Scientia Iranica E (2019) 26(2), 1023–1038

Sharif University of Technology
Scientia Iranica
Transactions E: Industrial Engineering
http://scientiairanica.sharif.edu

Redundancy allocation problem with a mixed strategy for a system with k-out-of-n subsystems and time-dependent failure rates based on Weibull distribution: An optimization via simulation approach

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Received 25 July 2017; received in revised form 7 January 2018; accepted 22 January 2018

KEYWORDS
Reliability;
Redundancy allocation problem;
Weibull distribution;
Time-dependent failure rates;
Optimization via simulation.

Abstract. Reliability improvement of electronic and mechanical systems is vital for engineers in order to design these systems. For this reason, there are many researches in this area to help engineers with real-world applications. One of the useful methods in reliability optimization is Redundancy Allocation Problem (RAP). In most previous related works, the failure rates of system components are considered to be constant based on negative exponential distribution, whereas nearly all real-world systems have components with time-dependent failure rates; in other words, the failure rates of system components will change from time to time. In this paper, we have worked on an RAP for a system under k-out-of-n subsystems that have components with time-dependent failure rates based on Weibull distribution. In addition, the redundancy policy of the proposed system is considered as a mixed strategy, and the optimization method is based on the simulation technique to obtain the reliability function as an implicit function. Finally, a branch and bound algorithm has been used to solve the model exactly.

1. Introduction

Reliability improvement is one of the most effective strategies for improving the quality level of electronic and mechanical systems. To this end, a common and useful method is RAP. The aim of this problem is to increase the redundant components in the system under some constraints, such as weight, volume, cost, etc., to help increase system reliability. Therefore, several studies have been carried out in this area. Fyffe et al. [1] proposed a mathematical model for RAP as the first research in this field and used dynamic programming to solve the model. Then, Nakagawa and Miyazaki [2] modified the model presented in [1] by considering the upper limit of system weight between 159 and 191. They showed that using a surrogate constraints algorithm leads to solutions with higher reliability in comparison with dynamic programming that solves 33 problems.

One of the most effective factors in reliability evaluation is component failure rates. In the literature of RAP, the failure rates of components are considered as follows:

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doi: 10.24200/sci.2018.20152

- Constant failure rates based on distributions such as exponential, etc. (in which the failure rate of components is constant over time);
- Time-dependent failure rates based on distributions such as Weibull, etc. (in which the failure rate of components is different from initial value over time).

When failure rates of components are constant, it is very easy to obtain the reliability function via statistical and mathematical relations. Therefore, most of research studies on RAP use this assumption for system components. Misra and Sharma [3] considered a system as series-parallel with k-out-of-n configuration, and assumed that the system could work as an active policy. In addition, a zero-one programming model was used to solve the model. Pham [4] solved RAP for a system with a single k-out-of-n subsystem and an active redundancy strategy to reduce the system cost as part of the objective function of their model. Considering multi-failure modes for components, Pham and Malon [5] extended Pham's model. Moreover, since it is proven that RAP belongs to the class of Np-hard problems [6], heuristics or meta-heuristic algorithms are essential to solve RAPs in large-scale problems. For this reason, in many papers after 1992, the researchers have used approaches based on heuristics or meta-heuristics in order to solve RAP models. Ida et al. [7] used Genetic Algorithm (GA) in a simple form for the first time to solve RAP of series-parallel systems with multiple failure state components. Painton and Campbell [8] presented a series-parallel RAP under risk and used GA to solve it. Coit and Liu [9] presented a series-parallel RAP with k-out-of-n subsystems. They considered both of active and cold-standby strategies as predefined for the whole subsystems at the start time of the process. Coit [10] proposed an optimal solution for RAP and solved the model by applying a zero-one integer programming. Moreover, the redundancy strategy is considered as an additional decision variable in RAPs for the first time in [10]. Moreover, Tavakkoli-Moghaddam et al. [11] solved the model proposed by Coit (2003) using GA with a new definition of chromosomes, crossovers, and mutations. In addition, there are many papers on RAP that use meta-heuristic methods like [12-15]. Garg et al. [16] presented a bi-objective reliability redundancy allocation problem for a series-parallel system, where reliability of the system and the corresponding designing cost are considered as two different objectives. They converted the developed fuzzy model to a crisp model to solve the problem. In addition, they solved the script optimization problem with swarm optimization. Garg et al. [17] provided a methodology to solve the multi-objective reliability optimization model. In their study, the model parameters are considered as imprecise in terms of triangular interval data. They converted the uncertain multi-objective optimization model to a deterministic one and used Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) to solve the model. A penalty-guided-based biogeography-based optimization model was used to solve the reliability redundancy allocation problems of series-parallel systems under various nonlinear resource constraints in [18]. Moreover, Garg [19] proposed a penalty-based Cuckoo Search (CS) algorithm to achieve the optimal solution to reliability-redundancy allocation problems (RRAP) with nonlinear resource constraints.

Recently, a new redundancy strategy for reliability optimization problems, which has a combination of active and standby strategies, was presented by Ardakan and Hamadani [20]. In this strategy, each subsystem can have different levels of active and cold-standby redundancies so that some components can be active, while others are kept in standby mode. This strategy is called "mixed strategy" and leads to higher system reliability in comparison to systems with active or standby strategy. Moreover, Ardakan et al. [21] used a mixed strategy in a Multi-Objective RAP (MORAP) as a nonlinear integer programming model and applied NSGA-II to solve the model. In addition, Gholinezhad and Hamadani [22] presented a new mathematical model for RAP with component mixing and mixed redundancy strategy considering the choice of a redundancy strategy as a decision variable.

In addition, if the failure rates of system components are constant based on an exponential distribution since this distribution has memory-less property, it is possible for researchers to use Markov process to obtain differential equations of a system to calculate its reliability function. Nourelfath et al. [23] proposed a combined method based on Markov processes, Genetic algorithm, and universal moment generating function in order to calculate multi-state systems availability. Pourkarim Guilani et al. [24] used Markov process to obtain differential equations that lead to reliability calculation of non-repairable three-state systems. Kim and Kim [25] proposed a new Markov chain approach to the standby RRAP problem. They used a Parallel GA to solve different test problems and showed the advantage of the proposed Markov-based approach in finding better structures. Chang and Kuo [26] considered Generalized Redundancy Allocation Problem (GRAP) in which a traditional RAP was extended to a more realistic situation where the system under consideration has a generalized network structure. They also proposed a partition-based simulation optimization method to solve GRAP. However, in the real-world problems, there are not many systems whose components are CFR. Therefore, it is more realistic for the failure rates of system components to be considered as time dependent. Azimi et al. [27] proposed a non-exponential redundancy allocation problem in series-parallel k-out-of-n systems with repairable components and used simulation techniques to estimate the system reliability.

One of the most applied distributions in reliability theory to model the real-world applications is Weibull distribution. This distribution is a good tool to formulate time-dependent failure rates due to its flexibility property to fit real-world stochastic events. However, when the components' failure rates are considered as time-dependent events, it is impossible for researches to evaluate system reliability through Markov process and differential equations, because, in this situation, the failure rates of components do not have memoryless property. The simulation technique is a powerful tool to model the stochastic events with any kind of distribution functions. This is the main reason why this technique is used in this research to model the objective function. However, using the simulation technique will not provide a closed form of the objective function; however, the simulation model is replaced instead of the mathematical form of the objective function (implicit form).

Ardakan et al. [28] proposed RAP considering time-dependent reliability of components. The reliability of components was considered as a function of time in their paper, and the RAP was reformulated by introducing "mission design life", defined as the integration of the system reliability function during the mission time.

In the case where the components' failure rates are time dependent, having an explicit function of the system reliability via mathematical and statistical relations is not possible. Therefore, in these circumstances, using the simulation technique is the only way to calculate the system reliability. Pourkarim Guilani et al. [29] proposed a RAP for a system with Increasing Failure Rates (IFR) based on the Weibull distribution. In their work, they used the simulation in order to estimate reliability function as implicit functions. The redundancy strategy for their proposed system was variable between active and cold-standby. A summary of literature review is provided in Table 1.

In this paper, it is intended to model a RAP for a system with k-out-of-n subsystems that have components with time-dependent failure rates based on the Weibull distribution. Moreover, the redundancy strategy for this system is mixed in accordance with [20], and a benchmark is provided to compare the present paper to the current literature. The innovation of this research is that, for the first time, a RAP under k-out-of-n configuration is considered in which the components' failure rates are time dependent based on the Weibull distribution, and the redundancy strategy is mixed simultaneously. With respect to many electronic and mechanical devices failed based on hazard function of the Weibull distribution during their lifetime, this study can achieve reliable results for managers and engineers.

The rest of this paper is organized as described below. In Section 2, the parameters, the variables, and the model of the problem are presented. Section 3 deals with the simulation approach along with a brief description about the Weibull distribution. The solution methodology is provided in Section 4. Numerical examples are given in Section 5 to demonstrate the verification of the suggested methodology. Finally, conclusions and directions for future research are presented in the last section.

2. Problem definition

In this section, a RAP for a system as series-parallel with k-out-of-n subsystems demonstrated in Figure 1 is studied, in which mixed strategies can be chosen for each subsystem. The basic assumptions of the proposed problem are listed as follows.

2.1. Problem assumptions

- System components are binary states (working or failed);
- The components' failure rates are time dependent based on the Weibull distribution;
- The redundancy strategy is mixed;
- Each subsystem is working as k-out-of-n;
- System components are non-repairable;
- The failed components will not damage the whole system;
- All parameters for components, including costs, weights, etc., are deterministic.

2.2. Notations of the model

The variables and parameters used in the mathematical model are listed as follows.

n_{ij} Number of the jth available component in subsystem i, $i = 1, 2, ..., s$, $j = 1, 2, ..., m_i$;

c_{ij} Cost for the jth available component in subsystem i;

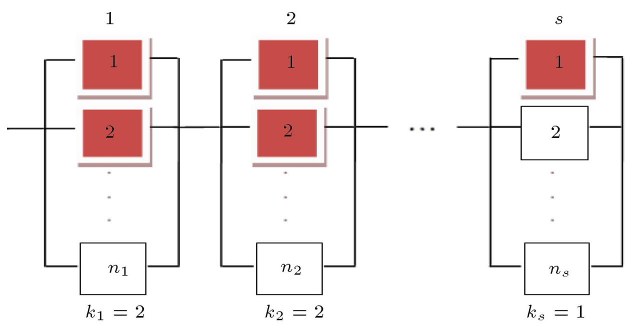

Figure 1. The system structure.

Table 1. A summary of literature review.

Authors	Failure rates	Description	Solving method
Fyffe et al. (1968)	Constant	Presented RAP for the first time	Dynamic programming
Nakagawa and Miyazaki (1981)	Constant	Model of Modified Fyffe et al.	Surrogate constraint
Misra and Sharma (1991)	Constant	Considered a series-parallel system with k-out-of-n subsystems under active redundancy policy	Zero-one programming
Pham (1992)	Constant	Proposed RAP for systems with a single k-out-of-n subsystem under active redundancy	Mathematical and statistical relations
Pham and Malon (1994)	Constant	Considered multi-failure mode for components in Pham (1992)	Mathematical and statistical relations
Chern (1992)	Constant	Proved that RAP is Np-hard	Mathematical and statistical relations
Ida et al. (1994)	Constant	Proposed RAP of series-parallel systems with multiple failure state components	Genetic algorithm
Painton and Campbell (1995)	Constant	Presented a series-parallel RAP under risk	Genetic algorithm
Coit and Liu (2000)	Constant	Proposed RAP with k-out-of-n subsystems under active and cold-standby redundancy	Zero-one integer programming
Coit (2003)	Constant	Considered redundancy strategy as a decision variable for Coit and Liu (2000)	Integer programming
Tavakkoli-Moghaddam et al. (2008)	Constant	Assumed that, the redundancy strategy for each subsystem is predetermined and fixed (active or cold-standby)	Genetic algorithm
Keshavarz Ghorabaee et al. (2015)	Constant	Presented BORAP for a system with k-out-of-n subsystems and non-identical components	Genetic algorithm and NSGA-II
Zhang and Chen (2016)	Constant	Worked on reliability redundancy allocation problems modeled in an interval environment	Particle swarm optimization algorithm
Teimouri et al. (2016)	Constant	Developed an electromagnetism to solve RAP	Memory-based electromagnetism-like mechanism algorithm
Pourkarim Guilani et al. (2017)	Constant	Proposed MORAP for three-state systems with k-out-of-n subsystems using Markov	NSGA-II and SPEA-II

Table 1. A summary of literature review (continued).

Authors	Failure rates	Description	Solving method
Garg et al. (2014)	General	Presented a BORAP in fuzzy environment	Swarm optimization
Garg et al. (2014)	General	Provided a MORAP in fuzzy environment	Genetic algorithm and particle swarm optimization
Garg (2015)	General	Proposed a RRAP under the various nonlinear resource constraints	Biogeography based optimization
Garg (2015)	General	Presented a RRAP with nonlinear resource constraints	Cuckoo search
Ardakan and Hamadani (2014)	Constant	Proposed RAP with the mixed redundancy strategy	Genetic algorithm
Ardakan et al. (2015)	Constant	Proposed MORAP with the mixed redundancy strategy	NSGA-II
Gholinezhad and Hamadani (2017)	Constant	Proposed RAP in which the redundancy strategy (active, cold standby, or mixed) is considered as a decision variable	Genetic algorithm
Nourelfath et al. (2012)	Constant	Used Markov and UGF to obtain multi-state system reliability	Genetic algorithm and Tabu search
Pourkarim Guilani et al. (2014)	Constant	Used Markov to obtain reliability function of three-state systems	Differential equations
Kim and Kim (2017)	Constant	Proposed a new Markov chain approach for standby RRAP	Genetic algorithm
Chang and Kuo (2018)	Constant	Considered generalized redundancy allocation problem	Simulation-based optimization
Azimi et al. (2017)	Time-dependent	Proposed a non-exponential redundancy allocation problem in k-out-of-n systems	Meta-heuristic and simulation
Ardakan et al. (2017) & Constant	Time-dependent	The reliability of components considered as a function of time	Genetic algorithm
Pourkarim Guilani et al. (2016)	Time-dependent	Used simulation to obtain reliability function as implicit	Genetic algorithm and random search
Current paper	Time-dependent	Used simulation to obtain reliability function as implicit	Integer programming

w_{ij}	Weight for the jth available component in subsystem i;
k_i	The minimum requirement amount of components that must be working in each subsystem ($n \geq k$);
n_{ij}^a	Number of the jth available component in subsystem i working as active;
n_{ij}^c	Number of the jth available component in subsystem i working as cold-standby;
$n_{\max,i}$	The upper bound on the assigned number of components in subsystem i;
z_{ij}	A binary variable equals 1 if the jth available component is placed in subsystem i and zero otherwise;
A	Shape parameters of the Weibull distribution;
B	Scale parameters of the Weibull distribution;
t	Mission time.

2.3. Mathematical model

The optimization model of the redundancy allocation problem is presented as follows:

$$\text{Max}R = \prod_{i=1}^{s} f(n_{ij}, {n^a}_{ij}, {n^c}_{ij}, z_{ij}, t, \theta), \quad (1)$$

s.t.:

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i} c_{ij}n_{ij} \leq C, \quad (2)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i} w_{ij}n_{ij} \leq W, \quad (3)$$

$$n_{ij} \leq z_{ij}n_{\max,i} \qquad \forall i = 1,...,s, \quad \forall j = 1,...,m_i, \quad (4)$$

$$\sum_{j=1}^{m_i} z_{ij} = 1 \qquad \forall i = 1,...,s, \quad (5)$$

$${n^c}_{ij} = n_{ij} - {n^a}_{ij} \qquad \forall i = 1,...,s, \quad \forall j = 1,...,m_i, \quad (6)$$

$$k_i \leq {n^a}_{ij} \leq n_{ij} \qquad \forall i = 1,...,s, \quad \forall j = 1,...,m_i, \quad (7)$$

$$z_{ij} \in \{0,1\}, \quad (8)$$

$${n^a}_{ij}, {n^c}_{ij}, n_{ij} \in \text{int}. \quad (9)$$

Eq. (1) is the objective function of the model which maximizes the system reliability. This function is obtained via the simulation replications as implicit for each subsystem based on several variables and parameters such as the number of available components, the number of components working in active and cold-standby modes, the type of selected components in subsystems, the mission time, and the error (θ) associated with estimating this function via the simulation technique. Finally, the reliability of system is obtained by multiplication of subsystem reliabilities. As previously mentioned, the components' failure rates are time dependent; therefore, calculating the system reliability is not an easy task. Thus, a simulation model is used to estimate the objective function. Inequalities (2) and (3) are cost and weight constraints, respectively. Constraint (4) puts an upper bound on the number of components of the subsystems if the jth available component is placed in subsystem i. Relation (5) ensures that there will be one type of components in each subsystem. In addition, Constraints (6)-(9) represent the conditions on model variables.

3. Simulation

In this section, a brief description of the Weibull distribution is provided. Then, the simulation steps are explained.

3.1. Weibull distribution

Weibull is one of the continuous probability distributions that has been widely used in modeling of system reliability. Time-dependent failure rates of system components can be modeled properly through this distribution. The hazard function of this distribution is $h(t) = At^B$. This hazard function can be used for both Increasing Failure Rate (IFR) and Decreasing Failure Rate (DFR) of system components so that if $A > 0$ and $B > 0$, then the hazard function is equal to an increasing function of t (the distribution is proper for modeling of the systems with IFR); if $A > 0$ and $B < 0$, then the hazard function is equal to an decreasing function of t (the distribution is proper for modeling of the systems with DFR) [29]. If the failure rates during time are considered as in the following function:

$$h(t) = \frac{B}{A}\left(\frac{t}{A}\right)^{B-1}; \quad B > 0; \quad A > 0; \quad t \geq 0, \quad (10)$$

then this hazard function belongs to the Weibull distribution; thus, its relationships such as reliability function, cumulative distribution function, and probability distribution function are calculated as follows:

$$R(t) = e^{-\int_{y=0}^{t} h(y).dy} = e^{-\int_{y=0}^{t} \frac{B}{A}\left(\frac{y}{A}\right)^{B-1}.dy} = e^{-\left({t}/{A}\right)^{B}}; \quad B > 0; \quad A > 0; \quad t \geq 0, \quad (11)$$

$$F(t) = 1 - e^{-\left({t}/{A}\right)^{B}}; \quad B > 0; \quad A > 0; \quad t \geq 0, \quad (12)$$

$$f(t) = R(t) \times h(t) = \frac{B}{A}\left(\frac{t}{A}\right)^{B-1} e^{-\left(t/A\right)^{B}};$$

$$B > 0; \quad A > 0; \quad t \geq 0. \tag{13}$$

In all of the above equations, B is the shape parameter, and A is the scale parameter of the Weibull distribution. In order to see further studies on the Weibull distribution, [30,31] are recommended.

3.2. Simulation implementation

As previously mentioned, the simulation has been always used to calculate reliability function of a subsystem with time-dependent components, explicitly. Therefore, the reliability function for a subsystem via simulation is estimated; then, the estimated function will be used for the whole system reliability. Since the redundancy policy is considered as the mixed one for a k-out-of-n system, number of components (n), minimum number of requirement component (k), number of active components (n^a), and number of cold-standby components (n^c) of each subsystem are required during each iteration of the simulation process. Because these variables are dependent on each other, in order to satisfy this dependency and create feasible designs for simulation, two new auxiliary variables are defined as follows: k' and $n^{a'}$, both of which are between 0 and 1. We know that in a k-out-of-n system, the amount of k should be between 1 and n ($1 \leq k \leq n$). In addition, k value is between 1 and n at all experiment levels; thus, it can be derived from Eq. (14):

$$k = round\left(1 + k' \times (n-1)\right). \tag{14}$$

According to Eq. (14), for different values of k' between 0 and 1, k can be between 1 and n. In addition, we know that in a k-out-of-n system, at least k components should work actively. Therefore, in each subsystem, we will have $k \leq n^a \leq n$. Therefore, for different values of $n^{a'}$ between 0 and 1, n^a is between k and n in Eq. (15):

$$n^a = round\left(k + n^{a'} \times (n-k)\right). \tag{15}$$

Then, the number of cold-standby components in each subsystem is calculated by $n^c = n - n^a$. The pseudo code and the flowchart of simulation are shown in Figures 2 and 3, respectively, and all the steps involved in the simulation experiments are coded in MATLAB 10 software.

As shown in Figures 2 and 3, details of the simulation experiments are described as follows. At first, the input parameters are added. These parameters include n, $n^{a'}$, k', A, B, and t. Then, by Eqs. (14) and (15), the values of n^a and k are obtained. Then, for each iteration, a number of the components working as active are inserted in w, and a number of the components reserved as cold-standby are inserted in st.

A. Estimating the subsystem reliability with mixed strategy

- *Set* n, $n^{a'}$, k', A, B, t, *iteration*
- *Calculate* $k = round\left(1 + k' \times (n-1)\right)$
- *Calculate* $n^a = round\left(1 + n^{a'} \times (n-k)\right)$
- *Calculate* $n^c = n - n^a$
- *Set* $num=0$
- *For i from 1 to iteration*
  Set $Tsys = 0$, $w = n^a$, $st = n^c$,
  Set $f = wblrnd\left(A, B, n^a, 1\right)$
- *While* $Tsys < T$ *and* $w \geq k$
  Set $tt = \min(f)$
  Set $w = w - 1$
  Set $Tsys = Tsys + tt$
- *If* $st > 0$
  Set $st = st - 1$
  Set $w = w + 1$
  Set $f = \left[f; wblrnd\left(A, B\right)\right]$
- *End If*
- *End While*
- *If* $Tsys > T$
  Set $num = num + 1$
- *End If*
- *End For*
- *Calculate subsystem reliability by* $R = {}^{num}\!/_{iteration}$

Figure 2. Pseudo code of simulation experiments.

In addition, random numbers are generated based on the Weibull distribution with parameters A and B for the number of active components and are inserted in f. In the simulation process, ($Tsys$) is the life time of the given subsystem. In this process, if $Tsys > t$, the given subsystem is working safely and there is no problem. However, when $Tsys < t$, it means that the system needs to be investigated. Indeed, when $Tsys < t$, the system performance has a problem that must be eliminated. While the subsystem life time ($Tsys$) is less than t and w is more than k, the minimum amount of f will be selected. This amount eliminates f and adds to $Tsys$. Therefore, one of the active working objects is reduced. Now, one of the cold-standby components will be deleted from st and added to w. To put it more delicately, if there is at least one component as reserved, this component enters the circuit. At the same time, a random number is generated based on the Weibull distribution with parameters A and B, and this number joins the existing values of set f.

This procedure continues until $w < k$, and there are some cold-standby components marked as reserved. In addition, if $Tsys > t$, it means that the subsystem works safely. At the end, the subsystem reliability

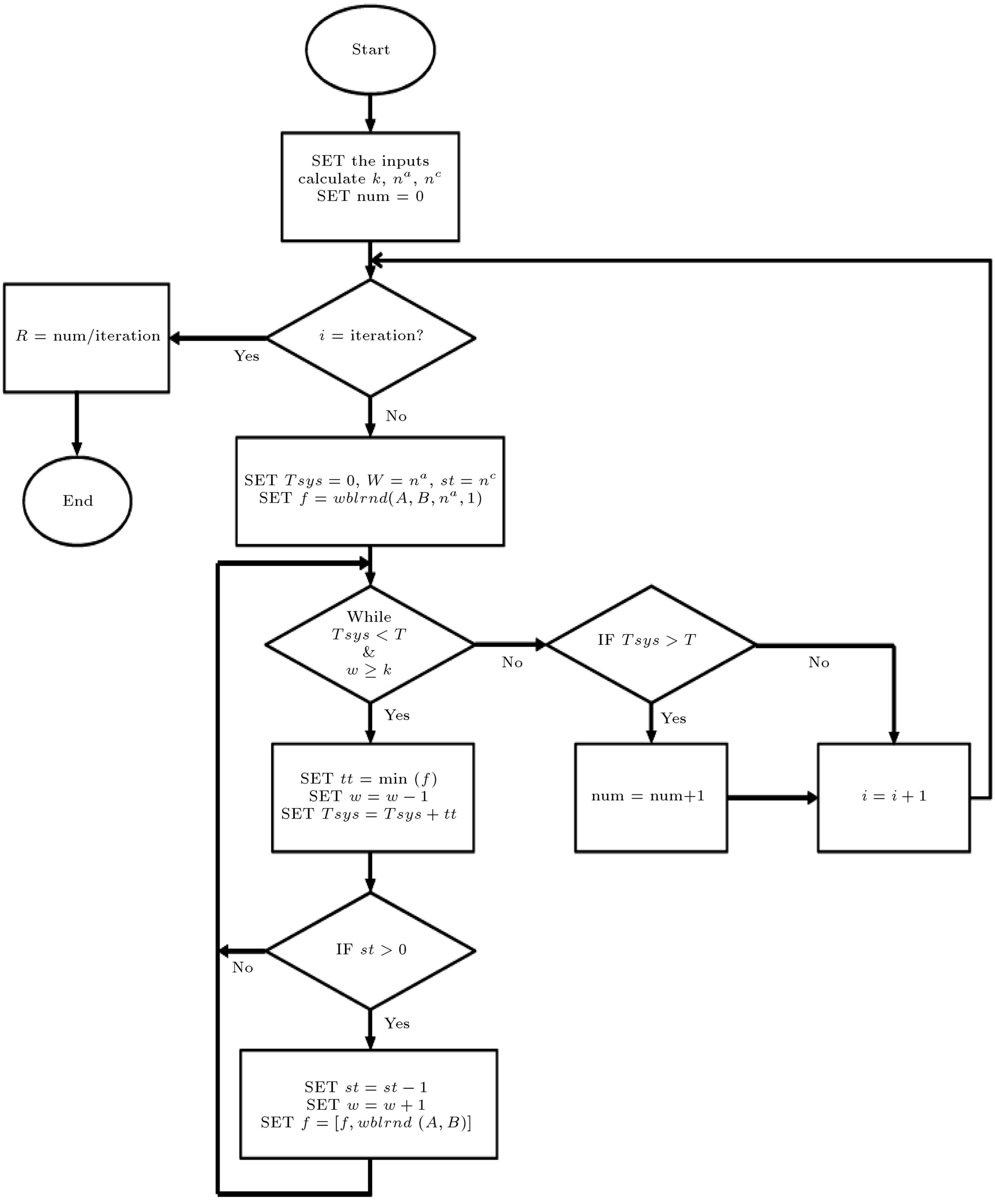

Figure 3. Flowchart of simulation experiments.

is estimated as the number of running loop processes divided by the number of iterations (num/iter).

3.3. Experiments of simulation

To estimate the objective function of the mathematical model presented in Section 2, it is required to conduct several experiments. For this purpose, a Box-Behnken design is performed in order to create different scenarios to estimate the reliability function in MINITAB 16 software. Here, six factors are considered in each scenario. The factor levels considered in the simulation experiments are demonstrated in Table 2, most of which have been adopted from Pourkarim Guilani et al. [29]. In addition, six central points were added in order to test a possible curvature involved in the response function. Moreover, the number of

Table 2. The lower and upper bounds of input parameters for numerical examples.

	Lower bound	Upper bound
n	1	7
A	100	200
B	0.8	1.25
k'	0	1
$n^{a'}$	0.8	1.25
t	100	200

replications is 30 for each design. The values of factors in each experiment and their results are shown in Table 3. Moreover, the estimated regression coefficients for reliability function of a subsystem are provided in Table 4. In all designs, $R-sq=91.46\%$, showing that the results are reliable.

According to the results in Table 3, the subsystem reliability function is estimated by implementing a non-linear regression in MINITAB 16 software as Eq. (16):

$$\begin{aligned}R =&1.79229-0.09272\times n-0.17104\times n^{a'}\\&-0.48286\times k'-0.01287\times A-1.04362\\&\times B+0.00468\times t+0.00695\times n\times n+0.16938\\&\times n^{a'}\times n^{a'}+0.42872\times k'\times k'+0.00005\\&\times A\times A+0.69108\times B\times B-0.00002\\&\times t\times t-0.00002\times n\times n^{a'}-0.16599\\&\times n\times k'+0.00015\times n\times A+0.08017\\&\times n\times B+0.00042\times n\times t+0.00010\times n^{a'}\\&\times k'-0.00020\times n^{a'}\times B+0.00002\times n^{a'}\\&\times t-0.06533\times k'\times B-0.00003\times k'\\&\times t+0.00040\times A\times B-0.00359\times B\times t.\end{aligned}\tag{16}$$

Therefore, Eq. (16) resulting from the simulation replications is used as the objective function (reliability function) in the mathematical model of the problem. The solution methodology will be presented in the next section.

4. Solution methodology

The presented model in Section 3 is an Integer Nonlinear Programming Model (INLP), and solving this problem via exact method is impossible [6]. However, it is possible to transform the problem to an Integer Linear Programming (ILP) based on [10]. Thus, the problem is linearized by taking the logarithm of the objective function to provide conditions to apply integer programming algorithms. Some new parameters and variables are defined in the following in order to change variables of the first RAP model, presented in Subsection 2.3.

- p — An index that takes value between k_i and $n_{\max,i}$;
- a_{ijp} — The cost of deployment p components of type j in subsystem i;
- b_{ijp} — The weight of deployment p components of type j in subsystem i;
- γ_{ijpq} — Logarithm of the reliability of p components of type j in subsystem i, in which q components are active and $p-q$ components are cold-standby;
- y_{ijpq} — A binary variable equals 1 if p components of type j used in subsystem i and zero otherwise, in which q components are active and $p-q$ components are cold-standby.

In addition, a_{ijp}, b_{ijp}, and γ_{ijpq} are expressed as functions of specified components and problem parameters. These values are obtained as follows:

$$a_{ijp}=c_{ij}\times p\qquad\forall i=1,...,s,\qquad\forall j=1,...,m_i\qquad\forall p=k_i,...,n_{\max,i},\tag{17}$$

$$b_{ijp}=w_{ij}\times p\qquad\forall i=1,...,s,\qquad\forall j=1,...,m_i\qquad\forall p=k_i,...,n_{\max,i},\tag{18}$$

$$\gamma_{ijpq}=Ln\left\{R^{sim}(p,q,p-q,z_{ij},t)\right\}\quad\forall i=1,...,s,\quad\forall j=1,...,m_i,\quad\forall p=k_i,...,n_{\max,i},\quad\forall q=k_i,...,p.\tag{19}$$

Finally, the new ILP is formed as follows:

$$\text{Max } R=\sum_{i=1}^{s}\sum_{j=1}^{m_i}\sum_{p=k_i}^{n_{\max,i}}\sum_{q=k_i}^{p}\gamma_{ijpq}y_{ijpq},$$

s.t.

$$\sum_{i=1}^{s}\sum_{j=1}^{m_i}\sum_{p=k_i}^{n_{\max,i}}\sum_{q=k_i}^{p}a_{ijp}y_{ijpq}\leq C,$$

$$\sum_{i=1}^{s}\sum_{j=1}^{m_i}\sum_{p=k_i}^{n_{\max,i}}\sum_{q=k_i}^{p}b_{ijp}y_{ijpq}\leq W,$$

$$\sum_{j=1}^{m_i}\sum_{p=k_i}^{n_{\max,i}}\sum_{q=k_i}^{p}y_{ijpq}=1\quad\forall i=1,...,s,$$

$$y_{ijpq}\in\{0,1\}.$$

Table 3. Experimental results obtained via simulation.

Exp.	n	$n^{a'}$	k'	A	B	t	R	Exp.	n	$n^{a'}$	k'	A	B	t	R
1	1	0	0.5	100	1.025	150	0.2217	**82**	7	0.5	0.5	200	0.8	150	0.7872
2	7	0	0.5	100	1.025	150	0.5459	**83**	1	0.5	0.5	100	1.25	150	0.1885
3	1	1	0.5	100	1.025	150	0.2199	**84**	7	0.5	0.5	100	1.25	150	0.6645
4	7	1	0.5	100	1.025	150	0.5442	**85**	1	0.5	0.5	200	1.25	150	0.4953
5	1	0	0.5	200	1.025	150	0.4736	**86**	7	0.5	0.5	200	1.25	150	0.962
6	7	0	0.5	200	1.025	150	0.9021	**87**	4	0	0.5	150	0.8	100	0.4222
7	1	1	0.5	200	1.025	150	0.4747	**88**	4	1	0.5	150	0.8	100	0.4219
8	7	1	0.5	200	1.025	150	0.903	**89**	4	0	0.5	150	1.25	100	0.6334
9	4	0	0	150	0.8	150	0.9636	**90**	4	1	0.5	150	1.25	100	0.6374
10	4	1	0	150	0.8	150	0.9633	**91**	4	0	0.5	150	0.8	200	0.1535
11	4	0	1	150	0.8	150	0.0182	**92**	4	1	0.5	150	0.8	200	0.1525
12	4	1	1	150	0.8	150	0.0187	**93**	4	0	0.5	150	1.25	200	0.2035
13	4	0	0	150	1.25	150	0.9926	**94**	4	1	0.5	150	1.25	200	0.2041
14	4	1	0	150	1.25	150	0.9927	**95**	1	0.5	0	150	1.025	100	0.516
15	4	0	1	150	1.25	150	0.0184	**96**	7	0.5	0	150	1.025	100	1
16	4	1	1	150	1.25	150	0.0179	**97**	1	0.5	1	150	1.025	100	0.5183
17	4	0.5	0	100	1.025	100	0.9831	**98**	7	0.5	1	150	1.025	100	0.01
18	4	0.5	1	100	1.025	100	0.0188	**99**	1	0.5	0	150	1.025	200	0.2607
19	4	0.5	0	200	1.025	100	0.9983	**100**	7	0.5	0	150	1.025	200	0.9997
20	4	0.5	1	200	1.025	100	0.1399	**101**	1	0.5	1	150	1.025	200	0.2625
21	4	0.5	0	100	1.025	200	0.8618	**102**	7	0.5	1	150	1.025	200	0.00006
22	4	0.5	1	100	1.025	200	0.00037	**103**	4	0.5	0.5	150	1.025	150	0.3128
23	4	0.5	0	200	1.025	200	0.9823	**104**	4	0.5	0.5	150	1.025	150	0.3125
24	4	0.5	1	200	1.025	200	0.0181	**105**	4	0.5	0.5	150	1.025	150	0.3143
25	1	0.5	0.5	100	0.8	150	0.2512	**106**	4	0.5	0.5	150	1.025	150	0.314
26	7	0.5	0.5	100	0.8	150	0.4224	**107**	4	0.5	0.5	150	1.025	150	0.3162
27	1	0.5	0.5	200	0.8	150	0.4524	**108**	4	0.5	0.5	150	1.025	150	0.3158
28	7	0.5	0.5	200	0.8	150	0.7885	**109**	1	0	0.5	100	1.025	150	0.219
29	1	0.5	0.5	100	1.25	150	0.1932	**110**	7	0	0.5	100	1.025	150	0.5462
30	7	0.5	0.5	100	1.25	150	0.6651	**111**	1	1	0.5	100	1.025	150	0.2214
31	1	0.5	0.5	200	1.25	150	0.4974	**112**	7	1	0.5	100	1.025	150	0.547
32	7	0.5	0.5	200	1.25	150	0.9609	**113**	1	0	0.5	200	1.025	150	0.4772
33	4	0	0.5	150	0.8	100	0.4242	**114**	7	0	0.5	200	1.025	150	0.9029
34	4	1	0.5	150	0.8	100	0.4199	**115**	1	1	0.5	200	1.025	150	0.4728
35	4	0	0.5	150	1.25	100	0.635	**116**	7	1	0.5	200	1.025	150	0.9018
36	4	1	0.5	150	1.25	100	0.6343	**117**	4	0	0	150	0.8	150	0.9626
37	4	0	0.5	150	0.8	200	0.1502	**118**	4	1	0	150	0.8	150	0.9636
38	4	1	0.5	150	0.8	200	0.1521	**119**	4	0	1	150	0.8	150	0.0177
39	4	0	0.5	150	1.25	200	0.2025	**120**	4	1	1	150	0.8	150	0.0182
40	4	1	0.5	150	1.25	200	0.2044	**121**	4	0	0	150	1.25	150	0.9919
41	1	0.5	0	150	1.025	100	0.5164	**122**	4	1	0	150	1.25	150	0.9922
42	7	0.5	0	150	1.025	100	1	**123**	4	0	1	150	1.25	150	0.0179
43	1	0.5	1	150	1.025	100	0.5155	**124**	4	1	1	150	1.25	150	0.0184
44	7	0.5	1	150	1.025	100	0.0101	**125**	4	0.5	0	100	1.025	100	0.9825
45	1	0.5	0	150	1.025	200	0.2598	**126**	4	0.5	1	100	1.025	100	0.0189

Table 3. Experimental results obtained via simulation (continued).

Exp.	n	$n^{a'}$	k'	A	B	t	R	Exp.	n	$n^{a'}$	k'	A	B	t	R
46	7	0.5	0	150	1.025	200	0.9995	**127**	4	0.5	0	200	1.025	100	0.9986
47	1	0.5	1	150	1.025	200	0.2608	**128**	4	0.5	1	200	1.025	100	0.1397
48	7	0.5	1	150	1.025	200	0.00011	**129**	4	0.5	0	100	1.025	200	0.8609
49	4	0.5	0.5	150	1.025	150	0.3119	**130**	4	0.5	1	100	1.025	200	0.00031
50	4	0.5	0.5	150	1.025	150	0.3138	**131**	4	0.5	0	200	1.025	200	0.9822
51	4	0.5	0.5	150	1.025	150	0.311	**132**	4	0.5	1	200	1.025	200	0.018
52	4	0.5	0.5	150	1.025	150	0.3133	**133**	1	0.5	0.5	100	0.8	150	0.2506
53	4	0.5	0.5	150	1.025	150	0.3151	**134**	7	0.5	0.5	100	0.8	150	0.4206
54	4	0.5	0.5	150	1.025	150	0.3112	**135**	1	0.5	0.5	200	0.8	150	0.4532
55	1	0	0.5	100	1.025	150	0.2183	**136**	7	0.5	0.5	200	0.8	150	0.7877
56	7	0	0.5	100	1.025	150	0.5473	**137**	1	0.5	0.5	100	1.25	150	0.1902
57	1	1	0.5	100	1.025	150	0.2188	**138**	7	0.5	0.5	100	1.25	150	0.665
58	7	1	0.5	100	1.025	150	0.5444	**139**	1	0.5	0.5	200	1.25	150	0.498
59	1	0	0.5	200	1.025	150	0.4752	**140**	7	0.5	0.5	200	1.25	150	0.9606
60	7	0	0.5	200	1.025	150	0.9023	**141**	4	0	0.5	150	0.8	100	0.4231
61	1	1	0.5	200	1.025	150	0.4739	**142**	4	1	0.5	150	0.8	100	0.4238
62	7	1	0.5	200	1.025	150	0.9019	**143**	4	0	0.5	150	1.25	100	0.637
63	4	0	0	150	0.8	150	0.9629	**144**	4	1	0.5	150	1.25	100	0.6333
64	4	1	0	150	0.8	150	0.9634	**145**	4	0	0.5	150	0.8	200	0.1497
65	4	0	1	150	0.8	150	0.0181	**146**	4	1	0.5	150	0.8	200	0.1528
66	4	1	1	150	0.8	150	0.019	**147**	4	0	0.5	150	1.25	200	0.2021
67	4	0	0	150	1.25	150	0.9926	**148**	4	1	0.5	150	1.25	200	0.2012
68	4	1	0	150	1.25	150	0.9927	**149**	1	0.5	0	150	1.025	100	0.5151
69	4	0	1	150	1.25	150	0.0179	**150**	7	0.5	0	150	1.025	100	1
70	4	1	1	150	1.25	150	0.0183	**151**	1	0.5	1	150	1.025	100	0.5191
71	4	0.5	0	100	1.025	100	0.9827	**152**	7	0.5	1	150	1.025	100	0.0098
72	4	0.5	1	100	1.025	100	0.0192	**153**	1	0.5	0	150	1.025	200	0.2601
73	4	0.5	0	200	1.025	100	0.9986	**154**	7	0.5	0	150	1.025	200	0.9996
74	4	0.5	1	200	1.025	100	0.1384	**155**	1	0.5	1	150	1.025	200	0.259
75	4	0.5	0	100	1.025	200	0.8596	**156**	7	0.5	1	150	1.025	200	0.00005
76	4	0.5	1	100	1.025	200	0.00027	**157**	4	0.5	0.5	150	1.025	150	0.311
77	4	0.5	0	200	1.025	200	0.9832	**158**	4	0.5	0.5	150	1.025	150	0.3131
78	4	0.5	1	200	1.025	200	0.0178	**159**	4	0.5	0.5	150	1.025	150	0.3182
79	1	0.5	0.5	100	0.8	150	0.253	**160**	4	0.5	0.5	150	1.025	150	0.3135
80	7	0.5	0.5	100	0.8	150	0.4211	**161**	4	0.5	0.5	150	1.025	150	0.3142
81	1	0.5	0.5	200	0.8	150	0.4503	**162**	4	0.5	0.5	150	1.025	150	0.3115

Since the second model is linear and is in the form of a standard zero-one integer programming, there are several algorithms to solve it exactly. In this paper, a Branch and Bound (B&B), as an exact algorithm, is used to solve the proposed model based on [10].

Of course, it is worth mentioning that in large-scale problems where integer programming approaches are inefficient, heuristic and meta-heuristic approaches are the best ways to achieve the near-optimum solutions.

5. Numerical example

In this section, a numerical example is presented to

Table 4. Estimated regression coefficients for R.

Term	Coef.	SE Coef.	T	P
Constant	1.79229	0.74675	2.4	0.018
n	-0.09272	0.048069	-1.929	0.056
$n^{a'}$	-0.17104	0.261405	-0.654	0.514
k'	-0.48286	0.282734	-1.708	0.09
A	-0.01287	0.00347	-3.71	0
B	-1.04362	0.916778	-1.138	0.257
t	0.00468	0.003481	1.344	0.181
$n \times n$	0.00695	0.002177	3.193	0.002
$n^{a'} \times n^{a'}$	0.16938	0.078386	2.161	0.032
$k' \times k'$	0.42872	0.078386	5.469	0
$A \times A$	0.00005	0.000008	5.981	0
$A \times B$	0.69108	0.387093	1.785	0.076
$t \times t$	-0.00002	0.000008	-1.927	0.056
$n \times n^{a'}$	-0.00002	0.014814	-0.002	0.999
$n \times k'$	-0.16599	0.014814	-11.205	0
$n \times A$	0.00015	0.000105	1.431	0.155
$n \times B$	0.08017	0.032919	2.435	0.016
$n \times t$	0.00042	0.000148	2.825	0.005
$n^{a'} \times k'$	0.0001	0.088882	0.001	0.999
$n^{a'} \times A$	0	0.000889	-0.005	0.996
$n^{a'} \times B$	-0.0002	0.139664	-0.001	0.999
$n^{a'} \times t$	0.00002	0.000889	0.019	0.985
$K' \times A$	0	0.000889	0.003	0.998
$K' \times B$	-0.06533	0.197515	-0.331	0.741
$K' \times t$	-0.00003	0.000628	-0.051	0.959
$A \times B$	0.0004	0.001975	0.201	0.841
$A \times t$	0	0.000009	0.019	0.985
$B \times t$	-0.00359	0.001975	-1.816	0.072

$S = 0.108858$, $PRESS = 2.40531$, $R - Sq = 91.46\%$,

$R - Sq(\text{pred}) = 87.06\%$, $R - Sq(\text{adj}) = 89.73\%$.

demonstrate the verification of the proposed methodology. A system with 14 subsystems is considered, in which the subsystems are connected serially to each other. The input parameters of the model are taken from [29]. The number of available types of components and the minimum requirement amount of components, which have to be working in each subsystem, are provided in Table 5. Moreover, the two parameters of the Weibull distribution of the available components in each subsystem are demonstrated in Tables 6 and 7. The cost and weight of each component are presented in Tables 8 and 9, respectively. In addition, the switch reliability is 1.

In addition, the maximum cost for the system is 300, the maximum system weight is 400, and the upper bound on the assigned number of components in each subsystem ($n_{\max i}$) is 5. In this system, the mixed strategy for each redundancy is used. The number of decision variables for the second model of this system is obtained from Eq. (20).

$$\Delta = \prod_{i=1}^{s} m_i (n_{\max,i} - k_i + 1)^2. \qquad (20)$$

Therefore, the total exact number of feasible and infeasible solutions is 2^{Δ}.

The problem is coded in MATLAB 10, and a Pentium IV computer with a core 2 CPU 2.4 GHz and 3 GB RAM under Windows 7 operating system is used in order to run the program. After solving the problem using a B&B algorithm, the results are summarized in Table 10.

In this table, the first column presents the type of selective component; the second column indicates the number of selective component in each subsystem. The third and fourth columns show the number of active and cold-standby components in each subsystem, respectively. Moreover, according to Table 10, the system reliability is 0.4779. In addition, sensitivity analysis has been carried out in order to investigate the effect of the number of components in each subsystem ($n_{\max i}$) on reliability function. For this purpose, 10 test problems are used to study the sensitivity analysis while keeping the other parameters stable, and the results are listed in Table 11 and Figure 4. As results show, any increase in $n_{\max,i}$ leads to an increase in system reliability, which shows a rational fact.

6. Conclusion and future researches

In many real-world systems, the components do not have a constant failure rate. Indeed, the components' failure rates change occasionally. Therefore, systems with time-dependent failure rates are considered more

Table 5. Number of available components for each subsystem.

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14
m	4	3	4	3	3	4	3	3	4	3	3	4	3	4
k	2	2	2	2	2	2	2	2	2	2	2	2	2	2

Table 6. The scale parameter (A) of the Weibull distribution for each type of components in each subsystem.

		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Components Type	**1**	114	143	196	183	144	149	174	162	169	143	103	107	148	193
	2	176	174	169	107	119	198	110	121	175	124	148	188	148	155
	3	155	102	182	126	171	140	200	142	197	186	106	175	181	191
	4	159	-	163	-	-	176	-	-	166	-	-	146	-	129

Table 7. The shape parameter (B) of the Weibull distribution for each type of components in each subsystem.

		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Components Type	**1**	1.07	1.04	0.85	0.95	1.17	0.83	0.89	1.05	1.13	1.12	1.24	1.16	1.04	1.22
	2	0.94	1.21	0.83	1.14	0.89	1.08	1.07	1.22	1.01	1.17	1	1.17	1.05	1.22
	3	1.23	1.16	1.13	1.23	1.22	1.17	1	0.84	1.17	0.94	0.99	1.04	1.22	1.16
	4	1.17	-	1.16	-	-	0.96	-	-	1.09	-	-	1.21	-	1.22

Table 8. The cost of each type of components in each subsystem.

		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Components Type	**1**	1	2	2	3	2	3	4	3	2	4	3	2	2	4
	2	1	1	3	4	2	3	4	5	3	4	4	3	3	4
	3	2	1	1	5	3	2	5	6	4	5	5	4	2	5
	4	2	-	4	-	-	2	-	-	3	-	-	5	-	6

Table 9. The weight of each type of components in each subsystem.

		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Components Type	**1**	3	8	7	5	4	5	7	4	8	6	5	4	5	6
	2	4	10	5	6	3	4	8	7	9	5	6	5	5	7
	3	2	9	6	4	5	5	9	6	7	6	6	6	6	6
	4	5	-	4	-	-	4	-	-	8	-	-	7	-	9

Table 10. The results of the numerical example.

i	z_i	n_i	n_i^a	n_i^c
1	3	5	5	0
2	2	5	2	3
3	3	5	2	3
4	1	5	2	3
5	3	5	2	3
6	2	5	2	3
7	3	5	2	3
8	1	5	5	0
9	3	5	2	3
10	3	5	2	3
11	1	5	5	0
12	2	5	2	3
13	3	5	2	3
14	1	5	2	3

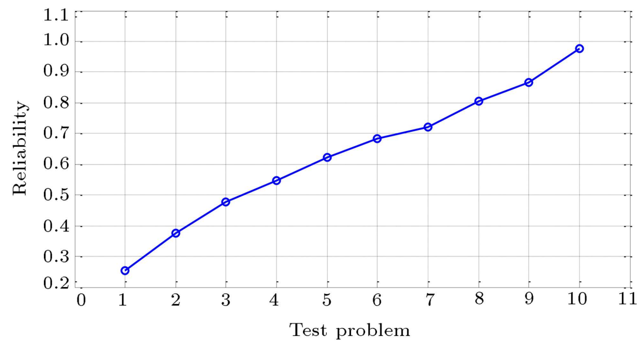

Figure 4. A graphical comparison to investigate effect of $n_{\max,i}$ on system reliability via 10 test problems.

realistic. The Weibull probability distribution is one of the proper distributions to model the time-dependent failure rates. Due to its hazard function, it has appropriate flexibility to model time-dependent failure rates and can be used in many real-world systems. This suitable property of the Weibull distribution can overcome the limitation of previous studies in which the component failure rates were considered as constant based on exponential distribution. This research study investigated a RAP under k-out-of-n subsystems with time-dependent failure rates based on this distribution. Moreover, it was assumed that the redundancy strategy was mixed. Since it is hard to obtain the reliability

Table 11. The results of test problems to investigate effect of $n_{\max,i}$ on system reliability .

		i														
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	***R***
Test problems	**1**	3	5	5	5	5	5	5	5	5	5	5	5	5	5	**0.254**
	2	4	5	5	5	5	5	5	5	5	5	5	5	5	5	**0.3764**
	3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	**0.4779**
	4	6	5	5	5	5	5	5	5	5	5	5	5	5	5	**0.5468**
	5	7	5	5	5	5	5	5	5	5	5	5	5	5	5	**0.6214**
	6	7	6	5	5	5	5	5	5	5	5	5	5	5	5	**0.6822**
	7	7	7	5	5	5	5	5	5	5	5	5	5	5	5	**0.7222**
	8	7	7	6	5	5	5	5	5	5	5	5	5	5	5	**0.8042**
	9	7	7	7	5	5	5	5	5	5	5	5	5	5	5	**0.8654**
	10	7	7	7	6	5	5	5	5	5	5	5	5	5	5	**0.9764**

function of such systems explicitly, a simulation-based optimization approach was developed to estimate the system reliability function. Then, due to the Np-hardness of the proposed RAP, the original INLP was replaced by an ILP problem. Finally, a B&B algorithm was developed to solve the ILP model exactly. In addition, a numerical illustration was presented to demonstrate the application of the proposed methodology. The presented idea in this paper can help managers and owners of electronic and mechanical systems make better decisions on their systems according to real-world issues and not merely based on academic assumptions. For future researches in this area, the following lines are recommended:

- Considering other probability distributions such as normal, lognormal, Gumbel, log-logistic, etc. to model the systems component life;
- Considering systems with failure rates which are dependent on working components;
- Considering repairable components;
- Considering failure rates of components as fuzzy variables;
- Studying reliability evaluation of systems with time-dependent failure rate via mathematical and statistical relations.

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