden layers, respectively;
- ψ_{k}^{t-1} and ψ_{k}^{t}, the net output values of the output nodes at times $t-1$ and t, respectively; and
- $f(.)$ and $g(.)$, the activation functions in the hidden and output layers, respectively.

From the analysis of the ERNNs model (Figure 8), we have:

$$\psi_{l1}{}^{t} = \psi_{j1}{}^{t-1} + \gamma\phi_{l1}{}^{t-1}, \tag{10}$$

$$\psi_{l2}{}^{t} = \psi_{l1}{}^{t-1} + \gamma\phi_{l2}{}^{t-1}. \tag{11}$$

The net input to the first hidden layer is given by using the following relation:

$$\psi_{j1}{}^{t} = \sum_{i=1}^{N} W_i\theta_{i,j1} + \gamma\psi_{j1}{}^{t-1} + \psi_{j2}{}^{t-1} + \psi_{l1}{}^{t} + \psi_{l2}{}^{t}. \tag{12}$$

The net input to the second hidden layer is:

$$\psi_{j2}{}^{t} = \sum_{j1=1}^{S} \psi_{j1}{}^{t}\theta_{j1,j2} + \gamma\psi_{j2}{}^{t-1} + \psi_{j3}{}^{t-1}. \tag{13}$$

The net input to the third hidden layer or to the network model is given by:

$$\psi_{j3}{}^{t} = \sum_{j2=1}^{S} \psi_{j2}{}^{t}\theta_{j2,j3} + \gamma\psi_{j3}{}^{t-1}, \tag{14}$$

$$net_j^t = \psi_{j3}^t = \psi_j^t = f(net_j^t), \tag{15}$$

$$net_k^t = \sum_{j3=1}^{S} \psi_{j3}^t\theta_{j3,k}. \tag{16}$$

The net output of the proposed network is given by:

$$\psi_k{}^{t} = g(net_k{}^{t}). \tag{17}$$

Each of the input and output layers contains six neurons, while each hidden or context layer has 18 neurons. The number of neurons or nodes in each of the hidden and context layers is chosen constant, because during the training process, the hidden and context layers can replicate the exact information from each other. The numbers of neurons are selected in an iterative manner during the training program. Considering different conditions of damage configuration in the structural system, 750 patterns are generated, out of which 650 are used in the training process and 100 in testing. Some of the patterns generated to train the network model are shown in Table 2. In Table 2, the output parameters with the value of zero for '*rcl*' and '*rcd*' clearly exhibit no presence of crack in the structure. Even the input parameters of '*rd*' with the value of 1 clearly indicate uncracked regions. The implemented activation function in the hidden and context layers is 'tan-sigmoid,' while 'purelin' is applied to the output layer. The L-M algorithm has been applied to the modified ERNNs model to estimate the position and severity of cracks in the structure. All the training and testing are carried out in a supervised manner to check the accuracy of the proposed RNNs model and L-M algorithm. The approximation error function (ε) has been employed in the input nodes of the ERNNs model to reduce the error value utilizing the updated

Table 2. Training patterns for the Elman's Recurrent Neural Networks (ERNNs) model.

Input parameters to ERNNs model						**Output parameters**					
***rd*1**	***rd*2**	***rd*3**	***rd*4**	***M* (kg)**	***v* (m/s)**	***rcl*1**	***rcl*2**	***rcl*3**	***rcd*1**	***rcd*2**	***rcd*3**
1	1.284	1.295	1.238	2	6	0.325	0.375	0.475	0.2	0.3	0.4
1.09	1.202	1.299	1.2484	1.8	5.8	0.3579	0.4211	0.5263	0.25	0.32	0.43
1	1.079	1.311	1.256	2.2	5.6	0.4286	0.5238	0.5714	0.3	0.4	0.5
1.012	1.115	1.36	1.296	1.4	6	0.48	0.56	0.64	0.4	0.4	0.4
1	1.05	1.377	1.306	1.2	10	0.5333	0.6333	0.7333	0.3	0.5	0.4
1	1	1.268	1.253	1.7	7	0.5385	0.6293	0.7692	0.4	0.5	0.4
1	1.153	1.419	1.911	3	12	0.55	0.65	0.75	0.5	0.5	0.5
1.191	1.281	1.622	3.5	3.4	12	0.4889	0.5778	0.6667	0.25	0.5	0.35
1	1	1	1	3.5	11	0	0	0	0	0	0
1	1	1	1	1	9	0	0	0	0	0	0

weight factors rule, i.e., $\theta^{new} = \theta^{old} + \zeta\Delta\theta$, where 'ζ,' the learning constant, varies from 0 to 1. The sum square error function has been employed to estimate the errors in the training process.

7. Results and discussion

The responses of a cracked cantilever structure subjected to transit load is analysed in this section. For the analysis of the forward and inverse problems, a numerical problem of a multi-crack cantilever beam under transit mass is exemplified (mild steel with dimensions of 125 cm $\times$ 6 cm $\times$ 0.5 cm, speed of 6.5 m/s, and mass of 2.5 kg). The deflections of the structure due to the movement of the mass are determined by both computation and FEA, as given in Figures 5 and 6. The results obtained by computation agree well with those of FEA. The feasible existence and positions of cracks are estimated from the measured dynamic response of the beam, as presented in Figures 7(a) and 7(b). The proposed ERNNs model has been trained by implementing L-M back propagation algorithm. The equations for the modified ERNNs were also developed. Several remedies were applied during the training process of the network model. A number of 750 patterns, including both damaged and undamaged, were developed for this problem, out of which 650 were used in the training process and 100 in testing. The relative crack depth and locations were predicted by training the network model. The results for the estimation of crack depth and locations by the ERNNs, FEA, and theoretical analyses are presented in Tables 3 and 4, respectively. All the training and testing procedures were conducted by supervised algorithm to check the accuracy of the implemented RNNs model and L-M algorithm. The results estimated by the ERNNs method were compared with FEA and theoretical results and they showed good agreement. The percentage deviation of the results between theoretical and FEA was about 2.3%, while with ERNNs, it was about 4.3%. The relation between the error value and the number of iterations is presented graphically in Figure 9. The conditions of cracked structures can be monitored online by employing the ERNNs

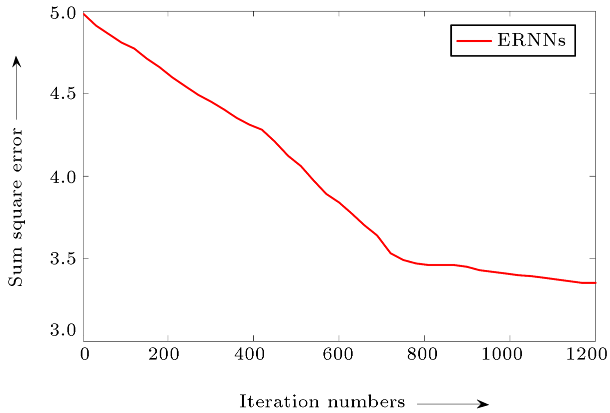


Figure 9. Graph of iterations vs. sum square error for Elman's Recurrent Neural Networks (ERNNs) approach.

Table 3. Comparison of the theoretical, Finit Element Analysis (FEA), and Elman's Recurrent Neural Networks (ERNNs) results for relative crack depth.

Theory			FEA			ERNNs		
α_1	α_2	α_3	α_1	α_2	α_3	α_1	α_2	α_3
0.25	0.52	0.35	0.244	0.5081	0.3411	0.2389	0.4994	0.3355
0.42	0.61	0.45	0.4117	0.5985	0.4404	0.4021	0.3855	0.4315
Average percentage of deviation			2.18	2.07	2.32	4.34	4.03	4.33
Total percentage of deviation				2.23			4.34	

Table 4. Comparison of the theoretical, Finit Element Analysis (FEA), and Elman's Recurrent Neural Networks (ERNNs) results for relative crack locations.

Theory			FEA			ERNNs		
β_1	β_2	β_3	β_1	β_2	β_3	β_1	β_2	β_3
0.384	0.48	0.64	0.3739	0.4676	0.624	0.3661	0.4582	0.6115
0.44	0.56	0.768	0.4291	0.549	0.751	0.4206	0.5379	0.7315
Average percentage of deviation			2.53	2.27	2.31	4.52	4.23	4.36
Total percentage of deviation				2.37			4.27	

based method to predict the faults in structures. The proposed ERNNs model can be very useful for fault detection in cracked structures.

8. Conclusions

Dynamic analysis of a cracked structure subjected to transit mass was carried out along with fault detection in the current study. The responses of the cracked beam under transit load were evaluated by both computational and Finite Element Analysis (FEA) methods. The potential existence and locations of cracks were determined from the observed dynamic responses of the structure. The severity of cracks was determined by FEA as a direct approach. In addition, a modified Elman's Recurrent Neural Networks (ERNNs) approach based on the L-M back propagation algorithm was developed to predict the locations and severity of faulty cracks in the structure as an inverse problem. The proposed ERNNs approach with Levenberg-Marquardt (L-M) back propagation algorithm was considered as a supervised process to check the accuracy of the implemented algorithm. The results estimated from the ERNNs analyses agreed well with the FEA and theoretical results. The present study showed that ERNNs could produce good prediction results and it could be very useful in monitoring unhealthy structures under transit mass. They might also be applied to fault detection in structures with an unsupervised algorithm.

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