Lion pride optimization algorithm: A meta-heuristic method for global optimization problems

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Abstract. This paper presents a new non-gradient nature-inspired method, Lion Pride Optimization Algorithm (LPOA), to solve optimal design problems. This method is inspired by the natural collective behavior of lions in their social groups "lion prides". Comparative studies are carried out using fifteen mathematical examples and two benchmark structural design problems in order to verify the effectiveness of the proposed technique. The LPOA algorithm is also compared with other algorithms for some mathematical and structural problems. The results have proven that the proposed algorithm provides desirable performance in terms of accuracy and convergence speed in all the considered problems.

1. Introduction

Real-world structural optimization problems are highly complex and multimodal due to effects of design constraints. The main objectives of the designers can be categorized into four groups: minimum weight, minimum cost, goal performance and a combination of some desired purposes. Design variables (e.g., cross-sectional areas and topology variables) are constrained by different accessible choices. Each decision has an impact on the structural responses such as internal forces, reaction forces, deflections, and natural frequencies, while no explicit relationship can easily be found.

Mathematical programming methods, such as Linear Programming (LP), combinatorial optimization, Quasi-Newton (QN), and nonlinear programming, can be applied to optimization problems. In this type of optimization methods, the results highly depend on the quality of the starting points. Besides, the feasible search space of a real-world design problem is characterized by a number of nonlinear constraints and contains plenty of local optimums. These methods in complex problems may fall into the trap of local optima, and they are not suitable for these problems.

The global search meta-heuristic optimization techniques, as an alternative approach to the conventional mathematical methods, can escape from being trapped in local minima and attempt to obtain near-global optimum solutions. However, the latter methods use non-deterministic approaches in which finding the optimum solutions is not guaranteed. Each meta-heuristic method has a unique behavioral base, and the performance of the algorithms is different in each problem space. For each algorithm, elevated performance over one class of problems comes with weak performance over other classes, and it is impossible for one metaheuristic algorithm to derive optimal solutions in all classes of problems. Thus, searching for new meta-heuristic optimization algorithms is an open problem [1].

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Nowadays, designers face new problems every day, and the complexity of the design optimization problems increases by recent advances in structural technologies. In order to tackle complex issues arising from the design problems, the necessity of proposing robust and new algorithms is evident.

The meta-heuristic methods consist of one or several search agents that explore the feasible region. The movements of each search agent are guided by a combination of randomness and one or more pre-defined rules. The behavior of search agents mainly has a natural inspiration. In the last decade, many new nature-based stochastic optimization methods have been developed, each of which mimics different sources of inspiration from nature; for instance, vibrating particle systems algorithm [2]; water evaporation optimization [3]; social spider algorithm [4]; dragonfly algorithm [5]; grey wolf optimizer [6]; charged system search [7]; and firefly algorithm [8]. However, inspiration has no border (e.g., artificial inspired mine blast [9] that mimics the explosion of mine bombs; and harmony search [10] that is inspired by the process of searching for better harmony [10]. A collection of 17 recently developed metaheuristics can be found in [11].

There are three different optimization algorithms in the literature, inspired by various aspects of a lion's life. Rajakumar [12] proposed the lion's algorithm based on the interaction between nomad and resident lions and the fight between resident lions and newly mature lions. Likewise, Lion Pride Optimizer (LPO) proposed by Wang et al. [13] and inspired by lions' life was developed. Yazdani and Jolai [14] proposed Lion Optimization Algorithm (LOA) that is based on the behavior of nomad and resident lions. Cooperative hunting of female lions in the pride, mating in pride, male lions defending against new mature resident males, mature lions defending against nomad males, and migratory behavior of female lions have been simulated in the mathematical concept of the algorithm [15].

This article proposes a new algorithm called Lion Pride Optimization Algorithm (LPOA) as an alternative approach to solving design optimization problems. As the name implies, the LPOA algorithm mimics some behaviors of lions in social groups called pride. A lion pride consists of lionesses, offspring, and a small number of male lions, each member having specific duties. Female lions have cooperative strategies for hunting. Male lions protect their territory against invading lions or other predators; for this reason, they usually move within their dominated areas.

In this paper, first, the main inspiration of the LPOA algorithm is presented. Then, the mathematical concept of the algorithm is introduced comprehensively. The relative efficiency of the proposed algorithm is next evaluated by solving some mathematical problems and compared to five modern meta-heuristic algorithms. Finally, two well-studied structural design problems are solved by the proposed method to demonstrate its effectiveness.

2. Inspiration from nature: background

2.1. Social groups

The lion (Panthera leo) is one of the largest and most powerful members of the Felidae family. Unlike other felids, they have a social innate behavior. Lions have two types of social behavior:

1. Residents that are members of social groups-called pride;

2. Nomads scattered in their habitats.

These categories are not mutually exclusive; a nomad may become a resident, and vice versa; however, there is a dichotomy between the two types of life [15].

All the members are engaged in a wide variety of group-leveling activities, including group hunting, communal club rearing, and group territoriality [15]; however, gender roles are almost specific. Each pride consists of 2 to 4 adult males, several adult females, and a number of sub adults and cubs [15]. It has been found that the average group size is 4 to 5 with a maximum of 15 [16].

2.2. Hunting

All the members typically hunt together as a team, and each member has a persistent division of labors during cooperative hunting. The hunting group generally involves a formation, whereby some lionesses (wings) circle prey, while others (centers) wait for prey to move towards them [17]. It should be stated that hunting groups are more cooperative when tackling difficult preys, such as elephants and buffalos.

The observations of hunting group provide evidence that not all lions present in the group participate in the activity; some in a larger group sizes may 'cheat' (exploit the foraging activities of conspecifics) during cooperative hunts [17]. An individual cheats only if it spots the prey first and stops hunting if a companion starts to hunt [18]. Therefore, individuals that do not participate in group hunts withhold effort that would increase the group's success rate [17].

2.3. Excursion

The territorial excursions of male lions provide effective protection for the whole pride territory. Lions leave scent marks within and on the borders of their territories in order to inform potential intruders, whose area is already occupied [19], and nomadic lions can warn by a sniff how recently a territorial male has passed on patrol and, thus, how safe it would be to stroll through the territory [20]. It can be mentioned that

there is a considerable amount of overlap between pride areas [15].

2.4. Mating

Lions have no particular breeding season. They mate at any time of the year and, also, when females do not rear cubs. A lioness may mate with multiple partners when she is in heat [21].

2.5. Intragroup interaction

Male lions do not behave compassionately towards the offspring, even towards their own offspring [22]. Males compete for the exclusive access to a pride and father, typically; only a single cohort of young females can negotiate long-term territorial boundaries and breed repeatedly over their lifetime [23]. Accordingly, mature resident males oust young mature male lions from their birth pride. A newly-matured lion is a new rival and must be eliminated. Mature lions fight each other, and weaker ones must leave their natal prides.

2.6. Migration

Some sub-adult lionesses leave the pride to become nomadic [15], and some of them migrate to other prides. Most of them remain with their natal pride, and just surplus lionesses are forced to leave their own group.

3. Lion Pride Optimization Algorithm (LPOA)

The LPOA is a population-based meta-heuristic algorithm that mimics some parts of the lion's life, and each agent is considered as a lion. In this technique, the search process is far dissimilar to all of the conventional optimization algorithms, such as Particle Swarm Optimization (PSO), Genetic Algorithm (GA), Cuckoo Search (CS), Simulated Annealing (SA), Harmony Search (HS), and Tabu Search (TS). In the proposed method, there are two different types of search agents with distinct types of defined rules. Moreover, they are divided into sub-groups, namely prides. In addition, each sub-group, which consists of females and males, probes into the search space unity. Each sub-group explores the defined space independently; therefore, the cumulative knowledge acquired by each pride's member is different from that by other groups' members. The migration mechanism engendering these distinctions reduces progressively. This part of the method urges the agents to probe into the search space better in the early stages of the optimization process and, then, to make them find near-global optimal solutions. Additionally, this mechanism can reduce the probability of being trapped due to transferring information between prides.

The mathematical model of the proposed algorithm is provided in the following.

3.1. The formation of pride groups

The lions and lionesses are the search agents of the proposed algorithm, and each agent explores the defined search space systematically.

The first step is to initialize the first population of the lions and form pride groups randomly:

$$a_{\min,i} < a_i < a_{\max,i} \qquad i = 1, 2, \cdots, nv, \qquad (1)$$

$$lion_j = [a_{1,j}, \cdots, a_{n,j}] \qquad j = 1, 2, \cdots, nl, \qquad (2)$$

$$Pride_k = \begin{pmatrix} lion_{1,k} \\ \vdots \\ lion_{nr,k} \end{pmatrix} = \begin{pmatrix} a_{1,1,k} & \cdots & a_{1,nv,k} \\ \vdots & \ddots & \vdots \\ a_{nr,1,k} & \cdots & a_{nr,nv,k} \end{pmatrix}$$

$$k = 1, 2, \cdots, np, \qquad (3)$$

where $lion_j$ determines the initial position of the jth agent; $a_{\min,j}$ and $a_{\max,j}$ represent the minimum and maximum permissible values for the ith variable; a_j indicates a random value between $a_{\min,j}$ and $a_{\max,j}$; nv is the number of design variables; nl is the number of total population; $pride_k$ includes the resident lions' position in pride k; nr is the population of residents; np represents the number of prides.

Fitness value of each lion is computed by evaluating the objective function as follows:

$$fit_j = objfun(lion_j = [a_{1,j}, \cdots, a_{n,j}]$$

$$j = 1, 2, \cdots, nl, \qquad (4)$$

where $objfun$ is the objective function, and fit_j is the fitness of the jth agent.

3.2. Main loop

Mathematical modeling of the LPOA with some characteristics of lions' life is presented in this subsection. Some variables that change in each iteration are described below:

$$D_i = DF \times amp_i \qquad i = 1, 2, \cdots, nv, \qquad (5)$$

$$amp_i = a_{\max,i} - a_{\min,i} \qquad i = 1, 2, \cdots, nv, \qquad (6)$$

where D represents diversification matrix; DF is the diversification factor that is constant in each iteration; however, it can be changed during the process of optimization. By examining different changes of this factor, it should decrease during the process in order to find near-global solutions. Figure 1 shows different types of changes of the diversification factor versus the number of iterations. Furthermore, the diversification factor can be changed adaptively according to the obtained fitness values or exploration success rate. In this study, the diversification factor declines rectilinearly with respect to the increasing number of iterations in the optimization procedure.
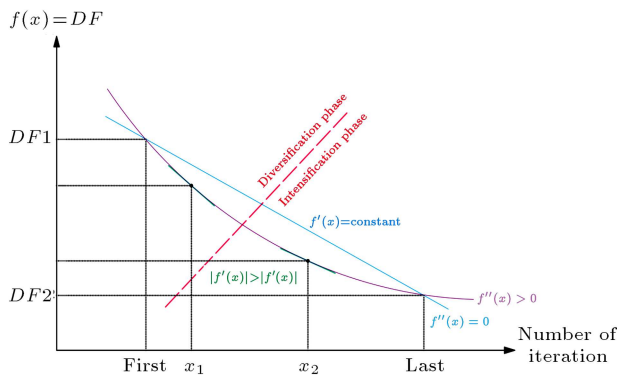
Figure 1. Diversification factor change diagram.

3.2.1. Hunting

The formulation of the cooperative hunting of lionesses is described as follows: All female lions in each pride are sorted as their fitness; after that, they are grouped into three groups. The best female lions' group members are named as "chasers". The second best group' members are labeled as "wingers", and the third group is called as "cheaters". Figure 2 shows the process of dividing female lions into these three general groups. Afterwards, the hunting subgroups are formed by three members, each of which is selected unmethodically from different general groups. In other words, each general group has a member in hunting subgroups. After the formation of three-member hunting groups, each group follows different prey to catch. The prey of each subgroup is selected randomly from the best position of all pride members.

Now, the position of each chaser changes using Eq. (7):

$$Chaser_{new} = Chaser + H_1 \times rand + (D)$$
$$\times (2 \ rand - 1), \tag{7}$$

$$H_1 = (Prey - Chaser), \tag{8}$$

where $Chaser_{new}$ and $Chaser$ are the new and existing positions of each chaser lion, respectively; $rand$ represents a random number between zero and one; and $Prey$ indicates the position of the prey.

The new position of wing hunters in each iteration follows the equation mentioned below:

$$Winger_{new} = Prey + H_2 \times |W| \times rand + (D)$$
$$\times (2 \ rand - 1), \tag{9}$$

where $Winger_{i,new}$ indicates the new position of Winger hunter lion, and H_2 is a random unit vector perpendicular to vector W considered as follows:

$$W = Prey - Winger, \tag{10}$$

$$|W| = \sqrt{W_1^2 + \cdots + W_n^2}. \tag{11}$$

Finally, Eq. (12) defines the movements of the cheater:

$$Cheater_{new} = Prey + H_3 \times rand + (D)$$
$$\times (2 \ rand - 1), \tag{12}$$

$$H_3 = (Prey - Cheater), \tag{13}$$

where $Cheater_{new}$ represents the new position of the Winger hunter lion, and $Cheater$ is the present position of the Winger hunter lion.

The schematic diagram of movements of each hunter is depicted in Figure 3.

3.2.2. Excursion

The simulation of male lions' excursion is formulated by the following equation:
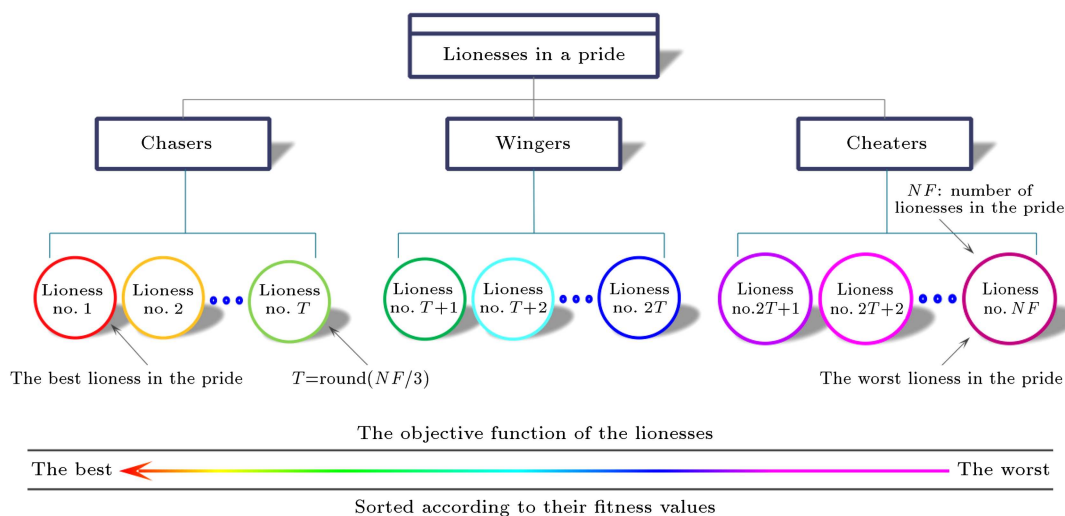

Figure 2. The formation of general hunting groups in a pride with twelve lionesses.
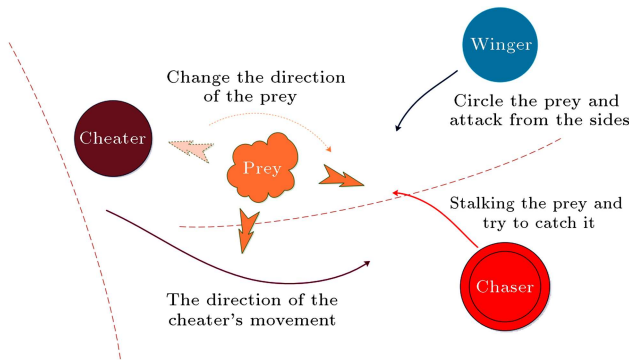
Figure 3. The schematic strategy of lionesses to catch the prey.

$$Male\ Lion_{new} = Territory + E \times (D) \times (2\ rand - 1), \tag{14}$$

where $Male\ Lion_{new}$ indicates the new position of the male lion, and $Territory$ is formulated as follows:

$$Territory_i = Best\ Positions\ (rand \times TR)_i, \tag{15}$$

$$Best\ Positions = sort\ [Male\ best\ Positions;$$

$$Female\ best\ Positions]. \tag{16}$$

$Best\ Positions$ represent the collective understanding of the pride's members about the search space. The cumulative best positions of the resident lions (including both males and females) are sorted from the best to the worst according to the corresponding fitness; E is an excursion constant; TR indicates the territory ratio. In accordance with Eq. (15), just $rand \times TR$ percent of the best memories are saved in $Territory$.

3.2.3. Mating
M percent of the lionesses (mating probability) in each pride mate with single or multiple resident lions. The lion(s) is/are selected randomly from the same pride as the selected female.

Two offspring breed in each mating according to the following equations:

$$Offspring_1 = \beta \times Female\ Lion + \sum_{l=1}^{nm} \frac{1-\beta}{\sum_{l=1}^{nm} S_l}$$

$$\times Male\ Lion_l \times S_l, \tag{17}$$

$$Offspring_2 = (1-\beta) \times Female\ Lion + \sum_{l=1}^{nm} \frac{\beta}{\sum_{l=1}^{nm} S_l}$$

$$\times Male\ Lion_l \times S_l, \tag{18}$$

where $Female\ Lion$ indicates the best position of the selected lioness; $Male\ Lion_l$ represents the best position of the lth lion in the pride; S_l equals 1 if male l is in the coalition; otherwise, it equals 0; nm is the number of resident males in the pride; β is a randomly generated

number with a normal distribution, mean value of 0.5, and standard deviation of 0.1. It is assumed that the chance of giving birth to a female or a male lion is 50:50.

3.2.4. Intragroup interaction
This behavior is simulated according to the following rule: The number of male lions in each pride is always in equilibrium; in each iteration, weaker males (according to their fitness values) must leave the pride.

3.2.5. Migration
The simulation of the natural phenomenon of the migration of resident lionesses is formulated as follows: The lionesses in each pride migrate with the probability of immigration rate (I) in every iteration. Further, the number of females in each pride is always constant. As a result, surplus female lions in each pride get out of the pride and become nomad. In this mechanism, the knowledge of the migratory lioness (the existing position and the best position of the agent so far) is transferred into the new pride.

Finally, the pseudo codes of the LPOA are defined in Figure 4, and a flowchart of the proposed algorithm is illustrated in Figure 5.

4. Performance evaluation in mathematical problems

It is common in optimization frameworks to evaluate the performance of the new algorithms in comparison with other algorithms on a set of mathematical functions. All 15 functions of benchmark problems based on artificially-created functions, such as the CEC 2015 [24], are considered to compare the presented algorithm with the known robust algorithms in structural problems.

Test functions are considered with different characteristics to test the performance of the LPOA algorithm from different points of view. These comparative functions can be divided into four types with respect to their definitions. Unimodal functions have single optimum and measure the exploitation and convergence of an algorithm. In hybrid functions, the variables are randomly divided into some subsets; then, different basic functions are used for different subsets [24]. For comparing the local optima relief in addition to other features, multimodal functions are defined with multiple optimum positions. The composite functions are mostly the combined, rotated, shifted, and biased versions of unimodal and multi-modal test functions.

The idea is to compose the standard benchmark functions to construct a more challenging function with a randomly located global optimum and several randomly located deep local optima [25]. The global optimum position shift in each type of functions according to the shift data is presented in Problem Definitions

Initialize the total population of lions and lionesses
The formation prides randomly
Calculate the fitness of all agents
While the end criterion is not satisfied
for each pride

Hunting
Sort all females according to their fitness values.
Group lionesses into three different general types (pursuers, wingers and cheaters)
Form the three members hunting subgroups by selecting one lioness in each general group as random
Select prey for each hunting subgroups
Update female lion positions using Eqs. (7) to (13)

Excursion
Create territory matrix using Eqs. (15) and (16) and then update lions' position using Eq. (14)

Mating
Each female mate with one or several resident males and create tow cubs using Eqs. (17) and (18) with the probability of $M\%$

Intragroup interaction
Merge new and old mature male lions
Sort all males according to their fitness values
Weaker (less fitness values) males must leave out of the pride and the number of male lions in each pride is always on equilibrium

Migration
Merge new and old mature female lions
1% of females in each pride, migrate to another prides. This process is randomly performed
Sort all females according to their fitness values
Weaker (less fitness values) females drive out of the pride and the number of lionesses in each pride is always on equilibrium

End for
End while

Figure 4. Pseudocode of the LPOA.

Table 1. Description of unimodal benchmark functions.

Category			
	Function name	**Function**	F_i^*
F1	Rotated high conditioned elliptic function	$F_1(x) = f_1(M(x - o_1)) + F_1^*$	100
F2	Rotated cigar function	$F_2(x) = f_2(M(x - o_2)) + F_2^*$	200

Table 2. Description of multimodal benchmark functions.

	Function name	**Function**	F_i^*
F3	Shifted and rotated Ackley's function	$F_2(x) = f_5(M(x - o_2)) + F_2^*$	300
F4	Shifted and rotated Rastrigin's function	$F_4(x) = f_8\left(M\left(\frac{5.12(x - o_4)}{100}\right)\right) + F_4^*$	400
F5	Shifted and rotated Schwefel's function	$F_5(x) = f_9\left(M\left(\frac{1000(x - o_5)}{100}\right)\right) + F_5^*$	500

and Evaluation Criteria for the CEC 2015 Competition on Learning-based Real-Parameter Single-Objective Optimization [24]. Mathematical test functions that are considered in this article are given in Tables 1-4. Different basic functions are used to construct the mentioned benchmark problems. These functions are depicted in Table 5. Furthermore, Figure 6 shows the perspective view of the fitness functions for the test cases considered in this section.

In this section, five modern optimization methods are compared to the LPOA consisting of Particle Swarm Optimization (PSO) algorithm accredited to Kennedy and Eberhart [26], colliding bodies optimization developed by Kaveh and Mahdavi [27], Enhanced Colliding Bodies Optimization (ECBO) algorithm proposed by Kaveh and Ilchi Ghazaan [28], water cycle algorithm proposed by Eskandar et al. [29], the Lion Optimization Algorithm (LOA) proposed
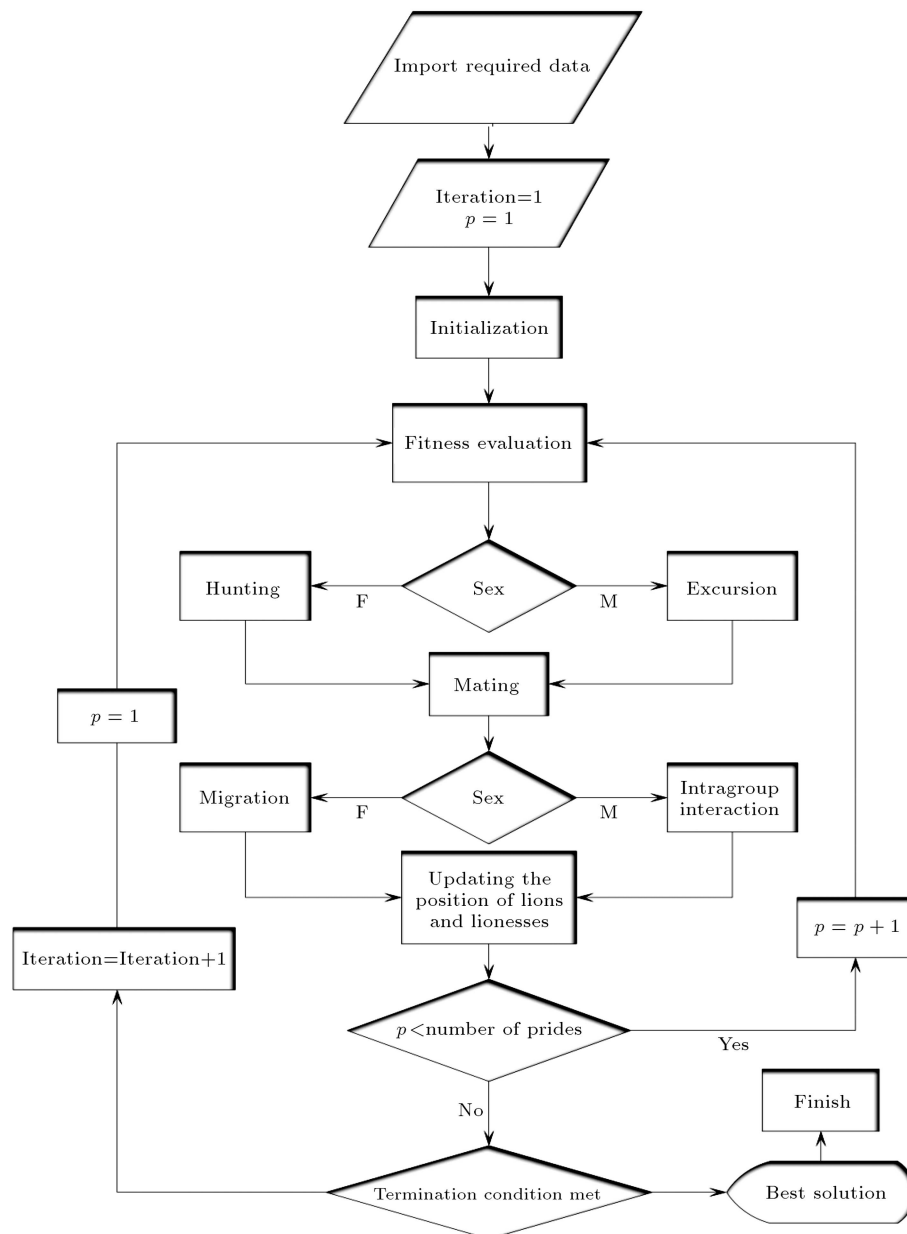
Figure 5. Simplified flowchart of the LPOA.

by Yazdani and Jolani [14], and The Whale Optimization Algorithm proposed (WOA) by Mirjalili and Lewis [30].

The same search ranges are defined for all test functions as $[-100, 100]^D$. In addition, all optimization parameters in each algorithm employed are assumed as recommended in the related articles. In all cases, the population size is set to 30, and 10-dimensional functions are used in this research. In addition, the maximum Number of Function Evaluations (NFE) in each function is set to 30000. Furthermore, 50 separate runs are carried out for each algorithm similar to the setting of the CEC 2015 single-objective optimization competition.

4.1. Adjusting LPOA parameters

Sensitivity analysis is performed in this section to evaluate how the parameters of the LPOA influence the results. Five related parameters in the proposed algorithm are analyzed in different mathematical functions to find a proper set of parameters due to the scatter plots of each parameter versus the fitness values and, eventually, have proper behavior in the process of optimization. In addition, the samples are taken from the marginal distribution of each factor.

Four mathematical test problems of F1, F3, F6, and F9 are used to tune the parameters of the proposed algorithm. These functions are representative of different types of the considered mathematical functions.

Table 3. Description of hybrid benchmark functions.

General definitions

$$F(x) = g_1(M_1 z_1) + g_2(M_2 z_2) + \cdots + g_N(M_N z_N) + F^*(x), \; z = [z_1, z_2, \cdots, z_N],$$

$$z_1 = [y_{s_1}, y_{s_2}, \cdots, y_{s_{n_1}}],$$

$$z_2 = \left[y_{s_{n_1+1}}, y_{s_{n_1+2}}, \cdots, y_{s_{n_1+n_2}}\right], \cdots, z_N = \left[y_{s_{\sum\limits_{i=1}^{N-1} n_i+1}}, y_{s_{\sum\limits_{k=1}^{N-1} n_i+2}}, \cdots, y_{s_D}\right],$$

$$y = x - o_i, \; S = \mathrm{randperm}(1, D),$$

$$n_i : \text{dimension for each basic function}, \; \sum_{i=1}^{N} n_i = D$$

$$n_1 = [p_1 D], \; n_2 = [p_2 D], \cdots, n_{N-1} = [p_{N-1} D], \; n_{N-1} = [p_{N-1} D], \; n_N = D - \sum_{i=1}^{N-1} n_i$$

$$p : \text{used to control the percentage of } g_i(x)$$

	Function name	Function	F_i^*
F6	Hybrid Function 1 $(N = 3)$	$g = \{f_9, f_8, f_1\}, \; p = [0.3, 0.3, 0.4]$	600
F7	Hybrid Function 2 $(N = 4)$	$g = \{f_7, f_6, f_4, f_{14}\}, \; p = [0.2, 0.2, 0.3, 0.3]$	700
F8	Hybrid Function 3 $(N = 5)$	$g = \{f_{14}, f_{12}, f_4, f_9, f_1\}, \; p = [0.1, 0.2, 0.2, 0.2, 0.1]$	800

The selected set of parameter values of LPOA is depicted in Table 6.

4.2. Numerical results and statistical test

The comparative results given from the mentioned algorithm are tabulated in Tables 7 and 8. Table 7 shows information about the minimum, maximum, and standard deviations of the corresponding objective function test, and the values of Table 8 indicate the average error rate derived from the considered algorithms. The average error rate datasets can be considered as a means for measuring the performance of optimization techniques. The results imply that the LPOA is able to generate competitive answers to its counterparts. It can be noted that the proposed optimization technique finds the best answers in 13 out of 15 function problems.

The comparative convergence curve of some objective function is illustrated in Figure 7. The convergence curves show that the LPOA works well; in most cases, the LPOA has higher efficiency than other considered algorithms do, for the benchmark functions.

For the validation of significance of results, it is assumed that all algorithms are performed on the null hypothesis, and all algorithms are similar when solving these unconstrained functions. Friedman rank test [31,32] and different post-hoc analyses were employed together to show the difference between algorithms in a statistically significant way. Suppose that a level of significance $\alpha = 0.05$ was selected, and the hypothesis would be rejected only if the value of p yielded by the test was not greater than the value of 0.05.

Table 9 depicts the ranks computed through the Friedman test, and the LPOA is the best performing algorithm in the study. In addition, p value obtained by the test is 0.00. Consequently, the Friedman test strongly suggests the existence of significant differences among the considered algorithms with a significance level of 0.05, as assumed.

The Friedman test only can detect significant differences over the whole multiple comparisons, being unable to establish proper comparisons between some of the algorithms considered [33]. For this reason, Holm [34], Rom [35], Hommel [36], Finner [37], Holland and Copenhaver [38] and Li [39] post-hoc procedures are performed in order to find the concrete pairwise comparisons, which produce differences.

Table 4. Description of composition benchmark functions.

General definitions			
$F(x) = \sum\limits_{i=1}^{N} \{\omega_i^* [\lambda_i g_i(x) + \text{bias}_i]\} + f^*, \; w_i = \dfrac{1}{\sqrt{\sum\limits_{j=1}^{D}(x_j - o_{ij})^2}} \exp\left(-\dfrac{\sum\limits_{j=1}^{D}(x_j - o_{ij})^2}{2D\sigma_i^2}\right), \; \omega_i = w_i / \sum\limits_{i=1}^{n} w_i$			

	Function name	**Function**	$\boldsymbol{F_i^*}$
F9	Composition Function 1 $(N = 3)$	$g = \{f_9, f_8, f_{12}\},$ $\sigma = [20, 20, 20],$ $\lambda = [1, 1, 1],$ $\text{bias} = [0, 100, 200] + F_9^*$	900
F10	Composition Function 2 $(N = 3)$	$g = \{F_6, F_7, F_8\},$ $\sigma = [10, 30, 50],$ $\lambda = [1, 1, 1],$ $\text{bias} = [0, 100, 200] + F_{10}^*$	1000
F11	Composition Function 3 $(N = 5)$	$g = \{f_{12}, f_8, f_9, f_6, f_1\},$ $\sigma = [10, 10, 10, 20, 20],$ $\lambda = [10, 10, 2.5, 25, 1e - 6],$ $\text{bias} = [0, 100, 200, 300, 400] + F_{11}^*$	1100
F12	Composition Function 4 $(N = 5)$	$g = \{f_9, f_8, f_1, f_{14}, f_{11}\},$ $\sigma = [10, 20, 20, 30, 30],$ $\lambda = [0.25, 1, 1e - 7, 10, 10],$ $\text{bias} = [0, 100, 100, 200, 200] + F_{12}^*$	1200
F13	Composition Function 5 $(N = 5)$	$g = \{F_8, f_8, F_6, f_9, f_{14}\},$ $\sigma = [10, 10, 10, 20, 20],$ $\lambda = [1, 10, 1, 25, 10],$ $\text{bias} = [0, 100, 200, 300, 400] + F_{13}^*$	1300
F14	Composition Function 6 $(N = 7)$	$g = \{f_{11}, f_7, f_9, f_{14}, f_1, f_2, f_8\},$ $\sigma = [10, 20, 30, 40, 50, 50, 50],$ $\lambda = [10, 2.5, 25, 10, 1e - 6, 1e - 6, 10],$ $\text{bias} = [0, 100, 200, 300, 300, 400, 400] + F_{14}^*$	1400
F15	Composition Function 7 $(N = 10)$	$g = \{f_8, f_6, f_{11}, f_9, f_4, f_3, f_{11}, f_{14}, f_{13}, f_5\},$ $\sigma = [10, 10, 20, 20, 30, 30, 40, 40, 50, 50],$ $\lambda = [0.1, 0.25, 0.1, 0.025, 1e - 3, 0.1, 1e - 5, 10, 0.025, 1e - 3],$ $\text{bias} = [0, 100, 100, 200, 200, 300, 300, 400, 400, 500] + F_{14}^*$	1500

Table 5. Description of the basic functions.

	Function name	Function
F1	High conditioned elliptic	$f_1(x) = \sum\limits_{i=2}^{D} (10^6)^{\frac{i-1}{D-1}} x_i^2$
F2	Cigar	$f_2(x) = x_1^2 + 10^6 \sum\limits_{i=2}^{D} x_i^2$
F3	Discus	$f_3(x) = 10^6 x_1^2 + \sum\limits_{i=2}^{D} x_i^2$
F4	Rosenbrock	$f_4(x) = \sum\limits_{i=1}^{D-1} \left(100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2 \right)$
F5	Ackley	$f_5(x) = -20 \exp\left(-0.2 \sqrt{\frac{1}{D} \sum\limits_{i=1}^{D} x_i^2} \right) - \exp\left(\frac{1}{D} \sum\limits_{i=1}^{D} \cos(2\pi x_i) \right) + 20 + e$
F6	Weierstrass	$f_6(x) = \sum\limits_{i=1}^{D} \left(\sum\limits_{k=0}^{k\,\max} \left[a^k \cos(2\pi b^k (x_i + 0.5)) \right] \right) - D \sum\limits_{k=0}^{k\,\max} [a^k \cos(2\pi b^k \cdot 0.5)],$ $a = 0.5,\ b = 3,\ k\,\max = 20$
F7	Griewank	$f_7(x) = \sum\limits_{i=1}^{D} \frac{x_i^2}{4000} - \prod\limits_{i=1}^{D} \cos\left(\frac{x_i}{\sqrt{i}} \right) + 1$
F8	Rastrigin	$f_8(x) = \sum\limits_{i=1}^{D} (x_i^2 - 10 \cos(2\pi x_i) + 10)$
F9	Modified Schwefel	$f_9(x) = 418.9829 \times D - \sum\limits_{i=1}^{D} g(z_i),\ z_i = x_i + 4.209687462275036e + 002$ $g(z_i) = \begin{cases} z_i \sin\left(|z_i|^{\frac{1}{2}} \right) & |z_i| \le 500 \\[2mm] (500 - \mathrm{mod}(z_i, 500)) \sin\left(\sqrt{|500 - \mathrm{mod}(z_i, 500)|} \right) - \frac{(z_i - 500)^2}{10000 D} & z_i > 500 \\[2mm] (\mathrm{mod}(|z_i|, 500) - 500) \sin\left(\sqrt{|\mathrm{mod}(|z_i|, 500) - 500|} \right) - \frac{(z_i + 500)^2}{10000 D} & z_i < -500 \end{cases}$
F10	Katsuura	$f_{10}(x) = \frac{10}{D^2} \prod\limits_{i=1}^{D} \left(1 + i \sum\limits_{j=1}^{32} \frac{|2^j x_i - \mathrm{round}(2^j x_i)|}{2^j} \right)^{\frac{10}{D^{1.2}}} - \frac{10}{D^2}$
F11	HappyCat	$f_{11}(x) = \left| \sum\limits_{i=1}^{D} x_i^2 - D \right|^{\frac{1}{4}} + \left(0.5 \sum\limits_{i=1}^{D} x_i^2 + \sum\limits_{i=1}^{D} x_i \right) / D + 0.5$
F12	HGBat	$f_{12}(x) = \left| \left(\sum\limits_{i=1}^{D} x_i^2 \right)^2 - \left(\sum\limits_{i=1}^{D} x_i \right)^2 \right|^{\frac{1}{2}} + \left(0.5 \sum\limits_{i=1}^{D} x_i^2 + \sum\limits_{i=1}^{D} x_i \right) / D + 0.5$
F13	Expanded Griewank plus Rosenbrock	$f_{13}(x) = f_7(f_4(x_1, x_2)) + f_7(f_4(x_2, x_3)) + \cdots + f_7(f_4(x_{D-1}, x_D)) + f_7(f_4(x_D, x_1))$ $f_{14}(x) = g(x_1, x_2) + g(x_2, x_3) + \cdots + g(x_{D-1}, x_D) + g(x_D, x_1)$
F14	Expanded scaffer	$g(x, y) = 0.5 + \frac{\left(\sin^2\left(\sqrt{x^2 + y^2} \right) - 0.5 \right)}{\left(1 + 0.001(x^2 + y^2) \right)^2}$

Table 6. Parameter values of LPOA in mathematical functions.

Parameter	Value
Number of prides	5
Lions in each pride	6
Male lions in each pride	2
Female lions in each pride	4
Territory ratio	0.4
Mating probability	0.6
Immigration rate	0.6
diversification factor	10e-03 to 0

Table 10 shows p values obtained through the application of the considered post-hoc methods. Adjusted p values can be compared directly with the chosen significance level. It can be seen that the null hypothesis is rejected subsequently as all output values are smaller than $\alpha = 0.05$, and the results of LPOA are significantly better than other techniques statistically.

5. Performance evaluation in constrained design problems

Design optimization problems are often constrained and complex with a lot of local optima and frequently limited by different constraints as material properties, feasible strength, displacements, load cases, and type or size of members. Thus, in order to demonstrate the efficiency of the proposed methodology, three well-studied design engineering benchmark problems are
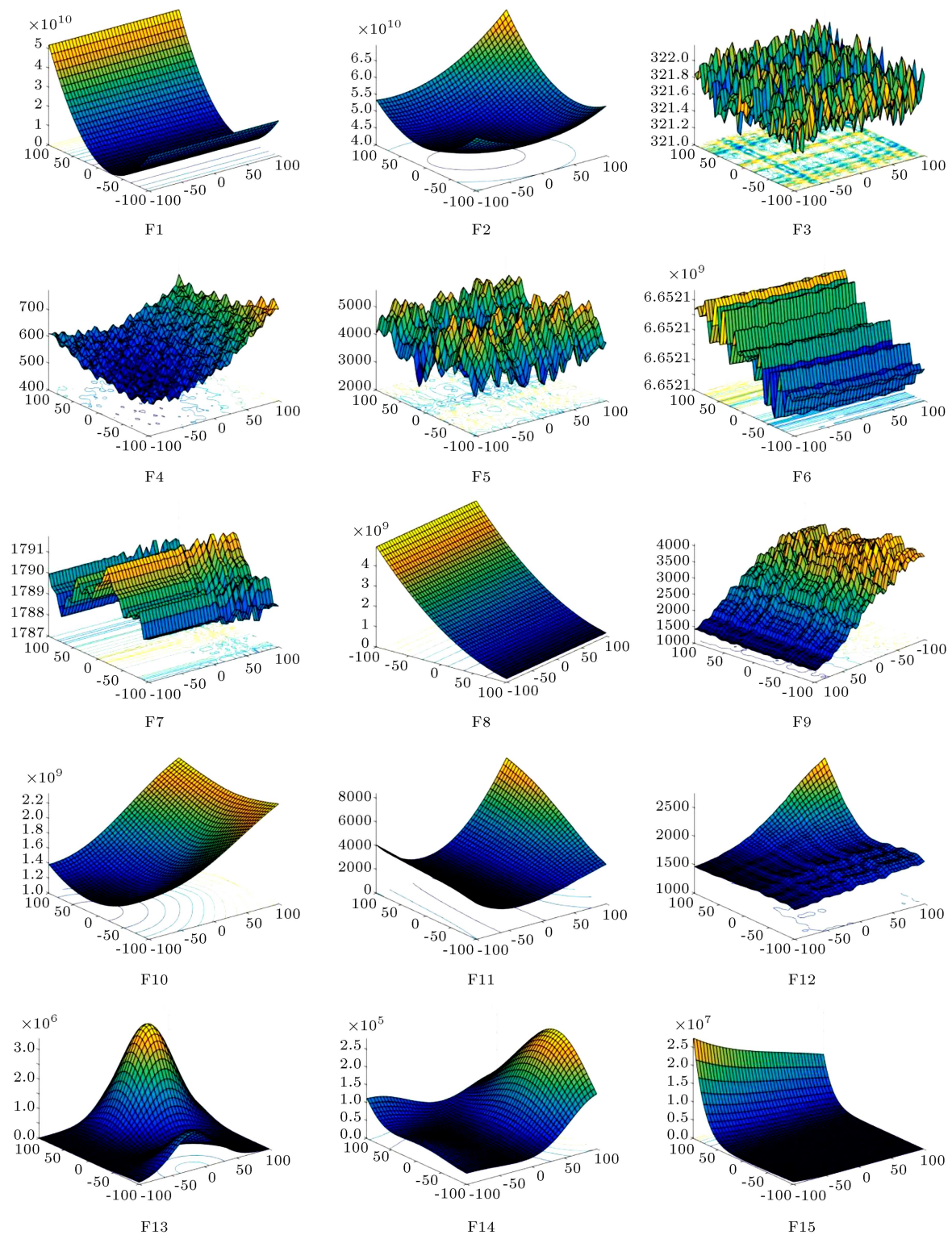
Figure 6. 2D Search space representations of benchmark mathematical functions.

solved with LPOA. In addition, the solutions are compared with different state-of-the-art approaches.

The LPOA algorithm is coded in MATLAB, and structural analysis is performed with the direct stiffness method.

The penalty approach is used for constraint handling. The penalty function method has been the most popular constraint-handling technique due to its simple principle and ease of implementation [40]. If the constraints are not violated, the penalty will be

Table 7. Comparative results on mathematical test functions.

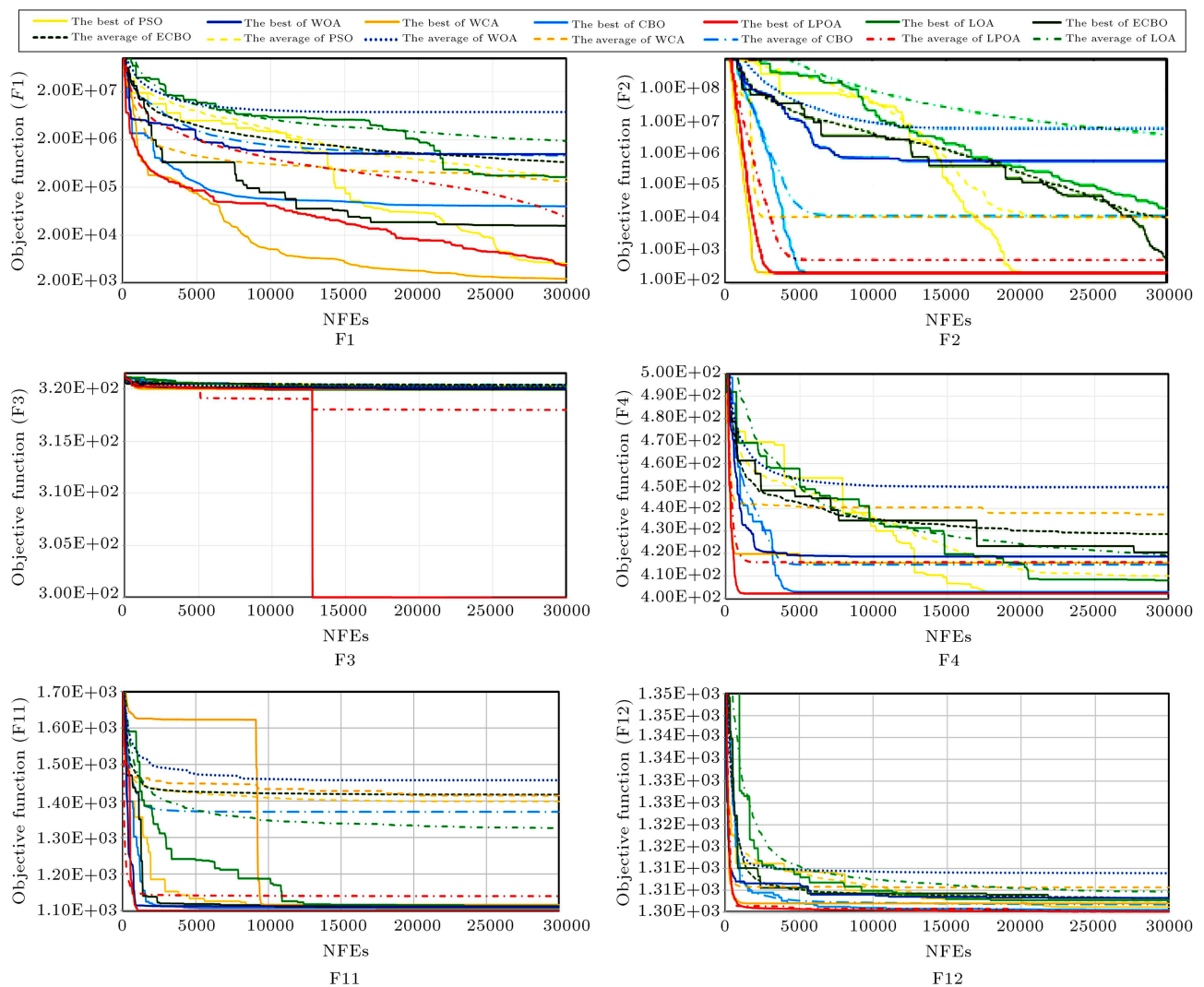| Function		Algorithm						
		Present work	PSO	CBO	ECBO	WCA	LOA	WOA
F1	Min.	4.67E+03	5.20E+03	8.14E+04	3.18E+04	2.47E+03	3.31E+05	9.99E+05
	Max.	3.18E+05	1.60E+06	3.41E+06	6.65E+06	2.91E+06	9.38E+06	2.38E+07
	St. Dev	9.64E+04	3.99E+05	7.68E+05	1.14E+06	6.03E+05	1.58E+06	5.21E+06
F2	Min.	2.00E+02	2.03E+02	2.05E+02	5.91E+02	2.03E+02	2.02E+04	6.03E+05
	Max.	1.09E+04	3.15E+04	3.36E+04	3.49E+04	3.40E+04	2.74E+07	1.77E+07
	St. Dev	3.95E+03	9.63E+03	1.15E+04	9.99E+03	1.37E+04	5.76E+06	4.37E+06
F3	Min.	3.00E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02
	Max.	3.20E+02	3.20E+02	3.21E+02	3.21E+02	3.20E+02	3.20E+02	3.21E+02
	St. Dev	3.03E+00	8.23E-02	1.27E-01	1.29E-01	5.91E-02	4.72E-02	1.30E-01
F4	Min.	4.02E+02	4.03E+02	4.03E+02	4.21E+02	4.16E+02	4.08E+02	4.19E+02
	Max.	4.23E+02	4.21E+02	4.31E+02	4.37E+02	4.75E+02	4.33E+02	5.04E+02
	St. Dev	3.76E+00	4.50E+00	7.94E+00	4.49E+00	1.34E+01	6.09E+00	1.74E+01
F5	Min.	5.07E+02	5.07E+02	5.07E+02	5.51E+02	6.22E+02	8.26E+02	1.00E+03
	Max.	2.52E+03	1.46E+03	1.46E+03	1.43E+03	2.09E+03	1.51E+03	2.17E+03
	St. Dev	1.65E+02	2.53E+02	2.17E+02	2.41E+02	3.32E+02	1.60E+02	2.82E+02
F6	Min.	7.59E+02	7.12E+02	7.98E+02	8.30E+02	6.50E+02	1.39E+03	2.80E+03
	Max.	1.15E+03	6.22E+03	8.29E+04	1.02E+04	1.10E+04	9.29E+03	1.38E+06
	St. Dev	5.05E+02	1.28E+03	2.02E+04	2.89E+03	3.94E+03	2.14E+03	4.13E+05
F7	Min.	7.00E+02	7.00E+02	7.01E+02	7.01E+02	7.02E+02	7.02E+02	7.03E+02
	Max.	7.02E+02	7.04E+02	7.05E+02	7.04E+02	7.06E+02	7.03E+02	7.10E+02
	St. Dev	2.95E-01	8.12E-01	7.62E-01	4.61E-01	1.23E+00	3.76E-01	1.40E+00
F8	Min.	8.18E+02	8.23E+02	1.09E+03	8.21E+02	8.23E+02	1.20E+03	1.44E+03
	Max.	3.91E+03	1.39E+04	5.39E+04	2.63E+04	2.56E+04	4.32E+03	2.62E+04
	St. Dev	7.18E+02	3.89E+03	1.36E+04	8.97E+03	5.78E+03	7.23E+02	5.92E+03
F9	Min.	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03
	Max.	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.11E+03	1.00E+03	1.00E+03
	St. Dev	4.58E-02	8.78E-02	1.83E-01	5.99E-02	1.80E+01	1.53E-01	2.94E-01
F10	Min.	1.33E+03	1.34E+03	1.75E+03	1.25E+03	1.45E+03	1.66E+03	1.81E+03
	Max.	3.09E+03	9.01E+03	3.72E+04	3.47E+04	1.65E+04	7.19E+03	5.49E+04
	St. Dev	1.24E+03	1.79E+03	8.24E+03	7.46E+03	4.28E+03	1.29E+03	1.41E+04
F11	Min.	1.10E+03	1.10E+03	1.11E+03	1.11E+03	1.12E+03	1.11E+03	1.11E+03
	Max.	1.30E+03	1.63E+03	1.59E+03	1.58E+03	1.65E+03	1.41E+03	1.77E+03
	St. Dev	2.33E+01	8.11E+01	1.19E+02	7.75E+01	8.24E+01	1.21E+02	1.41E+02
F12	Min.	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03
	Max.	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.31E+03	1.31E+03	1.32E+03
	St. Dev	8.08E-01	8.46E-01	8.74E-01	4.48E-01	2.07E+00	1.16E+00	3.96E+00
F13	Min.	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03
	Max.	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03
	St. Dev	1.76E-04	6.82E-03	1.47E-03	2.94E-04	3.64E-03	1.24E-03	5.48E-03
F14	Min.	1.50E+03	1.50E+03	4.33E+03	4.34E+03	1.50E+03	1.55E+03	4.37E+03
	Max.	7.15E+03	1.19E+04	1.19E+04	1.14E+04	1.29E+04	8.51E+03	8.57E+03
	St. Dev	9.88E+02	2.72E+03	2.27E+03	2.63E+03	2.91E+03	1.44E+03	1.44E+03
F15	Min.	1.60E+03	1.60E+03	1.60E+03	1.60E+03	1.60E+03	1.60E+03	1.60E+03
	Max.	1.60E+03	1.60E+03	1.60E+03	1.61E+03	1.60E+03	1.60E+03	1.61E+03
	St. Dev	8.41E-12	1.11E-11	3.41E-12	8.20E-01	5.20E-13	6.70E-01	3.37E+00

Table 8. Average error rates of the algorithms obtained in mathematical test functions.

Function	Algorithm						
	Present work	**PSO**	**CBO**	**ECBO**	**WCA**	**LOA**	**WOA**
F1	46755.922	316255.297	927865.185	681181.529	271115.021	1885622.710	7615642.072
F2	4802.406	10030.057	11700.583	10220.590	10831.886	4017996.482	6194263.355
F3	18.028	20.356	20.392	20.419	20.027	20.235	20.193
F4	15.972	10.070	15.060	28.708	37.446	19.249	49.496
F5	400.033	400.756	434.061	390.749	1019.646	761.286	1048.991
F6	429.211	1263.613	14949.533	3091.745	4815.625	3025.786	329233.543
F7	1.058	1.509	2.233	2.350	3.303	2.338	6.207
F8	1490.274	4504.074	13539.852	8455.109	4348.638	1470.065	6218.680
F9	100.211	100.281	100.473	100.416	103.648	100.574	100.723
F10	999.446	1882.728	9127.900	5307.194	3877.947	2091.119	12630.115
F11	43.055	299.089	270.649	317.711	315.165	226.392	357.387
F12	101.471	102.519	101.730	103.213	105.660	104.614	108.898
F13	0.030	0.069	0.031	0.032	0.032	0.032	0.036
F14	1954.700	6332.226	6068.181	5450.378	5491.305	2964.175	6374.899
F15	100.000	100.000	100.000	104.530	100.000	100.988	104.733

Figure 7. Comparative convergence curves obtained from the selected algorithms for mathematical functions.
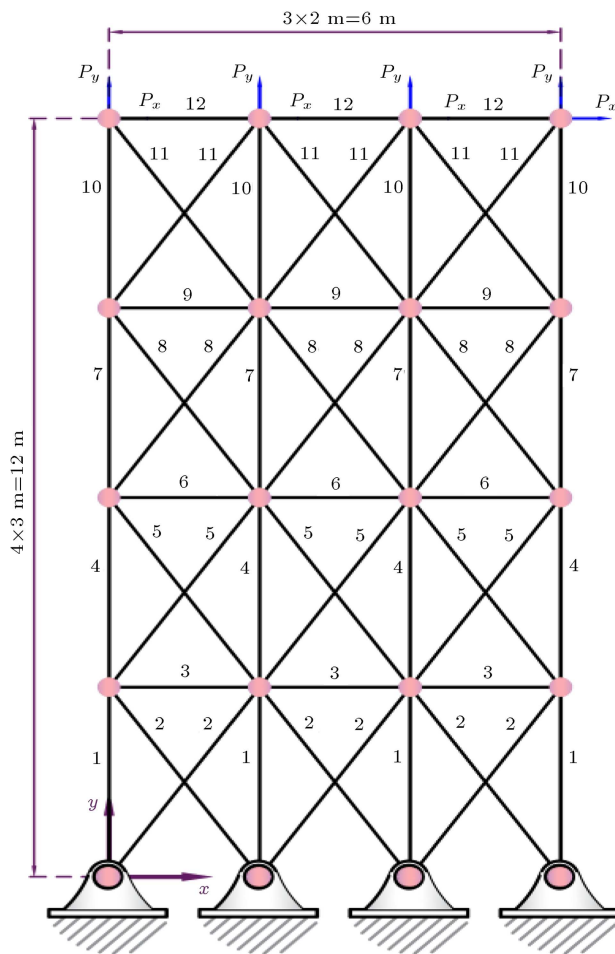
Table 9. Friedman test results for comparing the proposed methods with other algorithms on 15 mathematical datasets.

Algorithm	Rank
Present work	1.3667
PSO	3.2333
CBO	4.1667
ECBO	4.3333
WCA	4.4333
LOA	4
WOA	6.4667

Table 11. Parameter values of the LPOA in design functions.

Parameter	Value
Number of prides	3-7
Lions in each prides	4-7
Male lions in each prides	1-2
Female lions in each prides	3-6
Territory ratio	0.2
Mating probability	0.1
Immigration rate	0.2
Diversification factor	0.1 to 0.0001

zero; otherwise, the value of the penalty is calculated by dividing the violation of the allowable limit by the limit itself. The constraint violation penalty can be stated as follows:

$$f_{\text{penalty}}(x) = (1 + \varepsilon_1 \times v)^{\varepsilon_2}, \qquad v = \sum_{i=1}^{c} v_i. \qquad (19)$$

v is violation ratio that is defined as in Eq. (19), and c is the number of constraints where ε_1 and ε_2 are parameters that penalize the unfeasible solutions more severely. ε_1 is taken as unity, and ε_2 starts from 1.5 and linearly increases to 6 in all design problems.

If the constraint is satisfied : $v_i = 0$,

If the constraint is violated : $v_i = \left| 1 - \dfrac{b_i^*}{b_i} \right|, \qquad (20)$

where b_i^* is the upper bound constraint boundary value, and b_i is the value of the related property value of the structure. Penalized fitness is introduced as follows:

$$P(x) = f(x) \times f_{penalty}(x), \qquad (21)$$

where $f(x)$ is the fitness function, which generally is taken as the weight of the structure in a design optimization problem.

The examples considered from the literature include:

- A pressure vessel design;
- A 52-bar planar truss design;
- A 72-bar spatial truss design.

Parameter values of LPOA in the evaluation of structural benchmark problems are depicted in Table 11.

5.1. A 52-bar planar truss design

The 52-bar planar truss is depicted in Figure 8 schematically. The members of this structure are divided into 12 groups: (1) A1-A4, (2) A5-A10, (3) A11-A13, (4) A14-A17, (5) A18-A23, (6) A24-A26, (7) A27-A30, (8) A31-A36, (9) A37-A39, (10) A40-A43, (11) A44-A49, and (12) A50-A52. The material density is 7860.0 kg/m^3, and the modulus of elasticity is 2.07×10^5 MPa. The members are subjected to stress limitations of ± 180 MPa. Both loads, $p_x = 100$ kN and $p_y = 200$ kN, are considered, and the discrete variables are selected from Table 12.

This benchmark example has been optimized by different researchers; Lee and Geem [41] designed the problem by employing a harmony search algorithm. Li et al. [42] utilized the particle swarm optimization algorithm and the heuristic particle swarm optimizer to solve the design problem; Wu and Chow [43] found the solution of the problem by implementing the steady-state genetic algorithm; Sadollah et al. [9] carried out the optimized design using Mine Blast

Table 10. z-value, unadjusted and adjusted p-values for the Friedman test (LPOA is the control method).

Algorithm	z	Unadjusted p	pHolm	pRom	pHommel	pFinner	pHolland	pLi
WOA	6.4654	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
WCA	3.8877	0.0001	0.0005	0.0005	0.0004	0.0003	0.0005	0.0001
ECBO	3.7609	0.0002	0.0007	0.0006	0.0007	0.0003	0.0007	0.0002
CBO	3.5496	0.0004	0.0012	0.0012	0.0012	0.0006	0.0012	0.0004
LOA	3.3384	0.0008	0.0017	0.0017	0.0017	0.0010	0.0017	0.0009
PSO	2.3664	0.0180	0.0180	0.0180	0.0180	0.0180	0.0180	0.0180

Figure 8. Schematic of a 52-bar planar truss.

Table 12. The available cross-section areas of the AISC code.

No.	in.^{2}	No.	in.^{2}	No.	in.^{2}	No.	in.^{2}
1	0.111	17	1.563	33	3.84	49	11.5
2	0.141	18	1.62	34	3.87	50	13.5
3	0.196	19	1.8	35	3.88	51	13.9
4	0.25	20	1.99	36	4.18	52	14.2
5	0.307	21	2.13	37	4.22	53	15.5
6	0.391	22	2.38	38	4.49	54	16
7	0.442	23	2.62	39	4.59	55	16.9
8	0.563	24	2.63	40	4.8	56	18.8
9	0.602	25	2.88	41	4.97	57	19.9
10	0.766	26	2.93	42	5.12	58	22
11	0.785	27	3.09	43	5.74	59	22.9
12	0.994	28	1.13	44	7.22	60	24.5
13	1	29	3.38	45	7.97	61	26.5
14	1.228	30	3.47	46	8.53	62	28
15	1.266	31	3.55	47	9.3	63	30
16	1.457	32	3.63	48	10.85	64	33.5

Algorithm (MBA), while Cheng and Prayogo [44] as well as Mirjalili and Lewis [30] implemented symbiotic organisms (SOS) and Whale Optimization Algorithm (WOA) to solve the 52-bar planar truss design problem, respectively.

The LPOA proceeds until the maximum number of 300 iterations. Here, 20 search agents are utilized for solving the design problem, and 20 individual optimization runs have been carried out for this benchmark example. The corresponding results are presented in Table 13. The answers are rounded to the nearest integer number during the implementation of the optimization algorithm, since the variables of the problem are discrete. In accordance with the results, it can be seen that the best optimal answer is obtained by the LPOA, which is equal to 1902.28 kg achieved in 5980 analyses. In addition, the average and standard deviations of 20 individual optimal solutions exploited by the proposed algorithm are 1944.3478 kg and 54.8133 kg, respectively. The results show that the LPOA has better performance than other mentioned algorithms' performance in finding the best optimal solutions.

The stress ratios and stress values of two solutions

of LPOA and IMCSS and stress values in each member are shown in Figures 9 and 10, respectively. Moreover, the convergence curves of the LPOA are provided in Figure 11.

5.2. A 72-bar spatial truss

A 72-bar spatial truss structure is depicted in Figure 12. The material density is 0.1 lb/in^3 (2,767.990 kg/m^3), and the modulus of elasticity is 10,000 ksi (68,950 MPa) in this example. The nodes are subjected to the displacement limits of 0.25 in (0.635 cm), and the elements are subjected to the stress limits of 25 ksi (172.375 MPa).

Then, 72 members of the structure are classified into 16 variable groups: (1) A1-A4, (2) A5-A12, (3) A13-A16, (4) A17-A18, (5) A19-A22, (6) A23-A30, (7) A31-A34, (8) A35-A36, (9) A37-A40, (10) A41-A48, (11) A49-A52, (12) A53-A54, (13) A55-A58, (14) A59-A66 (15), A67-A70, and (16) A71-A72.

Two different optimization case studies are implemented as follows:

Case 1: The discrete variables are selected from the set D=[0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0, 3.1, 3.2] (in^2). In addition, Load Case 1 is applied to the truss, as depicted in Table 14;

Case 2: The discrete variables are selected from

Table 13. Optimized designs obtained for the 52-bar planar truss.

| Algorithm	Element group variables (mm^2)												Weight (kg)
	1	2	3	4	5	6	7	8	9	10	11	12	
Present work	4360.45	1161.29	285.16	3703.22	940.00	494.19	2290.32	1008.39	388.39	1283.87	1161.29	494.19	1902.283
HS [41]	4658.06	1161.29	506.45	3303.22	940.00	494.19	2290.32	1008.39	2290.32	1535.48	1045.16	506.45	1906.76
PSO [42]	4658.06	1374.19	1858.06	3206.44	1283.87	252.26	3303.22	1045.16	126.45	2341.93	1008.38	1045.16	2230.16
HPSO [42]	4658.06	1161.29	363.23	3303.22	940.00	494.19	2238.71	1008.39	388.39	1283.87	1161.29	792.26	1905.495
MBA [9]	4658.06	1161.29	494.19	3303.22	940.00	494.19	2238.71	1008.39	494.19	1283.87	1161.29	494.19	1902.605
SOS [44]	4658.06	1161.29	494.19	3303.22	940.00	494.19	2238.71	1008.39	494.19	1283.87	1161.29	494.19	1902.605
WOA [30]	4658.06	1161.29	494.19	3303.22	940.00	494.19	2238.71	1008.39	494.19	1283.87	1161.29	494.19	1902.605
GA [43]	4658.06	1161.29	645.16	3303.22	1045.16	494.19	2477.141	1045.16	285.16	1696.77	1045.16	641.29	1970
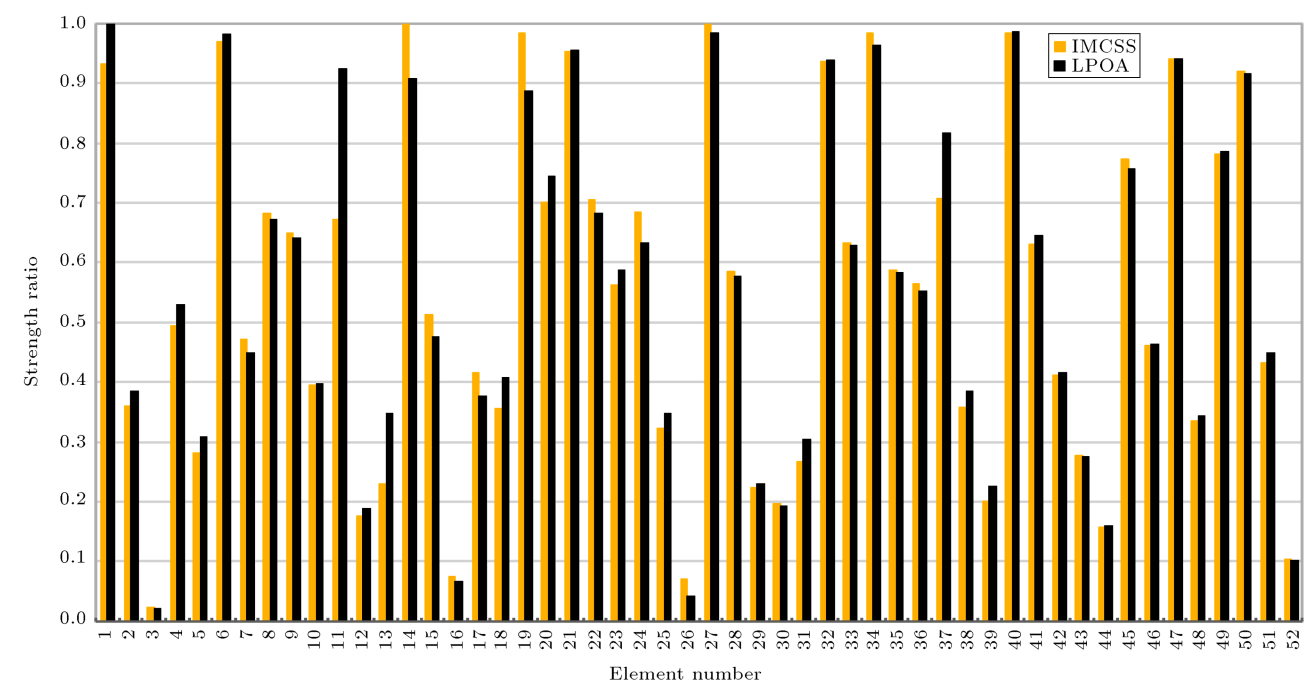
Figure 9. Member stress ratios of the best solution obtained by LPOA and IMCSS.

AISC code cross-sections as illustrated in Table 12, and Load Case 2 applied to the truss is shown in Table 14.

The 72-bar spatial truss problem has been studied by many researchers and, also, with different algorithms. Wu and Chow used Genetic Algorithm (GA) [43]; Li et al. [45] utilized Particle Swarm Optimization (PSO), Particle Swarm Optimizer with Passive Congregation (PSOPC), and Heuristic Particle Swarm Optimizer (HPSO); Kaveh and Farhoudi [46] employed Dolphin Echolocation (DE); Cheng et al. [47] used Hybrid Harmony Search (HHS) algorithm; Sadollah et al. [9] imposed Mine Blast Algorithm (MBA) to solve this benchmark problem.

Table 14. Load cases applied to the 72-bar spatial truss structure.

Node	Case 1 (kips)			Case 2 (kips)		
	P_X	P_Y	P_Z	P_X	P_Y	P_Z
17	5.0	5.0	5.0	0.0	0.0	5.0
18	0.0	0.0	0.0	0.0	0.0	5.0
19	0.0	0.0	0.0	0.0	0.0	5.0
20	0.0	0.0	0.0	0.0	0.0	5.0

It employed 50 search agents over 200 iterations and 20 independent runs for solving each case of the problem. The LPOA comparative solutions of the two loading cases are given in Tables 15 and 16, respectively.
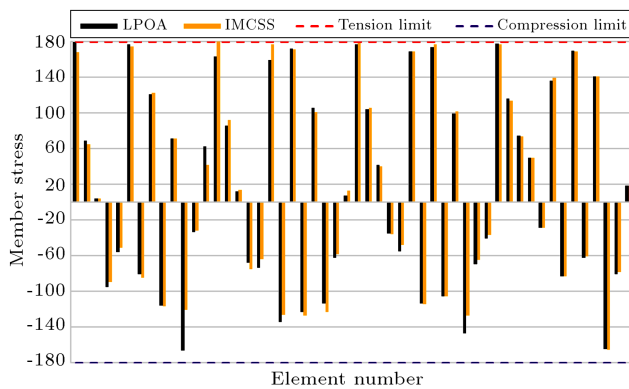
Figure 10. Member stresses of the best solution obtained by LPOA and IMCSS.
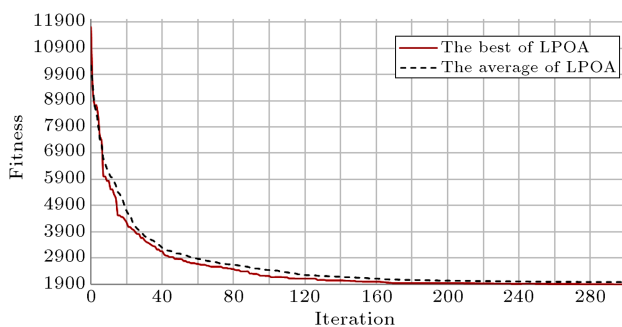

Figure 11. Convergence curves obtained for the 52-bar planar truss problem.

Figure 12. Schematic of a 72-bar spatial truss.

Based on Tables 15 and 16, LPOA found better results than the mentioned algorithms did, in these two case studies. The standard deviation and the average of the best solution obtained in the last iteration in the first case of study were 3.8167 lb and 373.6517 lb, respectively. In addition, in the second case, the standard deviation and average are obtained as 2.8360 lb and 389.5500 lb, respectively.

Of note, the standard deviation of the results by the LPOA in 20 independent runs is very small in both case studies, and the average of the solutions obtained by LPOA is less than those of the existing results from literature. The convergence histories of the LPOA are presented in Figures 13 and 14.

6. Conclusions

A new population-based optimization algorithm was introduced in this paper, called pride lion optimization algorithm, to provide a sufficiently good solution to mathematical and structural problems. The fundamental concepts and ideas for the codification of this method were derived from the behavior of lions in a pride. The proposed method contained two types of search agents (lions and lionesses) with a specific set of characteristics. Hunting, excursion of lions in territories, intra-group interactions, and mating behaviors were simulated in the proposed algorithm.
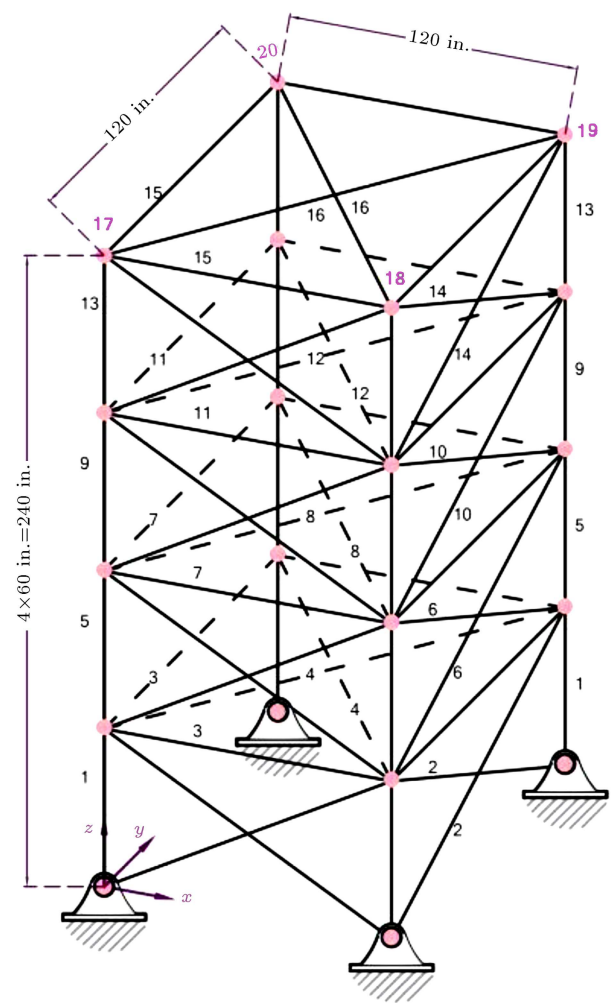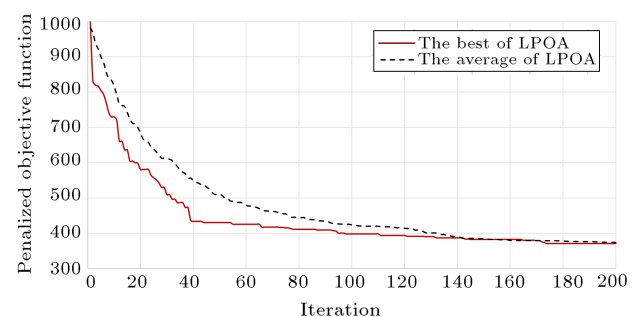

Figure 13. Convergence curves for the first case of the 72-bar spatial truss problem (Case 1).

In addition, a set of 15 benchmark-unconstrained functions with specific characteristics was tested and analyzed in comparison to other existing methods. The mathematical functions were categorized as four types of functions: unimodal, hybrid, multimodal, and composite.

In addition, sensitivity analysis of the parameters was performed to evaluate how these parameters influenced the performance of the algorithm. The Friedman

Table 15. Optimal design comparison for the spatial 72-bar spatial truss problem (Case 1).

Element group	Optimal cross-sectional areas (in²)							
	Present work	GA [43]	PSO [45]	PSOPC [45]	HPSO [45]	HHS [47]	DE [46]	MBA [9]
1	1.8	1.5	2.6	3	2.1	1.9	2	2.0
2	0.5	0.7	1.5	1.4	0.6	0.5	0.5	0.6
3	0.1	0.1	0.3	0.2	0.1	0.1	0.1	0.4
4	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.6
5	1.3	1.3	2.1	2.7	1.4	1.4	1.3	0.5
6	0.5	0.5	1.5	1.9	0.5	0.5	0.5	0.5
7	0.1	0.2	0.6	0.7	0.1	0.1	0.1	0.1
8	0.1	0.1	0.3	0.8	0.1	0.1	0.1	0.1
9	0.5	0.5	2.2	1.4	0.5	0.5	0.5	1.4
10	0.5	0.5	1.9	1.2	0.5	0.5	0.5	0.5
11	0.1	0.1	0.2	0.8	0.1	0.1	0.1	0.1
12	0.1	0.2	0.9	0.1	0.1	0.1	0.1	0.1
13	0.1	0.2	0.4	0.4	0.2	0.2	0.2	1.9
14	0.5	0.5	1.9	1.9	0.5	0.6	0.6	0.5
15	0.4	0.5	0.7	0.9	0.3	0.4	0.4	0.1
16	0.7	0.7	1.6	1.3	0.7	0.6	0.6	0.1
Weight (lb)	371.0037	400.66	1089.88	1069.79	388.94	385.54	385.54	385.54

Table 16. Optimal design comparison for the spatial 72-bar spatial truss problem (Case 2).

| Element group	Optimal cross-sectional areas (in²)						
	Present work	GA [43]	PSO [45]	PSOPC [45]	HPSO [45]	DE [46]	MBA [9]
1	2.13	0.196	7.22	4.49	4.97	2.13	0.196
2	0.563	0.602	1.8	1.457	1.228	0.442	0.563
3	0.111	0.307	1.13	0.111	0.111	0.111	0.442
4	0.111	0.766	0.2	0.111	0.111	0.111	0.602
5	1.563	0.391	3.09	2.62	2.88	1.457	0.442
6	0.442	0.391	0.79	1.13	1.457	0.563	0.442
7	0.111	0.141	0.56	0.196	0.141	0.111	0.111
8	0.111	0.111	0.79	0.111	0.111	0.111	0.111
9	0.563	1.8	3.09	1.266	1.563	0.442	1.266
10	0.442	0.602	1.23	1.457	1.228	0.563	0.563
11	0.111	0.141	0.11	0.111	0.111	0.111	0.111
12	0.111	0.307	0.56	0.111	0.196	0.111	0.111
13	0.141	1.563	0.99	0.442	0.391	0.196	1.800
14	0.563	0.766	1.62	1.457	1.457	0.563	0.602
15	0.391	0.141	1.56	1.228	0.766	0.307	0.111
16	0.563	0.111	1.27	1.457	1.563	0.563	0.111
Weight(lb)	386.4270	427.203	1209	941.82	933.09	391.329	390.73
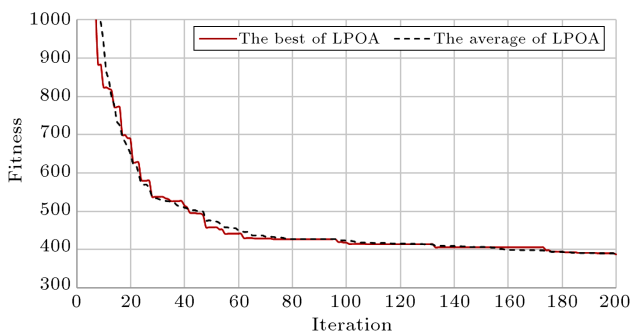
Figure 14. Convergence curves for the first case of the 72-bar spatial truss problem (Case 2).

test and 6 different post-hoc tests (Holm, Rom, Hommel, Finner, Holland and Li) were carried out to detect significant differences in the whole multiple comparison and pairwise comparisons consequently. According to the optimized solutions obtained, it appears that the performance of the proposed method is superior to those of the considered optimization algorithms in unconstrained problems.

Furthermore, two competitive design problems, consisting of the 52-bar planar truss, 72-bar spatial truss, and different constraint conditions (i.e., axial and shear stress limitations, deflection constraints) were investigated in order to show the performance of the LPOA algorithm. The competitive studies confirm the robustness of the proposed method for solving design problems.

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