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Static analysis of interaction between twin-tunnels using Discrete Element Method (DEM)

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Static analysis.

Abstract. The development of transportation in large cities requires the construction of twin-tunnels or the construction of new tunnels close to the existing ones. Since both the relative position of tunnels and the construction procedure affect the soil movement, this paper presents analysis of this issue with a particular interest for the optimization of both the relative position of the twin-tunnels and the construction procedure. Since the soil is composed of discrete particles with different sizes, modeling using finite element methods based on the mechanics of a continuous medium is not completely consistent with reality (especially, modeling of confining effect with depth). Therefore, in this study, discrete element method is used to model the discontinuum nature of soils. For these concerns, using software PFC^{2D} based on Discrete Element Method (DEM), the static analysis of circular twin-tunnels has been performed, and influence of the two factors on the soil settlement resulting from the tunnel construction has been investigated. Analyses were conducted for three configurations of the twin-tunnels: aligned-horizontally, vertically, and inclined. The results are compared with the FEM results. The comparison shows influence of modeling the discontinuous nature of coarse-grain alluvial soils with respect to continuous media modeling.

1. Introduction

The development of large cities requires utilizing underground area for the construction of transportation infrastructures and facilities. In some cities, the underground and geotechnical conditions impose the construction of new tunnels close to existing ones. In other cases, the solution of twin-tunnels presents major advantages, such as the reduction of the both tunnels diameters and the soil movement resulting from the tunnel construction. Both numerical modeling and in-situ observations were used to analyze the interaction between twin-tunnels [1]. Results show that in some configurations, the interaction could largely affect the soil settlement and the design of twin-tunnels requires numerical analyses associated to monitoring during the tunnel construction [1-16]. This paper presents analysis of the interaction between twin-tunnels with a particular interest for the optimization of both the relative position of the twin-tunnels and the construction procedure using the discrete element code, PFC^{2D}, and the results will be compared with the FEM results of $PLAXIS^{2D}$. The Discrete Element Method is selected for modeling of discontinuous nature of coarse-grain alluvial soils. Parametric study analyses were conducted for three configurations of twin-tunnels: aligned-horizontally, vertically, and inclined, to investigate their influence on the soil settlement resulting from the tunnel construction.

In Section 2, the computational flow chart of

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the Discrete Element Method (DEM) is described. In Section 3, numerical modeling parameters are obtained by the calibration of triaxial tests. Numerical results are presented in Section 4. Conclusions are given in the last section.

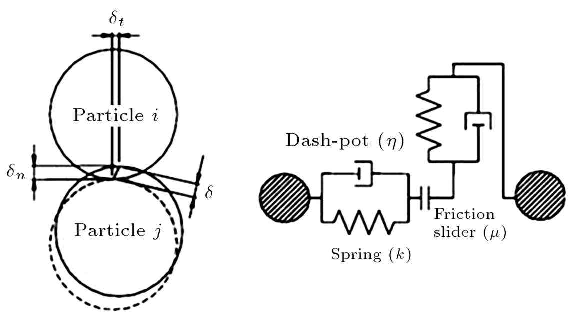
2. Discrete element method

The DEM was introduced by Cundall (1971) for the analysis of rock-mechanics problems [17], and then applied to soils by Cundall and Strack (1979) [18]. PFC^{2D} is classified as a discrete element code since it allows finite displacements and rotations of discrete bodies, including complete detachment, and recognizes new contacts automatically as the calculation progresses. The code can be viewed as a simplified implementation of the DEM because of the restriction to rigid circular particles [19].

The calculations performed in the DEM alternate between the application of Newton's second law to the particles and a force-displacement law at the contacts. Newton's second law is used to determine the motion of each particle arising from the contact and body forces acting upon it, while the force-displacement law is used to update the contact forces arising from the relative motion at each contact [18]. The presence of walls in the code requires only that the force-displacement law accounts for ball-wall contacts. Newton's second law is not applied to walls, since the wall motion is specified by the user [19].

When two particles collide, they actually deform. The overlap displacement δ is assumed as deformation in the DEM as shown in Figure 1(a). Then, the DEM contact is composed of a spring, a dash-pot, and a friction slider as shown in Figure 1(b) [20].

The calculation cycle in the code is an explicit time stepping algorithm that consists of the repeated application of the law of motion to each particle, a force-displacement law to each contact, and a constant updating of wall positions. Contacts, which may exist between two balls or between a ball and a wall, are formed and broken automatically during the course of a simulation. The calculation cycle is illustrated in Figure 2 [19].



(a) Displacement of two particles in contact (b) Spring-damper system to contact force

Figure 1. Contact force model [20].

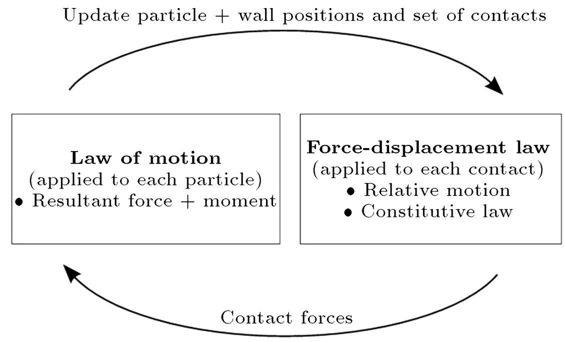


Figure 2. Calculation cycle in PFC^{2D} [19].

3. Numerical modeling

PFC^{2D} is a distinct element computer program designed to simulate the mechanical behavior of bonded or unbounded granular materials. The soil material is modeled as a collection of circular particles that can interact through normal and shear springs. Under the applied load, the bonds between particles can break and a small "crack" can form [19].

Although the code can simulate a particulate media, any circular element in this program does not necessarily model a particle in the real material as the two-dimensional nature of the program limits particles to disks or cylinders. Hence, the code can be considered as an attempt to mimic the basic mechanical features of the actual material. Therefore, it is not possible to use the geotechnical parameters directly in PFC^{2D}, and it is necessary to obtain the numerical modeling parameters using calibration of real triaxial tests. Followed in Sections 3.1 through 3.5, the most important specific aspects which have been considered in modeling using the code are given.

3.1. Particle generation

The particle radii were chosen to have a relatively uniform distribution in the sample. Particle in the code does not need to correspond to a real grain in the actual material [21]. In PFC^{2D}, particles are created at their final radii in sufficient numbers to achieve the desired porosity for the production of the real in-situ stresses. In this paper, the minimum radius of the particle assembly is $R_{\min} = 5$ cm and $R_{\max}/R_{\min} = 3$ [21]. They are placed at random positions in the given area, which leads to some large overlaps and correspondingly large forces. Then, the resulting large initial velocities may be sufficient to allow some particles to escape through the confining walls. To prevent this, the kinetic energy is reduced to zero several times during the first few cycles; then, convergence to equilibrium proceeds normally [19].

3.2. Boundary and initial conditions (initial stresses)

The soil layer is underlined at the depth of $H = 60$ m. The lateral extension of the soil mass is equal to 100 m. The extension ensures the absence of boundary effect on the numerical modeling of the tunnel construction. The number of particles in the domain is approximately 70000.

Normally, a particle assembly is created and compacted within a set of confining walls. These walls can continue to act as boundary constraints. The initial conditions in a granular assembly are partly inherited from the generation and compaction phases. The concept of stress in a discontinuous medium is different from that in a continuous. There is no meaning to the term "stress at a point", because forces may fluctuate wildly from point to point. An estimate of stress is only possible over a finite volume of space. A numerical servo-mechanism is applied iteratively to arrive at the radius expansion necessary to achieve the required mean stress [19]. There are various methods for achieving a desired initial stress state. Parameters such as density, porosity, and stiffness coefficients affect the initial stress, so if it turns out changes in parameters, it bring about changes in stress [19]. To obtain desired stress, the model is divided into six layers with 10 meters thicknesses (Figure 3), which in each layer, porosity, stiffness coefficients, and friction coefficient are fixed, and density and bond coefficients are different. Each layer is calibrated separately.

3.3. Determination of micro-parameters (calibration)

The overall constitutive behavior of a material is simulated in PFC^{2D} by associating a constitutive model with each contact. The constitutive model acting at a particular contact consists of three parts: A stiffness model, a slip model, and a bonding model. The stiffness model provides an elastic relation between the contact force and relative displacement. The slip model enforces a relation between shear and normal contact forces such that the two contacting balls may slip relative to one another. The bonding model serves to limit the total normal and shear forces that the contact can carry by enforcing bond-strength limits. The two basic bonding models supported in the code are: A contact-bond model and a parallel-bond model. Both bonds can be envisioned as a kind of glue joining the two particles. The contact-bond glue is of a vanishingly small size that acts only at the contact point, while the parallel-bond glue is of a finite size that acts over either a circular or rectangular cross section lying between the particles. The contact-bond can only transmit a force, while the parallel-bond can transmit both a force and a moment. The contact-bond and parallel-bond can be viewed as cohesion for soils and rocks, respectively. Hence, in this research, the contact-bond model is used. Constitutive behavior for contact occurring at a point is shown in Figure 4 regarding contact-bond model [19].

Keeping constant values of stiffness and friction coefficients, PFC^{2D}'s input parameters including bond coefficients, vary until the behavior of the numerical sample matches that of the physical sample. In PFC^{2D}, a biaxial test by confining a rectangular sample (comprised of a compacted particle assembly) within four walls is simulated. The top and bottom walls simulate loading platens, and the left and right walls simulate the confinement experienced by the sample sides. The sample is loaded in a strain-controlled mode by specifying the velocities of the top and bottom walls. The stresses and strains experienced by the sample are determined in a macro-mode by summing the forces acting upon, and relative distance between the appropriate walls. Material response is evaluated by tracking the various stress and strain quantities. Axial deviatoric stress versus axial strain for biaxial test on bonded granular material was drawn for each layer, and then Mohr's circle was drawn to reach failure envelope of laboratory. Table 1 summarizes the properties of the alluvial soil of Tehran city used in this study [22]. The determined micro-parameters (getting from calibration) are listed in Table 2 based on strength and stiffness criteria.

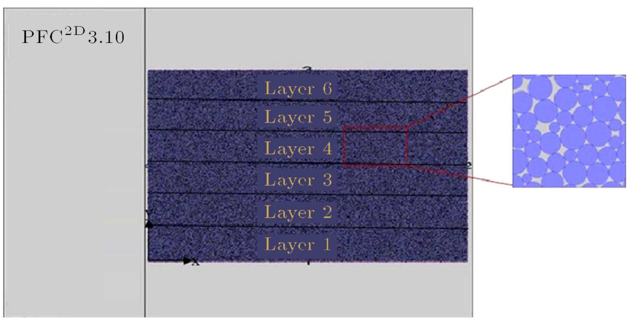


Figure 3. Configuration of particles for initial conditions (in-situ stresses).

3.4. Verification

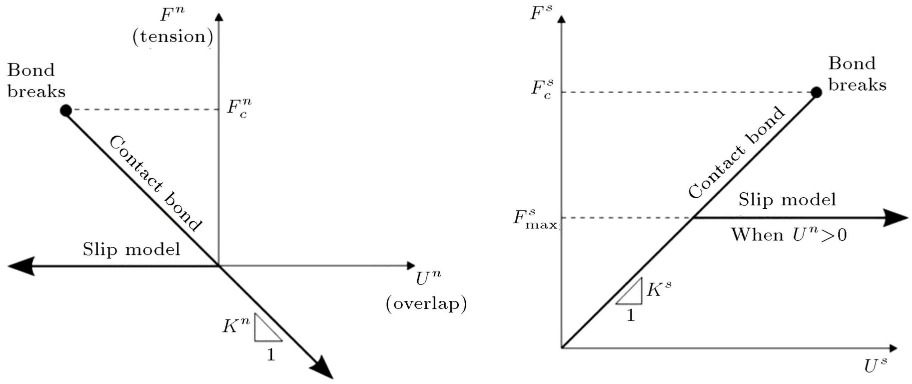
In order to validate the accuracy of calibration method for determination of micro-parameters, the verification based on field case study [23] is presented in the Appendix.

3.5. Tunnel modeling

The soil behavior is described using an elastic perfectly plastic constitutive model based on the non-associated Mohr-Coulomb criterion. The water table is considered

Table 1. Mechanical parameters of Tehran alluvium [22].

γ_{unsat} (kN/m^3)	γ_{sat} (kN/m^3)	E (kg/cm^2)	v	C (kg/cm^2)	φ (degree)
19	20	750	0.35	0.3	34



(a) Normal component of contact force (b) Shear component of contact force

Figure 4. Constitutive behavior for the contact occurring at a point regarding contact-bond model [19].

Table 2. Determined micro-parameters in PFC^{2D} (getting from calibration).

	k_n (N/m)	k_s (N/m)	n-bond (N/m)	s-bond (N/m)	Friction	Porosity	ρ (kg/m^3)
Layer 1	2e8	1e8	0.45e6	0.45e6	0.1	0.14	2650
Layer 2	2e8	1e8	0.42e6	0.39e6	0.1	0.14	2500
Layer 3	2e8	1e8	0.28e6	0.27e6	0.1	0.14	2450
Layer 4	2e8	1e8	0.24e6	0.22e6	0.1	0.14	2400
Layer 5	2e8	1e8	0.14e6	0.14e6	0.1	0.15	2400
Layer 6	2e8	1e8	0.04e6	0.04e6	0.1	0.15	2300

in the bottom of the model at depth of 60 m, and the influence of surface structures in comparison to tunnel overburden is negligible. It is defined a range identifying the space to be excavated for the tunnel, and another identifying the space which will be occupied by the lining. The particles of the domain will be used for modeling of lining by changing particles properties in the lining range. The thickness of the lining is equal to 0.5 m. Figure 5 shows the model used for the analysis of horizontally aligned tunnels with a spacing ratio of

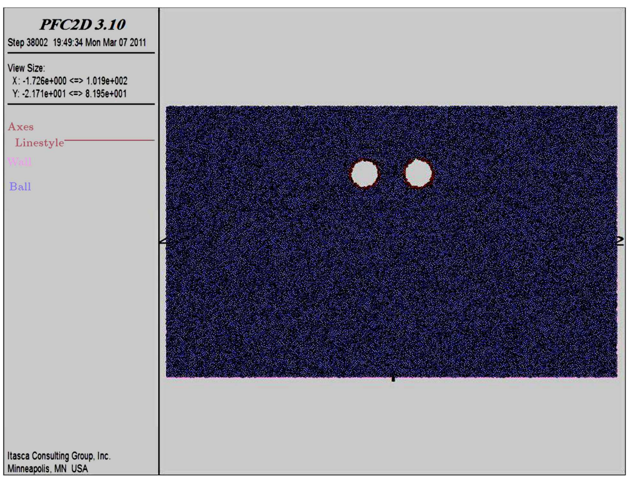


Figure 5. Model used in the analysis of twin-tunnels with horizontal alignment.

$dx/D = 2$ in PFC^{2D} (D and dx denote the tunnel diameter and the distance between tunnel axes, respectively. In this paper, the selected tunnel diameter equals to 6 m.) The determined micro-parameters of lining based on the guidelines are listed in Table 3.

4. Numerical results

Analyses are conducted for three configurations of the twin-tunnels: aligned-horizontally, vertically, and inclined (Figure 6, Table 4). The DEM results will be compared by the FEM results. The DEM and FEM models have the same dimensions. The soil behavior

Table 3. Lining parameters in PFC^{2D}.

k_n (N/m)	k_s (N/m)	n-bond (N/m)	s-bond (N/m)	Friction	ρ (kg/m^3)
2e10	2e10	1e15	1e15	0.1	2400

Table 4. Configurations of interaction analysis between twin-tunnels.

Configuration	dx/D	dy/D
Horizontal alignment	1.5, 2, 2.5, 3	0
Vertical alignment	0	2
Inclined alignment	2, 2.5	2
$D = 6$ m		

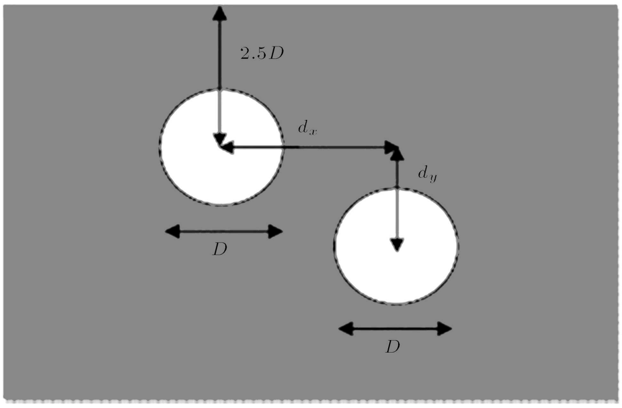


Figure 6. Configurations considered in the analysis of the interaction between twin-tunnels.

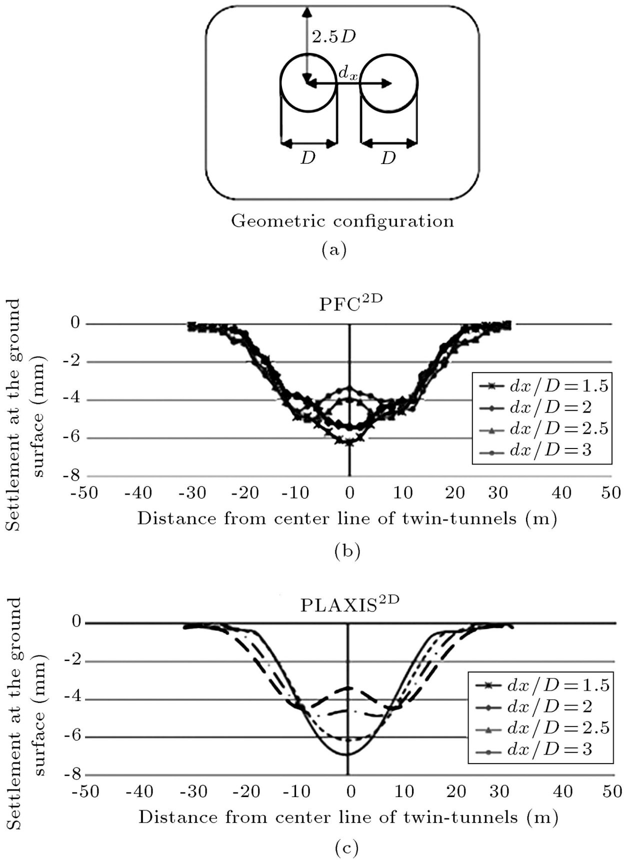


Figure 7. Tunnels with horizontal alignment: Surface settlement induced by the construction of the twin-tunnels.

is described using Mohr-Coulomb criterion and the behavior of lining is considered elastic in FEM. In the FEM model, using an automatic mesh generation, 15-node triangular elements are used.

4.1. Twin-tunnels with horizontal alignment

Analyses were conducted for four values of the tunnel spacing ratio dx/D (1.5, 2, 2.5 and 3) as shown in Figure 7(a). Figure 7(b) and (c) show the surface

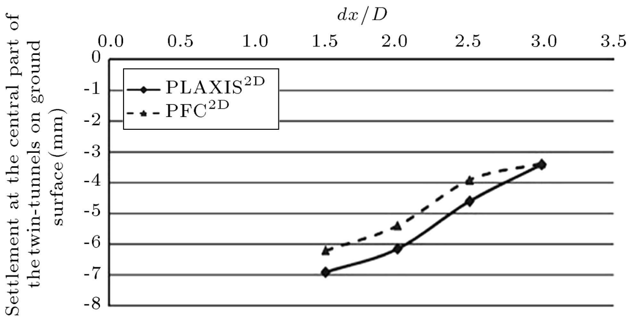


Figure 8. Variation of surface settlement at the central part of the twin-tunnels (horizontal alignment) with respect to distance between tunnels.

settlement pattern at the end of the construction of the second tunnel obtained from PFC^{2D} and PLAXIS^{2D}. It shows that both the settlement pattern and amplitude depend on the distance between tunnels. The maximum soil settlement is observed for the configuration of close tunnels ($dx/D = 1.5$) for both the DEM and FEM, but the results obtained from DEM analysis are lower than FEM analysis (about 10%). The maximum surface settlement for $dx/D = 1.5$ is induced between the two tunnels.

Increasing the distance between tunnels induces decreasing the settlement in the central part of the twin-tunnels and leads to changing of settlement pattern (it happens in $dx/D = 2 \sim 2.5$). Increasing dx causes changing the location of maximum surface settlement from central part of twin-tunnels to the center line of each tunnel. However, the disturbed surface area will be increased.

Figure 8 shows the rate of decreasing the settlement at the central part of the twin-tunnels on the ground surface versus increasing the distance between tunnels. It is seen that the rate of variation is approximately linear.

4.2. Twin-tunnels with vertical alignment

Figure 9(a) shows the tunnel configuration considered in this section. Two types of analyses were carried out with PFC^{2D} and PLAXIS^{2D}. In the first one, the upper tunnel is constructed at first (Case A), while in the second analysis, the lower tunnel is constructed first (Case B).

Figure 9(b) illustrates the influence of the construction procedure on the soil settlement. It shows that the construction of the upper tunnel at first leads to higher maximum settlement compared to that obtained by the construction of the lower tunnel at first (about 10%). However, it is limited to a central area between twin-tunnels, and for other locations it is reversely. It is seen again that results obtained from DEM analysis are lower than FEM analysis (about 20%). Based on the DEM results, in Case A, when the second tunnel is excavated, the maximum surface

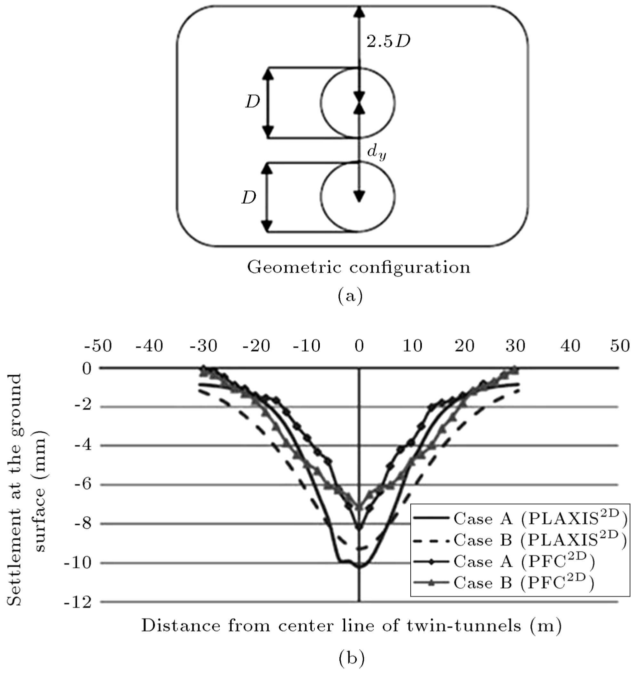


Figure 9. Tunnels with vertical alignment: Surface settlement induced by the construction of the twin-tunnels.

settlement is increased 160 percent while in Case B, it is increased 250 percent.

Comparison of surface settlements for tunnels with horizontal and vertical alignment shows the settlements for vertical twin-tunnels are higher and they are more critical.

4.3. Twin-tunnels with inclined alignment

Two configurations were analyzed (Figure 10(a)). The vertical distance between the tunnel axes is equal to $d_y/D = 2$. In the first configuration, the horizontal distance between the tunnel axes is equal to $d_x/D = 2$ ($\alpha = 45$); in the second configuration $d_x/D = 2.5$ ($\alpha = 39$). Figure 10(b) and (c) show the influence of both the tunnel configuration and construction procedure on the soil settlement. It can be observed that the construction of the upper tunnel at first (Case A) leads to higher maximum settlement than that induced when the lower tunnel is first constructed (Case B). This result is similar to that obtained for vertical aligned tunnels. Generally, the maximum displacement transmitted to the side of the first tunnel is constructed. Based on the DEM results, when the second tunnel is excavated, the maximum surface settlement is increased 130 percent for Case A when $\alpha = 45$, 110 percent for Case A when $\alpha = 39$, 225 percent for Case B when $\alpha = 45$, and 200 percent for Case B when $\alpha = 39$.

Figure 11 shows maximum settlement at the soil surface with respect to variation of α and construction process. It is seen that for $\alpha > 45°$, inclined tunnels behave similar to vertical tunnels.

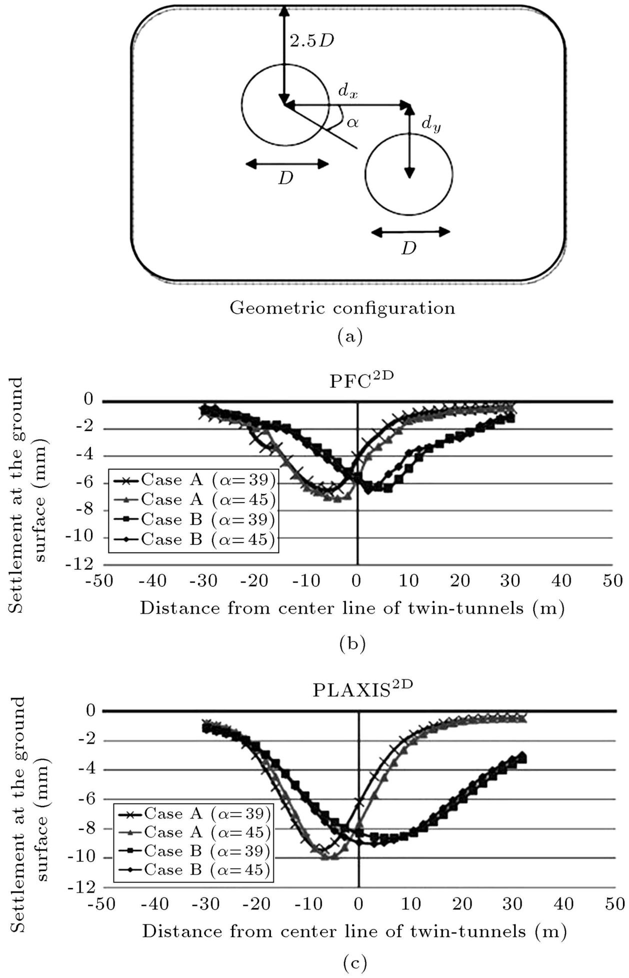


Figure 10. Tunnels with inclined alignment: Surface settlement induced by the construction of the twin-tunnels.

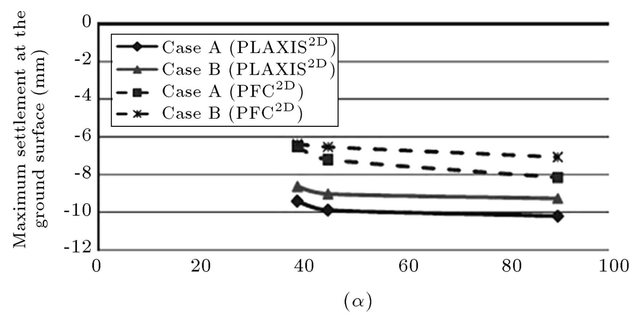


Figure 11. Variation of the maximum surface settlement vs. α.

4.4. Restriction

The main concern of DEM modeling is run-times of analyses. For the selected numbers of discs and elements in this paper, the DEM analyses take time approximately 15 times more than FEM analyses (ignoring the time for calibration in DEM). By increasing the numbers of discs and elements, the difference of run-times will be increased.

5. Conclusions

This paper included DEM analysis of the construction of twin-tunnels with a particular focus on the influence of both the construction procedure and geometric configuration on the soil settlement, compared with results of FEM analysis.

Based on the analyses results and findings, complicated DEM shows advantages and disadvantages with respect to conventional FEM:

The main advantages of DEM are modeling the discontiguous nature of soils (more consistent with reality), prevention of surface swelling during unloading in shallow tunnels, more accurate modeling of confining effect with depth (overburden or surcharge effect) and capability of modeling weak bond connections of particles (tensile and shear) in shallow layers.

However, the main disadvantages of DEM are higher computational effort and also need for determination of micro-parameters based on calibration.

Both DEM and FEM comparative analysis show that construction procedure affects the soil settlement while results obtained from DEM analysis are lower than FEM. This is due to confining effect. The construction of the upper tunnel at first leads to higher maximum settlement, compared to that obtained by the construction of the lower tunnel at first.

The highest surface settlement is obtained for vertical aligned tunnels, while horizontal aligned tunnels cause the lowest settlement. The maximum displacement for inclined aligned tunnels transmitted to the side of the first tunnel is constructed. For this type of tunnels, the inclination angle greater than 45° will cause inclined tunnels behave similar to vertical tunnels.

For twin-tunnels with horizontal alignment, increasing the distance between tunnels induces decreasing the settlement in the central part of the twin-tunnels and leads to changing of settlement pattern (it happens in $dx/D = 2 \sim 2.5$). Increasing dx causes the location of maximum surface settlement, from central part of twin-tunnels to the center line of each tunnel, to be changed. However, the disturbed surface area will be increased.

Partially nonsymmetrical settlement pattern in DEM analysis compared to FEM is another difference between two models. The mentioned difference is due to partially nonequivalent stresses in the defined depth that is more similar to real behavior of soil.

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Appendix

In order to validate the DEM results, a comparison between the numerical results (obtained in this study)

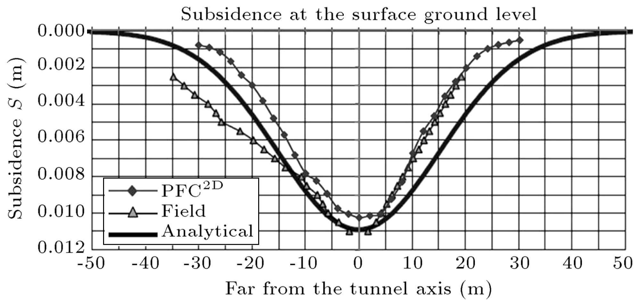

Figure A.1. The comparison of ground surface settlement profiles obtained from numerical, analytical and field methods.

and analytical and field monitoring [22] is presented in Figure A.1. The field case study has been selected for single tunnel of Porto subway in Portugal to validate the accuracy of calibration method.

The maximum surface settlement obtained from numerical analysis is 1.02 cm and 1.09 for field monitoring. There is a very good agreement between DEM and field results.

Biographies

Mohammad Oliaei is Assistant Professor of Civil Engineering. He joined the Geotechnical Group at the Department of Civil Engineering, Tarbiat Modares University, since 2008. He received his PhD degree from Sharif University of Technology (2007) as first rank student. In 2005, he was awarded a scholarship from British Council to continue his PhD study at Cambridge University. He specializes in the area of Geotechnical Engineering and Numerical Modelling (especially Meshless and DEM, the objective of this manuscript). He is the reviewer of several ISI papers in Scientific & Technical Journals.

Ehsan Manafi received his MSc degree in Geotechnical Engineering at the Department of Civil Engineering, from Tarbiat Modares University (2012). He specializes in the area of Numerical Modelling by Discrete Element Method.