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Research Note

Numerical analysis on crushing stress of sand in constitutive model with particle crushing

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Abstract. Estimation of crushing stress is important to understand the crushing mechanism and shear strength of granular material. One simple constitutive model using the reference crushing stress had been proposed for describing the variation in strength and deformation behavior of sand before and after particle crushing occurrence. The prediction capacity of the constitutive model is examined for representing the variation in mechanical behavior of relative high crushable sand. This study presents the parametric analysis on the reference crushing stress and examines its effect on the mechanical behavior of Cambria sand. Numerical results demonstrate that peak stress ratio increases and contractive behavior becomes less obvious with larger reference crushing stress. The stress ratio and dilatant ratio at failure with variable reference crushing stresses and confining pressures are also demonstrated. Predicted results indicate that reference crushing stress is basically dependent on the inherent feature and type of sand but has some relationship with the ultimate strength of a single particle. The linear relationship between the reference crushing stress and the yield stress decided in one dimensional compression test has been obtained on the logarithmic plot. The reference crushing stress can be applied as one effective strength index for sand in macro viewpoint.

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1. Introduction

Particle crushing occurs once external force overcomes its inherent material strength. It had been observed and confirmed in a series of laboratory experiments [1-4]. The effect of particle crushing on shear behavior can be understood as a decrease in positive dilatancy, which causes a decrease in the shear strength of sand simultaneously [5,6]. The strength and deformation behavior of sand is significantly affected by particle crushing occurrence. The coarse granular material such as rockfill similarly displays obvious crushable characteristics [7-9]. It is significant to evaluate the stress level at which the particles of granular material are crushed in infrastructure construction [10-13]. Study on the estimation of crushing stress for granular material had attracted many researchers' attentions. Different definitions and forms of the crushing stress were adopted in corresponding to the types of experiment. Single particle strength had been used as an index to describe the crushing strength of a single grain in micro viewpoint by many researchers [13-16]. It expressed the average tensile stress or force on the single grain without lateral confining reaction. Marketos and Bolton [17] investigated the crushing stress of particle using statistical processing. However, estimation of the crushing stress for specimen of assembled grains

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not the individual particle is quite important in both experimental and field practical aspects.

The crushing stress in macro viewpoint and its relationship with the relevant from of crushing stress in micro viewpoint needs further investigation. Just as the yield stress decided from the compression curve for specifying the crushing stress in one dimensional compression test which is commonly recognized as a simple method to examine the compressive and crushing features of granular material [18-21], it is necessary to clarify such kind of crushing stress for assembled grains subjected to more complex stresses. Triaxial compression test is an effective alternative to examine the mechanical behavior of assembled grains subjected to variable stress states and paths. Some constitutive models considering particle crushing had been established but a limited number of them directly introduced the crushing stress into the elasto-plastic theory.

Explanations of granular material behavior combining the critical state concept have been adopted by some researchers [22,23]. One constitutive model for sand using the concept of reference crushing stress had been proposed by Yao et al. [24] to represent the variation in strength and deformation at different confining pressures. It is necessary to understand the relationship among the different forms of crushing stresses in macro viewpoint under various loading patterns and confirm the rationality of the utilization of reference crushing stress for describing the strength for sand in triaxial test. To examine the prediction capacity of the constitutive model over a wide range of sand with different crushability, the constitutive model is validated by comparing the predicted values with experimental results for Masado and Cambria sand at first.

This study presents a parametric study about the effect of the reference crushing stress on the peak strength and deformation behavior of Cambria sand. Predicted results demonstrate that the peak stress ratio increases and contractive behavior becomes less obvious with larger reference crushing stress. The peak stress ratio at failure decreases and dilatant ratio at failure increases with increasing mean stress. The reference crushing stress expresses the linear relation with the yield stress decided in one dimensional compression test for five kinds of sand on the logarithmic plot. Therefore, the reference crushing stress can be applied as an effective strength index for sand in macro viewpoint and estimated from the experienced equation obtained in this study.

2. Constitutive model for sand with particle crushing

In recent decade, several constitutive models had been developed using elasto-plastic theory to include the characteristics of particle crushing. Daouadji et al. [25], Daouadji and Hicher [26] proposed a constitutive model and its enhanced version taking into account the particle breakage and proved its new theory on a basis of laboratory tests. Kikumoto et al. [27] also discussed the crushing behavior of sand with the revised Severn-Trent sand model based on the critical state theory. The grain size distribution was chosen as a parameter in model. The simple crushing model proposed by Yao et al. [24] could predict both positive and negative dilatancy of sand at failure and the variation in peak strength under different confining pressures. The reference crushing stress p_c is directly integrated with the expression of the hardening parameter, H, of the constitutive model. The evolution of characteristic state curve as well as failure state curve in the model is simultaneously affected by it. The constitutive model is successfully proved to predict the crushing behavior of soil in surrounding of pile tip [28,29].

2.1. Theory of the constitutive model with particle crushing and its reference crushing stress

The constitutive model is one of the members of modified Cam-clay model family. It is established by introducing the unified hardening parameter, H, and a reference crushing stress p_c. The unified hardening parameter, H, in Eq. (1) is capable of predicting both of the positive and negative plastic volumetric strain ε_v^p.

$$H = \int dH = \int \frac{M_c^4}{M_f^4}\frac{M_f^4-\eta^4}{M_c^4-\eta^4}d\varepsilon_v^p, \tag{1}$$

$$M_c = M\left(\frac{p}{p_c}\right)^n, \tag{2}$$

$$M_f = M\left(\frac{p}{p_c}\right)^{-n}. \tag{3}$$

The critical state line M involves two curves on the $p-q$ plane in Figure 1. Straight lines AB, CD and EF

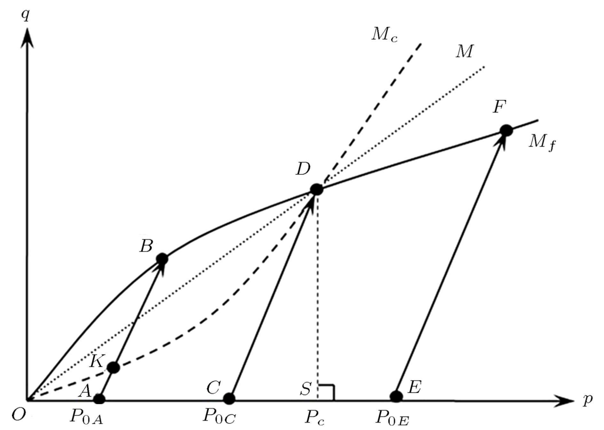

Figure 1. The M_f and M_c curves on $p-q$ plane.

denote different stress paths at low, medium and high initial confining pressures, respectively. p indicates mean stress, while q means deviatoric stress. $\eta = q/p$ is the stress ratio. The characteristic state curve M_c, in Eq. (2) is the differentiating curve of volumetric variation from contraction to expansion. Namely, the volume contracts continually provided that the stress ratio below M_c.M_f in Eq. (3) is the dividing line of material failure and takes the same value as the peak stress ratio. The indicia n is the material coefficient. Two curves start from the original point O and intersect at point D subsequently more again. The reference crushing stress p_c is corresponding to the point by drawing the straight line from the non-zero point perpendicular to the p-axis. The prediction theory of the constitutive model for volumetric variation is explained as follows. Path AB (low confining pressure); the volume initially contracts from A to K and expands in phase KB. Paths CD and EF (medium and high confining pressures); stress path reaches the characteristic state curve, M_c, and the failure state curve, M_f, simultaneously or later than that. Only the volumetric contraction is predicted.

The description of the constitutive model is reviewed and its tensor form for finite element analysis is specified in detail by Wu et al. [30]. The determination method of the reference crushing stress, p_c, is explained as the example of Cambria sand. Experiment results indicate that both of the stress ratio, q_f/p and $-(d\varepsilon_v/d\varepsilon_a)$, become the constant value when the Cambria sand is at failure under different confining pressures. Besides, connecting the points at the failure state in the q_f/p and $-(d\varepsilon_v/d\varepsilon_a)$ plane provides a straight line as shown in Figure 2. On the linear relation between these two ratio values, the elastic deformation part is ignored. The stress ratio at critical state, M, can be determined when the volumetric strain increment is zero, $-(d\varepsilon_v/d\varepsilon_a) = 0$. Utilizing

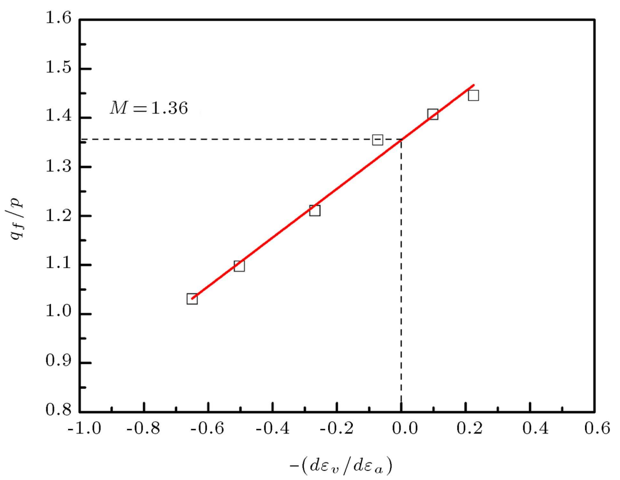

Figure 2. The relationship between q_f/p and $-(d\varepsilon_v/d\varepsilon_a)$ at different confining pressures.

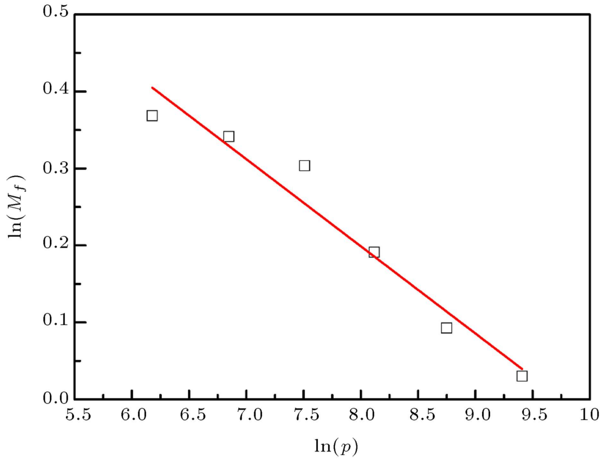

Figure 3. The relationship between $\ln(M_f)$ and $\ln(p)$.

the above linear relationship, we can determine M and then make rearrangement of Eq. (3), obtaining Eq. (4).

$$\ln M_f = -n \ln p + n \ln p_c + \ln M. \tag{4}$$

According to the relationship between the failure state curve, M_f, and the mean stress, p, under different confining pressure, we can draw the line in Figure 3 to express the relationship between $\ln(M_f)$ and $\ln(p)$. n is the gradient of the straight line, then we can decide the exact value of p_c.

$$\varepsilon_v^e = C_e \left[\left(\frac{p_x}{p_a} \right)^m - \left(\frac{p_o}{p_a} \right)^m \right], \tag{5}$$

$$\varepsilon_v^p = (C_t - C_e) \left[\left(\frac{p_x}{p_a} \right)^m - \left(\frac{p_o}{p_a} \right)^m \right]. \tag{6}$$

It had been revealed by Nakai [31] that the linear relation between the elastic volumetric stain, $d\varepsilon_v^e$, or plastic volumetric stain, $d\varepsilon_v^p$ and $(p/p_a)^m$, for granular material could be obtained based on the experimental results on Toyoura sand in Eq. (5) and Eq. (6). Herein, p_a is the atmosphere pressure. m is a coefficient for sand. Swelling index, C_e, and compression index, C_t, can be determined from the isotropic loading-unloading test. Poisson ratio, v, is assumed to be 0.3. There are seven parameters in the constitutive model in total. Six of them can be determined using the conventional isotropic and triaxial compression tests.

The constitutive for sand with particle crushing is established based on the critical state theory. The stress-dilatancy equation of this model is the same as that of modified Cam-clay model. The yield function of constitutive model for sand with particle crushing is given in Eq. (7).

$$f = \frac{C_t - C_e}{p_a^m} \left\{ \left[\frac{(2n+1)p_c^{2n}}{M^2} \times \frac{q^2}{p} + p^{2n+1} \right]^{\frac{m}{2n+1}} - p_o^m \right\} - H = 0, \tag{7}$$

where p_o is the initial mean stress. Crushing model takes the associated flow rule, so that plastic potential function is identical to the yield function.

2.2. Validation of constitutive model for high crushable sand

The constitutive model had been verified for describing the mechanical behavior of Toyoura sand in triaxial compression test considering particle crushing. Actually, many infrastructures are constructed on the relative crushable sand especially in coastal region. Therefore, it is necessary to examine the prediction capability of the constitutive model for high crushable sand. The numerical values from constitutive model are compared to the experimental results for another two kinds of relative high crushable sand in this study. The positive dilatancy of sand specimens in triaxial compression test disappears at confining pressure as 200 kPa, 1000 kPa and 4000 kPa for Masado, Cambria and Toyoura sand, respectively [32-34]. The criterion for evaluating the crushability of sand in this study is simply employed by comparing those confining pressures in triaxial test. Hence, Masado, Cambria and Toyoura sand are regarded as the high, medium, and low crushable sand. Table 1 shows the physical property for Masado and Cambria sand. The determination method of the reference crushing stress for Masado is the same as explained for Cambria sand. The seven parameters of constitutive model for both of the Masado and Cambria sand are shown in Table 2. Figure 4 represents the experimental and predicted relationship between stress ratio and axial strain under varying confining pressure from 60 kPa to 400 kPa in triaxial compression test for Masado. Prediction agrees well with the measured results except that the confining pressure is at low level. The predicted peak stress ratio exhibits the decreasing tendency as the confining pressure is increased. The constitutive model predicts the dilatancy from negative to positive at confining pressure as 60 kPa and 100 kPa, and only the negative dilatancy at confining pressure as 200 kPa and 400 kPa in Figure 5. The constitutive model is capable of representing the variation in deformation behavior and strength reduction for high crushable sand at relative low stress.

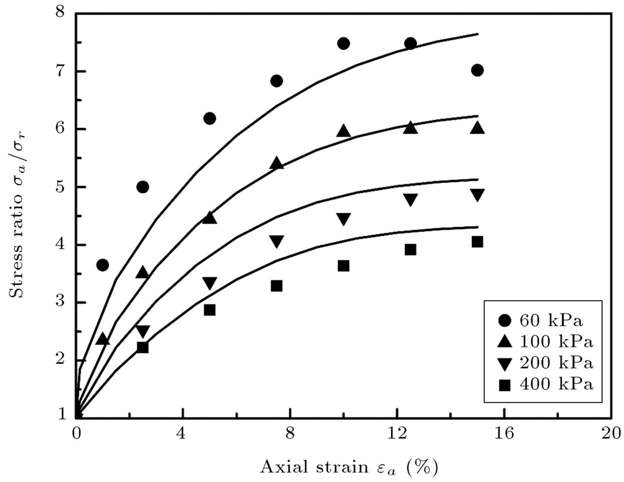

Figure 4. Experimental and predicted relationship between stress ratio and axial strain (Masado).

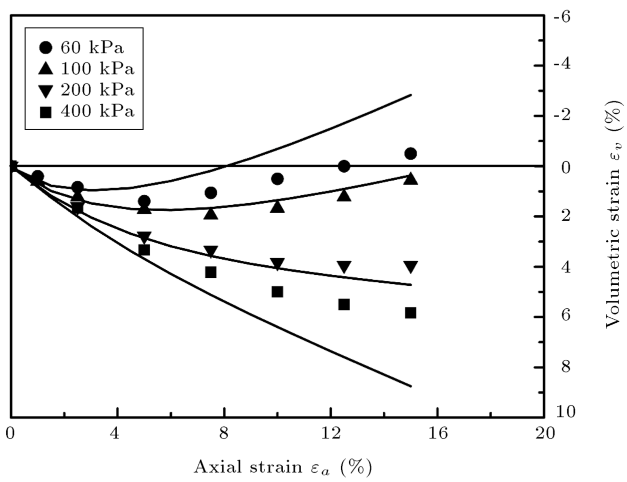

Figure 5. Experimental and predicted relationship between volumetric strain and axial strain (Masado).

The numerical and experimental relationship between the stress ratio and axial strain for Cambria sand is displayed in Figure 6. The peak strength gradually reduces from 3.8 to 2.5 as confining pressure

Table 1. Physical properties for two kinds of sand.

Name of sand	Specific gravity	$e_{\max}$	$e_{\min}$	d_{50} (mm)	Relative density (%)
Masado	2.62	0.967	0.491	0.760	90
Cambria sand	2.65	0.792	0.503	1.620	60

Table 2. Parameters of constitutive model with particle crushing for two kinds of sand.

Name of sand	C_e	C_t	m	p_c (MPa)	M	n	v
Masado	0.00637	0.01494	0.8	0.412	1.80	0.1782	0.3
Cambria sand	0.00640	0.00970	0.5	1.200	1.36	0.1132	0.3

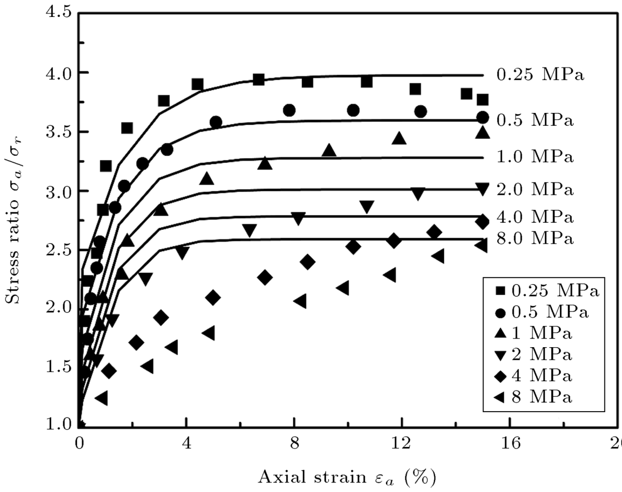

Figure 6. Experimental and predicted relationship between stress ratio and axial strain (Cambria sand).

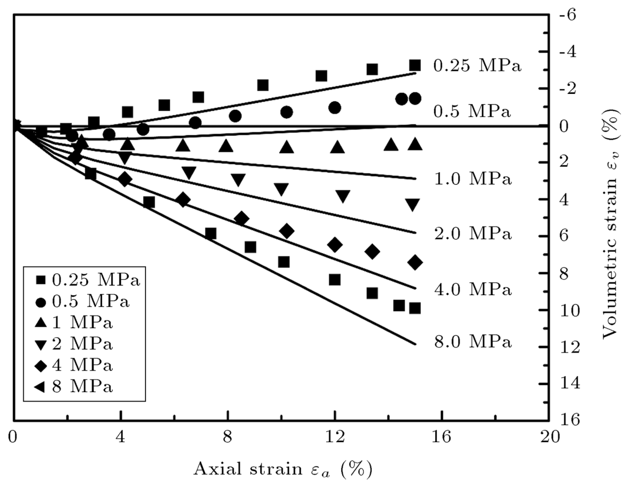

Figure 7. Experimental and predicted relationship between volumetric strain and axial strain (Cambria sand).

is increased from 0.25 MPa to 8 MPa. The predicted results of constitutive model exhibit dilatancy from negative to positive at confining pressures as 0.25 MPa and 0.5 MPa, and only negative dilatancy at confining pressures as 1 MPa, 2 MPa, 5 MPa and 8 MPa in Figure 7. Cambria sand displays markedly contractive behavior at high confining pressure and predicts volumetric strain attaining to 12% in compression side.

The constitutive model is proved to be capable of predicting the mechanical behavior of high and medium crushable sand over a wide range of confining pressures. It is noted that the reference crushing stress varies with the kind of sand and expresses some correlation with the peak strength of sand. To investigate the effect of reference crushing stress on its mechanical behavior, the parametric study on the reference crushing stress p_c is performed.

3. Numerical study on the reference crushing stress of Cambria sand

3.1. Parametric analysis of reference crushing stress p_c

The parametric study on the reference crushing stress p_c is carried out on Cambria sand. Cambria sand had been testified by Yamamuro and Lade and his collaborators [3,20,35] to investigate its compression and crushing features. The reference crushing stress p_c varies from 0.9 MPa to 1.8 MPa with the increment of 0.3 MPa. Other six parameters are kept as constant in the parametric analysis. The predicted mechanical relationship is expressed at confining pressure increasing from 0.25 MPa to 8 MPa in corresponding to the loading condition in triaxial test. The gradient of the failure state curve M_f decreases as the reference crushing stress becomes small. Oppositely, the gradient of the characteristic state curve M_c increases as the reference crushing stress decreases.

Owing to the completion of numerical analysis with reference crushing stress p_c as 1.2 MPa, only predicted relationship between the stress ratio and axial strain using the reference crushing stress 0.9 MPa, 1.5 MPa and 1.8 MPa are shown in Figure 8(a), (b) and (c). The predicted peak stress ratio increases as the reference crushing stress p_c becomes larger at the same confining pressure. The peak stress ratio is around 3.75 at confining pressure of 0.25 MPa with p_c as 0.9 MPa, while it reaches 4.2 at confining pressure of 0.25 MPa with p_c as 1.8 MPa. Numerical results show that the initial tangent modulus of stress-strain curve increases as the confining pressures increase. The predicted relationship between volumetric strain and axial strain when reference crushing stress takes value as 0.9 MPa, 1.5 MPa and 1.8 MPa is displayed in Figure 9(a), (b) and (c). The positive dilatancy becomes remarkable as the reference crushing stress p_c increases. It is summarized that the constitutive model adopting high reference crushing stress predicts larger peak stress ratio and less contractive volume. Reference crushing stress can be applied to be an index to describe the strength of assembled grains in macro viewpoint. It is noted that the scope between the predicted maximum volumetric expansion strain and maximum volumetric contraction strain is not affected by the reference crushing stress level.

3.2. The effect of reference crushing stress on mechanical behavior of Cambria sand

Figure 10 displays the predicted stress ratio at failure plotted against the mean stress when the reference crushing stress p_c is 0.9 MPa, 1.2 MPa, 1.5 MPa and 1.8 MPa, respectively. The stress ratio at failure M_{sf} significantly decreases with the increasing mean stress. The sand almost loses one third of the total strength

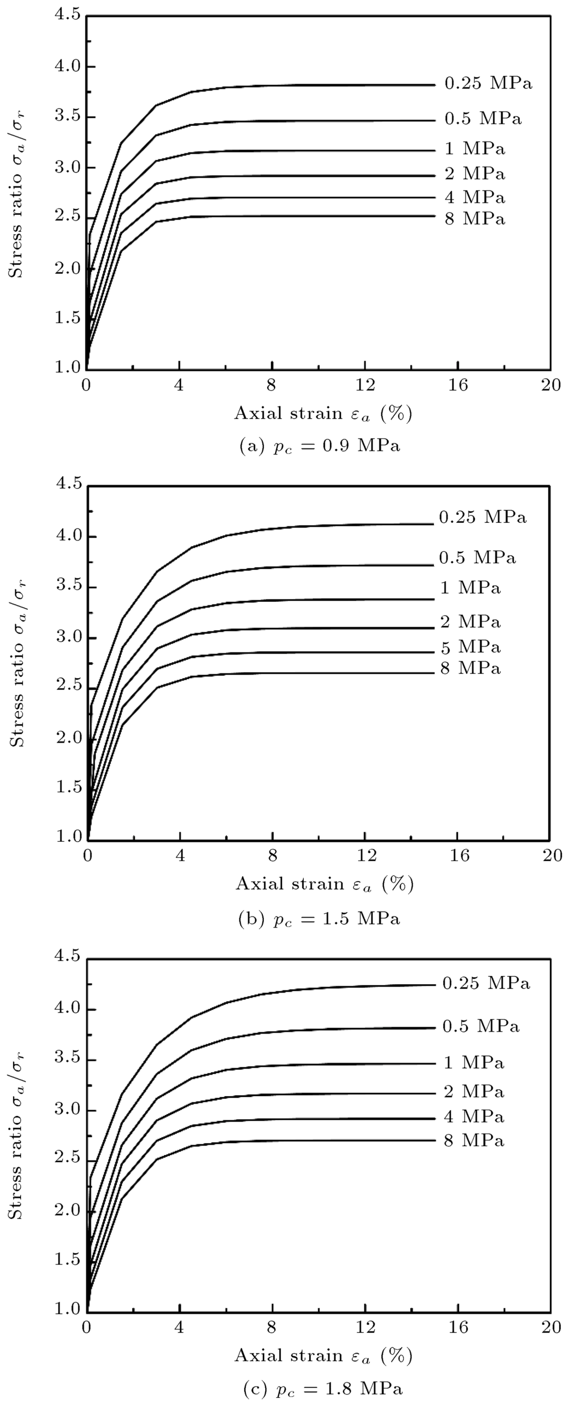

Figure 8. Predicted relationship between stress ratio and axial strain for Cambria sand with different reference crushing stresses.

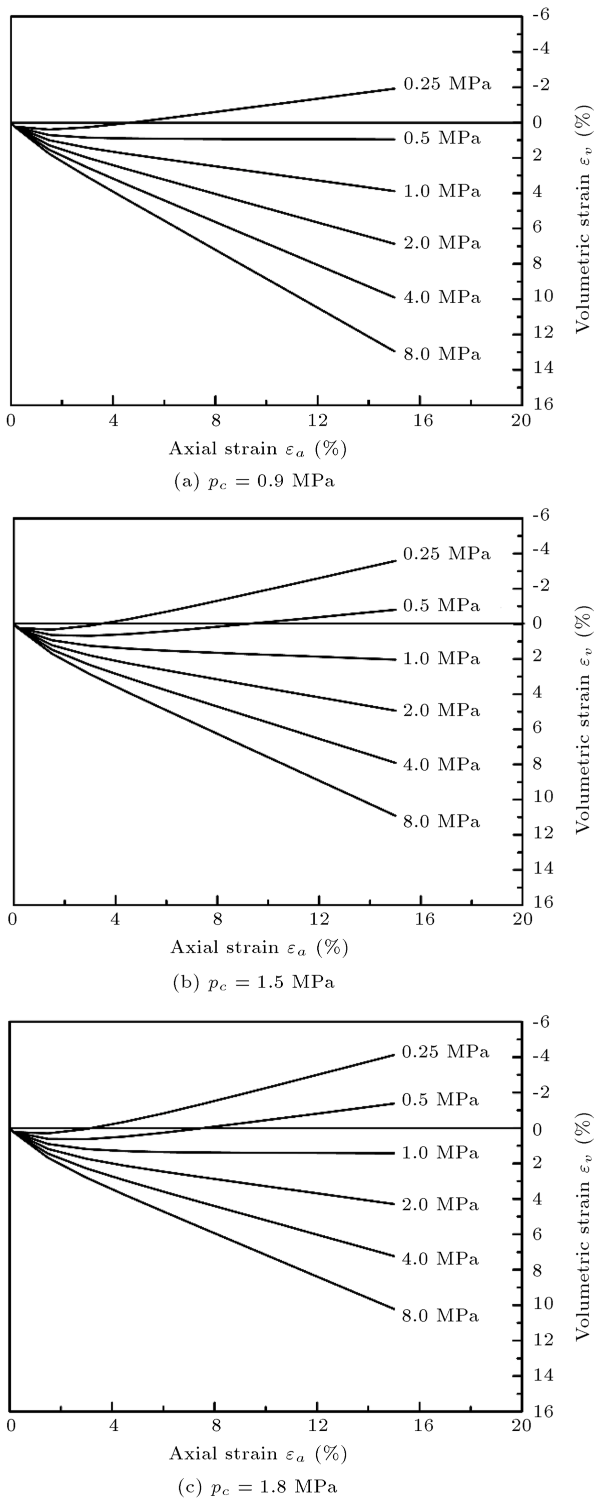

Figure 9. Predicted relationship between volumetric strain and axial strain for Cambria sand with different reference crushing stresses.

as the entire loading is applied. The progressive strength reduction is explained as the particle crushing and rearrangement in the specimen. However, the decreasing tendency and degree of strength at failure is less dependent on the reference crushing stress p_c. In addition, the stress ratio at failure slightly increases as the reference crushing stress p_c becomes large. However, the influence of reference crushing stress on the variation in the initial tangent modulus of stress-strain curve is minimal.

Figure 11 shows the predicted dilatant ratio at failure $-(d\varepsilon_v/d\varepsilon_a)$ plotted against the mean stress as the reference crushing stress varying from 0.9 MPa to

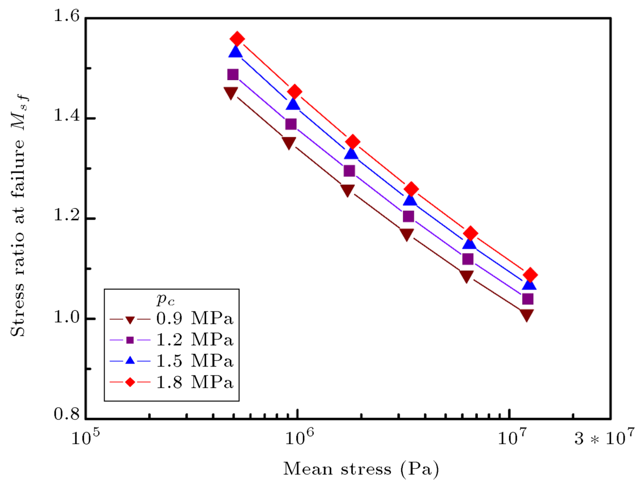

Figure 10. Relationship between stress ratio at failure M_{sf} and mean stress.

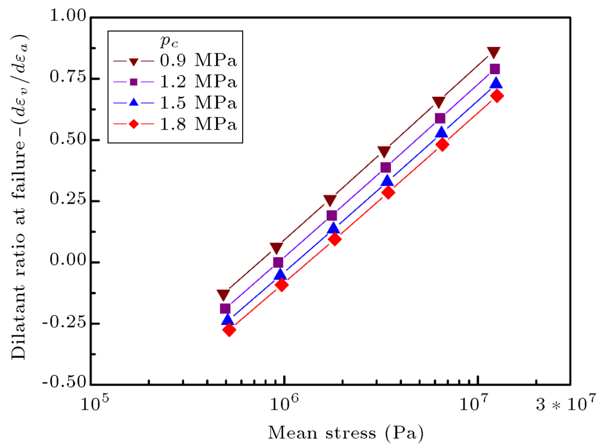

Figure 11. Relationship between dilatant ratios at failure $-(d\varepsilon_v/d\varepsilon_a)$ and mean stress.

1.8 MPa. The dilatant ratio $-(d\varepsilon_v/d\varepsilon_a)$ increases from the negative value to positive value with the increasing mean stress. As seen in Figure 11, the dilatant ratio at failure decreases reversely as the reference crushing stress p_c is increased. It is believed that constitutive model adopting larger reference crushing stress is believed to be difficult for being crushed. The predicted negative dilatancy for constitutive model with larger reference crushing stress is smaller under the same loading condition.

3.3. The effect of confining pressure on mechanical behavior of Cambria sand

Figure 12 demonstrates the predicted stress ratio at failure M_{sf} plotted against the reference crushing stress p_c. Confining pressure is increased from 0.25 MPa to 4 MPa. Predicted results show the stress ratio at failure M_{sf} slightly increases as the reference crushing stress increases. The stress ratio at failure under each confining pressure exhibits almost the similar increasing tendency and rate. It is noted

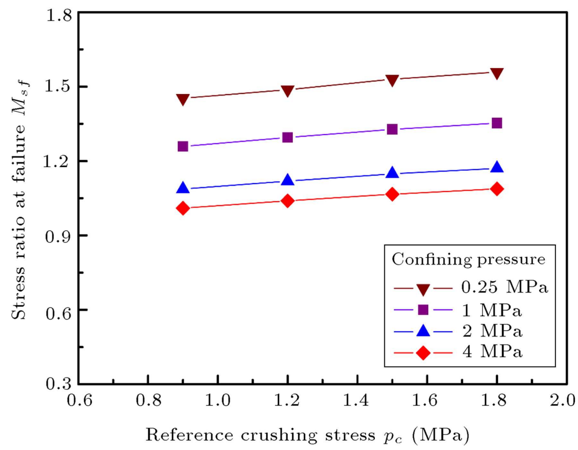

Figure 12. Relationship between stress ratio at failure M_{sf} and reference crushing stress p_c.

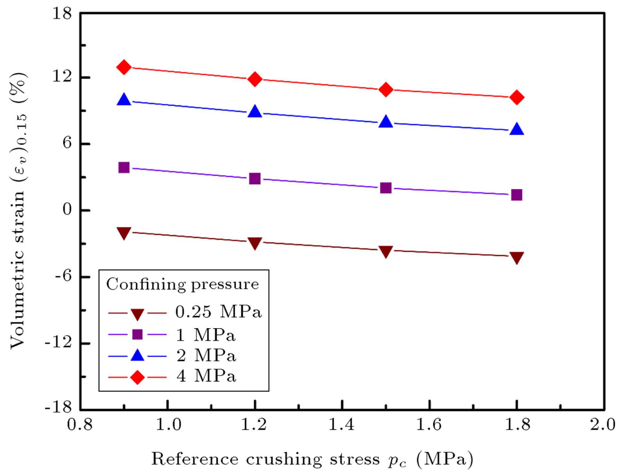

Figure 13. Relationship between volumetric strain $(\varepsilon_v)_{0.15}$ and reference crushing stress p_c.

that the reduction of stress ratio at failure with increasing confining pressure exhibits decreasing tendency compared to the incremental of confining pressure.

The relationship between the volumetric strain at failure $(\varepsilon_v)_{0.15}$ and the reference crushing stress is shown in Figure 13. The volume strain at failure $(\varepsilon_v)_{0.15}$ is defined as the value of volumetric stain ε_v when the axial strain is 0.15 in this study. The decreasing tendency of volumetric strain at failure is predicted with the increasing reference crushing stress p_c. The volumetric strain at failure increases as the confining pressure becomes large. The increasing tendency of volumetric strain at failure $(\varepsilon_v)_{0.15}$ becomes slower compared to the increment of confining pressure.

4. Relationship between the reference crushing stress and the yield stress in one dimensional compression test

One dimensional compression test is commonly performed to examine the crushing characteristics of sand

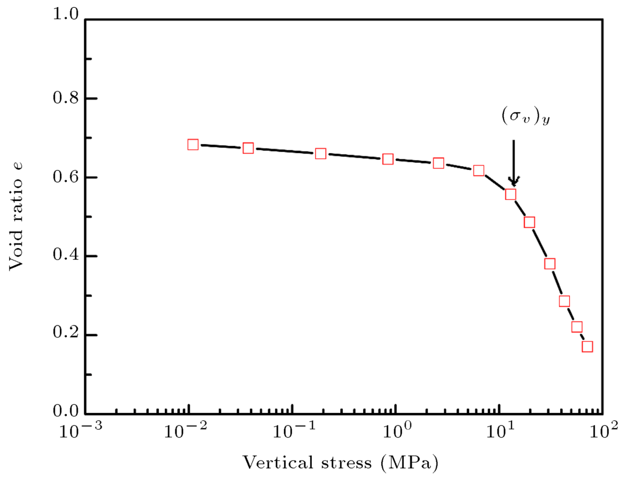

Figure 14. The yield stress $(\sigma_v)_y$ in the compression curve of one dimensional compression test for silica sand [15].

for simplicity. The compression curve always displays the obvious yield point as the applied vertical stress is increased on silica sand as shown in Figure 14. The yield point is related to the threshold stress for particle crushing when subjected to not only the one dimensional compression but also the isotropic pressures. The curve almost becomes the straight line after the yield point. The stress on the vertical stress axis in corresponding to the yield point is called the yield stress $(\sigma_v)_y$.

To examine the relationship between the reference crushing stress in triaxial compression test and that in one dimensional compression test, the reference crushing stress p_c and yield stress $(\sigma_v)_y$ on the logarithmic plot for five kinds of granular material express a linear relationship in Figure 15. The same sand specimens are prepared in both triaxial compression test and one dimensional compression test. The

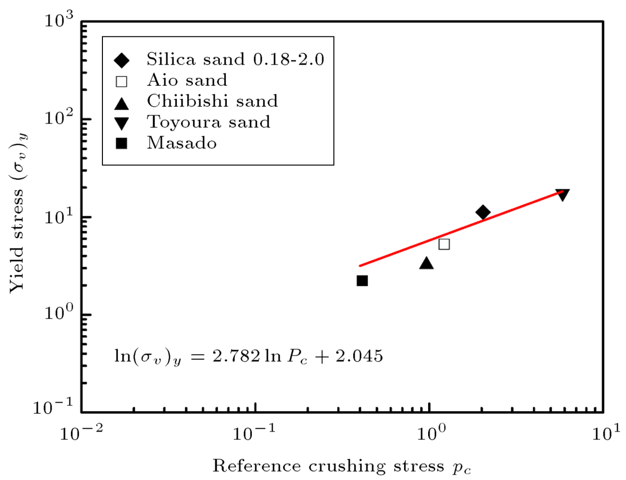

Figure 15. Relationship between yield stress $(\sigma_v)_y$ in one dimensional compression test and reference crushing stress p_c in constitutive model.

Table 3. Reference crushing stress, p_c, and the yield stress, $(\sigma_v)_y$, in one dimensional compression for sand [15].

Name of sand	Reference crushing stress p_c (MPa)	Yield stress $(\sigma_v)_y$ (MPa)
Silica sand (0.18-2.0 mm)	2.040	11.20
Aio sand	1.220	5.26
Chiibishi sand	0.965	3.26
Toyoura sand	5.850	17.46
Masado	0.412	2.23

specimens used for the triaxial compression test and one dimensional compression test are prepared at the same initial relative density. The values for these two kinds of crushing stresses are listed in Table 3. The yield stresses for five kinds of sand in one dimensional compression test are measured by Nakata et al. [12]. It is seen that the reference crushing stress p_c is smaller than the yield stress $(\sigma_v)_y$ in one dimensional compression test for the same kind sand. It is explained that the significant shear stress in triaxial compression test causes the particle to be crushed in early time at relative low stress level. It is shown that the reference crushing stress p_c and the yield stress $(\sigma_v)_y$ are the minimum for the Masado and maximum for the Toyoura sand. The difference of the reference crushing stress p_c for different kinds of sand is basically dependent on the mineral composition of material. The main reason for the difference between $(\sigma_v)_y$ and p_c is that reference crushing stress p_c does not represent the actual crushing stress in triaxial compression test but a strength parameter affecting the failure state curve during the entire loading process. It is well known that the crushing failure of grain is greatly affected by the confining pressure in triaxial compression test. While the confining pressures in the one dimensional compression test is proportional to the vertical stress. The crushing of sand grains in one dimensional compression test is mainly resulted from the compressive stress among particles. It indicates that the reference crushing stress could be applied as one strength index for sand in triaxial compression test just as the yield stress in one dimensional compression test. Meanwhile, the reference crushing stress could be estimated based on the experienced relationship established in Figure 15.

5. Conclusions

The constitutive model with reference particle crushing has been applied to predict the mechanical behavior of high and medium crushable sand. The constitutive model can predict the dilatancy from negative to positive at low confining pressure and only the negative dilatancy at high confining pressure. To investigate the effect of reference crushing stress, p_c, on the mechanical behavior of sand, a series of parametric study on p_c for Cambria sand is conducted. The variations in stress ratio and dilatant ratio at failure are also discussed in consideration of the influences of reference crushing stress and confining pressure. p_c is not the specific ultimate strength of the particle when particle crushing occurs in triaxial test but is dependent on the ultimate strength of a single particle to some degree in macro viewpoint. The major findings of the study are summarized below:

1. The constitutive model with reference particle crushing is verified to be applicable to a wide range of sand with different crushability. The constitutive model is capable of describing the mechanical behavior of relative high crushable materials and expressing the extremely large volumetric strain under high confining pressure for medium crushable sand.
2. The predicted peak stress ratio increases as the reference crushing stress, p_c, becomes larger at the same confining pressures level. The positive dilatancy becomes remarkable as the reference crushing stress, p_c, increases. It is noted that the scope between the predicted maximum positive strain and maximum negative strain is not affected by the reference crushing stress level.
3. The stress ratio at failure M_{sf} significantly decreases with the increasing mean stress. The progressive strength reduction is explained as the particle crushing and rearrangement. The dilatant ratio increases from the negative value to positive value with the increasing confining pressure. The dilatant ratio at failure decreases reversely as the reference crushing stress, p_c, is increased.
4. The reference crushing stress, p_c, for assembled grains in macro viewpoint and the yield stress, $(\sigma_v)_y$, in one dimensional compression test on the logarithmic plot expresses a linear relationship for five kinds of sand. The difference of the reference crushing stress, p_c, for different kinds of sand is basically dependent on the mineral composition of material. The reference crushing stress can be estimated from the experienced equation. The reference crushing stress, p_c, could be applied as one strength index for sand in triaxial compression test just as the yield stress in one dimensional compression test.

Acknowledgments

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