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Determination of optimum cross-section of earth dams using ant colony optimization algorithm

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Control variable.

Abstract. Earth dams are one of the most important and expensive civil engineering structures to which a considerable amount of budget is allocated. Their construction costs mainly correspond to the size of embankments, which in turn depends on their cross-sectional area. Therefore, reductions in cross-sectional areas of earth dams may cause a decrease in embankment volumes, leading to a significant reduction in the construction costs of these structures. On the other hand, it is almost impossible to obtain optimum cross-section in earth dams with desired stability and acceptable operational dimensions using traditional design methods. In this paper, Ant Colony Optimization algorithm (ACO), a well-known and powerful metaheuristic method for tackling problems in geotechnical engineering, was used to solve this complicated problem. The results showed that applying the ideal and optimum slope and berm arrangements resulting from ACO in designing embankments and earth dams with different heights could lead to a decrease in embankment volumes, compared to those without any berms or those with berms resulting from usual designs with trial and error.

1. Introduction

Over the years, different methods have been proposed to solve optimization problems, some of which have provided absolute optimum answers, while the others mainly seek good and suitable answers (not necessarily absolute optimum answers). The philosophy behind discovering methods to reach almost absolute optimum roots could be that some optimization problems tend to be NP-Hard. This means that, because of the magnitude of the problems, there may not be a possibility to reach an absolute optimum answer in a reasonable and limited period of time.

Nowadays, due to their size and complexity, most problems are not solvable using traditional optimization methods, although mostly good and suitable ones might be needed practically. In other words, non-optimum, yet suitable, answers could be trusted. Consequently, in recent decades, people have tried to make tools by which one can obtain almost optimum solutions, if not the most optimum ones. Metaheuristic algorithm is known in optimization methods resulting from scientists' efforts over several decades. These methods that are generally derived from nature are capable of solving complicated problems in a suitable time to reach almost optimum answers. In recent years, these algorithms have experienced an enormous growth in solving complicated and difficult optimization problems. At first, these algorithms

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included Simulated Annealing (SA), Genetic Algorithm (GA), Taboo Search (TS), and artificial neural network (ANN). Successful results of using these methods, all from nature, were so promising that natural systems were accepted as the basic source of modeling ideas. Some of these metaheuristic methods were based on scientific studies done on the behavior of social insects, e.g., Ant Colony Optimization (ACO), which was proposed by Dorigo in 1991. Despite being new, it was used much to deal with scientific and practical aspects of optimization problems, leaving good results [1]. Despite favorable results, it has not been applied seriously to solve optimization problems in geotechnical engineering. It might be due to the recent use of metaheuristic methods for solving optimization problems of geotechnical engineering, dating back to more than a decade ago. On the other hand, it could be because of ACO, a rather novel method, which dates back to about two decades ago. The present study analyzed the applicability of the latter as a valuable tool. In recent years, developing optimization methods has been qualitatively and quantitatively in progress. Different novel metaheuristic methods, such as Bee Colony Optimization (BCO), Charged System Search (CSS), Water Evaporation Optimization (WEO), Colliding Bodies Optimization (CBO), Enhanced Colliding Bodies Optimization (ECBO), and Vibrating Particles System (VPS), have also been introduced.

Using ant colony optimization to find the optimum cross-section of earth dams is considered one of the most complicated problems in geotechnical engineering. This could be due to the complex of several optimization problems, such as finding a critical slip surface for a certain cross-section of an earth dam as well as an optimum cross-section of the dam at a certain load of its related cases.

Although several studies have been done to find the geometry of a critical slip surface using some methods of optimization, ant colony optimization for analyzing slope stability seems to be rather new.

Baker and Garber [2] used the variation method, which was later questioned by Luceno and Castillo [3] who concluded that their variation relation was incorrectly formulated. Celestino and Duncan [4] as well as Li and White [5] used the alternating variable method to locate the critical noncircular failure surface in slope stability. This method was also disapproved as it became complicated, even for simple slope stability problems. Baker [6] used dynamic programming to locate the critical slip surface using Spence's [7] method of slope stability analysis. Other methods, such as the simplex method, steepest descent, and Davidson-Fletcher-Powell (DFP) method, have also been considered in the literature [8-10]. Cheng et al. [11] pointed out, at least, two broad demerits of the above-mentioned classical methods of optimization for slope stability analysis: (1) Classical methods are applicable mainly to continuous functions and are limited by the presence of the local minimum; (2) The global minimum within the solution domain may not be given by the condition of the gradient of the objective function $\nabla f' = 0$. To the above-mentioned two drawbacks, one may also add that many classical optimization methods usually rely on a good initial estimate of the failure surface in order to find the global minimum, which is often difficult to estimate for the general case.

With the advent of fast computers, modern metaheuristic optimization-based techniques have been developed to effectively overcome the drawbacks and limitations of the classical optimization methods in searching for the critical slip surface in slope stability analysis. In metaheuristic optimization, the solution is found among all the possible ones, and while there is no guarantee that the best solution is found, solutions close to the best are often obtained quite effectively. Monte-Carlo-based techniques have been successfully adopted for slope stability analysis through limit equilibrium methods. This method is essentially a randomized hunt within the search space, and finding the lowest factor of safety becomes a matter of pure chance. Greco [12] and Malkawi et al. [13,14] used the random walk-type Monte-Carlo technique for locating the critical factor of safety in a slope. Monte Carlo-based methods are simply structured optimization techniques, in which a large number of random trial surfaces are generated to ensure that the minimum factor of safety is found. This is advantageous because the possibility of finding a failure surface, which is different from what the designer originally expected, will be greater if the search space is not too tightly defined. However, the process involves the analysis of a large number of solutions, whereas the method does not guarantee the location of the minimum factor of safety. Fuzzy logic has also been used for locating the critical failure surface of several simple slope stability problems [15-17].

Metaheuristic optimization algorithms have evolved rapidly in recent years. These algorithms use some basic heuristic in order to escape local optima. Metaheuristic implies that low-level heuristics in the global optimization algorithm are allowed to obtain solutions better than those they could have achieved alone, even if iterated. The heuristic approach is usually controlled by one of the two general mechanisms: (1) constraining or randomizing the set of local neighbor solutions to consider in the local search; (2) combining elements taken by different solutions. Many metaheuristic algorithms have been developed in recent years that loosely imitate natural phenomena.

The simulated annealing method [18], which is based on the simulation of a very slow cooling process of heated metals, is perhaps one of the first methods used for determining the location of the critical failure surface in slope stability analysis. Cheng [19] applied the mentioned algorithm to slope stability analysis. The Genetic Algorithm (GA) developed by Holland [20] is one of the most popular metaheuristic methods used in slope stability analysis and is, also, based on the concepts of genetics and evolution of living creatures. The optimum solution in GA evolves through a series of generations. Genetic algorithm-based solutions have been reported in the literature by Zolfaghari et al. [21], MacCombie and Wilkinson [22], and Sengupta and Upadhyay [23], among others. Particle Swarm Optimization (PSO), first developed by Kennedy and Eberhart [24], is another method that has attracted attention in slope stability analysis in recent years. As described by Cheng et al. [25] who successfully applied the method and the modified form of the algorithm, Modified Particle Swarm Optimization (MPSO), to locate the critical non-circular failure surface in slope stability analysis, PSO is based on the simulation of simplified social models, such as bird flocking, fish schooling, and the swarming theory. Cheng et al. [26] also used the fish swarm algorithm for determining the critical slip surface in slope stability analysis. Other methods include the harmony search algorithm [27,28], which is based on the musical process of finding the state of perfect harmony, tabu search [29], and the leapfrog algorithm by Bolton et al. [30].

As a pioneer in the application of modern metaheuristic optimization algorithms to slope stability analysis, Cheng et al. [11] evaluated the performance of six metaheuristic global optimization methods in determining the location of critical slip surface in slope stability analysis, including ACO. As investigated by Cheng et al. [11], ACO performed efficiently for simple problems, while relatively poor performance was observed in cases where soft bands existed in the problem. It was also mentioned that all six methods studied, i.e., simulated annealing, genetic algorithm, particle swarm optimization, simple harmony search, modified harmony search, tabu search, and ant colony algorithm, could work efficiently and effectively, provided that the domain transformation technique as suggested by Cheng [19] is adopted in the optimization algorithms. It must be noted that, in the present study, in order to determine the critical slip surface, the simplest and earliest algorithm from the collection of ant colony algorithm called (AS) has been briefly surveyed. Here, the function of this algorithm has been evaluated without analyzing the function of other new, corrected and promoted algorithms of ant colony in solving the problem of determining the unsuitable critical slip surface [1].

Kahatadeniya et al. [31] conducted a study on ant colony optimization in 2009. In their work, although the simple Ant System (AS) algorithm was used, much time and care was allocated to setting some parameters to achieve better results with which ant colony optimization algorithm could be evaluated in solving the problem of slope stability, even in recognizing non-circular critical slip surface in slopes with complicated layering. In 2011, Rezaeean et al. [32] carried out research into determining critical slip surface using four algorithms selected from ant colony optimization algorithms, with Ant System algorithm (AS) being one of them. The study showed that the function of these new modified algorithms would be far better than ant system algorithm, which is the first, simplest, and rather most incomplete algorithm in known ant system algorithms. As a result, in addition to special care in setting parameters of the algorithm, using modified and newer algorithms of ant colony optimization could definitely be one of the most powerful tools to determine critical slip surfaces, even in complicated problems of slope stability. In addition, Kashani et al. [33] and Kang et al. [34] determined the critical slip surface of slopes in the non-circular mode using imperialistic competitive algorithm and bee colony algorithm, respectively. Kang et al. [35,36] also used support vector machines together with metaheuristic optimization methods to solve slope stability problems.

Many studies have been carried out on the use of metaheuristic methods to optimize the shape of concrete dams, e.g., gravity dams and arch dams. Kaveh and Zakian [37], for instance, worked on the optimization of cross-sectional areas of gravity dams using optimization algorithms (CSS, CBO, and ECBO) and, also, Deepika and Suribabu [38] applied Differential Evolution algorithm (DE). Kaveh et al. [39,40] and Talatahari et al. [41] also studied the efficiency of metaheuristic optimization methods in determining the optimum arch shape in arch dams. However, there has not been much research done on the optimization of embankments and earth dam cross-sections, whose principles of analysis and design are completely different from concrete dams. In 2000, Ponterosso and Fax [42] conducted a study on the optimization of the cross-section of constructed embankments with reinforced soil using genetic algorithm. In 2012, Rezaeeian et al. [43] researched the function of ant colony optimization in optimizing homogeneous symmetric embankments of different heights. However, the study did not analyze the optimization of heterogeneous and asymmetric earth dams with regard to different load cases.

2. Characteristics of determining optimum cross-section model

2.1. General model

Using ant colony optimization to find optimum cross

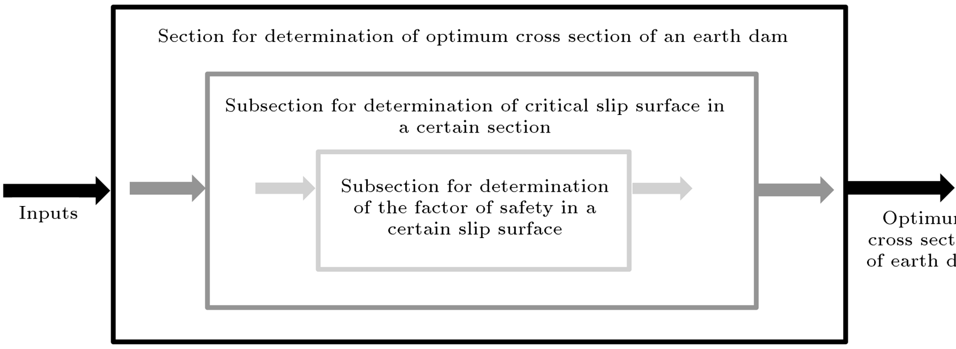

Figure 1. Function of the algorithm in determining the optimum cross-section of an earth dam in a certain load case.

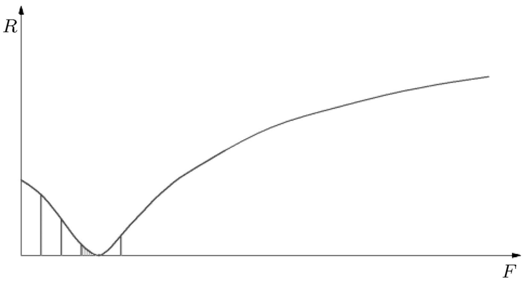

Figure 2. Determination of factor of safety in a slip surface with Newton's Method.

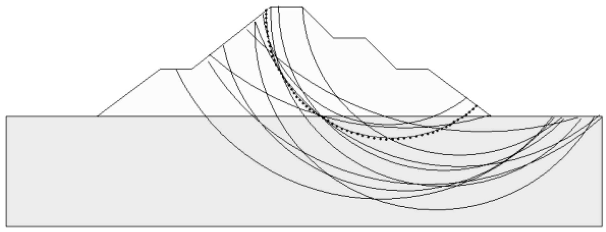

Figure 3. Determination of the critical slip surface in a certain cross-section of an earth dam.

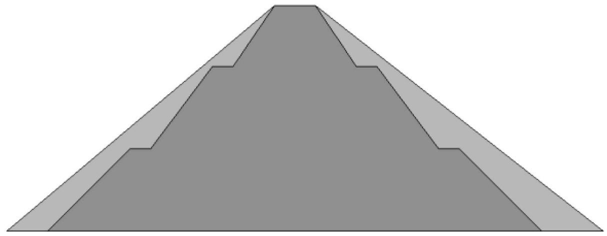

Figure 4. Determination of the optimum cross-section of an earth dam showing platform effects to decrease the volume of the dam in certain load cases.

-section of earth dams is considered one of the most complicated problems in geotechnical engineering. It generally consists of several optimization problems such as:

1. Finding safety factors in a certain slip surface based on the equilibrium method;
2. Finding the critical slip surface in a certain cross-section of an earth dam;
3. Finding the optimum cross-section of an earth dam in certain load cases in an earth dam;
4. Finding the critical load case of an earth dam dominant in choosing the final cross-section of an earth dam.

Subsections of determining critical slip surface, especially noncircular critical slip surface, and the optimum cross-section of an earth dam are considered as notable problems in optimization, which are too time-consuming to be solved by usual traditional methods. It would become more distinctive when observing these four steps at the same time. In Figures 1-4, a schematic of this problem is shown.

In the following, the problem of optimizing the cross-section of earth dams has been described after a brief description of the optimization process with the ant colony optimization algorithm. To avoid prolongation of the article, the subcategory of how to determine the critical slip surface of a specific cross-section of an earth dam by ACO algorithm has been considered; however, Reference [31] has been added for a further explanation of this point. ACO has been selected out of a large set of metaheuristic algorithms to solve this optimization problem due to the fact that, according to technical texts, it is one of the best algorithms for solving the problem of determining the critical slip surface of slopes. As described in Sections 1, it is regarded as one of the most important parts of the program of optimizing earth dams.

2.2. Biologic behavior of real ants and modeling artificial ants

Ant Colony Optimization (ACO) is a metaheuristic method, suggested in 1991 by Dorigo [1], which successfully passed its first test regarding the Traveling Salesman Problem (TSP). The algorithm is generally used to solve some problems, such as routing problems in transportation, time management in the management field, planning water supply systems, routing telecommunication networks, etc., leaving many successful results.

Figure 5. Indirect relationship of ants to pheromone to find shorter paths to food source.

In recent years, this optimization algorithm has been increasingly used to deal with different optimization problems in several scientific fields, too.

Ant colony optimization is modeled with a natural process of finding the shortest path between the nest and the food source by ants. Biologic bases of this process were discovered by biologic teams of Goss and Denenburg who answered the following question: "despite their weak eyesight, how do ants manage to find the shortest path between their nest and food source even with existing obstacles on their paths?" Here is where pheromone trails appear as the possible answer to it. Pheromone is a specific odorous substance, somehow considered to be ant footprint followed by the other ants to find their way. In other words, at the beginning of the food searching process, there would be no pheromone left in the environment; therefore, the pioneered ants tend to move randomly to find food and leave their pheromone trail in the environment. Having found food by specific coding, ants would return on the same way. Next ants would choose their paths by the routes being coded by the pioneered ants and leave pheromone on them. Although choosing a path tends to be done randomly, much pheromone could be left on the path. The more odorous the paths are, the more probable it will be for them to be chosen. However, at a shorter transition time, shorter paths (more optimum) are passed more quickly with more pheromone being left on them. Conversely, at a longer transition time and longer paths to the food source, less pheromone trail would be left. As a result of pheromone evaporation, the longer paths would gradually disappear, and almost all of the ants would move towards the shorter paths as a result (Figure 5) [1].

Immediately after this discovery by biologists in 1991, Dorrigo modeled, according to this natural optimization event, and invented ant colony optimization algorithm. In order to use the process of food finding by natural ants in optimization problems, the following equation was presented:

$$G = (D, L, C),$$

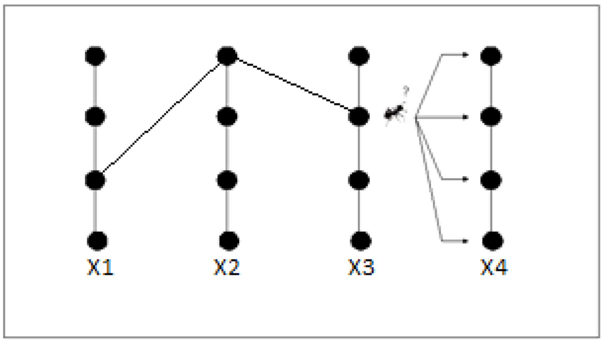

Figure 6. Graph of a general optimization case by ACO algorithm.

where $D = \{d_1, d_2, ..., d_n\}$ would represent a collection of decision points from which decision points could be decided. $L = \{l_{i,j}\}$ is a collection of destination points, $j = 1, 2, ..., n$ in the decision point, and $i = 1, 2, ..., m$ chosen as graph points by ants. Finally, $C = \{c_{i,j}\}$ represents different costs related to each destination point. Each possible path on the graph and a path with the minimum cost are generally called answer (φ) and optimum answer (φ^*), respectively. Members of D and L could be constrained, if necessary. In Figure 6, a general sample of a graph of optimization with variables $X1$, $X2$, $X3$, and $X4$ is presented [1].

2.3. Cross-section subdivision optimizer

The problem of finding optimum cross-section is presented in Eq. (1):

$$C = \min A(x), \quad (1)$$

where C is the objective function of the problem, $A(x)$ is the cross-sectional area of a certain section of the earth dam, and x is a set of variables in this problem, explained further.

In determining the optimum cross-section problem, the objective function was considered the cross-sectional area of certain sections which should be minimized. In other words, the objective was to find one cross-section with minimum area and, thereupon, minimum volume of earth works in certain earth dams. Variables used for determining the optimum cross-section problem generally include n, n', $b1_i$, $b2_i$, $h1_i$, $h2_i$, $I1_i$, and $I2_i$, represented in Figure 7. As shown in Figure 7, n and n' are the number of berms at upstream and downstream of an earth dam, $b1_i$ and $b2_i$ are widths of the ith berm at both upstream and downstream, $h1_i$ and $h2_i$ are the heights of the ith berm at upstream and downstream of it from the foundation level, and $I1_i$ and $I2_i$ are the slopes of the ith zone at upstream and downstream of the structure. In Figure 7, B, H, and H_f are constant parameters of the cross-section of the earth dam, which are the width of earth dam crest, the height of the dam, and the foundation depth, respectively.

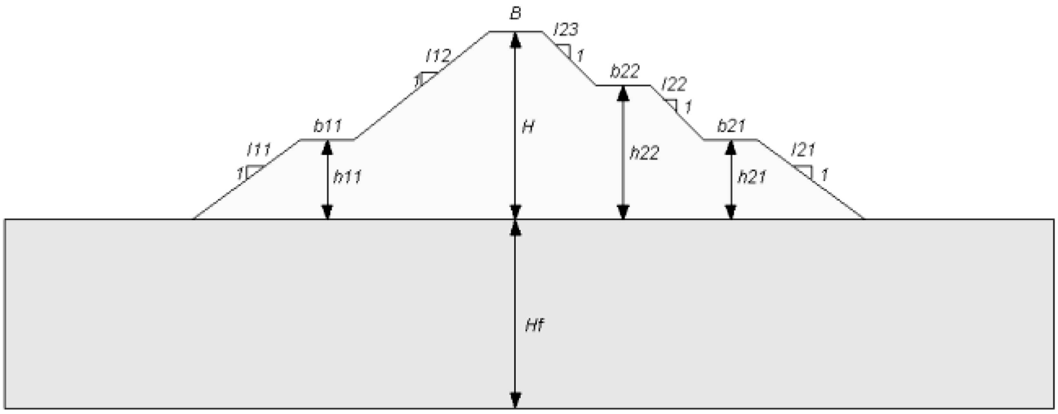

Figure 7. Optimization of the cross-section of an earth dam.

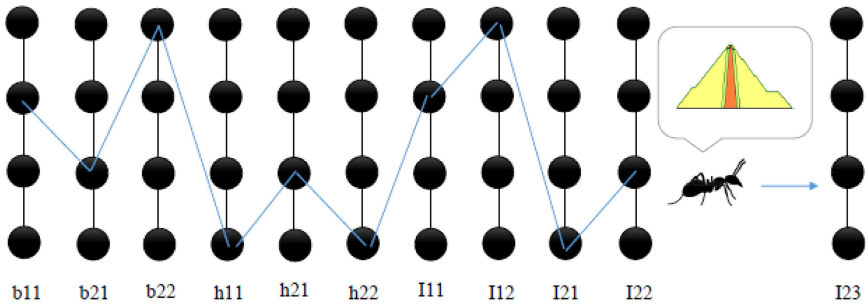

Figure 8. Optimization problem graph involving minimization of earth dam cross-section.

The variables, shown in Figure 8, are involved in finding optimum values of the cross-section with n berm at upstream and n' berm at downstream variables by moving artificial ants on this graph and depositing pheromone trail on the nodes of each path.

Two groups of independent constraints are considered in this problem. The first includes constraints that define the boundary of ant searching space, provided by searching space including possible solutions (possible paths). Zones with impossible solutions need to be avoided to determine searching space. To tackle this problem, variables $b1_i$ and $b2_i$ were chosen to be from 4 to 10-40 m (according to the height of the dam). The minimum width of the berm was determined according to administrative issues. Its maximum range was also measured by the maximum defined width of berms in earth dams with different heights. Values of $h1_i$ and $h2_i$ were chosen between 10 and H minus 10 m, randomly. $I1_i$ and $I2_i$ were also randomly, yet reasonably, selected, i.e., between response angle of soil material and slope of 1:4.

The second group of constraints includes conditional constraints, which would differentiate between possible solutions and impossible ones. To determine their impossibility and avoid them if encountered, one way could be punishment due to not following the constraints. Thus, instead of applying pheromone trial based on the objective function on these paths, pheromone trail was avoided by applying a penalty. This was exposed to the conditional constraints presented by Eqs. (2)-(4):

$$h1_{i+1} - h1_i \geq a, \tag{2}$$

$$h2_{i+1} - h2_i \geq a, \tag{3}$$

$$Fs \geq Fs_{allow}. \tag{4}$$

According to Eqs. (2) and (3), the height of each berm should be more than that of previous berms. The difference of the height of each berm from that of the previous one should also be more than or equal to a. Moreover, the minimum height of the first berm (lower berm) should be greater than or equal to foundation level a, unless this section was forgotten due to applying penalty because of not satisfying geometrical conditions. Eq. (4) also indicates that each section of an earth dam should meet slope stability conditions in all load cases of the earth dam. These load cases included the end of construction step, steady-state seepage of mid-level step, steady-state seepage of maximum level step, and rapid drawdown of the normal level step. In all load cases, the security factor of considered minimum cross-section should be more than the allowed security factor

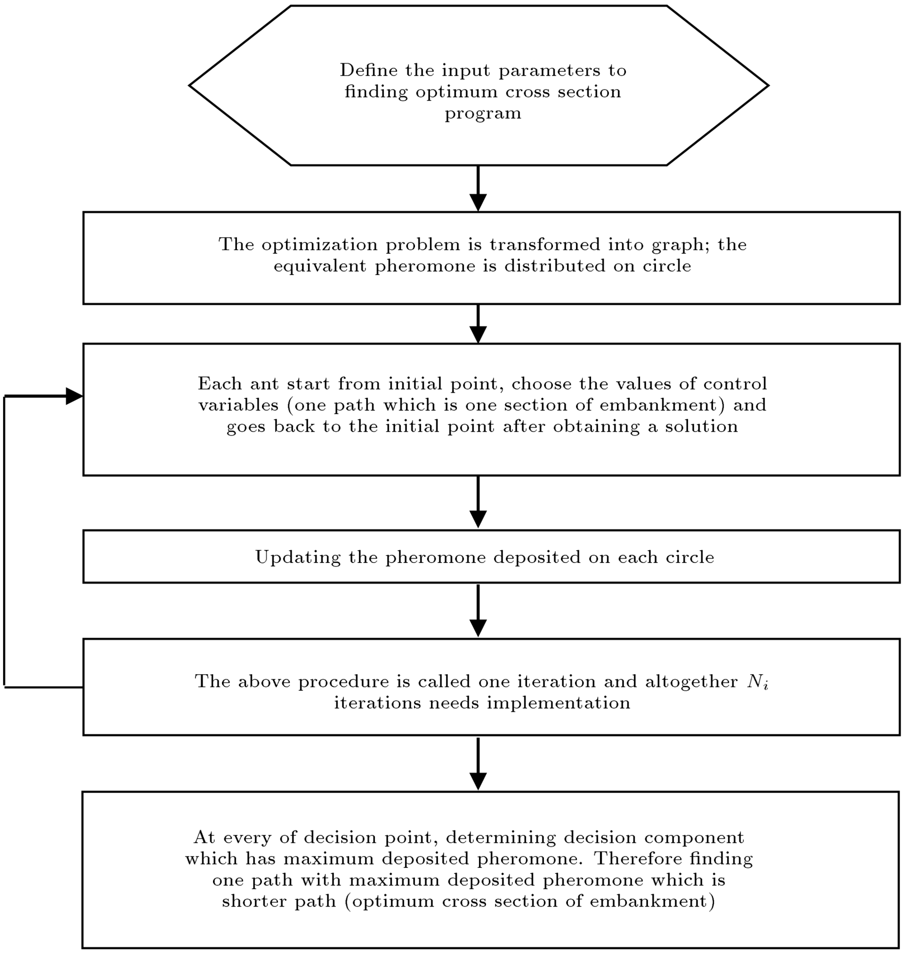

Figure 9. Flowchart of ant colony optimization algorithm for determining the optimum cross-section of earth dams.

while loading. Otherwise, if the security factor was less than the allowed security even in one of the load cases, it would be forgotten by applying penalty. In this case, the penalty would apply a cross-section of 10^{10} m^2. This large amount would cause pouring pheromone deposit to approach zero on unsuitable paths. As a result, impossible paths (solutions) would be forgotten, hence not effective in selecting further artificial ants in future iterations. In order to optimize earth dam cross-sections in all common load cases of earth dams, ODACO (Optimization of Dams by Ant Colony Optimization) was coded in more than 11000 lines by MATLAB. Figure 9 shows the flowchart of general stages of determining optimum cross-section of earth dam algorithm, ACO.

3. Review of ant colony optimization algorithms

Various improvements have been introduced to the original algorithm in recent years, aiming to make the search algorithm both more effective and more efficient. Accordingly, in addition to the Ants System (AS) algorithm, three other algorithms have been more successful and have been used in the present study: ranked ant system (AS_{rank}), elite ant system (AS_{elite}), and Maximum-Minimum Ant System (MMAS). The principal features of these algorithms are briefly discussed herein:

a. **Ants System (AS):** This is the simplest form of ACO first introduced by Dorigo et al. [44]. In AS, artificial ants choose their path according to the following probabilistic relation:

$$\rho_{i,j}(k,t)=\frac{[\tau_{i,j}(k,t)]^{\alpha}[\eta_{i,j}(k,t)]^{\beta}}{\sum_{j=1}^{j}[\tau_{i,j}(k,t)]^{\alpha}[\eta_{i,j}(k,t)]^{\beta}}, \tag{5}$$

where $\rho_{i,j}(k,t)$ is the probability of selecting the ith node of the jth column by the kth ant in the tth attempt. $\eta_{i,j}(k,t)$ in Eq. (5) represents the heuristic information, and the determination of its value is problem-specific. In some problems, the value of $\eta_{i,j}(k,t)$ is hard to determine and is, therefore, omitted from the equation. α and β in Eq. (5) are constants that determine the role of pheromone and heuristic information in the artificial ants' decision-making process. If $\alpha \gg \beta$, the role of pheromone is emphasized and heuristic information has less effect on the decision of the ants. Adversely, $\beta \gg \alpha$ means that the ants decide which node to move to base on the heuristic information, paying less attention to the pheromone deposited in the previous attempts [44].

Another important characteristic of ant colony algorithms is the way that pheromone update is defined in these algorithms. AS defines pheromone using Eq. (6) and $\Delta\tau_{i,j}$ is determined as follows:

$$\Delta\tau_{i,j}(t) = \sum_{k=1}^{m} \frac{Q}{f(S_k(t))} I_{s_k(t)}\{(i,j)\}, \tag{6}$$

where m is the number of artificial ants, or the number of solutions produced; Q is a constant named the pheromone return index, and its value depends on the amount of pheromone deposited; $S_k(t)$ represents all the nodes in which the kth ant has been chosen on the tth attempt; $I_{S_k(t)}\{(i,j)\}$ is a coefficient which is either zero or one, depending respectively on whether the kth ant has chosen the node (i,j) or not. In other words, $I_{S_k(t)}$ ensures that only the nodes towards which the kth ant has moved will be considered in depositing pheromone. It can be deduced from Eq. (8) that, in AS, solutions with a lower objective function will have more pheromone deposited, and vice versa [44].

b. **Elitist Ants System ($\mathbf{AS_{elite}}$):** In this algorithm, much attention is dedicated to the elite ant of the colony. The elite ant is the one which has produced the best answer in all previous attempts. Specifically, in AS_{elite}, extra pheromone is deposited on the path which the elite ant has produced. The ants decide which node to move towards using Eq. (7). The pheromone update rule in AS_{elite} is as follows:

$$\tau_{i,j}(t+1) = (1-\rho)\tau_{i,j}(t) + \Delta\tau_{i,j}(t) + \sigma\Delta\tau_{i,j}^{gb}(t), \tag{7}$$

where $\sigma\Delta\tau_{i,j}^{gb}(t)$ is the extra pheromone, deposited by the elite ant, and σ is the weight of the extra pheromone. AS_{elite} is an attempt to make a balance between exploration and exploitation in the algorithm [44].

c. **Ranked Ants System ($\mathbf{AS_{rank}}$):** The ranked ants system was first introduced by Bullnheimer et al. [45,46] as an extension of the elitist ants system. In this algorithm, unlike AS_{elite} in which all ants participate in the pheromone update process, only $\sigma - 1$ elite ants that have created better solutions are chosen to update the pheromone of the paths they have chosen. In AS_{rank}, following each attempt, the ants are lined up according to the solutions they have obtained, and pheromone update values are assigned to each ant; the most pheromone is assigned to the best solution and decreases thereafter to the last ant in the line. Thus, the pheromone update rule in AS_{rank} can be stated as in Eq. (8) [44]:

$$\Delta\tau_{i,j}^{rank}(t) = \sum_{k=1}^{\sigma-1} (\sigma-k) \frac{Q}{f(S_k(t))} I_{S_k(t)}\{(i,j)\}. \tag{8}$$

d. **Minimum-Maximum Ants System (MMAS):** Stutzle and Hoos [47-49] first reported the MMAS algorithm in a successful attempt to improve the efficiency of AS. The general structure of MMAS is similar to AS. However, only the path with the best solution in each attempt is chosen to deposit pheromone on its trail. In this way, the solution rapidly converges to the optimum. The danger always exists that the ants quickly move towards the first optimum solution achieved, before having the chance to explore other possibly better solutions in the search space. In order to prevent this danger, a restriction is placed on the minimum and maximum allowable net pheromone deposit on the trails, i.e., the deposited pheromone value is limited to $[\tau_{\min}, \tau_{\max}]$. Following each pheromone deposition step, all pheromone values are controlled to fit within the mentioned limit, and any node for which the pheromone value exceeds the limits is adjusted to the allowable limit. This is one way to promote the ants to explore new solutions in the search space. The maximum and minimum allowable pheromone values of the tth attempt are calculated as follows:

$$\tau_{\max}(t) = \frac{1}{1-\rho} \frac{Q}{f(S^{gb}(t))}, \tag{9}$$

$$\tau_{\min}(t) = \frac{\tau_{\max}(t)(1-\sqrt[n]{P_{best}})}{(NO_{avg}-1)\sqrt[n]{P_{best}}}, \tag{10}$$

where $f(S^{gb}(t))$ is the value of the objective function up to the tth attempt, P_{best} is the probability of the ants choosing the best solution once again, and NO_{avg} is the average of the number of decision choices in the decision points. It is noteworthy to mention that the initial pheromone value associated with the nodes, τ_0, is $\tau_{\max}(t)$ [47].

4. Implementation of ACO in optimization of cross-sections in embankments and earth dams

Three examples were analyzed in this study. In the first example, the optimization of a homogenous and symmetric cross-section of an embankment with different heights was analyzed in which, besides ant colony optimization (AS) as the earliest and simplest algorithm from the collection of any colony, three other algorithms were also used, e.g., elite Ant System (AS_{elite}), rank Ant System (AS_{rank}), and Maximum Minimum Ant System (MMAS), to make a comparison between the answers in this optimization problem. In the second and third examples, in order to show the effect of optimizer program on decreasing volume and costs of these dams, a case study regarding the optimization of cross-sections of two existing earth dams in Iran was investigated.

4.1. Embankments

Example 1

The first example represented homogeneous symmetric embankments composed of coarse soils with different heights, in addition to the number and arrangement of berms. Soil parameters were considered to be weight, cohesion, and effective friction angle (20.0 kN/m^3, 0 kPa, and 39°, respectively). The results showed that the effect of berms on embankments would be the decreasing volume of embankments, assuming that embankments were founded on bedrock. Figure 10 shows the layout of the embankments. Here, different heights were considered for embankments, and the effect of different berm numbers was considered in each embankment. Thus, not only the effect of berms on decreasing embankment volume was considered, but also the optimum number of berms in each specific embankment height was determined. Four different ant colony algorithms, i.e., ant system, elite ant system, ranked ant system, and maximum-minimum ant system, were employed to study the efficiency of ant colony optimization algorithms in reaching optimum cross-section. Cross-sectional areas obtained from different embankments using each algorithm are tabulated in Tables 1-7. Table 8 shows the amount of volume reduction at different embankment heights in a no-

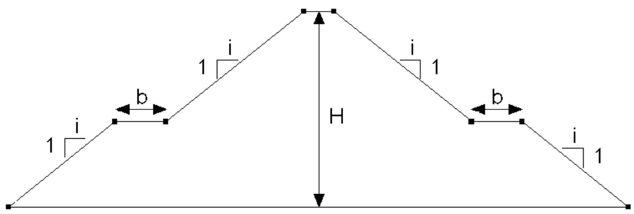

Figure 10. Configuration of variables controlling section-filling volume.

Table 3. ACO calculations of minimum filling volume regarding a 40 m high embankment.

Algorithms		Number of berms		
		Zero	One	Two
AS	Filling volume	2960	2739	2737
AS_{elite}		2800	2593	2775
AS_{rank}		2800	2723	2681
MMAS		2800	2507	2636

Table 4. ACO calculations of minimum filling volume of a 30 m high embankment.

Algorithms		Number of berms		
		Zero	One	Two
AS	Filling volume	1740	1501	1437
AS_{elite}		1560	1382	1488
AS_{rank}		1560	1350	1492
MMAS		1560	1390	1550

Table 1. ACO calculations regarding minimum filling volume of a 160 m high embankment.

Algorithms		Number of berms						
		Zero	One	Two	Three	Four	Five	Six
AS	Filling volume	63840	43499	45339	43592	49278	52745	68340
AS_{elite}		43360	43971	42602	44236	43118	43444	43456
AS_{rank}		43360	44336	42317	42316	43227	43523	42135
MMAS		43360	42306	42834	43071	42616	42877	42638

Table 2. ACO calculations of minimum filling volume of an 80 m high embankment.

Algorithms		Number of berms				
		Zero	One	Two	Three	Four
AS	Filling volume	12080	11596	11135	12163	12314
AS_{elite}		11440	11580	10960	11112	11007
AS_{rank}		11440	11121	11316	10842	11363
MMAS		11440	11213	11177	11426	11731

Table 5. ACO calculations of minimum filling volume of a 20 m high embankment.

Algorithms		Number of berms		
		Zero	One	Two
AS	Filling volume	760	656	697
AS_{elite}		760	672	686
AS_{rank}		760	728	787
MMAS		760	710	740

Table 6. ACO calculations of minimum filling volume of a 10 m high embankment.

Algorithms		Number of berms		
		Zero	One	Two
AS	Filling volume	200	170	243
AS_{elite}		200	157	220
AS_{rank}		190	166	182
MMAS		190	164	212

Table 7. ACO calculations of minimum filling volume of a 5 m high embankment.

Algorithms		Number of berms		
		Zero	One	Two
AS	Filling volume	55	50.5	59
AS_{elite}		55	51.5	52
AS_{rank}		55	47.5	54.25
MMAS		55	50	54

Table 8. Percent of reduction in filling volume of various heights.

Height of earth dam (m)	Reduction of embankment volume compared to without berm cross-sections (%)
5	14
10	17
20	13.7
30	13.5
40	10.6
80	5
160	3

berm condition [33]. This simple example was chosen to illustrate that even with the simplest problems, the AS is weak at finding the optimum cross-section of embankments, compared to the other algorithms. Moreover, a graph of the number of cycles against the cross-sectional area for this problem is drawn in Figure 11, suggesting that AS is not efficient in searching for the optimum solution, compared to the other ACO algorithms. The authors attribute this behavior to the fact that, in the pheromone-depositing process, AS does not support the optimum solutions. Therefore, it seems critical to point out that in order to properly assess ACO in solving geotechnical optimization problems such as determination of optimum cross-section of earth dams, it is necessary to evaluate at least several of the available ACAs before deducing a general judgment.

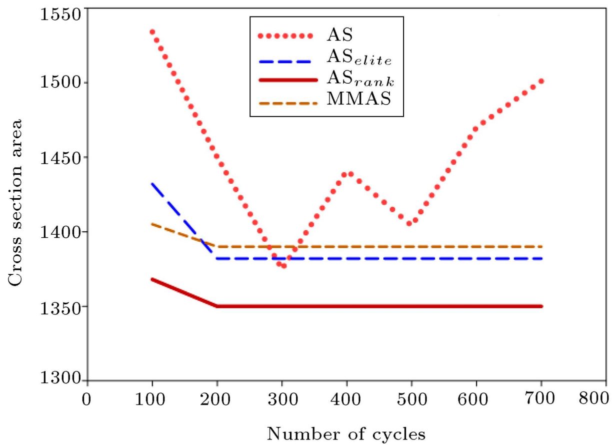

Figure 11. Comparison of the efficiency of the ACAs.

As clear in the table, in the case of homogeneous embankments, if the height of an embankment is less than 40 m, by using berms of suitable numbers, levels located in the body of them may reduce their volume by more than 10 percent, compared to an embankment without berms. Moreover, if the height of the embankment exceeds 40 m, as the height increases, the effect of berms may reduce. According to Tables 1-7, an optimum number of berms could be considered in the maximum reduction of filling volume in the embankment. The number could be three berms in an 80 m high embankment and one berm of 40 m, 30 m, 20 m, 10 m, and 5 m high embankments [42].

Following the first example, by deleting some of the geometrical constraints, a case study of a 40 m high homogenous symmetric embankment was investigated. The aim was to compare the effects of unequal slopes and berms with different widths on the decreasing volume of earthwork embankments and earth dams (Figure 12). The results achieved through the comparison are shown in Table 9. As can be seen, in a 40 m high embankment containing coarse-grained soil, the number of optimum berms is the same in both unequal and equal slopes. As can be also observed, in an embankment with unequal slopes, with optimum numbers, widths, and berm levels, the volume of earthwork decrease to around 14.3 percent, while in an embankment with equal slope, the reduction is 10.6 percent. Thus, compared to equal slopes, unequal slopes decrease the earthwork volume by 3.8 percent.

Table 9. Results of ant colony optimization algorithm to determine minimum cross-section of a 40 m high embankment with equal and unequal slopes.

The number of the berms	With unequal slopes		With equal slopes	
The name of the algorithm	One	Two	One	Two
AS	2930	2511	2739	2737
AS_{elite}	2398	2771	2593	2775
AS_{rank}	2403	2631	2723	2681
MMAS	2485	2805	2507	2636

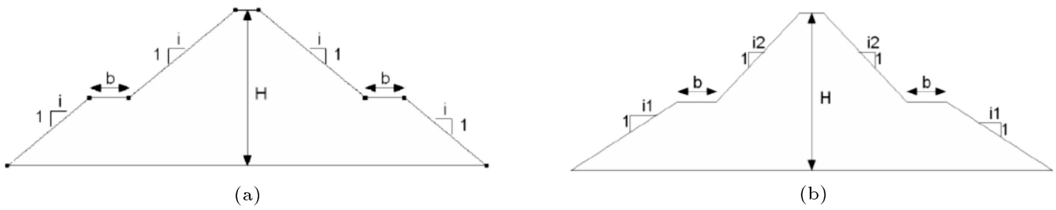

Figure 12. The General geometry of embankments: (a) With equal slope and (b) with unequal slope.

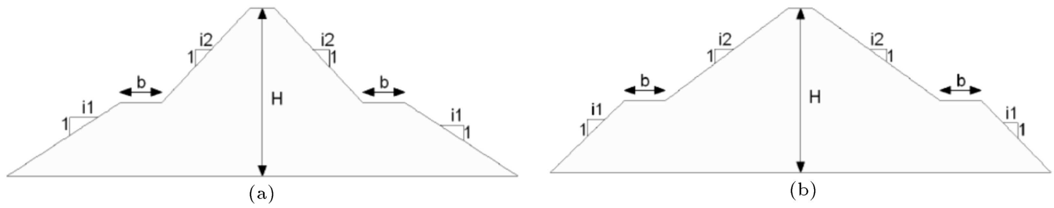

Figure 13. Slope changes in both modes of unequal slopes: (a) Optimum and (b) non-optimum.

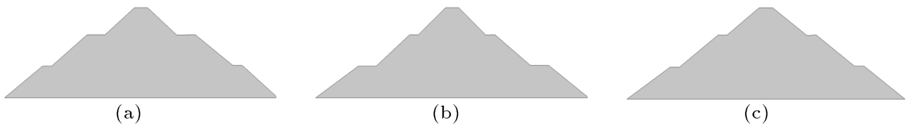

Figure 14. Effect of an unequal berm in the optimum cross-section of embankments: (a) Widths of berms are equal, (b) widths of berms decrease from bottom to top, and (c) widths of berms increase from bottom to top.

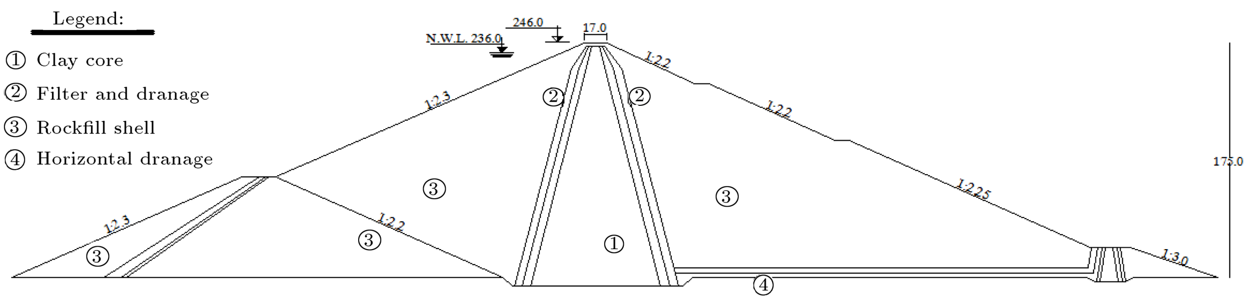

Figure 15. Critical cross-section of Gotvand Oliya Earth Dam.

Indeed, this may happen when the slopes increase from the bottom to the top of the body (Figure 13). Research has shown that the more the widths of the berms are at lower levels (and less at upper levels), the more the optimum cross-section of the embankments will be (Figure 14). Hence, using berms of suitable numbers, widths, and levels of unequal slopes results in more decreases in the volume of embankments because of declines in active force as well as increases in resisting force.

4.2. Earth Dams

Example 2

The second example is a case study on the effects of optimization of an earth dam cross-section by ant colony optimization on the Getvon-Olya Dam, a zoned earth dam with a central clay core considered as the highest earth dam in Iran, about 182 m high from the foundation. It is located in Khuzestan Province and was built in 2013 with the cost of more than 2000 billion Tomans. In Figure 15, the critical cross-

Table 10. Characteristics of the materials used in Gotvand Oliya Earth Dam.

Components	Y **(kN/M^3)**	C **(kPa)**	ϕ **(degree)**
Shell 1	21	0	40
Shell 2	20	0	35
Filter	20	0	30
Drain	20	0	35
Core	20.5	60	20

section of the dam is presented. Table 10 provides information on the characteristics of the materials used for constructing it. It was built based on minimum and maximum ant colony optimization algorithm (MMAS), one of the most powerful ant colony optimization algorithms surveyed and optimized under all common load cases of earth dams. Since this structure is one of the highest dams, the maximum width of berms was considered to be about 40 m. The results are shown in Figure 16 and Table 11. As can be seen, the applied program to optimize this earth dam led to a 5% decrease in the volume of embankment and, consequently, to a decrease in costs. To save some time, the analysis was done by ODACO to reach rather optimum answers. Undoubtedly, spending more time would result in achieving more optimum answers. On the other hand, the maximum number of berms was two, and several parameters of the optimum finding program were set experimentally with the least trial and error. Obviously, considering more berms and spending more time setting optimization parameters definitely lead to more optimum answers. However, this paper showed that by applying these tools without spending too much time and boring trial-and-error methods as well as arranging more suitable slopes and berms in earth dams considering administrative restraints in medium and huge projects of constructing dams, considerable amounts of money could be saved. For instance, in the Getvon-Olya Dam, the volume of earthwork and the cost of construction were considered to be more than 32 million cube meters and 2000 billion Tomans, respectively. This 5.8% decrease may

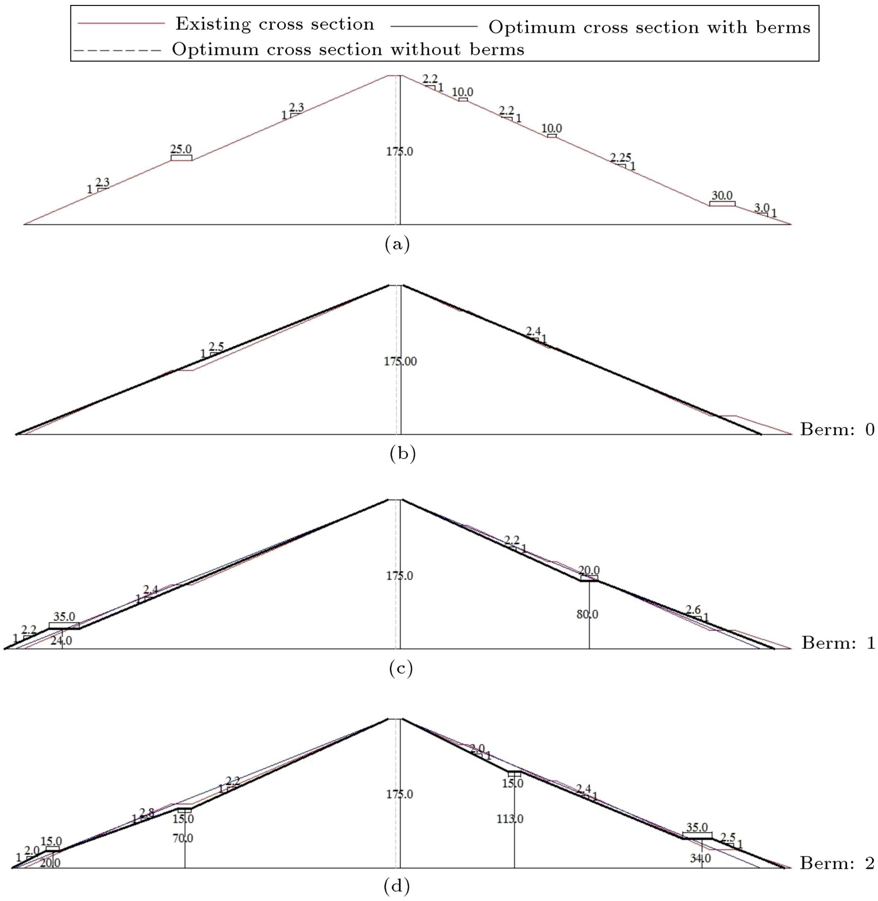

Figure 16. Results of the optimum cross-section of Gotvand Oliya Earth Dam by ant colony optimization algorithm: (a) Critical cross-section of Gotvand Oliya Earth Dam, (b) optimum cross-section of Gotvand Oliya Earth Dam without berms, (c) optimum cross-section of Gotvand Oliya Earth Dam with one berm, and (d) optimum cross-section of Gotvand Oliya Earth Dam with two berms.

Table 11. Results of the optimum cross-section of Gotvand Oliya Earth Dam by ant colony optimization algorithm without berms and with one or two berms.

Case of the cross-section	Cross-sectional area (m^2)	Percent of change in cross-section	Decrease of increase
Present	80087	—	—
Optimum without berm	78006	2.6	↓
Optimum with 1 berm	77075	3.8	↓
Optimum with 2 berms	75444	5.8	↓

Table 12. Characteristics of the materials used in Alborz Earth Dam.

Components	Y (kN/M^3)	C (kPa)	ϕ (degree)
Shell	21	0	42
Filter	19	0	35
Core	19.5	50	11

Table 13. Results of the optimum cross-section of Alborz Earth Dam by ant colony optimization algorithm without berms and with one berm or two berms.

Case of the cross-section	Cross-sectional area (m^2)	Percent of change in cross-section	Decrease of increase
Present	15274	—	—
Optimum without berm	14929	2.2	↓
Optimum with 1 berm	14795	3.1	↓
Optimum with 2 berms	14326	6.2	↓

save more than 100 billion Tomans in the cost of constructing dams.

Example 3

In the third example, a zoned earth dam with different heights, numbers, and different berm arrangements under all common load cases of earth dams was studied. A case study concerning the optimization of the Alborz Earth Dam cross-section, a 78 m high coral dam with a vertical clay core located in Mazandaran, Iran, was investigated first. The critical cross-section of the dam and the characteristics of the materials used for its construction are presented in Figure 17 and Table 12, respectively.

The dam was constructed by minimum and maximum ant system optimization algorithm (MMAS), one of the most powerful ant colony optimization algorithms surveyed and optimized under all common load cases of earth dams. Considering the height of this earth dam, the maximum width of berms was assumed to be about 20 m. The results are shown in Figure 18 and Table 13. As can be seen, applying this program in order to optimize the earth dam led to a 6% decrease in the volume of embankments and, consequently, a decrease in costs. The analysis was done by ODACO. Spending more time would result in more exact and probably more optimum answers as well as more decreases in costs. To compare how much the optimum cross-sectional area with berms decreased to cross-section without berms in earth dams of different heights, the Alborz Dam was scaled and analyzed at heights of 30, 50, 100, 150, and 200 m. Maximum widths of berms in dams of 30, 50, 100, 150, and 200 m high were considered to be 8, 10, 20, 30,

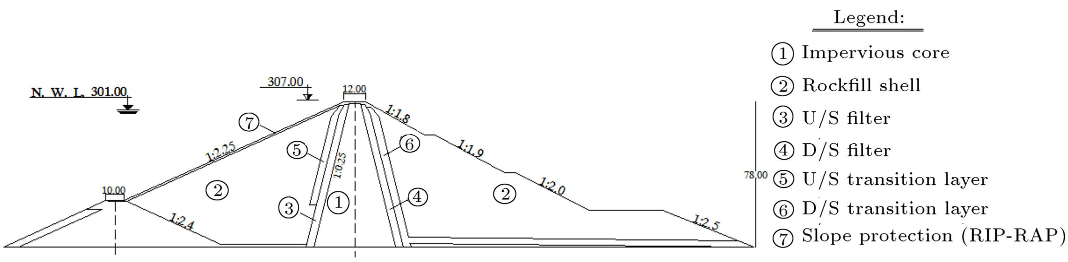

Figure 17. Critical cross-section of Alborz earth dam.

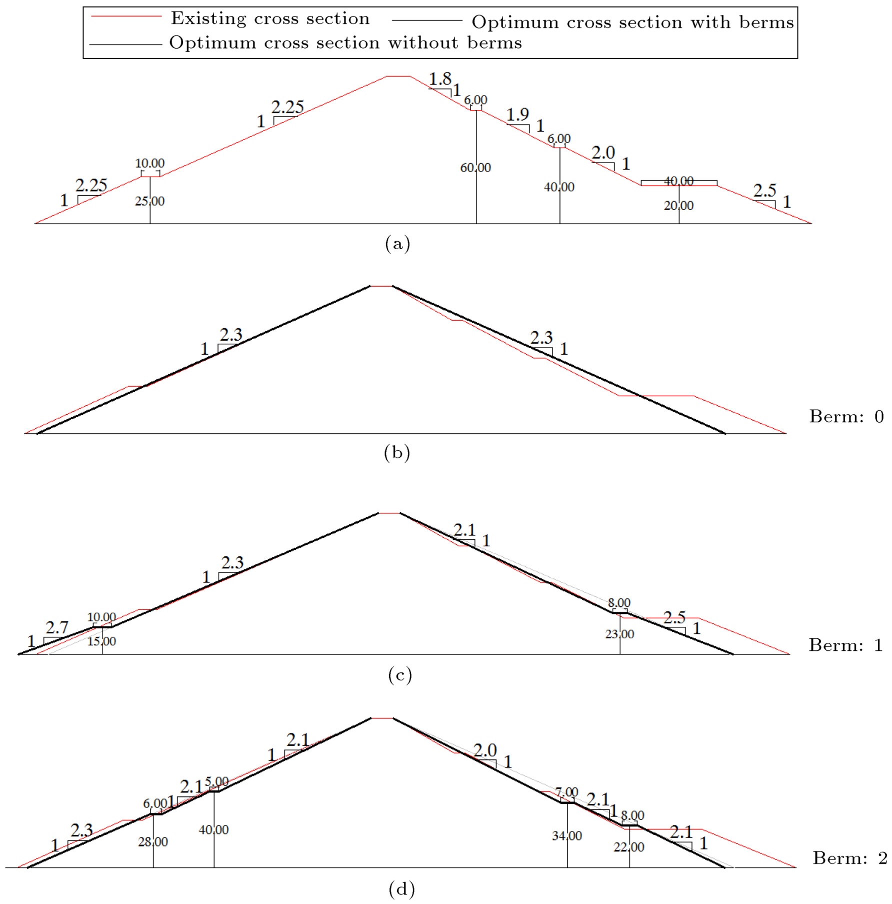

Figure 18. Results of the optimum cross-section of Alborz Earth Dam by ant colony optimization algorithm: (a) Critical cross-section of Alborz Earth Dam, (b) optimum cross-section of Alborz Earth Dam without berms, (c) optimum cross-section of Alborz Earth Dam with one berm, and (d) optimum cross-section of Alborz Earth Dam with two berms.

and 40 m, respectively. Characteristics of optimum cross-sections were analyzed in cases with and without berms, presented in Table 14. As the table shows, in comparison with cross-sections without berms, the greater the heights of the earth dams are, the more the lowering effects of berms in the decreasing cross-section will become. In other words, in shorter dams up to the height of 50 m, using berms in dams is administrative with no positive effects on decreasing embankment volumes. However, in higher dams of 50 m, the effect of using berms on decreasing embankment volumes is totally obvious, especially in much higher ones. In addition, in ranges with positive effects on decreasing embankment volumes (more than 50 m high), the higher the dams are, the fewer the number of needed berms to find more optimum cross-sections will be. Therefore, in earth dams of 78 and 100 m high, two berms are needed upstream and downstream; however, in earth dams of more than 150 and 200 m high, only one is required. Interestingly, the results are exactly the opposite of the first example, showing that the greater the height of the embankment, the less the effect of berms in the embankment volume. In other words, using berms in embankments of more than 40 m high might not lead to considerable decreases in embankment volumes. This considerable difference between embankments and earth dams could be due to the variety of load cases of earth dams (compared to embankments), the effect of full and half-full reservoir in earth dams, and encountering some constraints in analyzing the embankments, such as symmetry, equality of slopes, and limitation on the width of berms. The characteristics of the optimum cross-sections of earth dams in different modes are shown in Table 15, with parameters being presented from the lower to the upper levels.

5. Conclusion

In embankments lower than 40 m high with coarse-grained soil founded on hard rock, using common berms (maximum width of 10 m) of suitable numbers, widths, and arrangements in their cross-section could decrease embankment volumes more than 10%. In contrast, in embankments of more than 40 m high, the effect of small berms on decreasing embankment volumes

Table 14. Results of the optimum cross section of earth dams at the heights of 30, 50, 100, 150, and 200 m by ant colony optimization algorithm without berms and with one or two berms.

Cases		Decrease of increase	Percent of change in cross-section	Cross-sectional area (m^2)
Height (m)	Number of berms			
30	0	2130	—	—
	1	2275	7	↑
	2	2420	14	↑
50	0	6025	—	—
	1	6190	2.7	↑
	2	6133	1.8	↑
100	0	24600	—	—
	1	24519	0.3	↓
	2	24130	1.9	↓
150	0	55350	—	—
	1	53700	3	↓
	2	55770	0.8	↑
200	0	98400	—	—
	1	96929	1.5	↓
	2	99733	1.4	↑

Table 15. Details of the optimum cross-section of earth dams at the heights of 30, 50, 100, 150, and 200 m by ant colony optimization algorithm without berm and with one and two berms.

Cases		Upstream slopes	Downstream slopes	Berms width in upstream (m)	Berms width in downstream (m)	Berms level in upstream (m)	Berms level in downstream (m)
Height (m)	Number of berms						
30	0	1:2.2	1:2.2	—	—	—	—
	1	1:2.3 1:2.3	1:2.1 1:2.1	4.7	4	17	16
	2	1:2.4 1:2.1 1:2.0	1:2.2 1:2.1 1:2.2	4.3 4	4 5.7	13 21	12 28
50	0	1:2.3	1:2.2	—	—	—	—
	1	1:2.3 1:2.2	1:2.1 1:2.1	9	10	27	13
	2	1:2.4 1:2.3 1:2.0	1:2.7 1:2.0 1:2.3	9 4	5 4	11 27	11 38
100	0	1:2.3	1:2.3	—	—	—	—
	1	1:2.9 1:2.1	1:2.2 1:2.1	12	9	39	65
	2	1:2.7 1:2.4 1:2.1	1:2.5 1:2.4 1:2.2	7 4	4 4	18 42	16 43
150	0	1:2.3	1:2.3	—	—	—	—
	1	1:2.3 1:2.2	1:2.2 1:2.1	4	13	97	56
	2	1:2.5 1:2.2 1:2.2	1:2.2 1:2.2 1:2.2	26 8	21 4	34 72	24 81
200	0	1:2.3	1:2.3	—	—	—	—
	1	1:2.1 1:2.3	1:2.1 1:2.2	4	10	26	113
	2	1:2.4 1:2.2 1:2.2	1:2.3 1:2.4 1:2.2	22 10	10 16	30 92	38 121

would be less. Therefore, using these berms is proposed only in meeting executive needs of the embankments. In fact, berms with smaller embankment widths and average or more heights are considered grooves on embankments. Thus, to decrease the embankment volumes, berms with more widths are undoubtedly suggested.

In earth dams and other embankments, there are some optimum berms at each height. With the number of berms being more or less than needed, the volume of embankments would increase.

In determining the optimum cross-section of embankments and earth dams, MMAS, AS_{rank}, and AS_{elite} might do better than AS that, according to technical texts, proved to be the simplest and weakest algorithm. However, this weak function could not be enough to support AS against more optimum answers, which were obtained out of middle iterations; hence, in cases where only ACO is used or its function is compared to other algorithms, AS should not be used, and its results should not be trusted. However, its malfunction might not necessarily mean its rejection in that field.

Compared to cross-sections with equal slopes, in earth dams and embankments, using unequal slopes could cause decreases in embankment volumes and more optimum cross-sections. However, it seems necessary to carefully consider the fact that the slope should gradually increase from bottom to top. In addition, using berms of different widths could cause decreases in cross-section of embankments and earth dams. It happened when berms were of lower levels and their widths gradually decreased from bottom to top.

By using ODACO and an artificial intelligence method, like Ant Colony Optimization algorithm, (ACO) besides applying administrative constraints, instead of designers, a related software product responsible for implementing the trial-and-error method of finding either the critical slip surface or the optimum cross-section of earth dams was used. Interestingly, the obtained results were often much more optimum. Therefore, this method would not only simplify designing earth dams for counselors, but also cause saving and decrease high costs of construction and, finally, bring more success.

In earth dams with the heights less than 50 m, the effects of berms would not only be negative, but also cause opposite effects. In other words, in earth dams (up to 50 m high), using berms could only be justified in administrative requirements considered, which would not be recommended as an alternative to decreasing embankment volumes and optimization of earth dam bodies. However, to increase heights of earth dams, especially higher earth dams, using berms could have positive and considerable effects on the decreasing cross-section and volume of embankments and would even result in a decrease in the volume of embankments up to 6%. As a result, in higher earth dams which are costlier, using berms and finding suitable and optimum arrangements (definitely not possible without optimization tools) could lead to considerable decreases in construction costs. As mentioned earlier, the effect of berms on embankments could be exactly the opposite of earth dams, and the effect of decreases on embankment volumes could only be considerable in shorter embankments (from 50 m high). In these embankments, using berms with an optimum arrangement might cause a decrease in embankment volumes up to 15 percent. Interestingly, usually, defined embankments without earth dams, such as road embankments, tend to be located here.

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