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Designing an economical acceptance sampling plan in the presence of inspection errors based on maxima nomination sampling method

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Abstract. An acceptance sampling plan is one of the most useful and effective methods with an extensive application range in companies with the purpose of examining the quality of the raw material and final products. The inspection process is assumed free of errors in most designs of acceptance sampling plans. However, this assumption may not be true. In this research, an optimization model was presented for the acceptance sampling plan based on the Maxima Nomination Sampling (MNS) method, developed for the single acceptance sampling plan, in the presence of inspection errors. This study managed to propose an economical model which involved two types of inspection errors and investigated the impact of these errors from an economical point of view. Then, to prove its efficiency, it was compared with a classical method. Furthermore, the sensitivity analysis was carried out to analyze the behavior of the MNS scheme's optimal solution. The numerical studies indicated that the MNS method is always more economical than classical one is.

1. Introduction

The inspection is assumed to be perfect in most acceptance sampling plans. However, even under the ideal inspection conditions, the inspection tasks are generally disposed to errors of both Types I and II by the inspector. The probability of Type I and Type II errors and the cost objective functions can be suitable factors in estimating the statistical reliability of a sampling system. Accepting a nonconforming item or rejecting a conforming one are two kinds of errors considered in the objective function of this paper in order to model the probabilities associated with these two kinds of errors based on the MNS sampling method. Some expenses related to the cost of misclassifying a good component as bad one or misclassifying a bad item as a good one are categorized as misclassification costs. An accepted item by mistake may result in system failure and human loss; thus, the cost of false acceptance is more than that of false rejection.

In quality control, the acceptance sampling plan, which is one of the most important studies in quality assurance techniques, is a considerable tool applied in order to decide whether to accept or reject a manufactured lot of components. A batch can be accepted or rejected based on a chosen sample according to the acceptance sampling's rules. If the number of defective items is more than the acceptance number, then the lot is rejected; otherwise, it is accepted. Sampling plans are used by many people, such as manufacturers, suppliers, contractors and subcontractors, and service providers, in a wide range of industries. In this article,

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an acceptance sampling plan is represented to decide whether to accept or reject the received lot based on the cost objective function in the presence of inspection errors. Two concepts of Type I and Type II inspection errors, decisions of accepting the lot, rejecting the lot, and inspecting all items in the lot are considered in the cost objective function. The influence of inspection errors is considerable on the performance measures of the sampling plan. The source of these errors can be operational environment, inspector fatigue, and other operation and inspection-related factors. Therefore, there is a need to study the statistical and economical influence of these types of classification errors on the performance measures of an inspection scheme.

Khan and Duffuaa [1] considered the influence of inspection errors on the performance of different inspection plans. Duffuaa [2] explored the statistical and economical effects of inspector errors on the performance measures, i.e., Average Total Inspection (ATI), Average Outgoing Quality (AOQ), and Expected Total Cost (ETC). He considered the important effect of Type I and Type II errors on the performance measures of inspection plans. Tang and Schneider [3] explored the economical and statistical influence of inspection error on the complete inspection scheme. They developed two models in the presence of inspection errors under different rework plans and, then, compared them with the perfect inspection models. Raouf et al. [4] were the first to expand a model for determining the optimal number of inspections for multi-characteristic elements to minimize the total expected cost per accepted component by considering Type I and Type II errors and the cost of inspection. Bennet et al. [5] studied the influence of error on a single sampling plan with known incoming quality. Collins et al. [6] examined the effects of inspection error on the probability of acceptance, average total inspection, and average outgoing quality. They studied these measures under both replacement and non-replacement rectifying policies. Ayoub et al. [7] described the mean inspection error as the average number of defective items categorized as conforming items by the inspector. They proposed a formula for Average Total Inspection (ATI) in a single sampling plan in the presence of inspection error. Markowski and Markowski [8] discussed an attribute acceptance sampling problem in the presence of inspection errors and introduced alternative sampling plans designed to address the risk of statistical classification error. Their results showed that there are significant shortcomings in traditional sampling plans. The estimation of the rate of classification errors and plan parameters to reduce the potential impact of such errors are considered in this study. Ferrell and Chhoker [9] proposed a sequence of models that addressed 100% inspection and single sampling, with and without any inspection error by applying Taguchi-like loss function. Arshadi Khamseh et al. [10] proposed an economical model for a double-variable acceptance sampling plan in the presence of inspection errors. They applied Taguchi Loss function as the acceptance cost, while quality characteristics follow normal distribution with known variance. They also developed an optimization model for a double-variable acceptance sampling scheme in the presence of the inspection errors with either constant or monotone value functions. Hsu and Hsu [11] proposed an economical model to determine the optimal sampling plan in a two-stage supply chain that minimizes the total quality cost and failure costs while satisfying both the producer and consumer's quality and risk requirements. They figured that this optimal sampling plan is very sensitive to the producer's product quality. Jozani and Mirkamali [12] developed quality control charts for attributes applying the Maxima Nomination Sampling (MNS) method and compared them with the usual control charts based on the Simple Random Sampling (SRS) method. They studied the influence of the sample size, set size, and values of nonconforming proportion and analyzed the performance of MNS control charts using Average Run Length (ARL) curve, and demonstrated that MNS control chart can be used as an improvement method for quality inspection plan. They analyzed MNS charts from a cost perspective. Jozani and Mirkamali [13] proposed an acceptance sampling plan based on Maxima Nomination Sampling (MNS) technique in order to design and evaluate a single Acceptable Quality Level (AQL), Lot Tolerance Percentage Defective (LTPD), and Equilibrium Quality Level (EQL) acceptance sampling plans for attributes. They analyzed the Operating Characteristic (OC) function to exploit the influence of sample size and acceptance number on the performance of the MNS plans. They also compared Simple Random Sampling (SRS) schemes with MNS acceptance plans, and observed that MNS method has a smaller sample size and bigger acceptance number; thus, they stated that the MNS would perform better than the SRS sampling plans, and the OC curve of the MNS method would be much closer to the ideal OC curve.

Klufa [14] presented that the Average Outgoing Quality Level (AOQL) plan for inspection by variables is more economical than the corresponding Dodge-Romig AOQL attribute sampling plan (the inspection cost is saved about 48%) and also compared their OC curves. Klufa's results showed that the OC curve for the AOQL plan by variables is better than corresponding OC curve for the AOQL plan by attributes. Jamkhaneh et al. [15] developed an acceptance single sampling plan with inspection errors with fuzzy quality characteristics. Their proposed sampling plan, as compared to a traditional plan, is more robust and comprehensive. They also calculated the OC curve using the concept of fuzzy probability. They showed that the OC curve of the plan is like a band with high and low bounds, and Type I error reduces the fuzzy probability of acceptance. Fallah Nezhad et al. [16] proposed a novel acceptance-sampling plan in which the items in the receiving batch are inspected until a nonconforming item is found. The proposed model can be applied to group acceptance sampling plans, where simultaneous testing is not possible. They used Markovian approach for determining the probability of accepting the batch and the expected number of inspected items.

Duffuaa and El-Ga'aly [17] analyzed the impact of the inspection errors on the optimal parameters and objective functions' values of Duffuaa and El-Ga'aly's [18] multi-objective optimization model recently developed for process targeting. They extended this multi-objective optimization model by considering the inspection errors and penalties to reduce the effect of the errors. Fallah Nezhad and Hosseini Nasab [19] presented a new acceptance sampling plan in which it is assumed that every defective item cannot be detected with absolute certainty and the inspection process is imperfect. They developed a Bayesian method for evaluating the probability density function of the number of defective items and determining the value of the objective function for different decisions. They showed that a negative binomial prior is a suitable distribution for modelling the Bayesian acceptance sampling plan. Aslam et al. [20] proposed a new Repetitive Group Sampling (RGS) plan which has a better performance than the variables single sampling plan. These plans are developed for the Weibull and generalized exponential distributions. They showed that the variable RGS plans have a lower average sample number than single sampling plans.

Fallahnezhad and Qazvini [21] presented a new economical scheme of the acceptance sampling plan in a two-stage approach based on the Maxima Nomination Sampling technique. The objective of their model is to minimize the summation of costs and, then, to compare the proposed MNS economical sampling plan with the classical one. They developed an optimization model that involves three different costs consisting of inspection cost, internal failure cost, and outgoing defective item cost. They showed that the new method of sampling is more economical. Hsieh and Lu [22] represented a risk-embedded model via conditional value-at-risk that allows a decision-maker to select an acceptance sampling plan with minimal expected excess cost. They concentrated on Bayesian acceptance sampling under Type II censoring for Weibull distributed product lifetime with a known shape parameter. Yazdi and Fallahnezhad [23,24] proposed an acceptance sampling plan based on cumulative count of conforming using a minimum angle method. In addition, they compared counts of cumulative conforming sampling plans with Dodge-Romig single sampling plan. Their method had better performance in most of the cases. Fallahnezhad and Aslam [25] represented a new economical design of acceptance sampling models using Bayesian inference in order to decide on the received lot based on a cost objective function. They used Bayesian inference to determine the optimal decision along with backward induction. The sample information of this sampling plan was used to develop an economically optimal sampling system by considering different decisions for lot. The first- and second-type errors, which are important characteristics of a sampling plan, are not considered in this proposed plan.

The outline of this paper is as follows: Section 2 discusses the MNS method and its procedure. Section 3 presents model description acronyms, abbreviations, and notations. Then, Section 4 provides performance measures. Section 5 introduces the cost minimization model. Section 6 provides numerical examples and discussion. Section 7 gives sensitivity analyses. Finally, Section 8 presents conclusion remarks and future research topics.

2. MNS method

Willemain [26] first proposed the MNS approach for health care cases. The estimation of the actual cost of services at a nursing home was performed by this method. Since calculating the expenses for all patients at any nursing home was not economical, sampling approaches were applied. Nevertheless, estimating the total health care costs at a nursing home by using a random sample was not economical, because the patients with the most expensive cost might not be included in the sample. Therefore, in 1980, a method was explained by Willemain [26] called Maxima Nomination Sampling; in this way, the nursing home operator(s) could choose the patient with the most expensive care requirements among other patients. The maxima (minima) nomination sampling method has been of practical interest in numerous papers. As it has been studied in many researches, it is so common in most of the acceptance sampling plans to use a simple random sampling plan; however, it can be subject to some shortcomings; hence, there is a need to design another acceptance sampling plan capable of covering these shortcomings and providing a better result. The MNS acceptance sampling plan can be one of these plans. One of the concepts used in the MNS approach is Ranked Set Sampling (RSS). MNS method is a variation of Ranked Set Sampling (RSS), and its fundamental advantage is that it can be a suitable choice to establish better acceptance sampling plans and also select sample items for quality inspection in a more precise way, as compared to the usual ones. The applied procedure in the MNS method is based on a RSS technique because the sampled items in the MNS technique are selected according to the RSS technique criteria.

Ranked Set Sampling (RSS) is a sampling procedure that is suitable for situations where the inspection process of the items is difficult (destructive or very costly). The principal advantage of RSS policy is that an equal amount of information can be gained with the fewer number of observations.

This method has gained considerable popularity in applied statistics. Ranked set sampling is an efficient sampling design which is performed in two stages and causes a reduction in the number of samples needed using a more costly measurement, called "costly measurements". While ranking of components is easy in relation to the cost or difficulty of quality measurement, application of this method is quite advantageous. In this method, the economical measurement is used in order to rank the small sets of samples. On the other hand, in order to obtain the maxima nomination sample of size n, two stages of inspection should be applied. These two stages of sampling would cause a decrease in the overall cost of sampling by using a less costly measurement for the first stage. However, the costly measurement is applied to only one item from each set. The main contribution of this article is to apply a different method of sampling called MNS that results in decreasing the costs related to the quality. Since the lot defective fraction in the MNS method is different from the usual lot defective fraction, it can, thus, result in different solutions. Of course, the OC curve of MNS technique is much closer to the ideal OC curve; thus, it can be a suitable tool for the quality assurance. Another important point, which should be noticed, is that the model is developed in the presence of inspection errors. There is a major difference between the paper in [21] and this paper. The contribution of the paper in [21] is about designing an economical scheme based on maxima nomination sampling method; however, our main goal in this paper is to design a new acceptance sampling plan which results in decreasing the related costs. Hence, a model has been proposed to cover this requirement. The difference between our model and the one in [21] is that our proposed model has been developed in the presence of inspection errors. Three different costs are considered in the model's objective function that can be stated as follows.

$$\begin{aligned}\text{Objective function} &= \text{the cost of inspection}\\ &+ \text{the expected cost of accepting the lot}\\ &+ \text{the expected cost of rejecting the lot.}\end{aligned}$$

The optimized model will be obtained by minimizing the summation of the cost of inspection and total cost of misclassification resulting from the first- and second-type errors.

Therefore, a model has been proposed whose objective function consists of all the three probable different costs in an industrial environment by applying the MNS method.

MNS algorithm can be elaborated as follows [13] (suppose that the number of items in each period of manufacturing (lot size) is large):

Step 1: Choosing n^*k items by random from the lot and, then, dividing these items into n sets in which each set has k items classified as follows (k equals the set size):

$$\begin{aligned}&\text{Set } 1: \quad Y_{11}\ Y_{21}\ \dots\ Y_{k1}\\ &\text{Set } 2: \quad Y_{12}\ \ Y_{22}\ \dots\ Y_{k2}\\ &\vdots\\ &\text{Set } n: \ Y_{1n}\ \ Y_{2n}\ \dots\ Y_{kn}.\end{aligned} \tag{1}$$

Step 2: Applying a ranking mechanism like the one proposed by Terpstra and Nelson [27] and ordering the items from the best to the worst based on their quality level in each set from 1 to n.

Step 3: Categorizing the ordered set of items as follows:

$$\begin{aligned}&\text{Ordered set 1:}\ \ Y_{(1)1}\ \ Y_{(2)1}\ \dots\ Y_{(k)1}\\ &\text{Ordered set 2:}\ \ \ Y_{(1)2}\ \ Y_{(2)2}\ \dots\ Y_{(k)2}\\ &\text{Ordered set } n:\ \ Y_{(1)n}\ \ Y_{(2)n}\ \dots\ Y_{(k)n}\end{aligned} \tag{2}$$

As is shown above, there are two stages in order to obtain maxima nomination sample of size n. A two-phase sampling is an effective method to reduce the total cost of sampling; therefore, a less exact measurement, which is less costly, is applied in the first phase for sorting the items. The cost of inspection in the first stage is less than the inspection cost in the second stage, because an expensive quality measurement is used in the second phase. It should be noted that some errors in this ranking process cannot be ignored because judgmental order statistics may differ from real-order statistics. However, recent studies demonstrated that errors in ranking do not influence ranking mechanism considerably [13]. In the first stage of ranked set sampling, $Y_{(i)j}$ is the ith order statistic of the jth set. In the second phase, the kth order statistic of each set, $Y_{(k)j}$ $j = 1, ..., n$, will be selected as the final items for more inspection. Lot nonconforming proportion in the MNS method is different from nonconforming proportion in the classic acceptance sampling plan. In this method, the Maximum Likelihood (ML) method is used in order to estimate an unknown lot nonconforming proportion by applying the MNS method. Assume that $Y_{(k)1}, Y_{(k)2}, ..., Y_{(k)n}$ are Bernoulli variables in the ordered set where $Y_{ij} \sim Bin(1,p)$; $i = 1, ..., k$. Thus, the probability distribution of $Y_{(k)j}$ which represents random variables is $Bin(1, \pi_k(p))$ where $j = 1, ..., n$, and then probability $\pi_k(p)$ is obtained as follows:

$$\pi_k(p) = P\left[Y_{(k)j} = 1\right] = 1 - \prod_{i=1}^{k} P\left[Y_{ij} = 0\right]$$
$$= 1 - (1-p)^k, \tag{3}$$

where $\pi_k(p)$ is the lot nonconforming proportion in the second stage of the MNS method. The likelihood function of p is given as follows [13]:

$$L(p) = \prod_{j=1}^{n} P\left[Y_{(k)j} = y_{(k)j}\right]$$
$$= \prod_{j=1}^{n} \{\pi_k(p)\}^{y_{(k)j}} \{1 - \pi_k(P)\}^{1-y_{(K)j}}$$
$$= (1-p)^{nk - nk\bar{y}_{(k)}} (1 - (1-p)^k)^{n\bar{y}_{(k)}}. \tag{4}$$

Assuming that:

$$\overline{y}_{(K)} = \frac{1}{n} \sum_{j=1}^{n} y_{(k)j},$$

and solving the equation:

$$\left(\partial/\partial p\right) LnL(p) = 0,$$

then the ML estimator of nonconforming proportion, p, is obtained as follows:

$$\hat{p}_{ML,MNS} = 1 - \sqrt[k]{1 - \overline{y}_{(k).}}. \tag{5}$$

Let $c^* = [n\pi_k(p_0)]$, where [.] refers to the integer part and defines $Y_{(k).} = n\overline{Y}_{(K)} = \sum_{j=1}^{n} Y_{(k)j}$. Thus, the following can be obtained:

$$P\left[\hat{p}_{ML,MNS} \le p_0\right] = P\left[\overline{Y}_{(K)} \le \pi_k(p_0)\right]$$
$$= P\left[Y_{(k).} \le c^*\right], \tag{6}$$

where c^* is the acceptance number.

3. Model description

The mathematical formulation of the acceptance sampling plan can help investigate the influence of the inspection errors on the acceptance sampling plan.

The objective function is summarized as follows:

Objective function = the cost of inspection

+ the expected cost of accepting the lot

+ the expected cost of rejecting the lot.

In this model, there are five different types of costs categorized as follows:

- C_1: The inexpensive cost of sorting items in the first stage of the MNS method.
- C_2: The cost of expensive and accurate inspection in the second phase in the second stage of the MNS method.
- C_3: The cost of a nonconforming item in the accepted lot.
- C_4: The cost of one identified nonconforming item.
- C_5: The cost of classifying an item as nonconforming when it is conforming.

4. Performance measures

In an acceptance sampling plan, a sample size of n is selected from a lot with size N, and each item in the sample is inspected and categorized as either conforming or nonconforming. If the number of nonconforming items is more than the acceptance number, c, then the whole lot is inspected. Otherwise, it is accepted. The errors in an attribute sampling are classified into two types. Type I error is defined as a conforming item being classified as non-conforming, and Type II error is explained as a non-conforming item being classified as conforming. Therefore, we can state that:

$$e_1 = P\{\text{the item is classified as nonconforming}| \text{ the item is conforming}\},$$

$$e_2 = P\{\text{the item is classified as conforming}| \text{ the item is nonconforming}\}.$$

The apparent nonconforming proportion can be expressed as follows:

- A: The event in which an item is nonconforming;
- B: The event in which an item is classified as nonconforming

Then, the apparent nonconforming proportion, P', is obtained as follows:

$$P' = P(B) = P(B|A)P(A) + P(B|A')P(A')$$
$$= p(1-e_2) + (1-p)e_1, \tag{7}$$

where:

$p = P(A)$	True nonconforming fraction;
$p' = P(B)$	Apparent nonconforming fraction;
e_1	Type I error probability;
e_2	Type II error probability.

Thus, the apparent nonconforming fraction in the second stage of the MNS method is as follows:

$$\pi'_k(p) = \pi_k(p)(1 - e_2) + (1 - \pi_k(p))e_1. \qquad (8)$$

The probability of lot acceptance in the presence of inspection error is obtained by replacing the true nonconforming fraction, $\pi_k(p)$, with the apparent nonconforming fraction, $\pi'_k(p)$. Thus:

$$P_{\text{accept}} = \sum_{0}^{c} bin(n,\pi'_k(p)). \qquad (9)$$

5. Cost minimization model

In this section, the objective is to design the optimal sampling plan by minimizing the summation of the cost of inspection and total cost of misclassification resulting from errors of Types I and II. The mathematical formulation for the expected total cost will be represented by considering different events. The probabilities of errors of Types I and II are assumed to be known. Three different types of costs are considered:

1. Cost due to the false rejection of a conforming item;
2. Cost due to the false acceptance of nonconforming item;
3. Cost of inspection.

The new optimization model in the presence of errors is modelled as follows:

$$\begin{aligned}\text{Minimize } TC_1 =& knC_1 + n\pi_k(p)(1 - e_2)C_4 \\ &+ n(1 - \pi_k(p))e_1C_5 + n\pi_k(p)e_2C_3 \\ &+ nC_2 + P_{\text{accept}}[p(N - n)C_3] \\ &+ (1 - P_{\text{accept}})[(N - n)C_2 \\ &+ (N - n)p(1 - e_2)C_4 + (N - n) \\ &(1 - p)e_1C_5(N - n)pe_2C_3], \qquad (10)\end{aligned}$$

Subject to:

$$\sum_{c+1}^{n} bin(n,\pi_k(LTPD')) \geq 1 - \beta, \qquad (11)$$

$$\sum_{c+1}^{n} bin(n,\pi_k(AQL')) \leq \alpha. \qquad (12)$$

$$P\frac{N - n}{N}\sum_{0}^{c} bin(n, \pi_k(p')) \leq AOQL. \qquad (13)$$

It has been assumed that rectifying inspection is applied when the lot is rejected. Thus, when we reject the lot, then all items are inspected.

The components of this model can be expressed as follows:

- knC_1: This term denotes the cost of sorting k^*n items in the first stage of the MNS method;
- $n\pi_k(p)(1 - e_2)C_4$: This term denotes the cost of detected nonconforming items in the second stage of the MNS method;
- $n(1 - \pi_k(p))e_1C_5$: This term denotes the cost of classifying conforming items as nonconforming in the second stage of the MNS method;
- $n\pi_k(p)e_2C_3$: This term denotes the cost of classifying nonconforming items as conforming in the second stage of the MNS method;
- $p(N-n)C_3$: This term denotes the cost of classifying nonconforming items as conforming in the decision of accepting the remaining items in the lot;
- $(N-n)C_2$: This term denotes the cost of inspecting the remaining items of the lot in the decision of rejecting the lot;
- $(N-n)p(1-e_2)C_4$: This term is the cost of detected nonconforming items in the decision of inspecting remaining items of the lot;
- $(N - n)(1 - p)e_1C_5$: This term denotes the cost of conforming items classified as nonconforming in the decision of inspecting remaining items of the lot;
- $(N - n)pe_2C_3$: This term denotes the cost of classifying a nonconforming item as conforming in the decision of inspecting remaining items of the lot.

The classical model in the presence of errors is designed as follows [28]:

$$\begin{aligned}\text{Minimize } TC_2 =& nC_2 + P_{\text{accept}}[p(N - n)C_3] \\ &+ (1 - P_{\text{accept}})\Big[(N - n)C_2 \\ &+ (N - n)p(1 - e_2)C_4 \\ &+ (N - n)(1 - p)e_1C_5 \\ &+ (N - n)pe_2C_3\Big]. \qquad (14)\end{aligned}$$

Subject to:

$$\sum_{c+1}^{n} bin(n,LTPD') \geq 1 - \beta, \qquad (15)$$

$$\sum_{c+1}^{n} bin(n,AQL') \leq \alpha, \qquad (16)$$

$$\frac{N-n}{N}P\sum_{0}^{c} bin(n,p') \leq AOQL. \qquad (17)$$

6. Numerical example and discussion

The following numerical examples are studied to illustrate the application of the proposed methodology.

For more illustrations, assume that the following sets of input parameters are given:

$$AOQL = 0.03, \quad AQL = 0.01, \quad LTPD = 0.2,$$

$$\alpha = 0.04, \quad \beta = 0.1,$$

$$C_1 = 0.1, \quad C_2 = 1, \quad C_3 = 10, \quad C_4 = 5, \quad C_5 = 7.$$

In addition, the following sets are searched and investigated for determining the optimal solution:

$$k = \{1, 2, ..., 10\},$$

$$n = \{1, 2, ..., 20\},$$

$$c = \{1, 2, ..., 10\}.$$

The question is to find the minimum total cost by considering the optimal values of n, k, and c such that the constraints of producer's risk, the consumer' risk, and AOQL are satisfied simultaneously for the given values of $AOQL$, AQL, and $LTPD$. The MATLAB software has been used in order to obtain the optimal solution in this paper. As demonstrated in the numerical example and discussion, some assumed sets of input parameters and some following sets of n, k, and c which should be searched for determining the optimal solution are given. Then, a grid search procedure is applied to obtain the optimal solution.

The main goal is acquiring the minimum total cost by obtaining the optimal values of n, k, and c among those assumed sets for each decision variable. The procedure of obtaining the minimum total cost is as follows.

The optimal values for n, k, and c should be searched within the given assumed set. Every value of decision variable in the sets, which could satisfy the constraints of producer and consumer's risks and AOQL constraints simultaneously, is chosen, and then the feasible values of n , k and c are substituted into the objective function in order to achieve the minimum total cost. Hence, the solution method is a grid search algorithm among the feasible values of decision variables.

After obtaining the minimum value for the total cost, its result is compared with the classical model to figure out which one will include the minimum cost in the presence of errors. In this section, the optimized total costs for some selected different values of lot sizes and process averages (%) for both the MNS and classical models with the same input parameters are given in Tables 1 to 3.

Table 1. The optimal solutions for MNS scheme.

e_1	e_2	p	N	n	k	c	TC_{MNS}
0.05	0.1	0.03	251	7	7	3	106.6675
0.05	0.1	0.05	251	17	3	3	243.1044
0.05	0.1	0.1	251	3	10	1	405.2394
0.05	0.2	0.03	251	11	6	4	112.6175
0.05	0.2	0.05	251	6	10	2	248.6543
0.05	0.2	0.1	251	7	7	2	427.7599
0.1	0.1	0.03	251	9	9	5	114.5885
0.1	0.1	0.05	251	15	4	4	267.6025
0.1	0.1	0.1	251	5	8	2	447.3359
0.1	0.2	0.03	251	11	8	5	125.2864
0.1	0.2	0.05	251	11	5	3	266.3617
0.1	0.2	0.1	251	9	6	3	459.6267

Table 2. The optimal solutions for new values of α and β for MNS scheme.

e_1	e_2	p	N	n	k	c	TC_{MNS}
0.05	0.1	0.03	251	4	6	2	92.548
0.05	0.1	0.05	251	6	4	1	237.3584
0.05	0.1	0.1	251	3	10	1	405.2394
0.05	0.2	0.03	251	7	5	3	95.8777
0.05	0.2	0.05	251	8	3	1	241.5538
0.05	0.2	0.1	251	4	8	1	415.932
0.1	0.1	0.03	251	7	6	4	98.4889
0.1	0.1	0.05	251	3	9	1	260.6749
0.1	0.1	0.1	251	5	8	2	447.3359
0.1	0.2	0.03	251	7	8	4	103.7442
0.1	0.2	0.05	251	6	8	2	264.5352
0.1	0.2	0.1	251	5	10	2	459.5148

In the first case, the values of 0.03, 0.05, and 0.1 are assumed for the process averages. Two values of $e_1 = 0.05$, 0.1 and $e_2 = 0.1$, 0.2 are also considered. The obtained results are presented in Table 1.

In the classical model, since three considered constraints are not satisfied with the assumed parameters, there is no optimal solution for this model. Thus, in order to compare the optimal solution of MNS model with that of the classical one and to determine the more economical design, the values of α and β have increased to 0.2 and 0.4, respectively. The objective of analyzing the first set of parameters was to denote another advantage of the MNS plan which had feasible solutions in all of the simulated cases; however, the classical model of sampling did not have any feasible solutions in some cases.

Table 3. The optimal solutions for new values of α and β for classical scheme.

e_1	e_2	p	N	n	c	$TC_{\text{classical}}$
0.05	0.1	0.03	251	19	3	102.6903
0.05	0.1	0.05	-	-	-	-
0.05	0.1	0.1	-	-	-	-
0.05	0.2	0.03	-	-	-	-
0.05	0.2	0.05	-	-	-	-
0.05	0.2	0.1	-	-	-	-
0.1	0.1	0.03	251	20	4	122.283
0.1	0.1	0.05	-	-	-	-
0.1	0.1	0.1	-	-	-	-
0.1	0.2	0.03	-	-	-	-
0.1	0.2	0.05	-	-	-	-
0.1	0.2	0.1	-	-	-	-

The obtained optimal solutions by consideration of new values for α and β both for the MNS and classic methods are given in Tables 2 and 3.

The procedure of obtaining the minimum total cost is to search within the given assumed set of values for decision variables in order to obtain the optimal values for n, k, and c. Then, every value of n, k, and c, which could satisfy all three constraints simultaneously, can be used in the objective function to achieve the minimum total cost. If there is no solution in some combinations of e_1, e_2 and p, it means that the assumed set of values for n, k, and c is unable to find a feasible solution that simultaneously satisfies all the three constraints. Therefore, there are not any feasible values for n, k, and c to apply to the objective function to gain the minimum total cost. As seen in the tables, there are no solutions for some specified values of e_1, e_2 and p in the classic model. In addition, in cases that the classical method has the optimal solution, it is obvious that the MNS method is superior to the classical one.

7. Sensitivity analysis

In this section, the sensitivity analysis of parameter C_1 (the inspection cost in the first stage) is carried out. As is obvious, two mathematical models are proposed in this paper and the main purpose is to investigate the performance of these two models from an economical standpoint in order to distinguish the most economical scheme with the minimum total cost. Hence, the sensitivity analysis has been accomplished for the cost in the first stage of the proposed MNS plan in this section. The sensitivity analysis procedure has been carried out by considering some different levels of inspection cost in the first stage of MNS plan in order to determine the more appropriate method for the right cost value in the first stage.

Table 4. The sensitivity analyses for different values of $e_1 = 0.05$, $e_2 = 0.1$, and $p = 0.03$.

C_1	Total cost
0.1	92.548
0.2	94.9481
0.3	97.3481
0.4	99.7481
0.5	102.1481
0.6	104.5481

Table 5. The sensitivity analyses for different values of $e_1 = 0.1$, $e_2 = 0.1$, and $p = 0.03$.

C_1	Total cost
0.1	98.4889
0.2	101.8226
0.3	104.8226
0.4	107.8226
0.5	110.8226
0.6	113.8226
0.7	116.8226
0.8	119.35
0.9	121.75
1	124.15

Based on the sensitivity analysis results, it is better to use the MNS method up to a specified value for the inspection cost of the first stage where the value of the total cost in the MNS method is less than that of the total cost of the classical one. This increasing process continues until the cost of MNS plan is more than that of the classical one. Therefore, the extent of how economical the MNS is and for what value it will be can be concluded now.

The sensitivity analysis has been carried out for the specified values of $e_1 = 0.05$, $e_2 = 0.1$, and $p = 0.03$. As is demonstrated in Table 4, by increasing the value of C_1, the total cost of the MNS will increase too. As a result, it is better to use the MNS method up to the value of $C_1 = 0.5$ where the value of the total cost in the MNS method is less than the total cost of the classical one (the cost of MNS method = 102.1481; the cost of classic method = 102.6903). Then, sensitivity analysis has been performed for the values of $e_1 = 0.1$, $e_2 = 0.1$ with $p = 0.03$. The results are shown in Table 5.

In this table, the total cost in the classical method is 122.283; therefore, it is better to use the MNS method up to $C_1 = 0.9$; however, it is clear that C_1 is always smaller than $C_2 = 1$; thus, the MNS method is completely superior to the classical one. Moreover, as a conclusion, it is economical to apply the MNS scheme because the value of C_1 is much less than that of C_2.

Moreover, some sensitivity analyses have been

Table 6. The sensitivity analyses for different values of AOQL, AQL, and LTPD for Maxima Nomination Sampling (MNS) method.

AOQL	p	N	n	k	c	TC_{MNS}
0.015	0.03	251	9	5	1	232.422
0.03	0.03	251	4	6	2	92.5481
0.045	0.03	251	4	6	2	92.5481
AQL	p	N	n	k	c	TC_{MNS}
0.005	0.03	251	4	6	2	92.5481
0.01	0.03	251	4	6	2	92.5481
0.015	0.03	251	4	6	2	92.5481
LTPD	p	N	n	k	c	TC_{MNS}
0.1	0.03	251	8	10	4	114.0962
0.2	0.03	251	4	6	2	92.5428
0.3	0.03	251	4	4	2	86.8595

Table 7. The sensitivity analyses for different values of AOQL, AQL and LTPD for classical method.

AOQL	p	N	n	c	$TC_{\text{Classical}}$
0.015	0.03	-	-	-	-
0.03	0.03	251	19	3	102.6903
0.045	0.03	251	19	3	102.6903
AQL	p	N	n	k	$TC_{\text{Classical}}$
0.005	0.03	251	19	3	102.6903
0.01	0.03	251	19	3	102.6903
0.015	0.03	251	19	3	102.6903
LTPD	p	N	n	c	$TC_{\text{Classical}}$
0.1	0.03	-	-	-	-
0.2	0.03	251	19	3	102.6903
0.3	0.03	251	17	4	89.1793

Table 8. The sensitivity analyses for different values of α and β for Maxima Nomination Sampling (MNS).

α	p	N	n	k	c	TC_{MNS}
0.1	0.03	251	4	6	2	92.5481
0.2	0.03	251	4	6	2	92.5481
0.3	0.03	251	4	6	2	92.5481
β	p	N	n	k	c	TC_{MNS}
0.2	0.03	251	8	7	4	101.2436
0.4	0.03	251	4	6	2	92.5481
0.6	0.03	251	3	8	2	88.0097

performed for other parameters such as AQL, $LTPD$, $AOQL$, p, α, and β for different ranges of values with $e_1 = 0.05$, $e_2 = 0.1$, and $p = 0.03$. The results are shown in Tables 6 to 11 for both MNS and classical methods. As is obvious, the cost of the MNS approach is less than that of the classical method in all tables for different ranges of values except for different values of p when $e_1 = 0.1$ and $e_2 = 0.1$.

Table 9. The sensitivity analyses for different values of α and β for classical method.

α	p	N	n	c	$TC_{\text{Classical}}$
0.1	0.03	-	-	-	-
0.2	0.03	251	19	3	102.6903
0.3	0.03	251	19	3	102.6903
β	p	N	n	c	$TC_{\text{Classical}}$
0.2	0.03	-	-	-	-
0.4	0.03	251	19	3	102.6903
0.6	0.03	251	19	4	91.8493

Table 10. The sensitivity analyses for different values of p for Maxima Nomination Sampling (MNS) method.

p	e_1	e_2	N	n	k	c	TC_{MNS}
0.015	0.05	0.1	251	2	9	1	46.9289
0.03	0.05	0.1	251	4	7	2	90.6676
0.045	0.05	0.1	251	19	1	1	190.209
p	e_1	e_2	N	n	k	c	TC_{MNS}
0.015	0.1	0.1	251	7	6	4	98.4889
0.03	0.1	0.1	251	3	9	1	260.6749
0.045	0.1	0.1	251	5	9	2	228.9657

Table 11. The sensitivity analyses for different values of p for classical method.

p	e_1	e_2	N	n	c	$TC_{\text{Classical}}$
0.015	0.05	0.1	251	19	3	62.1301
0.03	0.05	0.1	251	19	3	102.6903
0.045	0.05	0.1	-	-	-	190.209
p	e_1	e_2	N	n	c	$TC_{\text{Classical}}$
0.015	0.1	0.1	251	20	4	79.0248
0.03	0.1	0.1	251	20	4	122.283
0.045	0.1	0.1	251	20	3	220.5295

8. Conclusion and future researches

The acceptance sampling plan is one of the significant methods applied to evaluate the quality of the raw material, semi-finished products, and final goods; moreover, it is used in almost any kind of industry. This paper has presented an acceptance sampling plan by considering inspection errors based on the MNS plan from an economical standpoint. Inspection errors can make classical plans useless for lot acceptance decisions. This new model is compared to the classical one. Numerical studies have shown that the MNS method is always more economical than the classical one is. In the following, some suggestions are made for future studies:

- Designing new economic models for double or multiple sampling plans based on the MNS method and comparing them with the proposed model of this paper.

- Considering errors in the ranking process in the MNS method and designing the economic models based on the MNS approach with ranking errors.

Nomenclature

Abbreviations

MNS	Maxima Nomination Sampling
SRS	Simple Random Sampling
RSS	Ranked Set Sampling
ATI	Average Total Inspection
LTPD	Lot Tolerance Percentage Defective
AQL	Average Quality Level
AOQL	Average Outgoing Quality Level

Parameters

N	The number of items in a whole lot
α	The consumer's risk
β	The producer's risk
e_1	The probability of a conforming item being classified as nonconforming (the first type error probability)
e_2	The probability of a nonconforming item being classified as conforming (the second type error probability)
$\pi_k(p)$	The actual nonconforming fraction in the MNS method
$\pi'_k(p)$	The apparent nonconforming fraction in the MNS method

Decision variables

n	The number of sets
c	The acceptance number
k	The set size

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