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Research Note

Proposing formulas for the parameters of suspension footbridge hanger systems to eliminate slackness, overstress and force oscillations problems

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Abstract. In suspension bridges both inclined and vertical hangers have their advantages and disadvantages. The inclined hangers are more prone to fatigue in comparison with vertical ones, but inclined hangers when encountering lateral loads (such as wind and earthquake loads) react better than vertical hangers. In some cases, some of the inclined hangers show slackness and some get too stressed. In this paper by using a new modification of the hangers system, while keeping the advantages of both systems, the disadvantages of both systems are resolved and slackness phenomenon is completely removed. The new arrangement of the hangers is formulated. Three different hanger systems are analyzed under nonlinear static analysis for symmetrical and asymmetrical live loads plus dead loads. Results show that the modified hangers system is improved considerably in comparison with vertical and inclined hangers.

1. Introduction

Suspension bridges are among the structures that can be constructed over long spans, and due to their high accuracy, performances, computing and control system after implementation, they are safe to use. Pedestrian suspension bridges have inclined or vertical hanger systems, which transfer forces from the deck to the main cables. Inclined hangers, due to the damping role against dynamic and lateral loads, act better than vertical ones. But inclined hangers, due to the slacking under excessive tension force, and also because of early fatigue in comparison with vertical hangers, require modifications in their systems to achieve the optimum system. In this way, the damping role is achieved, and excessive tensile stresses are reduced and finally slack problems are eliminated [1]. For this reason, to achieve an optimal system, modifications on inclined hangers system were recommended, for the first time, by Barghian and Moghadasi [2] only for a case study of bridges. Also, Moghadasi and Barghian [3] investigated the dynamic performances of suspension footbridges under the new model of hangers system. In this paper, the solution offered by them was carefully investigated, and the obtained results were improved. By analyzing different suspension footbridges, the parameters of the previous proposed method were generalized and formulated for the use of other pedestrian suspension bridges. Wu et al. [4] studied the possibility of cable slacking as well as displacements and internal forces in cable-stayed bridges. In another paper, Wu et al. [5] analyzed the slackness of cables in pre-stressed concrete cable-stayed bridges under the strong ground motion

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in which slacking had occurred. Also Wu, et al. [6] investigated the effect of slacked cables on nonlinear parametric vibrations of inclined cables under their support periodic (cycle) stimulations. They considered bending rigidity and damping equilibrium equations to solve diverging problems during slackness. Sigh and Tang [7] searched the growth of crack due to fatigue occurred because of the overstressing and slacking of cables. The nonlinear effect of slacked cables on suspension bridges was given in the papers by Laser and McKenna [8], Peterson [9], Sepe and Augusti [10]. Some researchers have studied the disadvantages of inclined hangers in suspension bridges, especially for Humber and Severn Bridges in England and Busphurus Bridge in Turkey, all of which are highway and have inclined hangers. In these bridges, early fatigue and fracture of hangers were observed [11]. Inclined hangers of Severn Bridge were designed for a period of 20 to 30 years. However, the existing wires in the hangers started to break eight years after the bridge opening. Connections between the deck and hangers were locally broken. Fracture was caused by fatigue due to local bending. There were some slacked and overstresses hangers due to heavy loads. The damage originally happened by longitudinal relative movement between the main cable and deck, when a change in stress in two parts of the hangers occurred. Hitherto, it suggested vertical hangers be replaced with inclined hangers in Severn bridge. However, the vertical system decreases the ability of a bridge to resist against oscillations caused by wind [12].

1.1. Analytical model

In this paper, several suspension footbridges with vertical, inclined and modified hangers were analyzed and nonlinear static analysis was considered. At first, for all bridges, vertical hangers were considered and then were replaced by inclined ones in the model. Table 1 shows information used for different examples. In Table 1, used parameters are as follows:

W : Deck width;

S : Minimum middle sag (vertical distance between deck and beneath the main cable);

N_1 : Number of divisions for left span;

N_2 : Number of divisions for middle span;

N_3 : Number of divisions for right span;

H_1 : Height of pier;

H_2 : Height of tower.

As shown in Table 1, three different suspension bridges were studied, Soti Ghat pedestrian bridge in Nepal as a case study, and two other bridges. The following properties in the three bridges were the same. The span was stiffened by two longitudinal pipe shaped beams. The diameter of the main cables was set to 120 mm and hangers to 26 mm. At every specified distance of the deck there was also transverse beam forming pinned connections between the longitudinal beams. The deck was stiffened by two horizontal pipe shaped braces laterally. The towers comprised steel pipes, braced laterally by diagonal braces. In the bridges model, steel (with the Young modulus of 2×10^{11} N/mm^2 and the density of 7850 kg/m^3) was chosen for all members. For main cables and hangers the following values were used: $fy = 1.18 \times 10^9$ N/m^2, $fu = 1.57 \times 10^9$ N/m^2 and the density of 7850 kg/m^3 where fy and fu are yield stress and tensile strength, respectively. The amount of pre-stressed load of cables was considered based on the weight of cables, sag and axial stiffness in cables. The view of vertical and inclined hanger systems of the Soti Ghat Bridge is shown in Figures 1 and 2.

1.2. A new arrangement for the bridge hangers

The vertical hangers have usually been used in most pedestrian bridges a few of which have been built with inclined hangers. The new model of hanger systems has been presented to remove the defects of both vertical and inclined hangers. In this model,

Table 1. Parameter values of some suspension footbridges.

Suspension bridge names	Parameters of suspension footbridges building (m)									
	H_1	H_2	S	L_1	L_2	L_3	N_1	N_2	N_3	W
Soti Ghat suspension footbridge	2	16	4	30	100	30	0	80	0	2
Suspension footbridge 1	2.5	15	3	25	120	25	0	96	0	2.5
Suspension footbridge 2	5	10	2	20	80	20	0	64	0	3

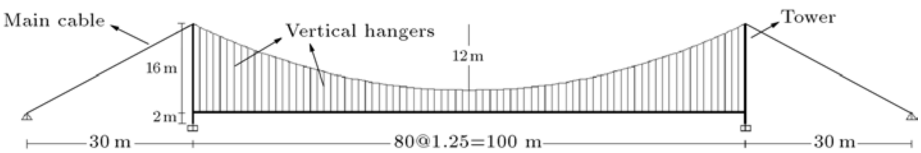


Figure 1. Soti Ghat pedestrian suspension bridge with vertical hangers.

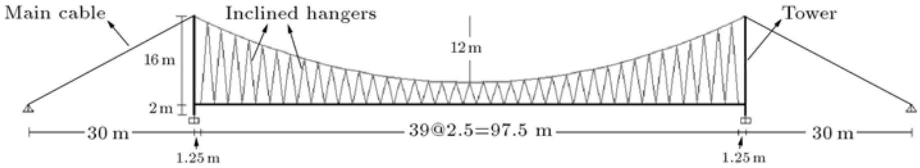


Figure 2. Soti Ghat pedestrian suspension bridge with inclined hangers.

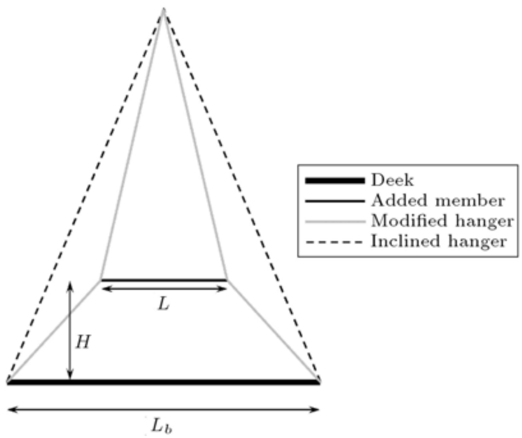


Figure 3. The specification of modified hangers.

a horizontal member is added between two adjacent inclined hangers as shown in Figure 3, so that the distribution of load between two adjacent hangers is done by the added member. The cross section and material used in the added member is the same as that used in hangers. The proposed model of hanger systems is shown as the modified hanger system in Figure 4. Formulas are proposed to determine the length (L) and the height (H) of the added member (L and H parameters are shown in Figure 3).

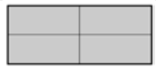
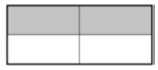
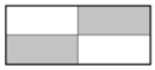
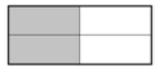
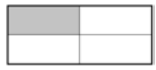
2. Materials and methods

Pedestrian suspension bridges usually experience several different loads such as the weight of pedestrians, bicycles, motorcycles and animals, or external loads such as earthquake and wind loads. In this study, the bridge was supposed to be subjected to live and dead loads statically. Live load was used symmetrically and asymmetrically as a distributed load with the amount of q (according to Iranian bridge design code):

$$q = \left(2 + \frac{150}{l + 150}\right), \tag{1}$$

where l is the loaded length (m), and q is the intensity

Table 2. Applied patterns of live loads.

Pattern names of load	Loaded length of deck (m)	Intensity of gravity loads (kN/m^2)	Load patterns
A	100	2.60	
B	100	2.60	
C	100	2.60	
D	50	2.75	
E	50	2.75	

of load (kN/m^2). The amount and patterns of loading are given in Table 2 in which it is shown that the critical conditions of live loads were due to patterns A and D. Due to this reason, the related charts to these loadings are provided in the results section. The A load is symmetrical and the D load is asymmetrical. In Table 2, load patterns are shown for the bridges plan.

3. Results and discussion

3.1. Comparison of the analysis results for vertical and inclined hanger systems

As mentioned before, different bridges with different dimensions were analyzed. Here, only the results of Soti Ghat bridge are given as an example. Dead and live loads were considered in this study. According to the results, it was observed that none of the vertical hangers were slacked under the load patterns D and A as seen in Figure 5. The inclined hangers were slacked under the load patterns D and A, so that, one of two adjacent hangers became slack alternatively under the load pattern D, as seen in Figure 6, and hangers were slacked under the load pattern A (Figure 14).

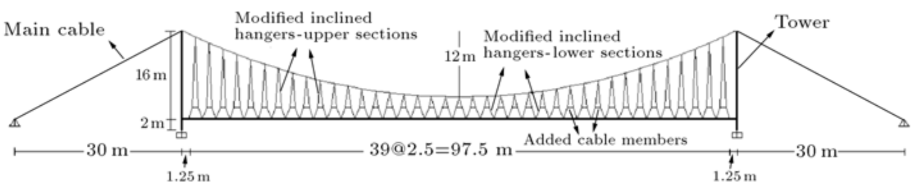


Figure 4. Soti Ghat pedestrian suspension bridge with modified hangers.

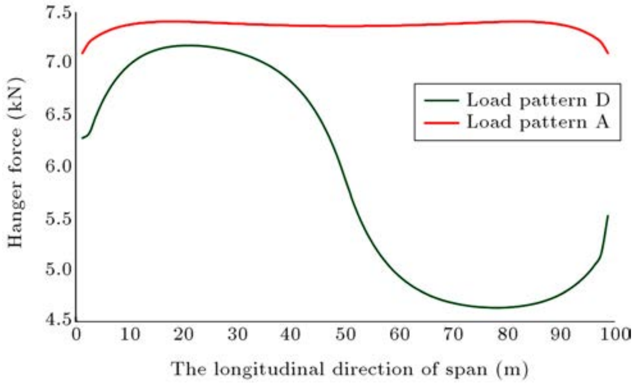


Figure 5. Vertical hangers forces due to the load patterns D and A.

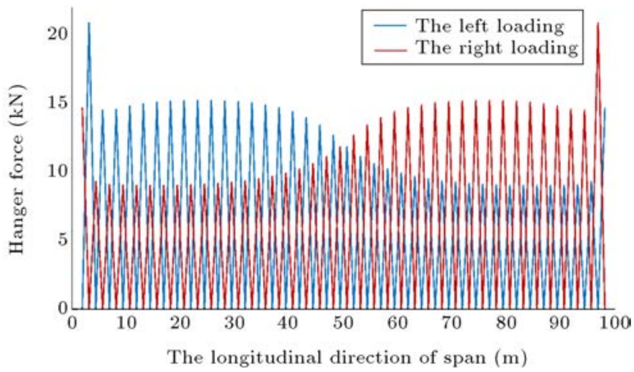


Figure 6. Inclined hangers forces due to the load pattern D.

For the load pattern D, when the live load moved from left to right on the deck, some inclined hangers became slack. Then, the movement direction was changed and the slacked hangers in the previous stage were subjected to tensile force. By repeating these actions, fatigue phenomenon occurs, which leads to the rupture of hangers.

3.2. Comparison of the analysis results for the modified hanger systems

Different H and L parameters (as shown in Figure 3) were used by try and error to get minimum force oscillation in two adjacent hangers along the bridge span. At the first step, different heights were used for H parameter while L parameter was kept constant. So, the value of H started from 10 cm and then was increased incrementally by 10 cm in each stage of analysis. When the suitable value of H was determined, the heights of 5 cm above and below the determined height were examined. It was found that $H = 0.8$ m gave good results (i.e., no hanger slackness, minimum force fluctuation and no overstress in hangers). Analysis results showed that in order to obtain the least force oscillation in upper and lower sections of the two adjacent modified hangers, $H = 0.8$ m could be chosen. Figures 7 and 8 show the modified hanger force in

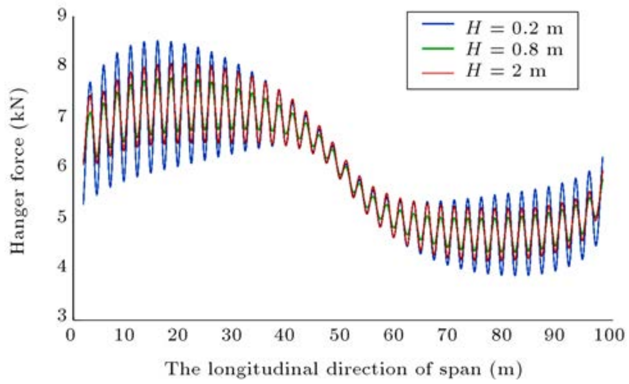


Figure 7. Hangers forces in the upper sections of modified hangers due to the load pattern D.

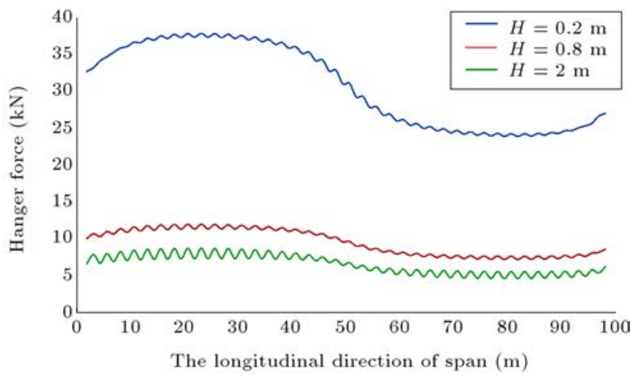


Figure 8. Hangers forces in the lower sections of modified hangers due to the load pattern D.

different levels of the added members. The amount of weight of hangers in bridges with vertical, inclined and modified hanger systems were 25.508 kN, 25.243 kN, and 27.173 kN, respectively. It was realized that the weight of modified hangers had been increased by about 7.6 percent compared with the bridge with inclined hangers.

Then, different lengths of added member were used for the L parameter while $H = 0.8$ m (the optimum height of added member) was kept constant. Initially the L parameter was taken as 2 m, and then it was decreased. It was realized that by decreasing the length of the added member (L), the amount of force oscillations decreased in two adjacent modified hangers. In extreme condition, when the length of the added member reached zero, the upper section of modified hangers tended towards vertical hangers position, so that, the force values in the upper sections of the modified and vertical hangers would be equal. According to the results of analysis, it was realized that the reduction of the amount of forces oscillation for lengths less than 40 cm are very little and their fluctuations are very close to $L = 40$ cm. Therefore, $L = 40$ cm as the shortest length of added member was elected. Figure 9 shows the upper section of modified hanger force for three

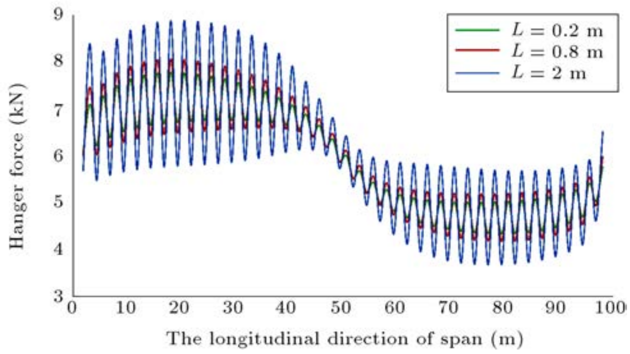


Figure 9. Hangers forces in the upper sections of modified hangers with constant $H = 0.8$ m due to the load pattern D.

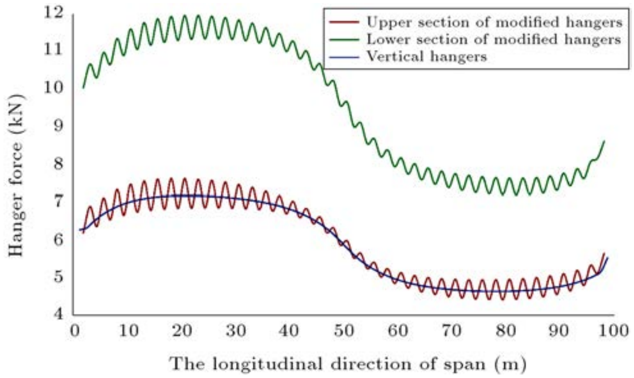


Figure 10. Vertical and modified hangers forces with $H = 0.8$ m and $L = 0.4$ m due to the load pattern D.

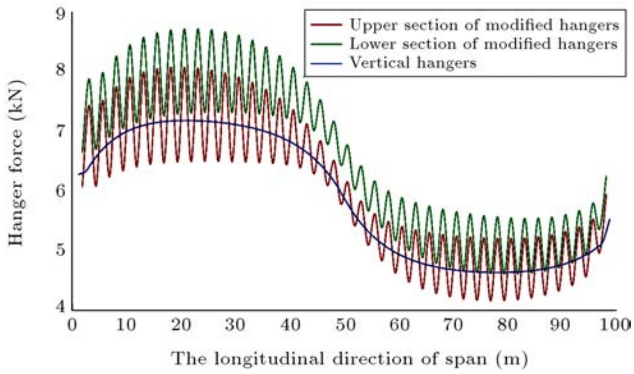


Figure 11. Vertical and modified hangers forces with $H = 2$ m and $L = 0.4$ m due to the load pattern D.

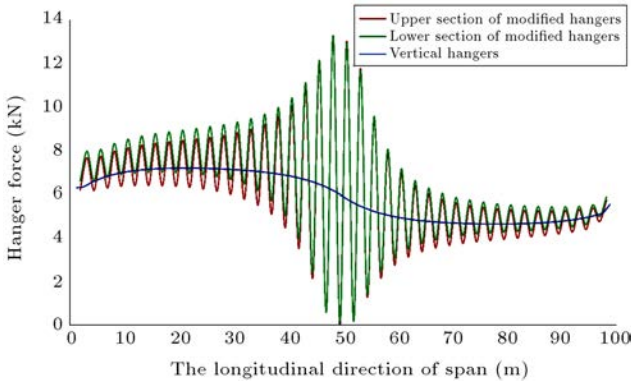


Figure 12. Vertical and modified hangers forces with $H = 2.27$ m and $L = 0.5$ m due to the load pattern D.

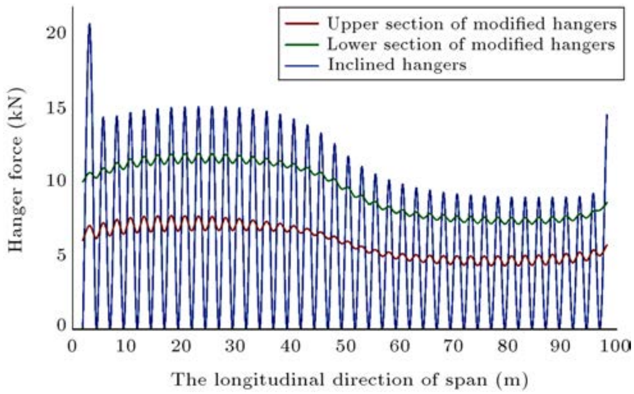


Figure 13. Inclined and modified hangers forces with $H = 0.8$ m and $L = 0.4$ m due to the load pattern D.

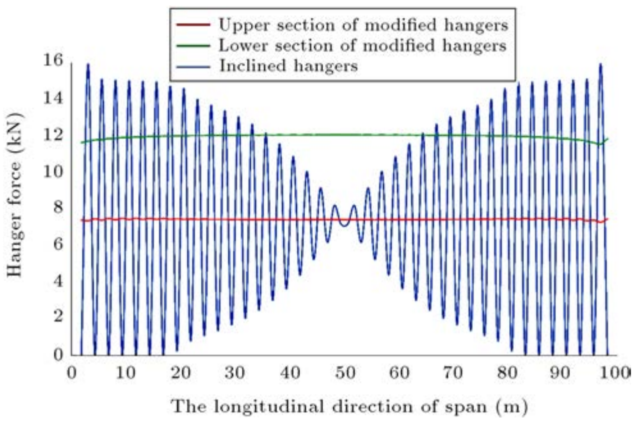


Figure 14. Inclined and modified hangers forces with $H = 0.8$ m and $L = 0.4$ m due to the load pattern A.

different lengths and the constant height of added member. As shown in Figures 10 to 12, in addition to increasing the force fluctuation, the forces in upper and lower sections of modified hangers gradually get closer together.

According to Figures 10 to 12, it was realized that, by increasing the value of the parameter H, the forces in the upper and lower sections of modified hangers were getting very close, while the oscillations of forces became high. In extreme case, when the height of the added member is equal to the vertical height of hangers (it means that there is no added member), the forces in the upper and lower sections of modified hangers will be equal. Figures 13 and 14 show inclined and modified hangers force due to the load patterns D and A. The summary of analysis results for three hanger systems is given in Table 3. According to Table 3, the modified models show that for the load patterns D and A, no slackness or tendency to slackness exists. Also, internal forces and their oscillations decrease

Table 3. The forces of vertical, inclined and modified hangers under the A and D load patterns.

Type and pattern of loads	Type of hangers	The number of slacked hangers	Maximum tensile of force (kN)	The amount of force oscillations in two adjacent hangers (kN)	
				Minimum	Maximum
Dead load	V	-	4.451	-	-
	I^b	-	7.447	-	-
	MU^c	-	4.274	-	-
	ML^d	-	6.834	-	-
Pattern A 2.60 kN/m^2	V	-	7.409	0	0.104
	I	14	15.944	0	15.944
	MU	-	7.508	0	0.109
	ML	-	12.068	0	0.085
Pattern D 2.75 kN/m^2	V	-	7.177	0	0.371
	I	39	20.83	8.964	20.83
	MU	-	7.778	0.108	0.819
	ML	-	11.966	0.091	0.700

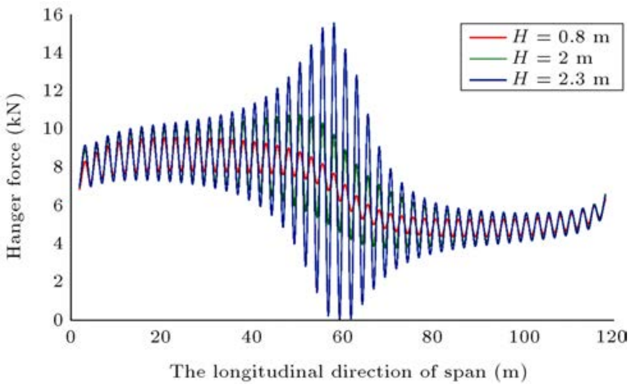


Figure 15. Hangers forces in the upper sections of modified hangers with $L = 40$ cm due to the load pattern D in suspension bridge 1.

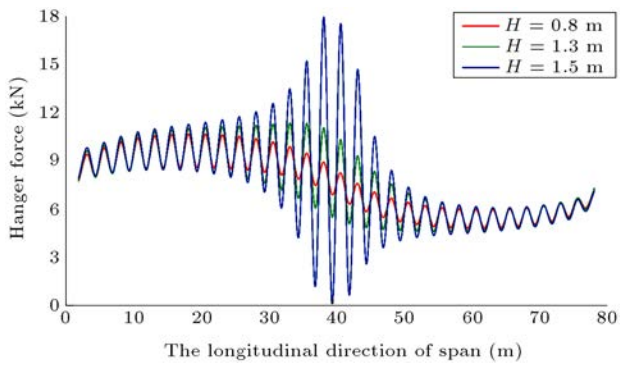


Figure 16. Hangers forces in the upper sections of modified hangers with $L = 50$ cm due to the load pattern D in suspension bridge 2.

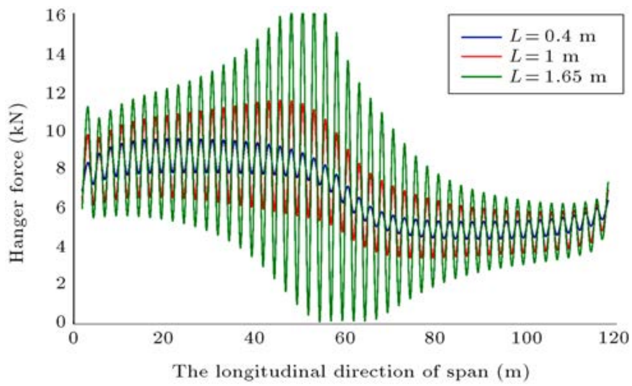


Figure 17. Hangers forces in the upper sections of modified hangers at $H = 80$ cm due to the load pattern D in suspension bridge 1.

significantly in modified hangers in comparison with the inclined hangers.

3.3. Study of the optimum height and length of the added member for different suspension footbridges

To obtain a mathematical relationship to use the added member for other suspension footbridges, several suspension footbridges with the desired structural dimensions and different from each other have been studied in this research. The optimum height of the added member in the other suspension footbridges was also obtained which was 80 cm. It was also realized that the smaller length of the added member gave better result. As seen in Figures 15 and 16 (related to the suspension footbridges 1 and 2, respectively), the hanger forces at $H = 80$ cm of the added member have less oscillations than the other heights. Figures 17 and 18 show that the minimum force oscillations of modified hangers occurs at $L = 40$ cm.

3.4. Proposing optimum height and length for the added member

3.4.1. General mathematical relationship for the length of the added member

The results showed that with shortening the length of the added member, force and oscillations of hangers

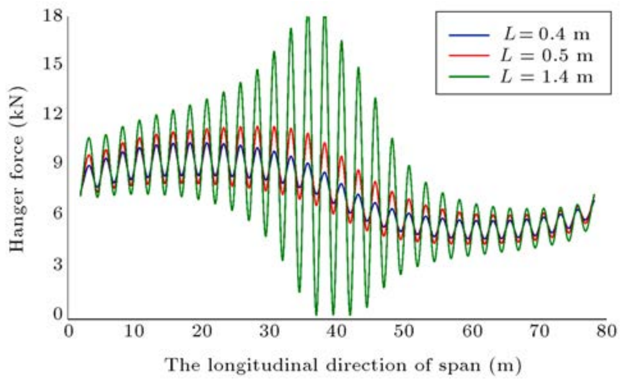


Figure 18. Hangers forces in the upper sections of modified hangers at $H = 80$ cm due to the load pattern D in suspension bridge 2.

were decreased while its force was increased. Therefore the length of the added member should be decreased as far as the force (in the added member) should not exceed its permissible tension.

As seen in Figure 12 and Figures 15 to 18, the slackness of hangers happens in the mid of bridge span. The reason is that (for an added member with a constant length) after a specific height, the added member has no effect for removing slackness. This phenomenon starts for the hangers with short height. Therefore, the hanger which has the least height acts as restrictive parameter for the added member. The mentioned explanations are shown in Figure 19(a) and (b).

Figure 20 shows adjacent hangers in the bridge mid span. According to this figure, the following linear equation can be written for AC line.

$$\frac{L - L_A}{L_C - L_A} = \frac{H - H_A}{H_C - H_A}. \tag{2}$$

Substituting the values from Figure 20 in Eq. (2) gives:

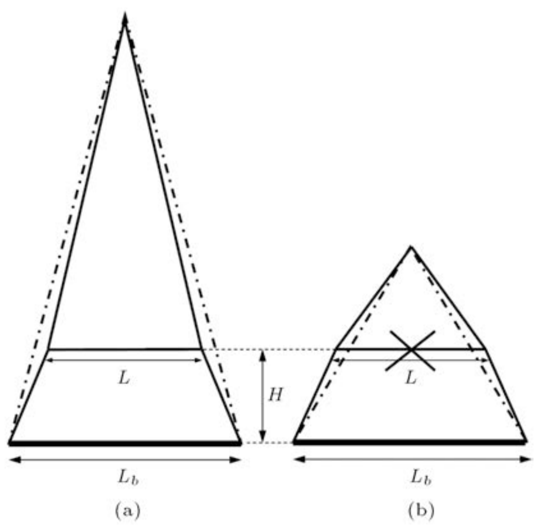


Figure 19. (a) Acceptable modified hanger. (b) Unacceptable modified hanger.

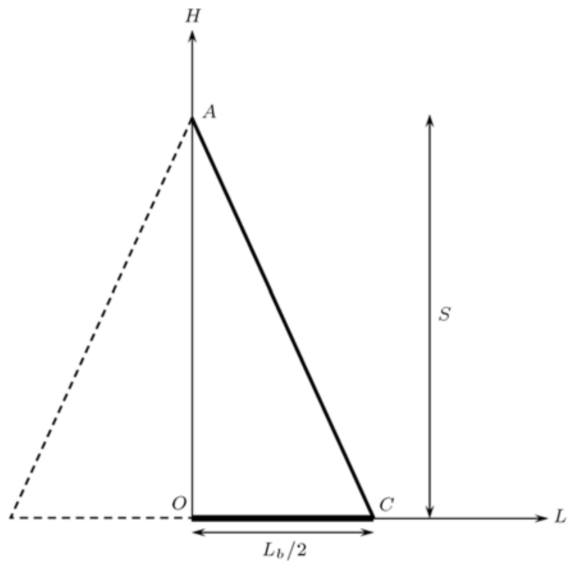


Figure 20. The shape of two adjacent hangers without adding the member.

$$\frac{L}{\frac{L_b}{2}} = \frac{H - S}{-S}, \tag{3}$$

where, S, H, L and L_b are the sag of main cables, the height of the added member, the length of the added member and the length of the beam between two adjacent hangers, respectively. The above equation represents the half length of the added member; the full length will have the following form:

$$L = \frac{L_b(S - H)}{S}. \tag{4}$$

Therefore, the following condition should be used:

$$L \leq \frac{L_b(S - H)}{S}. \tag{5}$$

Eq. (5) represents the length of the added member. In order to eliminate slackness problem completely in the hangers, the analyses showed that Eq. (5) should be modified. To achieve the state of Figure 4, 20 cm (=0.2 m) less than the amount of Eq. (5) is proposed. Thus, by considering the result obtained in Section 3.2. (i.e. $L = 40$ cm), the range of L is proposed as follows:

$$0.4\text{m} \leq L \leq \frac{L_b(S - H)}{S} - 0.2\text{m}. \tag{6}$$

By choosing the mentioned range, hanger forces did not fall below 0.5 kN; therefore there was no slack hanger.

3.4.2. General mathematical relationship for the height of the added member

The bridge dimensions were altered to obtain a general mathematical equation for the optimized height. It was again realized that 80 cm height for the added

Table 4. Obtained optimum height based on the beams length of the deck between two adjacent hangers.

L_b (m)	H (m)
1	0.30
2	0.65
2.5	0.80
3	0.95
4	1.30
5	1.60

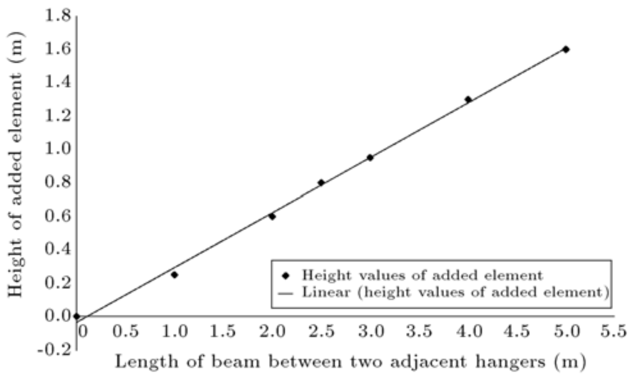


Figure 21. A linear regression for the obtained optimum height of the added member.

member was suitable. For different bridges, it was realized that optimal height can be achieved by changing the beams length of the deck between two adjacent hangers (L_b). To obtain a general equation, suspension footbridges with different L_b were analyzed. It was found that there was a unique value for each height of the added member depending on the related length (L_b). Some heights which depend on lengths are shown in Table 4.

To obtain a mathematical relationship from the amounts in Table 4, they were plotted in Figure 21 in which a linear regression was used.

Linear regression equations are given below:

$$y = ax + b, \tag{7}$$

$$a = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}, \tag{8}$$

$$b = \frac{\sum y_i - a \sum x_i}{n}, \tag{9}$$

$$\text{corr} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\left(\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right)\left(\sum y_i^2 - \frac{(\sum y_i)^2}{n}\right)}}. \tag{10}$$

The equation extracted from the linear regression can be written as follows:

$$H = 0.3228 \times L_b - 0.0071. \tag{11}$$

By ignoring the little amount of the H-intercept, Eq. (11) becomes:

$$H = \frac{8}{25} L_b, \tag{12}$$

Eq. (12) represents the lower bound and optimum value for the height of added member. According to Figure 20, the parameter H has a limited amount. The upper bound of H value is determined by considering: 1) the upper and lower bound of Eq. (6); and 2) that the maximum height of added member happens when L is minimum (from Figure 20); the upper limit of Eq. (6), which is equal to 40 cm, gives the upper limit for H (40 cm is the lower bound and optimum for L).

Thus the final relationship of height can be written as follows:

$$\frac{8}{25} L_b \leq H \leq \frac{S(L_b - 0.6)}{L_b}. \tag{13}$$

For example, the optimum height and length of the added member for Soti Ghat bridge, by considering $S = 4$ m and $L_b = 2.5$ m using Eqs. (13) and (6), are equal to:

$$0.8 \leq H \leq 3.04, \tag{14}$$

$$0.4 \leq L \leq 2.3 - 0.625H. \tag{15}$$

The above equations can be plotted as in Figure 22 to obtain the acceptable zone.

All points within the zone in Figure 22 are acceptable, but to get the lowest force oscillations in hangers, the optimum height is $H = 0.8$ m.

4. Conclusions

- The proposed modification in hangers reduces the tensile forces of inclined hangers significantly, and the possibility of slacking - for all hangers - is

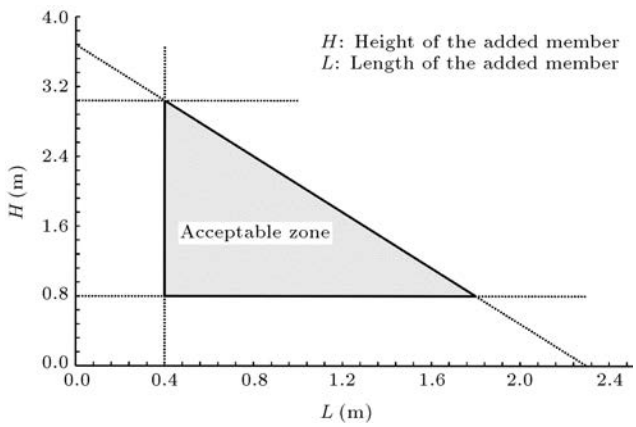


Figure 22. Relationship diagrams obtained from Soti Ghat suspension footbridge.

removed. The reason is that the overstressing forces transfer to the adjacent slacked hanger by the means of a horizontal member added between two adjacent hangers.

- The modification decreases oscillations in hanger forces significantly; therefore, it decreases fracture due to fatigue in hangers. Inclined hangers may be replaced after several years, but modified hangers can be at service for a longer period.
- For the new system, the disadvantages of both inclined and vertical systems have been removed and the advantages of both systems have been remained. In the new system, none of the hangers have been slackened and also hangers forces have been reduced significantly.
- In order to reduce force oscillation changes in two adjacent hangers, it is better that the added member to have the least length. But, by reducing the length of added member, its force is increased. Formulas are proposed to determine the length and height of added member between inclined hangers.

References

1. Gimsing, N. and Georgakis, C.T., *Cable Supported Bridges: Concept & Design*, 3th Edn., John Wiley and Sons, New York (2011).
2. Barghian, M. and Moghadasi, H. "Proposing a new model of hangers in pedestrian suspension bridges to solve hangers slackness problem", *Scientific Research*, **3**(4), pp. 322-330 (2011).
3. Moghadasi, H. and Barghian, M. "Improvement of dynamic performances of suspension footbridges by modifying the hanger systems", *J. Eng. Struct.*, **34**, pp. 52-68 (2012).
4. Wu, Q., Takahashi, K. and Nakamura, S. "The effect of loosening of cables on seismic response of the cable-stayed bridge", *Riron Oyo Rikigaku Koenkai Koen Ronbunshu Journal*, **50**, pp. 229-230 (2001).
5. Wu, Q., Takahashi, K. and Nakamura, S. "The effect of cable loosening on seismic response of a pre-stressed concrete cable-stayed bridge", *Journal of Sound and Vibration*, **268**(1), pp. 71-84 (2003).
6. Wu, Q., Takahashi, K. and Nakamura, S. "Influence of cable loosening on nonlinear parametric response of inclined cables", NAOSITE: Nagasaki University's Academic Output Site, Reports of the Faculty of Engineering, Nagasaki University, **35**(65), pp. 74-81 (2005).
7. Sih, G.C. and Tang, X.S. "Fatigue crack growth rate of cable-stayed portion of Runyang bridge. Part I-cable crack growth due to disproportionate cable tightening/loosening and traffic loading, multiscale fatigue crack initiation and propagation of engineering materials", Springer Science+business Media, Berlin, pp. 209-247 (2008).
8. Laser, A.C. and McKenna, P.J. "Large-amplitude periodic oscillations in suspension bridges: Some new connections with nonlinear analysis", *Siam Review*, **32**(4), pp. 537-578 (1990).
9. Peterson, I. "Rock and roll bridge: A new analysis challenges the common explanation of a famous collapse", *Science News*, **137**(22), pp. 344-346 (1990).
10. Sepe, V. and Augusti, G. "A deformable section model for the dynamics of suspension bridges, Part I: Model and linear response", *Wind and Structures*, **4**(1), pp. 1-18 (2001).
11. Suh, J. and Change, S.P. "Experimental study on fatigue behavior of wire ropes", *International Journal of Fatigue*, **22**(4), pp. 339-347 (2000).
12. Al-Khlili, M.A. "An investigation into the static behavior of the Severn bridge", M.Sc. Thesis, UMIST, Manchester (1986).

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